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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SPRING COLLEGE ON PLASMA PHYSICS

15 May - 9 June 1989

ALFVENIC TURBULENCE IN BEAM-PLASMA SYSTEMS (II)

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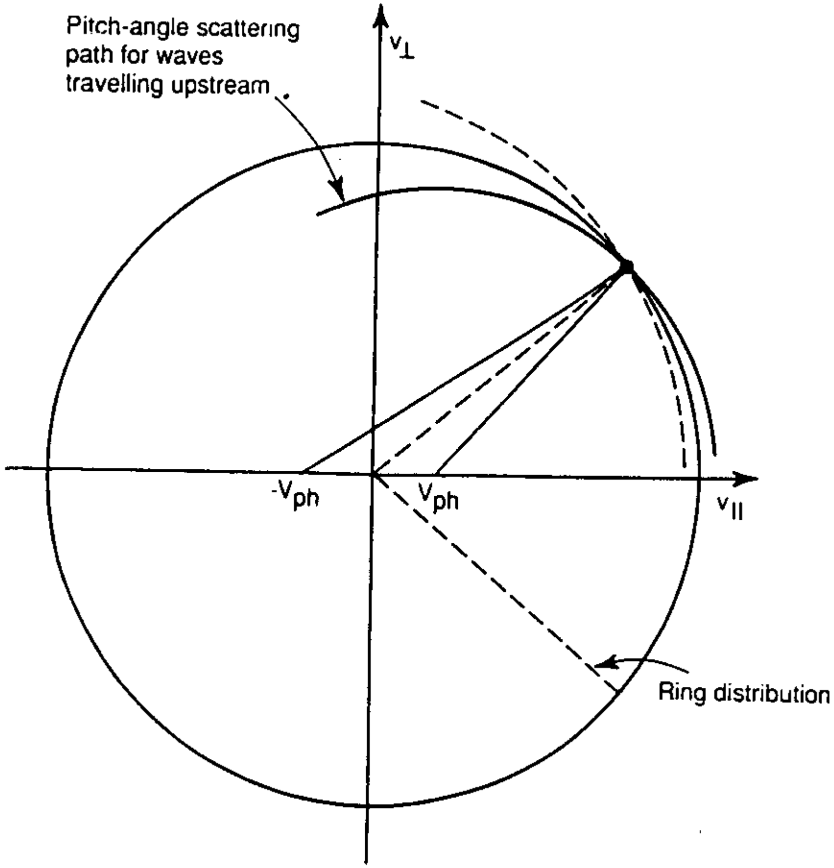


$$V_\perp^2 + (V_{||} \pm V_{ph})^2 = \text{const}$$

e.g. Alfvenic turbulence $V_{ph}=V_A$

Figure 1 - Schematic diagram showing the pitch-angle scattering path of injected ions from their initial ring distribution to a shell centred on the wave phase velocity.

DIFFUSION PATH IN VELOCITY SPACE

$$V_\perp^2 + (V_{||} \pm V_A)^2 = \text{constant}$$

- ⊛ particle energy conserved in the wave frame
- ⊛ energy and momentum conserved in wave-particle interaction

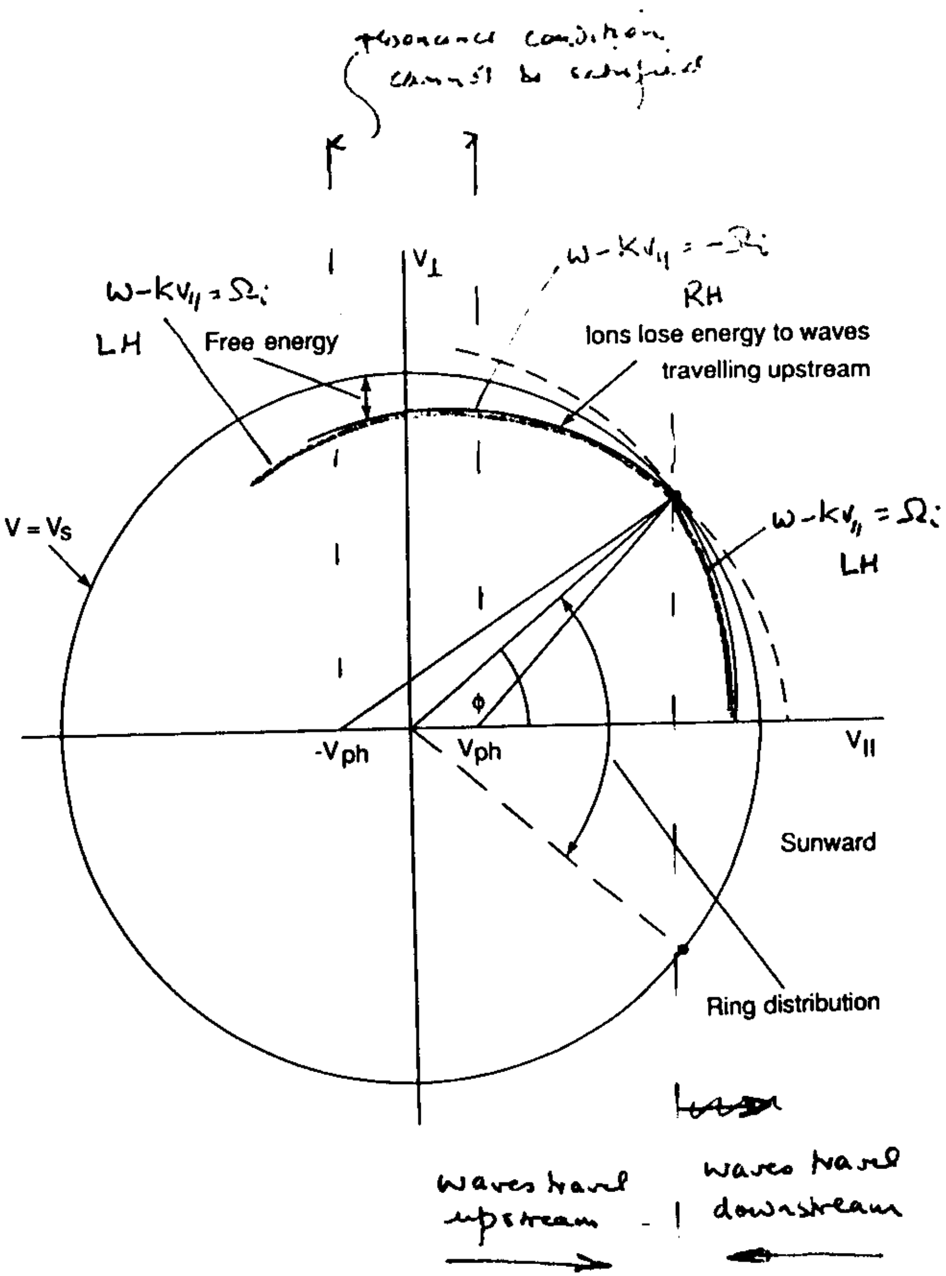
How does energy and momentum change in diffusion process?

Ring	bulk kinetic energy	$\frac{1}{2}\rho_i V^2 \cos^2\alpha$
	thermal energy	$\frac{1}{2}\rho_i V^2 \sin^2\alpha$
Shell	bulk kinetic energy	$\frac{1}{2}\rho_i V_A^2$
	thermal energy	$\frac{1}{2}\rho_i (V^2\sin^2\alpha + (V\cos\alpha - V_A)^2)$

$$E_{ring} - E_{shell} = \rho_i [V_A V\cos\alpha - V_A^2] \Rightarrow \text{"free energy"}$$

$$P_{ring} - P_{shell} = \rho_i (V\cos\alpha - V_A) = \frac{\Delta E}{V_A}$$

Acceleration requires waves travelling in both directions



POYNTING VECTOR $\vec{S}$

$$\vec{S} = \vec{E}_1 \times \vec{H}_1 = \vec{E}_1 \times \vec{B}_1/\mu_0$$

$\vec{B} = \vec{B}_0 + \vec{B}_1$ — B_0 — IMF; B_1 — wave field

$\vec{V} = \vec{V}_0 + \vec{V}_1$ — V_0 — solar wind velocity; V_1 — wave velocity.

$$\vec{E}_1 = -\vec{V}_1 \times \vec{B}_0$$

$$\vec{S} = (\text{wave energy density}) \times (\text{propagation velocity}) \times \vec{k}$$

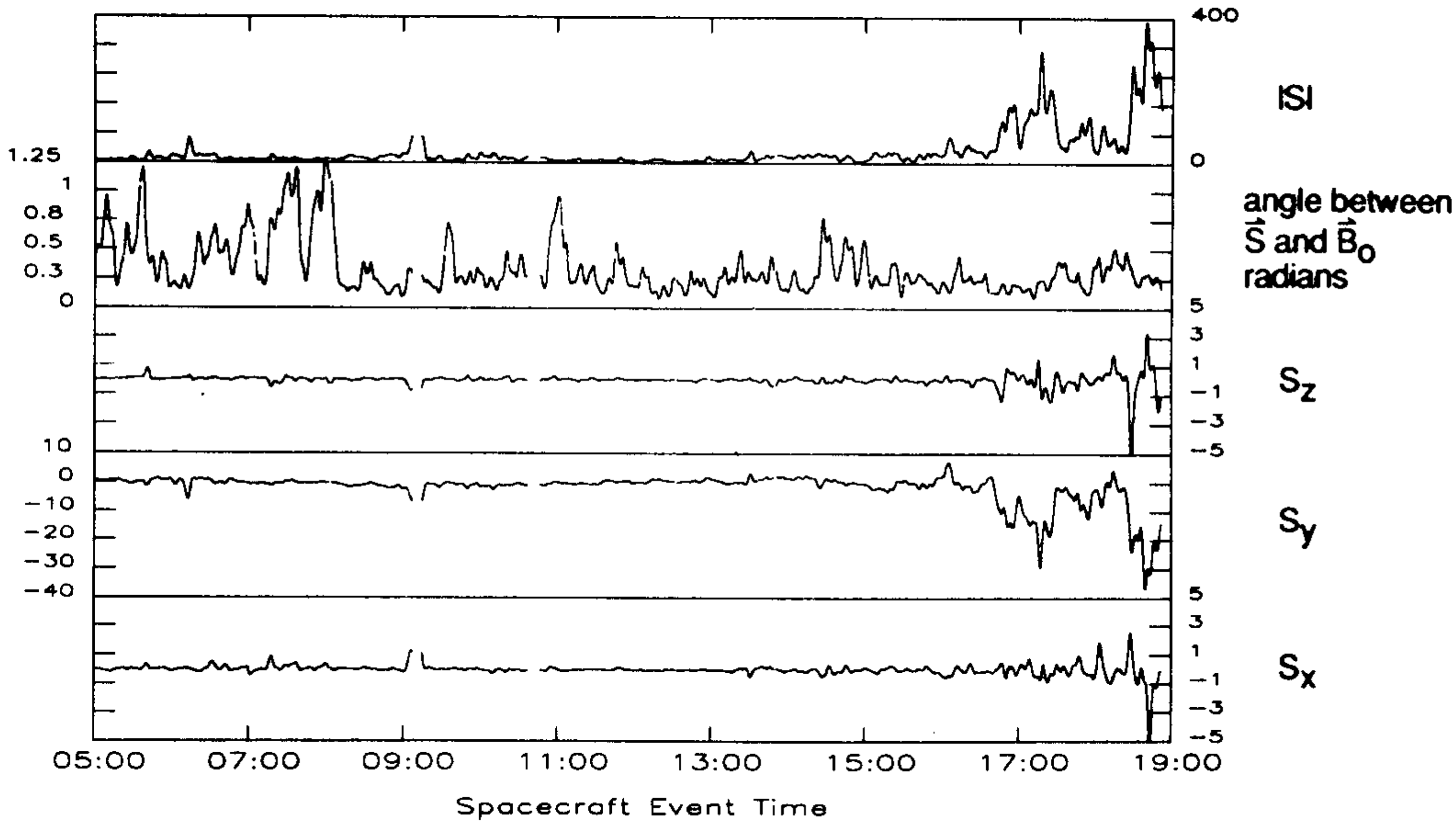
wave energy density $\frac{1}{2}\rho\langle V_1^2\rangle + \frac{1}{2}\langle B_1^2/\mu_0\rangle$

free energy for ion beam instability

$$\frac{1}{2}\rho_i V_0^2 \cos^2\phi$$

POYNTING VECTOR $\vec{S}$ (pJ/m^3 x km/s)

Year 1986 Day 72 FLD coordinates



TRANSPORT EQUATIONS

For particles, the Boltzmann equation

$$\frac{\partial f}{\partial t} + \vec{v}.\nabla f + \frac{\vec{F}}{m}\nabla_v f = \left[\frac{\partial f}{\partial t}\right]_{\text{collisions}} + \text{Source} - \text{Loss}.$$

For waves, we require a suitable kinetic equation which relates to the particle distribution

QUASI LINEAR THEORY

Lee + Ip JGR 92, 11041, 1987
Gaffey et al JGR 93, 5470, 1988

$$\frac{\partial F_i}{\partial t} = \frac{q_i^2}{2m_i^2 v_\perp} R_e\, i \int_D dk \sum_{p=\pm} \frac{(\omega_p)^2}{c^2k^2} G^p \left[\frac{v_\perp I_p(k)\, G^p F_i}{(\omega_p - kv_\parallel + \Omega_i)}\right]$$

$$\frac{\partial I_p}{\partial t} = 2\gamma_p I_p$$

If waves are propagating in one direction can transform to wave frame

- ⊛ diffusion path is circle centred on origin
- ⊛ G is simplified
- ⊛ pure pitch angle diffusion

$$\frac{\partial F_i}{\partial t} = \frac{\partial}{\partial \mu}\left[\frac{\pi q^2(1-\mu^2)}{2m^2c^2 v|\mu|} I_+\left(\frac{\Omega_i}{v\mu}\right)\frac{\partial F}{\partial \mu}\right]$$

$$\frac{\partial I_+}{\partial t} = I_+ \frac{4\pi^3 v_A q^2}{|k|\, c^2 m} \int dv\, d\mu\, v^2(1-\mu^2)\,\delta\left(\mu - \frac{\Omega_i}{kv}\right)\frac{\partial F_i}{\partial \mu}$$

$$\mu = \frac{v_\parallel - v_A}{v}$$

WAVE ENERGY CONSIDERATIONS

Integrating over k the total energy in the magnetic waves is

$$\frac{\langle \delta\vec{B}\cdot\delta\vec{B}\rangle}{8\pi} = \frac{1}{2} n_i m_i V_A v_0\mu_0$$

Total wave energy is twice this because equal amount in kinetic energy of the particles

Also $v_0\mu_0 = V\cos\alpha - V_A$ in solar wind frame

Hence $E_{waves} = n_i m_i V_A (V\cos\alpha - V_A)$

i.e. $E_{waves} = E_{ring} - E_{shell}$.

TIME ASYMPTOTIC DISTRIBUTIONS

Lee & Ip JGR 92, 11041, 1987

Put $F_i = \delta(v - v_0) f(\mu, t)$

Integrate both equations over time

$$f(\mu,\infty) - f(\mu,0) = \frac{\partial}{\partial\mu}\left[A(\mu)\int_0^\infty dt\, I_+\left(\frac{\Omega_i}{k v_0 \mu}\right)\frac{\partial f}{\partial\mu}\right]$$

$$I_+(k,\infty) - I_+(k,0) = B(|k|)\int_{-1}^{1} d\mu\,(1-\mu^2)\,\delta\left(\mu - \frac{\Omega_i}{k v_0}\right)\int_0^\infty dt\, I_+ \frac{\partial f}{\partial\mu}$$

$$A(\mu) = \frac{\pi}{2}\left(\frac{q}{mc}\right)^2 \frac{(1-\mu^2)}{v_0|\mu|} \qquad B(|k|) = \frac{4\pi^3 V_A q^3 v_0^2}{|k|\, m c^2}$$

$$f(\mu,0) = \frac{n_i}{2\pi v_0^2}\,\delta(\mu-\mu_0) \qquad f(\mu,\infty) = \frac{n_i}{4\pi v_0^2}$$

Then

$$I_+(k,\infty) - I_+(k,0) = \frac{2\pi n_i m_i V_A |\Omega_i|}{k^2}\left[\frac{\Omega}{k v_0} - \frac{\left(\frac{\Omega}{k v_0} - \mu_0\right)}{\left|\frac{\Omega}{k v_0} - \mu_0\right|}\right]$$

INTERPRETATION OF WAVE FREQUENCIES

1) The resonance condition is

$$\omega - kV_{\parallel 0} \pm \Omega_i = 0$$

where $V_{\parallel 0}$ is the ring beam velocity parallel to the solar wind.

2) Waves are propagating parallel to B in the moving plasma and are Doppler shifted in the spacecraft frame

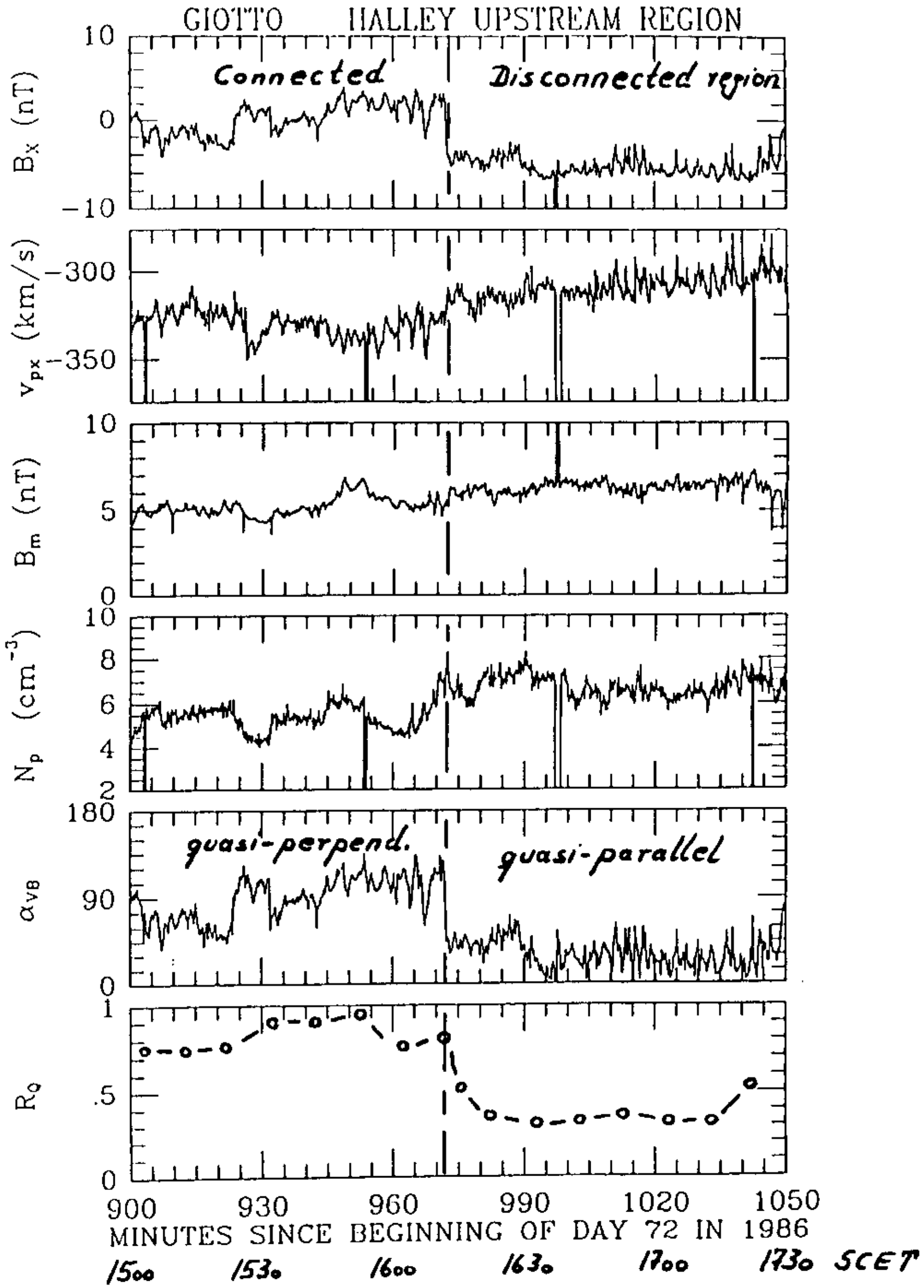
3) Spacecraft velocity is also $V_{\parallel 0}$ in solar wind frame

Waves are ∴ seen at $\omega - kV_{\parallel 0} = \Omega_i$

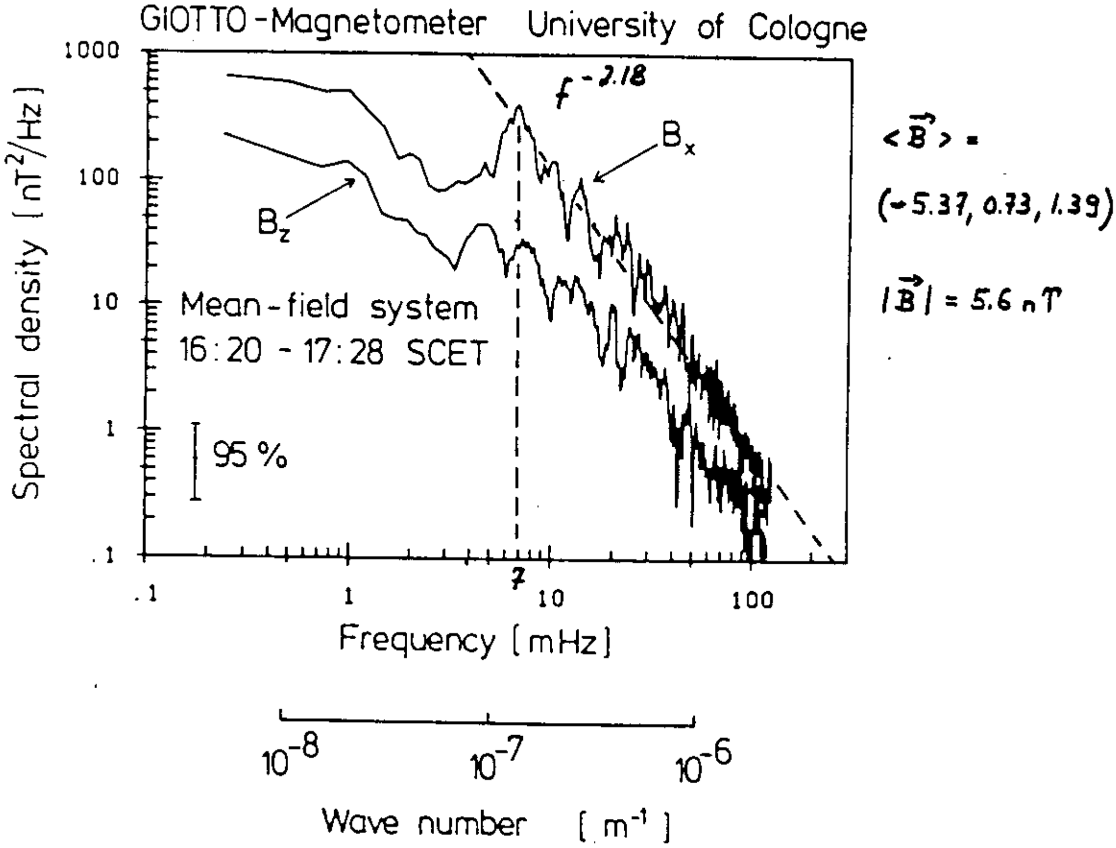
4) Relation between ω_{sc} (s/c frame) and k is

$$\omega = (V_{\parallel 0} + V_A)k$$

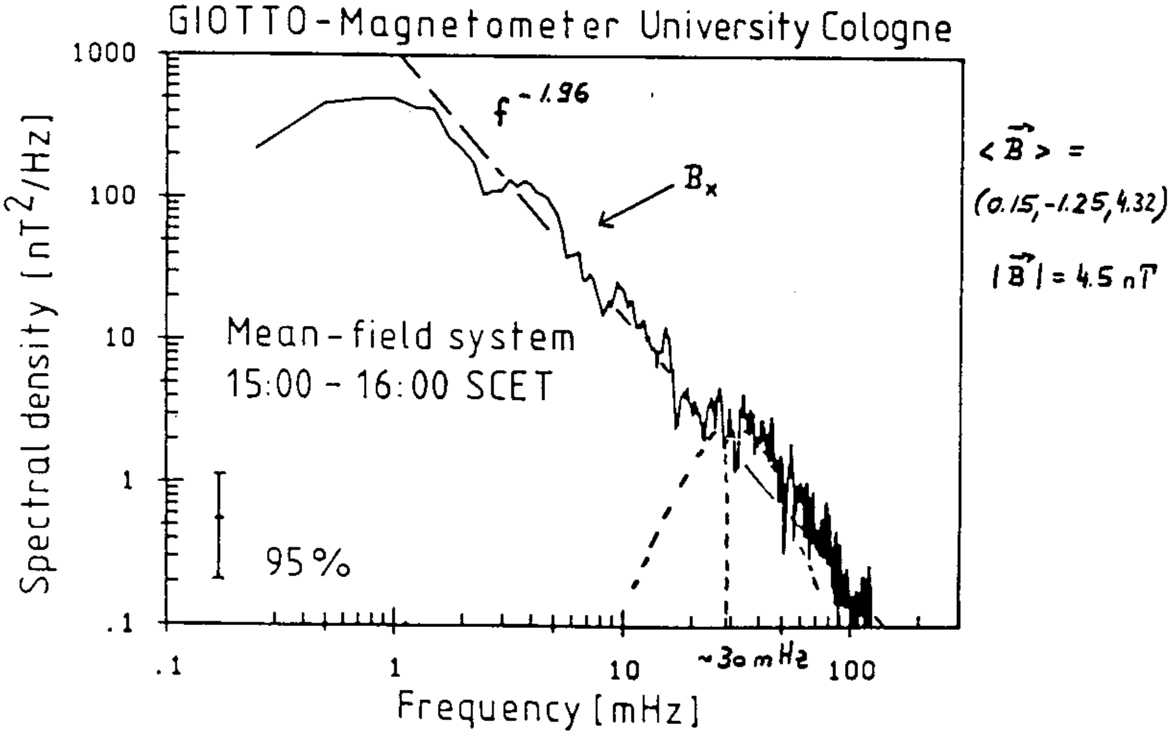
for waves propagating upstream

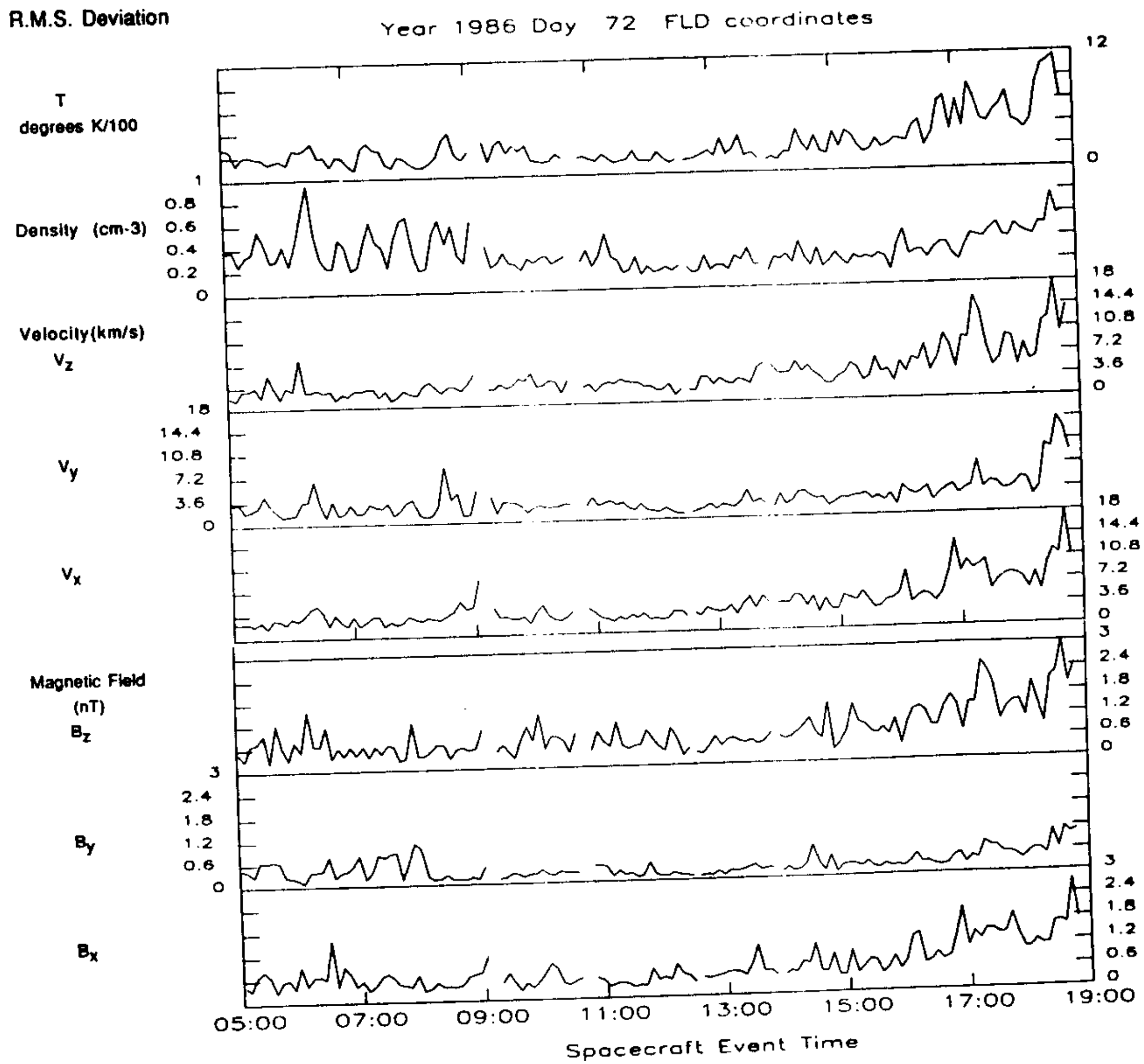


Disconnected, quasi-parallel region



Connected, quasi-perpendicular region




R.M.S. Deviation
Year 1986 Day 72 FLD coordinates
T
degrees K/100
Density (cm-3)
Velocity(km/s)
V_z
V_y
V_x
Magnetic Field
(nT)
B_z
B_y
B_x
05:00
07:00
09:00
11:00
13:00
15:00
17:00
19:00
Spacecraft Event Time

WAVE ENERGY DENSITY

The wave energy density is obtained by adding the energy densities of the magnetic and electric components :-

$$W = \frac{1}{2}\left(\rho < v_1^2 > + \frac{< B_1^2 >}{\mu_0}\right)$$

where $< v_1^2 >$ and $< B_1^2 > / \mu_0$ are the mean square amplitudes of the waves.

POYNTING VECTOR

The poynting vector, S, for the wave is defined as,

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

where $\mathbf{E}_1$ and $\mathbf{H}_1$ are the wave fields and may be obtained from the data by putting

$$\mathbf{E}_1 = \mathbf{v}_1 \times \mathbf{B}_0$$

where $\mathbf{B}_0$ is the interplanetary magnetic field and $\mathbf{v}_1$ is the velocity wave amplitude, and,

$$\mathbf{H}_1 = \frac{\mathbf{B}_1}{\mu_0}$$

giving,

$$\mathbf{S} = (\mathbf{v}_1 \times \mathbf{B}_0) \times \frac{\mathbf{B}_1}{\mu_0}$$

FINDING GROUP VELOCITY

It can be shown that S is equal to the wave energy density multiplied by the wave group velocity. i.e.

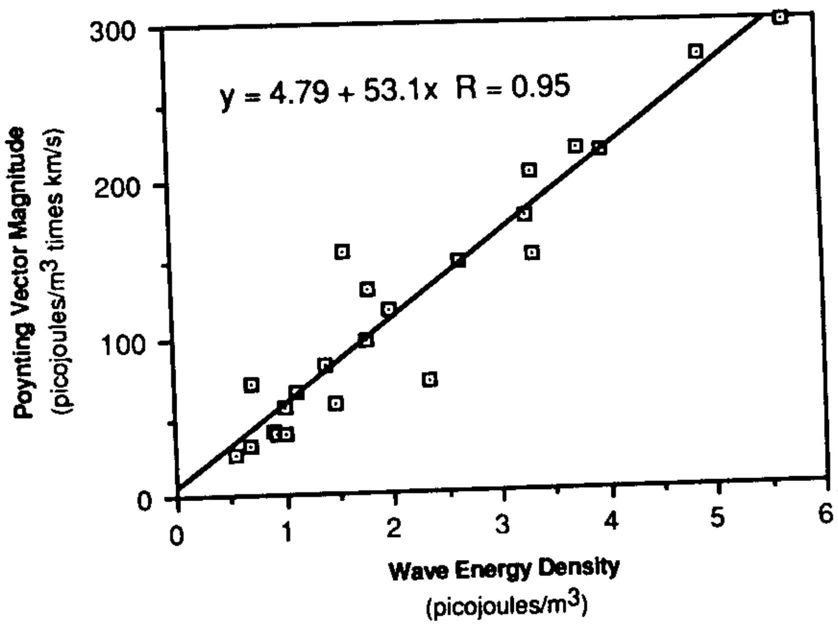
$$S = \frac{1}{2}\left(\varepsilon_0 E_1^2 + \frac{B_1^2}{\mu_0}\right) V_g\, k$$

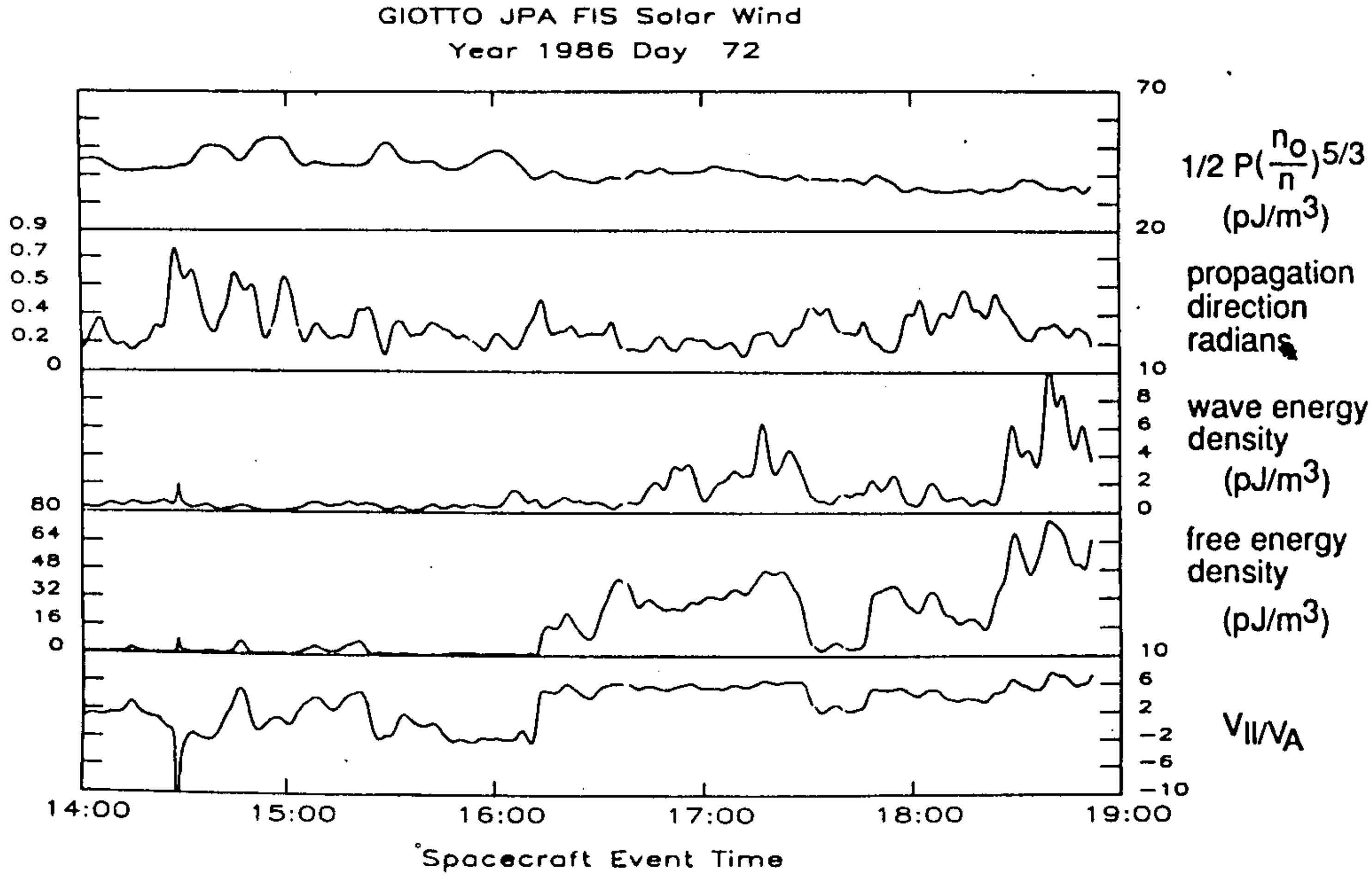
or, as measured,

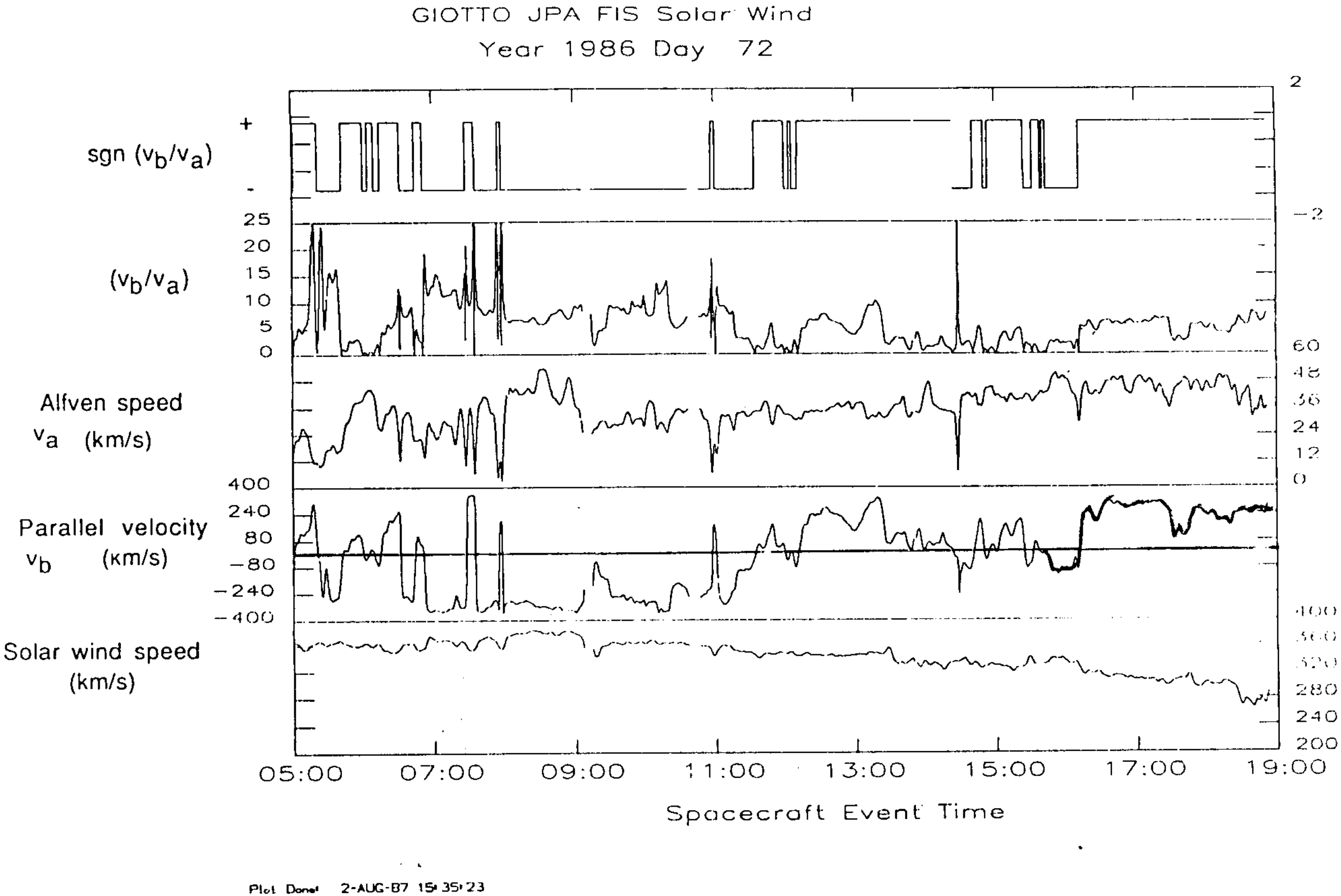
$$S = \frac{1}{2}\left(\rho < v_1^2 > + \frac{< B_1^2 >}{\mu_0}\right) V_g\, k$$

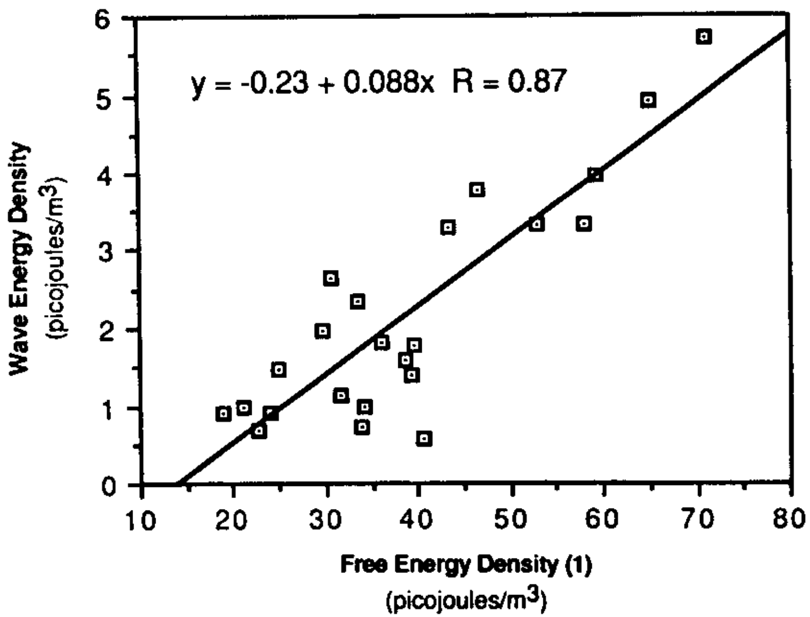
hence,

$$V_g = \frac{(v_1 \times B_0) \times \frac{B_1}{\mu_0}}{\frac{1}{2}\left(\rho < v_1^2 > + \frac{< B_1^2 >}{\mu_0}\right)}$$




GIOTTO JPA FIS Solar Wind
Year 1986 Day 72
70
20
1/2 P($\frac{n_o}{n}$)$^{5/3}$
(pJ/m^3)
0.9
0.7
0.5
0.4
0.2
0
propagation
direction
radians
10
8
6
4
2
0
wave energy
density
(pJ/m^3)
80
64
48
32
16
0
free energy
density
(pJ/m^3)
10
6
2
−2
−6
−10
V_{II}/V_A
14:00
15:00
16:00
17:00
18:00
19:00
Spacecraft Event Time


GIOTTO JPA FIS Solar Wind
Year 1986 Day 72
sgn (v_b/v_a)
+
-
2
-2
(v_b/v_a)
25
20
15
10
5
0
Alfven speed
v_a (km/s)
60
48
36
24
12
0
Parallel velocity
v_b (km/s)
400
240
80
−80
−240
−400
Solar wind speed
(km/s)
400
360
320
280
240
200
05:00
07:00
09:00
11:00
13:00
15:00
17:00
19:00
Spacecraft Event Time
Plot Done! 2-AUG-87 15:35:23



FREE ENERGY

$$E_{ring} - E_{shell} = 2V_A \left[V_s \cos\phi - V_A \right] \left(\tfrac{1}{2} n_i m_i \right)$$

if $V_s \cos\phi \gg V_A$

$$\Delta E = \frac{2V_A}{V_s \cos\phi} \left(\tfrac{1}{2} \rho_i V_s^2 \cos^2\phi \right)$$

$V_A \sim 50$ km/s $V_s \cos\phi \sim 250$ km/s

$\therefore \quad \Delta E \approx 0.4$ (kinetic energy of ion beam in solar wind frame)

Detected energy in waves

~ 0.09 (kinetic energy of ion beam

KINETIC EQUATION FOR WAVES

⊛ $$(V_{\parallel} - V_A)\frac{\partial U}{\partial x} = \frac{F n_c}{\tau_1} - \frac{U}{\tau_2}$$

F free energy per ion

n_c neutral particle density

$1/\tau_1$ ionisation rate

$1/\tau_2$ damping rate.

Assumes time independent situation

⊛

In our correlation

$$F \propto U$$

Where F is a function of α

⊛ What is the damping mechanism?

NON LINEAR DAMPING PROCESSES

Achterberg Ast. Astrophys 98, 161, 1981

⊛ Resonant three wave interaction

$\omega_1 + \omega_2 = \omega_3$ (conservation of energy)

$k_1 + k_2 = k_3$ (conservation of momentum)

Requires waves travelling in opposite directions

$k_1 \approx -k_2$

then ω_3, k_3 is an ion acoustic wave

⊛ Resonant interaction between thermal ions and the high + low frequency beat wave of two circularly polarised waves

$$(\omega_1 \pm \omega_2) - (k_1 \pm k_2) V_Z = 0$$

(non linear Landau damping)

⊛ Transfers momentum + energy to solar wind ions ⊛

Damping rate $\gamma_d = \frac{1}{\tau_2} = -\left(\frac{1}{32\pi}\right)^{1/2} \frac{e^2}{m^2} \frac{K}{V_A \Omega_p^2} \frac{T_p}{mc^2} \int dk |H_k|^2$

Sugden et al GRL 13, 85, 1986

RICHARDSON ET AL JGR (in press) 1988