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ACCELERATION OF CHARGED PARTICLES AT NONLINEAR STAGE OF LONGWAVELENGTH PLASMAS INSTABILITIES

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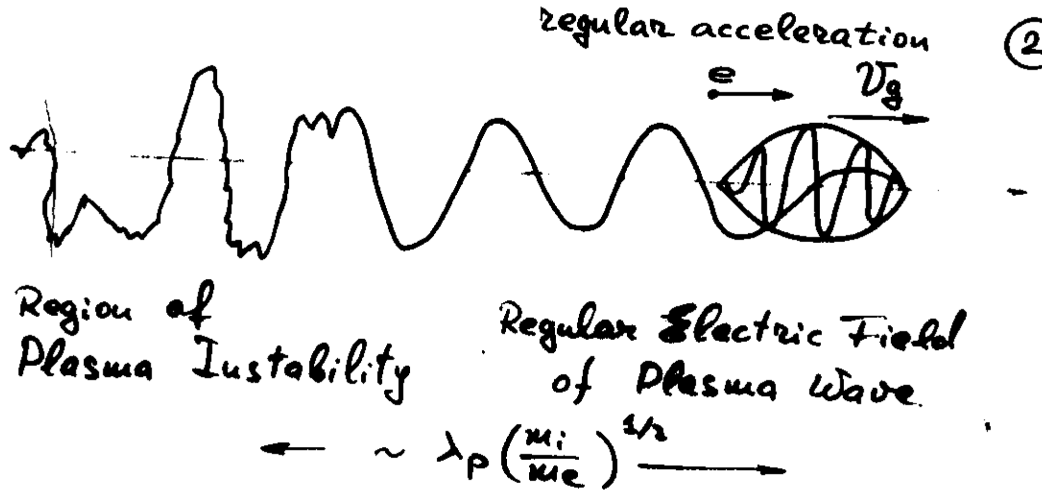
- rarefaction wave breaking
- fast ions $\mathcal{E}_i \gg \mathcal{E}_e \big|_{t=0}$



Plasma instability in Strong HF field $\rightarrow$ $\Delta\omega < \omega_{pe}$

$\rightarrow$ nonlinear regime $\rightarrow$

$\rightarrow$ dynamic electric double layers $\rightarrow$

$\rightarrow$ acceleration of ions

L.M. Kovrizhnykh, A.S. Sakharov (GPI)
P.V. Sasorov (ITEF)

Sov. J. Plasma Phys. (1986)
Comments on Plasma Phys. & Contr. Fus., (1988)

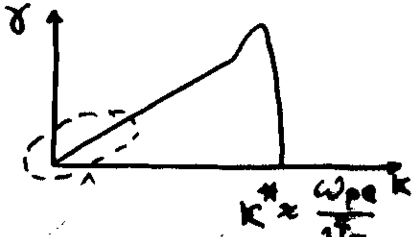
Two-stream instability in H F field

$\frac{m_e}{m_i} \ll 1$

Pump field frequency $\omega < \omega_{pe}$

Electron quiver velocity $v_E = \frac{eE}{m\omega} \gg v_{Te}$

Instability (V.P. Silin', 1965)



$k \to 0$ long-wavelength limit

for $k \ll \omega_{pe}/v_E$

$$\boxed{\gamma = C_b k} \qquad \boxed{C_b = \frac{v_E}{\sqrt{2}}\left(\frac{m_e}{m_i}\right)^{1/2}}$$

number of plasma instabilities have much in common if $\gamma \sim k$

nonlinear stage can be described by the system of nonlinear equations in partial derivatives, similar to those of gas dynamics.

$$\frac{\partial n}{\partial t} + \frac{\partial n v}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{1}{n}\frac{\partial P(n,v)}{\partial x}$$

n – "density"

v – "velocity"

P – "pressure"

These equations are linear with respect to the derivatives, that allows one finde their solution with the help of the hodograph-transformation

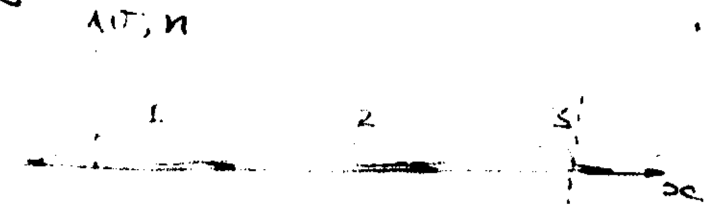
$$\begin{pmatrix} n(x,t) \\ v(x,t) \end{pmatrix} \longleftrightarrow \begin{pmatrix} x(n,v) \\ t(n,v) \end{pmatrix}$$

or another methods

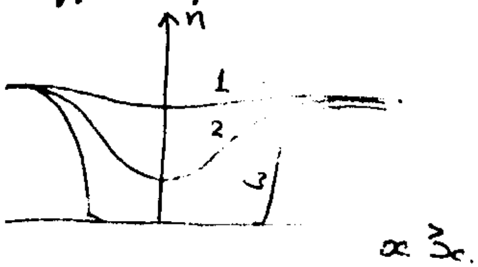
- Modulational instability of wave packet
- Self-focusing
- beam-plasma instability
- Buneman instability

The evolution of perturbations of finite amplitude in a unstable medium possesses some featured in comparision with stable one.

Stable medium – compression wave breaking – Riemann solution



Unstable medium – singularity of the type of rarefaction wave breaking



- Buneman istability
- Cavity formation
- nonstationary double layer
- ious acceleration

Alfvén & Carlquist (1972)
Galeev, Sagdeev, Shapiro, Shevchenko (1981)
Bulanov & Sasorov (1986)

Some typical features of singularity corresponding to breaking of rarefaction wave can be followed in linear approximation

$$\gamma = c_b k \longleftrightarrow \omega^2 = -c_b^2 k^2$$

for amplitude $a(x,t)$ we have

$$\boxed{\frac{\partial^2 a}{\partial t^2} + c_b^2 \frac{\partial^2 a}{\partial x^2} = 0}$$

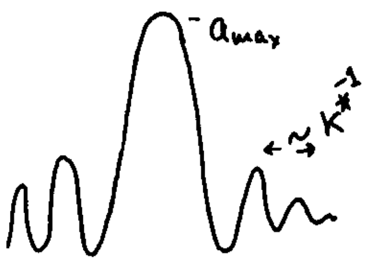
Laplace' equation

Solution of Caushi problem in terms of complex variables $z = x + i c_b t$

$$a(x,t) = \mathrm{Re}\left\{ f(z) \right\} =$$

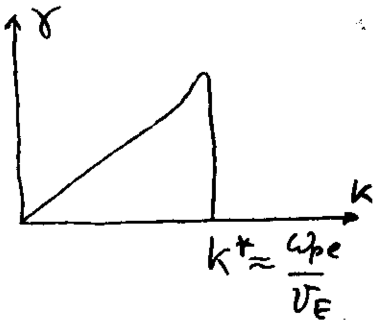
$$= \mathrm{Re}\left\{ a_0(x + i c_b t) + i \int_0^{x + i c_b t} \dot{a}_0(s)\, ds \right\},$$

where $a(x,t)\big|_{t=0} = a_0(x)$, $\left.\frac{\partial a}{\partial t}\right|_{t=0} = \dot{a}_0(x)$



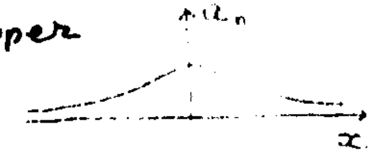
if we take into account that $\gamma(k)$ turns to zero at $k > k^*$

then $a_{max} \approx k^*$



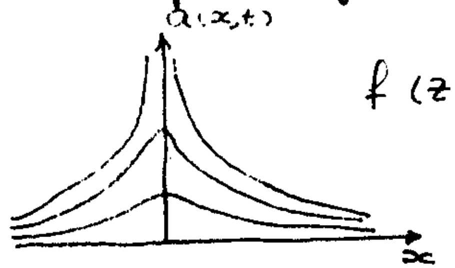
For typical initial conditions (for example $a_0 \sim \frac{1}{1+x^2}$)

$f(z)$ has singularity in upper half plane of varable z



that means that singularity of $a(x,t)$ will arise for finite time

Simplest singularity is the pole



$$f(z) \approx \frac{i}{z - z_0} = \frac{i}{(x + i\, c_b (t - t_0))}$$

Nonlinear stage of two-stream instability of plasma in HF field in longwavelength limit

$$\frac{\partial n_e}{\partial t} + \frac{\partial n_e v_e}{\partial x} = 0$$

$$\frac{\partial p_e}{\partial t} + v_e \frac{\partial p_e}{\partial x} = -\frac{eE}{m_e}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i v_i}{\partial x} = 0 \qquad \left(\begin{smallmatrix} & * & \\ * & & * \end{smallmatrix}\right)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = \frac{eE}{m_i}$$

$$\frac{\partial E}{\partial x} = 4\pi e (n_i - n_e)$$

$$p_e = \frac{m_e v_e}{\sqrt{1 - v^2/c^2}}$$

$$\frac{\partial E}{\partial t} = 4\pi e (n_e v_e - n_i v_i) - \frac{\partial D}{\partial t}$$

induction is defined by boundary conditions

if $D(t) = E_0 \sin \omega_0 t$, with frequency $\omega_{pi} \ll \omega \ll \omega_{pe}$

then $u(t) = v_e - v_i = u_0 \cos \omega_0 t$

where $u_0 = eE_0 \omega / m_e \omega_{pe}^2$

longwavelength perturbation can be regarded as quasineutral $(K u_0 \ll \omega_{pe})$

$$n_e = n_i = n \qquad v_i = v$$

hence $J(t) = \frac{1}{4\pi} \frac{\partial D}{\partial t}$ that is $v_e \sim \frac{1}{n}$

$$v_e = v_i - \frac{J(t)}{en}$$

i) excluding the electric field from $\left(\begin{smallmatrix} & * & \\ * & & * \end{smallmatrix}\right)$

ii) averagin over fast period $\frac{2\pi}{\omega}$ of oscillation

$$\boxed{\begin{aligned} &\frac{\partial n}{\partial t} + \frac{\partial n v}{\partial x} = 0 \qquad v = v_i, \; n = n_i \\ &\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = \frac{2 m_e c^2}{m_i} \frac{\partial K(n_R/n)}{\partial x} \end{aligned}} \qquad \left(\begin{smallmatrix} & * & \\ * & & * \\ & * & \end{smallmatrix}\right)$$

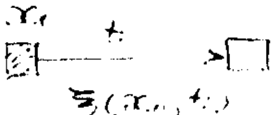
where $K(x)$ complite elliptic integral

$$n_R = \frac{n_0 u_0}{c} \qquad J = -e n_0 u_0 = -e n_R c$$

to solve $(*)$ trasfer to the Lagrange variables

$$x = x_0 + \xi(x_0, t)$$
$$v = \frac{\partial x}{\partial t}$$

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)_E = \left(\frac{\partial}{\partial t}\right)_L$$
$$\left(\frac{\partial}{\partial x}\right)_E = \left(\frac{\partial x_0}{\partial x}\right)\left(\frac{\partial}{\partial x_0}\right)_L$$



normalization

$\zeta = x_0/l_0$, $\tau = t\,(m_e/m_i)^{1/2}\left(\frac{u_0}{l_0\sqrt{2\pi}}\right)$

$\varkappa = n_R/n$, $\mu = \frac{\sqrt{2\pi}}{u_0}\left(\frac{m_i}{m_e}\right)^{1/2} v$,

then

$$\boxed{\begin{aligned} \frac{\partial\mu}{\partial\tau} &= -\frac{2}{\varkappa}\,K'(\varkappa)\,\frac{\partial\varkappa}{\partial\zeta} \\ \frac{\partial\varkappa}{\partial\tau} &= \frac{\partial\mu}{\partial\zeta} \end{aligned}}$$

$$K' \equiv \frac{dK}{d\varkappa}$$

For $\varkappa \to 0$ $\quad \frac{n}{n_R} \to \infty$ nonrelarivistic limit

$$K'(\varkappa) \simeq \frac{\pi}{4}\varkappa$$

For $\varkappa \to 1$ $\quad \frac{n}{n_R} \to 1$ ultrarelativistic limit of electron motion

$$K'(\varkappa) \approx \frac{1}{2(1-\varkappa)}$$

Nonrelativistic limit

$$K'(\varkappa) \simeq \frac{\pi}{4}\varkappa$$

$$\left.\begin{aligned} \frac{\partial\mu}{\partial\tau} &= -\frac{\pi}{2}\frac{\partial\varkappa}{\partial\zeta} \\ \frac{\partial\varkappa}{\partial\tau} &= \frac{\partial\mu}{\partial\zeta} \end{aligned}\right\}$$ Cauchi – Riemann relations for the real and imajinary parts of analitic function

$$F(z) = \mu(z) + i\,\varkappa(z) \sim \frac{1}{n}$$

where $z = \zeta + i\sqrt{\pi}\,\tau$

Simplest singularity corresponding to breaking of rarefuction disturbance

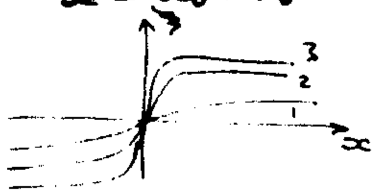
$$F(z) \simeq \frac{i l_0}{z - z_0}\ , \quad \text{where } l_0 = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{n_0 - n}{n_0}\,dx$$

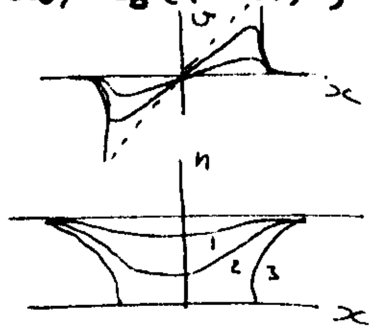
Hence we can write the expressions for

$$n(x_0,t) \simeq n_0\,(x_0^2 + C_b^2(t-t_0)^2)/(x_0^2 + C_b^2(t-t_0)^2 - l_0 C_b\,(t-t_0))$$

$$v(x_0,t) \simeq C_b\,l_0\,x_0/(x_0^2 + C_b^2(t-t_0)^2)$$

$$x = x_0 + l_0\,\mathrm{arctg}\,(x_0/\,C_b(t-t_0))$$





Energy and spectrum of fast ions

$$\mathcal{E}_{max} \approx \frac{1}{8}\frac{m_i l_0^2}{(t_0-t)^2} \quad \text{at } x_0 = \pm C_b (t_0-t)$$

$$dN = n_0\, dx_0 = n_0\left(\frac{\partial x_0}{\partial \mathcal{E}}\right) d\mathcal{E}$$

$$\frac{dN}{d\mathcal{E}} = \frac{2 n_0 l_0}{m_e u_0^2}\left(\frac{m_e u_0^2}{2\mathcal{E}}\right)^{3/2} \Big/ \left(1 - \frac{C_b^2 (t_0-t)^2\, 2\mathcal{E}}{l_0^2\, m_e u_0^2}\right)^{1/2}$$

qusineutrality approximation breaks at

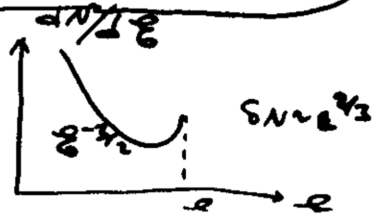
$$(t_0 - t) \approx \frac{l_0}{C_b}\,\varepsilon^{2/3}$$

where dimensionless parameter of the problem is

$$\varepsilon = \frac{u_0}{\omega_{pe} l} \ll 1$$

hence $\boxed{\mathcal{E}_{max} \approx \frac{m_e u_0^2}{2}\,\varepsilon^{-4/3} \gg \frac{m_e u_0^2}{2}}$

$$\langle \psi \rangle \propto J^{2/3}$$



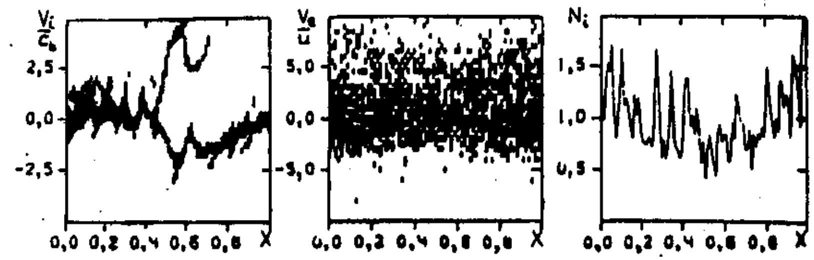


Fig. 1. Acceleration of ions as a result of the development of Buneman instability. The phase planes of the ions and electrons and the concentration of the ions prior to the time of $160\omega_{pe}^{-1}$. The initial value of the current velocity $u = \omega_{pe}L/80$ and the size of the calculation region is $L = 1$; $m_e/m_i = 10^{-2}$; and $c_b = (1/800)\omega_{pe}L$.

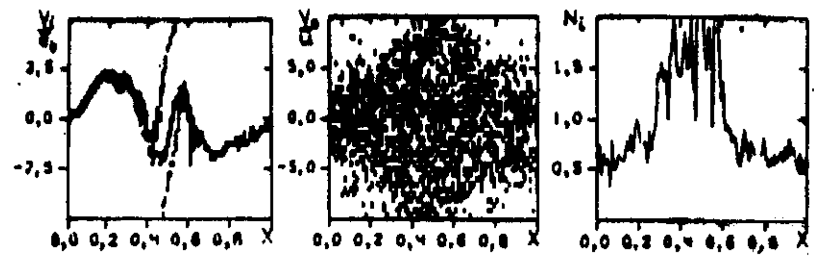


Fig. 2. Same as in Fig. 1 for the case of aperiodic instability of a plasma in a microwave field: $T = 160\omega_{pe}^{-1}$, the frequency of the external microwave field is $\omega = (\pi/10)\omega_{pe}$ and $c_b = (1/800)\omega_{pe}L$. The cited relation $N_i(x)$ corresponds to the stage in which the cavern was already shut and a thickening was formed in its place.

Ultrarelativistic (for electrons) limit

$\varkappa \to 1$, $n/n_R \to 1$, $K'(\varkappa) \approx 1/2(1-\varkappa)$

$\delta = 1 - \varkappa$, $\lambda = 2\delta^{1/2}$

$$\left.\begin{aligned} \frac{\partial \mu}{\partial \tau} &= -\frac{1}{\delta}\frac{\partial \delta}{\partial z} \\ \frac{\partial \delta}{\partial \tau} &= -\frac{\partial \mu}{\partial z} \end{aligned}\right\}$$

hodograph-transformation

$$\begin{pmatrix} \mu(\tau, z) \\ \delta(\tau, z) \end{pmatrix} \longleftrightarrow \begin{pmatrix} z(\mu, \delta) \\ \tau(\mu, \delta) \end{pmatrix}$$

$$\frac{1}{\lambda}\frac{\partial}{\partial \lambda}\left(\lambda \frac{\partial z}{\partial \lambda}\right) + \frac{\partial^2 z}{\partial \mu^2} = 0$$

$$z = c_1 \mu + c_2 \mu \left(\lambda^2 - \frac{2}{3}\mu^2\right)$$

$$\tau = -\frac{c_1}{4}\lambda^2 - \frac{c_2}{4}\lambda^2\left(\frac{\lambda^2}{4} - \mu^2\right)$$

$$n \approx n_R\left(1 + \frac{\lambda^2}{4}\right)$$

$$\langle \varphi \rangle \approx \frac{m_e c^2}{e\pi} \ln(1/\lambda)$$

$$\lambda \to 0$$

$$\lambda_{min} \approx \left(\frac{c}{\omega_{pe} l}\right)^{1/2} \left(\frac{c}{v_0}\right)^{1/4}$$

$$\mathcal{E}_{max} \approx mc^2 \ln(1/\lambda_{min})$$

$$\frac{dN}{d\mathcal{E}} \sim \exp\left(-\frac{3\pi \mathcal{E}}{m_e c^2}\right)$$

Beat-wave interaction with plasma and acceleration of ions

$\omega_1 - \omega_2 = \Delta\omega$, $\omega_1 \approx \omega_2 \approx \omega_0$

$K_1 - K_2 = \Delta K$. $E_1 \approx E_2 \approx E_0$

$$v_0 = \frac{eE_0}{m_e \omega_0}$$

! $\Delta\omega < \omega_{pe}$!

Ponderomotive force

$$F = -\frac{1}{2} m_e \frac{\partial}{\partial x} \langle v_e^2 \rangle$$

for longitudinal electron motion

$$\ddot{\tilde{v}} + \omega_{pe}^2 \tilde{v} = \frac{v_0^2 \Delta\omega \Delta k}{2} \cos(\Delta\omega t - \Delta k x)$$

$\Delta\omega < \omega_{pe}$ $$\langle \tilde{v}^2 \rangle \approx \frac{v_0^4}{8}\left(\frac{\Delta\omega \Delta k}{\omega_{pe}^2}\right)^2 \sim \frac{1}{n^2}$$

For longwavelength dynamics of plasma

$$\boxed{\begin{aligned} &\frac{\partial n}{\partial t} + \frac{\partial n v}{\partial x} = 0 \qquad\qquad \sim 1/n^2 \qquad \sim n \\ &\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{m_e}{m_i}\frac{v_0^2}{16}\frac{\partial}{\partial x}\left(\left(\frac{v_0(\Delta k \Delta\omega)^2}{\omega_{pe}^2} + 4\frac{\omega_{pe}^2}{\omega_0^2}\right)\right) \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad \parallel \qquad\qquad \perp \end{aligned}}$$

in linear approximation under the condition

$$\frac{v_0^2}{c^2} > \frac{2\omega_{pe}^2}{\omega_0^2 \Delta\omega^2 \Delta K^2 c^2}$$

instability growthrate

$$\gamma \approx K \frac{v_0}{4}\left(\frac{m_e}{m_i}\right)^{1/2}\left(2\frac{(\Delta\omega \Delta k v_0)^2}{\omega_{pe}^4} - 4\,\omega_{pe}^2/\omega_0^2\right)$$

$\xrightarrow{k_1}$ $\xleftarrow{k_2}$

Nonlinear regime

$$\mathcal{E}_{i\,max} = \frac{m_e}{4}\left(v_0^2 \Delta\omega \Delta k\, l_0^2\right)^{2/3}$$