INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4-SMR 393/48

SPRING COLLEGE ON PLASMA PHYSICS

15 May - 9 June 1989

PARAMETRIC PROCESSES IN PLASMA (III)

K. Nishikawa

Hiroshima University
1-1-89 Higashisends-Machi
Naka-Ku
Hiroshima 730
Japan

MAIN BUILDING STRADA COSTIERA, 11 TEL. 22401 TELEFAX 224163 TELEX 460392 ADRIATICO GUEST HOUSE VIA GRIGNANO, 9 TEL. 22424 TELEFAX 224531 TELEX 460449
MICROPROCESSOR LAB. VIA BEIRUT, 31 TEL. 22447 TELEFAX 224163 TELEX 460392 GALILEO GUEST HOUSE VIA BEIRUT, 7 TEL. 22401 TELEFAX 224559 TELEX 460392

Parametric Processes in Plasma

III. Finite Wavelength Weak Pump

pump: $\mathbf{E}_0(\mathbf{r},t) = \mathbf{E}_0 \exp[i(\mathbf{k}_0\cdot\mathbf{r} - \omega_0 t)] + c.c.$

excited waves:

h.f. $\mathbf{E}_\pm(\mathbf{r},t) = \mathbf{E}_\pm \exp[i(\mathbf{k}_\pm\cdot\mathbf{r} - \omega_\pm t)] + c.c.$

(either electromagnetic or electrostatic, weakly damped)

l.f. $\delta n_e \exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)] + c.c.$

(electrostatic, can be highly damped)

$\omega_\pm = \omega \pm \omega_0, \quad \mathbf{k}_\pm = \mathbf{k} \pm \mathbf{k}_0$

Physical mechanism

① h.f. wave coupled to pump wave

→ slow modulation of oscillating field

→ ponderomotive force on plasma particles

→ density perturbation δn_e

② density perturbation

→ modulation of pump current density

→ high freq. wave

Valid for

stimulated scattering or

excitation of long wavelength, low freq. density perturbation.

Ponderomotive Force as a potential force

(T. Watanabe : private communication)

Eq. of motion

$$m\frac{d\mathbf{v}}{dt} = q\left[\mathbf{E}(\mathbf{r},t) + \mathbf{v}\times\mathbf{B}(\mathbf{r},t)\right], \qquad \frac{d\mathbf{r}}{dt} = \mathbf{v}$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0(\mathbf{r}) + \tilde{\mathbf{E}}(\mathbf{r},t), \qquad \frac{\partial}{\partial t}\tilde{\mathbf{B}} = -\nabla\times\tilde{\mathbf{E}}$$

$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0(\mathbf{r}) + \tilde{\mathbf{B}}(\mathbf{r},t), \qquad \nabla\cdot\mathbf{B} = 0$$

at given $\mathbf{r}$: (static or l.f.) (h. f.)

$$\tilde{\mathbf{E}}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\exp[-i\omega_0 t] + \text{c.c.}$$

$$\tilde{\mathbf{B}}(\mathbf{r},t) = \mathbf{B}(\mathbf{r})\exp[-i\omega_0 t] + \text{c.c.}$$

Assume
- slow spatial dependence or weak field (variation negligible over excursion length)
- no wave-particle resonance

$$\mathbf{r}(t) = \underset{\substack{(\text{slow}\\ \text{dependence})}}{\mathbf{R}(t)} + \underset{\substack{(\text{driven}\\ \text{h.f. oscillation})}}{\tilde{\mathbf{r}}(t)} + \underset{(\text{to be neglected})}{\left[\begin{matrix}\text{higher harmonics and}\\ \text{cyclotron motion}\end{matrix}\right]}$$

$$\mathbf{E}_0(\mathbf{r}) = \mathbf{E}_0(\mathbf{R}) + \tilde{\mathbf{r}}\cdot\nabla\mathbf{E}_0(\mathbf{R}) + [\text{higher order terms}]$$

$$\mathbf{B}_0(\mathbf{r}) = \mathbf{B}_0(\mathbf{R}) + \tilde{\mathbf{r}}\cdot\nabla\mathbf{B}_0(\mathbf{R}) + [\text{higher order terms}]$$

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}(\mathbf{R}) + \tilde{\mathbf{r}}\cdot\nabla\mathbf{E}(\mathbf{R}) + [\text{higher order terms}]$$

$$\mathbf{B}(\mathbf{r}) = \mathbf{B}(\mathbf{R}) + \tilde{\mathbf{r}}\cdot\nabla\mathbf{B}(\mathbf{R}) + [\text{higher order terms}]$$

(slow dependence) (h.f. oscillation) (to be neglected)

$$\mathbf{v}(t) = \underset{\substack{(\text{slow}\\ \text{dependence})}}{\mathbf{V}(t)} + \underset{\substack{(\text{driven}\\ \text{oscillation})}}{\tilde{\mathbf{v}}(t)} + \underset{(\text{neglected})}{\left[\begin{matrix}\text{higher harmonics and}\\ \text{cyclotron motion}\end{matrix}\right]}$$

Driven oscillation

$$m\dot{\tilde{\mathbb{v}}} = q\,[\,\tilde{\mathbb{E}} + \tilde{\mathbb{v}}\times\mathbb{B}_0\,]\,,\quad \dot{\tilde{\mathbb{r}}} = \tilde{\mathbb{v}}$$

$$\tilde{\mathbb{v}}(t) = \frac{q}{m}\frac{\omega_c}{\omega_c^2-\omega_0^2}\,\tilde{\mathbb{E}}\times\mathbb{b} + \frac{q}{m}\left[\frac{\mathbb{b}\times(\dot{\tilde{\mathbb{E}}}\times\mathbb{b})}{\omega_c^2-\omega_0^2} - \frac{\mathbb{b}\mathbb{b}\cdot\dot{\tilde{\mathbb{E}}}}{\omega_0^2}\right]$$

(E×B drift) (polarization drift)

$$\tilde{\mathbb{r}}(t) = -\frac{q}{m}\left\{\frac{\omega_c}{\omega_0^2-\omega_c^2}\frac{\mathbb{b}\times\dot{\tilde{\mathbb{E}}}}{\omega_0^2} + \frac{1}{\omega_0^2-\omega_c^2}\,\mathbb{b}\times(\tilde{\mathbb{E}}\times\mathbb{b}) + \frac{\mathbb{b}\mathbb{b}\cdot\tilde{\mathbb{E}}}{\omega_0^2}\right\}$$

(E×B drift) (polarization drift)

where $\mathbb{b} = \mathbb{B}_0/|\mathbb{B}_0|$

Averaged force

$$m\dot{\mathbb{V}} = q\{\mathbb{E}_0(\mathbb{R}) + \mathbb{V}\times\mathbb{B}_0(\mathbb{R})\} + \mathbb{G}$$

$$\mathbb{G} = q\left\{\overline{\tilde{\mathbb{r}}\cdot\nabla\tilde{\mathbb{E}}} + \overline{\tilde{\mathbb{v}}\times\tilde{\mathbb{B}}} + \overline{\tilde{\mathbb{v}}\times[\tilde{\mathbb{r}}\cdot\nabla\mathbb{B}_0(\mathbb{R})]}\right.$$

(ponderomotive force)

Calculations:

$$\overline{\tilde{\mathbb{v}}\times\tilde{\mathbb{B}}} = \overline{\dot{\tilde{\mathbb{r}}}\times\tilde{\mathbb{B}}} = -\overline{\tilde{\mathbb{r}}\times\dot{\tilde{\mathbb{B}}}} = \overline{\tilde{\mathbb{r}}\times(\nabla\times\tilde{\mathbb{E}})}$$

$$\overline{\tilde{\mathbb{v}}\times[\tilde{\mathbb{r}}\cdot\nabla\mathbb{B}_0]} = -\overline{\tilde{\mathbb{r}}\times[\tilde{\mathbb{v}}\cdot\nabla\mathbb{B}_0]} = \frac{1}{2}\left\{\overline{\tilde{\mathbb{v}}\times[\tilde{\mathbb{r}}\cdot\nabla\mathbb{B}_0]} - \overline{\tilde{\mathbb{r}}\times[}\right.$$

Take G_x

$$\left\{\overline{\tilde{\mathbb{r}}\cdot\nabla\tilde{\mathbb{E}}} + \overline{\tilde{\mathbb{v}}\times\tilde{\mathbb{B}}}\right\}_x = \overline{\tilde{\mathbb{r}}\cdot\nabla\tilde{E}_x} + \overline{\tilde{y}\,\partial_x\tilde{E}_y} - \overline{\tilde{y}\,\partial_y\tilde{E}_x} - \overline{\tilde{z}\,\partial_z\tilde{E}_x} +$$

$$= \overline{\tilde{\mathbb{r}}\cdot\partial_x\tilde{\mathbb{E}}}$$

$$\left\{\overline{\tilde{\mathbb{v}}\times[\tilde{\mathbb{r}}\cdot\nabla\mathbb{B}_0}\right\}_x = \frac{1}{2}\left\{\overline{\dot{\tilde{y}}\,\tilde{\mathbb{r}}\cdot\nabla B_{0z}} - \overline{\dot{\tilde{z}}\,\tilde{\mathbb{r}}\cdot\nabla B_{0y}} - \overline{\tilde{y}\,\dot{\tilde{\mathbb{r}}}\cdot\nabla B_{0z}} + \overline{\tilde{z}}\right.$$

$$= \frac{1}{2}\left\{(\overline{\tilde{\mathbb{r}}\times\tilde{\mathbb{v}}})\cdot\partial_x\mathbb{B}_0 + (\overline{\tilde{\mathbb{r}}\times\tilde{\mathbb{v}}})_x\,\underset{0}{\underset{\parallel}{\nabla\cdot\mathbb{B}_0}}\right\}$$

$$= \frac{1}{2}\,(\overline{\tilde{\mathbb{r}}\times\tilde{\mathbb{v}}})\cdot\partial_x\mathbb{B}_0$$

Calculate $\overline{\tilde{\mathbf{r}}\cdot\partial_x m\dot{\tilde{\mathbf{v}}}} = \overline{m\dot{\tilde{\mathbf{v}}}\cdot\partial_x\tilde{\mathbf{r}}}$ (both sides)

$$\overline{\tilde{\mathbf{r}}\cdot\partial_x m\dot{\tilde{\mathbf{v}}}} = q\,\overline{\tilde{\mathbf{r}}\cdot\partial_x[\tilde{\mathbf{E}}+\tilde{\mathbf{v}}\times\mathbf{B}_0]}$$
$$= q\left\{\overline{\tilde{\mathbf{r}}\cdot\partial_x\tilde{\mathbf{E}}} + \overline{(\tilde{\mathbf{r}}\times\partial_x\tilde{\mathbf{v}})}\cdot\mathbf{B}_0 + \overline{(\tilde{\mathbf{r}}\times\tilde{\mathbf{v}}}\right.$$

$$\overline{m\dot{\tilde{\mathbf{v}}}\cdot\partial_x\tilde{\mathbf{r}}} = q\,\overline{(\partial_x\tilde{\mathbf{r}})\cdot[\tilde{\mathbf{E}}+\tilde{\mathbf{v}}\times\mathbf{B}_0]} = q\left\{\overline{\tilde{\mathbf{E}}\cdot\partial_x\tilde{\mathbf{r}}} - \overline{(\partial_x\tilde{\mathbf{v}}\times\tilde{\mathbf{r}})}\right.$$

$$-\quad 0 = q\left\{\overline{\tilde{\mathbf{r}}\cdot\partial_x\tilde{\mathbf{E}}} - \overline{\tilde{\mathbf{E}}\cdot\partial_x\tilde{\mathbf{r}}} + \overline{(\tilde{\mathbf{r}}\times\tilde{\mathbf{v}})}\cdot\partial_x\mathbf{B}_0\right\}$$

$$G_x = q\left\{\overline{\tilde{\mathbf{r}}\cdot\nabla\tilde{\mathbf{E}}} + \overline{\tilde{\mathbf{v}}\times\tilde{\mathbf{B}}} + \overline{\tilde{\mathbf{v}}\times[\tilde{\mathbf{r}}\cdot\nabla\mathbf{B}_0]}\right\}_x$$
$$= q\left\{\overline{\tilde{\mathbf{r}}\cdot\partial_x\tilde{\mathbf{E}}} + \frac{1}{2}\overline{(\tilde{\mathbf{r}}\times\tilde{\mathbf{v}})}\cdot\partial_x\mathbf{B}_0\right\}$$
$$= \frac{q}{2}\left\{\overline{\tilde{\mathbf{r}}\cdot\partial_x\tilde{\mathbf{E}}} + \overline{\tilde{\mathbf{E}}\cdot\partial_x\tilde{\mathbf{r}}}\right\} = \left\{\nabla\,\frac{q}{2}\overline{(\tilde{\mathbf{r}}\cdot\tilde{\mathbf{E}})}\right\}_x$$

$$\therefore\quad \mathbf{G} = \nabla\frac{q}{2}\left[\overline{\tilde{\mathbf{r}}(t)\cdot\tilde{\mathbf{E}}(t)}\right] \equiv -\nabla\Phi$$

$$\boxed{\begin{aligned}\Phi &= -\frac{q}{2}\,\overline{\tilde{\mathbf{r}}(t)\cdot\tilde{\mathbf{E}}(t)}\\ &= \frac{q^2}{2m}\left\{\frac{\omega_c}{\omega_0^2-\omega_c^2}\frac{(\dot{\tilde{\mathbf{E}}}_\perp\times\tilde{\mathbf{E}}_\perp)\cdot\mathbf{b}}{\omega_0^2} + \frac{\overline{|\tilde{\mathbf{E}}_\perp|^2}}{\omega_0^2-\omega_c^2} + \frac{\overline{|\tilde{\mathbf{E}}_\parallel|^2}}{\omega_0^2}\right\}\\ &\qquad\text{(E×B drift)}\qquad\qquad\text{(polarization drift)}\end{aligned}}$$

$$\tilde{\mathbf{E}}(\mathbf{r},t) = \mathbf{E}_0 e^{i[\mathbf{k}_0\cdot\mathbf{r}-\omega_0 t]} + \mathbf{E}_0^* e^{-i[\mathbf{k}_0\cdot\mathbf{r}-\omega_0 t]}$$
$$+\mathbf{E}_+ e^{i[\mathbf{k}_+\cdot\mathbf{r}-\omega_+ t]} + \mathbf{E}_+^* e^{-i[\mathbf{k}_+\cdot\mathbf{r}-\omega_+ t]}$$
$$+\mathbf{E}_- e^{i[\mathbf{k}_-\cdot\mathbf{r}-\omega_- t]} + \mathbf{E}_-^* e^{-i[\mathbf{k}_-\cdot\mathbf{r}-\omega_- t]}$$

$$\Phi(\mathbf{r},t) = \Phi(\mathbf{k},\omega)e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + \Phi(-\mathbf{k},-\omega)e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$\Phi(\mathbf{k},\omega)$ from $\mathbf{E}_0\mathbf{E}_-$ and $\mathbf{E}_0^*\mathbf{E}_+$

$\Phi(-\mathbf{k},-\omega)$ from $\mathbf{E}_0\mathbf{E}_+^*$ and $\mathbf{E}_0^*\mathbf{E}_-^*$

Low frequency equation

Coupled eq'ns for δn_e and δn_i in the presence of the ponderomotive potential, Φ_e and Φ_i :

$$-e\,\delta n_e(k,\omega) = -\chi_e(k,\omega)\varepsilon_0 k^2 \left[\varphi(k,\omega) - \Phi_e/e\right]$$

$$Ze\,\delta n_i(k,\omega) = -\chi_i(k,\omega)\varepsilon_0 k^2 \left[\varphi(k,\omega) + \Phi_i/Ze\right]$$

$$\varepsilon_0 k^2 \varphi(k,\omega) = e\left[Z\,\delta n_i(k,\omega) - \delta n_e(k,\omega)\right]$$

$$\Rightarrow \begin{cases} (1+\chi_e)e\delta n_e - \chi_e Ze\delta n_i = -\varepsilon_0\chi_e k^2 \Phi_e/e \\ -\chi_i e\delta n_e + (1+\chi_i)Ze\delta n_i = -\varepsilon_0\chi_i k^2 \Phi_i/Ze \end{cases}$$

$$\delta n_e(k,\omega) = -\frac{\chi_e(k,\omega)}{\varepsilon(k,\omega)}\frac{k^2}{e^2}\left\{\varepsilon_i(k,\omega)\Phi_e(k,\omega) + \varepsilon_0\chi_i(k,\omega)\Phi_i(k,\omega)/Z\right\}$$

High frequency equation

$$\varepsilon_0\left\{\left[\omega_\pm^2 - c^2k_\pm^2\right]\overleftrightarrow{1} + c^2 k_\pm k_\pm\right\}\cdot E_\pm = -i\omega_\pm J_\pm$$

$$J_\pm = \overleftrightarrow{\sigma}_\pm \cdot E_\pm + \frac{\delta n_e}{n_0}J_0 , \qquad J_0 = \overleftrightarrow{\sigma}_0\cdot E_0$$

(linear response)

$$\Rightarrow \overleftrightarrow{D}_\pm \cdot E_\pm = -i\omega_\pm \frac{\delta n_e}{n_0}\overleftrightarrow{\sigma}_0\cdot E_0$$

$$\overleftrightarrow{D}_\pm = \varepsilon_0\left[(\omega_\pm^2 - c^2k_\pm^2)\overleftrightarrow{1} - c^2 k_\pm k_\pm + i\frac{\omega_\pm}{\varepsilon_0}\overleftrightarrow{\sigma}_\pm\right]$$

$$\overleftrightarrow{\sigma}_\pm = -\varepsilon_0 i\omega_\pm \overleftrightarrow{\chi}_{e\pm} , \qquad \overleftrightarrow{\sigma}_0 = -\varepsilon_0 i\omega_0 \overleftrightarrow{\chi}_{e0}$$

$$\left.\begin{aligned} \overleftrightarrow{\chi}_{e\pm} &= \overleftrightarrow{\chi}_e(k,\omega\pm\omega_0) \\ \overleftrightarrow{\chi}_{e0} &= \overleftrightarrow{\chi}_e(k,\omega_0) \end{aligned}\right\}\ \text{susceptibility tensor}$$

Susceptibility tensor:

Let $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, $\mathbf{k} = k_\perp \hat{y} + k_\parallel \hat{\mathbf{z}}$

$$\overleftrightarrow{\chi}_e(\mathbf{k},\omega) = -\frac{\omega_{pe}^2}{\omega^2}\left\{1 - \frac{1}{n_0}\sum_{\ell=-\infty}^{+\infty}\int \frac{d^3v\ \overleftrightarrow{T}_\ell}{\omega - k_\parallel v_z - \ell\omega_c}\left[k_\parallel \frac{\partial f_0}{\partial v_z} + \frac{\ell\omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp}\right]\right.$$

where f_0: electron distribution function

$$\overleftrightarrow{T}_\ell = \begin{pmatrix} v_\perp^2 J_\ell'^2, & i v_\perp^2 \ell \frac{J_\ell J_\ell'}{\zeta}, & -i v_\perp v_z J_\ell J_\ell' \\ -i v_\perp^2 \ell \frac{J_\ell J_\ell'}{\zeta}, & v_\perp^2 \ell^2 J_\ell^2/\zeta^2, & -v_\perp v_z \ell J_\ell^2/\zeta \\ i v_\perp v_z J_\ell J_\ell', & -v_\perp v_z \ell J_\ell^2/\zeta, & v_z^2 J_\ell^2 \end{pmatrix}$$

$J_\ell = J_\ell(\zeta)$, $J_\ell' = dJ_\ell/d\zeta$, $\zeta = k_\perp v_\perp/\omega_c$

Electrostatic susceptibility

$$\chi_e(\mathbf{k},\omega) = \mathbf{k}\cdot\overleftrightarrow{\chi}_e(\mathbf{k},\omega)\cdot\mathbf{k}/k^2$$

$$= \frac{\omega_{pe}^2}{k^2 n_0}\sum_{\ell=-\infty}^{\infty}\int\frac{d^3v\ J_\ell^2(\zeta)}{\omega - k_\parallel v_z - \ell\omega_c}\left[k_\parallel \frac{\partial}{\partial v_z} f_0 + \frac{\ell\omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp}\right]$$

Solve for $\mathbf{E}_\pm$ as

$$\mathbf{E}_\pm = \overleftrightarrow{D}_\pm^{-1}\cdot\left[-i\omega_\pm \frac{\delta n_e}{n_0}\overleftrightarrow{\sigma}_0\cdot\mathbf{E}_0\right]$$

$$= -\varepsilon_0\omega_0\omega_\pm\frac{\delta n_e}{n_0}\overleftrightarrow{D}_\pm^{-1}\cdot\left[\overleftrightarrow{\chi}_{e0}\cdot\mathbf{E}_0\right]$$

and substitute into the expression for the ponderomotive potential, Φ_e and Φ_i.

⇨ Dispersion relation.

Unmagnetized plasma ($\omega_c = 0$)

ponderomotive potential $\Phi_e \gg \Phi_i$ (neglected)

$$\Phi(\mathbb{k},\omega) = \frac{e^2}{2m_e\omega_0^2}\left[\mathbb{E}_0^* \cdot \mathbb{E}_+ + \mathbb{E}_0 \cdot \mathbb{E}_-\right]$$

$$= \frac{m_e}{2}\left[\mathbb{v}_0^* \cdot \mathbb{v}_+ + \mathbb{v}_0 \cdot \mathbb{v}_-\right]$$

where $\mathbb{v}_0 = \frac{-ie}{m_e\omega_0}\mathbb{E}_0 = -\mathbb{v}_0^*$, $\mathbb{v}_\pm = \frac{-ie}{m_e\omega_\pm}\mathbb{E}_\pm \doteqdot \mp\frac{ie}{m_e\omega_0}\mathbb{E}_\pm$

low frequency equation

$$\delta n_e(\mathbb{k},\omega) = -\frac{k^2}{e^2}\frac{\varepsilon_i(\mathbb{k},\omega)\varepsilon_0\chi_e(k,\omega)}{\varepsilon(k,\omega)}\frac{m_e}{2}\left[\mathbb{v}_0^* \cdot \mathbb{v}_+ + \mathbb{v}_0 \cdot \mathbb{v}_-\right]$$

high frequency equation

$$\mathbb{E}_\pm = \overleftrightarrow{D}_\pm^{-1} \cdot \left[i\omega_\pm \frac{\delta n_e}{n_0} n_0 e \mathbb{v}_0\right]$$

$$\mathbb{v}_\pm = \mp\frac{ie}{m_e\omega_0}\mathbb{E}_\pm \doteqdot \pm\frac{e^2}{m_e}\overleftrightarrow{D}_\pm^{-1} \cdot \mathbb{v}_0\,\delta n_e(\mathbb{k},\omega)$$

substitute into low frequency equation → <u>dispersion relat</u>

$$1 = \frac{\varepsilon_i(\mathbb{k},\omega)\varepsilon_0\chi_e(\mathbb{k},\omega)}{\varepsilon(\mathbb{k},\omega)}\frac{k^2}{2}\mathbb{v}_0^* \cdot \left[\overleftrightarrow{D}_+^{-1} + \overleftrightarrow{D}_-^{-1}\right] \cdot \mathbb{v}_0$$

$$\overleftrightarrow{D}_\pm = D_{t\pm}\overleftrightarrow{I} + \varepsilon_0 c^2 \mathbb{k}_\pm \mathbb{k}_\pm$$

where $D_{t\pm} = \varepsilon_{e\pm}\omega_\pm^2 - \varepsilon_0 c^2 k_\pm^2$

$$\overleftrightarrow{D}_\pm^{-1} = \frac{1}{D_{t\pm}}\left[\overleftrightarrow{I} - \frac{\mathbb{k}_\pm \mathbb{k}_\pm}{k_\pm^2}\right] + \frac{1}{\omega_\pm^2 \varepsilon_{e\pm}}\frac{\mathbb{k}_\pm \mathbb{k}_\pm}{k_\pm^2}$$

Dispersion Relation:

$$1 = \frac{\varepsilon_i(\mathbb{k},\omega)\varepsilon_0\chi_e(\mathbb{k},\omega)}{\varepsilon(\mathbb{k},\omega)}\frac{k^2}{2}\left\{\frac{|\mathbb{k}_+\times\mathbb{v}_0|^2}{k_+^2}\frac{1}{D_{t+}} + \frac{|\mathbb{k}_-\times\mathbb{v}_0|^2}{k_-^2}\frac{1}{D}\right.$$

$$+ \frac{|\mathbb{k}_+\cdot\mathbb{v}_0|^2}{\omega_+^2 k_+^2}\frac{1}{\varepsilon_{e+}} + \frac{|\mathbb{k}_-\cdot\mathbb{v}_0|^2}{\omega_-^2 k_-^2}\frac{1}{\varepsilon_{e-}}$$

Stimulated scattering of laser light

pump: $(\mathbb{k}_0, \omega_0)$ [laser light or electromagnetic wave]

$$\omega_0^2 = k_0^2 c^2 + \omega_{pe}^2$$

scattered light: $(\mathbb{k}_s, \omega_s)$ [electromagnetic wave]

$$\omega_s^2 = k_s^2 c^2 + \omega_{pe}^2$$

scatterer: $(\mathbb{k}, \omega)$ [low frequency and electrostatic]

SRS: electron plasma wave (epw)

$$\omega^2 = \omega_{pe}^2 + 3k^2 v_{Te}^2 \equiv \omega_k^2$$

SBS: ion acoustic wave (iaw) [$T_e \gg T_i$]

$$\omega^2 = kC_s/[1+k^2\lambda_D^2] \equiv \Omega_k^2 \doteqdot kC_s \quad *$$

SCS or SBS at $T_e \sim T_i$

$$\omega \sim k v_{Ti} \quad *$$

$$* \quad k < 2k_0 \ll \lambda_D^{-1}$$

resonance condition:

$$\omega_0 = \omega_s + \omega, \quad \mathbb{k}_0 = \mathbb{k}_s + \mathbb{k}$$

$$\omega_0 \sim \omega_s \gg \omega \Rightarrow k_0 \sim k_s > k/2$$

backscattering: $\mathbb{k}_s \doteqdot -\mathbb{k}_0$, $\mathbb{k} \doteqdot 2\mathbb{k}_0$

forward scattering: $\mathbb{k}_s \doteqdot \mathbb{k}_0$, $\mathbb{k} \perp \mathbb{k}_0$

Scattered light equation

$$\left[(\omega-\omega_0)^2-\omega_s^2\right]\mathbb{v}_- = \omega_{pe}^2 \frac{\delta n_e}{n_0}\mathbb{v}_0$$

Low frequency equation

$$\varepsilon(\mathbb{k},\omega)\frac{\delta n_e}{n_0} = -\varepsilon_i(\mathbb{k},\omega)\chi_e(\mathbb{k},\omega)\frac{1}{\omega_{pe}^2}\frac{k^2}{2}\mathbb{v}_0\cdot\mathbb{v}_-$$

Dispersion relation

$$1=-\frac{\varepsilon_i(\mathbb{k},\omega)\chi_e(\mathbb{k},\omega)}{\varepsilon(\mathbb{k},\omega)}\frac{k^2}{2}\frac{|\mathbb{v}_0|^2}{\left[(\omega-\omega_0)^2-\omega_s^2\right]}$$

<u>SRS</u> (Stimulated Raman scattering)

epw $\quad \varepsilon(\mathbb{k},\omega) \doteq \varepsilon_e(\mathbb{k},\omega) \doteq \varepsilon_0\left[1-\omega_k^2/\omega^2\right]$

$$\varepsilon_i(\mathbb{k},\omega) \doteq \varepsilon_0, \quad \chi_e(\mathbb{k},\omega) \doteq -\omega_{pe}^2/\omega^2$$

low freq. equation

$$\left[\omega^2-\omega_k^2\right]\frac{\delta n_e}{n_0} = \frac{k^2}{2}\mathbb{v}_0\cdot\mathbb{v}_-$$

coupling constant

$$\varepsilon^2 = \frac{1}{2}k^2\omega_{pe}^2|\mathbb{v}_0|^2/\omega_0^2\omega_k^2$$

$$\doteq \frac{1}{2}\frac{k^2}{k_0^2}\frac{|\mathbb{v}_0|^2}{c^2} \le 2\frac{|\mathbb{v}_0|^2}{c^2} \quad \text{(for backscatter)}$$

maximum growth rate

$$\gamma_{max}^2 = \frac{\varepsilon^2\omega_0\omega_k}{4} \doteq \frac{k_0 c}{2}\omega_{pe}\frac{|\mathbb{v}_0|^2}{c^2} = \frac{k_0\omega_{pe}}{2c}|\mathbb{v}_0|^2$$

<u>SBS</u> (Stimulated Brillouin scattering) at $T_e \gg T_i$

i.a.w. $\varepsilon(\mathbb{k}, \omega) \doteqdot \varepsilon_0 \frac{1}{k^2\lambda_D^2} [1 - \frac{\Omega_k^2}{\omega^2}]$

$$\varepsilon_i(\mathbb{k}, \omega) \doteqdot -\varepsilon_0 \omega_{pi}^2/\omega^2, \quad \chi_e(\mathbb{k}, \omega) \doteqdot 1/k^2\lambda_D^2$$

low freq. equation

$$[\omega^2 - \Omega_k^2] \frac{\delta n_e}{n_0} = \frac{\omega_{pi}^2}{\omega_{pe}^2} \frac{k^2}{2} \mathbb{v}_0 \cdot \mathbb{v}_-$$

coupling constant

$$\varepsilon^2 = \frac{1}{2} k^2 \omega_{pi}^2 |\mathbb{v}_0|^2 / \omega_0^2 \Omega_k^2 \doteqdot \frac{1}{2k_0^2\lambda_D^2} \frac{|\mathbb{v}_0|^2}{c^2}$$

maximum growth rate

$$\gamma_{max}^2 = \frac{\varepsilon^2 \omega_0 \Omega_k}{4} \doteqdot \frac{k\omega_{pi}}{8 k_0 \lambda_D c} |\mathbb{v}_0|^2 \propto n_0$$

<u>SCS</u> (Stimulated Compton scattering) at $T_e \sim T_i$

$k\lambda_D \ll 1, \quad \chi_e(\mathbb{k}, \omega) \doteqdot 1/k^2\lambda_D^2$

$$\frac{\varepsilon_i(\mathbb{k},\omega)\chi_e(\mathbb{k},\omega)}{\varepsilon(\mathbb{k},\omega)} \doteqdot \frac{1}{k^2\lambda_D^2} \frac{\varepsilon_i(\mathbb{k},\omega)}{|\varepsilon(\mathbb{k},\omega)|^2} [\varepsilon_i^*(\mathbb{k},\omega) +$$

$$\mathrm{Im} \frac{\varepsilon_i(\mathbb{k},\omega)\chi_e(\mathbb{k},\omega)}{\varepsilon(\mathbb{k},\omega)} = \frac{\varepsilon_0 \mathrm{Im}\chi_i(\mathbb{k},\omega)}{|k^2\lambda_D^2 \varepsilon(\mathbb{k},\omega)|^2}$$

$$\omega \to \omega + i\gamma$$

$$\mathrm{Im}[(\omega + i\gamma - \omega_0)^2 - \omega_s^2] \doteqdot -2\omega_0\gamma$$

$$\Rightarrow \quad \gamma = \frac{k^2}{4\omega_0} \frac{\varepsilon_0 \mathrm{Im}\chi_i(\mathbb{k},\omega)}{|k^2\lambda_D^2 \varepsilon(\mathbb{k},\omega)|^2}$$

Stimulated scattering or decay of electron plasm wave (epw)

pump : $(\mathbf{k}_0, \omega_0)$ [epw] $k_0 \lambda_D < .3$

$$\omega_0^2 = \omega_{pe}^2 + 3 k_0^2 V_{Te}^2$$

decay wave : $(\mathbf{k}_s, \omega_s)$ [epw] $k_s \sim k_0 < .3 k_D$

$$\omega_s^2 = \omega_{pe}^2 + 3 k_s^2 V_{Te}^2$$

scatterer : $(\mathbf{k}, \omega)$ [low freq.] $k < 2k_0 < \lambda_D^{-1}$

$T_e \gg T_i$ ion acoustic wave

$$\omega^2 = \Omega_k^2 \doteq k^2 C_s^2$$

$T_e \sim T_i$ $\omega \sim k V_{Ti}$

[induced scattering on ions]

decay wave equation

$$[(\omega - \omega_0)^2 - \omega_s^2] \mathbb{V}_- = \omega_{pe}^2 \frac{\delta n_e}{n_0} \mathbb{V}_0$$

$T_e \gg T_i$: ion acoustic wave eq.

$$[\omega^2 - \Omega_k^2] \frac{\delta n_e}{n_0} = \frac{\omega_{pi}^2}{\omega_{pe}^2} \frac{k^2}{2} \mathbb{V}_0 \cdot \mathbb{V}_-$$

coupling constant

$$\varepsilon^2 \doteq \frac{k^2}{2} \omega_{pi}^2 |\mathbb{V}_0|^2 / k^2 C_s^2 \omega_{pe}^2 = \frac{1}{2} \frac{|\mathbb{V}_0|^2}{V_{Te}^2}$$

maximum growth rate

$$\gamma_{max}^2 = \frac{\varepsilon^2 \omega_{pe} \Omega_k}{4} = \frac{k C_s \omega_{pe}}{8} \frac{|\mathbb{V}_0|^2}{V_{Te}^2} < \frac{k_0 C_s}{4} \omega_{pe} \; !$$

(for backsca

$T_e \sim T_i$: same as SCS

$$\gamma = \frac{k^2}{4\omega_0} \frac{\varepsilon_0 \Im m \chi_i(\mathbf{k}, \omega)}{| k^2 \lambda_D^2 \varepsilon(\mathbf{k}, \omega)|^2}$$