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MUON CATALYZED COLD AND HOT FUSION

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ABSTRACT

Nuclear fusion reactions can be catalyzed in a suitable fusion fuel by muons (heavy electrons) which can form temporarily very tightly bound mu-molecules. The muons can be produced by the decay of negative pions, which in turn have been produced by an accelerated beam of light ions impinging on a target. Muon catalyzed fusion (MCF) is appropriately called "cold fusion" because the nuclear fusion occurs also at room temperature. For practical fusion energy generation it seems necessary to have a fuel mixture of deuterium (D)-tritium(T) at about liquid density and temperature of the order of 1000°K.

The present status of MCF is limited to demonstrate scientific break-even by showing that it is possible to sustain an energy balance between muon production (input) and catalyzed fusion (output). Conceptually an MCF reactor is seen to be an energy amplifier increasing by fusion reactions the energy invested in the nuclear-pion-muon beams. The important physical quantity determining this balance is X_μ, the number of fusion reactions each muon can catalyze before it is lost.

Feasibility for useful power production is equivalent to showing that X_μ can exceed a sufficiently large number, which is estimated to be about 10^4 if standard technology is used, or may be of the order of 10^3 if more advanced physics and technology can be developed. Since a muon can be produced with present technology for an expenditure of about 5000MeV and ~20 MeV are produced per fusion event, it follows that $X_\mu \simeq 250$ would be a significant demonstration of scientific break-even. Present experiments have measured $X_\mu \simeq 150$. It may be therefore that the energy cost of producing muons must be substantially reduced before MCF reactors could be seriously considered.

In this paper, the physics of muon-catalyzed fusion is summarized and discussed. The muon catalysis is surveyed for the following systems: proton-deuteron (p-d), deuteron-deuteron (d-d), deuteron-triton (d-t) and non-hydrogen elements.

The idea of muon catalysis in a plasma medium is also presented. The formation of mu-atoms, mu-molecules and their disintegration in a condensed plasma are calculated. It looks that in a homogeneous plasma there are no appropriate values of temperature and densities for achieving the desired $X_\mu \simeq 1000$.

New ideas that might lead towards the goal of 1000 fusions per muon are suggested by using laser beams or selective electromagnetic radiation.

The paper is concluded with the recent suggestion implying that cold fusion can also be achieved in a crystal. The first experiments by Fleischmann and Pons[1] and Jones *et al.*[2] are briefly summarized and a phenomenological estimate is given for the cold fusion rate in the condensed matter medium.

1. INTRODUCTION

The muon (μ) was discovered by Anderson & Neddermeyer[3] and by Street & Stevenson.[4] In 1947 after the discovery of the pion (π) and the π decay into μ, it was realized that the muon is actually a "heavy electron" with a mass (M_μ)

$$M_\mu = 206{\cdot}77\ M_e = 105{\cdot}66\frac{\mathrm{MeV}}{c^2} \tag{1.1}$$

where M_e is the electron mass. μ and e also differ by their lepton quantum number. Moreover, the muon is an unstable particle with a lifetime (τ_μ) of

$$\tau_\mu = 2{\cdot}2.10^{-6}\mathrm{sec} \tag{1.2}$$

The size of a " μ– atom " (e.g. the hydrogen atom where the electron is replaced by a muon) is of the order of the muon Bohr radius a_μ, given by

$$a_\mu = \frac{M_e}{M_\mu}a_e = 2{\cdot}6.10^{-11}\mathrm{cm} \tag{1.3}$$

where a_e is the Bohr radius of an electron atom.

When an energetic negative muon (μ^-) enters a compressed hydrogen gas (e.g. hydrogen liquid density defined by $n_o = 4{\cdot}25.10^{22}\mathrm{atoms/cm^3}$), the following chain of reactions occurs: (a) Slowing down of μ^-. (b) Capture of μ^- into atomic levels and the cascade to the atomic ground state (1s). (c) "μ-molecular" formation and the de-excitation to the molecular ground state. (d) Nuclear fusion of the nuclei (usually hydrogen isotopes) which are kept close together by the negative muon. The μ^- is either released or captured by the nuclear fusion products (μ^- sticking). If the lifetime of the muon is long compared to the time scale of the other processes ([a] to [d]), then many fusion reactions can occur during the lifetime of a muon. This chain of reactions, resulting in the nuclear fusion process, is usually called *muon catalyzed fusion*.

The idea that negative muons might be able to catalyze proton-deuteron (p-d) fusion was first considered by Frank[5] in 1947. Deuterium-deuterium and deuterium-tritium fusions catalyzed by muons were suggested by Sakharov & Lebedev[6] in 1948 and further rediscovered by Zeldovich[7] and analyzed by Jackson.[8] The first experimental observation by Alvarez *et al.*[9] in 1957 of ($pd\mu$) fusion

$$(pd\mu)\rightarrow {}^3\mathrm{He}+\mu^- \ (5{\cdot}4\mathrm{MeV}) \tag{1.4}$$

was a rediscovery and for a short period of time the Berkeley group thought that they had "solved all the fuel problems of mankind for the rest of time."[10] After this discovery the muon catalyzed fusion was studied in England[11] through the reaction

$$(pd\mu)\rightarrow {}^3\mathrm{He}+\mu^- + \gamma\,(5{\cdot}4\mathrm{MeV}) \tag{1.5}$$

These experiments were followed by a series of theoretical calculations, mainly by Soviet scientists.[12] The main research resulted in the prediction that one could not expect more than a few catalyzed fusions per muon during his lifetime. It thus appeared that muon-catalyzed fusion reactions were useless as an energy source. This conclusion was questioned after Dzhelepov *et al.*[13] experiments which showed that the rate of dd molecule formation depends strongly on temperature. Following this experiment it was suggested by Vesman[14] in 1967 that the muon-molecules can be formed resonantly if there are weakly

bound states of these molecules (i.e. the binding energy is smaller that the dissociation energy of the hydrogen molecules – few eV). This idea was already mentioned by Zeldovich[7] in 1954, however, Dzhelepov *et al.* experiment,[13] Vesman's paper[14] and the following calculations[15],[16] and experimental measurements[17],[18] of the energy levels of the muon molecules were crucial steps in renewing the interest in muon catalyzed fusion.

Using the same resonance theory formation for the dtμ molecule, as for the ddμ molecule, it was predicted[19] a very large dtμ formation rate which was confirmed experimentally.[20],[21] However, not everything seems to be understood on muon catalyzed fusion. For example, the crucial problem of muon sticking[22] as observed at Los Alamos, at high density is not explained theoretically in a satisfactory way. Before 1985 theorists[8],[23],[24] predicted muon alpha sticking loss fraction to be about 0.9% which imply the possible maximum numbers (i.e. upper limit) of d-t fusions per muon to be 110. Los Alamos experiments[21],[22] measured 150 d-t fusions per muon.

A comprehensive understanding of muon catalyzed fusion comprises the knowledge of atomic physics, molecular chemistry, nuclear and particle physics, accelerator science and possibly reactor technologies. At this stage one can conclude only that better theories and more experimental evidence are necessary to complete our understanding.

2. ENERGY LEVELS OF μ-ATOMS AND μ-MOLECULES

The binding energy of the muonic atoms in the simple Bohr model are given by

$$E_B = -\bar{m}_\mu c^2 \frac{(Z\alpha)^2}{2n^2} \tag{2.1}$$

where $\bar{m}_\mu$ is the muon-nucleus reduced mass, Z is the nucleus electric charge, n is the principal quantum number and α is the fine-structure constant $\alpha=\frac{1}{137}$. The corresponding Bohr radius is

$$r_n=\frac{h^2 n^2}{4\pi^2 \bar{m}_\mu c^2 Z} \tag{2.2}$$

The binding energies of the "μ-atom" are two orders of magnitude larger than the corresponding binding energies of the electronic atoms, while the radii of the Bohr orbits are two orders of magnitude smaller. For example, the radius of the 1s level of "lead muon" (Pbμ) is 4 fm (1 fm = 10^{-13} cm) which is smaller than the 7 fm nuclear radius of Pb. From this example it is evident that one has to take into account relativistic effects. In this case the Dirac equation yields, to first order in αZ, for the muonic energy levels the following expression,

$$E_{nj} = -\bar{m}_\mu c^2 \frac{(\alpha Z)^2}{2n^2}\left[1+\frac{(\alpha Z)^2}{n^2}\left(\frac{n}{j+\frac{1}{2}}-\frac{3}{4}\right)\right] \tag{2.3}$$

The binding energy of the 1s level of Pbμ is 21 MeV, while the fine structure splitting of its $2p$ level is 550 keV!

For the muon-hydrogen atoms the particle masses and the energy levels of the ground states are[15]

$$M_p = 1836.11\, M_e\ ;\ M_d = 3670.40\, M_e\ ;\ M_t = 5496.75 M_e$$

$$\varepsilon(p\mu) = -2528{\cdot}52\text{eV};\ \varepsilon(d\mu) = -2663{\cdot}23\text{eV};\ \varepsilon(t\mu) = -2711{\cdot}27\text{eV} \tag{2.4}$$

while the hyperfine splitting is given by

$$\Delta\varepsilon(p\mu) = 0{\cdot}183\text{eV};\ \Delta\varepsilon(d\mu) = 0{\cdot}049\text{eV};\ \Delta\varepsilon(t\mu) = 0{\cdot}241\text{eV} \tag{2.5}$$

In the study of "μ- molecules" one encounters a difficulty which is not met in the study of ordinary molecules, namely the relatively large value of the ratio M_μ/M, where M is the mass of the hydrogen (isotope) constituting the nuclei of the "μ- molecule". This fact implies that the Born-Oppenheimer approximation is not a good method of calculation for the μ-molecules. For these molecules one needs to know the binding energy of a muon up to 0.1 eV out of about 3 keV [see equation (2.4)]. Therefore a relative precision of better than 10^{-4} is needed. Moreover, since the muon is relatively "heavy" one must consider also the nuclear recoil due to the muon motion. These facts make the numerical calculations extremely difficult for these molecules. A comprehensive review on the methods and results of the theoretical calculations of μ-molecular binding energies can be found in Gershtein and Ponomarev[15] and Bracci and Fiorentini.[16]

We will discuss now the Coulomb three body problem which is relevant for the understanding of the molecular eigenstates and eigenfunctions. In considering a molecule one simplifies the problem by separating the nuclear motion from the electron motion. (First we analyze the "electron" molecule). This separation is usually justified in molecules by the small ratio fo the electron to the nuclear masses. In this case, the energy of the nuclei, E_N is significantly smaller than the electron energy $E_e \gg E_N$. The nuclear motion is usually separated into three terms: translations, vibrations and rotations. The translation energy of the nuclei is almost equal to the kinetic energy of the molecule as a whole and this is the same as the energy of a free particle. In considering the energy eigenvalues of the molecule this motion can be extracted and therefore we shall not consider it at this stage. Denoting by R_e the dimension of the molecule, one can estimate the order of magnitude of the electron energy E_e using the uncertainty principle: the momentum is of the order of h/R_e implying an energy

$$E_e \approx \frac{h^2}{2M_e R_e^2} \sim 10\text{eV} \tag{2.6}$$

The vibration energy of the nuclei in a molecule, E_V, is $\frac{h}{2\pi}(K_e/M)^{1/2}(V+\frac{1}{2})$ where V is the vibration quantum number, M is the molecular mass and K_e is a stiffness constant. For large vibrations (of the order of R_e) the molecule dissociates and in this case the energy of vibrations $K_e R_e^2$ is of the order E_e, implying $E_V \approx \frac{h}{2\pi}(E_e/MR_e^2)^{1/2}$. Using equation (2.6) one gets an order of magnitude estimate for E_V

$$E_V \approx \left[\frac{M_e}{M}\right]^{1/2} E_e \sim 0.1\text{eV} \tag{2.7}$$

where $M_e/M \approx 10^{-3}$ to 10^{-4}. The energy of rotation is estimated from its inertia, MR_e^2 implying $E_J \approx h^2/(4\pi^2 MR_e^2)$ or equivalently (using eq. (2.6))

$$E_J \approx \left[\frac{M_e}{M}\right] E_e \sim 10^{-3}\ \text{eV} \tag{2.8}$$

where J denotes the rotational quantum number.

The muon molecules are composed by two (or more) nuclei bound together by a negative muon. In this case the dimension of these molecules are of the order of

$$R_\mu \approx \left[\frac{M_e}{M_\mu}\right] R_e \approx \frac{R_e}{200}\ , \tag{2.9}$$

therefore the vibrational (E_V), the rotational (E_J) and the muonic (E_μ) energies in a muon molecule satisfy the following orders of magnitude

$$E_\mu \approx \left[\frac{M_\mu}{M_e}\right] E_e \sim 1000\text{eV}$$

$$E_V \approx \left[\frac{M_\mu}{M}\right]^{1/2} E_\mu \sim 300\text{eV} \tag{2.10}$$

$$E_J \approx \left[\frac{M_\mu}{M}\right] E_\mu \sim 100\text{eV}$$

where the proton mass M was taken for these numerical evaluations. Equations (2.10) are only an order of magnitude estimation but already indicate an important difference between the muon and the electron molecules. In the muon case, the vibrational and rotational energies are not very small perturbations to the main muonic energy and therefore the Born-Oppenheimer method is expected to be a poor approximation for the calculations of the mu-molecules eigenvalues. However, the Born-Oppenheimer approach is the simplest solutions and although the numerical eigenvalues in this approach are not to be taken too seriously, it is useful to start with this approximation in order to show the different aspects of the muon molecules. In particular, we are interested in H_2^+ types of molecules such as $(pp\mu)^+$,$(pd\mu)^+$,$(pt\mu)^+$,$(dd\mu)^+$ and $(dt\mu)^+$ where p,d and t denote the proton, deuteron and the triton nuclei. The (exact) Hamiltonian describing these H_2^+ ion systems is given by (we use here the units system: $\hbar = 2\pi$, $e = 1$).

$$H = -\frac{1}{2M_a}\nabla^2_{\vec{R}_a} - \frac{1}{2M_b}\nabla^2_{\vec{R}_b} - \frac{1}{2M_\mu}\nabla^2_{\vec{R}_\mu} - \frac{1}{|\vec{r}_\mu - \vec{R}_a|} - \frac{1}{|\vec{r}_\mu - \vec{R}_b|} + \frac{1}{|\vec{R}_a - \vec{R}_b|} \tag{2.11}$$

where a and b denote the two nuclei, their position is given by $\vec{R}_a$ and $\vec{R}_b$ and the coordinate of μ is $\vec{r}_\mu$. The center of mass coordinate $\vec{R}_{C.M.}$ is given by

$$\vec{R}_{C.M.} = (M_\mu \vec{r}_\mu + M_a \vec{R}_a + M_b \vec{R}_b)/(M_\mu + M_a + M_b). \tag{2.12}$$

It is convenient to define Jacobi coordinates $\vec{R}$ and $\vec{r}$,

$$\vec{R} = \vec{R}_b - \vec{R}_a$$
$$\vec{r} = \vec{r}_\mu - (\vec{R}_a + \vec{R}_b)/2 \tag{2.13}$$

where $\vec{R}$ is the internuclear coordinate and $\vec{r}$ is the muon coordinate relative to the midpoint between the

two nuclei. In these coordinates $(\vec{R},\vec{r})$ the Hamiltonian (2.11) is given by

$$H = -\frac{1}{2M_T}\nabla^2_{\vec{R}_{CM}} - \frac{1}{2M_a}\left(\nabla_R + \frac{\kappa}{2}\nabla_r\right)^2 - \frac{1}{2m_o}\nabla_r^2 - \frac{1}{r_a} - \frac{1}{r_b} + \frac{1}{R} \tag{2.14}$$

where (assuming $M_a \geq M_b$)

$$\vec{r}_a = \vec{r}_\mu - \vec{R}_a = \vec{r} + \vec{R}/2;\ \vec{r}_b = \vec{r}_\mu - \vec{R}_b = \vec{r} - \vec{R}/2$$

$$m_T = M_\mu + M_a + M_b\ ,\ M_a^{-1} = M_a^{-1} + M_b^{-1}\ , \tag{2.15}$$

$$m_o^{-1} = M_\mu^{-1} + (M_a + M_b)^{-1}, \kappa = (M_b - M_a)/(M_b + M_a)$$

After separating the center of mass motion one has to solve the following Schroedinger equation:

$$H\Psi(\vec{r},\vec{R}) = E\Psi(\vec{r},\vec{R}) \tag{2.16}$$

with

$$H = -\frac{1}{2M_a}\left(\nabla_R + \frac{\kappa}{2}\nabla_r\right)^2 - \frac{1}{2m_o}\nabla_r^2 - \frac{1}{r_a} - \frac{1}{r_b} + \frac{1}{R} \tag{2.17}$$

Equation (2.16) with the Hamiltonian (2.17) is an exact three body problem. As already mentioned above we now use the Born-Oppenheimer approximation. This approach assumes the nuclei to be infinitely heavy with respect to the negatively charged particle. Moreover, for simplicity we follow the solution for the $(dd\mu)^+$ molecule in complete analogy with the H_2^+ molecule. In this approximation the Schroedinger equation is

$$\left(-\frac{1}{2m_o}\nabla_r^2 - \frac{1}{r_a} - \frac{1}{r_b} + \frac{1}{R}\right)\beta_R(\vec{r}) = E(R)\beta_R(\vec{r})\ , \tag{2.18}$$

where R plays the role of a parameter. The eigenfunctions of this equation are classified according to the principal quantum number n, the projection of the angular momentum over the internuclear axis m and the symmetry with respect to inversion (in the midpoint) between the nuclei (the even and odd states are denoted by g-gerade and u-ungerade respectively). The solution to eq. (2.18) is obtained from that of H_2^+ by rescaling the length ($\sim 1/M_o$) and the energy ($\sim M_o$). The potential $E(R)$ for the ground state (denoted by $1\sigma g$, where $n=1$,$m=0$, even parity) has a minimum of 577eV at R_o=512fm(1fm=10^{-13}cm). One can approximate $E(R)$ by a harmonic potential around R_o so that the effective potential for the internuclear motion is described by

$$V_{eff} = E(R_o) + \tfrac{1}{2}M_o\omega^2(R - R_o)^2 + J^2/(2M_oR_o^2) \tag{2.19}$$

where M_o is the reduced mass of the nuclei (see eq. (2.15)) and ω is derived by rescaling the H_2^+ solution, ω=9.1·10^{17}sec^{-1}. The potential (2.19) yields the following energy levels:

$$E_{JV} = E(R_o) + \omega\,(V + \tfrac{1}{2}) + J\,(J+1)/(2M_oR_o^2) \tag{2.20}$$

For $E(R_o) = -577$eV one can see that bound states (i.e. $E_{JV} < 0$) exist only for $V=0$,$J=0$,$V=0$,and $J=1$ with eigenvalues $E_{00} = -278$eV and $E_{10} = -143$eV. Thus, the Born-Oppenheimer approximation for the muon molecule $(dd\mu)^+$ gives only tow bound states in sharp distinction with the case of electron molecules (where a large vibrational and rotational spectrum exists). Similar features, i.e. very few bound states , are obtained by solving more accurately the Shroedinger equation.

In the Born-Oppenheimer approximation, the three-body wave function is given by a product of a muonic wave function times a nuclear wave function:

$$\Psi(\vec{R},\vec{r}) = u(\vec{R})\beta_R(\vec{r}). \tag{2.21}$$

This description is inadequate for the muon molecules. One way out of this difficulty is to use the so-called "adiabatic representation". In this approach[25]-[26] the wave function Ψ is expanded over a complete set of solutions of the two-body Coulomb problem defined by the Hamiltonian $H_o(R)$:

$$H_o(R) = -\frac{1}{2m_o}\nabla_r^2 - \frac{1}{r_a} - \frac{1}{r_b}\ , \tag{2.22}$$

where R is the distance between the two body (note: $\vec{r}_a = \vec{r} + \vec{R}/2$ and $\vec{r}_b = \vec{r} - \vec{R}/2$). The complete set of the eigenfunctions of (2.22) are called the adiabatic basis. One has to solve the two body Schroedinger equation

$$H_o(R)\phi_n(\vec{r};R) = E_n(R)\phi_n(\vec{r};R) \tag{2.23}$$

and the total wave function $\Psi(\vec{r},\ \vec{R})$ of the three-body problem is expanded in terms of the two-centre basis

$$\Psi_i\ (\vec{r},\vec{R}) = \sum_n \phi_n(\vec{r}\,;R)\chi_{n,i}(\vec{R}). \tag{2.24}$$

Ψ satisfies the three-body Coulomb Schroedinger equation, while $\chi_{n,i}(\vec{R})$ satisfies the following equation:

$$\left(-\frac{1}{2M}\nabla_R^2 + \frac{1}{R} + E_n(R) - E_i\right)\chi_{n,i}(\vec{R}) =$$

$$= \frac{1}{2M}\sum_k \int d^3r(\phi_n^*\nabla_R^2\phi_k)\chi_{k,i} + \frac{1}{M}\sum_k \int d^3r(\phi_n^*\vec{\nabla}_R\phi_k)\cdot\vec{\nabla}_R\chi^i \tag{2.25}$$

where $M = M_o M_\mu / m_e$. The total angular momentum J and the vibration V quantum numbers are denoted by the index i while the index n describes the muon configuration of the molecule. The integro-differential equation (2.25) has to be solved with a high accuracy in order to obtain the weakly bound mu-molecular states. (See ref. 16 for a review of the different approximations.)

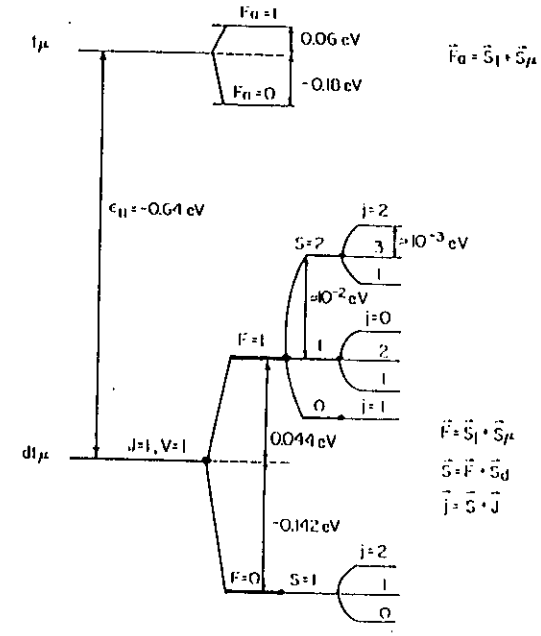
In table 1 we show the μ-molecular energy spectrum for the hydrogen isotopes. [28],[29] ε_{JV} (in electronvolt) is the difference between the molecular energy and the atomic energy of the heaviest hydrogen isotope. J and V are the rotational and the vibrational quantum numbers respectively. It is important to notice (from table 1) that ddμ and dtμ have weakly bound states with binding energies of -2.0 eV and -0.64 eV respectively. In these cases a resonance molecular formation is possible[14] i.e. the energy released from the μ-molecular formation is absorbed in the electronic molecule. Another interesting feature of the μ-molecules is the fact that very few levels can satisfy $\varepsilon_{JV} < 0$. This situation is significantly different to the case of ordinary molecules where a rich vibrational and rotational spectrum exists.

In calculating the spectrum of the μ-molecules the spin effects must also be taken into account. The hyperfine structure of the energy levels have been calculated[30] to lowest order in α. In figure 1 we show a schematic hyperfine structure of the levels tμ atom and the dtμ molecule.[31] Denoting by F the spin quantum number of the tμ system (in the dtμ molecule), by S the molecular total spin quantum number $\vec{S} = \vec{F} + \vec{S}_d = \vec{S}_t + \vec{S}_\mu + \vec{S}_d$) and by j the total angular momentum $\vec{J} = \vec{S} + \vec{J}$) , one can see from figure 1 that $\Delta\varepsilon_F \sim 10^{-1}$eV, $\Delta\varepsilon_S \sim 10^{-2}$eV and $\Delta\varepsilon_j \sim 10^{-3}$eV. ($\Delta\varepsilon_F = \varepsilon\ (J,V\ ;\ F=1) - \varepsilon(J,V\ ;\ F=0)$, etc.)

TABLE 1. Binding energy ε_{JV} (eV) of the μ-molecules of the hydrogen isotopes with respect to the appropriate mesic atom (i.e the difference between the molecular energy and the atomic energy of the heaviest isotope.)

Rotational (J) and Vibrational (V) quantum numbers	$-\varepsilon_{JV}$ (eV)					
	(ppμ)	(pdμ)	(ptμ)	(ddμ)	(dtμ)	(ttμ)
$J=1, V=1$				2.0	0.64	44.9
$J=3, V=0$						47.7
$J=0, V=1$				35.6	34.7	83.7
$J=2, V=0$				85.6	102.3	172.0
$J=1, V=0$	105.6	96.3	97.5	226.3	232.2	288.9
$J=0, V=0$	253.0	221.5	213.3	325.0	319.1	362.9

FIGURE 1. Schematic hyperfine structure of the levels of the atom tμ and the molecule dtμ.



In order to appreciate the difficulty of the energy level calculations we end this section pointing out the effects of vacuum polarization, the strong interactions between the nuclei of the μ-molecule and the electron screening effect. For example, the vacuum polarization shifts the energies upwards (i.e. increase the binding energy) for the 1 s states in pμ, dμ, and tμ, by 1.89 eV, 2.13 eV and 2.21 eV respectively.[32] The energy level shifts of the dtμ molecule induced by the nuclear interaction [16] are about 10^{-3} eV for the $(J,V)=(0,0)$ and $(0,1)$ states and are as small as 3.10^{-8} eV for the $(J,V)=(1,1)$ state. The electron screening effects may contribute a shift in the energy levels of about 10^{-2} eV for the dtμ and ddμ molecules. Since there are errors of the order of 0.1 eV in the calculations of the Coulomb energies, most of the non-Coulomb contributions just discussed are of academic interest only.

3. MUON-CATALYZED p-d FUSION

The slowing down and absorption of μ^- in hydrogen were calculated by Fermi & Teller[33] assuming a mechanism of adiabatic ionization where the electron leaves the atom and the negative muon is captured into a highly excited orbit of the pμ atom (see Fig. 2).

$$\mu^- + H \rightarrow p\mu + e^- \tag{3.1}$$

where H denotes a hydrogen atom. In this model[34] a μ^- slows down from relativistic velocities ($v \approx c$) to $v=\alpha c$ ($\alpha = \frac{1}{137}$) in a liquid hydrogen density ($n_{liq} = 4{\cdot}25\times10^{22}$ atoms/cm^3) and is captured into a pμ atom in less than about 10^{-9} sec. The deexcitation time of an isolated pμ atom is very large. However, in liquid hydrogen density the mu-hydrogen atom decays to its $n=4$ level state[35] during a time of about 10^{-12} sec due to Auger effect on the electrons of another atom. The μ^- decays from $n=4$ to the ground state by radiative transition during 10^{-9} sec.

The charge neutrality and the small dimensions of the pμ atom enable it to penetrate freely through the electron shells of atoms and molecules and to approach the nuclei within a distance of about the muon Bohr radius $a_\mu = 2{\cdot}6.\ 10^{-11}$ cm (eq. (1.3)). The typical elastic cross sections of these atoms are about $\sigma \approx 4\pi a_\mu^2 = 10^{-20}\text{cm}^2$.

The muon from the pμ atom can be transferred irreversibly to a deuteron

$$p\mu + d \rightarrow d\mu + p \tag{3.2}$$

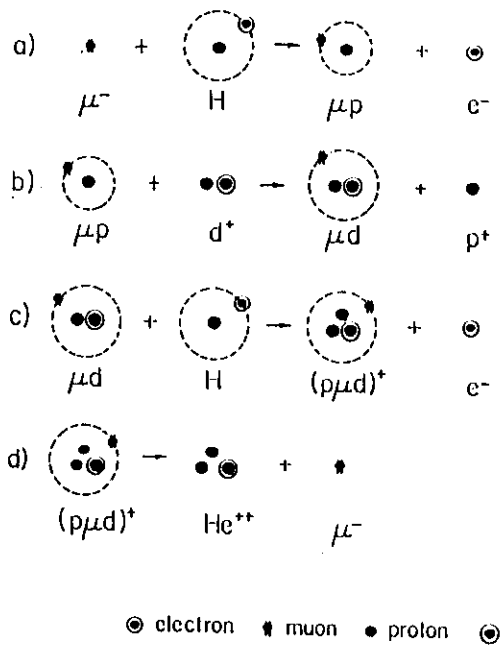
because the binding energy for the dμ ground state is 135 eV lower than that of the pμ atom. This transfer λ_{pd} is about 10^{10} sec^{-1}. Even for a very low deuteron concentration $C_d = 4\cdot10^{-5}$, the yield of equation (3.2) is 4.10^5 sec^{-1}, which is not too different than the μ^- decay probability rate $\approx\tau_\mu^{-1}\approx4{\cdot}5.10^5\text{sec}^{-1}$.

Mu-molecules (sometimes called mesonic molecules, however, since the muon is a lepton and not a meson, we avoid the nomenclature "mesoatoms" and "mesomolecules" for the muon atoms and the muon molecules) are created during collisions of dμ with the hydrogen nucleus.

$$\mu d + H \rightarrow (pd\mu)^+ + e^- \tag{3.3}$$

In the pdμ molecule, the proton and the deuteron are separated by a distance of about 10^{-10} cm (few times a_μ). Therefore, there is a large probability per unit time ($\lambda_f = 3.10^5\text{sec}^{-1}$) for the proton and deuteron to overcome the Coulomb barrier and to fuse into ^{3}He and μ^- (equation (1.4)) or into $^3\text{He}\mu + \gamma$ (eq. (1.5)). For a liquid hydrogen density with 1% deuterium the sticking probability (i.e. for reaction (1.5)) is

FIGURE 2. μ^- catalysis of p-d fusion.



85%! In this case only 15% of the muons which did not decay are available to repeat the muon catalysis cycle. The sequence of reactions leading to muon catalyzed pd fusion, in liquid hydrogen density containing 1% deuterium, can be repeated less than one fusion per muon.

4. MUON CATALYZED d-d FUSION

When a negative muon enters a condensed deuterium medium it quickly forms a dμ atom ($\lambda_\infty \approx 4.10^{12}\text{sec}^{-1}$ for liquid hydrogen density (l.h.d.) of deuterium). Subsequently a muonic molecular ion $(dd\mu)^+$ is formed with a rate $\lambda_{dd\mu}\approx3.10^6\text{sec}^{-1}$ (for l.h.d.). The deuterons are tightly bound (radius 5. 10^{-11} cm) and therefore the following muon induced fusions can occur:

$$(dd\mu)\rightarrow \begin{cases} ^3\text{He} + n + \mu(+3{\cdot}3\ \text{MeV}) \\ ^3\text{He}\mu + n \\ t + p + \mu(+4\text{MeV}) \\ t\mu + p \end{cases} \tag{4.1}$$

About 88% of the muons repeat the cycle while 12% are lost to ^{3}He by forming the $^3\text{He}\mu$ atom. This last effect is called the "sticking" loss.

The ddμ molecule has a rotational-vibrational level (J=1, V=1) with a very small binding energy (~ 2 eV). This fact causes a resonance formation of this molecule,[14] namely, during the E1 transition to the J=1, V=1 level, the binding energy (of ddμ) is transferred to the entire D_2 molecule. This phenomenon can happen as long as the binding energy of ddμ is larger that the dissociation energy of the D_2 molecule (4.7 eV).

The energy released in the formation of the ddμ molecule is transferred to the excited levels of the [(ddμ)d2e] molecule (an H_2 type of molecule), which is produced during the collision of a dμ atom with a D_2 molecule,

$$d\mu + D_2 \rightarrow [(dd\mu)d2e]^* \tag{4.2}$$

The rate of the resonant formation is given by

$$\lambda = n_0 \sigma \upsilon \ \text{sec}^{-1} \tag{4.3}$$

where n_0 is the D_2 density (of the order of the l.h.d.), υ is the relative velocity of the colliding particles [in (4.2)] and σ is the appropriate cross section. Following Vinitskii *et al.*[28] the rate for resonant production of ddμ or (dtμ) is given by

$$\lambda(\text{sec}^{-1}) = g\frac{64\pi^5 M_e e^4}{3h^3}(n_0 a_0^3)\left[\frac{M_e}{M_\mu}\right]^5\left[\frac{M_\mu}{m_a}\right]^2 I_v^2 |d_{fi}|^2 \gamma(\varepsilon_0, \varepsilon_T) \tag{4.4}$$

where g is a statistical factor which equals to ½ for ddμ formation (and 1 for dtμ), a_0 is the Bohr radius ($a_0 = h^2/m_e e^2$); $m_a^{-1} = M_\mu^{-1} + M_a^{-1}$ where M_a is the deuteron (or triton) mass; I_v is the dipole interaction matrix element describing the transition from the ground state $D_2(J=0,V=0)$ molecule to the excited [(ddμ)d2e] molecule ($J=1,V=8$); d_{fi} is the dipole matrix element calculated between the functions of the initial state i of the dμ + d system and the final state $f(J=1,V=1)$ of the ddμ molecule; γ is the Maxwellian distribution factor

$$\gamma(\varepsilon_0, \varepsilon_T) = \left[\frac{27\varepsilon_0}{2\pi}\right]^{1/2} \varepsilon_T^{-\frac{3}{2}} \exp(-3\varepsilon_0/2\varepsilon_T), \tag{4.5}$$

$\varepsilon_T = \frac{3}{2}kT$ is the average kinetic energy of the dμ atoms at temperature T and ε_0 satisfies the resonant condition

$$\varepsilon_0 = \varepsilon_{11} + E_v - E_0 \tag{4.6}$$

where ε_{11} is the binding energy of the molecule ddμ (or dtμ) for the $J=1,V=1$ state E_v is the vibrational energy of the [(ddμ)d2e] (or [(dtμ)d2e]) molecule. Equation (4.5) indicates a temperature dependence of λ(ddμ) as given in (4.4). The maximum value of λddμ is achieved at $\varepsilon_0=\varepsilon_T=0.053$eV (~ 600°K) and equals 0.8 x 10^6 sec^{-1} in good agreement with the measured experimental value. (The rates are calculated at l.h.d., $n_0=4.25\times10^{22} cm^{-3}$). Moreover, further experiments have confirmed the temperature dependence of the formation rate λ(ddμ); e.g. when the temperature of the deuterium changes from -160°C to +100°C (Δε≈ 0.04 eV corresponds to 260°C) the rate λ(ddμ) increases by a factor of four.[28,36] The nonresonant formation of the ddμ molecule occurs according to the reaction

$$d\mu + D_2 \rightarrow [(dd\mu)de]^* + e^- \tag{4.7}$$

and this process has a reaction rate of about 5×10^4sec^{-1}, which differs by a factor of 16 from the measured resonance formation rate.[13]

The muon catalysis in pure deuterium can be summarized by the following chain of reactions:

(a) μ^- is slowed down and captured by the deuterium (dμ) during a about 10^{-9} sec.

(b) The dμ atom collides with the l.h.d. of D_2 molecules and a [(ddμ)d2e] molecule is formed resonantly. The time scale for this process at ~600°K is about 10^{-6} sec.

(c) The dd induced fusion occurs in $\sim10^{-9}$ sec where ^{3}He+n (60%) and t + p (40%) are created. About 12% of the muons are lost by the sticking process to ^{3}Heμ, while 88% of the μ^- repeat the cycle.

5. MUON CATALYZED d-t FUSION

When a negative muon enters a dense deuterium (D)-tritium (T) target it starts a chain of reactions: (we denote by capital letters D and T the deuterium and tritium atoms while the small letter denotes the nuclei).

(a) Stopping and capture of μ^- by d or t and the cascade to the ground state of the hydrogen like atom,

$$\mu^- + D \xrightarrow{\lambda_a} (d\mu) + e^- \tag{5.1}$$

$$\mu^- + T \xrightarrow{\lambda_a} (t\mu) + e^- \tag{5.2}$$

where e^- denotes an electron and λ_a is the rate of the process (5.1) and (5.2). In general, the rate λ (sec^{-1}) is given by

$$\lambda = n\sigma\upsilon \ [\text{sec}^{-1}] \tag{5.3}$$

where n[cm^{-3}] is the density of the target, σ[cm^2] is the cross section describing the process under consideration and υ[cm/sec] is the velocity of the projectile. The rate of the muonic atomic formation[37] in equations (5.1) and (5.2) for the liquid hydrogen density is 4×10^{12}sec^{-1}

(b) μ^- transfer from the deuterium to the tritium

$$(d\mu) + t \xrightarrow{\lambda_{dt}} (t\mu) + d \tag{5.4}$$

The rate for this process at liquid density is estimated[21] to be 2×10^8sec^{-1}. The 48 eV difference in the binding energies of (dμ) and (tμ) causes the transfer of μ in an irreversible process (by equation (5.4)).

(c) At this stage, a (dtμ) molecule ion may be formed at the center of an H_2 type molecule. The formation of the relevant hydrogen type mumolecules are described by

$$(d\mu) + D_2 \xrightarrow{\lambda_{dd\mu}} [(dd\mu)d2e] \tag{5.5}$$

$$(t\mu) + D_2 \xrightarrow{\lambda_{dt\mu-d}} [(dd\mu)d2e] \tag{5.6}$$

$$(t\mu) + DT \xrightarrow{\lambda_{dt\mu-t}} [(dt\mu)t2e] \tag{5.7}$$

$$(t\mu) + T_2 \xrightarrow{\lambda_{t\mu t}} [(t\mu)t2e] \tag{5.8}$$

where in general the mumolecule is in an excited state. The values of the rates describing equations (5.5)-(5.8) were measured[21] to be $\lambda_{dd\mu}\approx 3\times10^6 sec^{-1}$, $\lambda_{t\mu t}\approx 3\times10^6 sec^{-1}$ and $\lambda_{dt\mu}\geq 10^8 sec^{-1}$, where $\lambda_{dt\mu}$ is defined by

$$\lambda_{dt\mu} = \lambda_{dt\mu-d} C_d + \lambda_{dt\mu-t} C_t \tag{5.9}$$

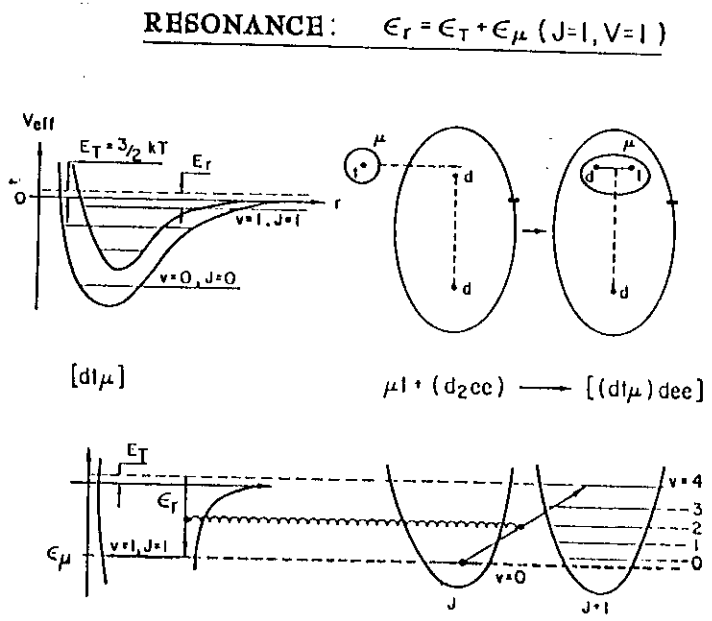
C_d and C_t are the concentrations of the deuterium and tritium nuclei, so that if the presence of He and other impurities are neglected one has,

$$C_d + C_t = 1 \tag{5.10}$$

The value of the $dt\mu$ rate is almost two orders of magnitude larger than the $dd\mu$ and $tt\mu$ rates. This phenomena can be explained by the resonant formation of these molecules.[28] A degeneracy in the excited state of the $dt\mu$ ion and the excited state of the electron molecular complex is causing a strong resonance effect (see Fig. 3). The rate $\lambda_{dt\mu}$ is found to be dependent on the temperature since the kinetic energy of the $t\mu$ atom, which forms the $dt\mu$ ion, is temperature dependent. This resonance condition can be described by the energy conversion

$$\epsilon_e = \epsilon_T + \epsilon_\mu \tag{5.11}$$

FIGURE 3. Resonance formation of the $dt\mu$ molecule.



where $\epsilon_\mu = 0.64$cV is the energy of $dt\mu$ ion in the quantum state $J=1, V=1$ (J is the rotational quantum number and V is the vibrational quantum number of the ion $(dt\mu)^+$) ϵ_e is the electron energy in the H_2^+-type molecule and $\epsilon_T = 3/2kT$ is the thermal energy of the $t\mu$ atom at a temperature T (for $T = 500°K$) one has $\epsilon_T\approx 0.04$eV). The energy released ($\epsilon_T + \epsilon_\mu$) during the formation of the $J=1, V=1$ $dt\mu$ ion is absorbed resonantly by the electronic molecular excited states of the mumolecule [(dtμ)d2e]. Since the thermal and the electronic molecular excitation have broad energy distributions, the resonance condition can be satisfied over a large range of temperatures. This effect has been observed experimentally.[22]

(d) Next in the chain of reactions one has the deexcitation of $dt\mu$, $dd\mu$, and $tt\mu$ ions to their ground states and the occurrence of the nuclear fusion reactions

$$d + t \xrightarrow{\lambda_f} {}^4He + n + 17.6MeV \tag{5.12}$$

$$d + d \xrightarrow{½\lambda_{fd}} {}^3He + n + 3.3MeV \tag{5.13}$$

$$d + d \xrightarrow{½\lambda_{fd}} t + p + 4MeV \tag{5.14}$$

$$t + t \xrightarrow{\lambda_{ft}} {}^4He + 2n + 10MeV \tag{5.15}$$

The fusion rates are estimated[29] to be $\lambda_f\approx 10^{12} sec^{-1}$ for the dt nuclear reaction while $\lambda_{fd}\approx\lambda_{ft}\approx 10^{11} sec^{-1}$.

(e) During the fusion reaction there is a possibility that the muon sticks to a charged product. In particular, in reaction (5.12), the muon can stick to the ^{4}He by forming a muonic helium ion (^{4}Heμ). In this case, the muon stays bound to the He particle and therefore it is lost for the chain of reactions described above [(a) to (d)]. The sticking probabilities for the reactions given by equations (5.12)-(5.15) were estimated to be[23] $\omega_s\approx 0.9\times10^{-2}$, $\omega_d\approx 0.13$ [for equation (5.13)], $\omega_d'\approx 0.003$ [for equation (5.14)] and $\omega_t\approx 0.05$, implying that the sticking fraction to the He particle in the dt fusion is about 0.9% of the events, in the dd fusion about 15% and in the tt fusion about 5%. For the most important dt case, the value of ω_s implies that the muon sticks to the He particle after catalyzing $1/\omega_s\approx 110$ fusions, no matter how fast are the other processes leading to the mesomolecule formation and fusion. The sticking process may be the bottle neck of the muon catalyzed fusion idea. However, new Los Alamos experiments[22] had measured 150 fusions per muon indicating that the sticking probability is about 0.6%. Moreover, a density dependent analysis[22] of the alpha-muon sticking might suggest $\omega_s = (0.2\pm0.1)\%$ for $\phi=1.2$ (i.e. 1.2l.h.d.). At present, neither the smallness of ω_s nor its density dependence is understood. Jackson[8] was the first to estimate $\omega_s\approx 0.9\%$ by using a Born-Oppenheimer approximation. A Monte Carlo calculation by Ceperley & Alder[37] yielded $\omega_s=0.6\%$. Mueller & Rafelski[38] suggested that the nuclear d-t interaction affects the wavefunctions describing the $dt\mu$ system in such a way that ω_s is reduced to 0.1%. It has also been suggested by Cohen & Leon[39] that a fraction of the muons may be delayed significantly during the deexcitation of the $dt\mu$ to the ground state, thus increasing ω_s (eff) above a very small initial value.[40]

A set of rate equations can be written to describe the kinetics[28] of the chain reactions described above from (a) to (e). The rates for these reactions and the sticking probabilities are summarized in

table 2 for liquid density of the hydrogen isotopes ($4.25\times10^{22}cm^{-3}$). The solution of the rate equations results in an expression for the fusion neutron yield— X_μ, namely the average number of dt fusion catalyzed by one muon. For high density mixtures of d and t and neglecting small effects, one obtains that the value of X_μ^{-1} is given by the sum of probabilities of muon decay during a catalysis cycle and the muon capture by ^{3}He or ^{4}He (sticking)

$$X_\mu^{-1} = \frac{\lambda_\mu}{\lambda_C} + W \tag{5.16}$$

where $\lambda_\mu = 1/\tau_\mu = 0.45\times10^6 sec^{-1}$, λ_C is the muon cycling rate, estimated to be

$$\frac{1}{\lambda_C} = \frac{C_d}{\lambda_{dt} C_t} + \frac{1}{\lambda_{dd\mu} C_d} \tag{5.17}$$

and W is the probability of muon capture by He, and can be approximated by

$$W = \omega_S + \frac{0.5\lambda_{dd\mu}\omega_d C_d}{\lambda_{dd\mu} C_d + \lambda_{dt} C_t} + \frac{\lambda_{tt\mu}\omega_t C_t}{\lambda_{dt\mu} C_d} \tag{5.18}$$

The rate equations are more complex if one takes into account the existence of ^{3}He and ^{4}He concentrations in the deuterium-tritium target. In this case equation (5.10) is replaced by

TABLE 2. Estimates of the rate of coefficients at hydrogen liquid density ($n=4.25\times10^{22}cm^{-3}$) normalized to the rate of muon decay ($\tau_\mu=2.2\times10^{-6}$sec). The sticking probabilities are given by their inverse ($1/\omega$).

Process	$\lambda\tau_\mu$	$1/\omega$
$\mu+D\rightarrow(d\mu)+e$	$\lambda_a\cdot\tau_\mu=8.8\times10^6$	
$\mu+T\rightarrow(t\mu)+e$	$\lambda_a\cdot\tau_\mu=8.8\times10^6$	
$(d\mu)+D_2\rightarrow(dd\mu)d2e$	$\lambda_{dd\mu}\cdot\tau_\mu=6.6$	
$(t\mu)+D_2\rightarrow(dt\mu)d2e$ + $(t\mu)+DT\rightarrow(dt\mu)t2e$	$\lambda_{dt\mu}\cdot\tau_\mu>2.2\times10^2$	
$(t\mu)+T_2\rightarrow(tt\mu)t2e$	$\lambda_{tt\mu}\tau_\mu=6\cdot6$	
$d+t\rightarrow{}^4He+n$	$\lambda_f\cdot\tau_\mu=2\cdot2\times10^6$	$\frac{1}{\omega_s}=300$
$d+d\begin{cases}50\%\rightarrow{}^3He+n\\50\%\rightarrow t+p\end{cases}$	$\lambda_{fd}\tau_\mu=2.2\times10^5$	$\frac{1}{\omega_d}=7.7$
$t+t\rightarrow{}^4He+2n$	$\lambda_{ft}\tau_\mu=2.2\times10^5$	$\frac{1}{\omega_t}=20$

$$C_d + C_t + C_{He} = 1 \tag{5.19}$$

where $C_{He} = C_{^3He} + C_{^4He}$.

Although the muon catalysis of d-t is not yet completely understood, the recent experiments suggest that about 200 fusions per muon may be possible to achieve in a deuterium-tritium mixture.

6. MUON-CATALYZED FUSION FOR NON-HYDROGEN ELEMENTS

It is very interesting to analyze the muon-catalysis of nuclei with charge $Z>1$ in general, and these leading to nuclear reactions without neutrons in the final state in particular. For example, the following reactions are of major interest in achieving "very clean" fusion:

$$d+{}^3He\rightarrow{}^4He+p+18.3MeV \tag{6.1}$$

$$d+{}^6Li\rightarrow2{}^4He+22.4MeV \tag{6.2}$$

$$p+{}^{11}B\rightarrow3{}^4He+8.7MeV \tag{6.3}$$

However, in order to achieve muon catalysis, the muon molecules must be formed and the nuclear fusion must occur faster than the muon decay time. It turns out that although the rates for some molecular formations have been calculated to be faster than the muon decay, the rates of the nuclear fusion reactions are too small.[41-46]

The formation rates $\lambda_p = \lambda(Zp\mu)$, $\lambda_d=\lambda(Zd\mu)$ and $\lambda_t=\lambda(Zt\mu)$ for the appropriate molecular formation were calculated to be[46]

$$\begin{aligned}
Z={}^3He &: \lambda_p = 0.87\cdot10^8 sec^{-1} ; \lambda_d = 1.48\cdot10^8 sec^{-1} ; \lambda_t = 5.62\cdot10^8 sec^{-1}\\
Z={}^4He &: \lambda_p = 0.44\cdot10^8 sec^{-1} ; \lambda_d = 2.03\cdot10^8 sec^{-1} ; \lambda_t = 1.98\cdot10^8 sec^{-1}\\
Z={}^6Li &: \lambda_p = 22.1\cdot10^6 sec^{-1} ; \lambda_d = 3.45\cdot10^6 sec^{-1} ; \lambda_t = 2.08\cdot10^6 sec^{-1}\\
Z={}^7Li &: \lambda_p = 10.8\cdot10^6 sec^{-1} ; \lambda_d = 1.85\cdot10^6 sec^{-1} ; \lambda_t = 0.81\cdot10^6 sec^{-1}
\end{aligned} \tag{6.4}$$

The rates are calculated for thermal collisions energy of $\varepsilon_0\approx0.04eV$ (~450°K).

Experimental data for ^{4}Hepμ, [47] ^{4}Hedμ, [48] ^{3}Hedμ and ^{3}Hetμ [21] are in agreement with the theoretical calculations summarized in (6.4).

The low rates of the fusion reactions of these molecules in comparison with the appropriate "hydrogen" muon molecules are primarily caused by the large nuclear separation—R. While $R\approx2a_\mu$ for muonic hydrogen molecules, $R\approx4a_\mu$ for the Hepμ molecule and $R\approx6a_\mu$ for the Lipμ molecule, where a_μ is the Bohr radius of muonic hydrogen. Due to the increase in the Coulomb barrier one gets for the Bpμ molecules a value of $R\approx15a_\mu$. It is estimated[46] that the fusion reaction rate for ^{3}Hedμ is about 100 sec^{-1}; while for ^{6}Lidμ it drops down to 0.01 sec^{-1}. Therefore due to the large dimensions of these molecules the fusion reaction rates are small in comparison with the muon decay rate. Moreover, the Zpμ molecules are formed in an excited state which dissociate very fast ($\approx10^{-12}$sec) by Auger or radiative transitions

$$p\mu + Z\rightarrow Zp\mu\rightarrow Z\mu + p. \tag{6.5}$$

In this respect, the Zdμ and Ztμ are distinguished from the Zpμ molecules.

In conclusion, muon catalyzed fusion for energy application might be used only for "clean" deuterium-tritium mixtures.

7. ENERGY PRODUCTION

Muon absorption in matter and the induced fusion of deuterium and tritium is a remarkable phenomenon since the negative muon is capable during its lifetime ($\tau_\mu = 2{\cdot}2 \times 10^{-6}$ sec) of inducing about 150 nuclear dt fusion reactions in a liquid density medium. This end result of nuclear fusion occurs after a chain of atomic and molecular processes. The most crucial step in the physics of muon catalysis cycle is the resonant formation of the dtμ molecule which increases the probability of the end result by at lest two orders of magnitude (relative to nonresonant dtμ formation). However, this high production rate of the dtμ molecule is disturbed by muons lost in the catalysis cycle. There seems to be a probability of about 0.3% that the helium ion created during the nuclear fusion interaction will capture the negative muon.

The "energy cost" for the creating of a muon is one of the most important practical parameters in analyzing the relevance of muon catalyzed fusion for energy production. The muons are produced during the decay of pions which can be created in nucleon-nucleon collisions. The energy threshold for pion generation is about 500 MeV of projectile kinetic energy and the process of negative muon generation seems to be effective for nucleonic projectiles with a kinetic energy of 1 GeV per nucleon. An optimistic estimate requires 5 GeV of energy to produce one negative muon. Therefore if one muon catalyzes about two hundred dt fusions the energy output is ≈3.5 *GeV* (one dt fusion gives 17.6 MeV) per muon, so that no energy gain seems to be possible from "pure' fusion nuclear reactions.

For a pure fusion reactor one can define the scientific gain G, as the ratio between the output energy $X_\mu q$ and the input energy E_μ

$$G = qX_\mu / E_\mu ,$$

where X_μ is the number of fusion reactions per muon, q = 17.6 MeV (for a dt reaction), and E_μ is the energy invested in creating one muon. Taking into account the thermal to electricity efficiency η_{th}, the accelerator efficiency η_A and the fraction η_r of the total electric power required to recirculate (in order to operate the accelerator and the auxiliary facilities) one gets the basic equation for the energy balance

$$X_\mu = \left(\frac{E_\mu}{17.6\text{MeV}} \right) (\eta_A \eta_r \eta_{th})^{-1} \qquad (7.1)$$

For example, taking optimistic efficiencies $\eta_A = 60\%$, $\eta_{th} = 35\%$ and assuming $\eta_r = 15\%$ is an economic possibility, one needs $X_\mu = 9000$ in a muon catalyzed reactor. This number seems to be unrealistic for the present status of knowledge. However, by improving technologies $\eta_{th} = 80\%$, $\eta_A = 70\%$ and $\eta_r = 20\%$ together with $E_\mu = 2$GeV, one reduces X_μ to 1000.

Since pure fusion devices using muon catalyzed fusion seem to lose rather than to gain energy, Petrov[49] suggested to use muon catalysis in a hybrid fusion-fission reactor. This reactor scheme includes *an accelerator* (of tritium or deuterium), *a target* (of tritium or beryllium) where the pions are created, *a synthesizer* with the d-t fuel and *a blanket* where the fusion and fissile materials are produced.

Following Petrov's concept of a muon catalyzed fusion-fission reactor, Eliezer, Tajima and Rosenbluth[50] suggested a reactor concept based on two new main ideas:

(a) The high energy beam of tritium or deuterium (~ 1 GeV/nucleon) is injected into a target of tritium (or beryllium) with dimensions smaller than the mean free path for strong interactions. After passage through the target, the bulk of the beam is collected for reuse and only the small portion of the beam which suffered strong interactions is directed into an electronuclear blanket.

(b) The pions created in the target are surrounded by the fuel of deuterium-tritium and are magnetically confined until they slow down and decay into muons which catalyze the fusion *in situ*. In this scheme *the converter and synthesizer are combined into one vessel*. The d-t fuel is the target and the produced pions are trapped in the fuel which slows down the pions, so that the necessary conditions for trapping the pions are sufficient conditions for stopping the muons. These muons cause the catalyzed chain reactions leading to nuclear dt fusion. In this way an efficient trapping of muons in a relatively small physical volume is achieved.

7.1. Petrov's Hybrid Reactor

In 1980, Petrov suggested a power reactor based on muon catalyzed fusion combined with nuclear fission processes. This reactor scheme includes *an accelerator* (of tritium or deuterium), a *target* where pions are created, a *converter* where the pions are confined in vacuum by strong magnetic fields, a *synthesizer* with the d-t fuel and *a blanket* where the fusion and fissile materials are produced. The converter is a cylinder about 40 m long and 20 cm radius (5 m^3 volume) having a longitudinal magnetic field of 11 to 16 Tesla and an applied d.c. electric field of 7.5×10^5 volt/m along the converter. Inside this vessel there is a cylinder 2 m long with a 2 cm radius target where the pions are created. The synthesizer is a second cylinder about 20 m long with an average radius of 20 cm surrounded by a (longitudinal) magnetic field coil of 11 Tesla. The density of the dt fuel is 0.5 liquid hydrogen density with 30% tritium, so that the synthesizer has 80 kg of tritium. The main result of the converter-synthesizer is the conversion of about 75% of the created pions into muons that participate in the catalyzed dt fusion. This means that the energy cost of a stopped negative muon in the dt mixture is about 6 GeV (using 4.5 GeV to create one negative pion) for tritium projectiles and 8 GeV for a deuterium projectile beam. These results seem to be optimistic in this model reactor, mainly because pion and muon drifts to the wall due to collisions were neglected. Also, the influence of the magnetic field on the scattered proton beam were not considered. In fact, the proton Larmor radius is of the same order of magnitude as that of pions, therefore a significant portion of the proton beam is lost in the vessel in this concept reactor. The fissions and fissile material are produced in the blanket not only by the neutrons derived from d-t fusion but also from the fast nucleons of the incident beam which have about 80% of their initial energy after the creation of the pions. Taking into account the losses of the projectiles in the converter-synthesizer vessels due to the magnetic field will reduce the energy and the fissile material obtained from direct beam-blanket collisions. Moreover, by taking into account the diffusion due to collisions, the necessary quantity of tritium might increase significantly.

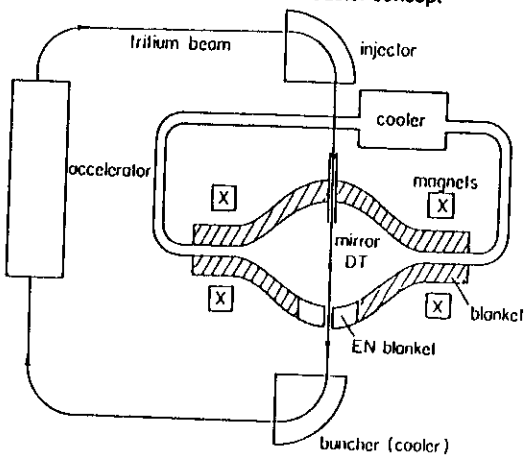
Petrov's reactor is actually a "neutron factory" which breeds a thermal fissile isotope ^{233}U or ^{239}Pu in order to use these materials in satellite fission reactors. In this scheme it was estimated that a commercial reactor would require about 100 fusion per muon. On the other hand, a pure fusion reactor seems to acquire 1000 fusion per muon in order to be economically viable.

7.2. Eliezer-Tajima-Rosenbluth (ETR) Hybrid Reactor

The ETR reactor concept is based on three ideas: (a) The high energy beam of tritium or deuterium is injected into the deuterium-tritium (d-t) fuel and, after passage through the fuel, part of the beam is collected for reuse while the portion of the beam which suffered a strong interaction is directed into an electronuclear blanket, (see figure 4). (b) The pions created in the target are surrounded by the fuel of deuterium and tritium and are magnetically confined until they slow down and decay into muons which catalyze the fusion in situ: (c) The fusion created neutrons are absorbed by blankets to breed fissile matter for energy production. In comparison to Petrov's concept the highlights of our concept are (a) and (b). Instead of a separate target, converter and synthesizer we combine these three functions into one. The d-t fuel is the target and the produced pions are trapped in the fuel which slows them down before they decay into muons. The muons are created in the fuel and trapped there, catalyzing the d-t fusion through the atomic and molecular processes until they decay. In this way, one of the most difficult problems of muon catalyzed fusion is solved, i.e., efficient trapping of muons in relatively small physical volume. The present concept solves this problem by creating mesons in the fuel and by confining them magnetically.

Figure 4 sketches a version of the reactor concept. The driver is a tritium (or deuterium) beam which is retrieved in part after passing through the target. The significantly scattered portion of the beam feeds into the electronuclear blanket. the fusion created neutrons are captured in the fissile blanket surrounding the fusion fuel. Figure 4 shows a magnetic mirror configuration. The mirror is filled with a pressurized gas mixture of deuterium and tritium gas. The gas is circulated through the mirror, with a cooling section between traversals. The mirror is enclosed by ^{238}U or ^{232}Th blankets with admixed lithium for breeding of Pu, ^{233}U and tritium. Magnets provide a magnetic field configuration with a mirror ratio R_m. The field at the mirror throats is typically 10 T.

FIGURE 4. The mirror reactor concept



An accelerated tritium beam of approximately one GeV per nucleon and a current of about 0.1 Ampere in injected through a small tube perpendicular to the axial magnetic field into the fuel. When the high energy beam (1 GeV/nucleon) strongly interacts with the target, in effect proton-proton, proton-neutron, neutron-neutron scatterings take place.

The total gain factor is given by

$$K = \frac{E_{\text{blanket,out}}}{E_{\text{target,in}}} = \frac{E_{\text{blanket,out}}(EN) + E_{\text{blanket,out}}(\mu c)}{y_\mu^{-1} y_\pi^{-1} T_0}$$

EN denotes the energy gain from "direct" beam-blanket interaction and μc denotes the energy gain from muon catalytic processes. The value of K is given in terms of the physical quantities X_μ, y_μ and y_π by the formula

$$K = \frac{Z_e T_{0f} \varepsilon_{fis}}{T_0} + \frac{y_\mu y_\pi X_\mu \delta_f \varepsilon_{fis}}{T_0} = K_{EN} + K_{\mu c} \tag{7.2}$$

where y_μ is the π^- to μ^- conversion efficiency (~50%); y_π^{-1} is the number of nucleonic projectiles necessary to produce a π^- , the *EN* contribution is $K_{EN} = Z_e T_{0f} \varepsilon_{fis}/T_0$ and the muon catalyzed gain is given by the second term. ε_{fis}≈0.2GeV is the uranium fission energy (for ^{238}U blanket), Z_e≈20 fissions/GeV in the "direct" beam-blanket interaction[49] (i.e., the *EN* process), T_{0f} is the beam kinetic energy before colliding with the blanket while T_0 is their initial kinetic energy before hitting the target (for pion → muon creation). δ_f≈1 is the number of fissions in the blanket caused by one 14 MeV neutron from the fusion process.[51] X_μ was measured experimentally to be about 150 for liquid hydrogen d-t targets and y_π and y_μ were estimated by Eliezer *et al.*[50] The above reactor produces a considerable power on its own. However, the fissile material produced in the blanket (e.g., ^{239}Pu) can be used to run between 3 to 6 satellite fission plants of equal output. X_R denotes the number of these satellite fission plants. The value of X_R, of course, depends on the design and breeding ratio of those plants and is thus somewhat arbitrary.

The total electrical gain K_{tot} is given by

$$K_{tot} = \frac{\eta_{th} P_{out}}{P_A} = \eta_{th}(1+X_R)K \tag{7.3}$$

where P_A is the accelerator power, η_{th}≈0.35 is the thermal to electric power conversion efficiency and K is given in equation (7.2). Defining η_r as the fraction of the total electric power required to recirculate in order to operate the accelerator and the auxiliary facilities, then the electric net output is

$$P_E = (1-\eta_r)\eta_{th}(1+X_R)KP_A \tag{7.4}$$

One gets the following relation

$$P_A/\eta_A = \eta_r \eta_{th} P_{out} \tag{7.5}$$

implying

$$\eta_r = \frac{1}{\eta_A \eta_{th} K(1+X_R)} = \frac{1}{\eta_A K_{tot}} \tag{7.6}$$

Substituting in equations (7.2) and (7.6) the following parameters for a fuel containing 50% tritium and 50% deuterium at 0.5 liquid hydrogen density, in the ETR reactor scheme it is conceivable to take:

$$Z_e=20\ GeV^{-1}\ ,\varepsilon_{fis}=0.2\ GeV\ ,\delta_f=1\ ,T_0=1\ GeV\ ,X_R=5\ ,X_\mu=200 \tag{7.7}$$

$$\eta_A=0.6\ ,\eta_{th}=0.35\ ,y_\mu=0.5\ ,y_\pi=0.35\ ,T_{0f}=0.8\ GeV$$

which yields

$$K_{EN}=3.2,\ K_{\mu c}=7.0,\ K=10.2,\ K_{tot}=21.4,\ \eta_r=7.8\% \tag{7.8}$$

An important result is the value of η_r, which can serve as a figure of merit in a driven nuclear reactor. If we assume as usual an output electricity of $P_E=(1+X_R)10^9 W$ we need an accelerator power of $P_A=3.0\ 10^8 W$ which for 1 GeV/nucleon projectile implies an average current of 0.3 Ampere.

In order to compare our scheme with other driven fusion-fission reactors we define as usual a gain Q for these reactors by (compare with K_{tot} in Equation (7.6))

$$Q=\frac{1}{\eta_{th}\eta_D\eta_r(1+X_R)} \tag{7.9}$$

where η_D is the driver efficiency (taken optimistically) as 0.6 for magnetic confinement and for ion fusion devices and 0.05 for laser inertial fusion. Equation (7.9) implies that the μCFF (muon catalyzed fusion-fission) reactors with $X_\mu\approx200$ are equivalent to MFF (magnetic fusion-fission) or ion ICFF (inertial confinement fusion-fission) which have a gain of $Q=10$ or a laser ICFF with a gain of $Q=120$.

7.3. Tajima-Eliezer-Kulsrud (TEK) Fusion Reactor[52]

In order to achieve a thermodynamically meaningful energy gain by muon catalyzed DT fusion alone without resorting to breeding fissile matter by fusion neutrons, it is believed that the fusion catalysis cycling rate in a muon lifetime, X_μ, needs to exceed 1000. Neglecting faster intermediate processes, nowadays we can identify two crucial bottlenecks for muon catalyzed fusion energy production. The one is the dtμ mu-molecular formation process (rate $\lambda_{dt\mu}$ and the other is the muon sticking with fusion α particles (probability ω_{s0}):

$$X_\mu^{-1}\approx\omega_s+(\lambda_{dt\mu}\tau_\mu)^{-1}\ , \tag{7.10}$$

where

$$\omega_s=\omega_{s0}(1-R)\ , \tag{7.11}$$

τ_μ is the lifetime of a muon, R is the stripping rate of muon from the $(\alpha\mu)^+$ atom, and $\omega_{s0}\approx0.008$, $R\approx40\%$, $\lambda_{dt\mu}\approx10^9 sec^{-1}$. The sticking rate has now been experimentally and theoretically settled to $\omega_s=0.0045$. This means that even if $\lambda_{dt\mu}\to\infty$, the catalysis cycle rate X_μ is merely 220.

The question is: if the nature gives us a value of $X_\mu\approx220$, is it possible to enhance it by an ingenious method? Roughly speaking, the determination of ω_{s0} involves nuclear physics along with dtμ molecular dynamics; that of R and $\lambda_{dt\mu}$ involves atomic and molecular physics. Thus it seems prudent to us to try to "manipulate" or improve the processes that pertain to lower energy physics, i.e. atomic and molecular physics, as it is easier (or "cheaper") to influence lower energy physics. This is one of our guidelines. In the following we consider the enhancement of stripping by one way or another that does not involve nuclear physics.

The effective sticking probability ω_s can be reduced by increasing the stripping coefficient R [see Eq. (7.11)]. The stripping (or reactivation) coefficient R may be written as

$$R=1-\exp\left[-n\int_{E_i}^{E_b}\frac{\sigma_{st}(E)dE}{F(E)}\right], \tag{7.12}$$

where σ_{st} is the stripping cross section, F is the stopping power of $(\alpha\mu)^+$ in matter, and E_b and E_i are the birth energy of $(\alpha\mu)^+$(≈3,5MeV) and the $(\alpha\mu)^+$ ionization potential (≈ 10 keV). The cross section peaks around ≈ 10 keV and quickly decreases below this energy. In order to make the reactivation coefficient large (i.e., close to unity), Bracci and Fiorentini[53] argued that $(\alpha\mu)^+$ should be kept as long as possible in the higher velocity range where the stripping probability is higher. They suggested to apply accelerating electric fields in the matter. Unfortunately, the effective slowdown field in the solid matter is of the order of 45 MeV/cm, with which, either by dc or ac fields, breakdown of the target arises.

Kulsrud[54] proposed a clever scheme to reduce the sticking coefficient by reaccelerating the $(\alpha\mu)^+$ ions by ion cyclotron resonance mechanism. In this scheme the target D-T is in solid (or liquid) form, say at about 25°K, and the target is divided into many rods (or pellets) of the order of 100 microns (each) in radius. By imposing a magnetic field and a rotating electric field the $(\alpha\mu)^+$ ion is kept at a constant (average) velocity until stripping occurs. The drag force in the rods (while the stripping occurs) is balanced by the acceleration force in the vacuum between the rods (or the pellets). The acceleration is achieved by the ion cyclotron resonance heating with the frequency $\omega=\Omega_{\alpha\mu}=eB/4m_pc$, where m_p is the proton mass. In his calculation the necessary strength of rf electric field E_{st} for stripping is

$$eE_{st}\approx5\times10^4 f\ \mathrm{kV/cm}\ , \tag{7.13}$$

where f is the filling factor of the DT target. On the other hand, the breakdown field for ac field E_{bd} (which is much higher than the dc case) is approximately given[55] by

$$eE_{bd}\approx m\omega c\ , \tag{7.14}$$

where m is the electron mass and ω is the rf frequency. At the frequency range of $\Omega_{\alpha\mu}\approx10^{10}$ Hz (with 1M Gauss magnetic field) the field in Eq.(7.13) is tolerable against breakdown. Unfortunately, Kulsrud's[54] idea does not work as he put it, since the alpha heating of the target cannot be tolerated, for the following reason: Imagine that we want to let fusion happen in DT ice bars (below 25°K). Per a dt fusion reaction, a neutron of 14.1 MeV and an alpha particle of 3.5 MeV are created. The neutron leaves the ice and presumably hits a surrounding blanket (or worse, a metal), while the alpha particle is likely to stay in the DT ice and heats it. If we are to insist on keeping the low temperature of the specimen, every time a fusion alpha particle heats the target, we have to cool (or extract the energy of that amount from) the material. That is, we have to remove 3.5 MeV energy per fusion. With the cooling efficiency η_c (because of the second law of thermodynamics, it is not unity and in fact a miniscule number typically of the order of) ≈ 0.1, the necessary energy to keep the material at a desired cool temperature is 3.5MeV/η_c (≈35MeV) per fusion. On the other hand the total fusion energy production is of course only 17.6 MeV per fusion. No energy gain is possible. We call this problem *"too cold fusion - no energy production"*.

The TEK fusion reactor concept is based on the following configuration. A simple mirror ratio R_m with magnetic field B generated by superconductors is surrounded by neutron blankets such as Li etc. Although the strength of the magnetic field is not uniquely fixed, we may use a typical number of 1 MG. A pellet (or pellets) of DT ice is repetitively injected either mechanically or gravitationally into the mirror vessel. The pellet consists of a series of ribbons of DT ice. The minimum overall size is $\approx(2\text{cm})^3$, which is determined by the two conditions, the inertial confinement condition of the overall pellet and the magnetic confinement condition of π^-. These DT ice ribbons fill the pellet with the filling factor f in order to accommodate sufficient acceleration time, while keeping enough matter to strip $(\alpha\mu)^+$ according to Kulsrud's mechanism of stripping. These DT ice specimens are penetrated by a high energy beam of d or t particles ($\approx$ 1 GeV/nucleon) injected perpendicular to the mirror axis, which coincides with the (longitudinal) axis of the specimens. Beam particles collide with the target nuclei, creating π^-'s. Pions are created *in situ* and confined by the mirror magnetic field, as in the earlier hybrid reactor concept (see section (7.2)). The portion of the beam that did not suffer strong interaction will be collected, cooled, accelerated, and reused for the next pass. At this energy of the d or t beam may be appropriately provided by Budker's electron cooling method [56] or Van der Meer's stochastic cooling method. [57] The energy cost of this cooling will be studied in the future. The beam energy has to be boosted as well.

The ribbons have the following characteristics. The thickness l_1 of a ribbon should be smaller than the range of $(\alpha\mu)^+$ at the 3.5 MeV energy at birth. Typically we choose $100\mu m$. The width l_2 is typically several times l_1. The length l_3 is determined by the larger of the inertial confinement length and a few π^- Larmor radius:

$$l_3 = \max(c_s\tau_\mu,\ 4\rho_\pi)\,, \tag{7.15}$$

where c_s is the sound speed of the DT specimen and $\rho_\pi = \beta c/\Omega_\pi$. For 1 MG field and 1 eV DT specimen, these two are in the same range of $\approx 10^0$ cm. The distance d_2 between two ribbons in the width direction is

$$d_2 = \min\,(2\rho_{\alpha\mu}\,, l_2/\sqrt{f})\,, \tag{7.16}$$

and the distance d_1 between two ribbons in the thickness direction is

$$d_1 = \max\left[l_1/\sqrt{f}, \frac{l_1 l_2}{2f\rho_{\alpha\mu}}\right], \tag{7.17}$$

where $\rho_{\alpha\mu}$ is the Larmor radius of $(\alpha\mu)^+$. Typical numbers for these are 0.4 cm and 0.2 cm, respectively. Radio frequency electromagnetic waves with frequency $\omega = \Omega_{\alpha\mu}$ are applied with the electric field rotating.

When the beam is injected into these ribbons and fusion reactions begin, fusion alpha particles are created and all their energy is deposited in the heating DT ice. In order to avoid the dilemma of too cold fusion — no energy production mentioned before, the DT ribbons are heated up to temperature T_h which is much higher than the room temperature T_r, so that we can now *extract* energy from the DT ribbon instead of investing energy to cool it. This condition is written as

$$\eta_h\Delta T_h - \Delta T_c/\eta_c > 0\,, \tag{7.18}$$

where η_h and η_c are the thermodynamical efficiencies of the heat energy extraction from the hot material (T_h) into electricity and the cooling, and $\Delta T_h = T_h - T_r$ and $\Delta T_c = T_r - T_{ice}$. From this T_h has to

be larger than 1–2eV for η_c=0.3 and η_c=0.1. This will resolve the difficulty of Kulsrud's idea. Of course, this brings in a new difficulty of confining hot DT ribbons at nearly the solid density to cope with an enormous pressure.

In order to confine these DT ribbons, we introduce the concept of coronal confinement. We surround each ribbon with a coat of coronal plasma with density n_c and temperature T_c such that

$$n_h T_h = n_c T_c\,, \tag{7.19}$$

where n_h and T_h are the heated DT ribbon density ($\phi\approx 1$) and temperature (=< 1eV). The thickness l_c of the coronal plasma is

$$l_c = \frac{1}{4} d_1\,. \tag{7.20}$$

We choose the density of the corona such that the majority of $(\alpha\mu)^+$ of stripping does not take place there:

$$n_c \ll \frac{l_1}{l_c} n_h \ \text{ and } \ T_c \gg \frac{l_c}{l_1} T_h. \tag{7.21}$$

Equation (7.21) can be written as

$$n_c \ll \frac{4l_1}{d_1} n_h = 4\sqrt{f}\, n_h \ \text{ and } \ T_c \gg \frac{1}{4\sqrt{f}} T_h\,, \tag{7.22}$$

where the equality in Eq.(7.22) applies when the first term in the parenthesis in Eq.(7.17) applies. T_c typically is 10eV. When the DT ribbon expands after alpha heating, the coronal pressure balances it, while the coronal pressure is now borne by the magnetic pressure outside of it. Thus, the corona is magnetically confined:

$$n_c T_c + \frac{B_c^2}{8\pi} = \frac{B^2}{8\pi}\,, \tag{7.23}$$

where B_c is the magnetic field which penetrates into the corona and B is the external field. Details of the creation of such a corona will be discussed elsewhere. Some possibilities include the alpha heating of the ribbon itself with or without a sponge-like surface, injection of very hot and very tenuous plasma to evaporate the surface, surface current application, laser surface ablation, etc.

The magnetic field tends to penetrate into the corona (and the ribbon) by the resistive process. The velocity of such penetration v_p is given by

$$v_p = \frac{\eta c \nabla P_c}{B^2} \approx \frac{\eta_0 c (n_h T_h)/l_c}{B^2 (T_c/T_0)^{3/2}}\,, \tag{7.24}$$

where η is the collisional resistivity of the corona and an explicit dependence of the coronal temperature is written in. This velocity should be sufficiently small such that

$$l_c/v_p > \tau_\mu\,. \tag{7.25}$$

This equation imposes a constraint on B, l_c, T_h, and T_c. Besides the classical resistive penetration, there may arise anomalous penetration. A most prominent example is due to the Rayleigh-Taylor instability developing at corona-vacuum interface, as the expanding corona decelerates due to the magnetic pressure. A simple estimate of the growth rate of the R-T instability is

$$\gamma=\alpha\sqrt{g/l_c}=\alpha c_s/l_c\ , \tag{7.26}$$

where α is a geometrical factor ordinarily less than unity. There may be a possibility of developing an electric field and the coronal plasma penetrate through the magnetic field with $cE\times B/B^2$ drift velocity. In this case it may become necessary to short out the electric field. Finally, the collection of all these ribbons is confined inertially

$$l_3/c_s\approx a/c_s>\tau_\mu\ , \tag{7.27}$$

where a is the size of the pellet.

The above described TEK fusion reactor concept via muon catalysis tried to reduce the effective sticking probability to be less than 10^{-3} by using ion cyclotron resonance heating. The necessary structure of the DT fuel was elaborated. This particular structure allows the above operation, simultaneously keeping the necessary criteria for the mu-molecule formation processes. A number of parameters such as the strength of magnetic field, the filling factor etc. have yet to be optimized. It is entirely possible, however, that a comfortable (for physics) choice of parameters, may lead to the requirement of a fairly large magnetic field of the order of 1 MG. Many anomalous plasma related processes may come into play. The rf power may be dissipated by extraneous resonances and other unwanted heating and thus the Q-value of rf source may not be as high as we wish. All these physical as well as technical problems can be formidable and certainly provide an immense challenge.

8. THE PRODUCTION OF μ^-

The μ^- particles are produced by the decay of the negative pion, $\pi^-\to\mu^-+\bar{\nu}_\mu$. The π^- mesons are generated by using proton, deuterium or tritium accelerators. For kinetic energies per nucleon T_0 smaller than 0.7 GeV/nucleon, the probability of inelastic nucleon-nucleon collisions, $V_{ab}(T_0)$, $[a,b=p$ (proton) or n (neutron)] is very small so that a π^- cannot effectively be produced. V_{ab} is usually defined by the ratio of inelastic cross-section, σ_{ab}^{in}, to the total cross-section, σ_{ab}^{tot}, for strong interactions ($\sigma_{ab}^{tot}=\sigma_{ab}^{in}+\sigma_{ab}^{el}$ where σ^{el} is the strong elastic cross section). We do not consider initial energies larger than 2 GeV per nucleon since in this case the number of undesirable particles (e.g., neutral pions) that are created increases. Therefore, in the energy domain under consideration, $0{\cdot}7<T_0<2$GeV, the negative pions are produced as shown in figure 5, with cross sections $\sigma_{pp}^{\pi^-}$, $\sigma_{pn}^{\pi^-}$ and $\sigma_{nn}^{\pi^-}$. The probabilities W_{ab} of producing a π^- once the inelastic collision occurs are given by $W_{ab}(T_0)$. The multiplicities Y_{AB}^{π} were calculated[58] to be

$$Y_{AB}^{\pi}=C_{pp}^{AB}V_{pp}W_{pp}+C_{pn}^{AB}V_{pn}W_{pn}+C_{nn}^{AB}V_{nn}W_{nn} \tag{8.1}$$

Where C^{AB} give the fraction of pp, pn and nn collisions between a projectile A and a target B (A,B = p,d,t). $(Y_{AB}^{\pi})^{-1}$ equals the number of nucleonic projectiles (passing an ∞ target) necessary to create one π^- in a reaction $A+B\to\pi^-+\cdots$. $(y_{AB}^{\pi}(x))^{-1}$ is the number of nucleonic projectiles necessary to create one pion while the projectile A passes through x cm of target B, and $(y_{AB}^{\pi})^{-1}T_0$ is the energy yielded for this type of π^- creation. $y_{AB}^{\pi}(x)$ is given by

$$y_{AB}^{\pi}(x)=n_0\phi x[Z_AZ_B\sigma_{pp}^{\pi^-}+[Z_A(A_B-Z_B)+Z_B(A_A-Z_A)]\ \sigma_{pn}^{\pi^-}+(A_A-Z_A)(A_B-Z_B)\sigma_{nn}^{\pi^-}] \tag{8.2}$$

where Z and A are the number of protons and nucleons respectively for the appropriate collisions (A-target; B-beam). ϕ is the density in liquid hydrogen units $\phi=n/n_0$, where $n_0=4{\cdot}25\ 10^{22}$atoms/cm^3. $\sigma_{ab}^{\pi^-}(a,b=p$ or $n)$ is calculated from figure 5 by using $\sigma_{pn}^{tot}\approx0.8\sigma_{pp}^{tot}$ (which is satisfied for our range of

FIGURE 5. The probabilities of μ^- production.

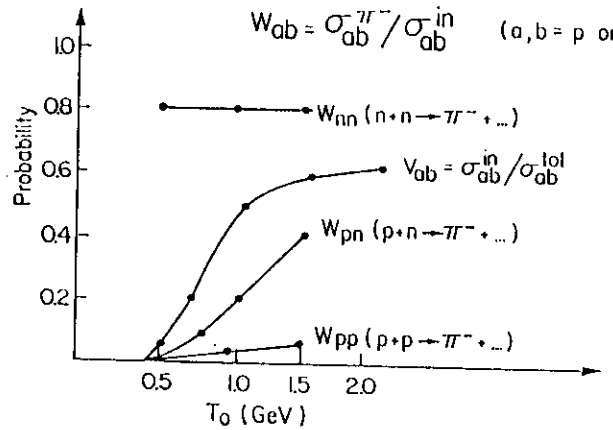
PRODUCTION OF μ^-

$$\pi^-\to\mu^-+\bar{\nu}_\mu$$

$$p+p\to p+p+\pi^++\pi^-$$

$$p+n\to p+p+\pi^-$$

$$n+n\to n+p+\pi^-$$



energies), $\sigma_{pp}^{tot}=\sigma_{nn}^{tot}$ and $\sigma_{pp}^{tot}(1\text{GeV})\approx45$ mb(1mb = 10^{-27}cm^2). The mean free path $l_{s,AB}$ is calculated from

$$l_{s,AB}^{-1}=n\sigma_{AB}^{tot}=\phi n_0\sigma_{AB}^{tot} \tag{8.3}$$

where σ_{AB}^{tot} is given by

$$\sigma_{AB}^{tot}=Z_AZ_B\sigma_{pp}^{tot}+(A_A-Z_A)(A_B-Z_B)\sigma_{nn}^{tot}+ \tag{8.4}$$

$$[(Z_A(A_B-Z_B)+Z_B(A_A-Z_A)]\sigma_{pn}^{tot}$$

One obtains, for example, from (8.2)-(8.4) that in tritium-tritium collisions along 64 cm ($x=l_{s,u}=64$ cm), one π^- is created by four nucleons having a total kinetic energy of 4 GeV. The number of nucleonic projectiles necessary to create one π are (for a single collision)

$$[Y^{\pi}(\text{pp})]^{-1}=\infty$$

$$[Y^{\pi}(\text{pd})]^{-1}=20;\ [Y^{\pi}(\text{dd})]^{-1}=6 \tag{8.5}$$

$$[Y^{\pi}(\text{pt})]^{-1}=14.3;\ [Y^{\pi}(\text{dt})]^{-1}=4.8;\ [Y^{\pi}(\text{tt})]^{-1}=4$$

where the projectile has a kinetic energy of 1 GeV/nucleon. This situation is achieved for target dimensions much smaller than the mean free path for strong interaction. For large target dimension, the nucleon projectile has multiple collisions in the target, about 50% of the incident nucleons have an elastic scattering in the first collision. These nucleons can have in their second or third scattering an inelastic collision if the transverse and longitudinal dimensions are large enough. However, since the probability of a strong inelastic interaction (where a π^- can be created) is very small for energies less than 0.6 GeV, all the nucleons with energies below this value are ineffective and their energy is actually lost as far as the creation of negative pions is concerned. Therefore the multiplication factor is calculated by adding the probabilities of the first collision being inelastic and the appropriate probabilities that a possible second and third collisions are inelastic. Taking into account multiple scattering we obtain the following energies E_π required to create one π^- (for projectile kinetic energy of 1 GeV/Nucleon).

$$E_\pi(\text{d–d})\approx 4.5\ GeV,\quad E_\pi(\text{d–t})\approx 3.7\ GeV, \tag{8.6}$$

$$E_\pi(\text{t–t})\approx 2.0\ GeV,\quad E_\pi(\text{t–50\%d+50\%t})=2.85\ GeV.$$

In the ETR scheme (see section 7.2), the π^- mesons are created inside the d-t fuel and therefore the pions created by multiple collisions are not lost into an undesirable surrounding material.

We end this section by pointing out the possibility to create π^- by triton-triton colliding beams. A laboratory kinetic energy of 1 GeV per nucleon corresponds to a center of mass energy of 225 MeV/nucleon. In this case one need 8×225 MeV = 1.8 GeV of energy in order to produce a π^- in triton-triton colliding beams. In this scheme the production of desirable numbers of π^-, e.g. 10^{18}sec^{-1} will require very large aperture storage rings, something similar to Tokamak rings. The minimum luminosity in this case is about 10^{44}cm^{-2}sec^{-1} (number of events = luminosity (L)× cross section (σ)). This luminosity is many order of magnitude larger than that of any existing storage ring. However, it is interesting to point out that such a luminosity is anticipated in tokamak devices.

9. Mu MOLECULAR FORMATION UNDER LASER IRRADIATION

The muon catalysis leading to nuclear fusions depends on the density of the hydrogen isotopes mixture (e.g. D-T) as well as on its temperature and composition (e.g. the percentage of T in the D-T mixture). The experimental data and our present theoretical knowledge suggest that even by optimizing the density, temperature and concentrations one cannot increase the value of X_μ above 220 due to the sticking problem. In section (7.3) we have discussed the possibility of eliminating the sticking problem by the ion cyclotron resonance acceleration of $(\alpha\mu)^+$. Another new direction of research is suggested by using laser beams.[59-61] The aim of this research is to substitute the high density needed for mu-molecular formation by the laser beams. The laser photons can play the role of a "new" energy balancing body or altering the density of states for the atomic and molecular process.

We consider a three energy-level system of a μ molecule such as dtμ, ddμ, dpμ etc., as illustrated in fig. 6. The quantum levels are denoted by (a), (b) and (c) where $E_a > E_c > E_b$ are the appropriate energies. The lifetimes of these levels satisfy the condition $\tau_a \gg \tau_c$. For example for the dtμ molecule these levels are: a state in the continuum of the system tμ+d, the bound state $(J,V)=(1,1)$ of the molecules dtμ, and the bound state $(J,V)=(0,1)$ of dtμ.

The system is initially in the state (a). Under the influence of an external field with frequency ω we can induce a two-step decay of level (a). During the first step a virtual transition level (c) is induced by absorption of a single photon while the second step is a transition to level (b) through a spontaneous decay or an Auger process. The electromagnetic interaction between the external field and the system is described by the Hamiltonian $H = V\cos\omega t$. V will be specified later. Our formal approach is analogous to the treatment of Raman scattering. The Schroedinger equation yields a system of equations for the amplitudes ψ_a and ψ_c of the levels (a) and (c). The transitions from (a) to (c) without the laser fields and the decay level (c) are included phenomenologically by the rates γ_a and γ_c. Within the rotating wave approximation these equations are

$$\dot{\psi}_a = -\frac{1}{2}\gamma_a\psi_a + \frac{i\pi}{h}V_{ca}e^{i\Delta t}\psi_c\ , \tag{9.1}$$

$$\dot{\psi}_c = -\frac{1}{2}\gamma_c\psi_c + \frac{i\pi}{h}V_{ac}e^{i\Delta t}\psi_a\ . \tag{9.2}$$

$|V_{ac}|$ denotes the transition matrix element between the states (a) and (c) and Δ is the detuning given by

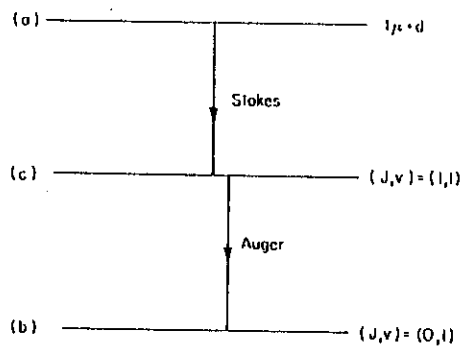
$$\Delta = \frac{2\pi(E_a - E_c)}{h} - \omega\ . \tag{9.3}$$

The solution of eqs. (9.1) and (9.2) for the initial conditions $\psi_a(0)=1$ and $\psi_c(0)=0$ is

$$|\psi_c(t)|^2 = \left[\frac{2\pi|V_{ca}|}{h}\right]^2 \frac{e^{-\gamma t}}{\mu\mu^*}\sin\tfrac{1}{2}\mu t\ ,\ \sin(\tfrac{1}{2}\mu^* t) \tag{9.4}$$

where γ is

FIGURE 6. Stokes (resonance) formation of dtμ.



$$\gamma=\tfrac{1}{2}(\gamma_a+\gamma_c)\,, \tag{9.5}$$

μ is the complex Rabi flopping frequency,

$$\mu=|(\Delta-i\delta)^2+(2\pi|V_{ac}|/h)^2|^{1/2} \tag{9.6}$$

and δ is given by

$$\delta=\tfrac{1}{2}(\gamma_a-\gamma_c)\,. \tag{9.7}$$

The probability for the Stokes process $P_{St}(T)$ up to time T is given by

$$P_{St}(T)=\gamma_c\int_0^T dt\,|\psi_c(t)|^2\,. \tag{9.8}$$

Since for our cases of interest $\tau_a \gg \tau_c$ one can obtain the probability $P_0(T)$ for the transition from (a) to (b) by switching off the laser during a time T by

$$P_0(T)=\gamma_a\int_0^T dt\,|\psi_a(t)|^2\,. \tag{9.9}$$

The efficiency η for Stokes transitions up to time T is given by

$$\eta=\frac{P_{St}(T)}{P_0(T)}\,. \tag{9.10}$$

By integrating (9.4) one obtains the following expression for $P_{St}(T)$:

$$P_{St}(T)=\frac{\gamma_c}{2\mu\mu^*}\left[\frac{2\pi|V_{ca}|}{h}\right]^2$$

$$\times\left|e^{-\mu^*}\frac{-\gamma\cosh yT-y\sinh yT}{\gamma^2-y^2}-\frac{\gamma\cos xT+x\sin xT}{\gamma^2+x^2}+\frac{2\gamma^2+x^2-y^2}{(\gamma^2-y^2)(\gamma^2+x^2)}\right|, \tag{9.11}$$

x and y are the real and imaginary parts of μ (see eq. (9.6)).

To first order one finds a simplified solution of eqs. (9.1) and (9.2):

$$|\psi_a(t)|^2=e^{-\gamma_a t}\,, \tag{9.12}$$

$$|\psi_c(t)|^2=\left[\frac{\pi|V_{ac}|}{h}\right]^2\times\frac{e^{-\gamma_a t}+e^{-\gamma_c t}-2e^{-\gamma t}\cos\Delta t}{\Delta^2+\delta^2}\,. \tag{9.13}$$

We are interested in a system where (c) decays much faster than (a), i.e. $\gamma_c \gg \gamma_a$. Then γ and δ reduce to $\gamma=-\delta=\frac{1}{2}\gamma_c$. According to (9.12) and (9.13) the population of the excited level $|\psi_c(t)|^2$ decays to first order for large times on the same slow time scale γ_a^{-1} as $|\psi_a(t)|^2$. We assume that the pulse of the external field is long compared to the lifetime of (c) and small compared to the lifetime of (a) ($\gamma_c^{-1} \ll T \ll \gamma_a^{-1}$). By integrating (9.12) and (9.13) one gets the following expressions for $P_{St}(T)$ and η:

$$P_{St}(T)=\left[\frac{\pi|V_{ac}|}{h}\right]^2\frac{\gamma_c T}{\Delta^2+(\frac{1}{2}\gamma_c)^2}\,, \tag{9.14}$$

$$\eta=\frac{\pi^2|V_{ac}|^2\gamma_c}{h^2\gamma_a(\Delta^2+\frac{1}{4}\gamma_c^2)}\,. \tag{9.15}$$

Consider the resonance case, $\Delta=0$. One gets from eqs. (9.14) and (9.15)

$$P_{St}(T)=\left[\frac{2\pi|V_{ac}|}{h}\right]^2\frac{T}{\gamma_c}\,, \tag{9.16}$$

and the induced Stokes efficiency:

$$\eta=\left[\frac{2\pi|V_{ac}|}{h}\right]^2\frac{1}{\gamma_a\gamma_c}\,. \tag{9.17}$$

In order to calculate the interaction matrix element $|V_{ac}|$ we assume a dipole transition between quantum levels (a) and (c):

$$|V_{ac}|^2=\frac{1}{3}(eE_L)^2|d_{ac}|^2\,, \tag{9.18}$$

E_L is the electric field of the laser, related to the flux F by $E_L^2=4h^2F/\lambda_L$, where λ_L is the laser wave-length. In terms of the laser intensity defined by $I_L=cE_L^2/8\pi$, the matrix element $|V_{ac}|$ is given by

$$|V_{ac}|^2=\frac{8\pi}{3}\frac{e^2}{c}I_L|d_{ac}|^2\,. \tag{9.19}$$

We use the matrix element $|d_{ac}|^2$ as calculated in ref. 28

$$|d_{ac}|^2=\frac{1}{4\pi}\sum_{mJ}|dR\,dr\,\psi_c(r,R)d(r,R)\psi_a(r,R)|^2\,, \tag{9.20}$$

where R is the radius vector joining the nuclei t and d, r is the radius vector of μ with origin at the geometric center of segment R and d is the dipole moment of the system comprising the mu-atom and the nucleus of the D_2 molecule, relative to the mass center of $dt\mu$.

TABLE 3. Quantities that determine the efficiency of resonant Stokes induced molecular formation of $dd\mu$ and $dt\mu$.

Quantity	$dd\mu$	$dt\mu$
γ_a	$3\times10^6 s^{-1}$	$10^9 s^{-1}$
γ_c	$3\times10^8 s^{-1}$	$10^{12} s^{-1}$
$\lvert d_{ac}\rvert$	1.058×10^{-8}cm	1.005×10^{-8}cm
$\eta(\Delta<\frac{1}{2}\gamma_c)$	$70 I_L(\mathrm{W/cm^2})$	$5\times10^{-5}/I_L(\mathrm{W/cm^2})$
$\eta(\Delta>\frac{1}{2}\gamma_c)$	$10^{-11} I_L(\mathrm{W/cm^2})/\alpha^2$	$10^{-11} I_L(\mathrm{W/cm^2})/\alpha^2$

$$\vec{d} = -\frac{1}{2}\frac{M_d - M_t}{M_d + M_t}\left[1 - \frac{m_\mu}{M_{tot}}\right]\vec{R} - \left[1 + \frac{m_\mu}{M_{tot}}\right]\vec{r}. \tag{9.21}$$

M_μ, M_d, M_t are the masses of μ, d, t and $M_{tot} = M_\mu + M_d + M_t$. Table 3 shows the numerical values of the dipole matrix element $|d_{ac}|$ as calculated in ref. 28, the lifetimes of the levels [31,62] (a) and (c) and the estimated efficiency for resonant Stokes transitions as a function of the laser intensity, for the molecules dtμ and ddμ. The results show that for moderate laser intensities of 10^6 W cm^{-2} the efficiency for resonance formation of dtμ by induced Stokes transitions is about 50, while for ddμ it has the huge value of 10^7.

However it must be emphasized that the resonance condition is extremely important for the above estimation of η. The resonance condition $\Delta < \frac{1}{2}\gamma_c$ is equivalent to $\alpha < 3\times10^{-16}\gamma_c/(E_a - E_c)$ (eV) where

$$\alpha = \frac{h\Delta}{2\pi(E_a - E_c)} = 1 - \frac{h\omega}{2\pi(E_a - E_c)} \tag{9.22}$$

In particular for the dtμ case the resonance condition implies $\alpha < 5\times10^{-4}$, while for ddμ, $\alpha < 5\times10^{-8}$. Moreover if $\Delta > \frac{1}{2}\gamma_c$ then the efficiency of the Stokes induced transitions is reduced significantly. In this case one obtains $\eta = 10^{-11} I_L(W/cm^2)/\alpha^2$. For example requiring a resonance detuning of 10% ($\alpha = 0.1$) and a Stokes efficiency $\eta = 10$ one needs a laser field of 10^{+11} W/cm^2.

10. MUON CATALYZED FUSION IN PLASMA

10.1. Formation of mu-Atoms in Plasma

By shooting a muon beam into an ionized dense plasma the μ^- particles stop and thermalise into the medium at a temperature T. The formation of mu-atoms in such a plasma is possible through the following processes:

(a) radiative recombination

$$\mu^- + X^+ \rightarrow X\mu + \gamma \tag{10.1}$$

where X denotes an hydrogen isotope; p, d or t (or in general any other ion in the plasma),

(b) three-body collisional recombination

$$\mu^- + X^+ + Y \rightarrow (X\mu) + Y \tag{10.2}$$

$$\mu^- + X^+ + e \rightarrow (X\mu) + e \tag{10.3}$$

where Y denotes another ion and e an electron.

The processes of radiative (10.1) and the three-body (10.2) recombinations seem to be negligible relative to the electron three- body collisional recombination (10.3). Therefore, we estimate now [63] only the rate for the Xμ formation through the process (10.3) in a deuterium-tritium plasma.

The rate of mu-atom formation is determined in analogy with the theory of the three-body electron-ion recombination,

$$\lambda = \lambda_{d\mu} + \lambda_{t\mu} = \alpha n \ (sec^{-1}) \tag{10.4}$$

where a fully ionised plasma is considered with an ion density (n_i) equal to the electron density (n_e)

$$n_e = n_i \equiv n, \tag{10.5}$$

and $\alpha(cm^3 sec^{-1})$ is the collisional recombination coefficient. α is derived by using the principle of detailed balancing

$$\frac{1}{p_f^2}\frac{d\sigma_{if}}{d\Omega} = \frac{1}{p_i^2}\frac{d\sigma_{fi}}{d\Omega} \tag{10.6}$$

where i and f denote the initial and final state and $d\sigma/d\Omega$ is the differential cross section. Equation (10.6) relates the processes $i \rightarrow f$ and $f \rightarrow i$. Note however that eq. (10.6) is correct to the first order in the perturbation theory. The exact relation is between $i \rightarrow f$ and $f^* \rightarrow i^*$ where i^* and f^* denote states which differ from i and f by a change in the signs of the velocities and angular momentum components of the particles under consideration. Using eq. (10.6) one has to calculate the rate $X\mu + e \rightarrow \mu^- + X^+ + e$ instead of that described by eq. (10.3). Thus α in eq. (10.4) is given by

$$\alpha = \frac{1}{4}\left[\frac{\pi}{T}\right]^{\frac{3}{2}} < \frac{(\Delta\varepsilon_\mu)^2}{\delta t} > I < \varepsilon_\mu^2 > \tag{10.7}$$

where ε_μ is the total energy of the muon in the mu-atom, $\Delta\varepsilon_\mu = \varepsilon_\mu' - \varepsilon_\mu$ is the change in ε_μ due to a collision

$$(X\mu)_{\varepsilon_\mu} + e \rightarrow (X\mu)_{\varepsilon_\mu'} + e, \tag{10.8}$$

$(\delta t)^{-1}$ is the frequency of these collisions and

$$< \varepsilon_\mu^2 > = \int_0^\infty d\,|\varepsilon_\mu|\,\varepsilon_\mu^{\frac{5}{2}} e^{-\varepsilon_\mu/T} = \frac{15\sqrt{\pi}}{8} T^{\frac{7}{2}}. \tag{10.9}$$

In this chapter we are using the atomic units (a.u.)

$$e = \frac{h}{2\pi} = m_e = K_B = 1$$

$$1\ a.u.\ of\ length = \frac{h^2}{4\pi^2 mc^2} = 5.29\cdot10^{-9}\text{cm}$$

$$1\ a.u.\ of\ mass = m = 9.11\cdot10^{-28}\text{g}$$

$$1\ a.u.\ of\ time = \frac{h^3}{8\pi^3 mc^4} = 2.43\cdot10^{-17}(\text{sec}) \tag{10.10}$$

$$1\ a.u.\ of\ energy = \frac{4\pi^2 mc^4}{h^2} = 4.36\cdot10^{-11}\ erg = 27.21\ \text{eV}.$$

e and m_e are the charge and the mass of the electron, h is Plank's constant divided by 2π and K_B is Boltzman's constant. In these units the length is measured in Bohr radii, energy has the dimension of $(length)^{-1}$ and temperature has the energy dimension.

It is assumed that the main contribution to $<(\Delta\varepsilon_\mu)^2>$ in the process (10.8) comes from the collisions with a high impact parameter (ϱ) relative to the mu-atom radius, $\rho \gg r_\mu$, where $r_\mu = 1/|\varepsilon_\mu|$ is the characteristic size of the mu atom. $\Delta\varepsilon_\mu$ is estimated to be

$$\Delta c_\mu \simeq \frac{1}{2m_\mu}[(\vec{p}_\mu+\Delta\vec{p})^2-\vec{p}_\mu^{\,2}] \simeq \frac{\vec{p}_\mu}{m_\mu}\cdot\Delta\vec{p} = \vec{v}_\mu\cdot\Delta\vec{p}$$

$$\Delta\vec{p} = \int \vec{F}_{e\mu}\,dt \simeq \left[\frac{\vec{\rho}}{\rho^3}\right]\cdot\frac{2\rho}{v_e}, \tag{10.11}$$

where $F_{e\mu}$ is the Coulomb repulsive force between the electron and the muon. The electron-muon collision frequency is estimated by

$$\frac{1}{\delta t} = n\sigma v_e \simeq n v_e 2\pi\rho d\rho\,. \tag{10.12}$$

Inserting (10.9), (10.11) and (10.12) into (10.7) one gets [63] the mu-atom formation rate in a plasma

$$\lambda = \frac{16\sqrt{2}\pi^{\frac{3}{2}} n^2 \Lambda_D}{9 m_\mu T^{9/2}} \;(a.u.) = \frac{3.2\cdot 10^{17}\phi^2\Lambda_D}{[T(\mathrm{eV})]^{9/2}}\,\mathrm{sec}^{-1} \tag{10.13}$$

where $\phi = n/n_0$ is the plasma density in liquid hydrogen units, Λ_D is the logarithmic term ($\Lambda_D \gg 1$) which can be approximated by

$$\Lambda_D = \ln(\rho_{max}/\rho_{min}) \simeq \ln(r_D/r_\mu) \simeq \ln(T r_D) \simeq \frac{1}{2}\ln\left[\frac{3\cdot 10^{-4} T(eV)^3}{\phi}\right] \tag{10.14}$$

where r_D is the Debye radius, $r_D = [T/(8\pi n)]^{\frac{1}{2}}$. From eqs. (10.13) and (10.14) one gets

$$\lambda(\phi=1,\ T=10^3 eV) \simeq 6\cdot 10^4\,\mathrm{sec}^{-1}\ ,\ \lambda(\phi=0.1\ ,T=10^3 eV) \simeq 7.5\cdot 10^2\,\mathrm{sec}^{-1},$$

$$\lambda(\phi=1, T=10^2 eV) \simeq 9\cdot 10^8\,\mathrm{sec}^{-1}\ ,\ \lambda(\phi=0.1, T=10^2 eV) \simeq 1.3\cdot 10^7\,\mathrm{sec}^{-1}, \tag{10.15}$$

$$\lambda(\phi=1, T=30 eV) \simeq 10^{11}\,\mathrm{sec}^{-1}\ ,\ \lambda(\phi=0.1, T=30 eV) \simeq 1.5\cdot 10^9\,\mathrm{sec}^{-1}.$$

These rates λ must be compared with the quantity $(\tau_\mu\omega_e)^{-1} \simeq 10^8\,\mathrm{sec}^{-1}$. For $\lambda \le (\tau_\mu\omega_e)^{-1}$ the process lowers the value of X_μ in comparison to a liquid (D-T) density (see chapter 5), while for $\lambda \gg (\tau_\mu\omega_S^{-1})$ it does not influence X_μ. Therefore, a practical requirement for choosing a proper density and temperature in the plasma is

$$\lambda = \lambda_{dt} + \lambda_{1\mu} \gg 10^8\,\mathrm{sec}^{-1}. \tag{10.16}$$

10.2. Formation of mu-Molecules in a Plasma

In a cold (T < 1 eV) mixture of D-T, the dtμ molecule is formed resonantly (see section 5)

$$t\mu + D_2 \rightarrow [(dt\mu)_{Jv} dee]_*, \tag{10.17}$$

where the resonance energy of dtμ is absorbed by the vibrational degree of freedom (v) of the $[(dt\mu)dee]$ complex. This resonance formation is not possible in a plasma due to the dissociation of the large H_2 molecule.

In a plasma the resonance energy is absorbed by electrons, ions or neutral atoms. The mu-molecule has charge +1 so that inside the plasma medium it can be found either as an ion $(dt\mu)^+$ or as a nucleus of a neutral atom $(dt\mu)e$. Therefore the dtμ formation in a plasma medium composed of deuterium and tritium can be induced by collisions with electrons,

$$t\mu + d + e \underset{\lambda_{br}}{\overset{\lambda_{form}}{\rightleftarrows}} d\mu + e' \tag{10.18a}$$

$$t\mu + D + e \Leftrightarrow (d\mu)e + e' \tag{10.18b}$$

by neutral-neutral transitions

$$t\mu + D + X \Leftrightarrow (d\mu)e + X' \tag{10.19}$$

or by ion-ion or neutral-ion transitions

$$t\mu + d + X \Leftrightarrow d\mu + X' \tag{10.20a}$$

$$t\mu + D + X \Leftrightarrow (d\mu)e + X^+ + e \tag{10.20b}$$

where X = D or T. λ_{form} is the rate (sec^{-1}) for the formation of the dtμ molecule while λ_{br} is the breakup rate of dtμ. λ_{form} and λ_{br} are related through the principle of the detailed balance

$$\frac{\lambda_{form}}{\lambda_{br}} = 3 n_b \left[\frac{2\pi}{\mu_a T}\right]^3 \exp(|\varepsilon_{Jv}|/T) \tag{10.21}$$

where $n_b(\mathrm{cm}^{-3})$ is the density of the species b (b=d or D), $\mu_a^{-1} = M_{t\mu}^{-1} + M_b^{-1}$, ε_{Jv} is the binding energy of dtμ in the state Jv ($\varepsilon_{11} \simeq -0.64$eV). The densities n_b are given by

$$n_d = n_0\phi c_d\alpha_D\alpha_i \tag{10.22}$$

$$n_D = n_0\phi c_d\alpha_D(1-\alpha_i) \tag{10.23}$$

where c_d is the deuterium concentration, α_D is the degree of dissociation ($D_2 \rightarrow D+D$) and α_i is the degree of ionization ($D \rightarrow d+e$) and are determined by the Saha type of equations [64,65]

$$\frac{\alpha_D^2}{1-\alpha_D} = \frac{2K^{1/2}\exp(-D/T)}{(2\pi)^{3/2} r_0^3 n_0\phi c_d T^{1/2}} \tag{10.24}$$

$$\frac{\alpha_i^2}{1-\alpha_i} = \frac{2T^{3/2}\exp(-I/T)}{(2\pi)^{3/2} n_0\phi c_d}. \tag{10.25}$$

$D \simeq 4.5eV$ is the dissociation energy for the hydrogen molecules, $r_0 \simeq 1.4 a.u.$ is the distance of equilibrium between the nuclei in the molecule, $K \simeq 0.062$ ($a.u.$) is the rigidity coefficient of the molecule (potential energy $\simeq \frac{1}{2}K(r-r_0)^2$ at $r \simeq r_0$) and I is the ionization energy ($I \simeq 13.6$eV).

Using the detailed balance it is convenient to calculate λ_{br} rather than λ_{form}. The interaction causing the disintegration of dtμ is given by

$$H = \vec{d}\cdot\vec{E} \tag{10.26}$$

where $\vec{d}$ is the dipole moment of the mu-molecule and $\vec{E}$ is the electric field acting on the molecule. For the processes (10.18), (10.19) and (10.20) this electric field can be approximated by

$$\vec{E}=\frac{\vec{r}}{r^3} \quad \text{(for (10.18)} \tag{10.27}$$

$$\vec{E}=\frac{\vec{r}}{r}\frac{dU(r)}{dr}\begin{cases} U(r)\simeq -\dfrac{6.5}{r^6} & \text{(for (10.19)).} \\ U(r)\simeq \dfrac{-2.2}{r^4} & \text{(for (10.20))} \end{cases} \tag{10.28}$$

where $\vec{r}$ is the distance between the dμ center of mass and the electron in (10.18) or the X particle (D or T) in (10.19) and (10.20). $U(r)$ in (10.28) is the potential energy of a hydrogen atom in the field of an H_2 molecule; the r^{-6} potential is a Van der Waals interaction and the r^{-4} is a polarization potential in the field of an ion. The dipole moment in eq.(10.26) can be approximated by

$$\vec{d}=\frac{M_t\mu}{M_{dt\mu}}\vec{R}. \tag{10.29}$$

($\vec{R}$ and $\vec{r}$ are Jacobi coordinates) since the principal contribution from $\vec{d}$ to the transition matrix come from large distance R between $t\mu$ and d.

The formation rates for the processes (10.18), (10.19) and (10.20) were calculated by Menslikov[63] by using the Fermi Golden Rule for the break up processes,

$$\text{Probability per unit time} = \lambda(\text{sec}^{-1})=\frac{(2\pi)}{h}|R_{fi}|^2\delta(E_i-E_f) \tag{10.30}$$

where the R_{fi} matrix is given in the first order of perturbation theory by $\langle f|H|i\rangle$, $|i\rangle$ and $|f\rangle$, are the initial and final states and H is given by (10.26). This yields to the following results:

(a) Electron induced collisions (eq. (10.18))

$$\lambda_{form}=\alpha_i\alpha_D\phi^2 c_d\lambda^{(0)}_{form} \tag{10.31}$$

$$\lambda^{(0)}_{form}(\text{sec}^{-1}) = 1.9\cdot 10^{10}T(eV)^{-2}\exp[(0.64/T(\text{eV})] \tag{10.32}$$

(b) Neutral-neutral transitions (eq.(10.19))

$$\lambda_{form} = \alpha_D^2(1-\alpha_i^2)\phi^2 c_d\lambda^{(0)}_{form} \tag{10.33}$$

$$\lambda^0_{form}(\text{sec}^{-1}) = 3n_0^2(2\pi/\mu_\alpha T)^{\frac{3}{2}}\exp(0.64/T(\text{eV}))\langle v\sigma_{c1}W_p\rangle \tag{10.34}$$

where σ_{c1} is the capture cross section (due to Van der Waals potential) for molecules X approaching $(d\mu)_{2s}$.

$$\sigma_{c1}(E) = \pi\left[\frac{3}{2}\right]^{\frac{2}{3}}\left[\frac{19.5}{E}\right]^{\frac{1}{3}} \tag{10.35}$$

$E=\frac{1}{2}Mv^2$, $M^{-1}=M_{d\mu}^{-1}+M_X^{-1}$, and W_p is the breaking probability of the mu-molecule by a collision.

(c) Ion-neutral transitions (eqs.(10.20))

$$\lambda_{form}=\alpha_i(1-\alpha_i)\phi^2 c_d\lambda^0_{form} \tag{10.36}$$

$$\lambda^{(0)}_{form}(\text{sec}^{-1})=6n_0^2(2\pi/\mu_\alpha T)^{\frac{3}{2}}\exp(0.64/T(\text{eV})\langle v\sigma_{c2}W_p\rangle, \tag{10.37}$$

where σ_{c2} is the capture cross section due to the polarization potential

$$\sigma_{c2}=\frac{2\pi}{v}\left[\frac{4.5}{M}\right]^{\frac{1}{2}}. \tag{10.38}$$

It is worthwhile to point out that for $T\geq 10$eV and $\phi\leq 1$ the formation rate satisfy

$$\tau_\mu\lambda_{d\mu}\ (T\geq 10\text{eV},\ \phi<1) < \tau_\mu\lambda^{(o)}_{d\mu} < 1000 \tag{10.39}$$

where τ_μ is the muon lifetime. Therefore a significant molecular formation rate in a plasma medium might be achievable only for $T\leq 10$eV (or for $T\geq 10$eV but with extremely dense plasmas, $\phi\gg 1$).

So far we have discussed the processes (10.18), (10.19) and (10.20) where the resonance energy of the dμ released during its formation is transferred to a single particle. In a plasma however there is also a possibility to transfer the "resonance energy" into a collective degree of freedom such as "ion-acoustic" excitation or "electron oscillation mode" (Langmuire wave). It looks that the Langmuire mode is dominant

$$t\mu+d\rightarrow d\mu+h\nu \tag{10.40}$$

where $\omega=2\pi\nu$ is the plasma frequency and $\vec{k}$ is its wave vector. The interaction inducing this transition is given by (10.26) where the electric field is described by

$$\vec{E}=\sum_{\vec{k}}\left[\frac{2\pi\omega}{\Omega}\right]^{\frac{1}{2}}\frac{\vec{k}}{k}\left[C_k e^{i\vec{k}\cdot\vec{R}_0}+C_k^+ e^{-i\vec{k}\cdot\vec{R}_0}\right] \tag{10.41}$$

where Ω is the normalization volume, C_k and C_k^+ are the plasma annihilation and creation operators and $\vec{R}_0$ is the coordinate of the dμ center of mass. Since $k<1/r_D$ (r_D = Debye radius) and $P_T r_D\gg 1$ (P_T= thermal momentum of the nuclei in the plasma) one has $k\ll P_T$. Therefore the plasma momentum is negligible and therefore it is justified to take $\vec{R}_0=0$ in eq. (10.41). Using the Fermi Golden Rule the $\lambda_{d\mu}$ can be calculated in the plasma model.[63] The plasma mechanism yields a maximum value at $T\simeq 2.6$eV (for $\phi\leq 1$)

$$\tau_\mu\lambda^{\max}_{d\mu}(\phi=1,\ C_d=0.7)\simeq 10^3. \tag{10.42}$$

The dependence of λ_{form} on ϕ in the plasman models is much stronger than ϕ^2 (which is characteristic for reactions (10.18) -(10.20)).

The probability ω_f for the nuclear fusion reaction of dt (or dd, etc.) before the disintegration of the dμ (or ddμ, etc.) molecule in the plasma medium is given by

$$\omega_f=\lambda_f/(\lambda_f+\lambda_{br}) \tag{10.43}$$

where λ_f is the fusion rate ($\lambda_f(dt)\simeq 1.2\cdot 10^{12}\text{sec}^{-1}$, etc.) and λ_{br} is related to λ_{form} as previously described. The number X_μ of nuclear fusion catalyzed by a single muon in the plasma medium can be approximated by

$$X_\mu \simeq [\omega_S + (\tau_\mu \lambda_{d\mu}(eff))^{-1}]^{-1}, \tag{10.44}$$

ω_S is the sticking probability and the effective rate for the dtμ formation is

$$\lambda_{d\mu}(eff) = \lambda_{form}\,\omega_f = \frac{\lambda_{form}\lambda_f}{\lambda_f+\lambda_{br}} \simeq \lambda_f \frac{\lambda_{form}}{\lambda_{br}} \tag{10.45}$$

where λ_{br} is assumed to be much larger than λ_f. If $\lambda_f \gg \lambda_{br}$ then $\lambda_{d\mu}(eff) \simeq \lambda_{form}$. From eq.(10.21) one gets for the processes (10.18)-(10.20) the following relation:

$$\frac{\lambda_{form}}{\lambda_{br}} \simeq 3.2\cdot 10^{-4} \phi c_d \,(T(\mathrm{eV}))^{\frac{-3}{2}} \exp(0.64/T(\mathrm{eV})) \tag{10.46}$$

In particular for $\phi = 1$, $c_d = 0.7$, $T = 3$eV one gets $\lambda_{form}/\lambda_{br} \simeq 5.3\cdot 10^{-5}$. In this case one gets $\lambda_{d\mu}(eff)\,\tau_\mu \simeq 140$ implying from (10.44) a value of X_μ smaller than 140.

Considering all possible mechanisms for dtμ formation one has to define

$$\lambda_{form} = \sum_j \lambda^j_{form} \tag{10.47}$$

$$\lambda_{br} = \sum_j \lambda^j_{br} \tag{10.48}$$

and to recalculate (10.43)-(10.46) accordingly. However, the above analysis suggest that the dtμ effective formation rate for $T \geq 3$eV is small comparatively to the case of a condensed gas or a condensed cold plasma ($T \leq 3$eV).

10.3. Muon stripping in a Plasma Medium

The ions $(\alpha\mu)^+$ are formed during the nuclear fusion reactions with an initial probability $\omega_{S0} = 0.0085$. The initial energy of the ion is about 3.5 MeV. During its collision with nuclei

$$\alpha\mu + X \rightarrow \begin{matrix} \alpha + \mu + X \\ \alpha + X\mu \end{matrix} \tag{10.49}$$

the muon can be stripped from its α host with a probability R

$$R = 1 - \exp(-P) \tag{10.50}$$

$$P = \int_{E_i}^{E_b} \frac{\sigma_{st}(E)dE}{\kappa(E)} \tag{10.51}$$

where σ_{st} is the stripping cross section, $\kappa(E)$ is related to the friction force (F_{fr}) acting on the ion $\alpha\mu$

$$F_{fr} = n\kappa \tag{10.52}$$

where $n(cm^{-3})$ is the plasma density (or in general the medium density), E_b and E_i are the "birth" energy of $\alpha\mu$($\simeq$3.5MeV) and the ionization energy ($\simeq 10KeV$). The stripping cross section σ_{st} peaks arpund 10 KeV and quickly decreases below this energy. The effective sticking probability ω_S is given by

$$\omega_S = \omega_{S0}(1-R) = \omega_{S0}\exp(-P)\,. \tag{10.53}$$

ω_S can be reduced by increasing the stripping coefficient R to its maximum value ($R \rightarrow 1$ or equivalently $P \rightarrow \infty$).

For a fully ionized plasma one can calculate $\kappa(E)$ from the following approximation [63]

$$\kappa = -\frac{4\pi e^2}{m_e} \ln(a\nu) \frac{d\psi}{d\nu} \tag{10.54}$$

where ν is the $\alpha\mu$ velocity

$$\nu = (2E/M_{\alpha\mu})^{\frac{1}{2}}, \tag{10.55}$$

$a = 2m_e r_D/e_0$, r_D is the Debye length and $e_0 = 2.718\cdots$, and ψ is an effective potential created by a charge density $f(\nu)$:

$$\psi(\nu) = \int \frac{f(\nu)d^3\nu_e}{|\vec{\nu}-\vec{\nu}_e|}, \tag{10.56}$$

$f(\nu_e)$ is the normalised electron distribution function $\int f d^3\nu_e = 1$. Assuming a simple model

$$4\pi\nu_e^2 f(\nu_e) = \begin{cases} \delta[\nu_e - (\nu-\nu_0)] & \text{for } \nu<\nu_0 \\ \delta(\nu_e) & \text{for } \nu>\nu_0 \end{cases} \tag{10.57}$$

one gets from eqs. (10.56) and (10.57) the following "potential"

$$\psi(\nu) = \begin{cases} \dfrac{1}{\nu_0} & \text{for } \nu<\nu_0 \\ \dfrac{1}{\nu} & \text{for } \nu>\nu_0 \end{cases} \tag{10.58}$$

implying from eq. (10.54)

$$\kappa = \begin{cases} \dfrac{4\pi e^4}{m_e\nu^2} ln(a\nu) \equiv \kappa_0 & \text{for } \nu>\nu_0 \\ 0 & \text{for } \nu<\nu_0 \end{cases} \tag{10.59}$$

From eq.(10.51) it is clear that the $\kappa \rightarrow 0$ contribution is the dominant one in calculating P. Equation (10.59) is an approximation. The more accurate behavior at $\nu<\nu_0$ is $\kappa/\kappa_0 \simeq (\frac{\nu}{\nu_0})^3$ (instead of the zero in (10.59)). However, in general one can see that for $\nu \ll \nu_0$, $\kappa(E)$ falls sharply with the decrease in ν.

For a Maxwellian distribution of electrons in the plasma one gets

$$\kappa(E) \equiv \kappa_0(E)\mu(E/E_T) \tag{10.60}$$

$$\mu(X) = \frac{4}{\sqrt{\pi}} \int_0^X e^{-t^2} t^2 dt\,,\; E_T = \left[\frac{M_{\alpha\mu}}{m_e}\right] T.$$

As explained above, the main contribution to P comes from $E \ll E_T$, and in this limit

$$\mu\left[\frac{E}{E_T}\right] \simeq \frac{4}{3\sqrt{\pi}} \left[\frac{E}{E_T}\right]^3. \tag{10.61}$$

From the neutral gas data one calculate P_0 to be

$$P_0 \simeq 0.63\,,(\omega_S = \omega_{S0}\exp(-P_0) = 0.0045). \tag{10.62}$$

Assuming for a plasma the approximation

$$P \approx P_0 + \Delta P \tag{10.63}$$

and using (10.61) we get

$$\Delta P \approx \frac{P_0}{E_i}\int_{E_i}^{\infty} dE\left[\frac{1}{\mu}-1\right] = \frac{P_0}{\mu\left[\frac{E_i}{E_T}\right]} = 5.3\cdot 10^{-4}[T(\mathrm{eV})]^3 \tag{10.64}$$

Using this result in eqs. (10.53), (10.62) and (10.63) we get the effective sticking in a plasma

$$\omega_S = 4.5\times 10^{-3}\exp[-5.3\cdot 10^{-4}[T(\mathrm{eV})]^3] \tag{10.65}$$

From this result one can see that at a sufficiently high temperature there occurs practically complete stripping of the muons from their $\alpha\mu$ ions. In particular for T = 10 eV ω_S is reduced by a factor of 0.6 while for T larger than 20 eV ω_S is effectively zero.

In summary, in a plasma with a temperature larger than 20 eV the "sticking problem" disappears. In this case the mu-catalysis is limited by the $d\mu$ (or $dd\mu$) fusion probability during the molecular stability. From the previous section (10.2) one can see that for T > 20 eV the effective rate for fusion is small because the $d\mu$ molecules break up. Thus it looks that in an homogeneous plasma there are no appropriate values for temperature and density for which, at the same time, ω_s is small and $\lambda_{d\mu}$(eff) is large. The main problem is to find a mechanism where a high rate of mu-molecular formation exists for which the break up rate is small in the domain where the effective sticking is negligible. We suggest that it might be possible to use powerful lasers or ion-resonance cyclotron heating in a plasma medium in order to get a positive answer to this problem.[66]

11. COLD FUSION IN A CRYSTAL

On March 23, 1989, during a press release at the University of Utah in Salt Lake City, U.S.A., it was disclosed that Dr. Martin Fleischmann, (Professor of Electrochemistry at the University of Southhampton) and Dr. Stanley Pons (Professor of Chemistry and Chairman of the Department of Chemistry at the University of Utah), have successfully created a sustained nuclear fusion reaction at room temperature in a chemistry laboratory at the University of Utah. In ref. 1 the authors claim to have achieved nuclear fusion in an electrolytic cell containing heavy-water (D_2O). Apparently the fusion occurs when the deuterium atoms accumulate in the palladium electrode (the cathode) and a rate of about 10^4 fusions/sec were reported.[1] The energy released from the fusion reactions seems to be less by nine to ten orders of magnitude in comparison with the calorimeter measurements. The most astonishing result of ref. 1 is the claim that during the (120 hours) experiments an energy of 4MeggaJoule per cm^3 of electrode (Pd) volume was liberated. It is too soon to proper analyze this data and to argue with Prof. Fleischmann statement: "What we have done is to open the door of a new research area". However this experiment might suggest that cold fusion is possible in a crystal. The role of μ is played by a crystal.

In a preprint (released at the same time with the University of Utah press release) written by a second Utah group (led by Professor Steven E. Jones of Brigham Young University in Provo), there is also evidence for nuclear fusion inside the palladium (or titanium) lattice. Jones *et al.* have observed nuclear fusion at room temperature during electrolytic infusion of deuterons into Ti and Pd electrodes.

This experiments measured about 0.5 fusions per second implying

$$\lambda_f \approx 10^{-23}\,\mathrm{fusions/(deuteron\times sec)} \tag{11.1}$$

The neutrons were measured with a very sensitive spectrometer in the energy domain of 2.5 MeV where the neutrons from d-d fusion are expected.

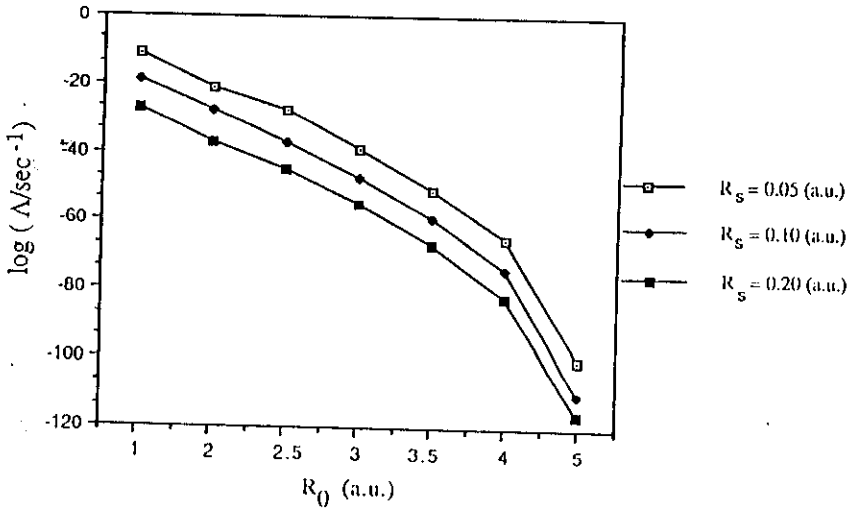
The rate of nuclear fusion of deuterons is estimated now by considering a phenomenological model[67] which might resemble the physical conditions arising in condensed matter. The interaction between two deuterons is described by tailoring a screened Coulomb potential (with a screening length R_S due to the electron gas in the hybride environment) with an harmonic potential. This potential might simulate phenomenologically the confinement of nuclei within a lattice cell. In atomic units the potential describing the deuteron-deuteron interaction is given by:

$$V(R) = \frac{\exp(-R/R_S)}{R} + V_o + \tfrac{1}{2}K(R-R_o)^2 \tag{11.2}$$

where R is the relative position of the deuterons, K is the harmonic oscillator constant given by $K = \mu(2\pi\nu)^2$, μ is the reduced mass of the deuterons. The ground state energy of the deuteron is taken from Ref. 68: $h\nu/2 = 0.072$eV. V_o is of order of several eV (the dissociation energy of the lattice). R_S is the screening length (of the Coulomb potential) due to the electrons in the lattice.

The fusion reaction rate Λ is proportional to the probability of quantum-mechanical tunneling through the barrier, or the probability of finding the nuclei at strong interaction distance (of the order of one Fermi):

FIGURE 7. Nuclear fusion reaction rates as function of the internuclear separation for different values of the screening length R_S.



$$\Lambda = A\,|\psi(0)|^2 (sec^{-1}), \tag{11.3}$$

ψ is the solution of the Schroedinger's equation for the potential given in (11.2), and Λ is the nuclear reaction constant obtained from measured cross sections for low energy collisions between hydrogen nuclei. For the d-d reaction, A is given by [69]

$$A = 2.10^{16} cm^3 sec^{-1} \tag{11.4}$$

A semiclassical caluclation of $\psi(0)$ is done following Ref. (8) for mesomolecular ions (two isotopic nuclei bound together by a negative muon) and Ref. (69) for hydrogen molecules. For the potential (11.2) we get [67]

$$\Lambda = 10^9 \frac{1}{R_a(a.u.)^2} e^{-\lambda} (sec-1) \tag{11.5}$$

The turning point R_a is obtained from $V(R) = E$ and is given by:

$$R_a = R_o + (K\mu)^{-1} \tag{11.6}$$

The barrier penetration factor was calculated by connecting the screened Coulomb potential with the harmonic potential at a radius ρ:

$$\lambda = \int_R^{\rho} \left\{ 2 \left[2\mu \left[E - \frac{\exp(-R/R_S)}{R} - \frac{1}{4R^2} \right] \right]^{1/2} - \frac{1}{R} \right\} dr +$$

$$\int_{\rho}^{R_a} \left\{ 2 \left[2\mu \left[E - V_o - \tfrac{1}{2} K (R-R_o)^2 - \frac{1}{4R^2} \right] \right]^{1/2} - \frac{1}{R} \right\} dr \tag{11.7}$$

Fusion reaction rates were calculated as a function of the equilibrium distance between the nuclei R_o for different values of the screening length R_S. The results are summarized in Fig. 7.

In summary, we estimated nuclear fusion reaction of d-d in condensed matter by modeling the d-d interaction by the combination of a Yukawa and an harmonic potential. The results show that fusion rates larger than 10^{-23}/ (sec x deuteron pair) can arise in the domain given approximately by $R_o + 14.3R_S < 2.9$ in atomic units. In static conditions, Palladium (Pd) is a face centered cube with lattice spacing of 7.35 a.u. When hydrogen is squeezed into the Pd it goes into octahedral sites and then into tetrahedral sites. In the steady state configuration the distance between two deuterons is 5.38 a.u. for the octahedral geometry. The distance between two deuterons in the octahedral-tetrahedral positions is 3.28 a.u. In the above calculations, we do not suggest any ideas or model causing the condensed matter to be in the domain of parameters where fusion rates are $\Lambda > 10^{-23} sec^{-1}$ (per deuteron pair) as recently suggested by the cold fusion experiments.

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