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# Cerenkov Counters

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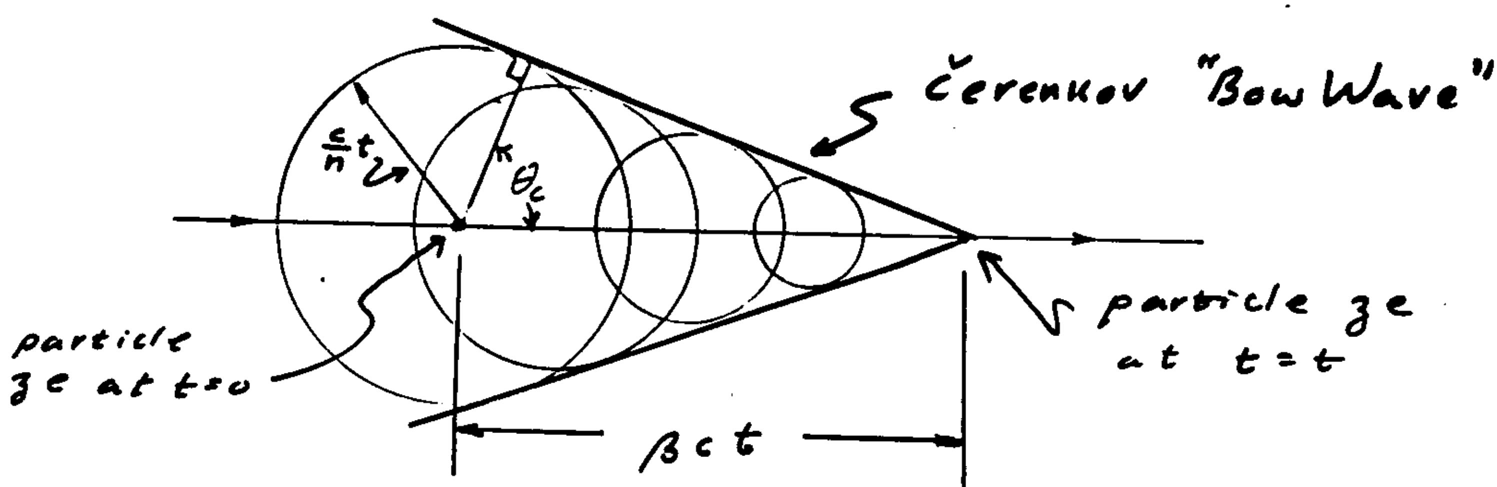
## Čerenkov Counters

These are important devices in High Energy Physics. They are used to measure particle velocity with great resolution, and in concert with other measurements, e.g. momentum, to identify particle type ($\pi, K, p, \dots$). A good reference is:

J. Litt and R. Meunier, "Čerenkov Counter Technique in High Energy Physics", Ann. Rev. Nuc. Sci. 23 (1973)

## Čerenkov Effect

The Čerenkov effect is the emmission of light by a relativistic particle due to the fact that it moves faster than light in the detector medium.



A formula for the Čerenkov angle follows from this simple picture:

$$\cos\theta_c = \frac{(c/n)\,t}{\beta c t}$$

$$\boxed{\cos\theta_c = \frac{1}{\beta n}}$$

where: $n = n(\lambda)$ is index of refraction

$\lambda$ = wavelength of Č radiation

$\therefore\ \theta_c = \theta_c(\lambda)$

The theory of the Čerenkov radiation is intimately connected to the theory of the density effect in the $dE/dx$ calculation. Both are aspects of the coherent response of the medium to the passage of a relativistic particle. More details and further refs. are given in:

J.D. Jackson, "Classical Electrodymics" (Wiley, 2nd Ed. 1975)

W.W.W. Allison, "The Interaction of charged Particles and Photons in Matter," ICFA School on Inst. (World Sci. 1987).

Time does not permit giving the details of the theory. The basic result is that the $dE/dx$, due to emmission of Čerenkov radiation, for a particle of charge $\underline{Ze}$, velocity $\underline{\beta c}$, moving in a medium whose dielectric constant is $\underline{\epsilon(\omega)}$, is

$$\left(\frac{dE}{dx}\right)_{\check{C}\,Rad} = \left(\frac{Ze}{c}\right)^2 \int\limits_{\substack{\omega \text{ such} \\ \text{that } \epsilon(\omega) > \frac{1}{\beta^2}}} \omega \left[1 - \frac{1}{\beta^2 \epsilon(\omega)}\right] d\omega$$

where $\omega$ $(= 2\pi f)$ is the angular frequency of a spectral component of the Čerenkov radiation.

If the radiation is detected by a device which has a response range $\omega_1 \rightarrow \omega_2$ both of which lie within the range $\epsilon(\omega) > 1/\beta^2$, the detectable energy loss will be:

$$\left(\frac{dE}{dx}\right)_{\substack{\text{Detectable} \\ \check{C}\text{ radiation}}} = \left(\frac{Ze}{c}\right)^2 \int_{\omega_1}^{\omega_2} \omega \left[1 - \frac{1}{\beta^2 \epsilon(\omega)}\right] d\omega$$

If we now use $n = \sqrt{\epsilon}$

and photon energy $E_\gamma = \hbar\omega$

we can readily convert this formula to a formula for $\frac{dN_\gamma}{dx}$, the number of detectable photons emmitted per unit length

$$\frac{dN_\gamma}{dx} = \left(\frac{ze}{c}\right)^2 \int_{\omega_1}^{\omega_2} \frac{\cancel{\omega}}{\hbar \cancel{\omega}} \left[1 - \frac{1}{\beta^2 n^2}\right] d\omega$$

writing in terms of wavelength $\lambda$ and recalling the fine structure constant $\alpha = e^2/\hbar c$ ($\approx 1/137$) :

$$\frac{dN_\gamma}{dx} = 2\pi\alpha z^2 \int_{\lambda_2}^{\lambda_1} \left[1 - \frac{1}{\beta^2 n^2}\right] \frac{d\lambda}{\lambda^2}$$

or in terms of $\theta_c$ :

$$1 - \frac{1}{\beta^2 n^2} = 1 - \cos^2\theta_c = \sin^2\theta_c$$

$$\boxed{\frac{dN_\gamma}{dX} = 2\pi\alpha z^2 \int_{\lambda_2}^{\lambda_1} \sin^2\theta_c \, \frac{d\lambda}{\lambda^2}}$$

Čerenkov Counters can be broadly grouped into three categories:

Threshold counters

Differential Counters

Ring Imaging Čerenkov Counters

or on West coast (USA) CRIDs

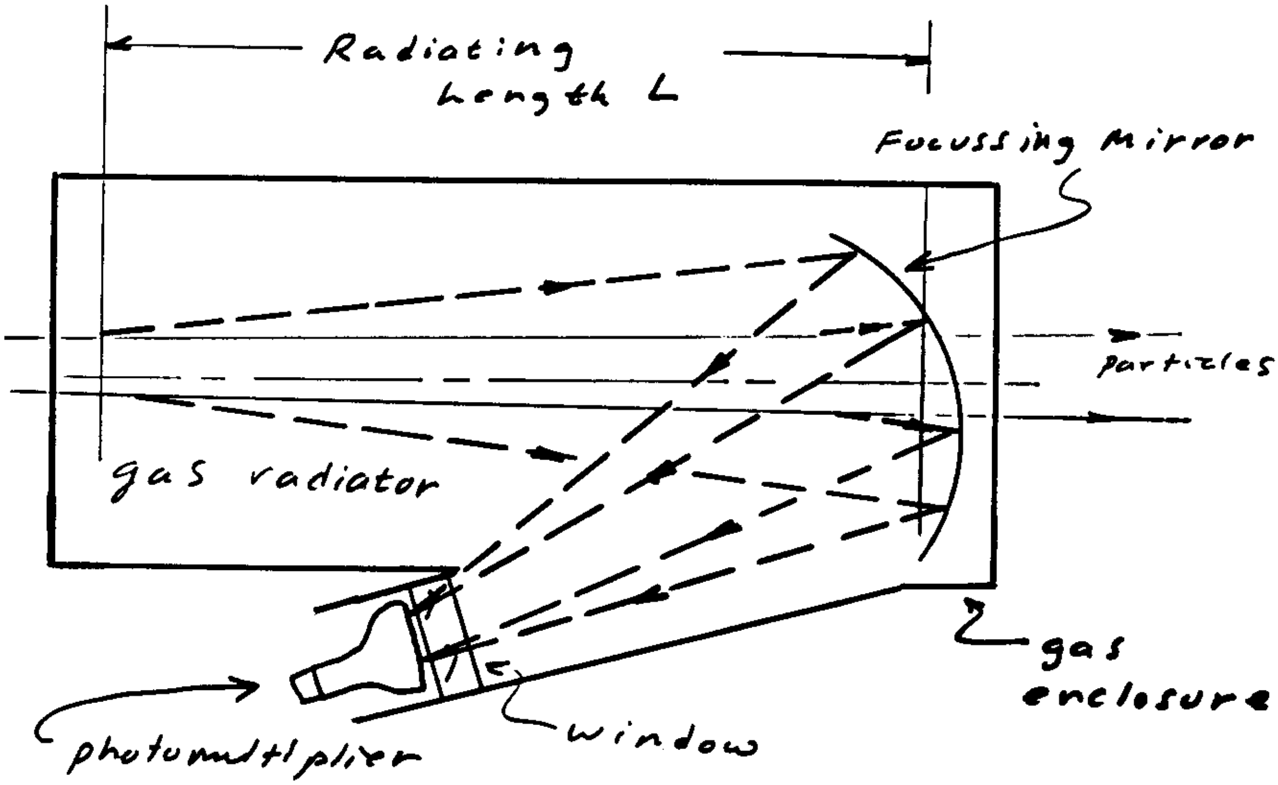
## Threshold Counters

These are designed to detect particles that have a velocity sufficient to produce Č light in the detector while (just) not detecting a slower species of particle whose velocity, $\beta_0$, the threshold velocity is such that $\theta = 0$ for such $\beta_0$.

$$\text{i.e. } \frac{1}{\beta_0 n} = 1$$

These counters are used in two general applications a) in beam lines

b) in large acceptance counters in (fixed target) spectrometers.




These counters can be tuned to the proper $\beta$ by the choice of gas, usually at 1 atmosphere, as is typical for the large acceptance threshold counters used in Spectrometers.

or

by varying the pressure of the gas as is typical for counters used in beam line systems.

For efficient detection, an adequate number of photoelectrons must be produced at the photocathode of the PM tube.

Typically, the range of sensitivity of a PM is ~ 400 nm → 700 nm which for $z = 1$ gives:

$$\frac{dN_\gamma}{dx} = \frac{2\pi}{137} \sin^2\theta_c \left[ \frac{1}{.4} - \frac{1}{.7} \right] \times 10^4$$

$$\frac{dN_\gamma}{dX} = 491 \text{ cm}^{-1}$$

If the Quantum efficiency of the photocathode and the losses at the mirror and window surfaces are taken into account (Bi-alkali p.c.) One typically finds:

$$\boxed{\frac{dN_{p.e.}}{dX} = A \sin^2\theta_c \qquad A \approx 100 \text{ to } 150 \text{ cm}^{-1}}$$

$\theta_c$ and the radiator length, $L$, must be chosen so that the number of photoelectrons gives efficient detection.

If the threshold is set at 1 p.e.

$$Eff = 1 - e^{-N}$$

where $N$ = mean no. of p.e.

we consider the momentum, $p$, and the masses $m_1$, $m_2$ $(m_2 > m_1)$ to be given.

we also limit ourselves to the high energy case: $\Delta\beta = 1-\beta \ll 1$

Now $$p^2 = m^2\beta^2\gamma^2 = m^2 \frac{\beta^2}{1-\beta^2}$$

$$(1-\beta)(1+\beta) = \frac{m^2\beta^2}{p^2}$$

with hi E approx:

$$\Delta\beta = \frac{m^2}{2p^2}$$

$$\Delta\beta_{1,2} \equiv \beta_1 - \beta_2 = \frac{m_2^2 - m_1^2}{2p^2}$$

Recall basic $\check{C}$ equation:

$$\cos\theta_c = \frac{1}{\beta n}$$

at high E $\beta \approx 1$, $n \approx 1$, $\theta_c \ll 1$

$$1 - \frac{\theta_c^2}{2} = \frac{1}{(1-\Delta\beta)(1+\Delta n)}$$

$$\theta_c^2 = 2[\Delta n - \Delta\beta]$$

By definition $m_2$ is at threshold

$\theta_c = 0$ when $\Delta\beta = \Delta\beta_{m_2}$

so $\Delta n = \Delta\beta_{m_2}$

$$\therefore \quad \theta_c^2(m_1) = 2\left[\Delta\beta_{m_2} - \Delta\beta_{m_1}\right] = 2\,\Delta\beta_{1,2}$$

$$\boxed{\theta_c^2(m_1) = \frac{m_2^2 - m_1^2}{p^2}}$$

Take a concrete example:

$\pi, p$ Separation $\begin{cases} m_\pi = .14 \text{ GeV} \\ m_p = .938 \text{ GeV} \end{cases}$

$$\theta_c^2 = \frac{(.938)^2 - (.14)^2}{p^2} = \frac{.86}{p^2_{(\text{GeV})}}$$

<u>10 GeV/c</u> $\quad \theta_c^2 = \dfrac{.86}{(10)^2} = .0086$

$$\theta_c = .0927 \text{ Radians}$$

Take const. $A = 100\ \text{cm}^{-1}$ calculate $N_{p.e.}$ or rather $L$ for $\underbrace{N_{pe} = 7}_{\Rightarrow\ \varepsilon = 99.9\%}$

$$\frac{dN_{pe}}{dx} = 100 \times .0086 = .86/\text{cm}$$

$$L = \frac{7}{.86} = \underline{8.1 \text{ cm}}$$

<u>100 GeV/c</u> $\quad L = 8 \times (10)^2 = \underline{8 \text{ meters}}$

<u>400 GeV/c</u> $\quad L = 8 \times (40)^2 = \underline{128 \text{ meters}}$

The acceptance of such a counter in any realistic design goes down as it gets very long. This length scaling - whose origin is in the basic physics of Čerenkov radiation and in relativistic kinematics - is the dominant constraint on the range of applicability of threshold counters.

In spectrometer applications, momenta up to $\sim 50$ GeV/c or so can be studied and in beam applications momenta up to $\sim 100$ GeV/c can be used with threshold technique.

This situation has led to new techniques in which the beam and the spectrometer applications diverge.

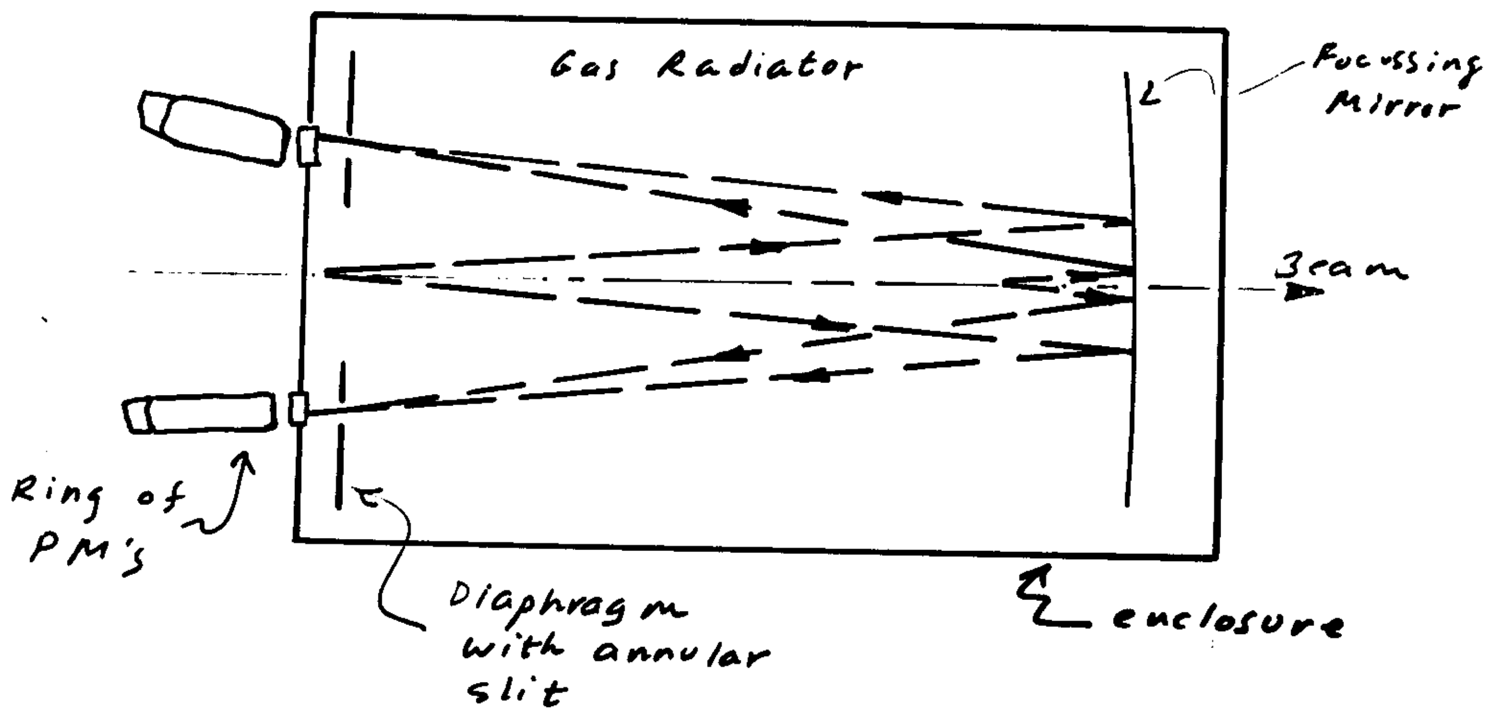
Differential Counter (Beam)

Ring Imaging Counter (Spect.)

We consider these next.

## Differential Counters

Here we try to improve the situation by measuring the $\check{C}$ angle rather than by using a simple light - no light approach. Again, we consider high energies, $\Delta\beta \ll 1$. Such counters are typically used in beam lines to select or "tag" the different types of particles in the beam ($\pi$, $K$, $p$ ...)




A major constraint in these counters is caused by <u>OPTICAL DISPERSION</u>

we will compare the change in $\check{C}$ angle due to $\Delta\beta_{1,2}$ with that due to dispersion ($\Delta\theta_{Disp}$).

Calculate $\Delta\theta_{Disp}$ :

$$\cos\theta = \frac{1}{\beta n}$$

Differentiate wrt $n$ holding $\beta$ fixed

$$-\sin\theta \, d\theta = -\frac{1}{\beta n^2} \, dn = -\frac{1}{\beta n} \frac{dn}{n} = -\cos\theta \frac{d}{r}$$

$$\therefore \quad \operatorname{Tan}\theta \, d\theta = \frac{dn}{n}$$

we now consider $\Delta n$ due to a range of wavelengths $\lambda_1$ to $\lambda_3$ with mean $\lambda_2$.

$$\Delta n = n(\lambda_1) - n(\lambda_3)$$
$$n \equiv n(\lambda_2)$$

Define the dispersion parameter $\nu$ :

$$\nu = \frac{n(\lambda_2) - 1}{n(\lambda_1) - n(\lambda_3)}$$

$$\operatorname{Tan}\theta \; \Delta\theta_{Disp} = \frac{n(\lambda_1) - n(\lambda_3)}{n(\lambda_2)}$$

$$\Delta\theta_{Disp} = \frac{n(\lambda_2) - 1}{\nu \; n(\lambda_2)} \; \frac{1}{\operatorname{Tan}\theta}$$

But $n(\lambda_2)-1$ is related to the $\check{C}$ angle $\theta$ :

recall $\dfrac{\theta^2}{2} = \Delta n - \Delta\beta$

so $\Delta n = \dfrac{\theta^2}{2} + \Delta\beta$

here $\Delta\beta = 1-\beta \cong \dfrac{1}{2\gamma^2}$

Take $n \approx 1$, $\tan\theta \approx \theta$

$$\Delta\theta_{Disp} = \frac{1}{\nu}\left[\frac{\theta^2}{2} + \frac{1}{2\gamma^2}\right]\frac{1}{\theta}$$

$$\Delta\theta_{Disp} = \frac{\theta}{2\nu}\left[1 + \frac{1}{\gamma^2\theta^2}\right]$$

we now calculate $\Delta\theta_{1,2}$ ($\delta\theta$ due to the difference $\Delta\beta_{1,2} = \beta_1 - \beta_2$ between the two velocities we wish to separate)

$$\cos\theta = \frac{1}{\beta n}$$

differentiate wrt $\beta$ holding $n$ constant:

$$-\sin\theta\, d\theta = -\frac{1}{\beta^2 n}\, d\beta = -\cos\theta\,\frac{d\beta}{\beta}$$

$$\tan\theta\, d\theta = \frac{d\beta}{\beta}$$

or $$\Delta\theta_{1,2} \cong \frac{\Delta\beta_{1,2}}{\theta}$$

So to just separate the rings due to $\beta_1$ and $\beta_2$ in the presence of dispersion

$$\Delta\theta_{1,2} = \Delta\theta_{Disp}$$

$$\frac{\Delta\beta_{1,2}}{\theta} = \frac{\theta}{2\nu}\left[1 + \frac{1}{\gamma^2\theta^2}\right]$$

we can solve for $\theta$ – clearly this is maximum $\theta$ allowed:

$$\frac{\theta^2}{\nu} \leq 2\Delta\beta_{1,2} - \frac{1}{\nu\gamma^2}$$

If we have the case where the momentum is known, $p$, then as we saw

$$2\Delta\beta_{1,2} = \frac{m_2^2 - m_1^2}{p^2}$$

whence:

$$\boxed{\frac{\theta^2}{\nu} \leq \left[\frac{m_2^2 - m_1^2}{p^2} - \frac{1}{\nu\gamma^2}\right]}$$

Recalling the steps of the derivation, it is obvious that $\gamma$ is the $\gamma$ of the particle ($m_1$ or $m_2$) which the counter is set to detect.

To carry on with a numerical example, we first list some relevant gas properties:

| gas | $\nu$ | $[n(\lambda_2)-1]\times 10^6$ |
|---|---|---|
| He | 54.5 | 32.90 |
| Ne | 52.2 | 63.37 |
| $H_2$ | 15.7 | 135.3 |
| $N_2$ | 22.5 | 287.0 |
| $CH_4$ | 15.3 | 430.3 |
| $CO_2$ | 19.5 | 433.3 |
| $SF_6$ | 35.5 | 727.3 |

Here the numbers are for 1 atm, 20 °C with $\lambda_2 = 350$ nm

$\lambda_1 = 280$ nm

$\lambda_3 = 440$ nm

For definiteness let us say $m_1$ is detected then $\gamma \cong \frac{p}{m_1}$

$$\frac{\theta^2}{\nu} \leq \left[ \frac{m_2^2 - m_1^2(1+\frac{1}{\nu})}{p^2} \right]$$

Let us take He gas as the radiator (best value of $\nu$) and try to separate $\underline{\pi}$ and $\underline{K}$ at $\underline{p = 50}$ GeV/c

$$\frac{\theta^2}{54.5} \leq \left[ \frac{(.5)^2 - (.14)^2\left(1 + \frac{1}{54.5}\right)}{(50)^2} \right]$$

$$\theta^2 \leq 9.2 \times 10^{-5}$$

$$\theta \leq 7.08 \times 10^{-2} \text{ Rad.}$$

Now if we want clean separation (and one component is large compared to the other) we would like to be separated by several resolution widths.

Typically one chooses $\theta \sim \frac{1}{4}\theta_{MAX}$

so $\theta = 1.77 \times 10^{-2}$

for $A = 100 \text{ cm}^{-1}$

$$N_{pe} = 100\, L\, (1.77 \times 10^{-2})^2 = 3.1 \times 10^{-2} L$$

for $N_{pe} = 10$, $\underline{L = 3.2 \text{ meters}}$

This is a manageable design for a beam line counter.

**Note:** the beam particles themselves must have an angular spread small compared to $\Delta\theta_{1,2}$.

<u>Also</u> one must be careful about optical abber.

## The Chromatically Corrected Counter

If we try to design a Diff. $\check{C}$ counter for $p = 300$ GeV/c, it will clearly be $(300/50)^2 = 36$ times longer!

This problem led to the refinement of the chromatically corrected counter. The basic idea is to insert a chromatic lens into the system which "undoes" the effect of the gas dispersion $n(\lambda)$. Such systems (DISC counters) have been constructed and have separated $\pi$ and $K$ at $p \sim 250$ GeV/c. ($L = 5$ m, $\theta = 24.5$ mr)

In general, the correction schemes (till now) only work for beam line counters because of the stringent limits on the angular spread of the beam particles which are required.

There is a limit to the "perfection" of the correction and beyond "that",* the only recourse for better resolution is again to increase counter length ($L \sim p^{4/3}$).

* correction design is, of course, reoptimized at each momentum range.

# Ring Focussing Č Counters

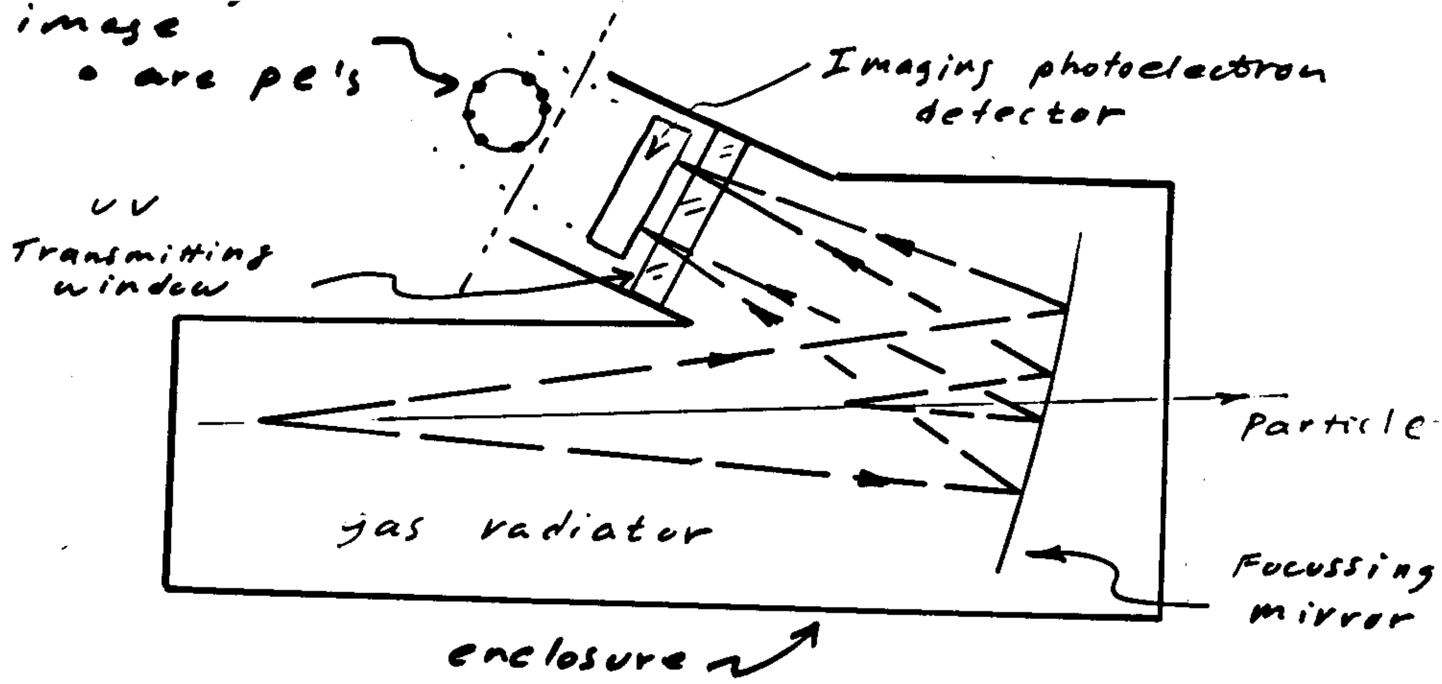
Two physics needs have driven the development (still ongoing) of these counters which were first suggested in their "modern" form by Seguinot and Ypsilantis (1977).

a) The need for larger angular acceptance, while maintaining the resolution + compactness of differential counters, in fixed target experiments.

b) The need for compact, ~ $4\pi$ acceptance, Č counters in $e^+$-$e^-$ high energy collider experiments.

## Basic Idea

Measure the radius of the ring of Č light formed at the focal plane of the focussing mirror. This is done by using a position sensitive photo electron detector.




Schematic Design
of Ring Imaging Counter

The RICH has much the same phenomenology as the differential $\check{C}$ counter with of course the added resolution factors arising from the accuracy of the imaging detector.

The technique has evolved and photosensitive gases (e.g. TMAE) used in a drift chamber detector allow detection of photons farther into the U.V. [TMAE I.P. 5.4 eV and quartz windows allow 170 nm to 220 nm] with increased $\check{C}$ yield (recall $\frac{1}{\lambda^2}$).

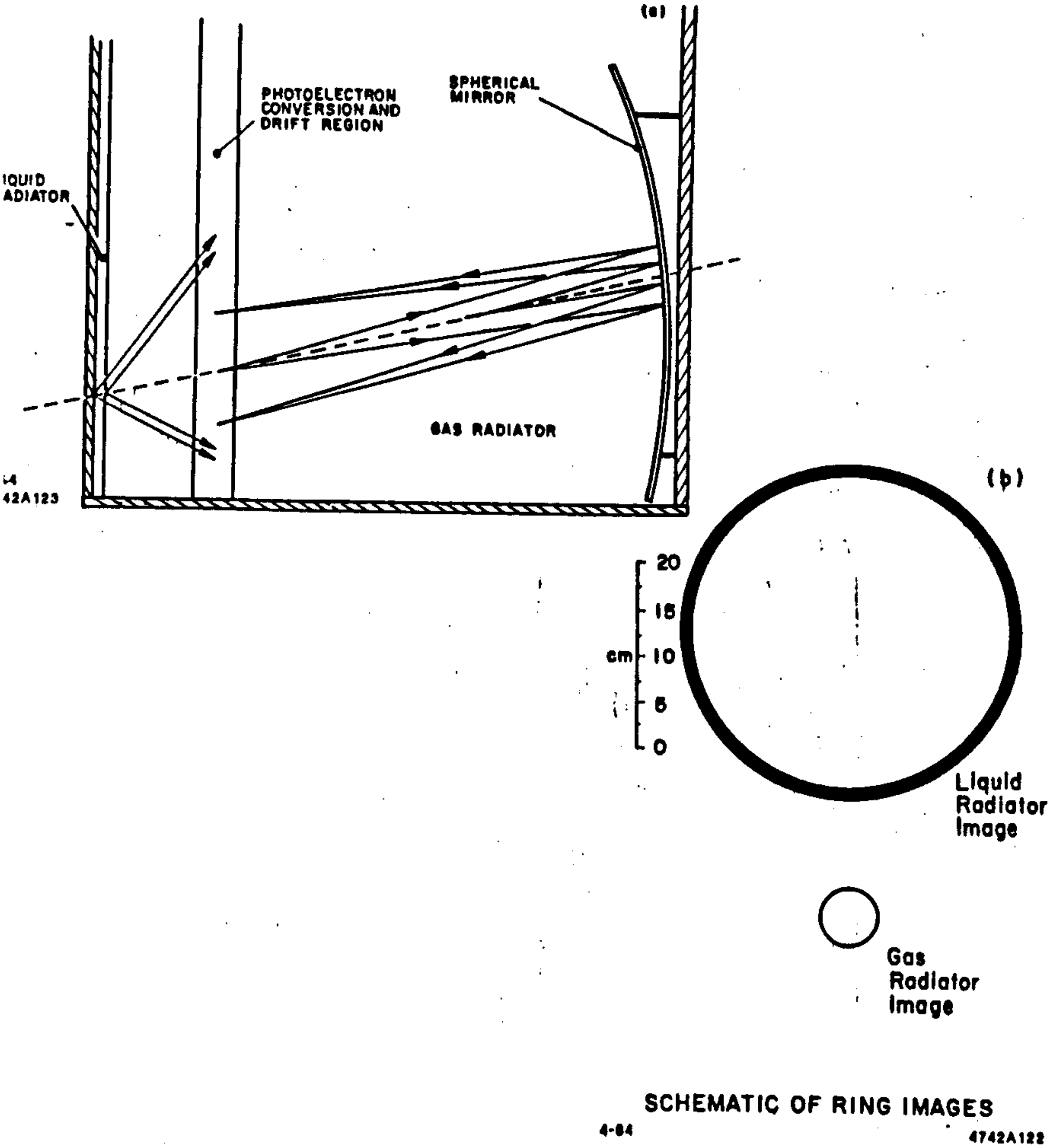



**Figure 6.4.** (a) The *CRID* principle for a two-radiator system. (b) Size and thickness of ring images produced at 90° in a liquid and gas radiator.

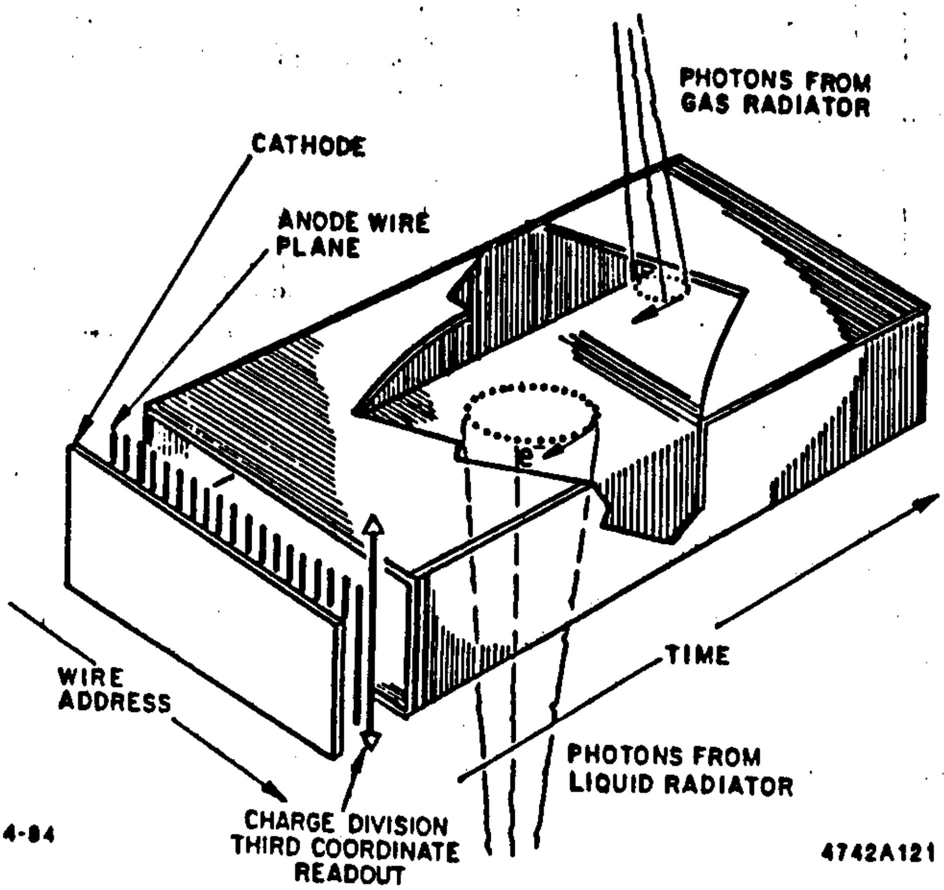



**Figure 6.5. Photon detector schematic.**

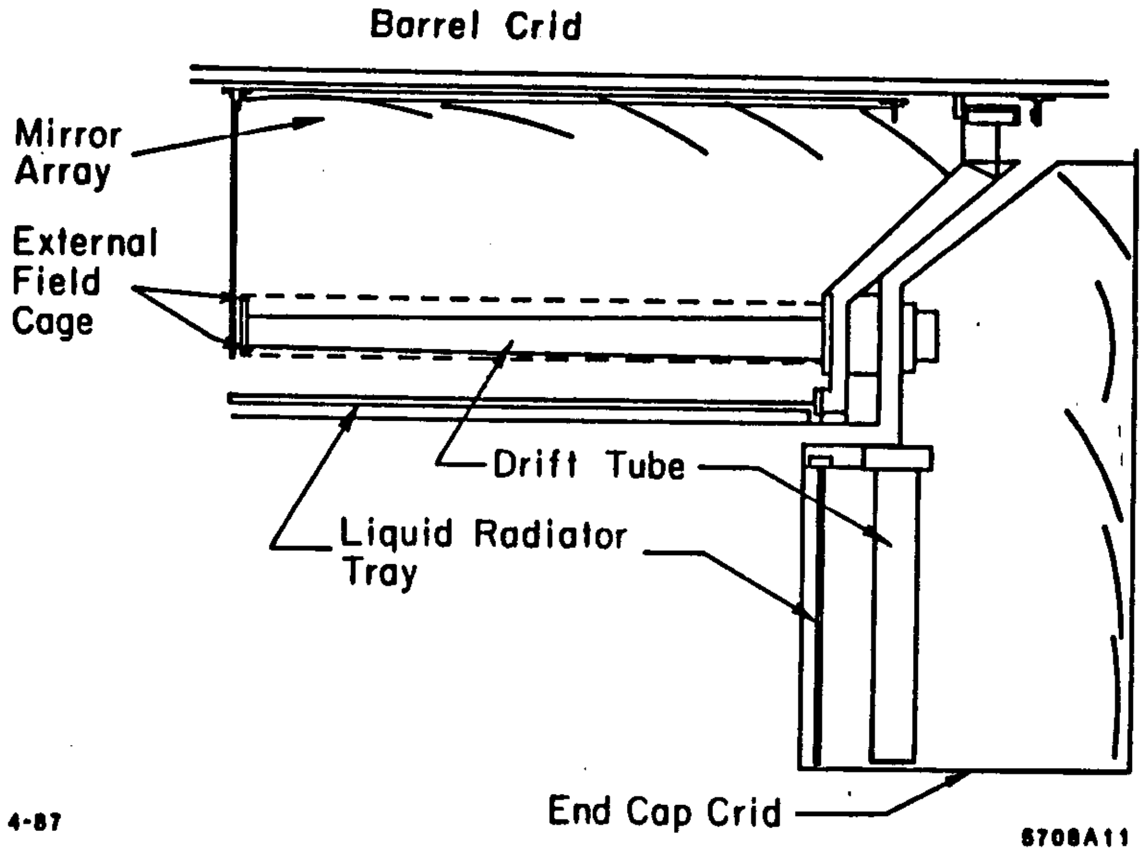



Figure 6.11. Schematic of barrel and endcap longitudinal section.

Table 6.1.
Summary of Parameters of *SLD CRID*

| | | | Liquid | Gas |
|---|---|---|---|---|
| 1. | Solid Angle Coverage | | 88% | 93% |
| 2. | Angular Coverage | (Endcap) | 9.7°—34.8° | 8.5°—42.1° |
| | | (Barrel) | 40.2°—89.1° | 46°—89.5° |
| 3. | Radiator Material | | $C_6F_{14}$ | $C_5F_{12}$ |
| 4. | Index of Refraction[8] | (at 6.5 eV) | 1.277 | 1.001725 |
| 5. | Thickness of Radiator | | 1 cm | ~ 45 cm |
| 6. | Focusing Method | | proximity | spherical mirror |
| 7. | Čerenkov Threshold $\gamma$ for pions | | 1.61 | 17.05 |
| 8. | Čerenkov Angle | (for $\beta = 1$) | 672 mrad | 59 mrad |
| 9. | Radius of Čerenkov Ring | (for $\beta = 1$) | 17 cm | 2.9 cm |
| 10. | Number of Photoelectrons | (for $\beta = 1$) | 14 (at 64°) | 14 |
| 11. | Momentum Threshold | (for 3 p.e.) | | |
| | $e$ | | ~ 1 MeV/c | ~ 9.5 MeV/c |
| | $\pi$ | | 0.23 GeV/c | 2.6 GeV/c |
| | $K$ | | 0.80 GeV/c | 9.1 GeV/c |
| | $p$ | | 1.50 GeV/c | 17.3 GeV/c |
| 12. | Particle Separation Range at 90° (3 $\sigma$ Level) | | [both radiators] | |
| | $e/\pi$ | | (0.2 to 6.2 GeV/c) | |
| | $\mu/\pi$ | | (0.2 to 1.1 GeV/c) (2.1 to 3.8 GeV/c) | |
| | $\pi/K$ | | (0.230 to 23 GeV/c) | |
| | $K/p$ | | (0.800 to 37 GeV/c) | |

The radiator lengths required to keep the dispersion effect acceptable will scale with energy in the same way as the (uncorrected) differential $\check{C}$ counter.

So $L \propto p^2$. Since these counters must lie inside the calorimeter layer, as $L$ increases the volume and weight of the "outside" detectors increase as $L^2$ or as $p^4$!

This is of course highly oversimplified but nevertheless has the "germ" of truth in it. Indeed one does not (yet) see $\check{C}$ counters in the $4\pi$ collider experiments at the $Sp\bar{p}S$, TEV I or proposed for LHC or SSC.

Physics <u>may</u> be such that $\pi, K, p$ separation is not so important at very high energies. However, we believe that $e, \mu$ separation – from each other and from hadrons – is likely to be important. Fortunately, this can be done with other techniques (Transition radiation, shower counters, muon filters, ...) But that is a story for another day.