INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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H4-SMR 393/8

SPRING COLLEGE ON PLASMA PHYSICS

15 May - 9 June 1989

PLASMA TURBULENCE IN THE EQUATORIAL IONOSPHERE

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PLASMA TURBULENCE IN THE EQUATORIAL IONOSPHERE

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Spring College
May 1989

Int. Ctr. Th... Physic
TRIESTE

Outline of Topics for the Lectures

Introductory Remarks

Equatorial Ionosphere
Electrojet
Electron density Irregularities : Type I , Type II
Spectral measurements by radar back-scatter
Rocket borne measurements
Spread F

Mathematical Model for Weakly ionized collisional Plasm

Linear Theory of Low frequency electron density fluctuati

$\underset{\sim}{E} \times \underset{\sim}{B}$ Gradient-drift Instability (Simon , Hoh)

Two-Stream Instability (Farley , Buneman)

Rayleigh - Taylor Instability

Drift - Waves

Nonlocal linear Theory of $\underset{\sim}{E} \times \underset{\sim}{B}$ Instability in Electrojet

Quasi-linear theory of $\underset{\sim}{E} \times \underset{\sim}{B}$ modes in the Electrojet

Saturation of Type I Two-Stream Instability

Plasma Turbulence in the Equatorial Electrojet

Non-linear Equation
Direct Interaction Approx. for the Power Spectrum and Responce Function

Kolmogorov cascade
Transport in $\underset{\sim}{k}$-space
Numerical Studies
Spread in $k_{\parallel}$: Almost 2-d turbulence
Observations

Kilometer Scale Irregularities in the Equatorial Electroje

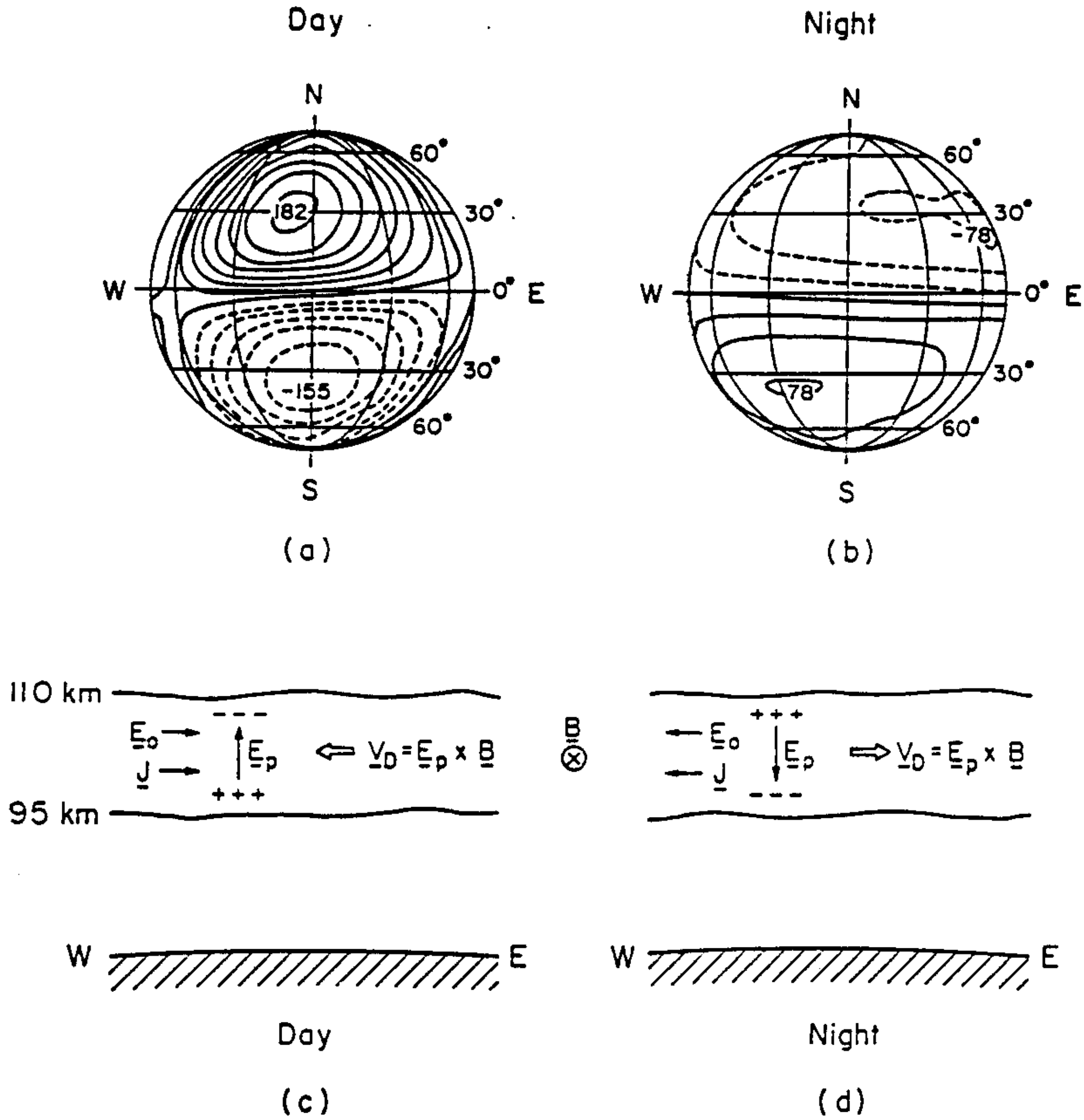
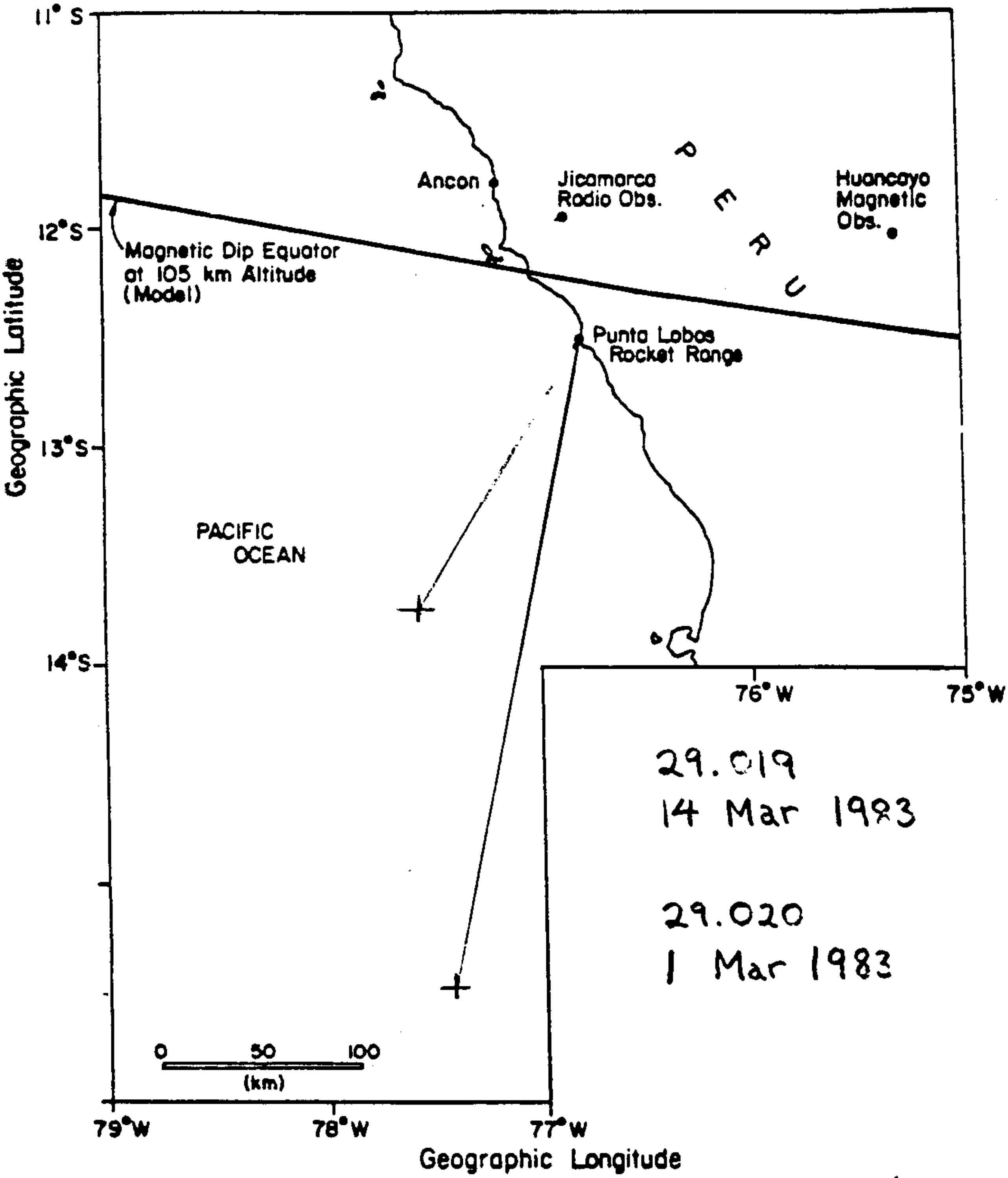
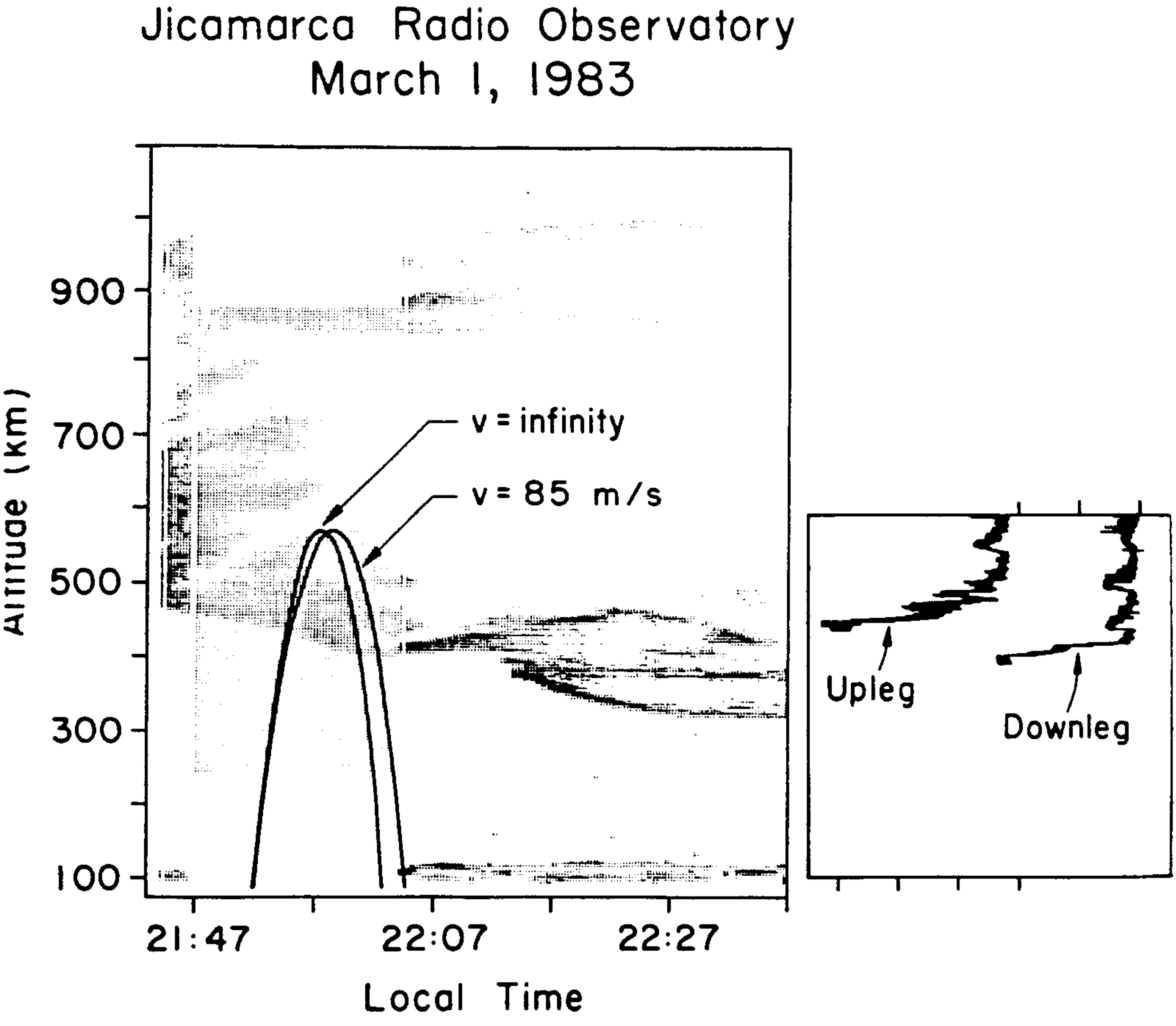


Figure 2.3 Current patterns set up by the mid-latitude atmospheric dynamo for the day (a) and night (b) hemispheres [after Matsushita, 1965]. The dawn-dusk dynamo horizontal electric fields create vertical polarization fields (E_p) at the magnetic dip equator, as shown in (c) and (d). The E_p x B drifts drive the daytime and nighttime electrojets.



PROJECT CONDOR, Kelley et. al. JGR 91, 5487, '86


Jicamarca Radio Observatory
March 1, 1983
v = infinity
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700
500
300
100
Altitude (km)
21:47
22:07
22:27
Local Time
Upleg
Downleg

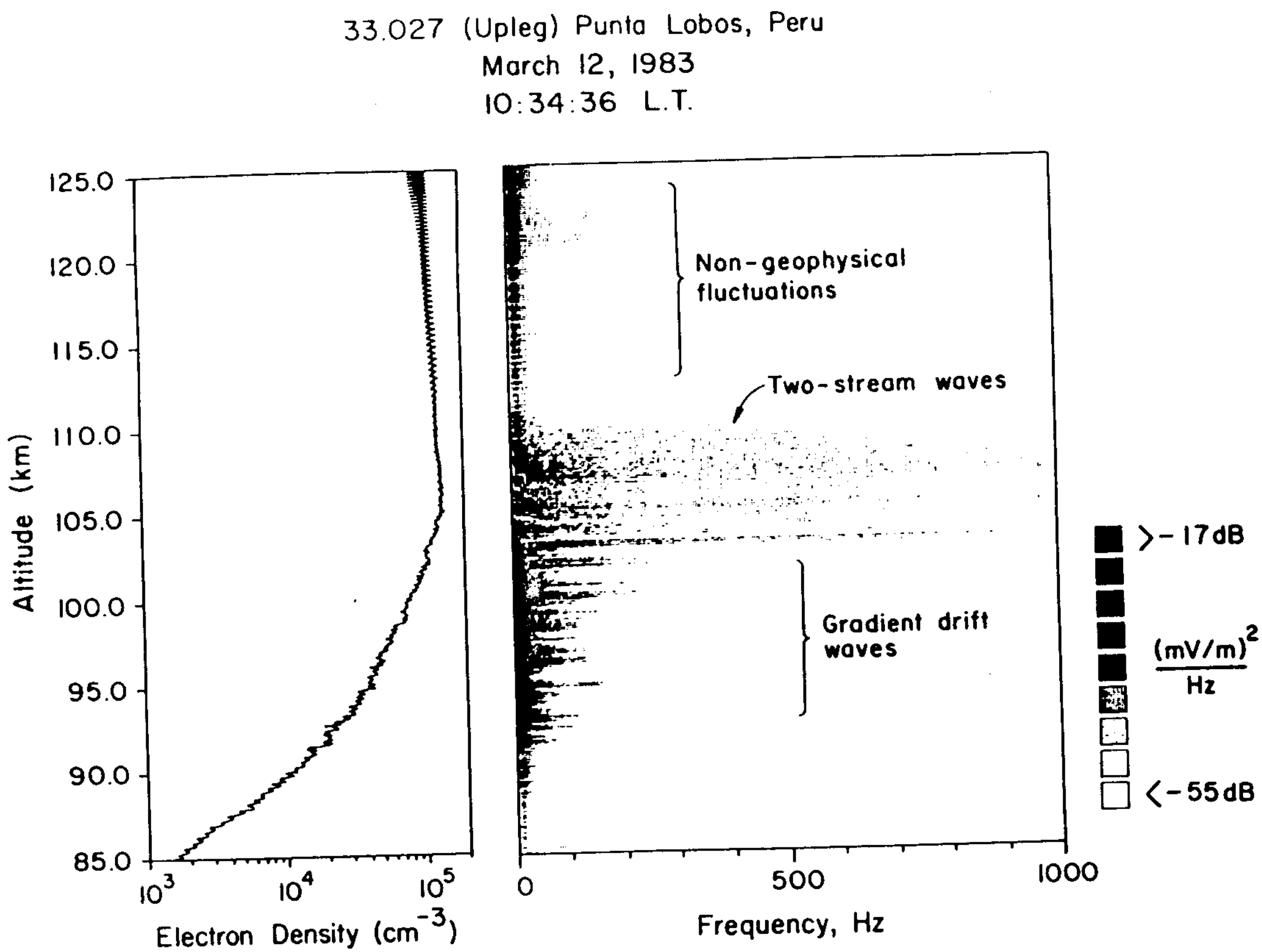
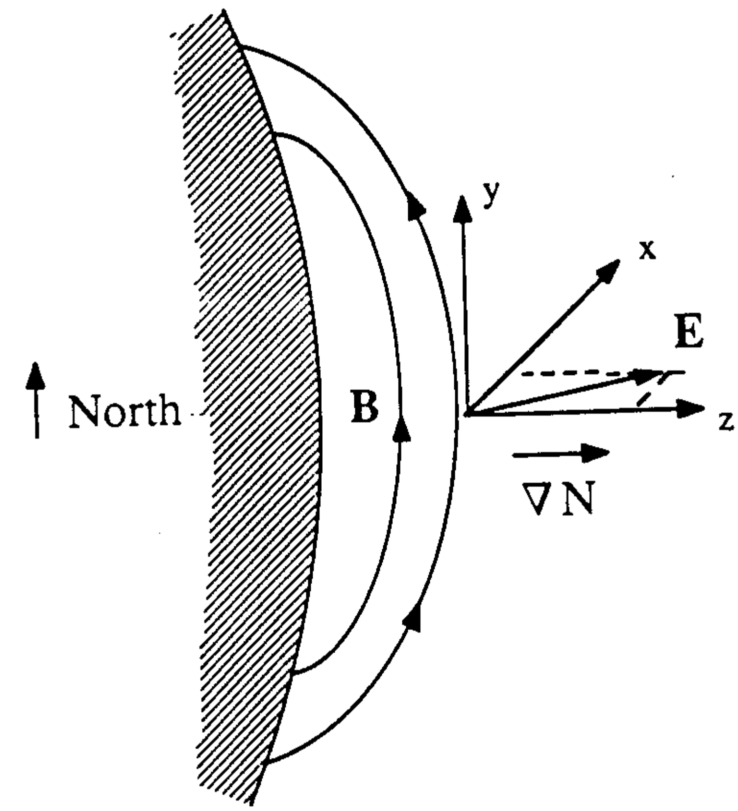


Fig. 4.27. Frequency-height sonogram of the horizontal component of the irregularities measured during the upleg of rocket 33.027 during strong electrojet conditions. This instrument had a low frequency roll-off (3 dB) at 16 Hz. The electron density profile (on the left) shows the presence of large scale irregularities. Both panels show non-geophysical "interference" above about 110 km.

7


y
x
E
North
B
z
∇N

MATHEMATICAL MODEL

- Weakly ionized Collisional plasma

  $n_N \gg n_e$

- Collisions predominantly with neutrals

- Quasi-neutrality $\quad n_e = Z n_i \qquad Z = 1$

- TWO-Fluid Model for Plasma

  In the frame of the neutral fluid

$$\frac{\partial n}{\partial t} + \nabla \cdot n \underset{\sim}{v}_e = Q - \alpha n^2$$

$$n m_s \left(\frac{\partial}{\partial t} \underset{\sim}{v}_s + \underset{\sim}{v}_s \cdot \nabla \underset{\sim}{v}_s \right) = n q_s \left(\underset{\sim}{E} + \frac{\underset{\sim}{v}_s}{c} \times \underset{\sim}{B} \right) - T_s \nabla n - n m_s \nu_s + n m_s \underset{\sim}{g}$$

- Low β plasma : $\beta < \frac{m_e}{m_i}$ $\qquad \underset{\sim}{E} = - \nabla \varphi + \underset{\sim}{E}_{ex}$

  $\delta \underset{\sim}{B} = 0$

- Low frequency : $\omega \ll \Omega \qquad \omega \ll \nu_e, \nu_i$

- Isothermal

$$\nabla \cdot \underset{\sim}{j} = \nabla \cdot e n \left(\underset{\sim}{v}_i - \underset{\sim}{v}_e \right) = 0$$

In E region $\qquad \nu_e \ll \Omega_e$

$$\nu_i > \Omega_i$$

In F region $\qquad \nu_e \ll \Omega_e$

$$\nu_i \ll \Omega_i$$

E region, for $\omega < \nu_i$ neglect inertial term

$$\underset{\sim}{B} = B\,\hat{b}$$

$$\underset{\sim}{V}_s = \mu_{\perp s}\left(\underset{\sim}{E} - \frac{T_s}{q_s}\nabla \ln n\right) + \mu_{Hs}\left(\underset{\sim}{E} - \frac{T_s}{q_s}\nabla \ln n\right) \times \hat{b}$$

$$\mu_\perp = \frac{q}{m}\frac{\nu}{\Omega^2 + \nu^2} \qquad \text{Pedersen}$$

$$\mu_H = \frac{q}{m}\frac{\Omega}{\Omega^2 + \nu^2} \qquad \text{Hall}$$

$$\boxed{\begin{aligned} &\frac{\partial n}{\partial t} + \mu_{He}\,\hat{b} \times \nabla\varphi \cdot \nabla n = \nabla\cdot\left(\mu_{\perp e}\, n \nabla\varphi + D_e \nabla n\right) + Q - \alpha n \\ &\mu_H^- \nabla n \cdot \nabla\varphi \times \hat{b} + \nabla\cdot \mu_\perp^- n \nabla\varphi + \nabla\cdot (D_i - D_e)\nabla n = 0 \end{aligned}}$$

$$\nabla\varphi \to \nabla\varphi - E_{0x}$$

$$\mu_\perp^\pm = \mu_{\perp i} \pm \mu_{\perp e}$$

$$\mu_H^\pm = \mu_{Hi} \pm \mu_{He}$$

$$D_i = \frac{T_i}{q_i}\mu_{\perp i}$$

external electric field $\quad \underset{\sim}{E}_{ex} = \hat{x}\, E_0$

$$\hat{b} = \hat{y}$$

<u>vertical Polarization field</u> : $\boxed{J_z = 0}$

$$E_z = -\frac{\mu_H^-}{\mu_\perp^-} E_0 + \frac{D_i - D_e}{\mu_\perp^-}\frac{1}{n_0}\frac{dn_0}{dz}$$

<u>Horizontal current density</u>

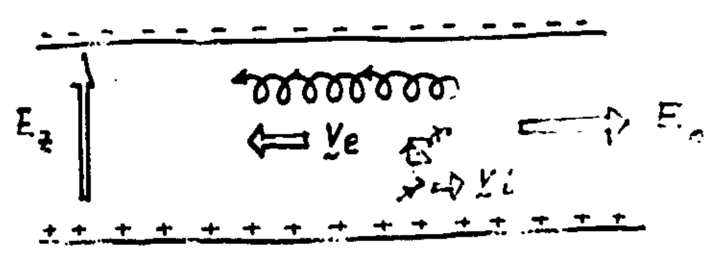
$$J_x = e n_0 \left(\mu_\perp^- E_0 - \mu_H^- E_z\right) + T \mu_H^+ \frac{dn_0}{dz}$$

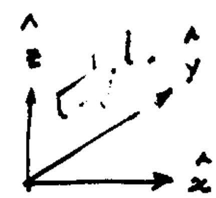
In E region : $\mu_{\perp e} = -\dfrac{e}{m_e}\dfrac{\nu_e}{\Omega_e^2}$, $\mu_{\perp i} = \dfrac{e}{m_i \nu_i}$

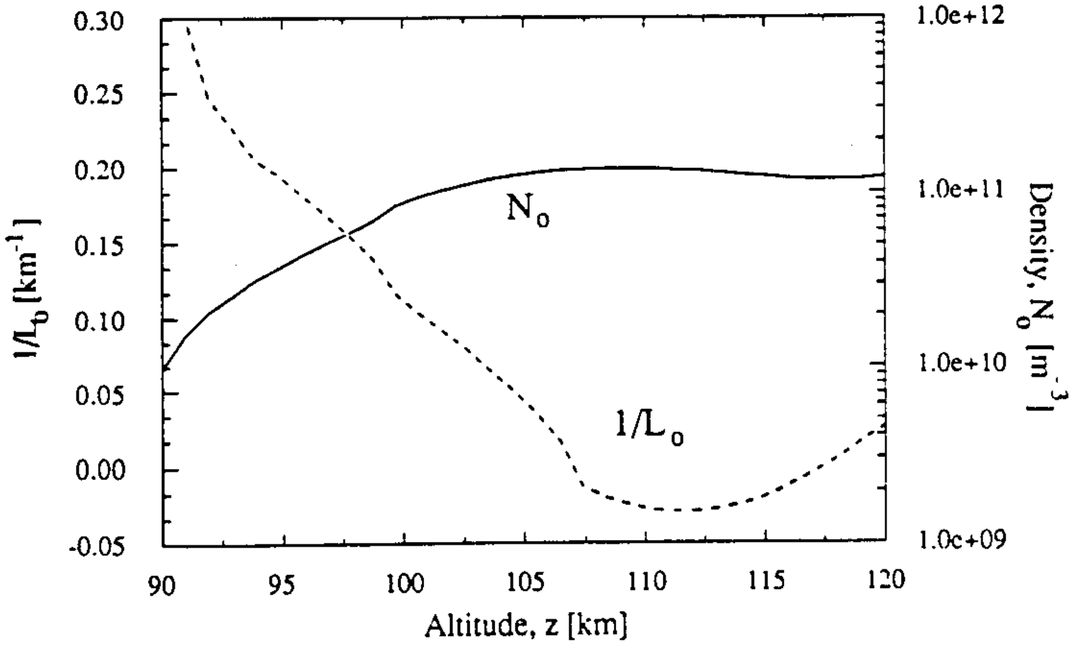
$$\mu_\perp^- = \frac{e}{m_i \nu_i}(1 + \psi)$$

$$\mu_H^- = \frac{c}{B_0} \quad ; \quad \psi = \frac{\nu_e \nu_i}{\Omega_i |\Omega_e|}$$

$$\frac{\mu_H^-}{\mu_\perp^-} = -\frac{\nu_i}{\Omega_i}\frac{1}{(1+\psi)}$$






0.30
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
1/L0 [km-1]
90
95
100
105
110
115
120
Altitude, z [km]
1.0e+12
1.0e+11
1.0e+10
1.0c+09
Density, No [m-3]
No
1/Lo

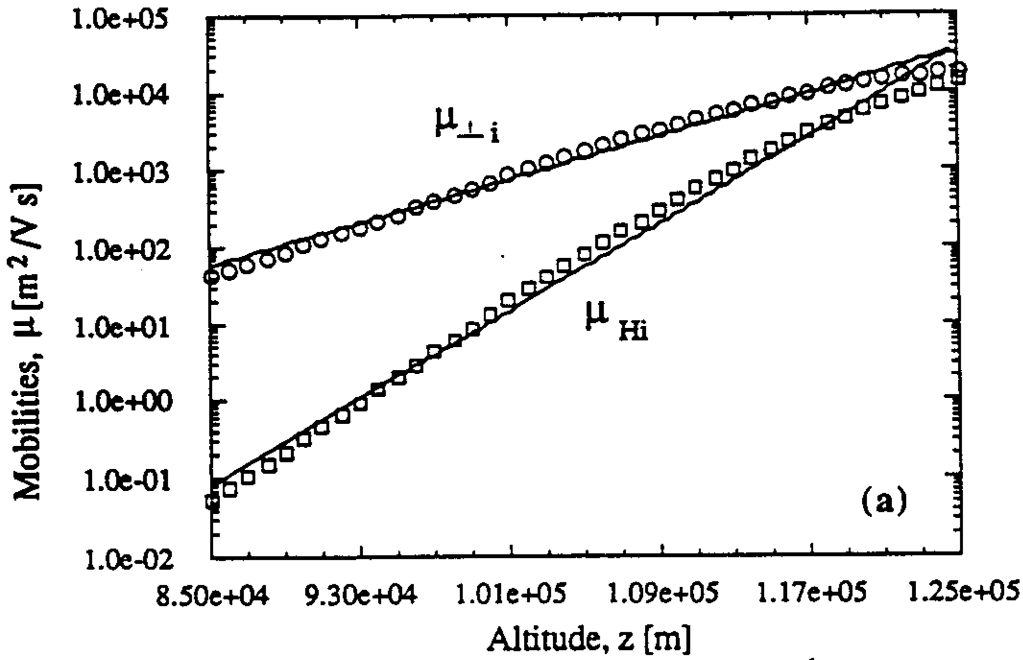


Figure 2a

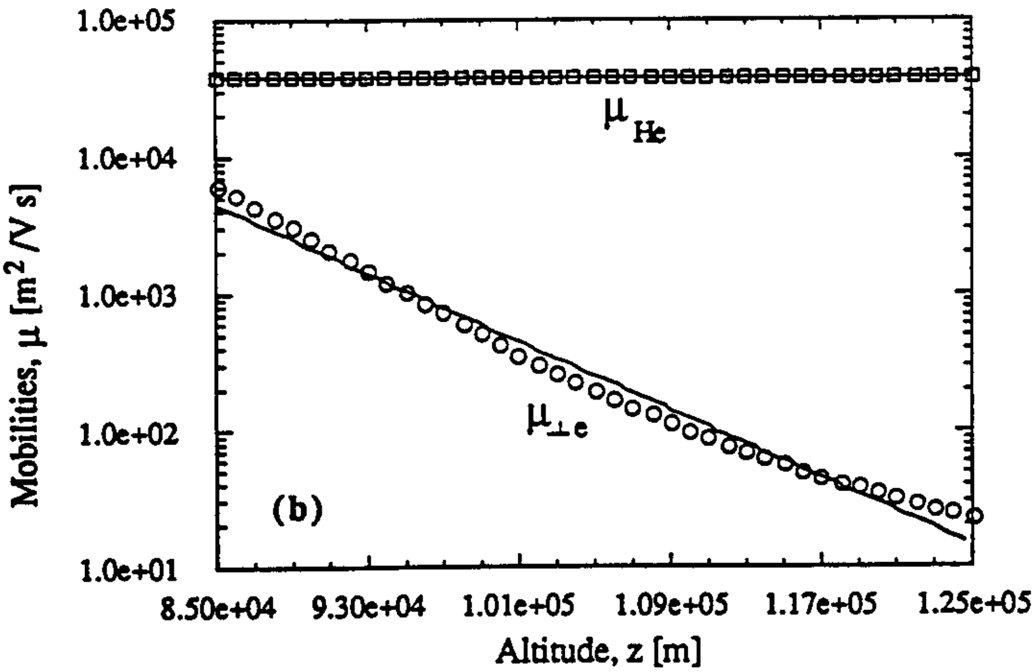


Figure 2b

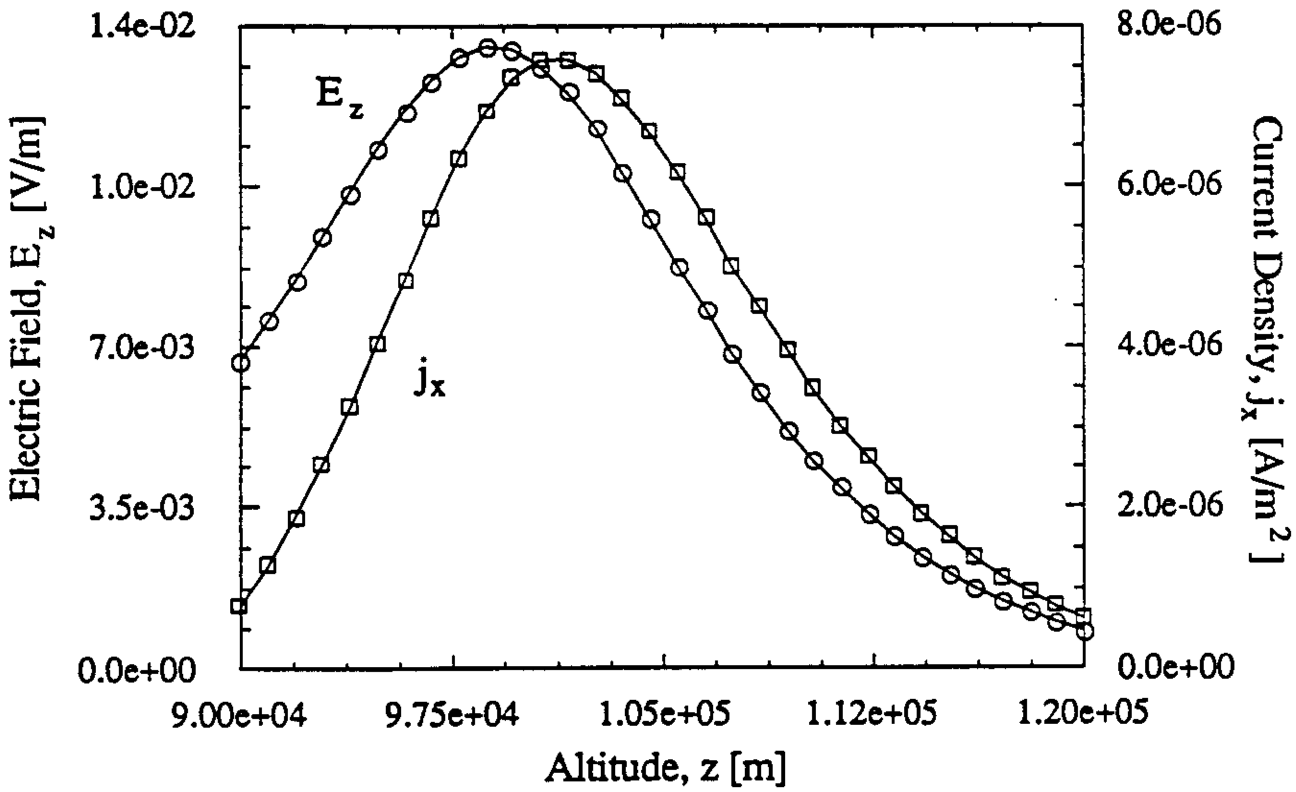


Figure 4 $E_x = 0.4 mV/m$

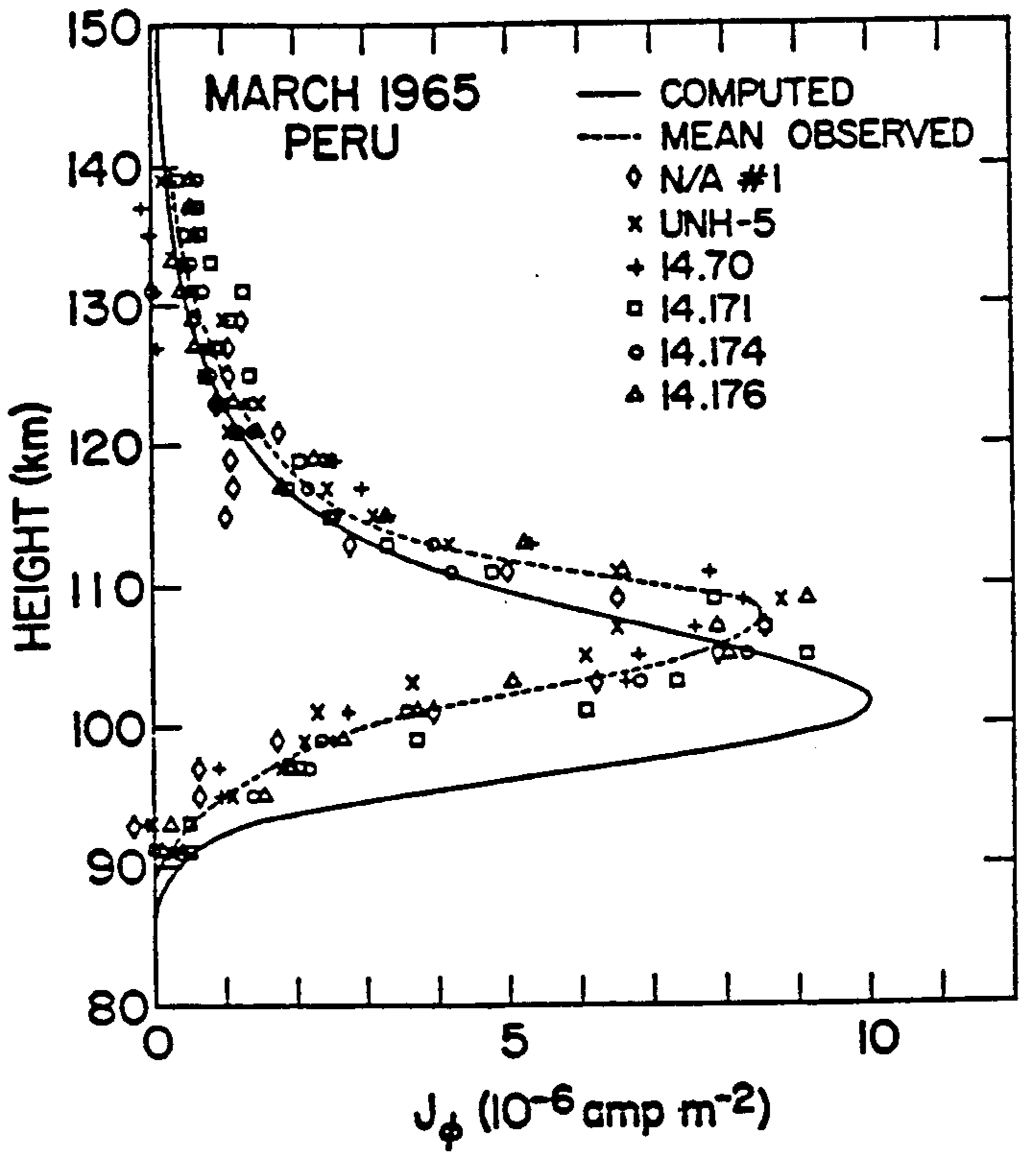


Figure 2.4 Observations from 6 rocket flights of the equatorial electrojet current density at midday off the coast of Peru. The values have been adjusted so that their simultaneous ground magnetometer variation scales to 100γ at Huancayo. The solid curve is the predicted current density from a theoretical model. [After Richmond, 1973b.]

Small amplitude perturbations : neglect nonlinear terms

$$n = n_0(z) + \delta n(x, z, t)$$

$$\varphi = \varphi_0(z) + \delta\varphi(x, z, t)$$

$$\delta n = \widetilde{\delta n} \exp i(\underset{\sim}{k} \cdot \underset{\sim}{x}_\perp - \omega t) \qquad \text{etc.}$$

$$\widetilde{\delta\varphi} = \left\{ \frac{\omega + \frac{c}{B_0} \underset{\sim}{k} \cdot \nabla\varphi_0 \times \hat{y} + i k^2 D_e}{\frac{c}{B_0} \underset{\sim}{k} \cdot \nabla n_0 \times \hat{y} - i k^2 \mu_{\perp e} n_0} \right\} \widetilde{\delta n}$$

∇n_0 and $\nabla\phi_0$ independent of z.

From $\nabla \cdot \underset{\sim}{j} = 0$

$$\widetilde{\delta\varphi} = \widetilde{\delta n} \left\{ \frac{\frac{d\varphi_0}{dz}\left(k_x - k_z \frac{\Omega_i}{\nu_i}(1+\psi)\right) + i k^2 \frac{T}{e} \frac{\Omega_i}{\nu_i}(1+\psi)}{\frac{1}{n_0}\frac{dn_0}{dz}\left(k_x + k_z \frac{\Omega_i}{\nu_i}(1+\psi)\right) + i k^2 \frac{\Omega_i}{\nu_i}(1+\psi)} \right\}$$

Short wavelength limit : $\Omega_i/\nu_i \ll k_x/k_z$, [illegible]

$$\omega_r = -\frac{k_x c E_z/B_0}{1+\psi} + \frac{1-\psi}{(1+\psi)^2}\left(\frac{T}{e}\frac{c}{B_0 L_0}\right) k_x \equiv \frac{\underset{\sim}{k} \cdot \underset{\sim}{V}_T}{1+\psi}$$

$$\gamma = \frac{\psi}{(1+\psi)^2} \frac{|\Omega_e|}{\nu_e} \left(\frac{c E_z}{B_0 L_0}\right) \frac{k_x^2}{k^2} - k^2 \left(\frac{2 D_e}{1+\psi}\right)$$

$$\gamma = \frac{\Omega_e/\nu_e}{(1+\psi)}\left(\frac{cE_z}{B_0 L_0}\right)\frac{1}{1+(k_0/k_x)^2} - 2\alpha n_0^2$$

$$k_e L_0 = \Omega_e/\nu_e$$

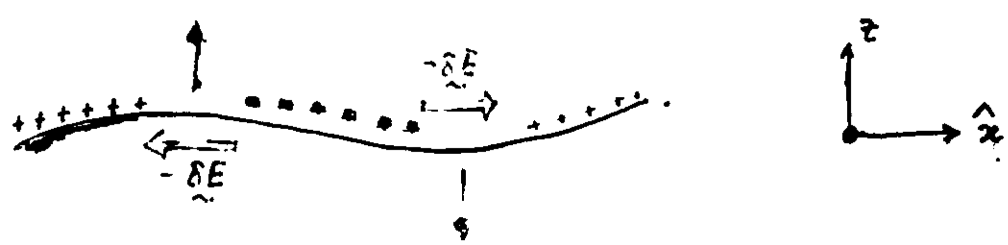
Growth occurs for $E_z/L_0 > 0$ i.e.

$$\boxed{\nabla n_0 \, \nabla(-\phi_0) > 0}$$

Equivalent to Rayleigh-Taylor instability criterion

$$\boxed{\nabla n_0 \, \nabla \Psi > 0}$$ Ψ gravitational potential.

$\underset{\sim}{E} \times \underset{\sim}{B}$ inst. $\equiv$ R-T inst. !



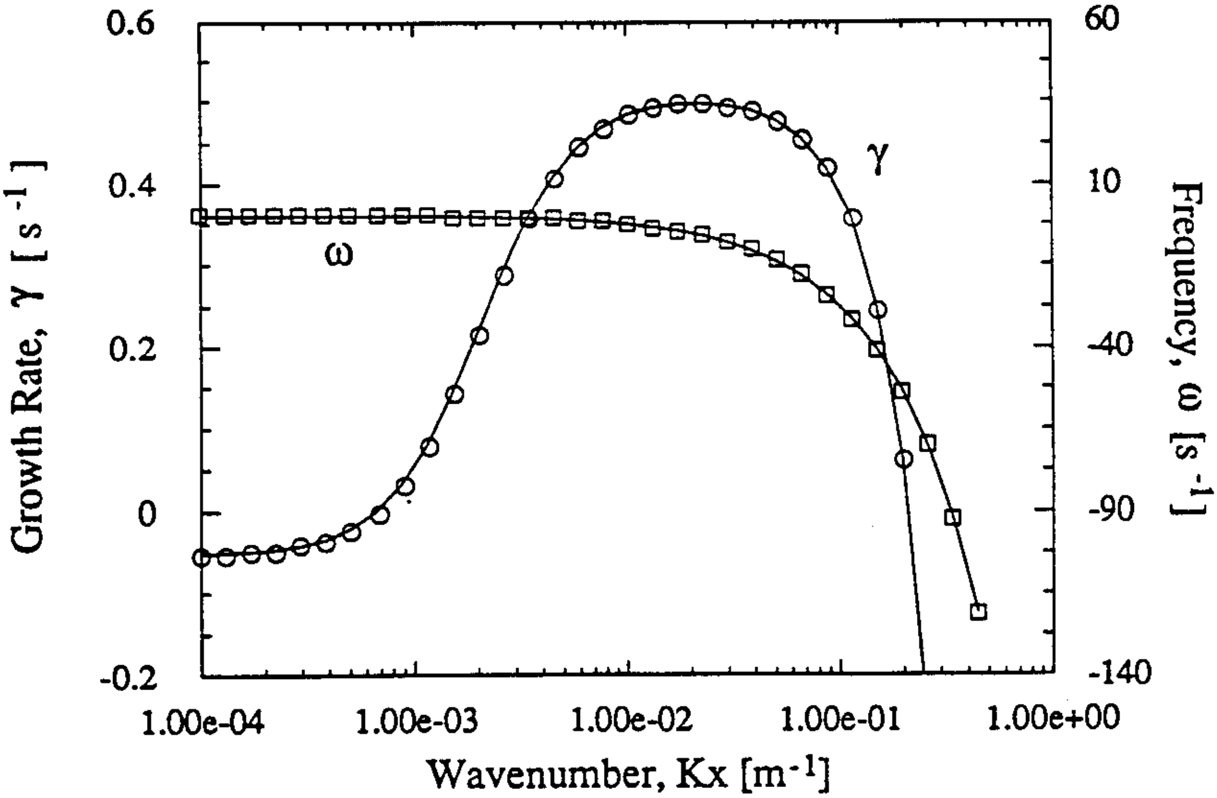


Figure 5

Z = 19.7 km

Two-Stream Farley-Buneman Instability

$[illegible] > \omega > \nu_i \quad ; \quad \nabla n_0 \to 0 \, , \quad L \to \infty$

Electrons : $\quad -e\left(\underset{\sim}{E} + \frac{\underset{\sim}{v}_e}{c} \times B_0\right) - m_e \nu_e \underset{\sim}{V}_e = 0$

Include Ion inertia $\qquad \frac{\partial}{\partial t}\delta n + \underset{\sim}{v}_D \cdot \nabla \delta n + n_0 \nabla \cdot \delta \underset{\sim}{v}_e$

[illegible] = [illegible] $\underset{\sim}{E}$ − [illegible]

Dispersion Relation

$$\omega(1+\psi) - \underset{\sim}{k} \cdot \underset{\sim}{V}_D - \frac{i\psi}{\nu_i}\left(\omega^2 - k^2 C_s^2\right) = 0$$

$$\omega_{\underset{\sim}{k} r} = \frac{\underset{\sim}{k} \cdot \underset{\sim}{V}_D}{1+\psi}$$

$$\gamma_{\underset{\sim}{k}} = \frac{\psi}{\nu_i(1+\psi)}\left(\omega_{\underset{\sim}{k} r}^2 - k^2 C_s^2\right)$$

$$\psi = \nu_e \nu_i / |\Omega_e| \Omega_i$$

$$C_s^2 = (T_e + T_i)/m_i$$

$$\frac{V_D}{1+\psi} > C_s$$

Nonlocal Stability Analysis of $\underset{\sim}{E} \times \underset{\sim}{B}$ Instability

n_0 and φ_0 arbitrary functions of height z

Linearised Eqns written formally as :

$$\underline{\underline{D}}\left(z, -i\frac{\partial}{\partial z} ; k_x, \omega\right) \cdot \underline{\Psi} = 0 \qquad \underline{\Psi} \equiv (\delta n, \delta\varphi)$$

Let $\underset{\sim}{\Psi} = \underset{\sim}{\hat{\Psi}}(z) \exp i S(z)$

$$\underline{\underline{D}}\left(z, k_z - i\frac{\partial}{\partial z} ; k_x, \omega\right) \cdot \underset{\sim}{\hat{\Psi}} = 0$$

$$k_z(z) \equiv \frac{\partial S}{\partial z}$$

Require $|\partial S/\partial z| \gg |\partial \underset{\sim}{\hat{\Psi}}/\partial z|$

To lowest order

$$\underset{\approx}{D}(z, k_z ; k_x, \omega) \cdot \underset{\sim}{\hat{\Psi}} = 0$$

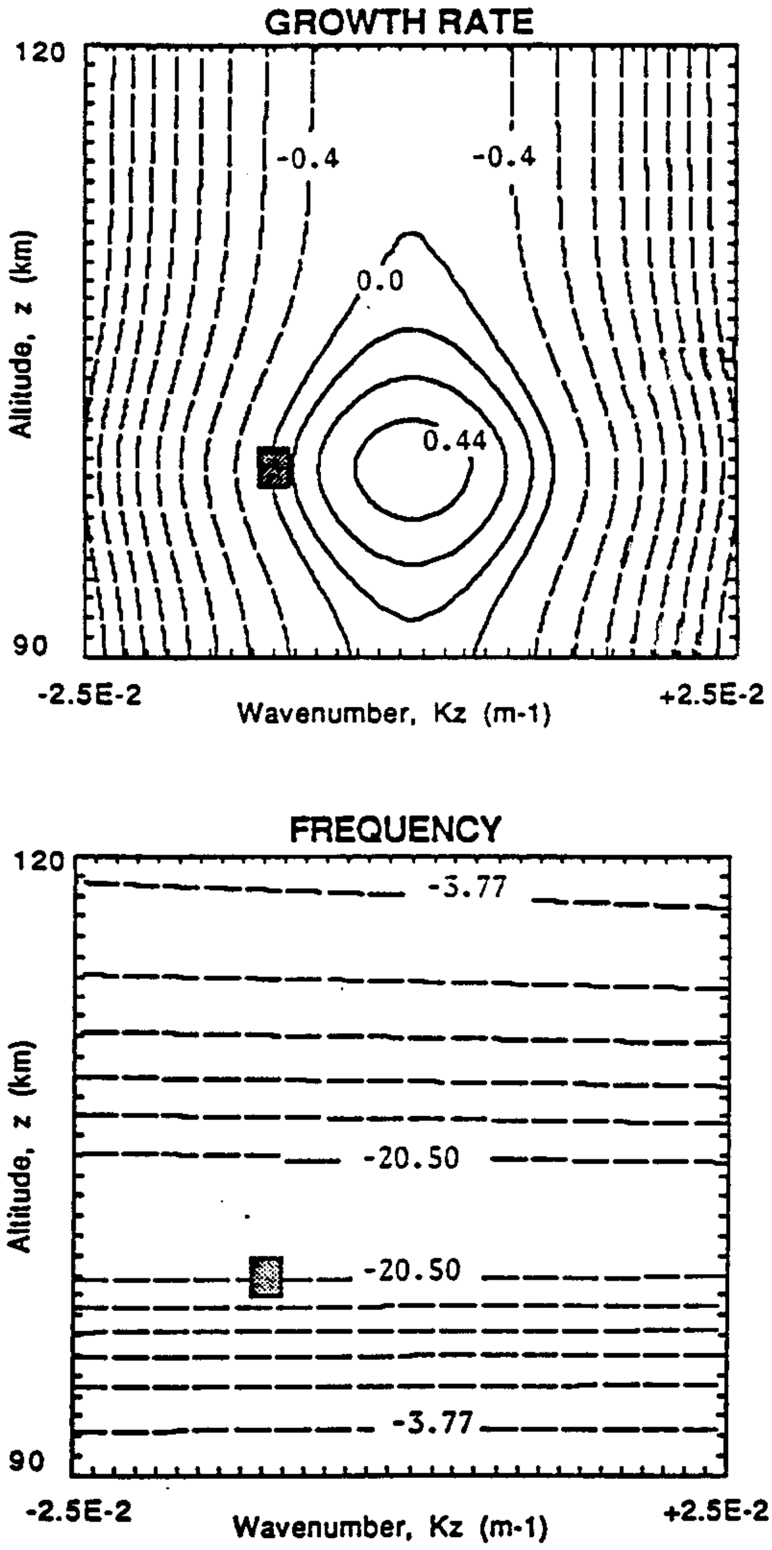
Solubility condition : $\Delta = \det \underline{\underline{D}}(z, k_z ; k_x, \omega) = 0$

Ray Equations

$$\frac{dz}{dt} = \frac{\partial \omega}{\partial k_z} = \frac{\partial \Delta}{\partial k_z}\left(\frac{\partial \Delta}{\partial \omega}\right)^{-1}$$

$$\frac{dk_z}{dt} = -\frac{\partial \omega}{\partial z} = -\frac{\partial \Delta}{\partial z}\left(\frac{\partial \Delta}{\partial \omega}\right)^{-1}$$

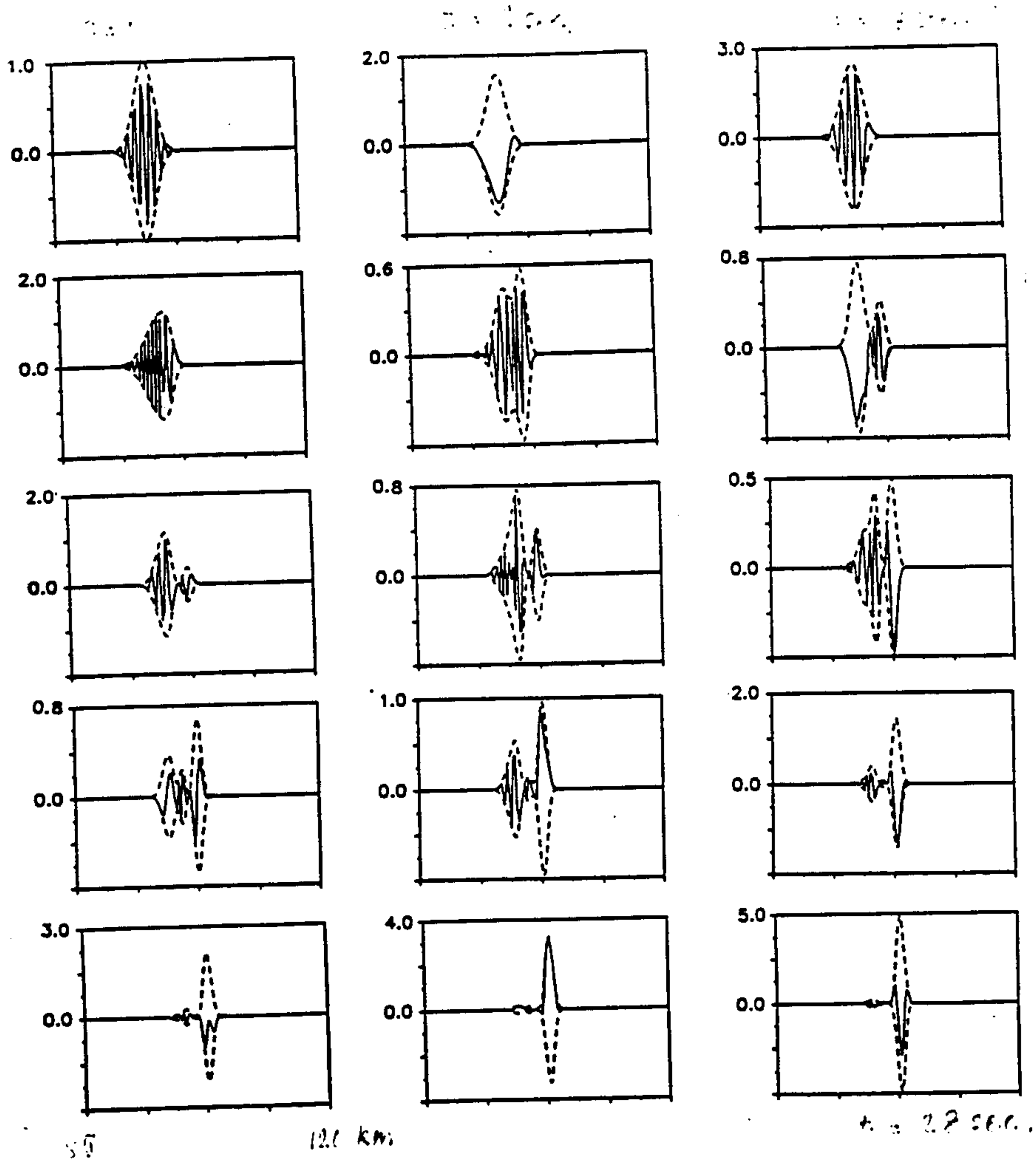
Plot in (k_z, z) plane contours of constant ω_r and constant γ for fixed k_x.



$k_x = 7.0 \times 10^{-2}\ m^{-1}$

$\lambda \sim 10^2 m$

Figure 6



1.0
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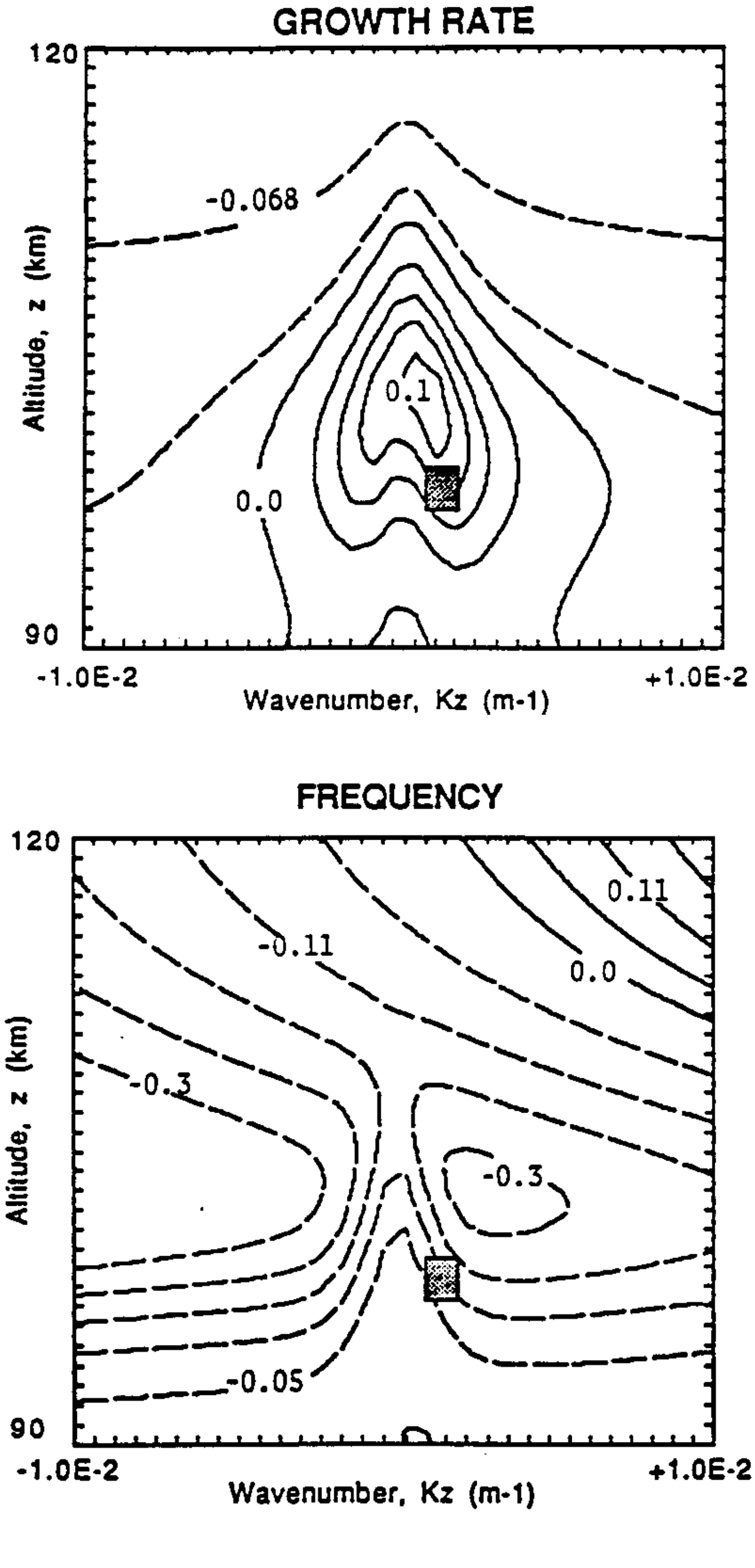
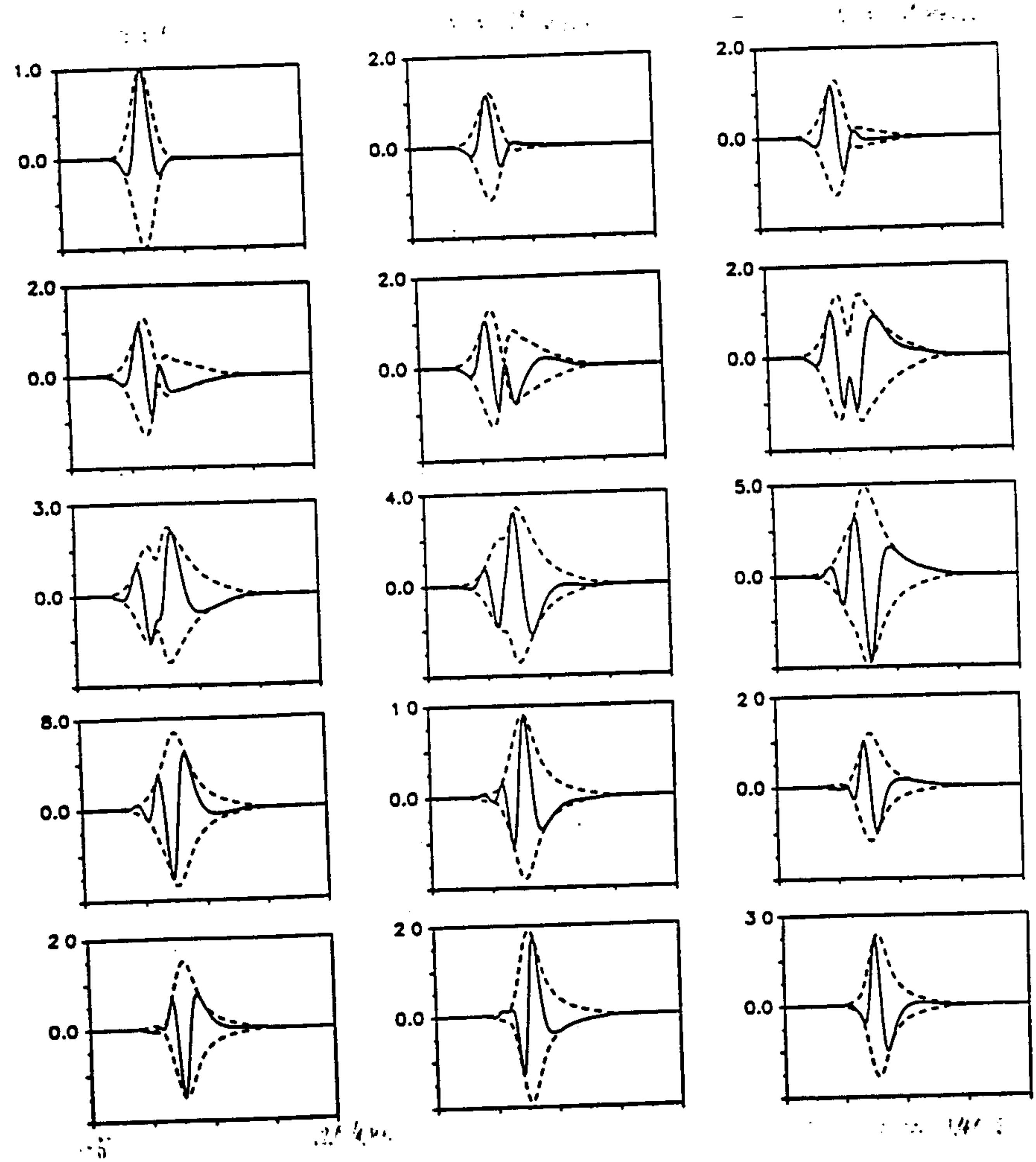
Figure 8



$k_z = 1.0 \times 10^{-3}\ m^{-1}$

$\lambda \sim 6$ km

Figure 7


1.0
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5.0
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8.0
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1 0
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2 0
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2 0
0.0
2 0
0.0
3 0
0.0

Figure 9