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SPRING COLLEGE ON PLASMA PHYSICS

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EXCITATION OF A LARGE AMPLITUDE PLASMA WAVE BY A SHORT LASER PULSE

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1979 T. TAJIMA & J. DAWSON

1987 L. GORBUNOV & V. KIRSANOV

Solution for the weakly-relativistic limit ~~in~~ the envelope approx'n.

1988 P. SPRANGLE, E. ESAREY, A. TING & G. JOYCE

Same solution + Optical driving

1989 V. TSYTOVICH, R. BINGHAM, U. DE ANGELIS

Fully relativistic wake / Beat excitation

(Take into account pump spectrum

$\Delta\omega \sim 1/\Delta t$)

COMPARE

PLASMA BEAT-WAVE ACCELERATOR PBWA

PLASMA WAKE-FIELD ACCELERATOR PWFA

LASER WAKE-FIELD ACCELERATOR LWFA

LASER WAKE-BEAT ACCELERATOR LWBA

I

PROBLEMS WITH PBWA & PWFA:

1) RESONANT CONDITION $\omega_1 - \omega_2 = \omega_p$

- High Plasma Uniformity
- Detuning → Saturation
- Fine tuning of laser frequencies
- Long pulse lenghts

2) LASER BEAMS MUST PROPAGATE LONG DISTANCES WITHIN PLASMA

- diffraction
- instabilities & ion dynamics
- phase detuning
- energy depletion

3) DIFFICULT BEAM TECHNOLOGY

- driving beam (PWFA)
- accelerated beam (both)

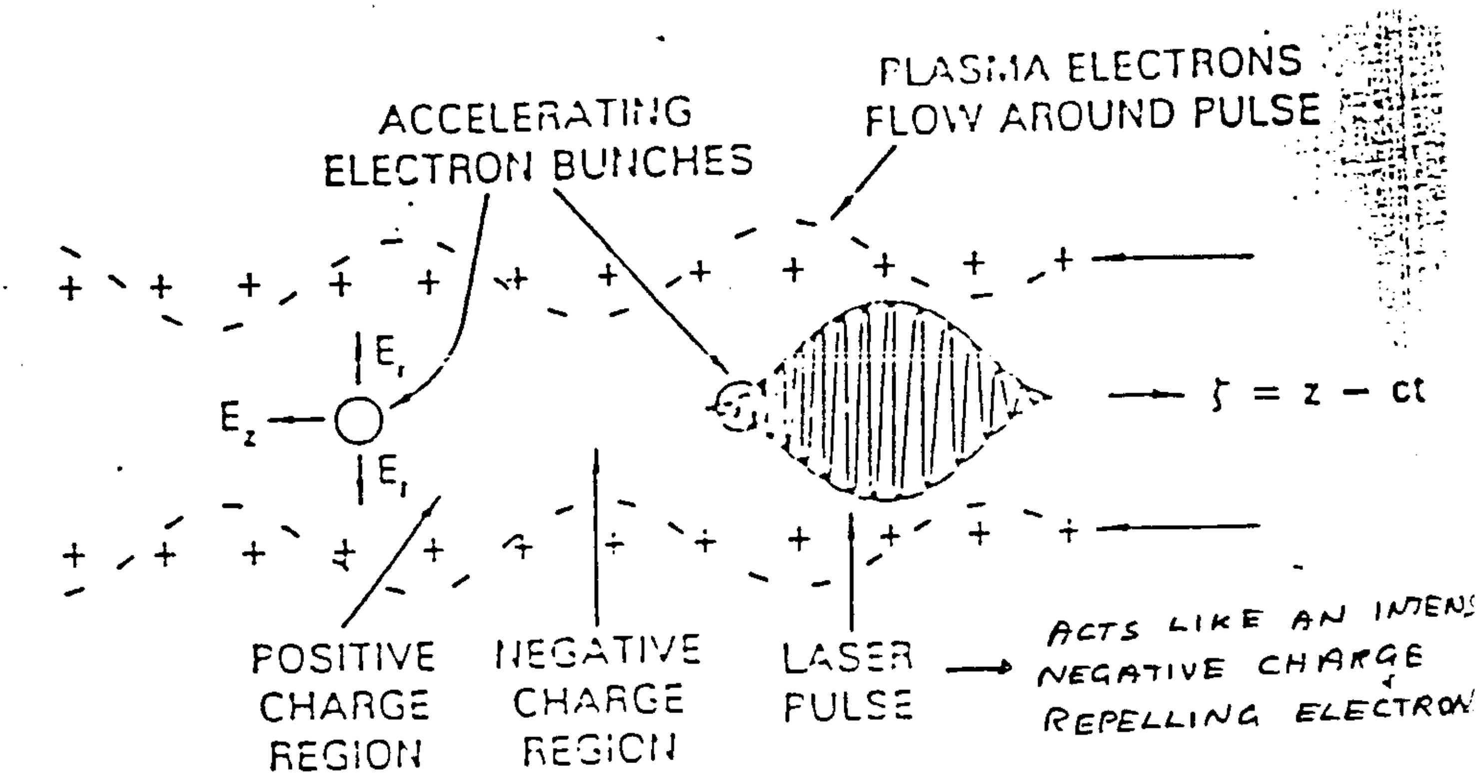
II

ADVANTAGES WITH LWFA:

- Single pump: avoid "resonance" problems
- Plasma wave can grow to higher ampl.
- Relativistic Optical driving
- Intrinsic large $\Delta\omega$

SCHEMATIC OF THE LWFA

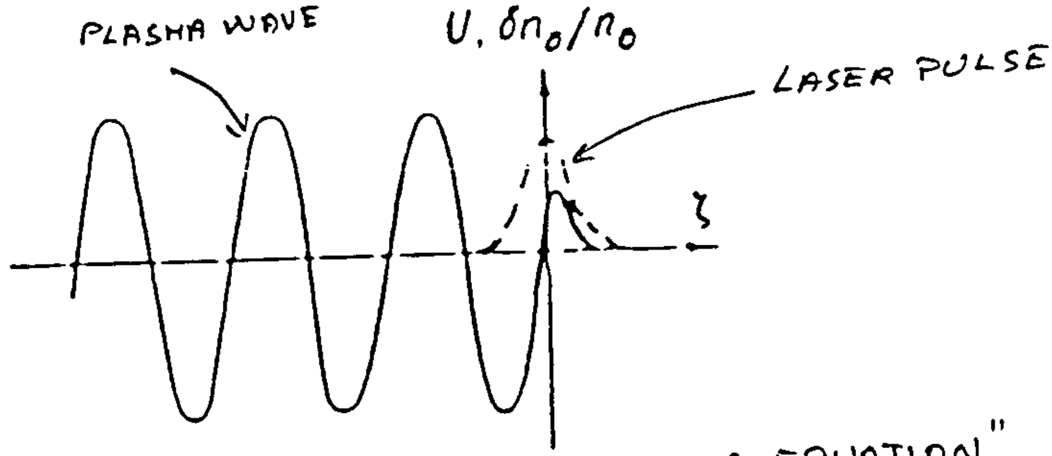
(from E. Esarey et al., 1988)



$\ell_L = \lambda_p = 0.03$ cm

$\frac{\delta n}{n_0} \simeq 0.2$

$E_z \sim 2$ GeV/m



SOLUTION OF THE "FORCED OSCILLATOR EQUATION"

(from L. Gorbunov & V. Kirsanov, 1987)

$$\frac{\partial^2 \delta n}{\partial t^2} + \omega_p^2 \delta n = \frac{n_0}{2} \frac{\partial^2 V_\perp^2}{\partial x^2} \quad ; \quad V_\perp = \frac{eA}{mc}$$

i) CLASSICAL LIMIT : $V_\parallel \ll V_\perp$; $\frac{V_\perp}{c} \ll 1$

ii) "ENVELOPE" APPROXIMATION : $V_\perp = \frac{1}{2} a\, e^{-i\omega t + ikx} + c.c.$

SLOWLY VARYING

WAKE/BEAT EXCITATION.

- Fully relativistic (strongly non linear wave)
- Broad pump (short pulse)
- Combination of wake (initial) + beat (final) mechanisms.

Relativistic 1-D cold fluid equations:

$$\frac{\partial}{\partial x_0}\left(\gamma_a\sqrt{\gamma^2-1}\right)+\frac{\partial}{\partial z}\gamma\gamma_a=\frac{\partial\varphi}{\partial z}$$

$$\frac{\partial}{\partial x_0}\left(n\gamma\gamma_a\right)+\frac{\partial}{\partial z}\left(n\gamma_a\sqrt{\gamma^2-1}\right)=0$$

ANY FRAME

$$\frac{\partial^2\varphi}{\partial z^2}=\frac{\omega_{p0}^2}{c^2}\left(\gamma\gamma_a\frac{n}{n_0}-\gamma_0\gamma_a\right)$$

$x_0=ct$, $\gamma=(1-V_n^2/c^2)^{-1/2}$, $\gamma_a=\sqrt{1+a^2}$

$\vec{a}=\frac{e\vec{A}}{m_0c^2}$; $\vec{E}^{HF}=-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}$, $\vec{B}^{HF}=\nabla\times\vec{A}$

$\varphi=\frac{e\phi}{m_0c^2}$, ϕ ambipolar potential

n is proper electron density

$\gamma_0=(1-V_0^2/c^2)^{-1/2}$ Lorentz factor

$n_0\gamma_0=n^0$ ion background in L frame

NOTICE:

γ_a contains all $\Delta\omega$ components of pump spectrum: $\omega=\omega_0\pm\Delta\omega$

RELATIVISTIC OPTICAL DRIVING

Refractive driving (∴ exceed Rayleigh lenght)

$$n = n(r) \, , \quad \frac{\partial n}{\partial r} < 0$$

Laser beam:

$$n = \frac{ck}{\omega} = \left(1 - \omega_p^2/\omega^2\right)^{1/2}$$

Relativistic electrons:

$$m_e = m_{e0}\gamma \, , \quad \gamma = \left(1 - v_\perp^2/c^2\right)^{-1/2}$$

$$P_\perp = \frac{e}{c} A \rightarrow \gamma = \left(1 + a^2(r)\right)^{1/2}$$

$$a(r) = e A(r)/m_{e0} c^2$$

∴

$$n(r) = \left[1 - \frac{\omega_{p0}^2}{\omega^2}\left(1 + a^2(r)\right)^{-1/2}\right]^{1/2}$$

$\frac{\partial n}{\partial r} < 0$ for beam peaked on axis.

CONDITIONS:

1) Fast time scale (ω^{-1}) → SHORT PULSE

2) High power: $P > P_{cr} = 17\left(\frac{\omega}{\omega_p}\right)^2 GW$

∴ $a \gtrsim 1$ → HIGHLY RELATIVISTIC

1) → Envelope approx'n fails

2) → Weakly Relativistic limit fails

3) 3-D effects ?!

- Broad spectrum

For maximum amplitude $1-W^2 \sim 1/\gamma_0^2$

$$\omega_{min} = \frac{\pi \omega_{p0}}{\sqrt{\gamma_0}}, \quad \omega_{max} \simeq \omega_{p0}\sqrt{\gamma_0}$$

"RESONANCE" CONDITION (inside pump)

$$\frac{\Delta\omega_{pump}}{\omega_0} \simeq \frac{\omega_{max}}{\omega_0} = \frac{\omega_{p0}}{\omega_0}\sqrt{\gamma_0}$$

For $\gamma_0 = \omega_0/\omega_p \Rightarrow \frac{\Delta\omega_{pump}}{\omega_0} \simeq \sqrt{\frac{\omega_{p0}}{\omega_0}}$

e.g. $n = 10^{17}\,cm^{-3}$ ($\omega_{p0} \simeq 10^{13}\,s^{-1}$)

$\omega_0 \simeq 10^{15}\,s^{-1}$

$$\frac{\Delta\omega_{pump}}{\omega_0} \simeq 10^{-1}$$

BUT : wave must be strongly NL

wake excitation
(charged sheet σ)

$$\sigma^2 \gg c^2 m_0 n_0 / 2\pi$$

For

$$\sigma \simeq e n_0 l_L (\gamma_a - 1)$$

CHECK FROM SIMULATION, (Katsouleas, Mori, Darrow 1989)

$\therefore$

$$\frac{\omega_{p0}^2}{c^2} l_L^2 (\gamma_a - 1)^2 \gg 1$$

For $l_L \sim c/\omega_{p0} \rightarrow \gamma_a \gtrsim 2$ ok

$$\frac{\partial^2 \gamma}{\partial \xi^2} - \left(\gamma \frac{n}{n_0} - \gamma_0\right) = \langle \text{driving term} \rangle$$ (1)

$$\frac{\partial}{\partial \xi}\left(n\sqrt{\gamma^2 - 1}\right) = \langle \text{driving term} \rangle$$

Averaging period $T = \frac{2\pi}{\Delta\omega} = \frac{2\pi}{\omega_{p0}\sqrt{\gamma_0}} = T_0$

$$\langle f \rangle = \frac{1}{\xi_0}\int_{\xi - \xi_0/2}^{\xi + \xi_0/2} f \, d\xi \longrightarrow d\xi = \left(\frac{d\gamma}{d\xi}\right)^{-1} d\gamma$$

$$\gamma_{Free}(\omega, \omega_0, \xi) \longrightarrow \gamma(\omega(\xi), \omega_0(\xi), \xi)$$

$$\frac{d\gamma}{d\xi} = \frac{\partial\gamma}{\partial\xi} + \frac{\partial\gamma}{\partial\omega}\frac{d\omega}{d\xi} + \frac{\partial\gamma}{\partial\omega_0}\frac{d\omega_0}{d\xi}$$

$\therefore$ System (1) $\longrightarrow$ System for $\frac{d\omega}{d\xi}, \frac{d\omega_0}{d\xi}$

e.g. for strongly N.L. case

$$\frac{d\omega_0}{d\xi} \simeq -2(1-\omega_0)\left\langle \frac{1}{\gamma_a}\frac{d\gamma_a}{d\xi} \right\rangle$$

$$\frac{d\omega}{d\xi} \simeq 48(1-\omega)^{3/2}\left\langle \frac{1}{\gamma_a}\frac{d\gamma_a}{d\xi}\left(\frac{d\gamma}{d\xi}\right)^2 \right\rangle$$

Function of ω, ω_0

AMPLITUDE CHANGES FASTER THAN ω_0

DEPLETION LENGHT

$$Z \sim 6\frac{c}{\omega_{p0}}\sqrt{\gamma_0}$$

Reduce to driven system:

$$\frac{\partial}{\partial z}\left(\frac{\partial}{\partial x_0}\sqrt{\gamma^2-1}+\frac{\partial\gamma}{\partial z}\right)-\frac{\omega_{p0}^2}{c^2}\left(\gamma\frac{n}{n_c}-\gamma_0\right)=$$

$$=-\frac{\partial}{\partial z}\ln\gamma_a\left(\frac{\partial}{\partial x_0}\sqrt{\gamma^2-1}+\frac{\partial\gamma}{\partial z}\right)-\frac{1}{\gamma_a}\frac{\partial}{\partial z}\left(\sqrt{\gamma^2-1}\frac{\partial\gamma_a}{\partial x_0}+\gamma\frac{\partial\gamma_a}{\partial z}\right)$$

$$\frac{\partial}{\partial x_0}n\gamma+\frac{\partial}{\partial z}\left(n\sqrt{\gamma^2-1}\right)=-\frac{1}{\gamma_a}\left(n\gamma\frac{\partial\gamma_a}{\partial x_0}+n\sqrt{\gamma^2-1}\frac{\partial\gamma_a}{\partial z}\right)$$

RHS = driving terms (to average over pump period)

Use Bogoliubov method based on "free" solution (RHS = 0).

Properties of "free" solution:

- Periodic, wave energy concentrated in soliton-like structures (∴ appearance of HF components)
- Period increased with amplitude

$$T=\frac{2\pi}{\omega},\quad \omega=\omega_{p0}\frac{\pi}{2}(1-W^2)^{1/4}/E(W)$$

W related to wave amplitude

$$\gamma_{min}^{max}=\frac{\gamma_0+\sqrt{\gamma_0^2-1}}{2\sqrt{1-W^2}}\pm\frac{\sqrt{1-W^2}}{2(\gamma_0+\sqrt{\gamma_0^2-1})}$$

Maximum potential well:

$$\Delta\phi_{max}=2\gamma_0^2\phi_D\gg\phi_D\ \text{(cold wave-breaking)}$$

e.g. $V_0=V_{ph}\Rightarrow\gamma_0=\omega/\omega_{p0}\gg1$

PROBLEMS (Some)

- CAN WE FIND "OPTIMUM" PUMP FOR MAXIMUM GROWTH?

- DEPHASING EFFECTS ($V_0 \neq V_g$)

- 3-D EFFECTS $\rightarrow$ OPTICAL DRIVING

- COUPLING OF WEAKLY N-L REGIME TO STRONGLY N-L