INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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H4-SMR 393/51

SPRING COLLEGE ON PLASMA PHYSICS

15 May - 9 June 1989

RADIO FREQUENCY HEATING IN TOKAMAKS

ION CYCLOTRON RESONANCE HEATING (ICRH)

D. F. Start

Jet Joint Undertaking
Oxfordshire
Abingdon OX14 3EA
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Radio Frequency Heating in Tokamaks

DFH Start

JET Joint Undertaking

- Ion Cyclotron Resonance Heating (ICRH)
- Electron Cyclotron Resonance Heating (ECRH)
- RF Current Drive

Ion Cyclotron Resonance Heating

RF frequency = Ion gyrofrequency.

$$2\pi f = \frac{q\,(\text{coul.})}{M\,(\text{kg})} \cdot B(\text{T})$$

Ex

$B = 3\text{T}$ $\quad f = \underline{46\text{ MHz}}$ for H^+

$\underline{\lambda_{\text{free space}} = 6.6\text{m}}$

In tokamaks RF used to excite plasma wave - fast magnetosonic wave, $\tilde{E} \perp r$ to B

Propagation at sound speed $v_S \approx c\,\omega_{ci}/\omega_{pi}$

ω_{ci} = ion cyl. freq, ω_{pi} = ion plasma freq.

$\omega_{pi}^2 = n_i q_i^2 / m_i \varepsilon_0$

Ex $\quad H^+$ with $n_i = 3\times10^{19}\text{m}^{-3}$ $\quad f_{pi} = 1200\text{MHz}$

$v_s = c/26$ $\quad \underline{\lambda = 25\text{cm.}}$

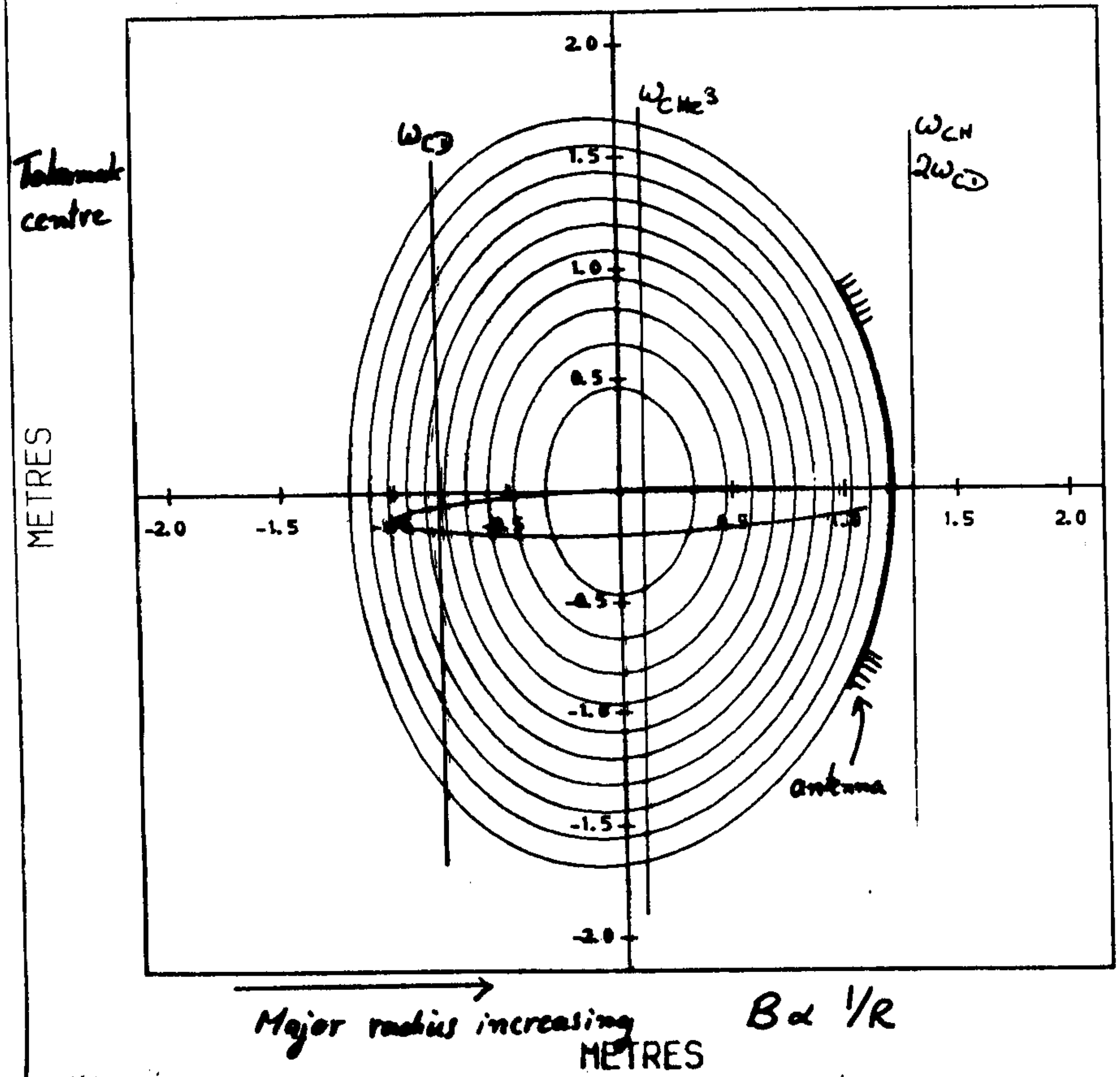
Fundamental freq. ICRH - minority ions + majority ions

eg $(He^3)D$, $3\%\ He^3$

2nd harmonic ICRH - majority ions

BBBBBBBB JJJJJJJJ EEEEEEEE TTTTTTTT PPPPPPPP LLLLLLLL

RAY POLOIDAL PROJECTION



Pure D plasma

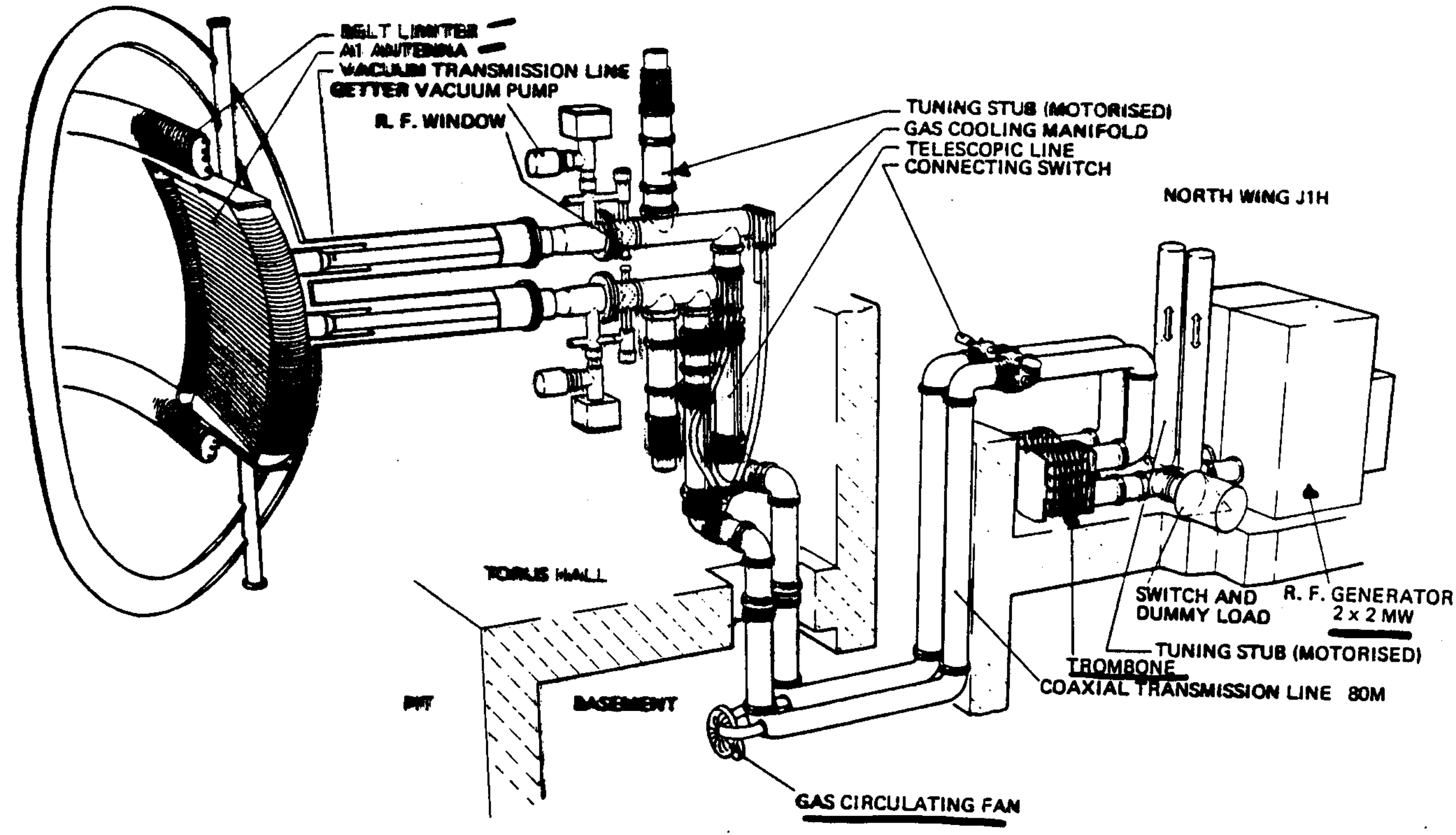
$f = 32\,MHz$

$B_T = 3.1\,T$

$k_{\parallel} = 4\,m^{-1}$

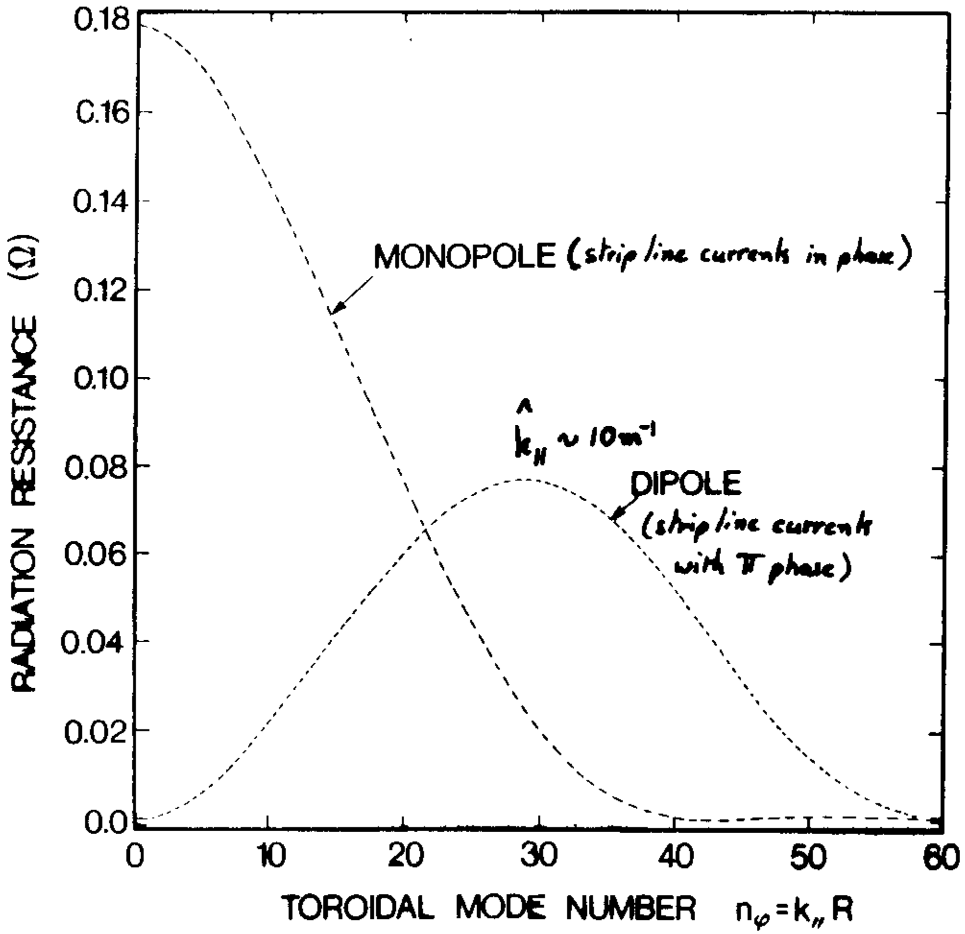
REFERENCE

JETPLB 15:16:41 06/03/89

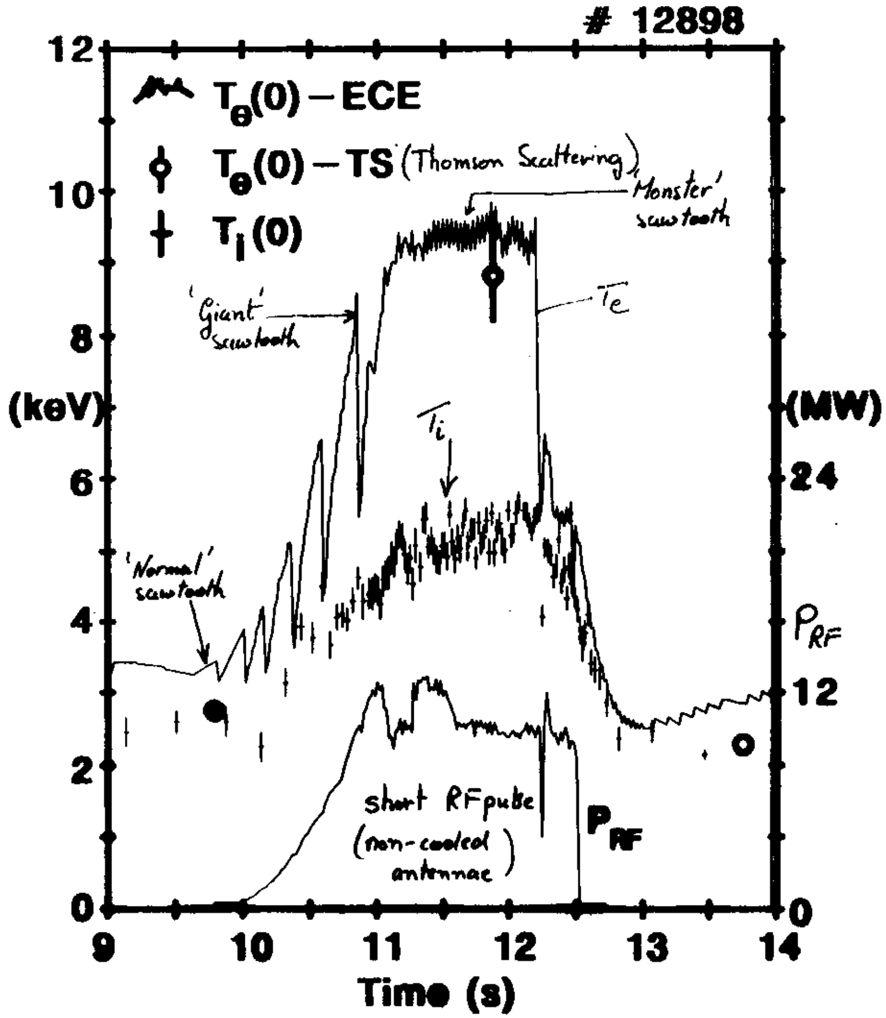


Layout of an ICRF unit in its upgraded version. The additions to the present system include phase shifters, capacitive stubs, forced air cooling, water cooling and new antenna design.

Radiated Spectrum for Monopole and Dipole Antennae



High Electron Temperature Regime



- $He^3(H)$
- $I_p = 2.45$ MA, $B_\phi = 2.4$ T, K = 1.5, $q_{psi} = 5.3$
- $\bar{n}_e = 2.0 \times 10^{19}\ m^{-3}$, $\bar{T}_e = 3$ keV

Electron Heating by Minority Ions

Ratio of power transfer from minority to bulk ions or electrons depends on m-i and m-e collision frequencies and hence on energy of minority ions

Low energy → bulk ion heating

High energy → electron heating.

Minority Ion Distribution Function $f(v, \xi = \frac{v_\parallel}{v})$

M.I. Fokker Planck Equation: heating effect balanced by collisions

Formally $\left(\frac{\partial f}{\partial t}\right)_{ICRF} = \left(\frac{\partial f}{\partial t}\right)_c$

For $v_i < v < v_e$

$$\tau_s\left(\frac{\partial f}{\partial t}\right)_c = \frac{1}{v^2}\frac{\partial}{\partial v}\left[\left(v^3 + v_c^3\right)f\right] + \beta\frac{v_c^3}{v^3}\frac{\partial}{\partial \xi}\left[\left(1-\xi^2\right)\frac{\partial f}{\partial \xi}\right]$$

collisions with electrons — collisions with ions — pitch angle scattering

$v_c^3 = 0.75\pi^{1/2}\frac{m_e}{m_i}\bar{Z}\, v_e^3$

Cox, Nu Fusion 24(1984)399

high v, low T_e, m-e collisions dominate

low v, high T_e m-i collisions dominate

$\left(\frac{\partial f}{\partial t}\right)_{ICRF}$ causes diffusion of minority ions in velocity space

Launched wave is lin polarized ≡ LH + RH circular polarizat'

LH component rotates in same sense as ion gyration

Produces $\perp$r accel or decel depending on relative wave phase and gyrophase.

Successive passes of ion through resonance produces 'random walk' in $v_\perp$

Diffusion coefficient $D = \frac{(\Delta v_\perp)^2}{\tau}$, τ = mean time between accelerations

$$\left(\frac{\partial f}{\partial t}\right)_{ICRF} = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left(v_\perp D\frac{\partial f}{\partial v_\perp}\right) \quad \text{for fundamental}$$

$D \propto |E_+|^2 \propto P_{ICRF}$

Solution of F-P Equation

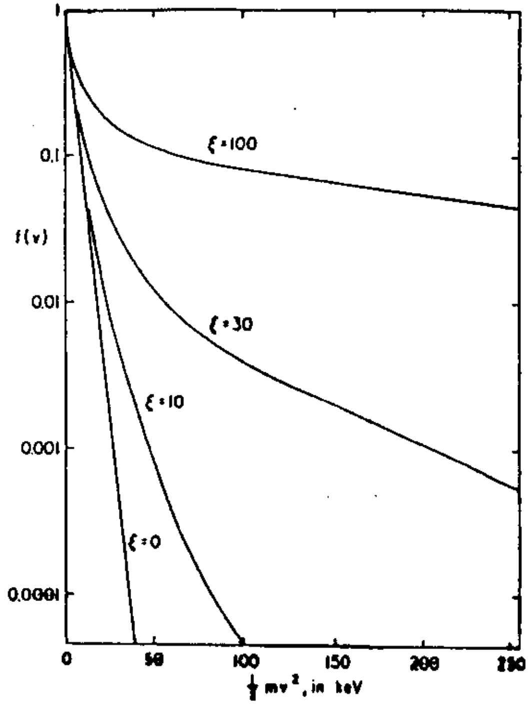
Stix – Nu Fusion 15(1975)737

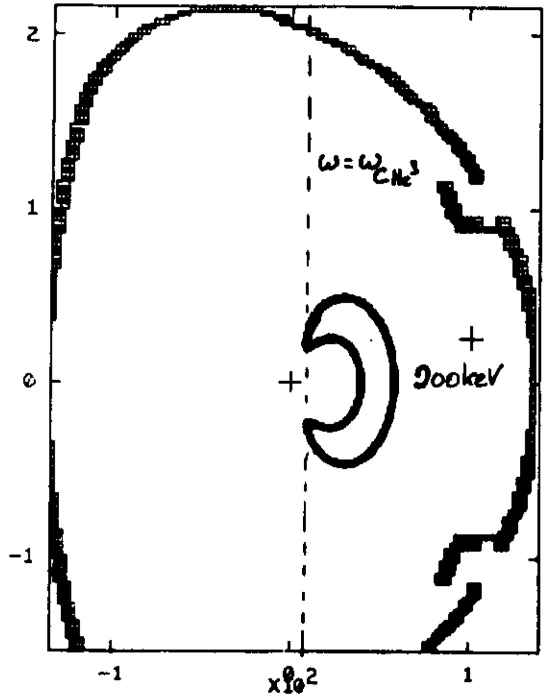
RF produces high energy tail in $V_\perp$

Tail temp $T_t = (1+\xi)T_e \sim \frac{P_d\tau_s}{n}$ (Power per minority ion × slowing down time,

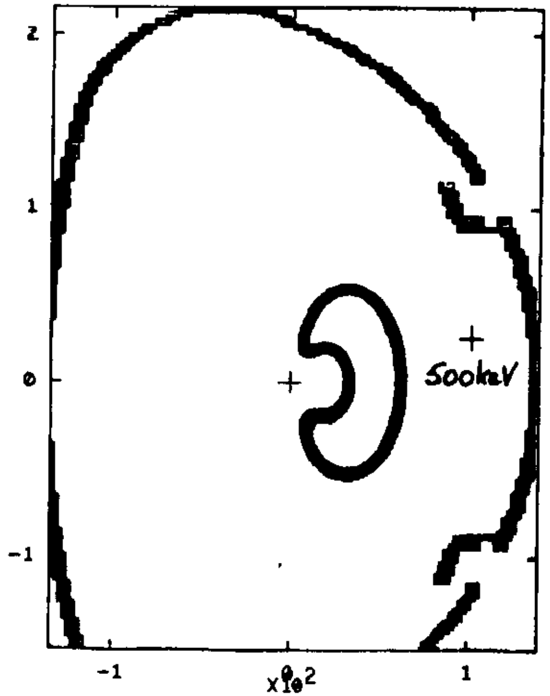
$$\xi = \frac{m P_d v_e}{8\pi^{1/2} n_e n Z^2 e^4 \ln\lambda}$$

Banana orbits turning at resonance — He^3 ions





BTOR = 3.40 T
IPLAS = 2000.0 kA
XUPPER= 0.0 kA
XLOWER= 0.0 kA
PEAK.F= 3.50
BETAI = 0.50
LI = 1.00
K = 1.60
E/M = 0.667 e/amu
ENERGY= 0.200 MeV
ATT = 90.0 deg
X0 = 10.0 cm
Z0 = 30.0 cm
V(RAD)= 3586.7 km/s
V(UP) = 0.0 km/s
V(TOR)= 0.0 km/s
BP/BT = 0.055
X-END = 15.8 CM
Z-END = 35.7 CM
TIME = 88.015 US



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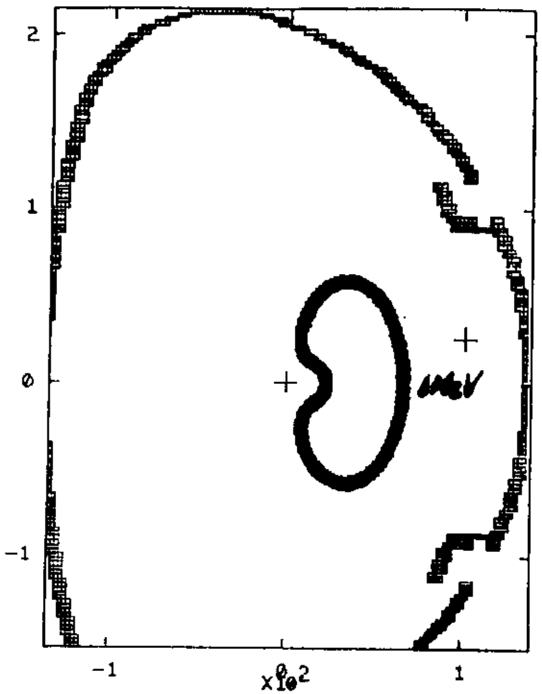
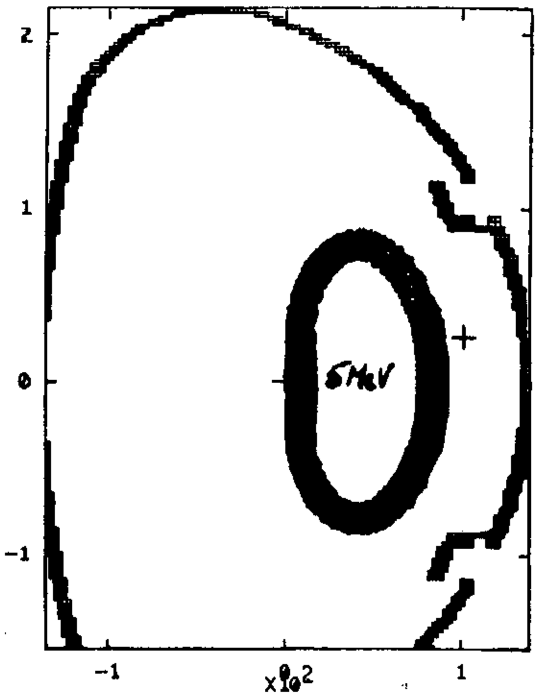
IPLAS = 2000.0 kA
XUPPER= 0.0 kA
XLOWER= 0.0 kA
PEAK.F= 3.50
BETAI = 0.50
LI = 1.00
K = 1.60
E/M = 0.667 e/amu
ENERGY= 0.500 MeV
ATT = 90.0 deg
X0 = 10.0 cm
Z0 = 30.0 cm
V(RAD)= 5671.1 km/s
V(UP) = 0.0 km/s
V(TOR)= 0.0 km/s
BP/BT = 0.055
X-END = 12.4 CM
Z-END = 41.6 CM
TIME = 58.677 US



BTOR =	3.40	T
IPLAS =	2000.0	kA
XUPPER=	0.0	kA
XLOWER=	0.0	kA
PEAK.F=	3.50	
BETAI =	0.50	
LI =	1.00	
K =	1.60	
E/M =	0.667	e/amu
ENERGY=	1.000	MeV
ATT =	90.0	deg
X0 =	10.0	cm
Z0 =	30.0	cm
V(RAD)=	8020.2	km/s
V(UP) =	0.0	km/s
V(TOR)=	0.0	km/s
BP/BT =	0.055	
X-END =	22.7	CM
Z-END =	58.5	CM
TIME =	44.007	US



BTOR =	3.40	T
IPLAS =	2000.0	kA
XUPPER=	0.0	kA
XLOWER=	0.0	kA
PEAK.F=	3.50	
BETAI =	0.50	
LI =	1.00	
K =	1.60	
E/M =	0.667	e/amu
ENERGY=	5.000	MeV
ATT =	90.0	deg
X0 =	10.0	cm
Z0 =	30.0	cm
V(RAD)=	[illegible]	km/s
V(UP) =	0.0	km/s
V(TOR)=	0.0	km/s
BP/BT =	[illegible]	
X-END =	84.1	CM
Z-END =	[illegible]	CM
TIME =	[illegible]	US

Fokker - Planck Calculations with Trapped Particles

Kerbel et al

Contours of f_i for 2nd harmonic ICRF

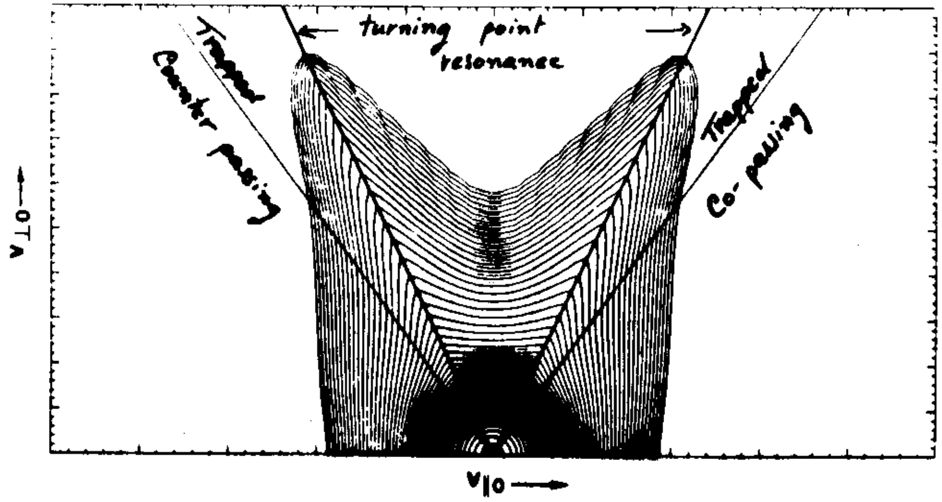
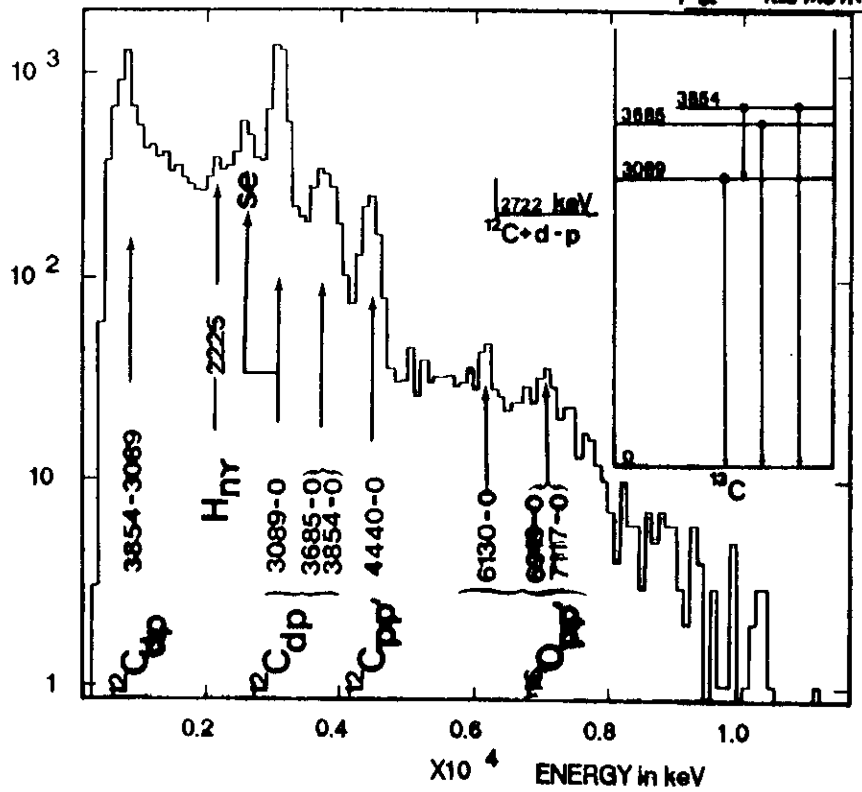


Figure 15

Evidence for High Energy Ions

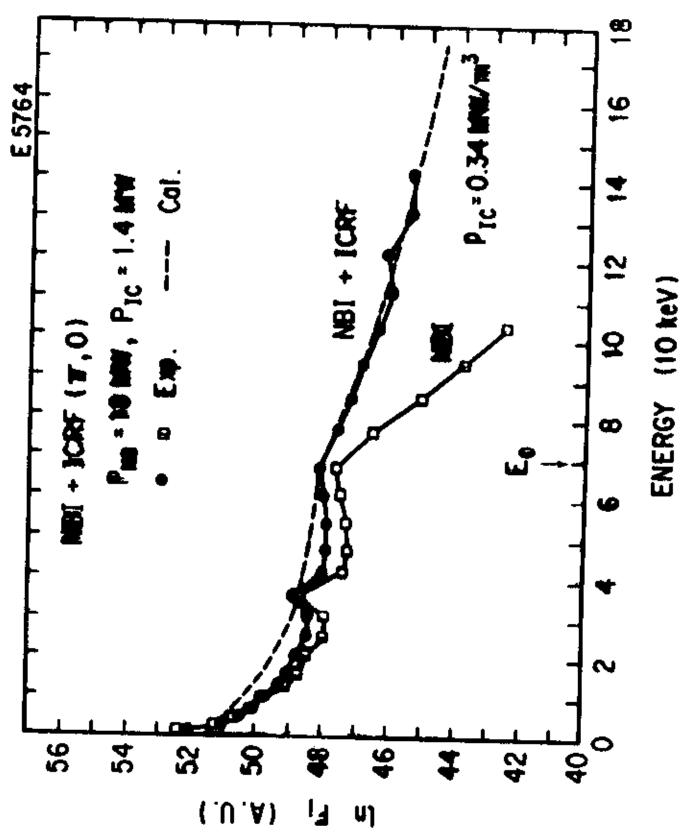
Gamma radiation due to energetic minority ions

COUNTS IN SPECTRUM vs ENERGY **Fig 1** (H)D + 2nd harmonic D



JT60 ICRF Acceleration of H^+ Beam Ions at $2\omega_{CH}$

Charge Exchange Energy Spectra of Shot E5764

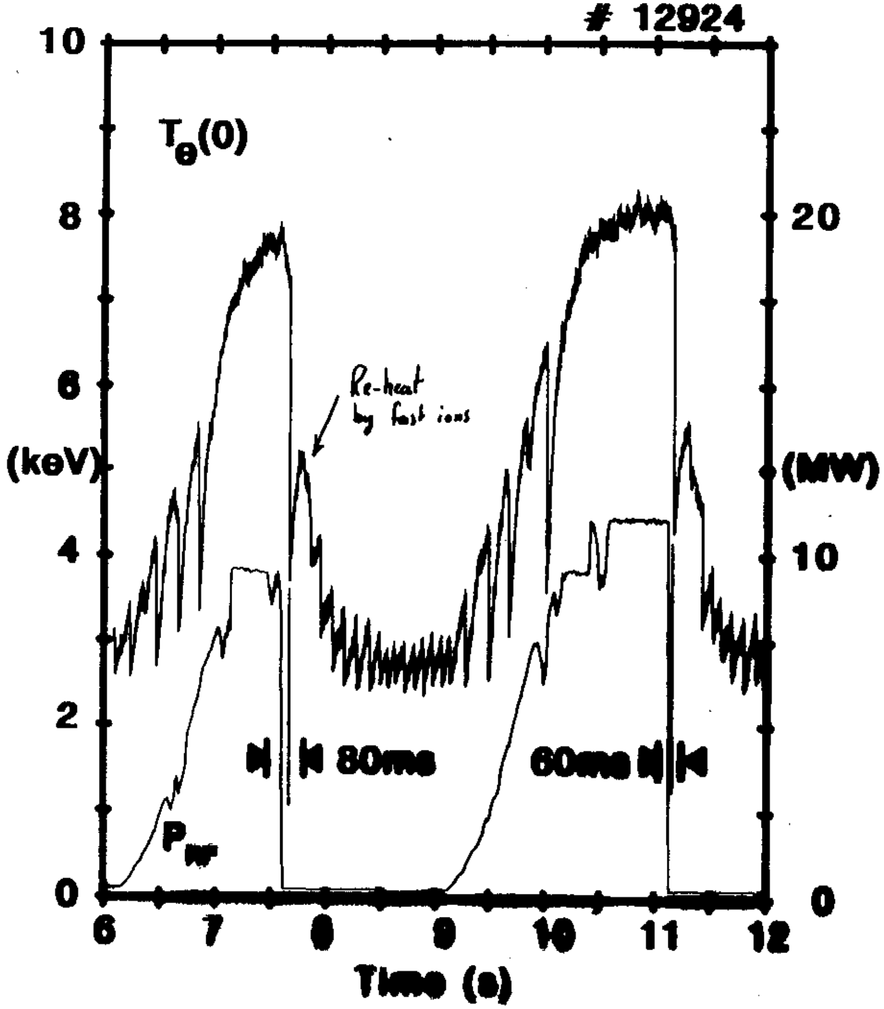


Ion average energy as an effective ion temperature ;

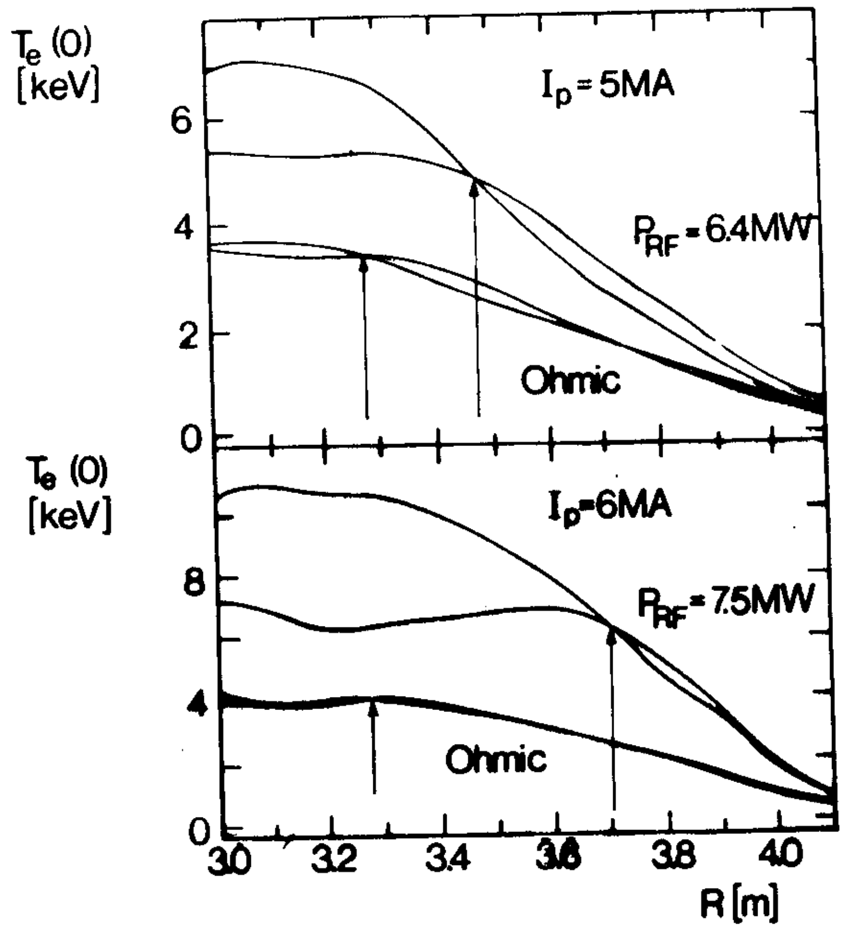
$$T_i^{eff} = \frac{2}{3}\int_0^\infty \frac{1}{2}Mv^2F_0(v)v^2dv \Big/ \int_0^\infty F_0(v)v^2dv$$

$\underline{T_i^{eff} = 12\ \text{keV}}$ (NBI only) $\longrightarrow$ $\underline{22\ \text{keV}}$ (NBI+ICRF)

ICRH Switch – off Experiment

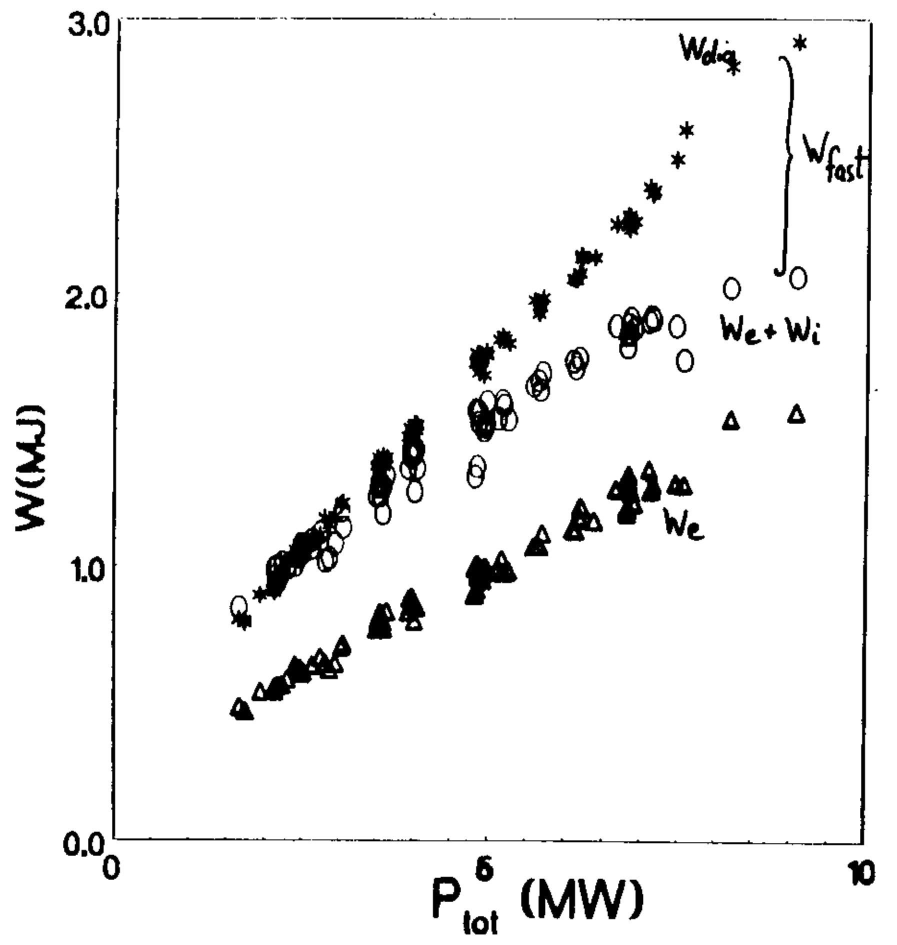


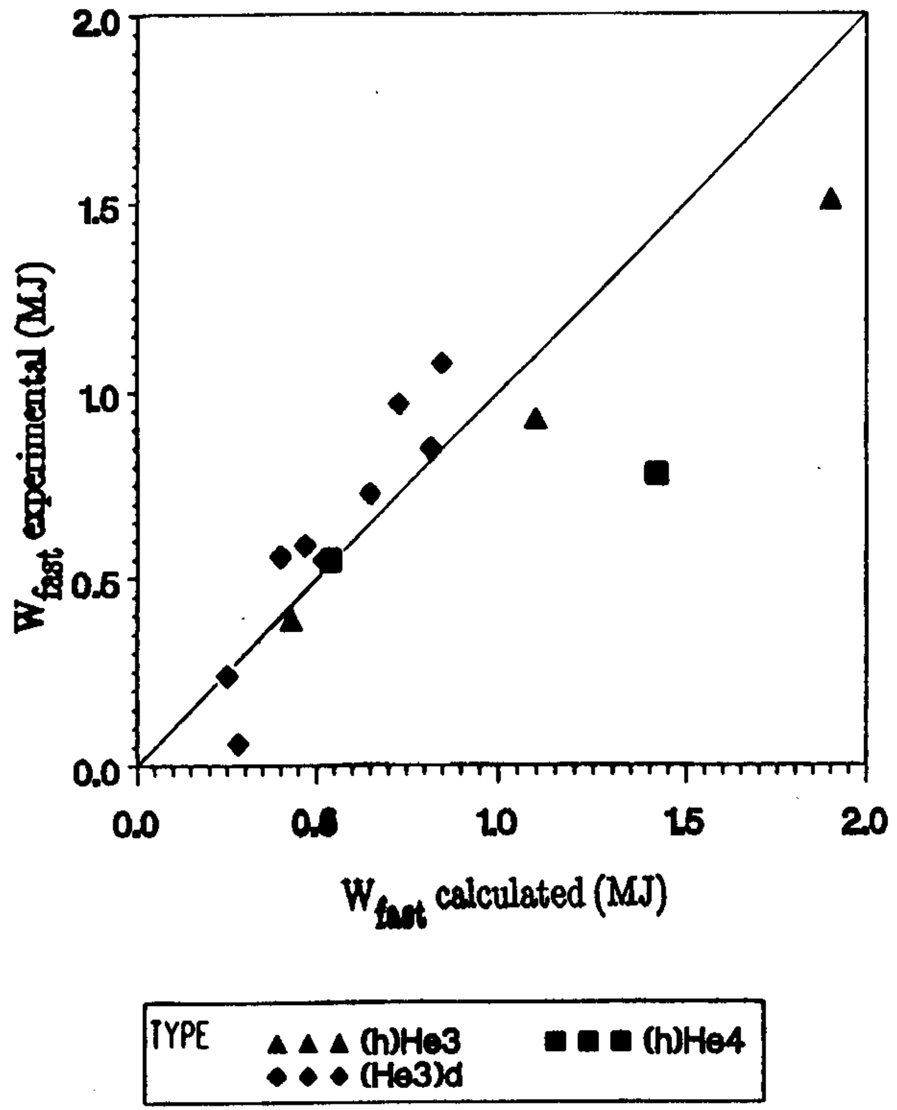
- I_p = 2.45 MA, B_0 = 2.4 T, $\bar{n}_e = 2\times10^{19}$ m^{-3}
- When RF power switched – off before crash of 'monster', the crash occurs within a fraction of the fast ion slowing – down time

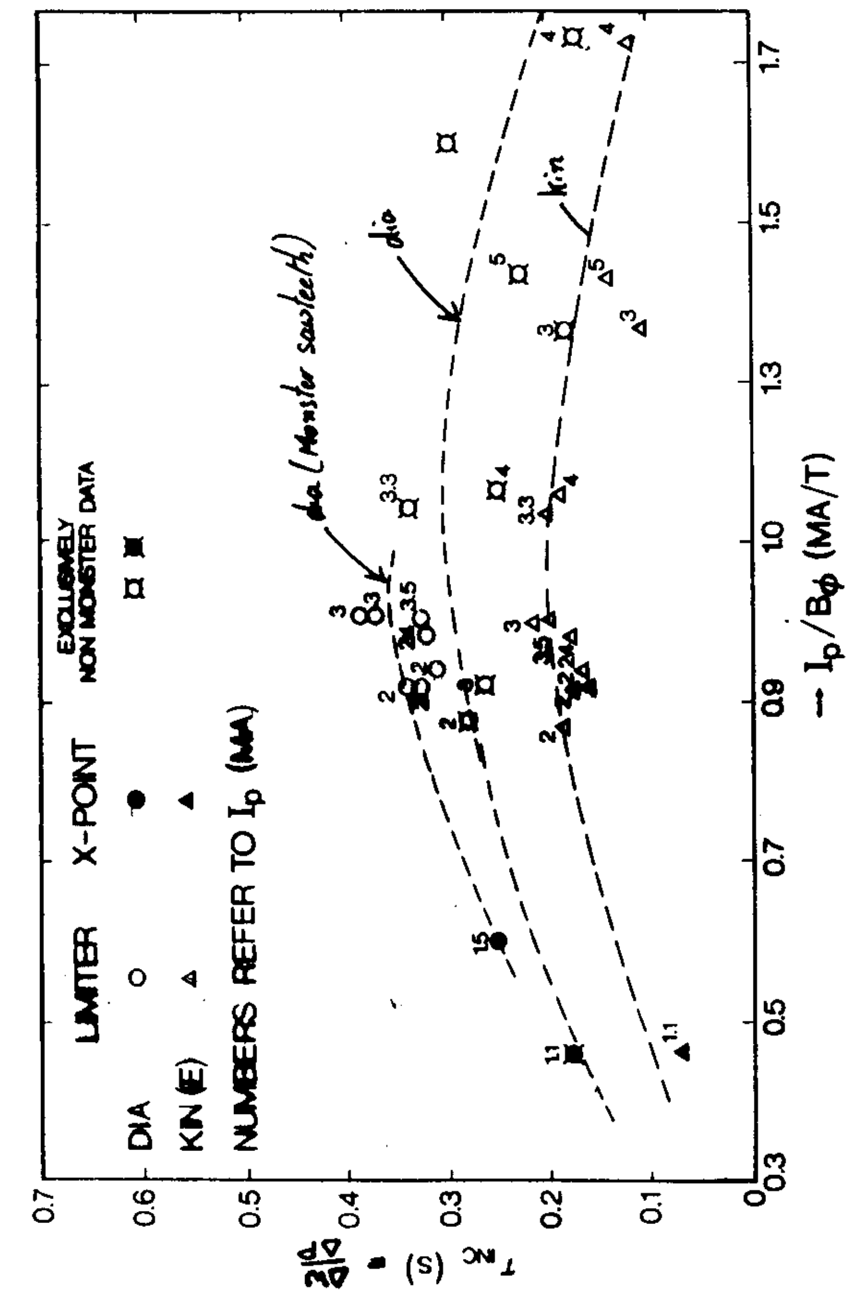


Energy in H-minority

Wdia, Wkin, We, versus Power



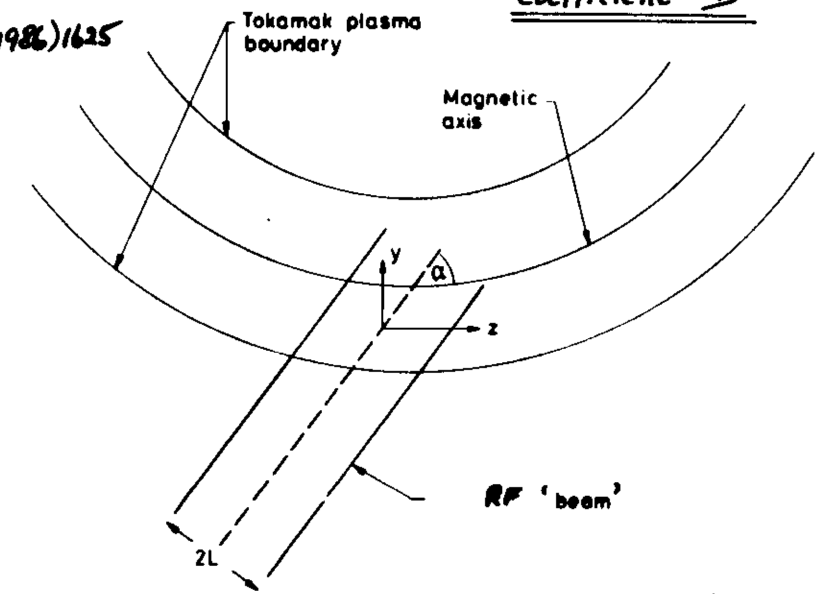




Single Particle Model for Quasi-Linear Diffusion

O'Brien et al
Nu Fusion 26(1986)1625

Coefficient D



Eq. of motion in RF field

$$\frac{dv}{dt} = \frac{q}{m}\left\{\tilde{E} + v \times (B + \tilde{B})\right\}$$

$\perp$r components are real parts of

$$\frac{dV_x}{dt} = \frac{q}{m}\tilde{E}_x - i\Omega V_y \quad \text{and} \quad \frac{dV_y}{dt} = \frac{q}{m}\tilde{E}_y + i\Omega V_x$$

$V_x, V_y, \tilde{E}_x\ \tilde{E}_y$ complex, $\Omega = qB/m$

$$v_x = v_\perp \cos\psi = Re(V_x) \qquad v_y = v_\perp \sin\psi = Re(V_y)$$

ψ is gyrophase $\qquad \psi(t) = \int_{-\infty}^{t} \Omega\, dt + \psi_0$ - initial phase

define $\quad \tilde{E} = E(r)e^{(-i\omega t + ik.r)}$

$$V_\pm = V_x \pm iV_y$$

$$E_\pm = E_x \pm iE_y$$

E_+ is LH circularly pol wave

Eqs of motion become

$$\frac{dV_\pm}{dt} \mp i\Omega V_\pm = \frac{q}{m} E_\pm e^{(-i\omega t + ik.r)}$$

Integrate for resonant interaction

$$V_+(\infty) e^{i\psi(\infty)} = V_+(-\infty) e^{i\psi_0} + \underbrace{\frac{q}{m}\int_{-\infty}^{\infty} E_+ e^{(-i\omega t + ik.r + i\psi)} dt}_{G}$$

Calc. $\Delta v_\perp$

$$\Delta (v_\perp)^2 = \frac{1}{4}\left[V_+(\infty) V_+^*(\infty) - V_+(-\infty) V_+^*(-\infty)\right]$$

$$\Delta v_\perp = \frac{1}{2}\Delta(v_\perp)^2/v_\perp = \left[V_+(-\infty) G^* e^{i\psi_0} + V_+^*(-\infty) G e^{-i\psi_0}\right]/8v_\perp$$

Evaluate G

$$k.r = k_\parallel \int_{-\infty}^{t} v_\parallel dt' + \frac{k_\perp v_\perp}{\Omega} \sin\psi$$

expand $\exp(i\frac{k_\perp v_\perp}{\Omega}\sin\psi)$ in terms of Bessel fns.

$$G = \frac{q}{m}\sum_{\ell=1}^{\infty} e^{i\ell\psi_0}\int_{-\infty}^{\infty} J_{\ell-1}\left(\frac{k_\perp v_\perp}{\Omega}\right) E_+ e^{i(-\omega t + k_\parallel\int_{-\infty}^{t} v_\parallel dt' + \ell\int_{-\infty}^{t}\Omega dt)} \cdot dt$$

Sub in expression for $\Delta v_\perp$, use $V_+(-\infty) = 2v_\perp e^{-i\psi_0}$ and the small argument expansion of the Bessel fn to obtain

$$[(\Delta v_\perp)^2] = \frac{1}{8}\left(\frac{q}{m(\ell-1)!}\right)^2 \left|\int_{-\infty}^{\infty}\left(\frac{k_\perp v_\perp}{\Omega}\right)^{2(\ell-1)} E_+(r)\, e^{i(-\omega t + k_\parallel\int_{-\infty}^{t} v_\parallel dt' + \ell\int_{-\infty}^{t}\Omega dt')} \cdot dt\right|^2$$

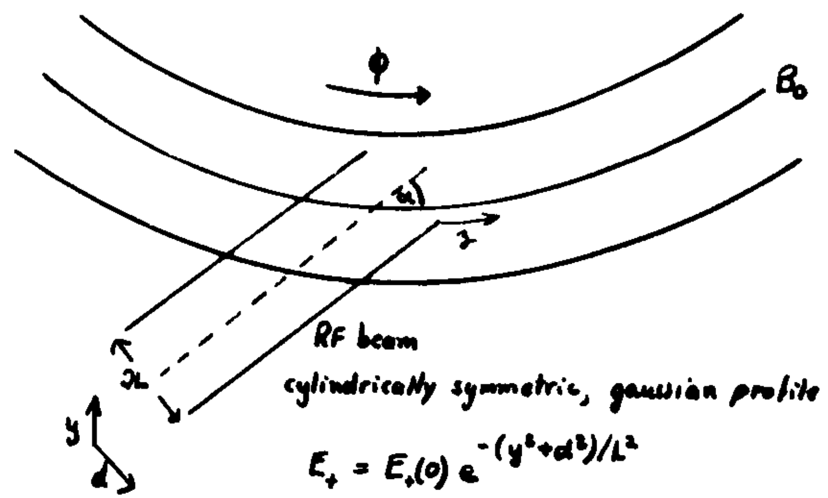
Resonance when phase is stationary $\boxed{\omega - k_\parallel v_\parallel = \ell\Omega}$

ℓ = harmonic number

To evaluate $[(\Delta v_\perp)^2]$ we need

- spatial dependence of $E_+(r)$
- variation of gyrofrequency and $v_\parallel$ along orbit

RF Injection Geometry



$$E_+ = E_+(0)\, e^{-(y^2+d^2)/L^2}$$

Gyrofrequency $\Omega(t) = \Omega_0 + v_\parallel \Omega' t \qquad \Omega' = \frac{d\Omega}{dz} = \frac{\Omega_0 r \sin\chi}{q R^2}$

and $v_\parallel \qquad v_\parallel(t) \approx v_\parallel(0)$

Analytic evaluation of phase integral along orbit

$$[(\Delta v_\perp)^2] = \frac{\pi}{8}\left(\frac{q}{m}\frac{1}{(\ell-1)!}\right)^2\left(\frac{k_\perp v_\perp}{2\Omega_0}\right)^{2(\ell-1)} |E_+(0)|^2 e^{-y^4/L^4}$$

$$\times \frac{1}{v_\parallel^2}\cdot\frac{1}{\left(\frac{\sin^4\alpha}{L^4}+\left(\frac{\ell\Omega'}{2v_\parallel}\right)^2\right)^{1/2}} \exp\left[\frac{-\frac{\sin^2\alpha}{2L^2}\left(\frac{\omega-\ell\Omega_0}{v_\parallel}-k_\parallel\right)^2}{\frac{4\sin^4\alpha}{L^4}+\left(\frac{\ell\Omega'}{v_\parallel}\right)^2}\right]$$

resonance condition maximises $(\Delta v_\perp)^2$

finite beam width effect

Ω variation

$D = [(\Delta v_\perp)^2]/2\tau$, τ = circulation time

Model does not apply to banana orbits turning at resonance
For modifications see M Cox et al Budapest

Determination of E_+ from Dispersion Relation

Maxwells Eqns lead to

$$\underline{n}\times(\underline{n}\times\underline{E}) + \underline{K}\cdot\underline{E} = 0 \qquad \text{for plane waves}$$

or

$$\begin{pmatrix} S-n_z^2 & -iD & n_x n_z \\ iD & S-n^2 & 0 \\ n_z n_x & 0 & P \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

see Theory of Plasma Waves by Stix

For MHD modes (low freq. high conductivity approximation)
$E_z \ll E_x, E_y$, $\perp$r wave fields (E_x, E_y) satisfy

$$\begin{pmatrix} S-n_z^2 & -iD \\ iD & S-n^2 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix} = 0$$

Stix, Nu Fusion 15 (1975) 73

Non trivial sols for det of square matrix = 0 ⇒ dispersion relation

$$n_\perp^2 = \frac{(R-n_\parallel^2)(L-n_\parallel^2)}{(S-n_\parallel^2)}$$

Takahashi J. de Physique 3rd Int Congress on Waves + Instabilities Palaiseau 1977 p C6-171 $\quad n_\parallel \equiv n_z \quad n_\perp \equiv n_x$

where

$$R = 1 - \sum_j \frac{\Omega_{pj}^2}{(1+\varepsilon_j\Omega_j)} \qquad \Omega_{pj} = \omega_{pj}/\omega$$

$$L = 1 - \sum_j \frac{\Omega_{pj}^2}{(1-\varepsilon_j\Omega_j)} \qquad \Omega_j = \omega_{cj}/\omega$$

$$S = \tfrac{1}{2}(R+L), \quad D = \tfrac{1}{2}(R-L)$$

Also $$\frac{E_+}{iE_y} = 1 + \frac{D}{(S-n_\parallel^2)}$$

Cut-offs for $(R-n_\parallel^2)=0$, $(L-n_\parallel^2)=0$. Two-ion hybrid resonance for $(S-n_\parallel^2)=0$

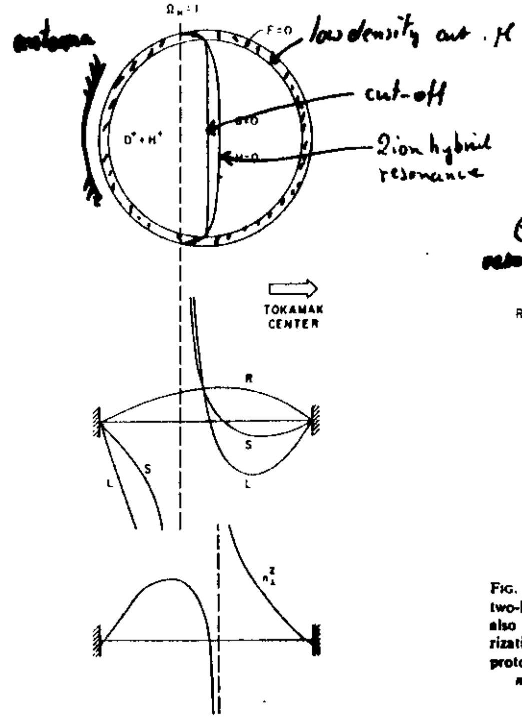


FIG. 6. — (a) Geometry of two-ion hybrid resonance/cut-off pair in a D-H plasma ($C_H = 10$ %). The Tokamak center is to the right of the figure. In the shaded regions the fast wave is evanescent. (b) Variation of the cold plasma dielectric tensor elements along the meridian chord for the case shown in (a). (c) Variation of the square of the perpendicular refractive index.

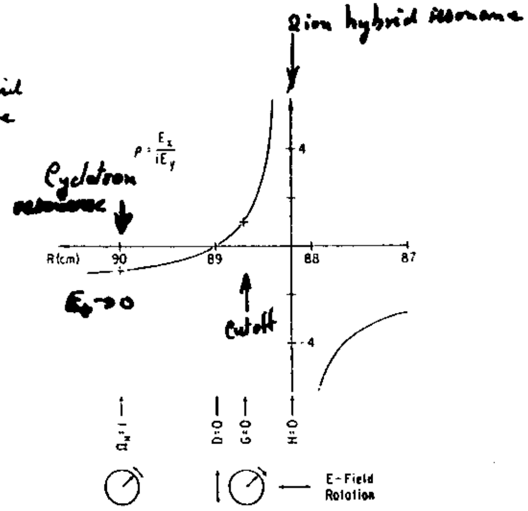


FIG. 7. — Polarization of the fast wave electric field near two-ion hybrid resonance. Rotation of the E-field vector is also indicated (B-field is upward). Rapid variation of the polarization over a small distance between the hybrid and proton cyclotron resonance is notable. A cold plasma with $n_{e0} = 2.5 \times 10^{19}/m^3$, $C_H = 3$ % and $n_\parallel = 12$ are assumed.

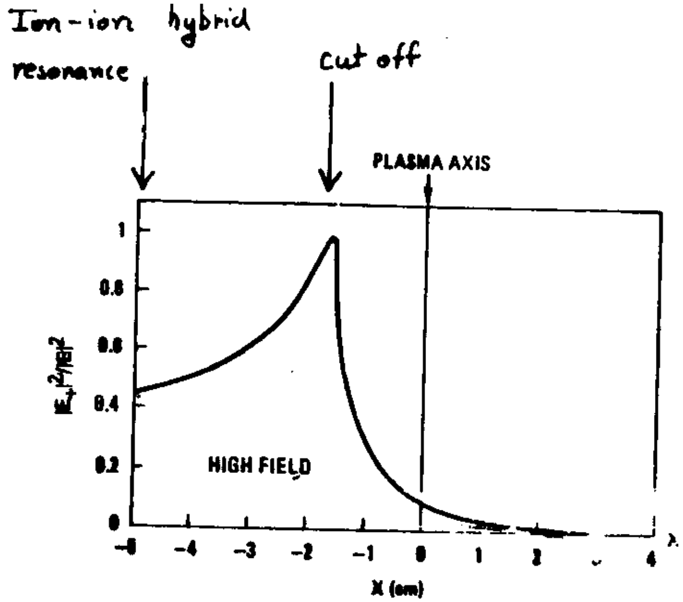
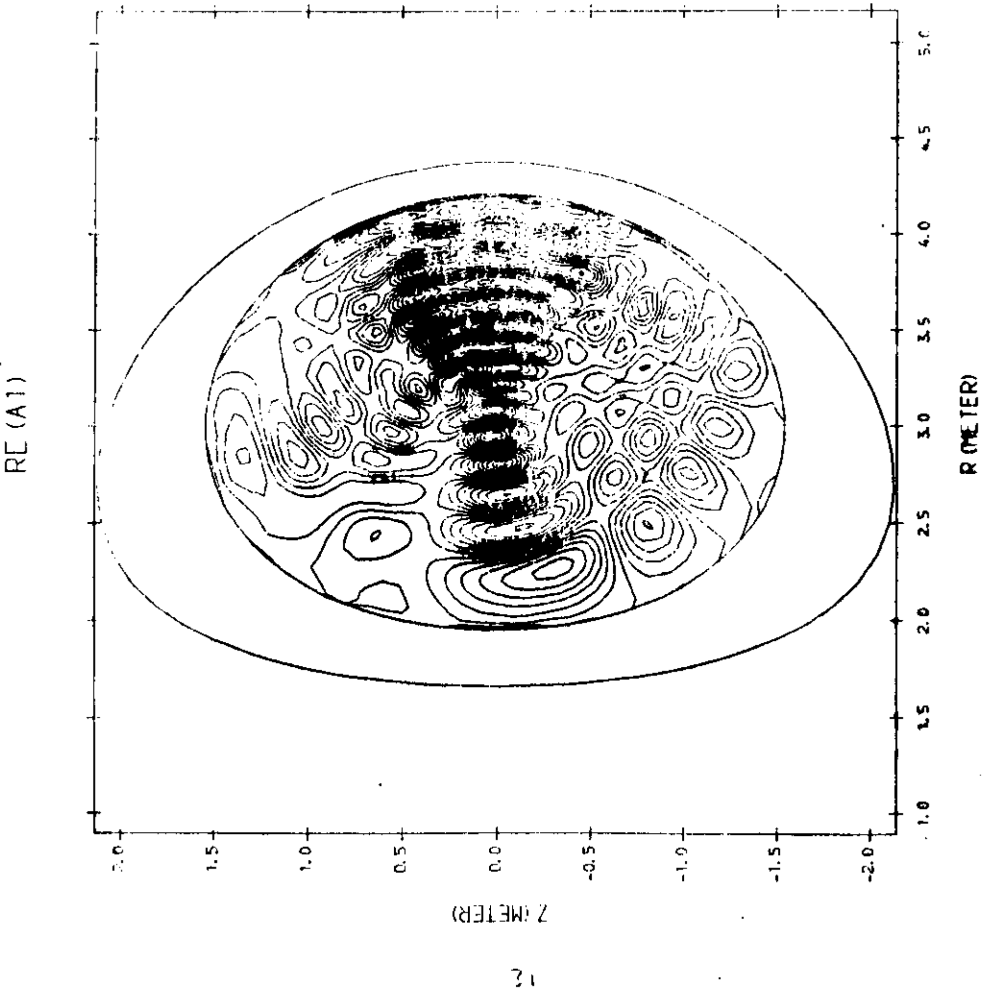


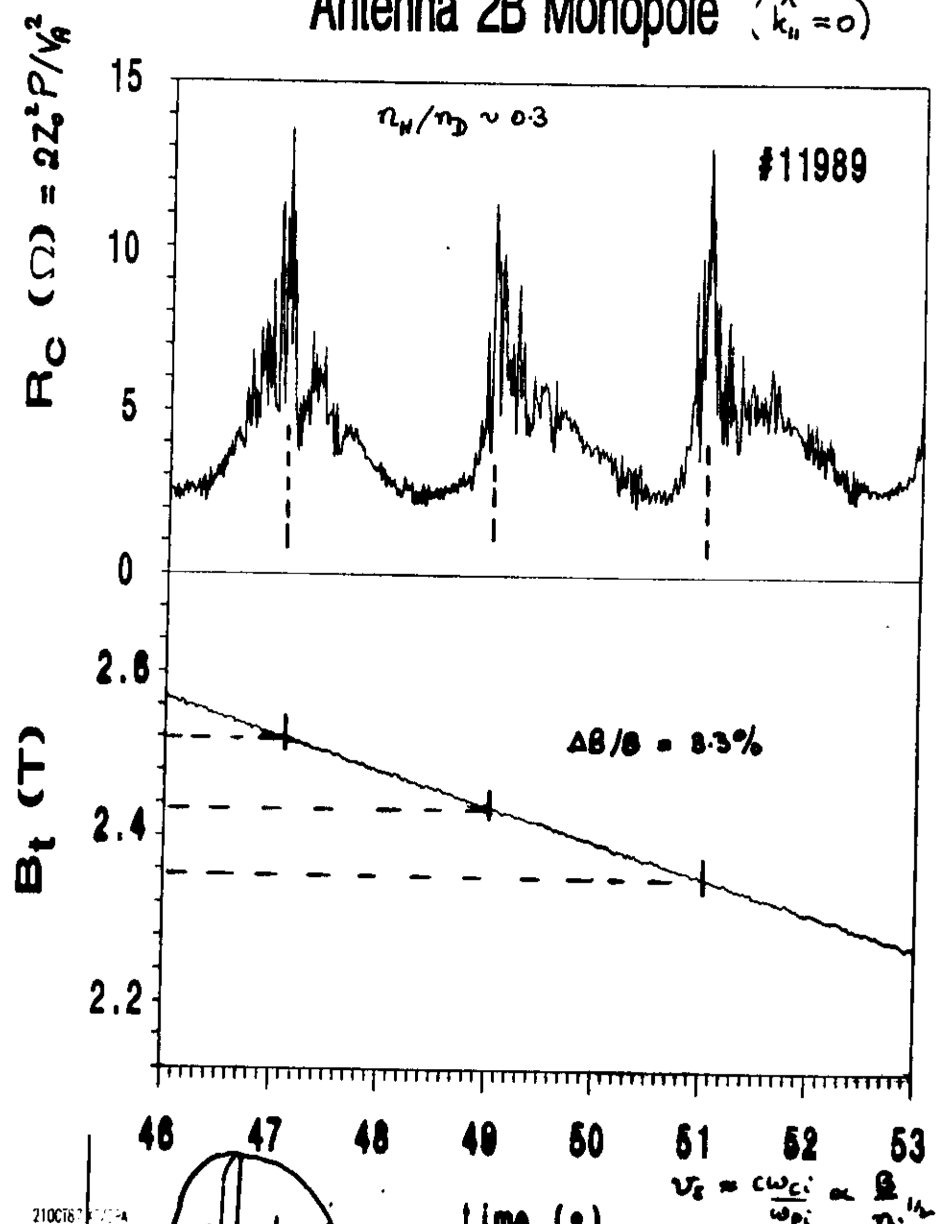
FIG. 5. Fraction of left-hand component of the total power as a function of major radius for a D-H plasma. Plasma parameters are as follows: $T_D = T_h = 4$ keV, $n_D = 10^{14}/cm^3$, $n_h = 5 \times 10^{12}/cm^3$, $B_0 = 20$ kG, and $R_0 = 200$ cm. $x < 0$ points to the high-field side; plasma axis is at $x = 0$.

JET FULL WAVE CODES
ALCYON (CADARACHE)
LION (LAUSANNE)

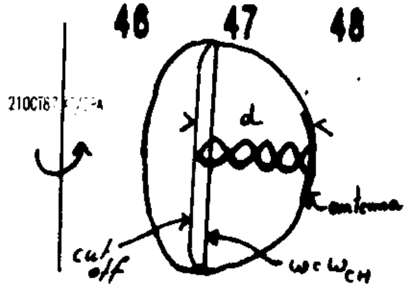


First observation of radial eigenmodes

Antenna 2B Monopole ($\hat{k}_\parallel = 0$)



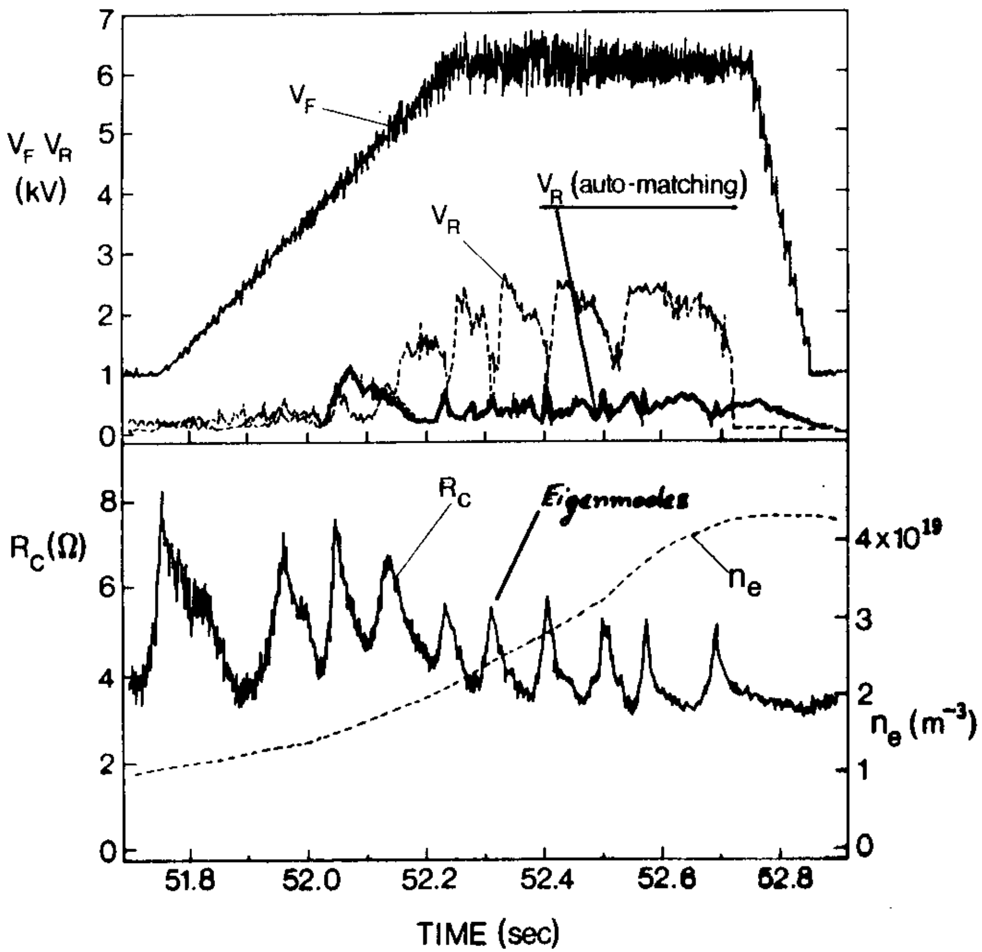
$v_g = \frac{c\omega_{ci}}{\omega_{pi}} \propto \frac{B}{n_i^{1/2}}$



$\Delta\phi = \pi = \Delta k_\perp d + k_\perp \Delta d$
$= k_\perp \frac{\Delta B}{B}(R_0 + d)$

$0.9\% < \Delta B < 3.7\%$ slab model

Fast Auto-matching via Frequency Control



Fast magnetosonic wave

Mode conversion $\boxed{\text{zone 2}}$

5%

to e up to 10% when $|k_\parallel| < 2m^{-1}$

Cyclotron resonance $\boxed{\text{zone 1}}$

80% — to H or He^3 minority resonance — $T_{tail} \sim 1MeV$

coll. 5% — bulk ions

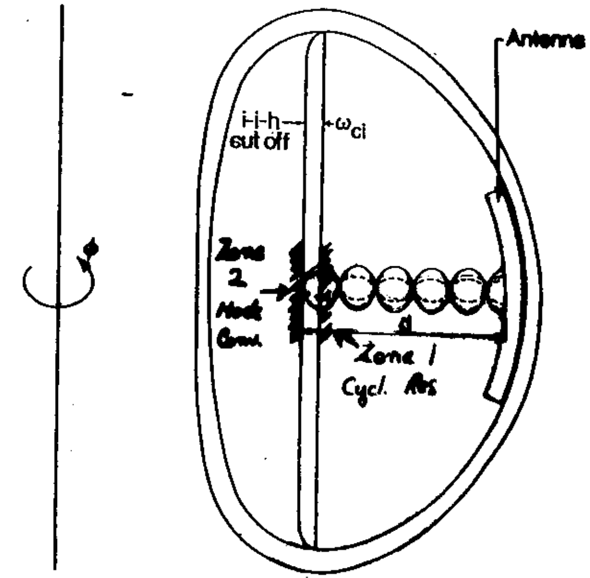
drag 75% — e

10% — to D 2nd harmonic

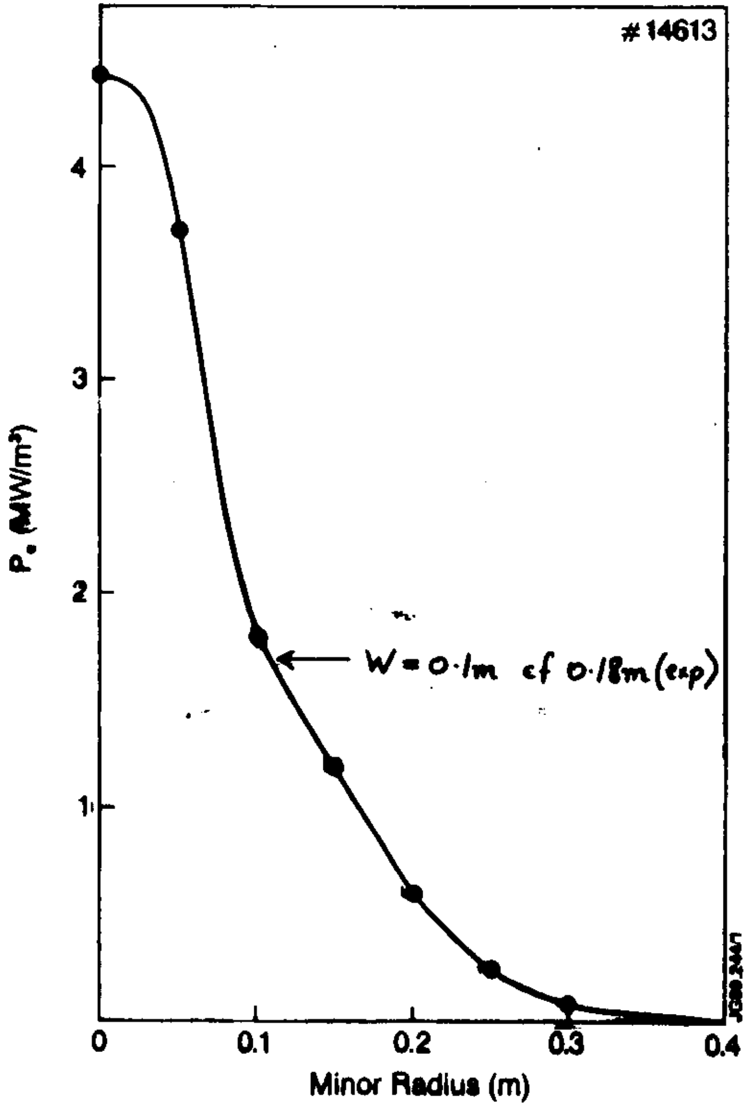
5% — to e landau damping

85% to electrons

15% to ions

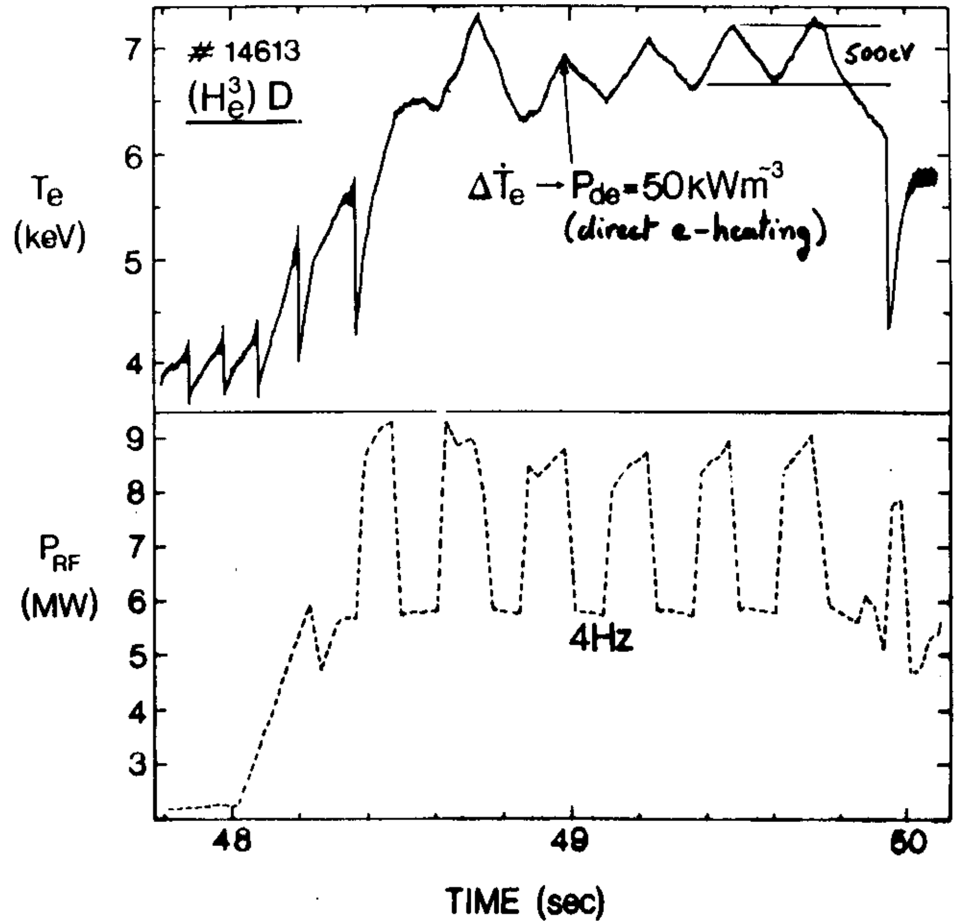


Theoretical Minority → Electron Power Transfer Profile

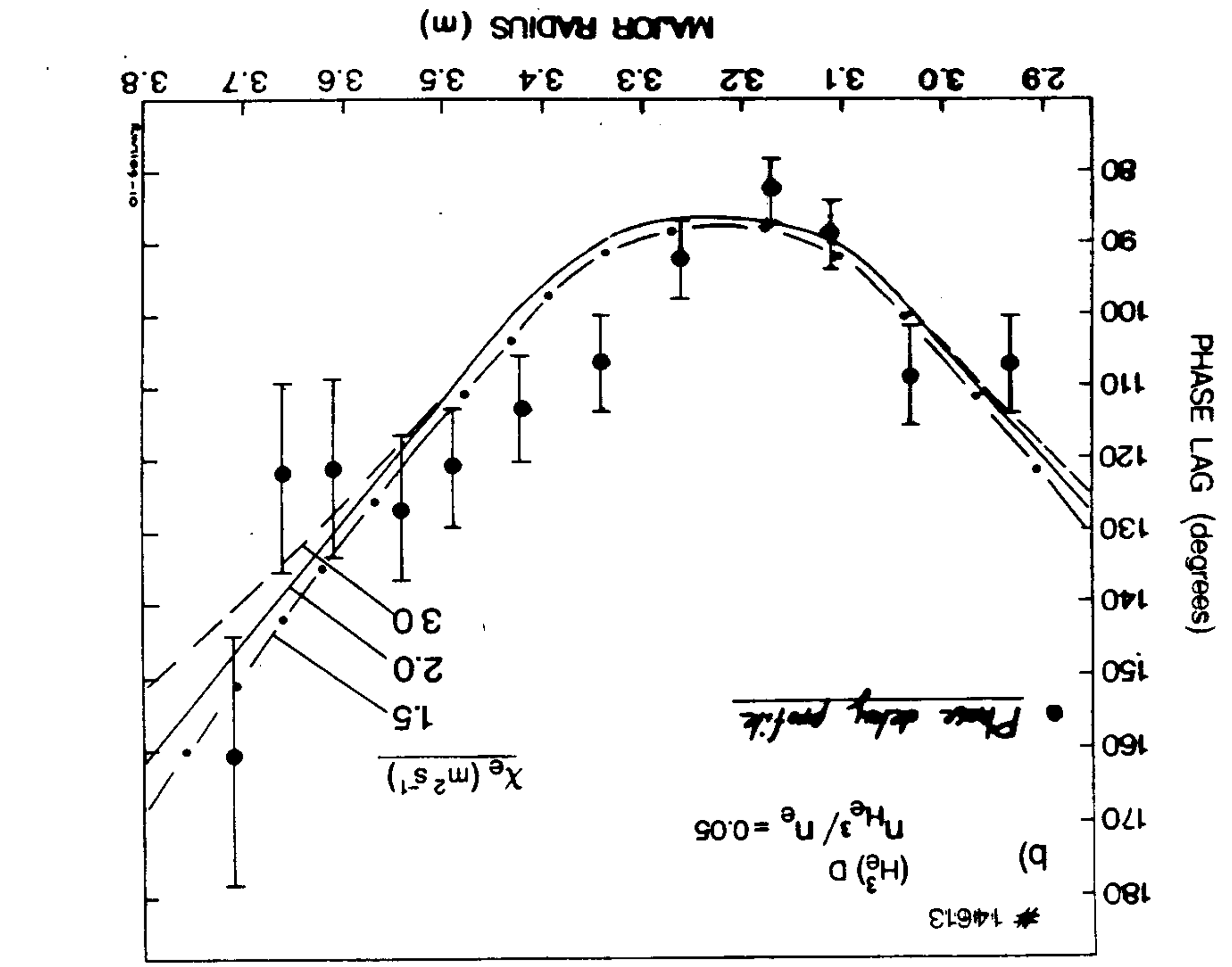


ICRH Power Modulation Experiments during Monster Sawteeth

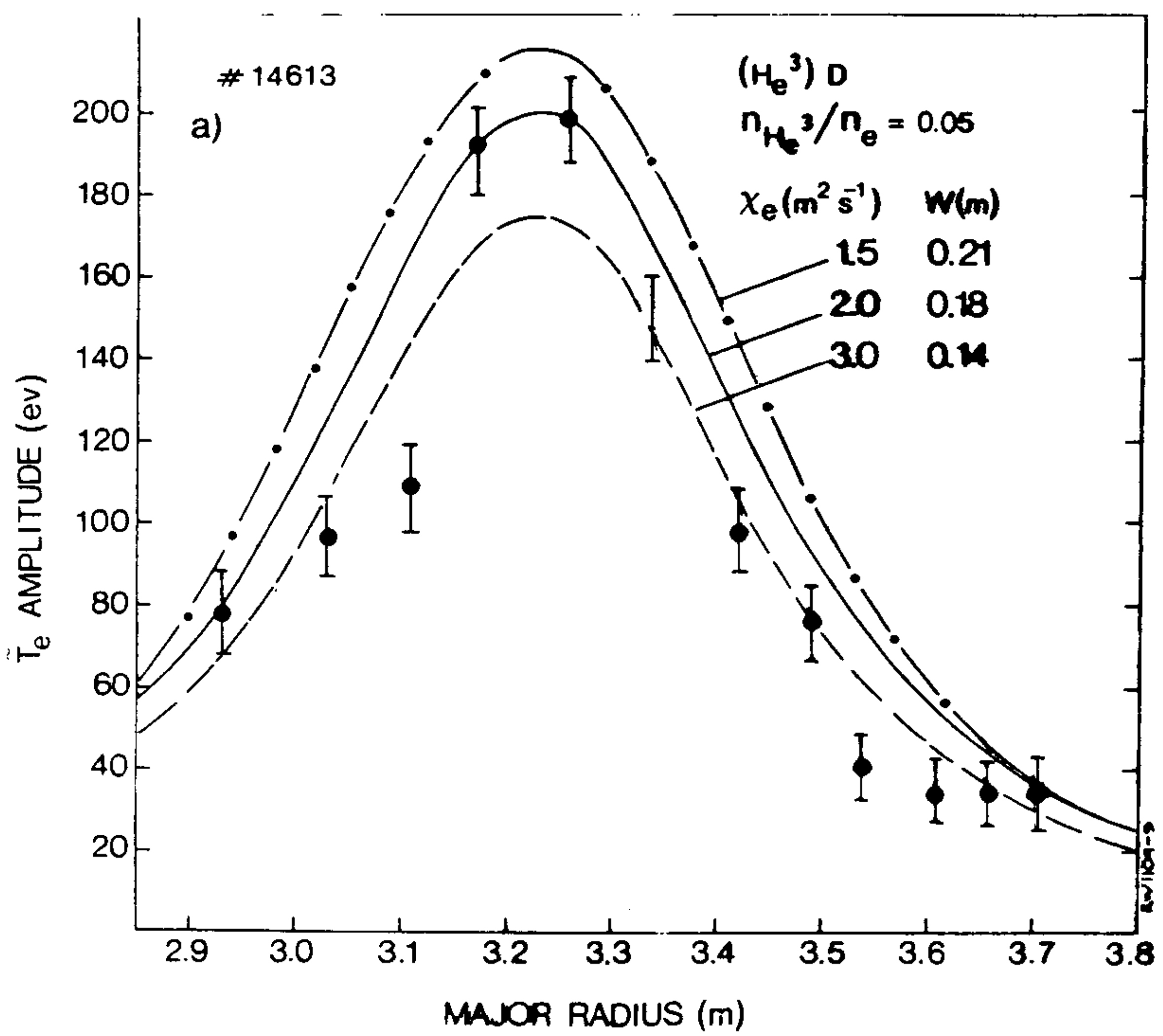
RF Modulation



Measurements of
- χ_e
- *Power deposition*
- *Direct electron heating*



14613
b)
(H_e^3) D
$n_{H_e^3}/n_e = 0.05$
● Phase delay profile
$\chi_e\,(m^2 s^{-1})$
1.5
2.0
3.0
PHASE LAG (degrees)
MAJOR RADIUS (m)



14613
a)
(H_e^3) D
$n_{H_e^3}/n_e = 0.05$
$\chi_e\,(m^2 s^{-1})$ W(m)
1.5 0.21
2.0 0.18
3.0 0.14
$\tilde{T}_e$ AMPLITUDE (ev)
MAJOR RADIUS (m)

HEAT DIFFUSION MODEL

Electron Heat Diffusion Eqn.

$$\frac{3}{2}\frac{\partial}{\partial t}(n_e \tilde{T}_e) = \frac{1}{r}\frac{\partial}{\partial r}\left(r n_e \chi_e \frac{\partial \tilde{T}_e}{\partial r}\right) + \tilde{P}_d$$

$$\tilde{P}_d = \frac{\tilde{P}_{RF}\, e^{-r^2/w^2}}{\pi w^2 (1+R)}\left[\underbrace{\cos\omega t}_{\text{direct electron heating}} + \underbrace{\frac{R}{\left[1+\left(\frac{\omega\tau_s}{2}\right)^2\right]^{1/2}}\cos(\omega t-\theta)}_{\text{collisional heat transfer from minority}}\right]$$

2 free parameters

χ_e, W

$R = \tilde{P}_{em}/\tilde{P}_{ed}$, $\quad \theta = \arctan\left(\frac{\omega\tau_s}{2}\right)$

$n_e \chi_e = \text{const}$ $\quad n_e(r)$ parabolic

Solution

$$\tilde{T}_e(r,t) = K\tilde{T}_{e0}(r)\cos(\omega t - \phi(r) - \theta')$$

$$K = \sqrt{\frac{\left(1+\left(\frac{\omega\tau_s}{2}\right)^2+R\right)^2 + R^2\left(\frac{\omega\tau_s}{2}\right)^2}{1+\left(\frac{\omega\tau_s}{2}\right)^2}}$$

$$\theta = \arctan\left[\frac{\frac{\omega\tau_s}{2}\cdot R}{1+\left(\frac{\omega\tau_s}{2}\right)^2+R}\right]$$

GR87.926

Applications

- High Fusion Yield Experiments with $(He^3)D$ minority ICRH
- Heating of Peaked Density Profile Discharges
  — especially suited to localized power deposition.
- Heating H-mode Plasmas

High Fusion Yield Experiments using $(He^3)D$ minority ICRH

$$He^3 + D \rightarrow He^4 + p$$

$$\rightarrow Li^5 + \gamma$$

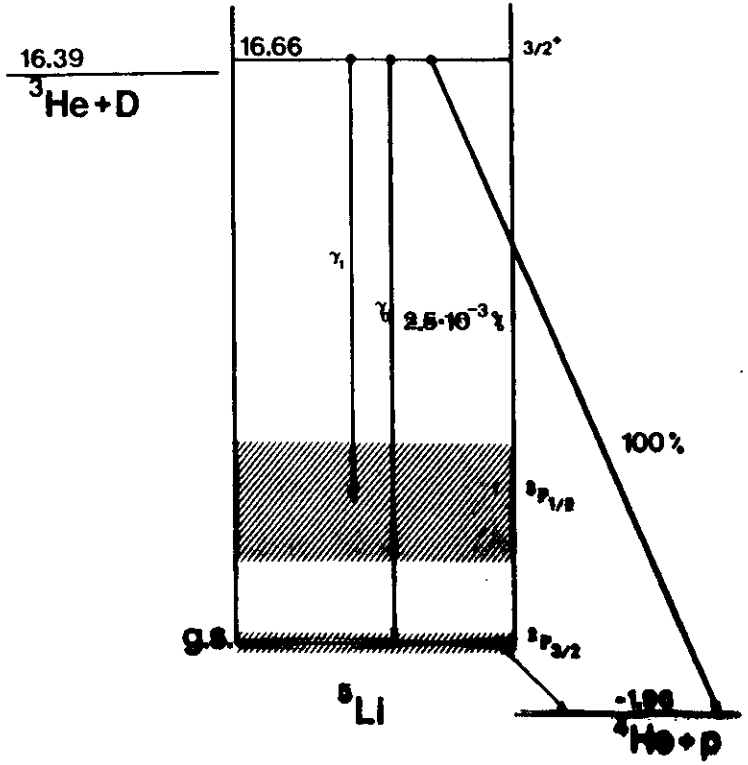


Fig 3a

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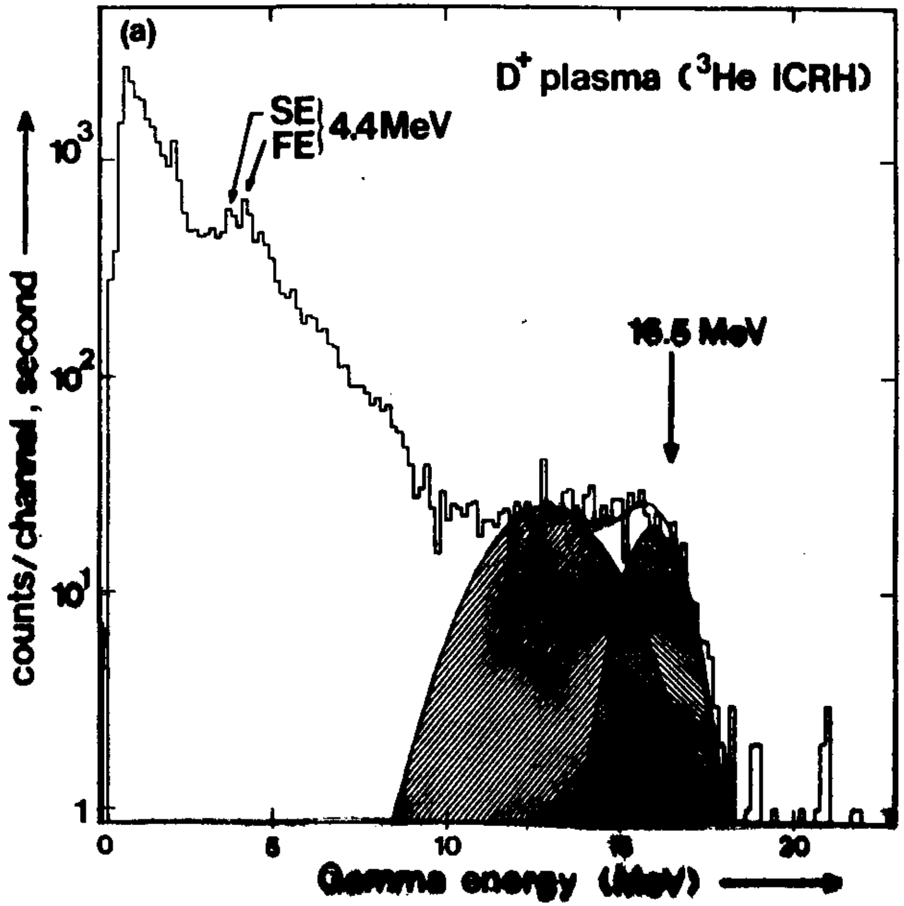
He3-D Fusion Studies



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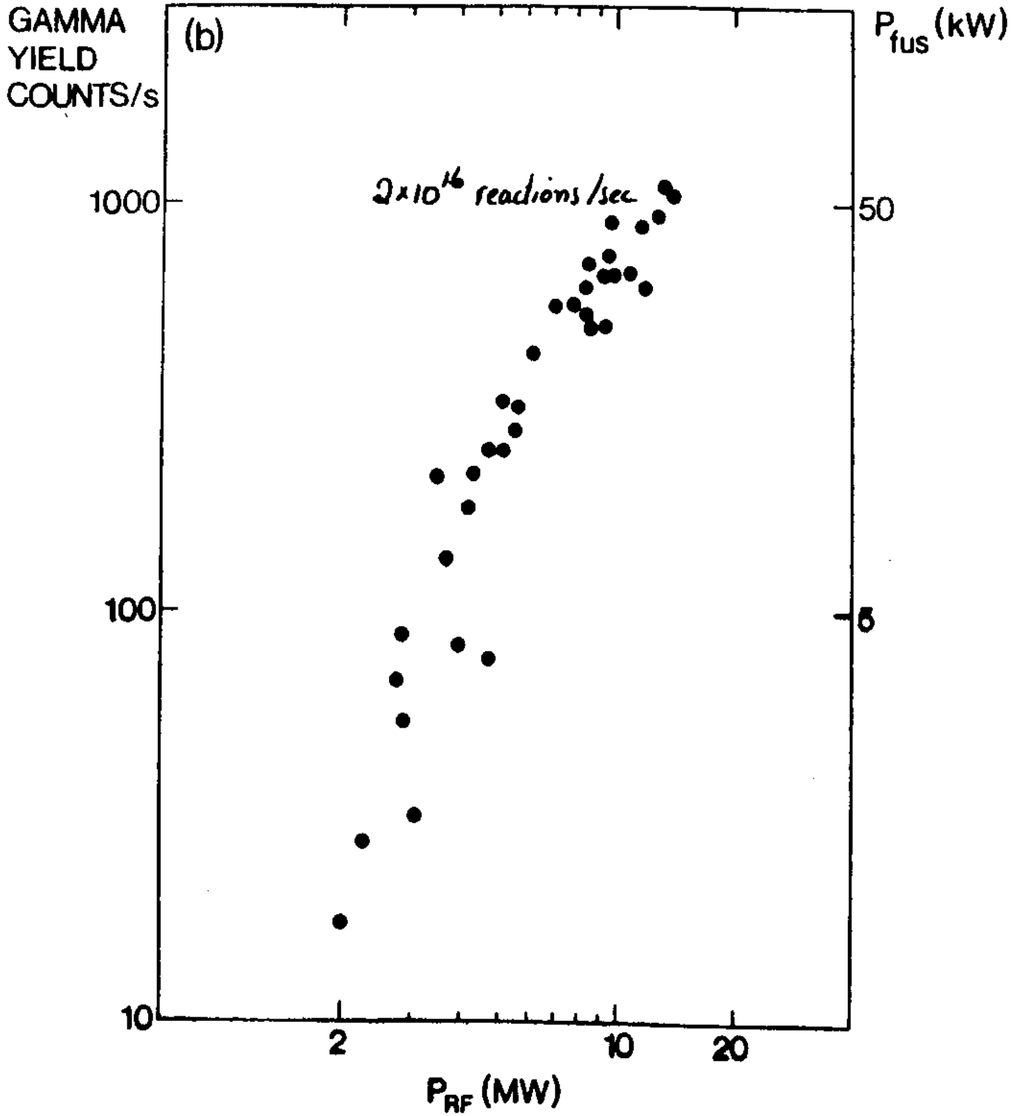
He^3-D Fusion Yield from Fundamental (He^3)D ICRH



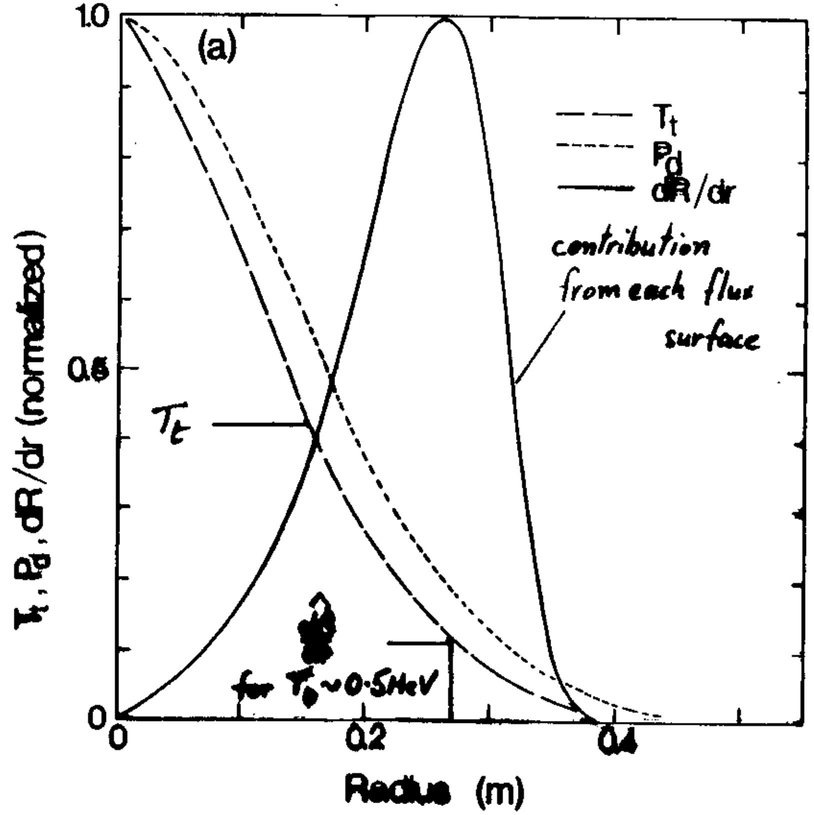
Comparison with Theory

Stix model for f_{min} and P_{fusion}

Stix Model

Reactions/sec inside vol of radius r

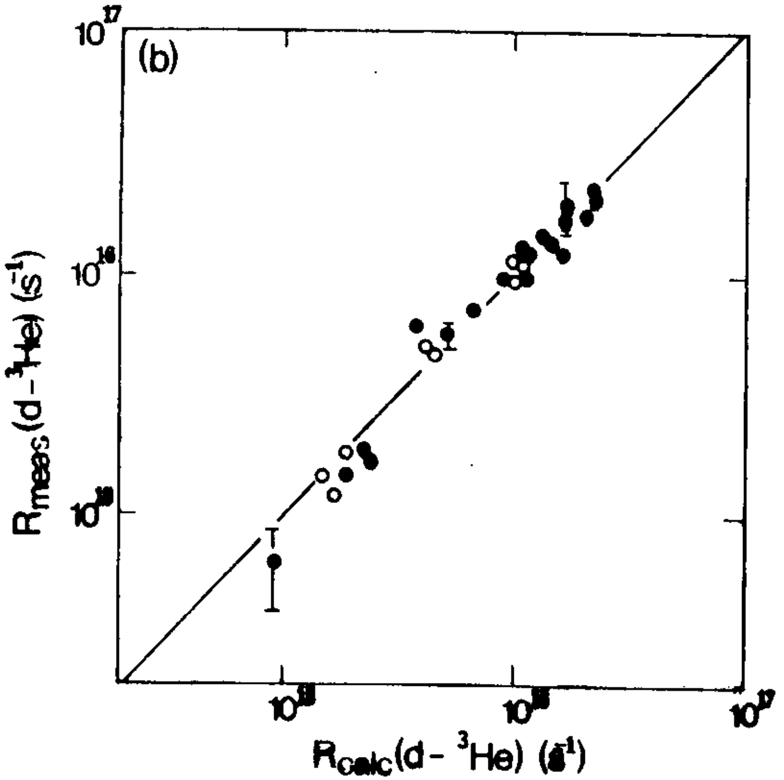
$$R(s^{-1}) = 2.10^{15}\int_0^r AZ^2 \ln\lambda \cdot \underbrace{\frac{n_D}{n_e}}_{\text{dilution factor}} \cdot P_{RF}\, T_e^{3/2} \langle\sigma v\rangle / T_t \cdot r\,dr$$



Substantial overheating of He^3 — profile tailoring for improved yield

Stix Model Versus Experiment

Non-thermal fusion using $(He^3)D$ ICRF



Non-Thermal D-T Fusion from (D)T ICRH

15% – 30% D concentration

Simulation based on:-

1) Stix model for F_{min} and P_{fusion} validated against $(He^3)D$ fusion yield experiments.

2) JET High Performance Plasmas

   a) Peaked density profiles, pellet fuelled (3MA)

   b) Current rise heating (5MA)

   c) Monster sawteeth (3MA)

Extrapolated to $P_{RF} \sim 25MW$ using exp off-set linear scaling

$T_e(0) = \alpha + \beta P_{total}/n_e(0)$ etc

3) P_{min}/P_{RF} from ray tracing.

Ex $f = 25MHz$, $B_T = 3.55T$, 30% D, pellets (a)

$P_{min}/P_{RF} \sim 80\%$

$Q_{RF} = P_{fus} / P_{RF}$

a) *Peaked density* (PD) 0·75

b) Current rise (CR) 0·68

c) *Monster sawteeth* (MS) 0·55

$n_D / n_e = 30\%$, $Z_{eff} = 2$, ICR 30 cm off axis

$P_{RF} = 24\,MW$, $P_{ic} = 20\,MW$ actually achieved in CR