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REDUCED KINETIC DESCRIPTIONS:
GYROKINETICS AND QUIVER KINETICS (II)

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TOPIC: **Reduced Kinetic Descriptions: Gyrokinetics and Quiver Kinetics**

Lecture #3: Quiver Kinetics

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Lecture #3 QUIVER KINETICS* ICTP
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Abstract

Intense, applied radio frequency (rf) fields can cause one or more charged species to oscillate with a quiver (or jitter) speed comparable to its thermal speed. The near fields in the edge plasma region of rf heated tokamaks are an example of such fields, and may be responsible for impurity generation, edge heating, and edge profile modification. When collisional, non-resonant wave particle processes dominate over collisionless, resonant interactions (as in the tokamak edge region) a reduced kinetic description, quiver kinetics, can be obtained by employing fast time and gyroaverages. The slow time and gyrophase independent portion of the distribution function will be found to satisfy a quiver kinetic equation, which once solved, can be used to evaluate the radial particle fluxes.

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*Excerpts from the paper "A Quiver Kinetic Formulation of Radio Frequency Heating and Confinement in Collisional Edge Plasmas" by Peter J. Catto and J. R. Myra, Lodestar Research Corporation, Boulder, Colorado, which is to be published in Phys. Fluids.

I. INTRODUCTION

The cool edge plasma of a radio frequency (rf) heated tokamak, i.e. the region near the last closed flux surface, is subjected to intense rf fields, particularly in the vicinity of the antennas, and is strongly influenced by atomic processes. These facts suggest that there are a wealth of physical phenomena occurring at the edge which merit close scrutiny. Furthermore, this region is important because it fuels the scrape off layer (SOL) on the exterior and acts as a boundary condition for the hot core plasma contained inside.

Many ion cyclotron heating (ICRF) experiments observe edge heating[1,2] edge potential changes[2], and/or edge density modifications[2,3]. Often confinement is found to be degraded by ion cyclotron heating, but the observation in two independent ion Bernstein wave heating experiments[4,5] of <u>enhanced</u> particle confinement provide a notable exception. From these experiments it is clear that ICRF can dominate edge physics phenomena.

Interaction of the edge plasma with the ICRF antenna Faraday shield and launched waves is an important consideration for both global wave coupling and impurity generation[6]. The latter has often been enhanced during intense ICRF heating. The development of a first principles theory which can describe the self-consistent equilibrium and confinement of the edge plasma in the presence of ICRF is thus of considerable interest. This is the goal of the present work. It should be noted in passing that while ICRF systems have provided the immediate motivation for our study, and will dominate our discussions, the theoretical tools developed herein apply equally well to the case of lower hybrid[7] and electron cyclotron heating[8] (ECH), though the practical importance of the same mechanisms remains to be addressed for these cases.

Previous investigations of rf induced transport[9–13] have focused on resonant quasilinear processes (i.e. wave–particle heating regimes) which are not usually relevant to the bulk edge plasma since the rf is deliberately chosen to resonantly interact with the core plasma, not with the main species at the edge. The approach developed herein is nonlinear but does not employ a resonant quasilinear operator since collisional, nonresonant, wave particle interactions dominate an edge plasma subject to intense rf fields. Instead the formalism is developed by adopting an ordering that permits the species quiver (or jitter) velocity u_j in the intense rf fields to be on the order of the species thermal speed v_j and the wave frequency ω to be on the order of the cyclotron frequency Ω_j. As a result of the $u_j \sim v_j$ ordering neoclassical effects[14,15] are negligible as long as $1 \tilde{>} (u_j/v_j)^2 \gg \rho_j/\ell$, where $\rho_j = v_j/\Omega_j$ is the species gyroradius and ℓ the scale length. The neglect of electron neoclassical effects is reasonable in the edge plasma since $(u_e/v_e)^2 \sim 1$ near the antenna[16], whereas $\rho_e/\ell \tilde{<} 10^{-3}$. For the ions $(u_i/v_i)^2$ again drops off rapidly from order unity as one moves away from the antenna, so that sufficiently close to the antenna, the edge ions will also be rf dominated[16]. The description developed in the following sections focuses on rf modifications. If it becomes

necessary to include $\rho_i/\ell \sim (u_i/v_i)^2$ effects, they can be retained straightforwardly.

In the edge plasma the mean free path for the electrons and ions, λ can be larger, smaller, or comparable to the antenna dimensions along a field line, ℓ_a, so that Coulomb collisions can be retained for all regimes of collisionality by adopting the most general ordering $\lambda \sim \ell_a \sim \ell$. In most present day experiments[2,3,5] edge densities are on the order of $N \sim 10^{12}$ cm^{-3} and edge temperatures are approximately $T \sim 25$ eV so that $\lambda \sim 600/Z_{eff}$ cm (where Z_{eff} is the effective ionic charge). Thus, λ is large compared to the length scales associated with the antenna and these experiments are collisionless to moderately collisional from the present point of view. For some high density tokamaks[1,4], and for future tokamaks which approach the reactor regime, edge plasmas become collisional ($\lambda < \ell_a$) since the edge density N tends to rise with the core density while edge temperatures tend to remain more or less the same because they are greatly affected by atomic physics processes.

In summary, therefore, the treatment herein is characterized by intense rf fields driving large quiver motion at the edge near the antenna, and by collisional effects which disrupt the organized motion.

Using the ordering $1 \tilde{>} (u_j/v_j)^2 \gg \rho_j/\ell$ and $\lambda \sim \ell_a \sim \ell$, a reduced kinetic equation is derived in Sec. II by developing a small quiver amplitude and gyroradius ordering and averaging over fast time as well as gyrophase. This "quiver kinetic" formalism permits a rigorous treatment of the full Vlasov operator (with ponderomotive effects[17-21] retained) and the retention of the relevant Coulomb and atomic processes via Fokker–Planck[14,15] and Boltzmann[22] (excitation/line radiation, charge exchange, ionization and ion–neutral energy equilibration) collision operators. The quiver kinetic procedure is developed for arbitrary geometries and so can be used to investigate non–axisymmetric effects in a tokamak edge plasma. Some of the details are relegated to Appendix A.

In Sec. III a moment formalism[14,15,23] is developed within the same general ordering scheme so that once the quiver kinetic equation (31) of Sec. II is solved the particle fluxes and current can be evaluated in terms of the self–consistently determined density, temperature, and electrostatic potential gradients create by the applied rf. Appendix C presents a proof of an important identity employed to rewrite the ponderomotive force in its standard form. The nonlinear formalism of Secs. II and III is valid for general geometry; however, tokamak applications are of particular interest so the flux surface operation is defined in Appendix B for a torus (which need not be axisymmetric).

The intense fields in the edge plasma region typically result in $(u_e/v_e)^2 \gg \rho_e/\ell$, particularly in the vicinity of an antenna, so that neoclassical and classical effects which depend on the electron's banana motion and its gyroradius ρ_e will be neglected in Sec. II and III. However, local collisional heating by the applied rf occurs because of the collisional disruption of the quiver motion by unlike particle collisions. The heating by this inverse bremsstrahlung[19,24,25] process must normally be balanced by line radiation losses[26] to prevent

secular heating. The quiver kinetic description permits the electrons to be cooled by the inelastic scatterings whereby the energy removed changes the internal state of partially stripped impurities and/or neutrals which then radiate the energy away. Electron–ion equilibration is only effective at removing energy from the rf heated electrons at extremely small quiver speeds; $(u_e/v_e)^2 \sim M_e/M_i$.

II. Quiver Kinetic Equation for RF Induced Transport

Quiver kinetic equations differ from their drift kinetic and gyrokinetic counterparts by retaining both the quiver (or jitter) motion of the charges in the applied rf field and the gyromotion in the unperturbed magnetic field. Such a description is useful when an rf plasma interaction is characterized both by rf quiver amplitudes that can be comparable to the gyroradius and by the absence of cyclotron and/or Landau resonances.

To develop a quiver kinetic equation for rf effects in magnetized plasmas it is convenient to define the species quiver or jitter velocity $\vec{u}$ via the linear equation

$$\frac{\partial \vec{u}}{\partial t} = \frac{Ze}{M}(\vec{e} + \frac{1}{c}\vec{u}\times\vec{B}) \tag{1}$$

where $\vec{e}$ is the applied rf electric field, $\vec{B}$ the unperturbed magnetic field, and species subscripts are suppressed on $\vec{u}$, the charge number Z, and the mass M.

In the derivation that follows all terms in Eq. (1) are assumed to be the same order so that the orderings

$$\vec{u} \sim \frac{Ze}{M\omega}\vec{e} \quad \text{and} \quad \omega \sim \Omega \tag{2}$$

are adopted, where fast time derivatives are taken to be of the order of the wave frequency ω and $\Omega \equiv ZeB/Mc$ is the species gyrofrequency with $B \equiv |\vec{B}|$. In this section and the next it is

convenient to treat the species quiver velocity $\vec{u} = \vec{u}(\vec{r},t)$ and the species thermal speed $v_j \equiv (2T/M)^{1/2}$ as the same order,

$$\vec{u} \sim v_j , \tag{3}$$

so that $\vec{e}\times\vec{B}$ drifts are the same order as v_j, where T is the species temperature.

In order for Eq. (1) to contain only the dominant forces the species gyroradius $\rho \equiv v_j/\Omega$ and quiver or jitter amplitude $\xi \equiv |\vec{u}|/\omega$, must be assumed small compared to the smallest scale or wave length ℓ,

$$\rho/\ell \sim \xi/\ell \ll 1 . \tag{4}$$

Then the Lorentz forces due to the applied rf magnetic field $\vec{b}$, as given by Faraday's law, and the unperturbed electrostatic potential Φ, with ordering $Ze\Phi \sim T$, are small in $\rho/\ell \sim \xi/\ell$. Any induced electric field generated by the applied rf will be assumed to be at least ρ_i/ℓ smaller than the electrostatic field so that

$$\vec{E} = -\vec{\nabla}\Phi \tag{5}$$

can always be employed to lowest order.

As an aside, it should be noted that while the orderings given by Eqs. (2)–(4) permit large amplitude rf waves, they exclude wave–particle trapping effects because of the ordering of the space scales. This is evident from considering two specialized trapping limits. For motion parallel to $\vec{B}$, the well known trapping frequency $\omega_{tr} = (Ze|\vec{e}|/\ell M)^{1/2}$ (where here ℓ is the wavelength) is of order $\omega_{tr}/\omega \sim (\xi/\ell)^{1/2} \ll 1$ in the present scheme. Similarly, for cyclotron trapping in perpendicularly polarized $\vec{e}$ fields[27], the characteristic trapping frequency $\omega_{tr} = (Ze|\vec{e}|v_\perp/M\omega \ell^2)^{1/2}$ is of order $\omega_{tr}/\omega \sim (\rho\xi)^{1/2}/\ell \ll 1$. Implicit in Eq. (2) is the assumption that $|\omega - \Omega|/\omega \gg (\rho\xi)^{1/2}/\ell$.

To study the effects on transport of collisions and atomic processes, in a quivering plasma, the temporal evolution of the unperturbed fields is taken to be much slower than the species collisional rate ν. In addition, ν is assumed small compared to the gyrofrequency. Therefore the orderings

$$\frac{\rho}{\ell} \sim \frac{\nu}{\Omega} \gg \frac{1}{\Omega}\frac{\partial}{\partial t}\Big|_{slow} \sim \left(\frac{\rho}{\ell}\right)^2 \tag{6}$$

are employed.

In the remainder of this section the preceding orderings are employed to derive a reduced kinetic equation from the full kinetic equation

$$\dot{f} = C, \tag{7}$$

where a dot is used to denote the unperturbed Vlasov operator

$$\dot{f} = \frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla} f + (\vec{A} + \vec{a})\cdot\vec{\nabla}_v f, \tag{8}$$

the unperturbed and rf portions of the acceleration are defined as

$$\vec{A} \equiv \frac{Ze}{M}(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}) \tag{9}$$

and

$$\vec{a} \equiv \frac{Ze}{M}(\vec{e} + \frac{1}{c}\vec{v}\times\vec{b}) \tag{10}$$

respectively, and the operator C is used to denote atomic and Coulomb collision processes, as well as any sources and sinks.

It is convenient to consider the distribution function f as a function of the velocity $\vec{w}$ in the quivering frame, where

$$\vec{w} \equiv \vec{v} - \vec{u}(\vec{r},t), \tag{11}$$

such that

$$f \equiv f(\vec{r},\vec{w},t). \tag{12}$$

Using the preceding two equations to transform to the quivering frame and recalling the definition of $\vec{u}$ from Eq. (1) gives to lowest order that

$$\begin{aligned}\dot{f} &= \frac{\partial f}{\partial t}\Big|_w - \frac{\partial \vec{u}}{\partial t}\cdot\vec{\nabla}_w f + \frac{Ze}{M}(\vec{e} + \frac{1}{c}\vec{v}\times\vec{B})\cdot\vec{\nabla}_w f + \ldots \\ &= \frac{\partial f}{\partial t}\Big|_w + \frac{Ze}{Mc}\vec{w}\times\vec{B}\cdot\vec{\nabla}_w f + \ldots\end{aligned} \tag{13}$$

since $\vec{\nabla}_v = \vec{\nabla}_w$, $\partial/\partial t|_v \equiv \partial/\partial t = \partial/\partial t|_w - (\partial\vec{u}/\partial t)\cdot\vec{\nabla}_w$, and $\vec{v}\cdot\vec{\nabla}$ is next order in ρ/ℓ.

Equation (13) suggests splitting $\vec{w}$ into gyromotion and streaming along $\vec{B}$ and introducing the velocity variables E and μ such that

$$\vec{w} \equiv \vec{w}_\perp + w_\parallel\vec{n} \equiv w_\perp(\vec{e}_1\cos\phi + \vec{e}_2\sin\phi) + w_\parallel\vec{n},$$

$$E \equiv \frac{1}{2}w^2 + \frac{Ze\Phi + \Psi}{M} \equiv \mu B + \frac{1}{2}w_\parallel^2 + \frac{Ze\Phi+\Psi}{M}, \tag{14}$$

and $\mu \equiv |\vec{n}\times\vec{w}|^2/2B \equiv w_\perp^2/2B$, with $\vec{e}_1$, $\vec{e}_2 \equiv \vec{n}\times\vec{e}_1$, and $\vec{n} \equiv \vec{B}/B$ an orthogonal set of unit vectors and Ψ a velocity independent potential to be defined shortly containing only slow time dependence. Using the energy E, the magnetic moment μ, and the gyrophase ϕ as the velocity variables for $\vec{w}$, Eq. (13) becomes

$$\dot{f} = \frac{\partial f}{\partial t}\Big|_{E,\mu,\phi} - \Omega\frac{\partial f}{\partial \phi} + \dots \quad . \tag{15}$$

To satisfy the lowest order kinetic equation $\partial f/\partial t|_{\vec{w}} - \Omega\partial f/\partial\phi = 0$, Eq. (15) suggests writing $f = f(\vec{r},\vec{w},t) = f(\vec{r},E,\mu,\phi,t)$ as

$$f \equiv \bar{f}(\vec{r},E,\mu,t) + \tilde{f}(\vec{r},E,\mu,\phi,t) \tag{16}$$

with $\tilde{f}$ containing either the gyrophase or fast time dependence or both,

$$\tilde{f}/\bar{f} \sim \rho/\ell, \quad \frac{\partial \bar{f}}{\partial\phi} = 0, \quad \frac{1}{\Omega}\frac{\partial \bar{f}}{\partial t}\Big|_{E,\mu} \sim \frac{\rho^2}{\ell^2}\bar{f}, \tag{17}$$

and the ordering on the time dependence of $\bar{f}$ following from (6). Then to next order in the $\rho/\ell \sim \xi/\ell \sim \nu/\Omega$ expansion, Eq. (7) becomes

$$\partial\tilde{f}/\partial t - \Omega\partial\tilde{f}/\partial\phi + \dot{\bar{f}} = C \tag{18}$$

where in the operator C only $\bar{f}$ enters. To the requisite order in the quivering frame

$$\begin{aligned}\dot{\bar{f}} = {} & (\vec{w} + \vec{u})\cdot\vec{\nabla}\bar{f} + (\vec{w} + \vec{u})\cdot[M^{-1}\vec{\nabla}(Ze\Phi + \Psi) - \vec{\nabla}\vec{u}\cdot\vec{w}]\partial\bar{f}/\partial E \\ & -(\vec{w} + \vec{u})\cdot[\mu\vec{\nabla}\ln B + (w_\|/B)\vec{\nabla}\vec{n}\cdot\vec{w}_\perp + B^{-1}\vec{\nabla}\vec{u}\cdot\vec{w}_\perp]\partial\bar{f}/\partial\mu \\ & + (Ze/M)[\vec{E} + c^{-1}(\vec{w} + \vec{u})\times\vec{b}]\cdot[\vec{w}\partial\bar{f}/\partial E + (\vec{w}_\perp/B)\partial\bar{f}/\partial\mu] + \dots\end{aligned} \tag{19}$$

since $\vec{\nabla}_v E = \vec{w}$, $\vec{\nabla}_v \mu = B^{-1}\vec{w}_\perp$, $\vec{\nabla}_v \phi = w_\perp^{-2}\vec{n}\times\vec{w}$, and

$$\vec{\nabla}E = -\vec{\nabla}\vec{u}\cdot\vec{w} + M^{-1}\vec{\nabla}(Ze\Phi + \Psi)\,,$$

$$\vec{\nabla}\mu = -(\mu/B)\vec{\nabla}B - (w_\|/B)\vec{\nabla}\vec{n}\cdot\vec{w} - B^{-1}\vec{\nabla}\vec{u}\cdot\vec{w}_\perp\,,$$

$$\vec{\nabla}\phi = \vec{\nabla}\vec{e}_2\cdot\vec{e}_1 - (w_\|/w_\perp^2)\vec{\nabla}\vec{n}\cdot\vec{n}\times\vec{w}\,,$$

where the gradients of E, μ, and ϕ are performed holding $\vec{v}$ fixed and $\vec{\nabla}\bar{f}$ in (19) is carried out at fixed $\vec{w}$.

To obtain the reduced kinetic equation from Eq. (18) for applied rf of period τ, it must be gyroaveraged

$$\langle\cdots\rangle_\phi \equiv (2\pi)^{-1}\oint d\phi(\cdots)\,, \tag{20}$$

and fast time averaged

$$\langle\cdots\rangle_t \equiv \tau^{-1}\int_{t-\frac{1}{2}\tau}^{t+\frac{1}{2}\tau} dt\,(\cdots)\,. \tag{21}$$

Both averages are carried out in the quivering frame so that $\vec{r},E,\mu,\phi$ or $\vec{r},\vec{w}$ are held fixed during fast time averaging and the gyroaverages are performed holding $\vec{r},E,\mu,t$ or $\vec{r},w_\perp,w_\|,t$ fixed. Notice in particular that $\langle\vec{w}_\perp\rangle_\phi = 0 = \langle\vec{u}\rangle_t = \langle\vec{e}\rangle_t = \langle\vec{b}\rangle_t$ while $\langle\tilde{f}\rangle_\phi$ and $\langle\tilde{f}\rangle_t$ need not vanish but $\langle\tilde{f}\rangle_{\phi,t} \equiv 0$. Carrying out the gyro and fast time average of Eq. (19) as shown in Appendix A yields

$$\begin{aligned}\langle\dot{\bar{f}}\rangle_{\phi,t} = {} & w_\|\vec{n}\cdot\vec{\nabla}\bar{f} + w_\|\vec{n}\cdot[M^{-1}\vec{\nabla}\Psi - \langle\vec{u}\cdot\vec{\nabla}\vec{u}\rangle_t \\ & + (Ze/Mc)\langle\vec{u}\times\vec{b}\rangle_t]\partial\bar{f}/\partial E\,,\end{aligned} \tag{22}$$

where $\vec{\nabla}\bar{f}$ is performed at fixed E, μ, and t and $\langle\dot{\mu}\rangle_{\phi,t} = 0$.

To simplify Eq. (22) further, define the displacement $\vec{\xi} = \vec{\xi}(\vec{r},t)$ as the periodic solution of

$$\partial\vec{\xi}/\partial t = \vec{u} \tag{23}$$

having $\langle\vec{\xi}\rangle_t = 0$, and integrate Faraday's law

$$\vec{\nabla}\times\vec{e} = -\frac{1}{c}\frac{\partial\vec{b}}{\partial t} \tag{24}$$

and Eq. (1) over fast time to obtain

$$\vec{b} = -c\vec{\nabla}\times(\int^{t} dt\,\vec{e}) \tag{25}$$

and

$$\vec{u} - (Ze/M)\int^{t} dt\,\vec{e} = \Omega\vec{\xi}\times\vec{n}\,, \tag{26}$$

where $\int^{t} dt\,\vec{e}$ is made periodic by the choice of the lower limit and $\langle\int^{t} dt\,\vec{e}\rangle_t = 0$. Then expanding the triple cross products that follow both ways and taking half the sum gives

$$\vec{n}\cdot\langle\vec{u}\times[(Ze/Mc)\vec{b} + \vec{\nabla}\times\vec{u}]\rangle_t$$

$$= \vec{n}\cdot\langle\vec{u}\times[\vec{\nabla}\times(\Omega\vec{\xi}\times\vec{n})]\rangle_t$$

$$= \tfrac{1}{2}\vec{n}\cdot\langle[\vec{\nabla}(\Omega\vec{\xi}\times\vec{n})\cdot\vec{u} - \vec{u}\cdot\vec{\nabla}(\Omega\vec{\xi}\times\vec{n})] + \vec{u}\times[\Omega\vec{n}\cdot\vec{\nabla}\vec{\xi} - \vec{\nabla}\cdot(\vec{\xi}\vec{n}\Omega)]\rangle_t$$

$$= \tfrac{1}{2}\vec{n}\cdot\vec{\nabla}\langle\Omega\vec{\xi}\times\vec{n}\cdot\vec{u}\rangle_t\,, \tag{27}$$

since $\langle\vec{u}\vec{\xi}\rangle_t + \langle\vec{\xi}\vec{u}\rangle_t = 0 = \langle(\vec{\nabla}\vec{u})\vec{\xi}\rangle_t + \langle(\vec{\nabla}\vec{\xi})\vec{u}\rangle_t$. Using (27), defining Ψ as the ponderomotive potential[17,18,20,21]

$$\Psi \equiv \tfrac{1}{2}M\langle\vec{u}\cdot(\vec{u} - \Omega\vec{\xi}\times\vec{n})\rangle_t = -\tfrac{1}{2}Ze\langle\vec{\xi}\cdot\vec{e}\rangle_t \tag{28}$$

and using $\vec{u}\times(\vec{\nabla}\times\vec{u}) = \tfrac{1}{2}\vec{\nabla}u^2 - \vec{u}\cdot\vec{\nabla}\vec{u}$ gives

$$\vec{n}\cdot\vec{\nabla}\Psi = M\vec{n}\cdot[\langle\vec{u}\cdot\vec{\nabla}\vec{u}\rangle_t - (Ze/Mc)\langle\vec{u}\times\vec{b}\rangle_t]\,. \tag{29}$$

Thus Eq. (22) reduces to the simple result that

$$\langle\dot{\bar{f}}\rangle_{\phi,t} = w_{\parallel}\,\vec{n}\cdot\vec{\nabla}\,\bar{f}\,, \tag{30}$$

since $\langle\dot{E}\rangle_{\phi,t} = 0$.

The reduced kinetic equation obtained by gyro and time averaging Eq. (18) and employing Eq. (30) is the slow time, quiver kinetic equation:

$$w_{\parallel}\vec{n}\cdot\vec{\nabla}\,\bar{f} = \bar{C}\,, \tag{31}$$

where the overbar on the operator C denotes that it is a gyro and fast time average of the operator C acting on $\bar{f}$.

Only rf modifications large compared to classical and neoclassical effects are retained in Eq. (31) because of the ordering

$$u^2/v_j^2 \gg \rho/\ell\,. \tag{32}$$

The classical and neoclassical corrections that enter via $\tilde{f}$ and the order ρ/ℓ correction in $\bar{f}$, respectively, may be neglected as small even when $1 \gg u^2/v_j^2$ as long as inequality (32) pertains.

By employing the appropriate moments of the full kinetic equation (7), a moment approach is developed in the next section in such a way that only the quiver kinetic equation (31) need be solved to evaluate the time average particle flux across the magnetic field. This feature is similar to that of conventional neoclassical transport formulations[14,15] except that the particle flux is induced by the rf and can be convective in nature.

Before developing the moment approach some mention must be made of the properties of $\overline{C}$ since it should not be thought of as just the Fokker-Planck collision operator. Consider a toroidal magnetic field (which may be non-axisymmetric) and label the magnetic surfaces by the poloidal flux $2\pi\psi$ as indicated in Appendix B. Then, using $\langle\cdots\rangle_\psi$ to denote the flux surface averaging operation defined by (B5), Eq. (31) is multiplied by $B/w_\parallel$ and flux surface averaged, using property (B6), to obtain the solubility constraint

$$\langle \frac{B}{w_\parallel} \overline{C} \rangle_\psi = 0 \ . \qquad (33)$$

Multiplying (33) by 1 and ME, and integrating the results over all E, μ, and ϕ gives

$$\langle \int d^3w \ \overline{C} \rangle_\psi = 0 \qquad (34)$$

and

$$\langle M \int d^3w \ E\overline{C} \rangle_\psi = 0 \ . \qquad (35)$$

Equation (34) simply requires that the particle sources and sinks balance in $\overline{C}$ such that it conserves number on a flux surface to lowest order.

Equation (35) is more interesting because a flux surface averaged energy conservation property is not satisfied by a Fokker-Planck collision operator in the presence of sufficiently intense rf. In particular, collisional heating of electrons occurs in their quiver frame because their quiver motion is randomized by pitch angle scattering

collisions with the ions (inverse bremsstrahlung).[19,24,25] As a result, an electron energy loss mechanism is required to prevent a secular heating which would violate (6). Since equilibration with the ions is too slow for sufficiently intense rf, the most likely energy loss mechanism at the edge is electron excitation of neutrals and charged impurities, which then radiate the energy away almost instantaneously.[26]

III. Moments and Ponderomotive Force

The orderings outlined at the start of Sec. II permitted the quiver kinetic equation (31) to be derived by an expansion of the full kinetic equation (7). However, both the time averaged particle flux across the magnetic field,

$$\vec{\Gamma}_\perp \equiv \langle \int d^3v\, f\vec{n}\times(\vec{v}\times\vec{n})\rangle_t = \int d^3w\, \langle \tilde{f}\, \vec{n}\times[(\vec{u}+\vec{w})\times\vec{n}]\rangle_t\,, \quad (36)$$

and the net radial, time averaged particle flux out of a flux surface (labeled by the poloidal flux $2\pi\psi$),

$$\Gamma \equiv \langle \int d^3v\, f\vec{v}\cdot\vec{\nabla}\psi\rangle_{t,\psi} = \langle \int d^3w\, \langle \tilde{f}\,(\vec{u}+\vec{w})\rangle_t\cdot\vec{\nabla}\psi\rangle_\psi\,, \quad (37)$$

require some knowledge of $\tilde{f}$, the portion of the distribution function that is rapidly varying in time and/or gyrophase. The flux surface average $\langle\cdots\rangle_\psi$ is defined in Appendix B for a general, non–axisymmetric toroidal magnetic field. The condition for well defined edge flux surfaces in the presence of strong rf is discussed by Myra.[28]

The equation for $\tilde{f}$ can be obtained by subtracting Eq. (31) from(18). However, solving for $\tilde{f}$ is a formidable task for a general magnetic field when $C - \overline{C} \sim (u/v_j)\nu$ must be retained. Fortunately it is also unnecessary since a moment approach allows $\vec{\Gamma}_\perp$ and Γ to be evaluated by finding alternate expressions for the desired moments of $\tilde{f}$ in terms of moments of $\overline{f}$, the solution of Eq. (31).

In carrying out the moment approach in the presence of an applied rf field, ponderomotive effects will be encountered once

again. It is this feature that complicates the procedure and thereby distinguishes the moment description that follows from the usual strong B moment approaches.[23]

Moments of the full kinetic equation are most conveniently formed by multiplying (7) by any quantity $X \equiv X(\vec{r},\vec{w},t)$, integrating over all $\vec{w}$, and time averaging over the fast time (recall $d^3v = d^3w$) to obtain

$$\frac{\partial}{\partial t}\left(\int d^3w\, \langle Xf\rangle_t\right) + \vec{\nabla}\cdot\left(\int d^3w\, \langle \vec{v}Xf\rangle_t\right) = \int d^3w\, \langle XC + f\dot{X}\,\rangle_t\,. \quad (38)$$

In the preceding expressions $\dot{X}$ is the Vlasov operator defined by Eq. (8) acting on X and the fast time averages in Eq. (38) are to be performed holding $\vec{w}$ fixed as in Eq. (21).

Using X equal to 1 and ME in Eq. (38) gives to lowest significant order the corresponding moments of the quiver kinetic equation (31), namely,

$$\vec{\nabla}\cdot\left(\vec{n}\int d^3w\, w_\parallel \overline{f}\,\right) = \int d^3w\, \overline{C}\,, \quad (39)$$

$$\vec{\nabla}\cdot\left(\vec{n}\int d^3w\, MEw_\parallel \overline{f}\,\right) = \int d^3w\, ME\overline{C}\,, \quad (40)$$

where $\overline{C}$ again denotes the gyro and fast time average of C operating on the solution to (31).

The leading odd $\vec{w}$ moment is obtained for X equal to $M\vec{w}$, for which (38) yields

$$\vec{\nabla}\cdot[(\overline{I} - \vec{n}\vec{n})p_\perp + \vec{n}\vec{n}p_\parallel] = \int d^3w\, M\vec{w}\langle C\rangle_t + N[(Ze/c)\langle\vec{u}\times\vec{b}\rangle_t - M\langle\vec{u}\cdot\vec{\nabla}\vec{u}\rangle_t - Ze\vec{\nabla}\Phi] + M\Omega\int d^3w\, \langle\tilde{f}\rangle_t\, \vec{w}\times\vec{n}, \quad (41)$$

with $\overleftrightarrow{I}$ the unit dyadic, $\langle C\rangle_t$ the time average of C operating on $\bar{f}$,

$$N \equiv \int d^3w\, \bar{f}, \quad p_\perp \equiv \int d^3w\, \tfrac{1}{2}Mw_\perp^2 \bar{f}, \quad \text{and} \quad p_\parallel \equiv \int d^3w\, Mw_\parallel^2\, \bar{f} . \quad (42)$$

To obtain Eq. (41), $\dot{w} = \Omega\vec{w}\times\vec{n} - (Ze/M)\vec{\nabla}\Phi + (Ze/Mc)(\vec{w} + \vec{u})\times\vec{b} - (\vec{w} + \vec{u})\cdot\vec{\nabla}\vec{u}$ and its gyro and fast time average $M\langle\dot{\vec{w}}\rangle_{\phi,t} = (Ze/c)\langle\vec{u}\times\vec{b}\rangle_t - M\langle\vec{u}\cdot\vec{\nabla}\vec{u}\rangle_t - Ze\vec{\nabla}\Phi$ are employed.

Next, note that the ponderomotive force[17,18,20,21] density $\vec{F}$, can be written in the following two forms:

$$\vec{F} \equiv N[(Ze/c)\langle\vec{u}\times\vec{b}\rangle_t - M\langle\vec{u}\cdot\vec{\nabla}\vec{u}\rangle_t] + M\Omega\langle\vec{u}\times\vec{n}\vec{\nabla}\cdot(N\vec{\xi})\rangle_t$$

$$\equiv -N\vec{\nabla}\Psi + \vec{B}\times(\vec{\nabla}\times\vec{M}) \quad (43)$$

with the ponderomotive potential Ψ defined by Eq. (28) and $\vec{M}$ the species magnetization defined as

$$\vec{M} \equiv (Ze/2c)N\langle\vec{\xi}\times\vec{u}\rangle_t \ . \quad (44)$$

The proof that the two forms of $\vec{F}$ are equivalent requires the identity proven in Appendix C.

To obtain the desired form of Eq. (41), (43) is inserted to find

$$M\Omega\langle\vec{\xi}\times\vec{n}\vec{\nabla}\cdot(N\vec{u})\rangle_t = ZeN\vec{\nabla}\Phi - \vec{F} - M\int d^3w\, \vec{w}\langle C\rangle_t$$

$$+ \vec{\nabla}\cdot[(\overleftrightarrow{I} - \vec{n}\vec{n})p_\perp + \vec{n}\vec{n}p_\parallel] - M\Omega\int d^3w\, \langle\tilde{f}\rangle_t\vec{w}\times\vec{n} \ . \quad (45)$$

Next, the fast time continuity equation is formed by subtracting the time averaged form from the unaveraged version to obtain

$$\frac{\partial}{\partial t}\left(\int d^3w\, \tilde{f}\right) + \vec{\nabla}\cdot(N\tilde{\vec{u}}) = \int d^3w\, (C - \bar{C}) \ , \quad (46)$$

where the right side vanishes unless C operating on $\bar{f}$ contains fast time variation. Using (46) to eliminate $\vec{\nabla}\cdot(N\tilde{\vec{u}})$ from (45) yields the desired result

$$(Ze/c)\int d^3w\, \langle\tilde{f}(\vec{u} + \vec{w})\rangle_t\times\vec{B} = ZeN\vec{\nabla}\Phi - \vec{F} - \int d^3w\, M\vec{w}\langle C\rangle_t$$

$$+ \vec{\nabla}\cdot[(\overleftrightarrow{I} - \vec{n}\vec{n})p_\perp + \vec{n}\vec{n}\,p_\parallel]$$

$$+ (Ze/c)\vec{B}\times\langle\vec{\xi}\int d^3w\, C\rangle_t . \quad (47)$$

Equation (47) can be used to form $\vec{\Gamma}_\perp$ by crossing it with $\vec{n}$ and dividing by ZeB/c to obtain

$$\vec{\Gamma}_\perp = \frac{cN}{ZeB}\vec{n}\times\vec{\nabla}(Ze\Phi + \Psi) + \frac{c}{Ze}(\overleftrightarrow{I} - \vec{n}\vec{n})\cdot\vec{\nabla}\times\vec{M} - \frac{1}{\Omega}\int d^3w\, \vec{n}\times\vec{w}\langle C\rangle_t$$

$$- (\overleftrightarrow{I} - \vec{n}\vec{n})\cdot\langle\vec{\xi}\int d^3w\, C\rangle_t + \frac{c}{ZeB}\vec{n}\times\vec{\nabla}p_\perp + \frac{c(p_\parallel - p_\perp)}{ZeB}\, \vec{n}\times(\vec{n}\cdot\vec{\nabla}\vec{n})$$

$$= \frac{c}{ZeB}\vec{n}\times[N\vec{\nabla}(Ze\Phi + \Psi) + p_\perp\vec{\nabla}\ell nB + p_\parallel\vec{n}\cdot\vec{\nabla}\vec{n}]$$

$$+ \frac{c}{Ze}(\overleftrightarrow{I} - \vec{n}\vec{n})\cdot\vec{\nabla}\times(\vec{M} - \frac{p_\perp}{B}\vec{n}) - \frac{1}{\Omega}\int d^3w\, \vec{n}\times\vec{w}\langle C\rangle_t$$

$$- (\overleftrightarrow{I} - \vec{n}\vec{n})\cdot\langle\vec{\xi}\int d^3w\, C\rangle_t \ , \quad (48)$$

where the second form shows that the normal diamagnetism of the plasma is modified by the rf induced species magnetization $\vec{M}$. Equation (48), unlike Eq. (36), allows $\vec{\Gamma}_\perp$ to be evaluated without knowing $\tilde{f}$.

The first form of Eq. (48) is also convenient for forming Γ. Dotting it by $\vec{\nabla}\psi$, using $\vec{n}\cdot\vec{\nabla}\psi = 0$, flux surface averaging, and using property (B6) to find $\langle\vec{\nabla}\psi\cdot\vec{\nabla}\times\vec{M}\rangle_\psi = 0$ gives

$$\Gamma = \langle \frac{c}{ZeB}\vec{n}\times[N\vec{\nabla}(Ze\Phi + \Psi) + \vec{\nabla}p_\perp + (p_\parallel - p_\perp)\vec{n}\cdot\vec{\nabla}\vec{n}]\cdot\vec{\nabla}\psi\rangle_\psi$$

$$- \langle \int d^3w\, (\vec{n}\times\frac{1}{\Omega}\vec{w}\langle C\rangle_t + \langle\vec{\xi}C\rangle_t)\cdot\vec{\nabla}\psi\rangle_\psi \quad . \qquad (49)$$

Equation (49) shows that the flux through a flux surface is driven by the usual magnetic and electric flows plus a ponderomotive flow which is the only remnant of the $\vec{B}\times\vec{F}$ velocity.

Often C is at least locally number conserving to lowest order in ρ/ℓ so that $\int d^3w\,(C - \overline{C}) = 0$ in (46) and the right side of (39) vanishes (giving $B^{-1}\int d^3w\, w_\parallel \overline{f}$ to be a lowest order flux function). The results of this section do not employ these simplifications. Only the general properties of local total, charge, mass, momentum, and energy conservation, $\sum \int d^3v\,(Ze, M, M\vec{v}, \frac{1}{2}Mv^2)C = 0$, are used, where $\sum$ denotes a sum over all species.

If desired, the local slow time evolution of N could be found by first determining $\Gamma_\parallel \equiv \int d^3w\,(w_\parallel \overline{f} + \vec{n}\cdot\langle\vec{u}\tilde{f}\rangle_t)$ from the solution of (31) and by using (46) to find

$$\langle\vec{n}\cdot\vec{u}\int d^3w\,\tilde{f}\rangle_t = \langle\vec{n}\cdot\vec{\xi}[\vec{\nabla}\cdot(N\vec{u}) - \int d^3w\, C]\rangle_t \quad .$$

Therefore, to determine $\Gamma_\parallel$ to the same order as $\vec{\Gamma}_\perp$ requires $\langle\vec{n}\cdot\vec{\xi}\vec{\nabla}\cdot(N\vec{u})\rangle_t$ be known as well as Ψ and $\vec{M}$ for each species. Letting $\vec{\Gamma} = \vec{\Gamma}_\perp + \Gamma_\parallel\vec{n}$ and using (38) to form the time averaged continuity equation to one order higher gives the equation for $\partial N/\partial t$ to be

$$\frac{\partial N}{\partial t} + \vec{\nabla}\cdot\vec{\Gamma} = \int d^3w\,\langle C\rangle_t \equiv S \quad . \qquad (50)$$

Subtracting (39) from (50) shows that the right side of (50) must be retained to one order higher since $\int d^3w\,(\langle C\rangle_t - \overline{C}) \sim \nu\rho/\ell$.

Flux surface averaging (50) and using (B6) yields

$$\frac{\partial\langle N\rangle_\psi}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial\psi}(V'\Gamma) = \langle S\rangle_\psi \quad , \qquad (51)$$

where $V' = \langle 1\rangle_\psi$ as defined in Appendix B. Equation (51) is of the standard form for a transport description and does not require $\Gamma_\parallel$. The radial flux can contain both convective and diffusive contributions.

When Eqs. (50) and (51) are multiplied by Ze and summed over all species, the conservation of charge equations are obtained,

$$\frac{\partial}{\partial t}(\sum ZeN) + \vec{\nabla}\cdot\vec{J} = 0 \qquad (52)$$

and

$$\frac{\partial}{\partial t}\langle\sum ZeN\rangle_\psi + \frac{1}{V'}\frac{\partial}{\partial\psi}(V'\sum Ze\Gamma) = 0 \quad , \qquad (53)$$

since $\sum ZeS = 0$. In principle, Eq. (52) can be solved for the charge density build-up $\partial(\sum ZeN)/\partial t$ because $\vec{J} = \vec{J}_\perp + J_\parallel\vec{n}$ can be formed from $J_\parallel \equiv \sum Ze\Gamma_\parallel$ and $\vec{J}_\perp \equiv \sum Ze\vec{\Gamma}_\perp$. Quasi-neutrality, $\sum ZeN = 0$, simplifies Eqs. (52) and (53) to the ambipolarity equations $\vec{\nabla}\cdot\vec{J} = 0 = \sum Ze\Gamma$ and removes the $\vec{E}\times\vec{B}$ drifts from $\vec{J}_\perp$.

Parallel momentum conservation for each species can be obtained from the parallel component of (47)

$$\vec{n}\cdot\vec{\nabla}p_\perp + \vec{B}\cdot\vec{\nabla}(\frac{p_\parallel - p_\perp}{B}) + N\vec{n}\cdot\vec{\nabla}(Ze\Phi + \Psi) = M\int d^3w\, w_\parallel\overline{C} \quad , (54)$$

since $\int d^3w\ w_\parallel \langle C \rangle_t = \int d^3w\ w_\parallel \overline{C}$ to the requisite order. Equation (54) can also be obtained by multiplying Eq. (31) by $w_\parallel$ and integrating over all velocity space. Summing (54) over all species and using collisional momentum conservation gives the parallel momentum conservation equation.

The preceding set of moment equations can be employed when only perpendicular particle fluxes need be evaluated. However, higher order moment equations advancing $p_\perp$ and $p_\parallel$ in time need to be derived to determine if a closed system of equations exists. Unfortunately, using X equal to μ and $w_\parallel^2$ or E) in Eq. (38) results in higher moments involving $\tilde{f}$ which must then be eliminated by employing the fast time moment equations. Such complications are beyond the scope of the present seminal treatment and are likely to result in non-standard (and perhaps not useful) forms of the transport equations[29] for $\vec{u} \sim v_j$.

The quiver kinetic description of this section and the last is most easily derived using the $\vec{u} \sim v_j$ ordering rather than assuming $(u/v_j)^2 \ll 1$ and iterating. However, the results obtained remain valid and are more tractable when $1 \gg (u/v_j)^2 \gg \rho/\ell$ so that the rf induced flows are larger than their classical and neoclassical counterparts. Since $\Psi/Ze\Phi \sim (u/v_j)^2 \sim B\vec{M}/p_\perp$, ponderomotive effects will then enter as small perturbations in the moment equations. Consequently, it is best to use the moment equations and moments of Eq. (31) to obtain as much information as possible. Typically other expansions (i.e. collisionality) are required to obtained tractable results in complicated magnetic fields. Simple models can be solved

to illustrate a collision dominated and weak collisionality applications of the results of Secs. II and III in a tokamak.

IV. Discussion

In the preceding sections a completely new description of the interaction of applied rf fields with the edge plasma is developed by assuming the quiver motion dominates over drift and gyro motion. The nonlinear formulation of Secs. II and III retains Coulomb collisions and the relevant atomic processes and is valid for general magnetic field geometries, although tokamak applications are of particular interest.

The principal results of Sec. II are the quiver kinetic equation (31) and the continuity and energy balance equations (34) and (35) which follow from it. The most important results in Sec. III are the expressions for the perpendicular particle flux (48), the net radial particle flux through a flux surface (49), and the perpendicular current density $\vec{J}_\perp = \sum Ze\vec{\Gamma}_\perp$, all of which follow from perpendicular momentum conservation and the general proof in Appendix C for the form of the ponderomotive force (43). The quiver-kinetic description of Sec. II and III is capable of retaining induced electrostatic potentials, convective and/or diffusive fluxes, collisional heating, and atomic processes in an rf heated edge plasma. The formalism differs from other approaches in several ways, but the two most notable differences are the following: (i) an ordering is adopted which makes rf effects dominate over, rather than compete with, neoclassical (and classical) effects; and (ii) the model is collisional and by necessity retains atomic processes as well as Coulomb scattering in a magnetized plasma. Because of (i) the

transport need not be intrinsically ambipolar as can be seen from (49).

A moment description is adopted in Sec. III to obtain the expressions for the particle fluxes and currents. The formalism is developed in such a way that only the lowest order quiver kinetic equation (31) need be solved to evaluate the particle fluxes and current. This feature is similar to that of conventional neoclassical transport formulations except that the fluxes are driven by the rf inducing density, temperature, and electrostatic potential gradients.

The applied rf generates gradients by collisionally heating the electrons. The heating occurs because the organized quiver motion of the electrons is randomized by electron-ion collisions. To prevent secular electron temperature increases with time due to this inverse bremsstrahlung[19,24,25] process, a large energy loss mechanism must be retained for the electrons. Equilibration with the ions is too slow for the intense rf fields of interest. Consequently, the likely loss mechanisms are ionization and line radiation caused by electron excitation of neutrals and impurity ions, which then radiate almost instantaneously.[22,26] Ionization is a mechanism that can remove excess energy, but since it is not number conserving it does not satisfy the flux surface average constraint (34). Therefore, line radiation appears to be a necessary feature of the electron model.

The solution of the quiver kinetic equation for several species, arbitrary collisionality, and with various atomic processes retained is a formidable undertaking. To gain insight into the relevant physical processes simple, analytically tractable models must be investigated. Once such models are understood more complicated ones can be used to make quantitative predictions.

At present the edge plasma is assumed to contain only electrons, a single cold ion species, and neutrals. The neutrals are needed to remove energy from the plasma via line radiation. However, Coulomb collisions are assumed to be the dominant collisional effect and electron impact excitation is the only other collisional process retained. The applied rf field is assumed to be of moderate strength so that $1 \gg (u_e/v_e)^2 \gg \rho_e/\ell_a$ and the ions enter only to maintain quasi-neutrality.

Within the preceding framework two limits of collisionality are amenable to an analytic treatment: (i) very short mean free paths, $\lambda \ll \ell_a$; and (ii) very long mean free paths, $\lambda \gg qR \gg \ell_a$, where R is the major radius and q the safety factor. In both limits the lowest order electron distribution is Maxwellian to lowest order. However, for $\lambda \ll \ell_a$ it is a local Maxwellian, while for $\lambda \gg qR$ the spatial dependence of the Maxwellian is that of a flux function. As a result, in the collisional limit inverse bremsstrahlung heats the electrons locally (that is, in the vicinity of the antenna), while in the collisionless limit the collisional heating is spread over the entire flux surface. In both cases the energy balance is maintained by the electrons giving up energy to the neutrals by electron impact excitation, and the neutrals then removing the energy from the system instantaneously by line radiation.

The fluxes generated in the two limits of collisionality have quite different features even though they are <u>convective</u> in both cases. In the collisional limit the perpendicular electron flux has a diamagnetic component in addition to the $\vec{E}\times\vec{B}$ contribution it has in common with the cold ions. The electrostatic potential adjusts to keep the parallel current force free, thereby causing the perpendicular potential and temperature variations to be the same order. As a result, the radial particle flux is locally the same order as that of Bohm diffusion. Moreover, a net radial flux is possible if up-down symmetry is broken (for example, by a divertor).

In the long mean free limit, the flux function character of the Maxwellian causes the radial flux to depend on the <u>local</u> rf induced magnetization (rather than the $\vec{E}\times\vec{B}$, diamagnetic, and ponderomotive fluxes) and a <u>nonlocal</u> collisionally constrained, rf induced contribution. Because of its nonlocal character, this second piece is small by roughly the ratio of the antenna area over the surface area of the tokamak. Furthermore, the rf magnetization flux is somewhat smaller than the collisional fluxes <u>and</u> neither collisionless flux leads to a <u>net</u> radial particle flux. As a result long mean free path operation appears to be preferable, in which case the edge heating is spread over the entire flux surface.

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Appendix A: Gyro and Fast Time Averaging Procedure

To evaluate the gyro and fast time average of $\dot{f}$ as given by Eq. (19), recall that $\bar{f}$ possesses only slow time variations. Then making use of $\langle\vec{w}\rangle_\phi = w_\parallel \vec{n}$, $\langle\vec{u}\rangle_t = 0 = \langle\vec{b}\rangle_t$, $\langle\Phi\rangle_t = \Phi$, $\langle\Psi\rangle_t = \Psi$, $\langle\vec{w}\,\vec{w}\rangle_\phi = \frac{1}{2}w_\perp^2(\vec{I} - \vec{n}\vec{n}) + w_\parallel^2\,\vec{n}\vec{n}$, $\vec{\nabla}\cdot\vec{B} = 0 = \vec{n}\cdot\vec{\nabla}B + B\vec{\nabla}\cdot\vec{n}$, and $\vec{E} = -\vec{\nabla}\Phi$, the terms in Eq. (19) may be evaluated to find the following results:

$$\langle(\vec{w} + \vec{u})\cdot\vec{\nabla}\,\bar{f}\rangle_{\phi,t} = w_\parallel \vec{n}\cdot\vec{\nabla}\,\bar{f} \ ,$$

$$\langle(\vec{w} + \vec{u})\cdot[M^{-1}\vec{\nabla}(Ze\Phi + \Psi) - \vec{\nabla}\vec{u}\cdot\vec{w}]\partial\bar{f}/\partial E\rangle_{\phi,t}$$
$$= w_\parallel \vec{n}\cdot[M^{-1}\vec{\nabla}(Ze\Phi + \Psi) - \langle\vec{u}\cdot\vec{\nabla}\vec{u}\rangle_t]\partial\bar{f}/\partial E \ ,$$

$$\langle(\vec{w} + \vec{u})\cdot[\mu\vec{\nabla}\ell nB + (w_\parallel/B)\vec{\nabla}\vec{n}\cdot\vec{w}_\perp + B^{-1}\vec{\nabla}\vec{u}\cdot\vec{w}_\perp]\partial\bar{f}/\partial\mu\rangle_{\phi,t} = 0,$$

$$\langle(Ze/M)\vec{E}\cdot[\vec{w}\partial\bar{f}/\partial E + (\vec{w}_\perp/B)\partial\bar{f}/\partial\mu]\rangle_{\phi,t} = -(Ze/M)(w_\parallel \vec{n}\cdot\vec{\nabla}\vec{\Phi})\partial\bar{f}/\partial E \ ,$$

and

$$\langle(Ze/Mc)(\vec{w} + \vec{u})\times\vec{b}\cdot[\vec{w}\partial\bar{f}/\partial E + (\vec{w}_\perp/B)\partial\bar{f}/\partial\mu]\rangle_{\phi,t}$$
$$= (Ze/Mc)w_\parallel \vec{n}\cdot\langle\vec{u}\times\vec{b}\rangle_t\partial\bar{f}/\partial E \ ,$$

where $\vec{\nabla}\,\bar{f}$ is performed holding E, μ, and t fixed.

Appendix B: Flux Coordinates

The general representation of the magnetic field in a torus[30]

$$\vec{B} = \vec{\nabla}\psi \times \vec{\nabla}[\Theta - q(\psi)\chi] \qquad \text{(B1)}$$

is employed, with $2\pi\psi$ the poloidal flux, Θ a toroidal angle variable, χ a poloidal angle variable, and $q(\psi) = \vec{B}\cdot\vec{\nabla}\Theta/\vec{B}\cdot\vec{\nabla}\chi$. Because $\vec{B}\cdot\vec{\nabla}\psi = 0$, ψ is a flux surface label.

The incremental length $d\vec{r}$ in the ψ,χ,Θ coordinates may be written as

$$d\vec{r} = (\vec{B}\cdot\vec{\nabla}\chi)^{-1}[d\psi\vec{\nabla}\Theta\times\vec{\nabla}\chi + d\chi\vec{\nabla}\psi\times\vec{\nabla}\Theta + d\Theta\vec{\nabla}\chi\times\vec{\nabla}\psi] \qquad \text{(B2)}$$

so that $d\vec{r}\cdot\vec{\nabla}\psi = d\psi$, $d\vec{r}\cdot\vec{\nabla}\chi = d\chi$, $d\vec{r}\cdot\vec{\nabla}\Theta = d\Theta$, and $d^3r = d\psi d\chi d\Theta/\vec{B}\cdot\vec{\nabla}\chi$. Incremental areas on surfaces of constant ψ, χ, or Θ may then be formed by cross products. For example, the incremental area on a constant ψ surface is

$$d^2\vec{r}\big|_\psi = (\vec{B}\cdot\vec{\nabla}\chi)^{-1}\, d\chi d\Theta\vec{\nabla}\psi \ . \qquad \text{(B3)}$$

The incremental volume between two neighboring flux surfaces is $dV = V'd\psi$ where

$$V' \equiv \oint \frac{d\chi d\Theta}{\vec{B}\cdot\vec{\nabla}\chi} \qquad \text{(B4)}$$

with both angle integrals over a complete circuit. A flux surface average of any quantity Q may then be defined via

$$\langle Q\rangle_\psi \equiv \frac{1}{V'}\oint \frac{d\chi d\Theta Q}{\vec{B}\cdot\vec{\nabla}\chi} \ . \qquad \text{(B5)}$$

When the flux surface average as defined by (B5) operates on the divergence of any quantity, $\vec{\nabla}\cdot\vec{Q}$, it gives

$$\langle\vec{\nabla}\cdot\vec{Q}\rangle_\psi = \frac{1}{V'}\frac{\partial}{\partial\psi}(V'\langle\vec{Q}\cdot\vec{\nabla}\psi\rangle_\psi) \ , \qquad \text{(B6)}$$

as can be verified by considering the integral of $\vec{\nabla}\cdot\vec{Q}$ over the incremental volume between two neighboring flux surfaces.

Appendix C: Proof of Ponderomotive Force Identity

The proof that the two forms of (43) are identically equal is only slightly more complicated than that given in Sec. II which led to the choice of Ψ as the ponderomotive potential given by Eq. (28). It appears to be a minor extension of previous proofs[17,18,20,21] because it permits $\vec{u} \sim v_j$ and only requires fast quantities ($\vec{e}$, $\vec{b}$, $\vec{u}$, and $\vec{\xi}$) to be periodic with vanishing time averages on the fast time scale. The proof begins as before by using (23)–(26) to write

$$(Ze/Mc)\langle\vec{u}\times\vec{b}\rangle_t - \langle\vec{u}\cdot\vec{\nabla}\vec{u}\rangle_t = \langle\vec{u}\times[\vec{\nabla}\times(\Omega\vec{\xi}\times\vec{n})]\rangle_t - \tfrac{1}{2}\nabla\langle u^2\rangle_t \; . \qquad (C1)$$

Expanding cross products both ways and taking half the sum, and using $\vec{\nabla}\cdot\vec{B} = 0$ and $\langle\vec{u}\,\vec{\xi}\rangle_t + \langle\vec{\xi}\,\vec{u}\rangle_t = 0$ to remove $\vec{\nabla}\cdot(\Omega\vec{n})$ and $\vec{\nabla}(\Omega\vec{n})$ terms, leaves

$$\langle\vec{u}\times[\vec{\nabla}\times(\Omega\vec{\xi}\times\vec{n})]\rangle_t = \tfrac{1}{2}\vec{\nabla}\langle\Omega\vec{\xi}\times\vec{n}\cdot\vec{u}\rangle_t - \tfrac{1}{2}\Omega\langle\vec{\nabla}\vec{u}\cdot\vec{\xi}\times\vec{n} + \vec{n}\cdot\vec{\nabla}\vec{\xi}\times\vec{u}$$
$$+ \vec{u}\cdot\vec{\nabla}\vec{\xi}\times\vec{n} + \vec{u}\times\vec{n}\,\vec{\nabla}\cdot\vec{\xi}\rangle_t \; . \qquad (C2)$$

To simplify (C2) further, utilize an identity obtained by expanding the cross products in $\tfrac{1}{2}\langle\vec{n}\times[\vec{\nabla}\times(\vec{u}\times\vec{\xi})]\rangle_t$ both ways:

$$\tfrac{1}{2}\langle\vec{\nabla}(\vec{u}\times\vec{\xi})\cdot\vec{n} - \vec{n}\cdot\vec{\nabla}(\vec{u}\times\vec{\xi})\rangle_t = \tfrac{1}{2}\langle[\vec{\nabla}\cdot(\vec{u}\,\vec{\xi} - \vec{\xi}\,\vec{u})]\times\vec{n}\rangle_t$$
$$\langle\vec{\nabla}\vec{u}\cdot(\vec{\xi}\times\vec{n}) + \vec{n}\cdot\vec{\nabla}\vec{\xi}\times\vec{u}\rangle_t = \langle[\nabla\cdot(\vec{u}\,\vec{\xi})]\times\vec{n}\rangle_t \; , \qquad (C3)$$

where $\langle\vec{u}\,\vec{\xi} - \vec{\xi}\,\vec{u}\rangle_t = 2\langle\vec{u}\,\vec{\xi}\rangle_t$ and $\langle(\vec{\nabla}\vec{u})\vec{\xi} - (\vec{\nabla}\vec{\xi})\vec{u}\rangle_t = 2\,\langle(\vec{\nabla}\vec{u})\vec{\xi}\rangle_t = -2\langle(\vec{\nabla}\vec{\xi})\vec{u}\rangle_t$ are employed. Using (C3) along with $\langle\vec{u}\cdot\vec{\nabla}\vec{\xi}\times\vec{n}\rangle_t = -\langle\vec{\xi}\cdot\vec{\nabla}\vec{u}\times\vec{n}\rangle_t = -\langle\nabla\cdot(\vec{\xi}\,\vec{u})\times\vec{n}\rangle_t + \langle\vec{u}\times\vec{n}\,\vec{\nabla}\cdot\vec{\xi}\rangle_t$ in Eq. (C2) gives

$$\langle\vec{u}\times[\vec{\nabla}\times(\Omega\vec{\xi}\times\vec{n})]\rangle_t = \tfrac{1}{2}\vec{\nabla}\langle\Omega\vec{\xi}\times\vec{n}\cdot\vec{u}\rangle_t - \tfrac{1}{2}\Omega\langle[\nabla\cdot(\vec{u}\,\vec{\xi} - \vec{\xi}\,\vec{u})]\times\vec{n}$$
$$+ 2\,\vec{u}\times\vec{n}\,\vec{\nabla}\cdot\vec{\xi}\rangle_t$$
$$= \tfrac{1}{2}\vec{\nabla}\langle\Omega\vec{\xi}\times\vec{n}\cdot\vec{u}\rangle_t - \frac{\Omega}{2\eta}\langle\vec{n}\times[\vec{\nabla}\times(\eta\vec{u}\times\vec{\xi})]\rangle_t$$
$$- \frac{\Omega}{\eta}\langle\vec{u}\times\vec{n}\,\vec{\nabla}\cdot(\eta\vec{\xi})\rangle_t \qquad (C4)$$

where η is an arbitrary function of space and slow time. Combining (C1) and (C4) with η equal to N establishes the equivalence of the two forms of Eq. (43).

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