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SECOND ICFA SCHOOL ON INSTRUMENTATION IN  
ELEMENTARY PARTICLE PHYSICS

12- 23 JUNE 1989

**LECTURE NOTES**

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SECOND ICFA SCHOOL ON INSTRUMENTATION  
TRIESTE ICTP, June 12-23, 1989

P.G. MANFREDI  
LECTURE NOTES

CHAPTER 1 :  
FRONT END ELECTRONICS

THE DESIGN OF A FRONT-END FOR ENERGY AND POSITION MEASUREMENTS IS SUBJECT TO THE MOST SEVERE REQUIREMENTS IF THE DETECTOR HAS NO INTERNAL MULTIPLICATION :

IN THIS CASE THE ACCURACY IN THE MEASUREMENT IS REDUCED BY THE STOCHASTIC NOISE IN THE FRONT-END AMPLIFIERS. AS AN EXTREME SITUATION, THE NOISE MAY IMPAIR THE DETECTION PROCESS.

EVENTS THAT HAVE STIMULATED  
A CONSIDERABLE EVOLUTION  
IN THE DESIGN OF LOW NOISE  
FRONT END ELECTRONICS

- 1] FIXED TARGET EXPERIMENTS  
AT TeV ENERGIES

- 2] DETECTOR DEVELOPMENT FOR  
APPLICATIONS IN COLLIDING  
BEAM MACHINES EXPECTED  
TO BECOME OPERATING SOON

- 3] PRELIMINARY INSIGHT INTO  
THE DETECTOR FEASIBILITY  
IN EXPERIMENTS AT FUTURE  
LARGE HADRON COLLIDERS

DETECTORS WITHOUT  
INTERNAL MULTIPLICATION

SILICON MICROSTRIP DETECTORS  
(POSITION MEASUREMENTS)

SOLID STATE (SILICON, GERMANIUM  
.....)

IONIS. CHAMBER  
(ENERGY MEASUREMENTS)

LIQUID (CRIOGENIC AND ROOM  
TEMP)

IONIS. CHAMBER:  
(ENERGY MEASUREMENTS,

UPPER LIMIT TO THE TIME  
AVAILABLE FOR SIGNAL PROCES-  
SING IN DIFFERENT EXPER.  
SITUATIONS

FIXED TARGET  $\sim 100$  ns

PRESENT HADRON  
COLLIDERS  $\sim 1$   $\mu$ s

LEP, SLC  $\sim 10$   $\mu$ s

HERA, VEP-4  $\sim 100$  ns

FUTURE LARGE  $\sim 10$  ns

HADRON COLLIDERS

DETECTORS WITH A COLLECTION  
TIME SHORT ENOUGH TO COPE  
WITH THE REQUIREMENTS SET  
BY LARGE HADRON COLLIDERS DO  
EXIST (SILICON, FOR INSTANCE).

FRONT-END DESIGN IS FAR  
BEHIND

# EVOLUTION IN THE FRONT-END ELECTRONICS FOR MICROSTRIP READOUT

DISCRETE-THICK FILM DESIGN  
USING STATE-OF-THE-  
ART LOW NOISE COMPONENTS

SETTLED

MONOLITHIC DESIGN FOR LEP  
EXPERIMENTS MOS-VLSI

STARTED  
1983

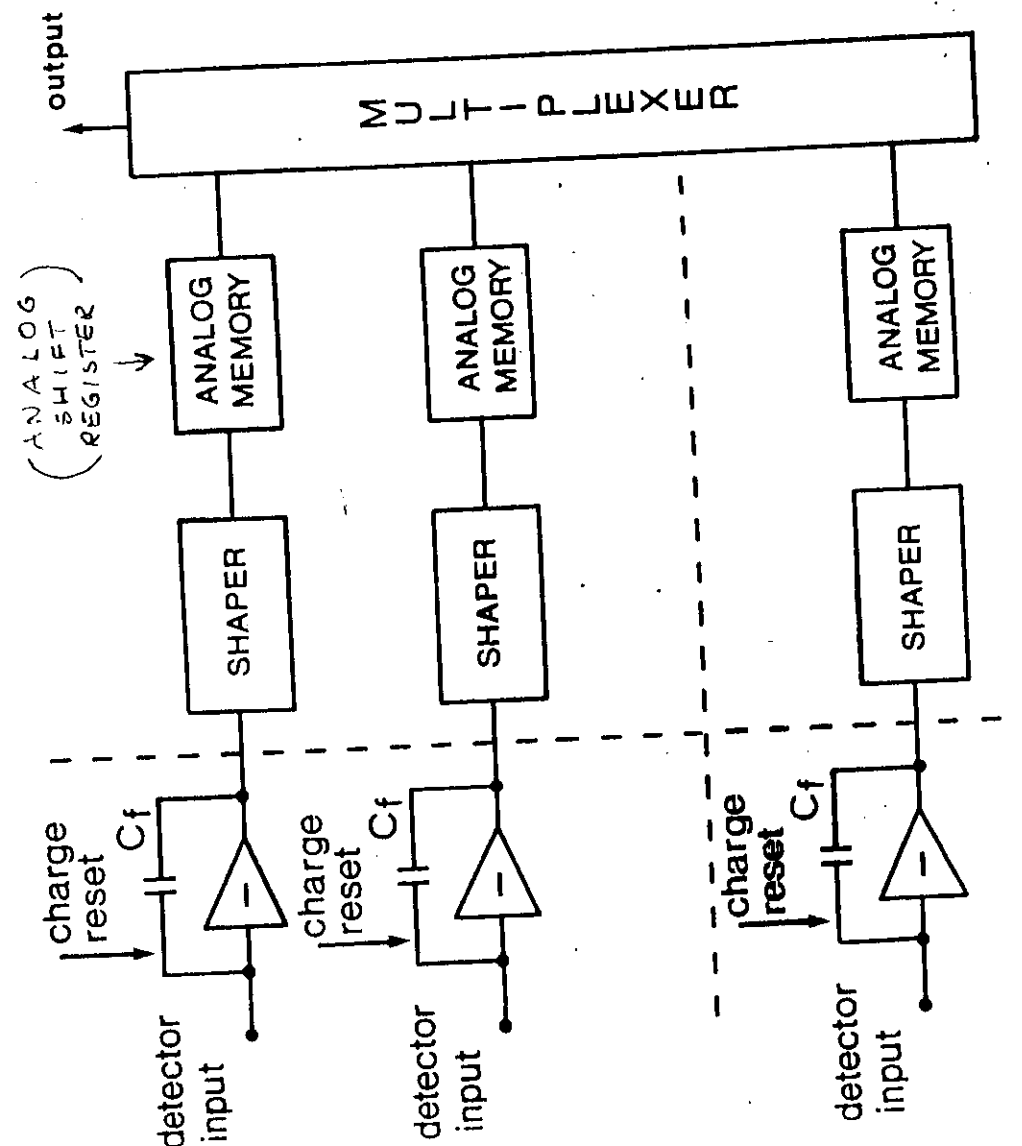
STILL DEVELOPING, FURTHER  
STEPS FORESEEN: RADIATION  
HARDENED DEVICES, JFET-  
CMOS.....

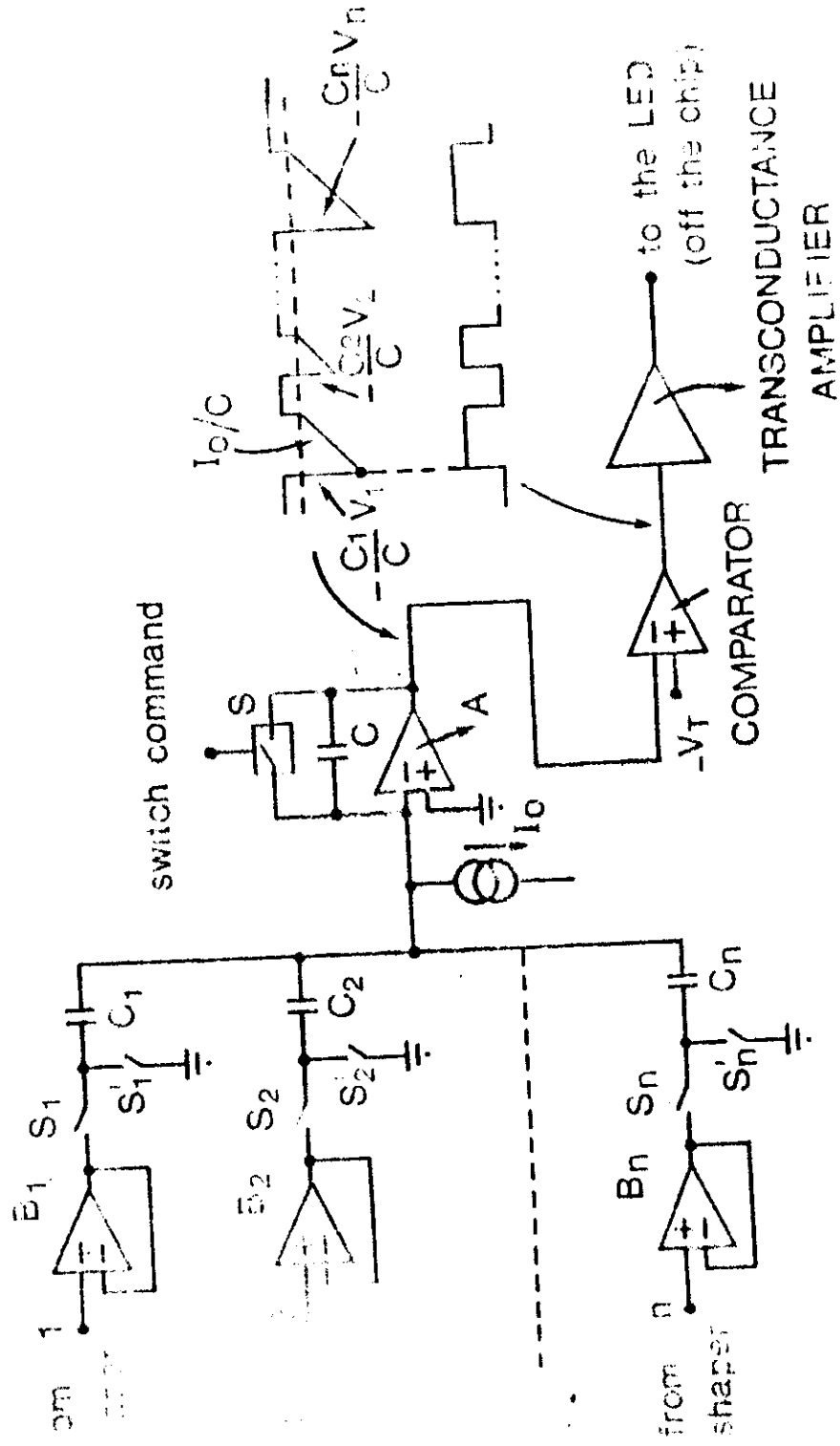
EXPECTED  
TO SETTLE  
BY 1990

TECHNOLOGICAL BREAKTHROUGH  
MAY BE REQUIRED TO COPE WITH  
HERA, VEP-4

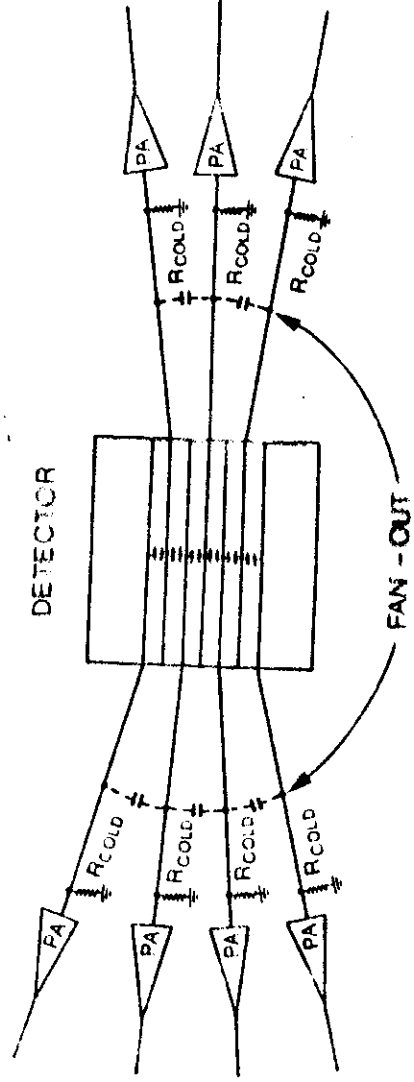
GaAs MESFET VLSI  
MICROWAVE BIPOLAR

TECHNOLOGICAL BREAKTHROUGH  
WILL BE REQUIRED BY EXPERIMENTS  
AT LARGE HADRON COLLIDERS

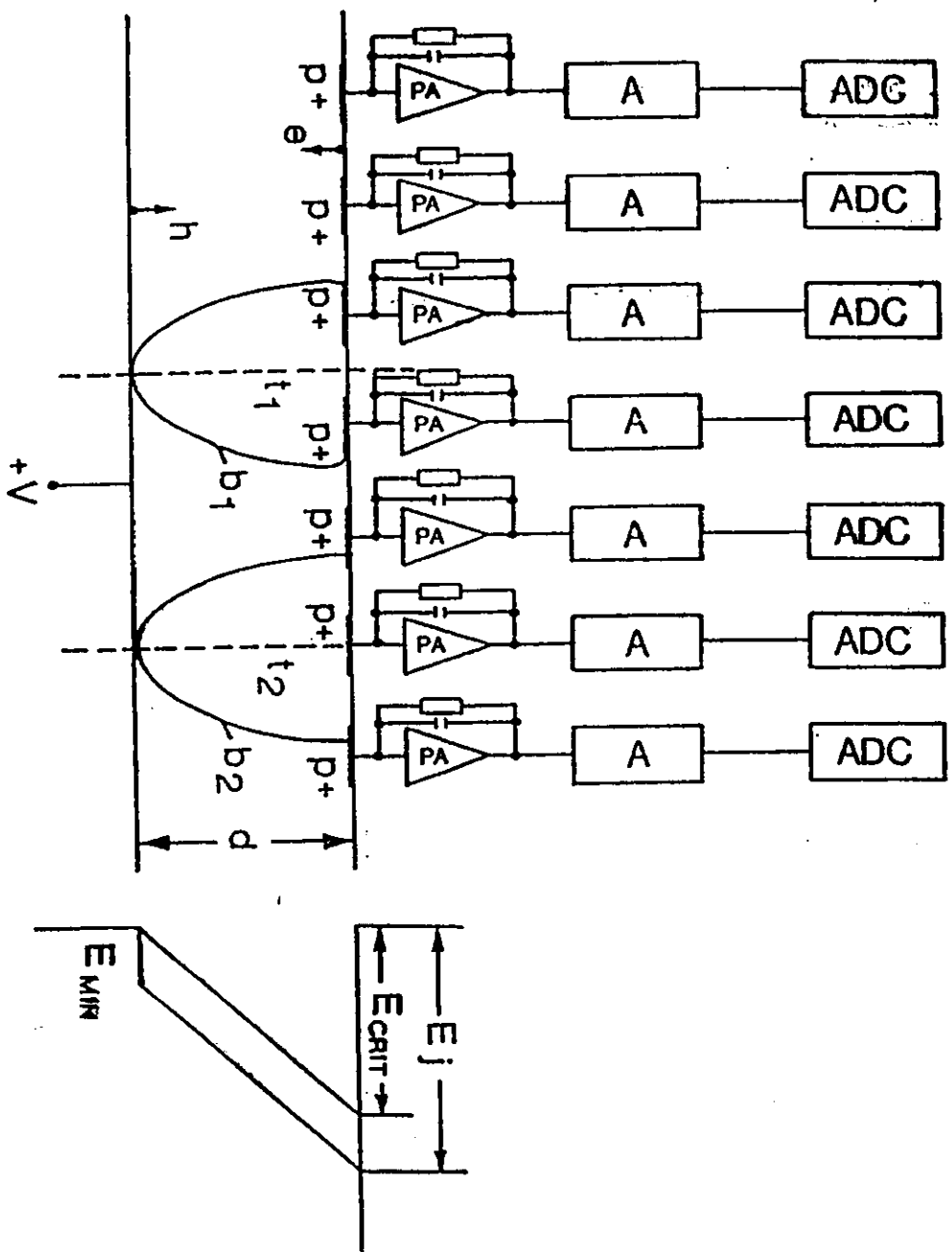


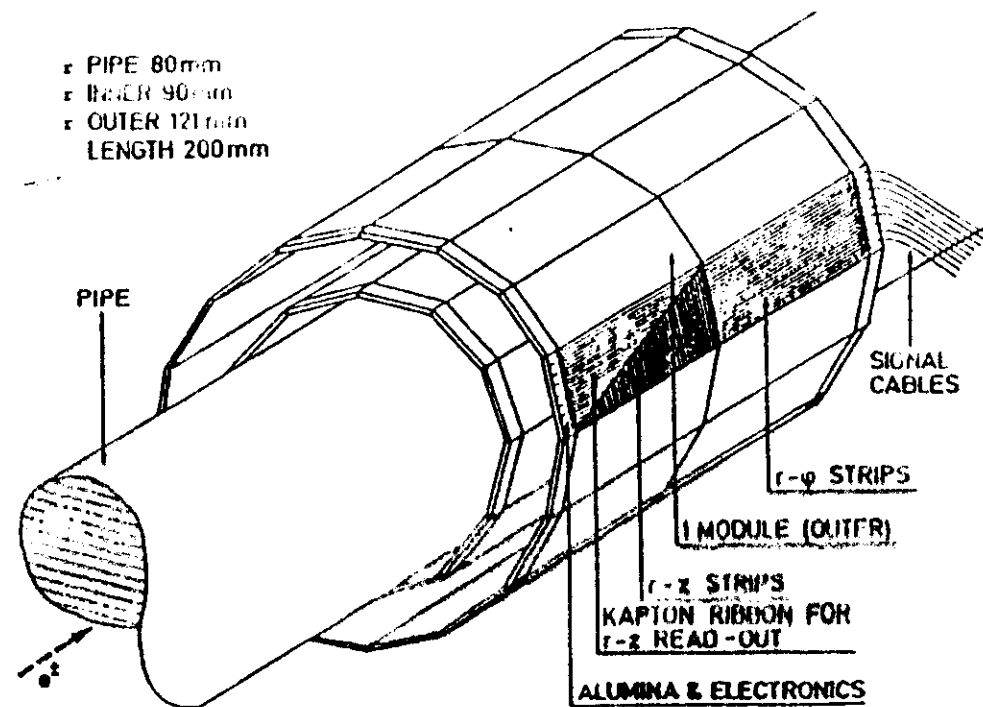
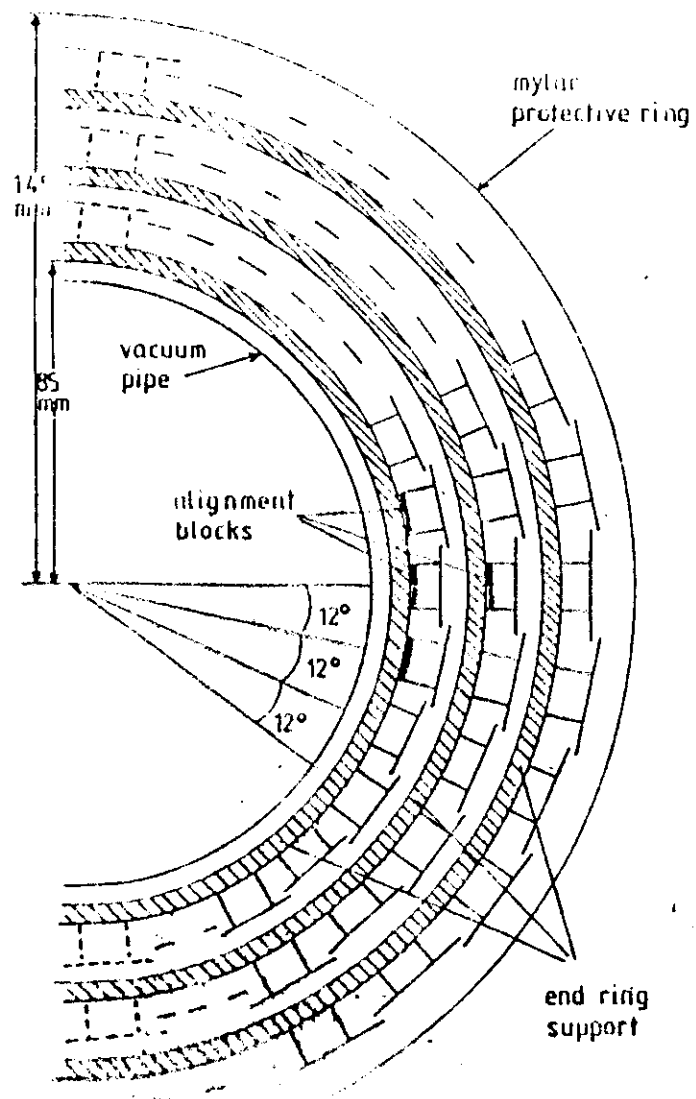


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9

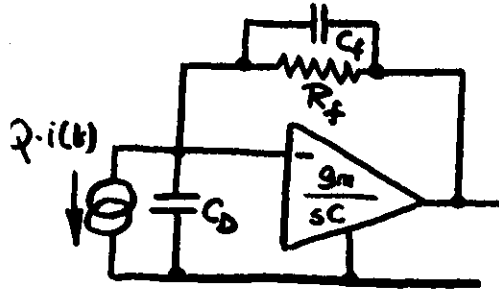




# HEAD AMPLIFIER CONFIGURATIONS

(9)

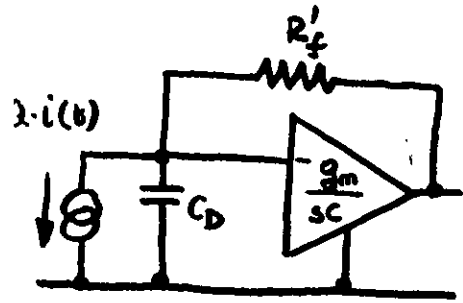
## 1) CHARGE - SENSITIVE LOOP



- NEGLIGIBLE NOISE CONTRI- BUTION FROM SOURCES OUTSIDE THE ACTIVE BLOCK
- IMPEDANCE PRESENTED TO THE DETECTOR IS RESISTIVE ON A BROAD FREQUENCY RANGE

$$Z_i = \frac{C}{g_m g_f}$$

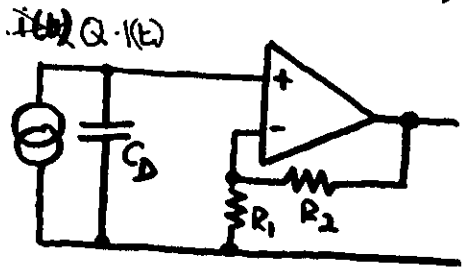
## 2) CURRENT SENSITIVE



- PARALLEL NOISE CONTRIBU- TION FROM  $R'_f$
- IMPEDANCE PRESENTED TO THE DETECTOR OF INDUCTIVE NATURE

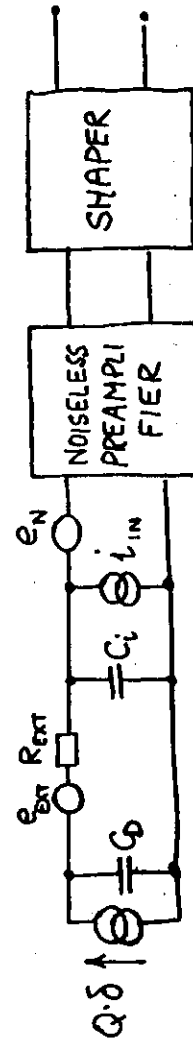
## 3) VOLTAGE SENSITIVE

$$(R_2 \gg R_1)$$



- SERIES EQUIVALENT NOISE RESISTANCE INCREASED OF  $R_1$ .
- IMPEDANCE PRESENTED TO THE DETECTOR OF CAPACITIVE NATURE

(13)



$$ENC^2 = 2kT R_{EXT} C_D^2 \frac{a_{F1}}{\tau_R} + \frac{2kT}{\omega_T} \cdot \gamma \cdot \left[ \left( \frac{C_D}{C_i} \right)^2 + \left( \frac{C_i}{C_D} \right)^2 \right] \cdot C_D \cdot \frac{a_{F1}}{\tau_R} + 4 H_f \left[ \left( \frac{C_D}{C_i} \right)^2 + \left( \frac{C_i}{C_D} \right)^2 \right] C_D + q I_{A2}$$

THERMAL NOISE IN  $R_{EXT}$       COLLECTOR SHOT NOISE      THERMAL NOISE IN THE CHANNEL      PARALLEL NOISE

MICROWAVE BIPOLAR TRANSISTOR •  $R_{EXT} = R_{BB}$  •  $\gamma = 5$  •  $I_{A2} = I_{C2}$

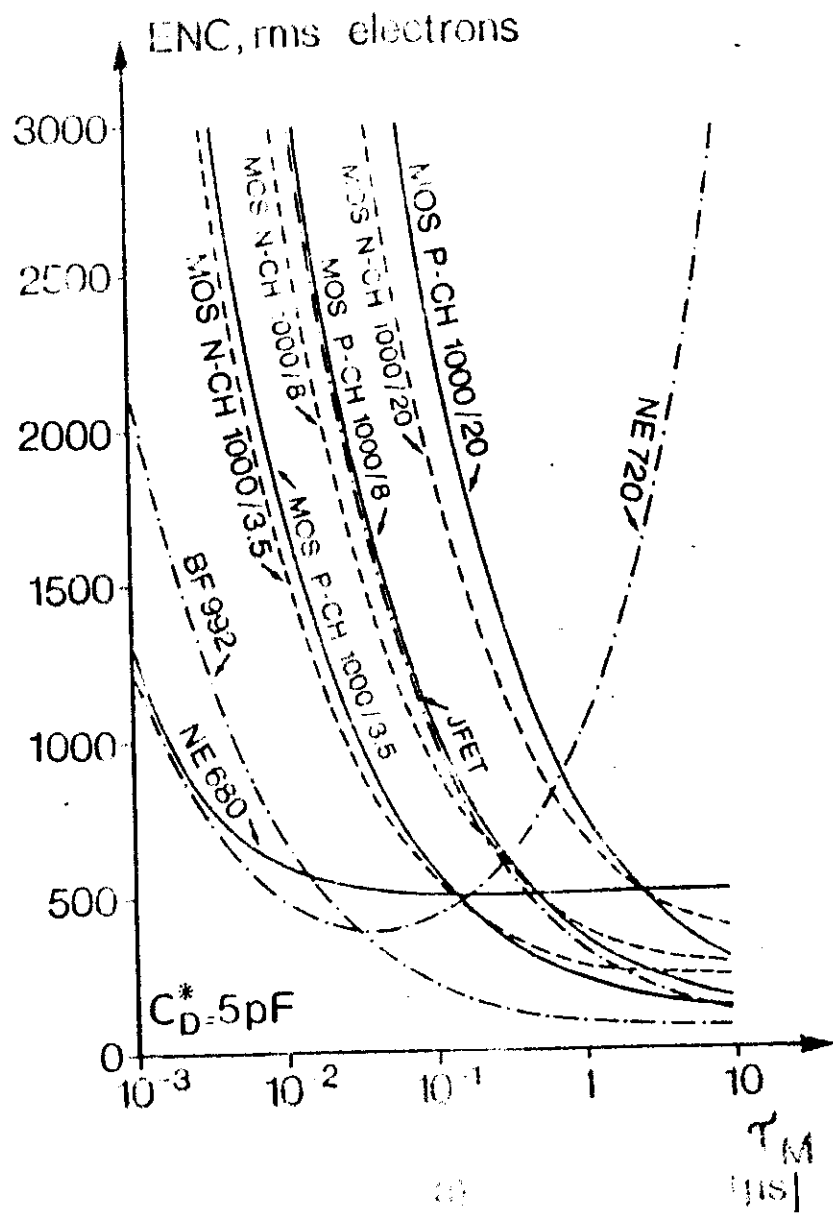
SILICON JFET •  $\gamma = 7$

SB MESFET •  $\gamma = 1.3$  •  $I_{A2} = I_{D2} = 10^{-9} \cdot 10^{-6} A$  •  $H_f = 5 \cdot 10^{-4}$

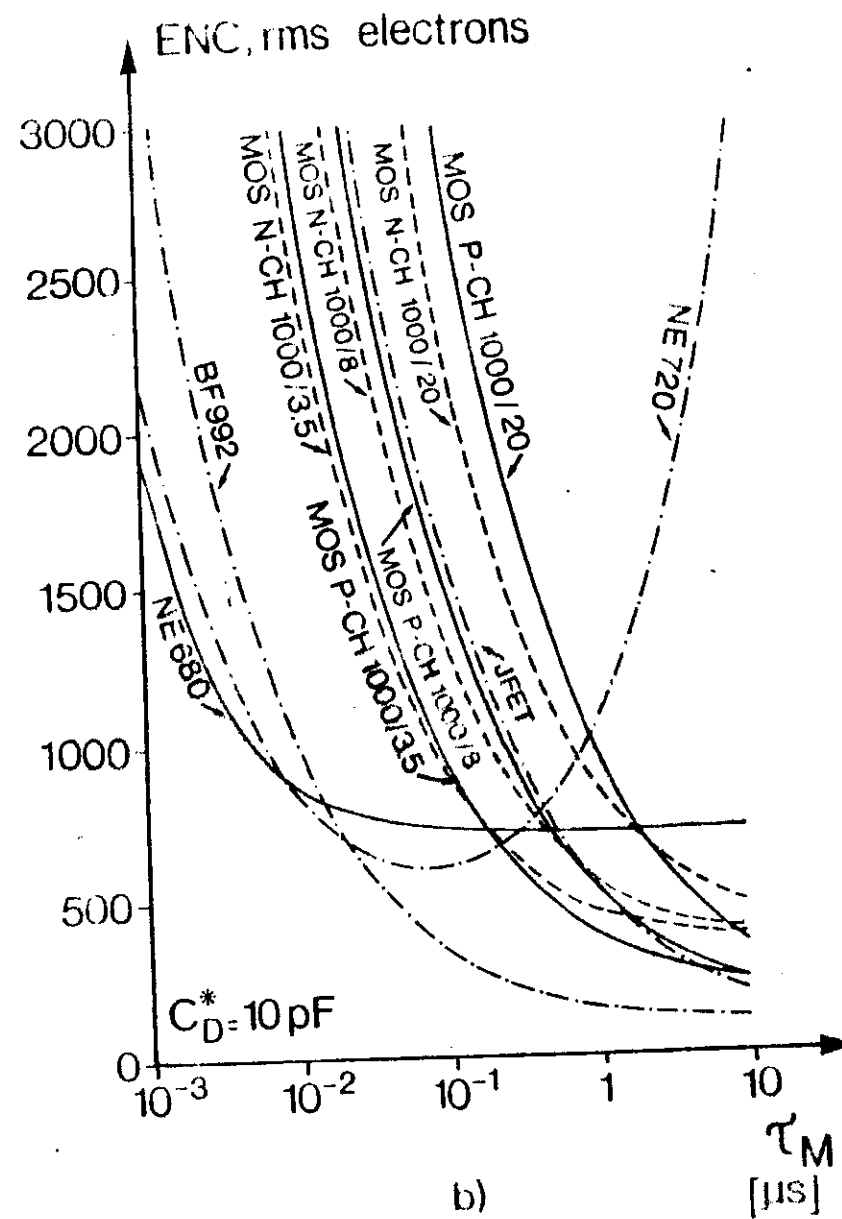
IGFET, MONOLITHIC PROCESS •  $\gamma = 7$  •  $H_f = 10^{-4}$

IGFET, DUAL GATE BF992 •  $\gamma = 7$  •  $H_f = 5 \cdot 10^{-4}$

$ENC_{TOT}^2 = ENC^2 + a_{F2}^2 \left[ \frac{q I_{A2}}{R_f} \right]$

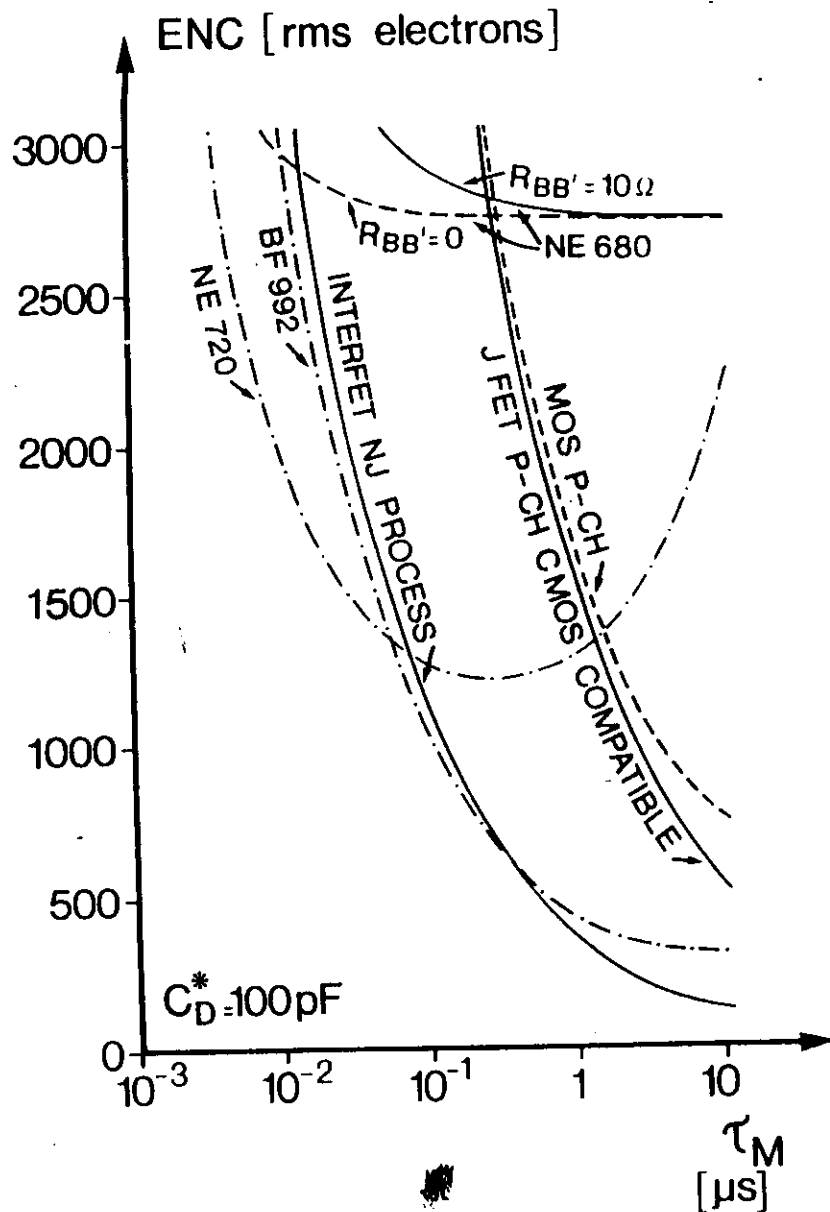


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b)

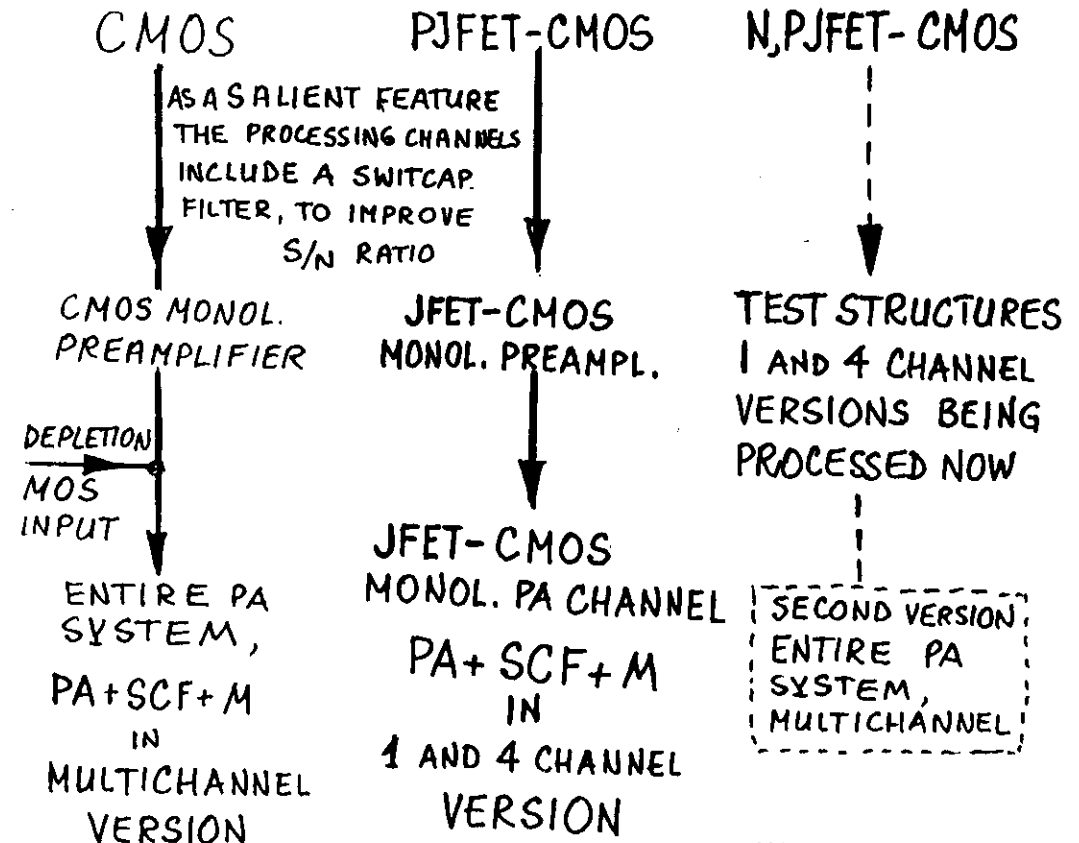
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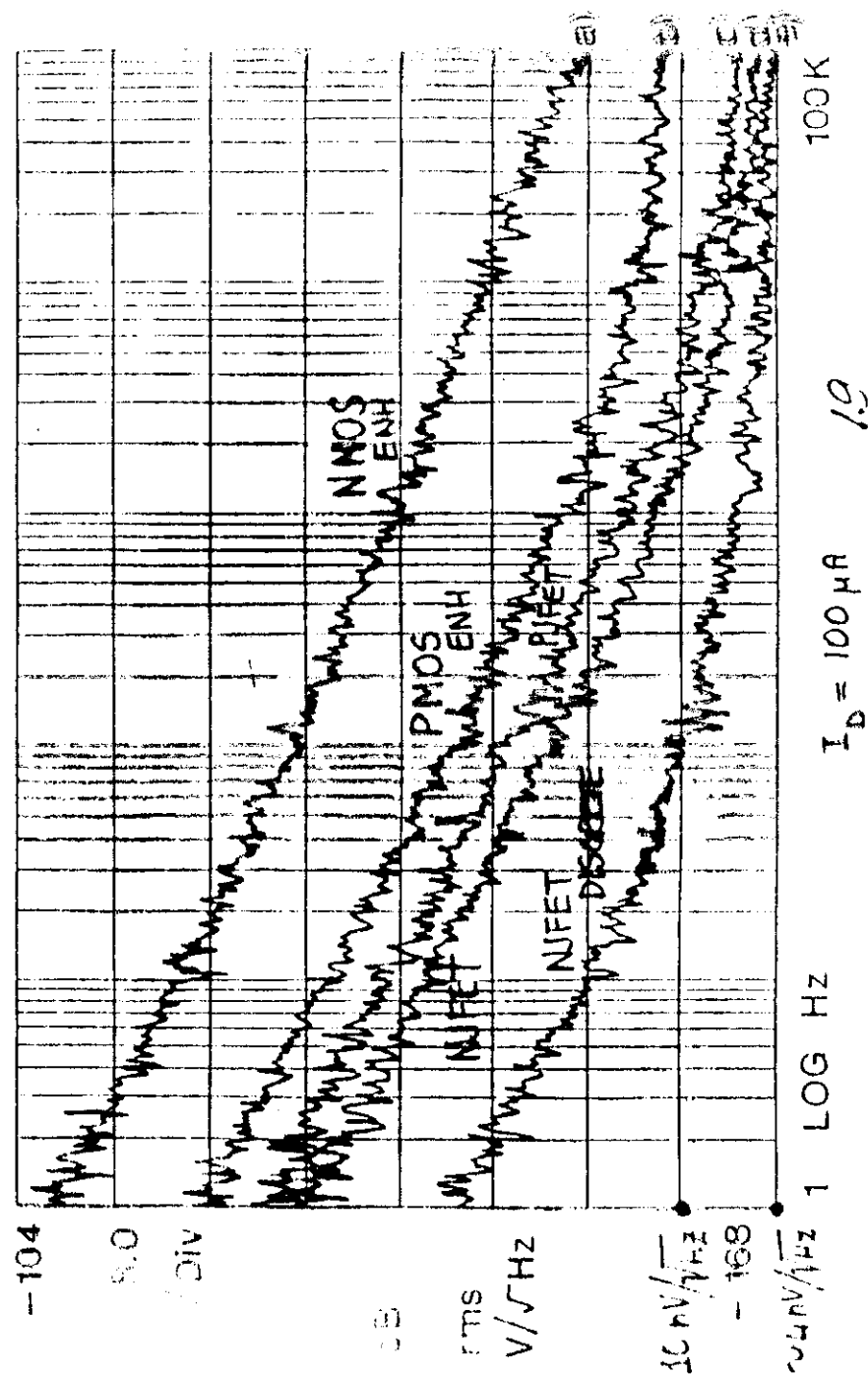
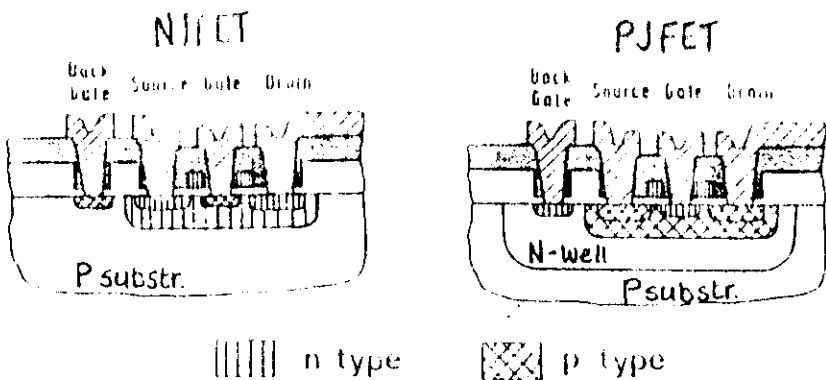


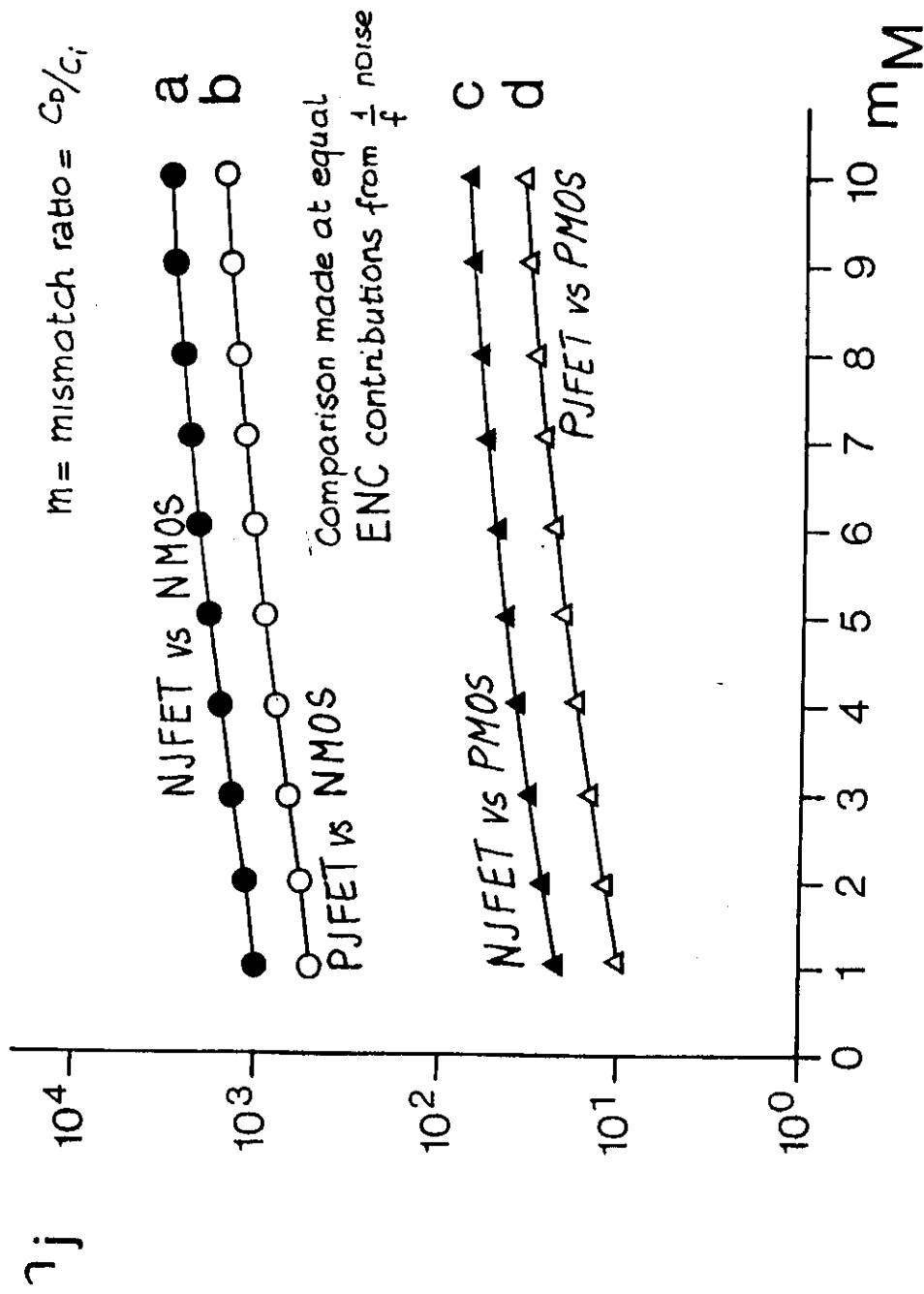
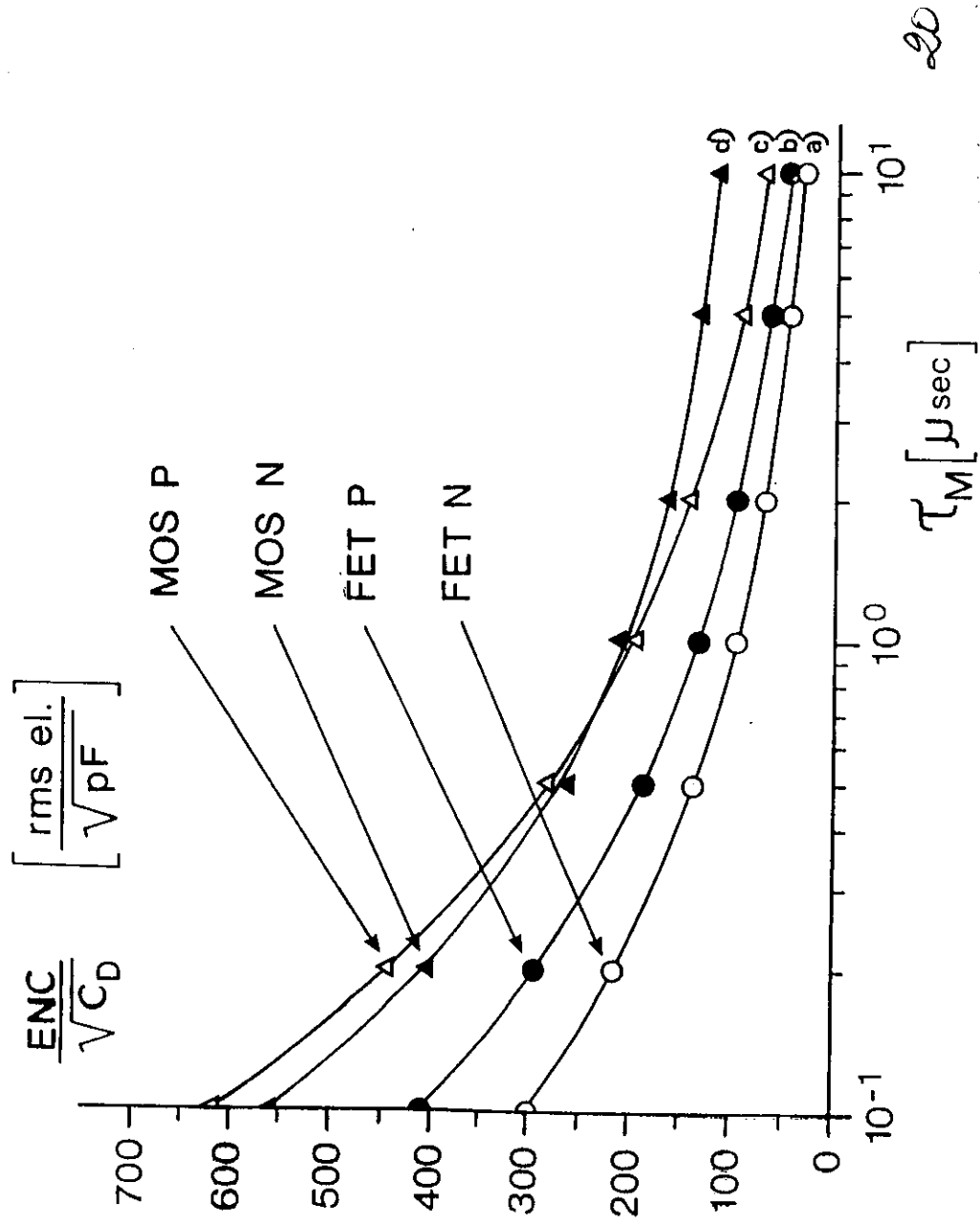
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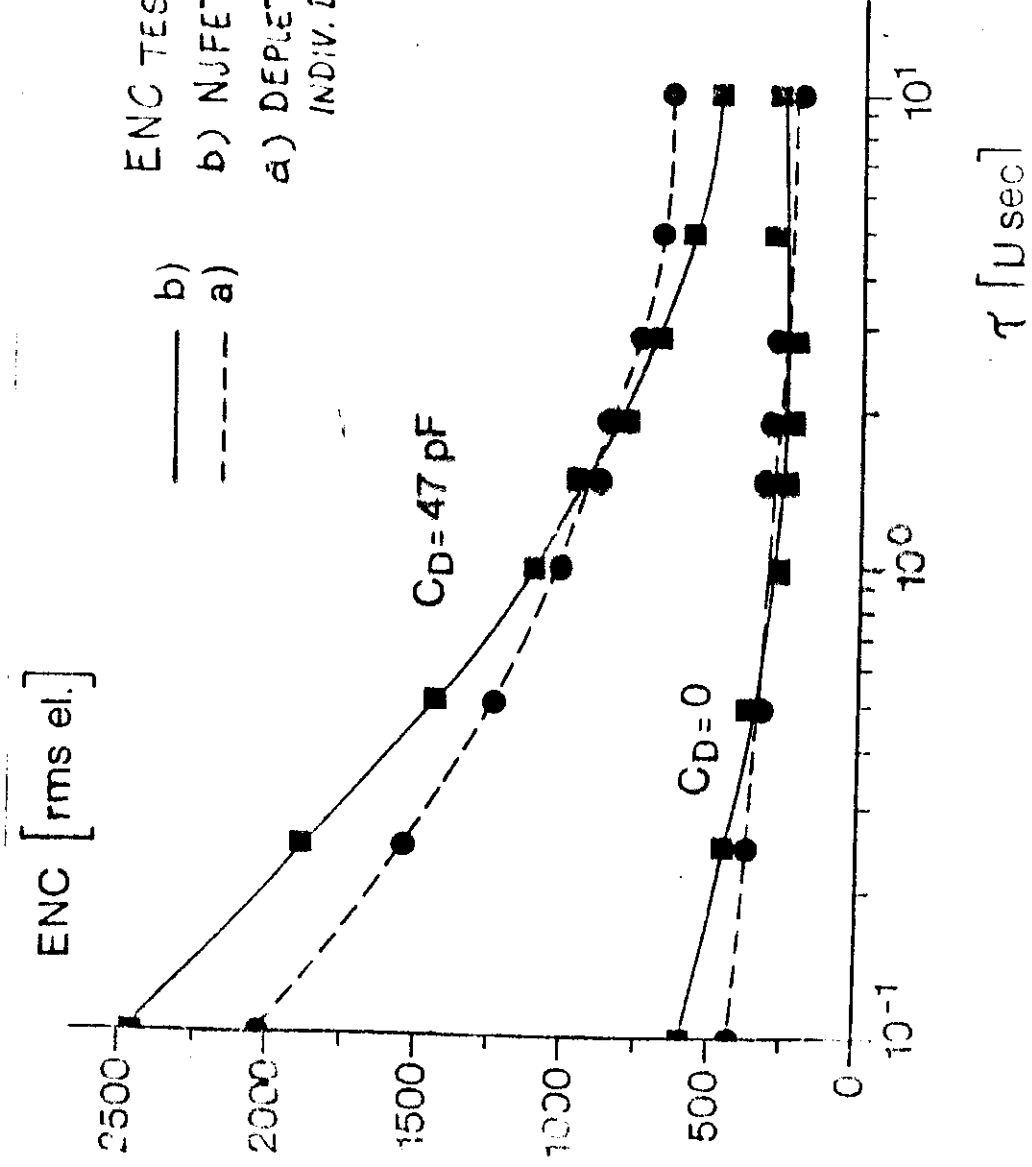
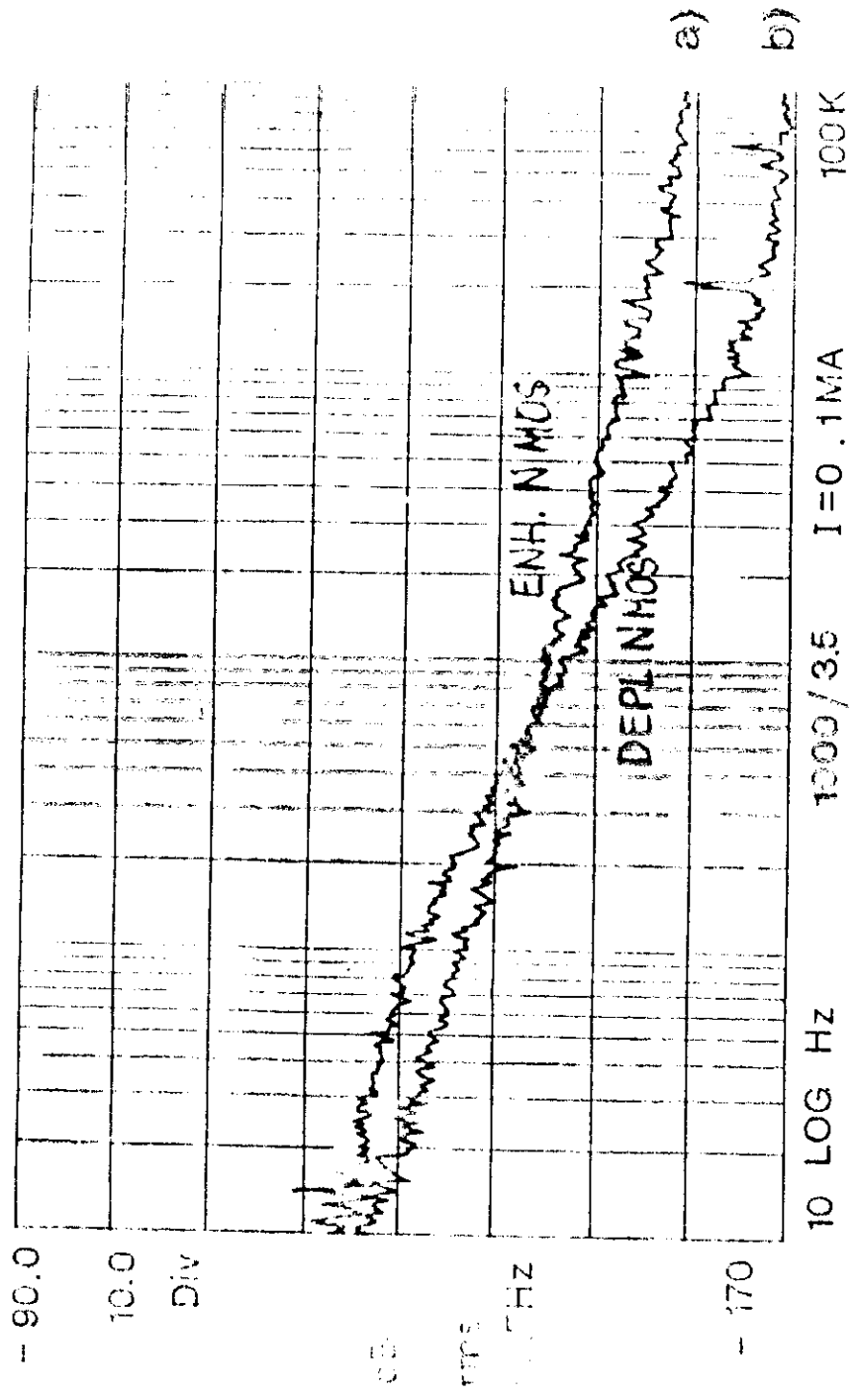
RESULTS ACHIEVED SO FAR -

- TECHNOLOGICAL COMPATIBILITY ON THE SAME SUBSTRATE OF:  
ENHANCEMENT P AND N-CHANNEL MOS (CMOS,  
DEPLETION N-CHANNEL MOS  
JUNCTION P AND N-CHANNEL FETs
- LOW NOISE MONOLITHIC CIRCUITS





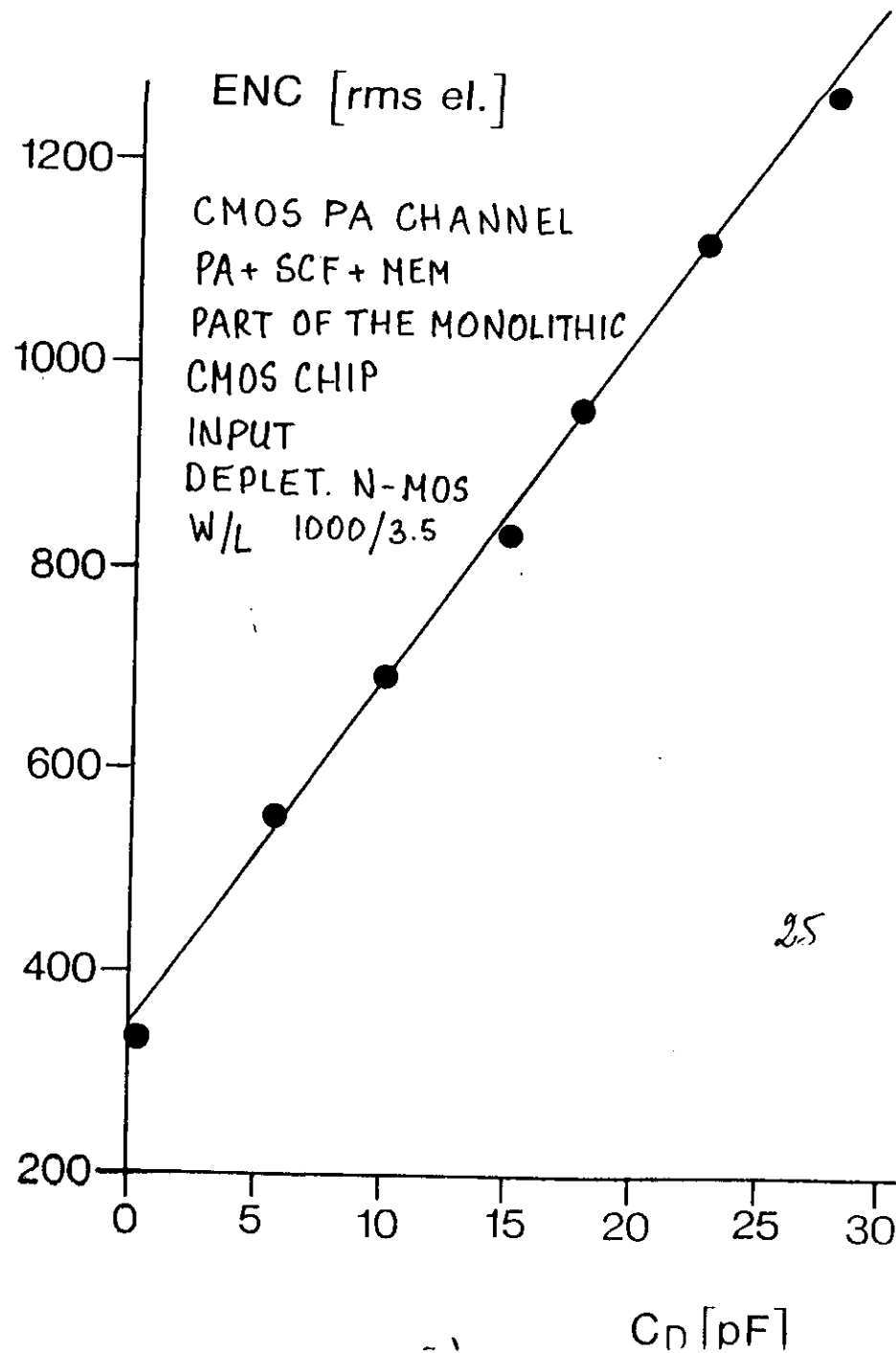
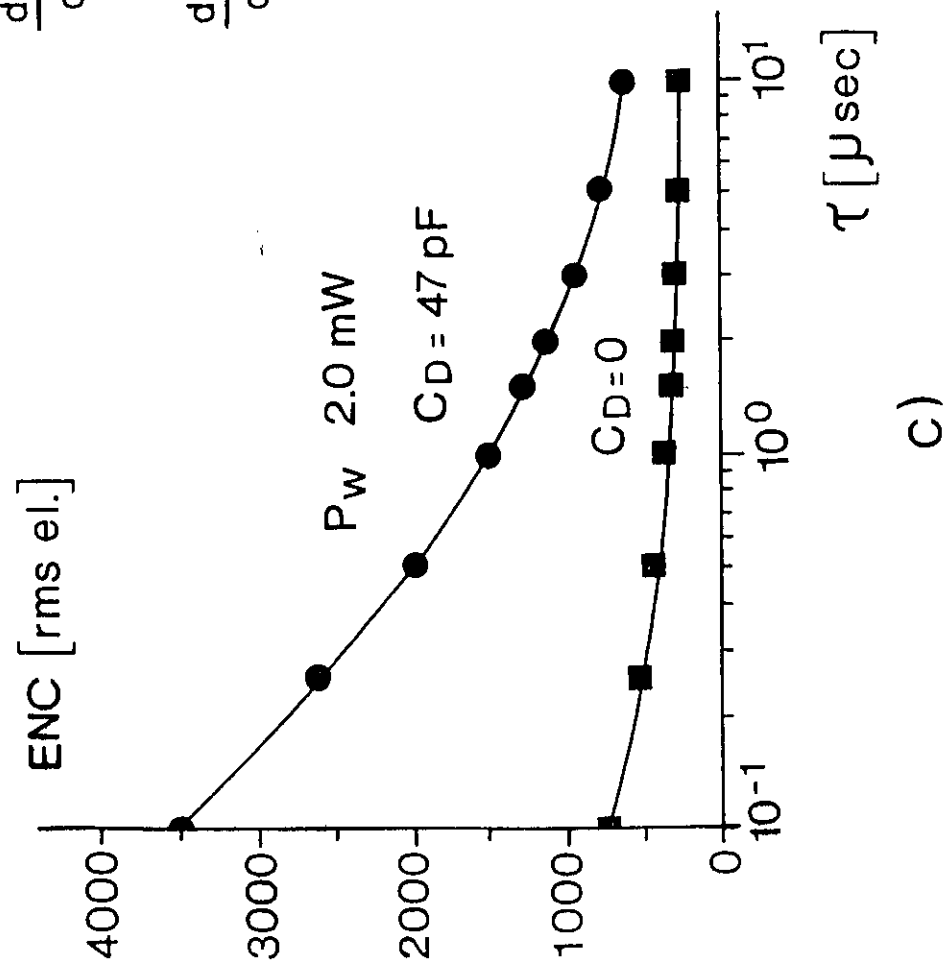


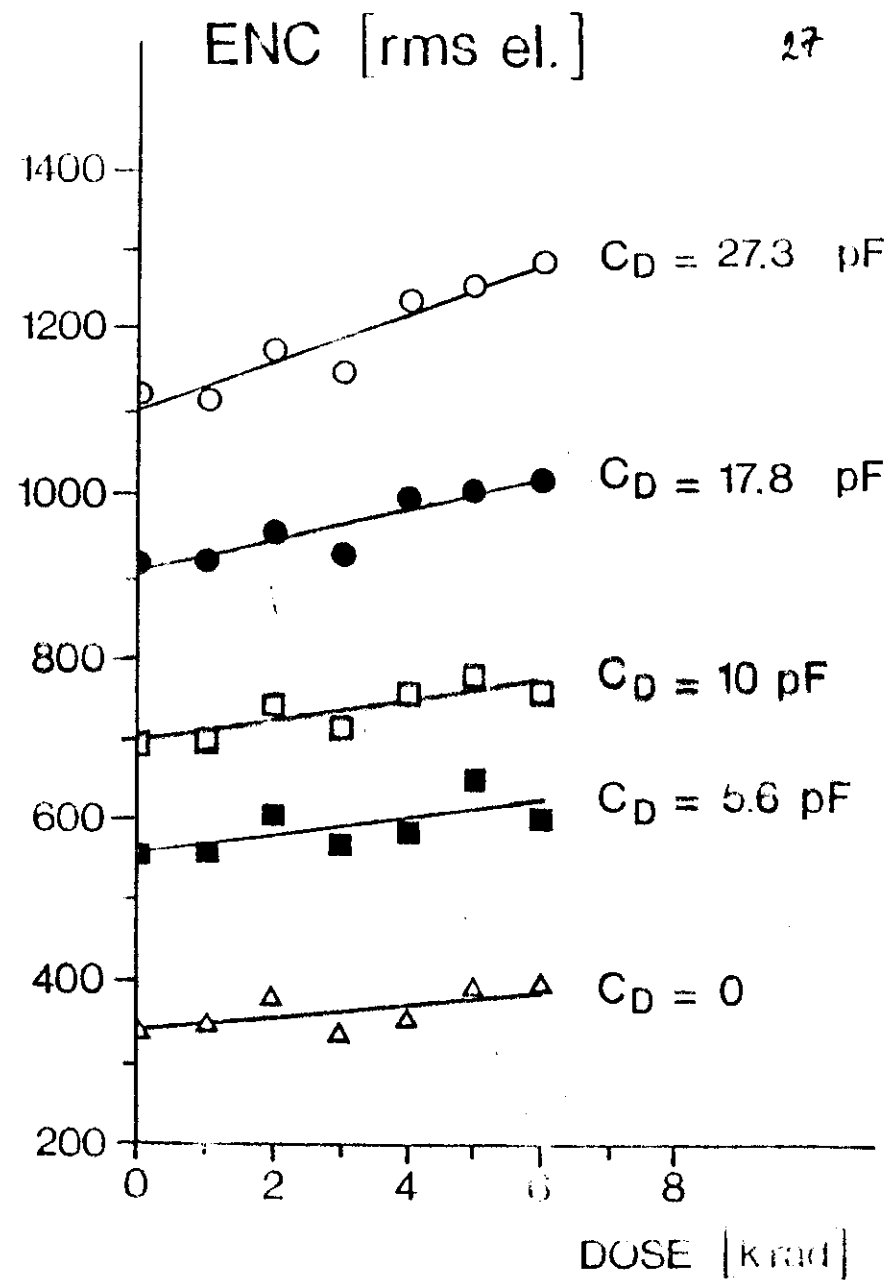
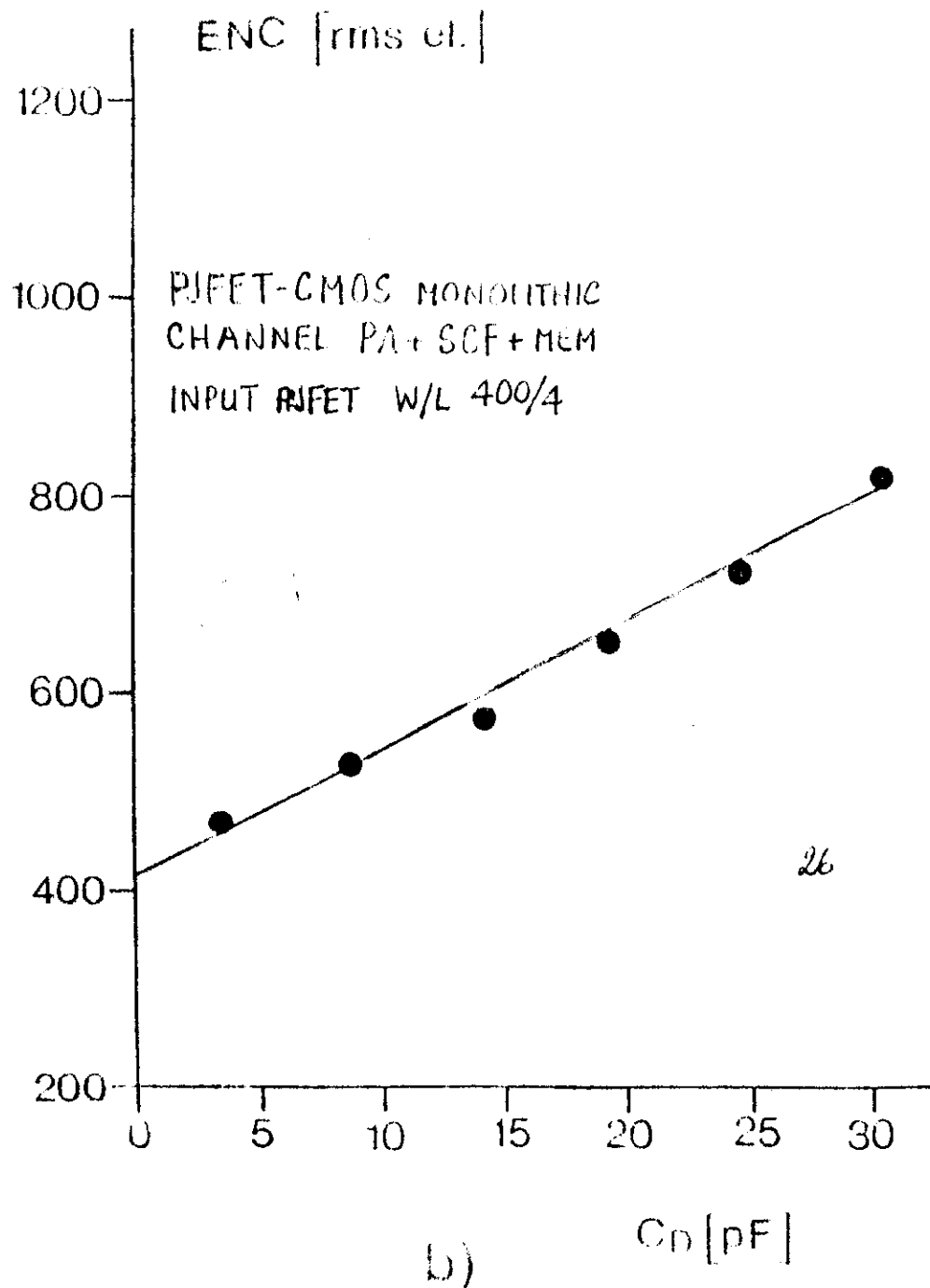


$$\left. \frac{dENC}{dC_D} \right|_{\tau=10\mu s} = 9.2 \frac{\text{el}}{\text{pF}}$$

$$\left. \frac{dENC}{dC_D} \right|_{\tau=3\mu s} = 14 \frac{\text{el}}{\text{pF}}$$

MONOLITHIC  
PJFET- CMOS PA  
INPUT PJFET  
W/L 200/4





ENC [rms el.]

$C_D = 10 \text{ pF}$

■ Dose 0  
△ 10 k rad  
□ 20 k rad  
◆ 30 k rad

28

10<sup>-1</sup> 10<sup>0</sup> 10<sup>1</sup>  
 $\tau$  [ $\mu\text{s}$ ]

a)

ENC [rms el.]

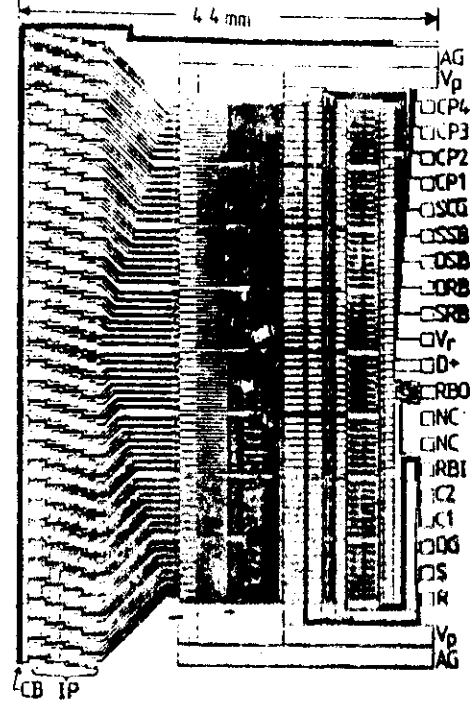
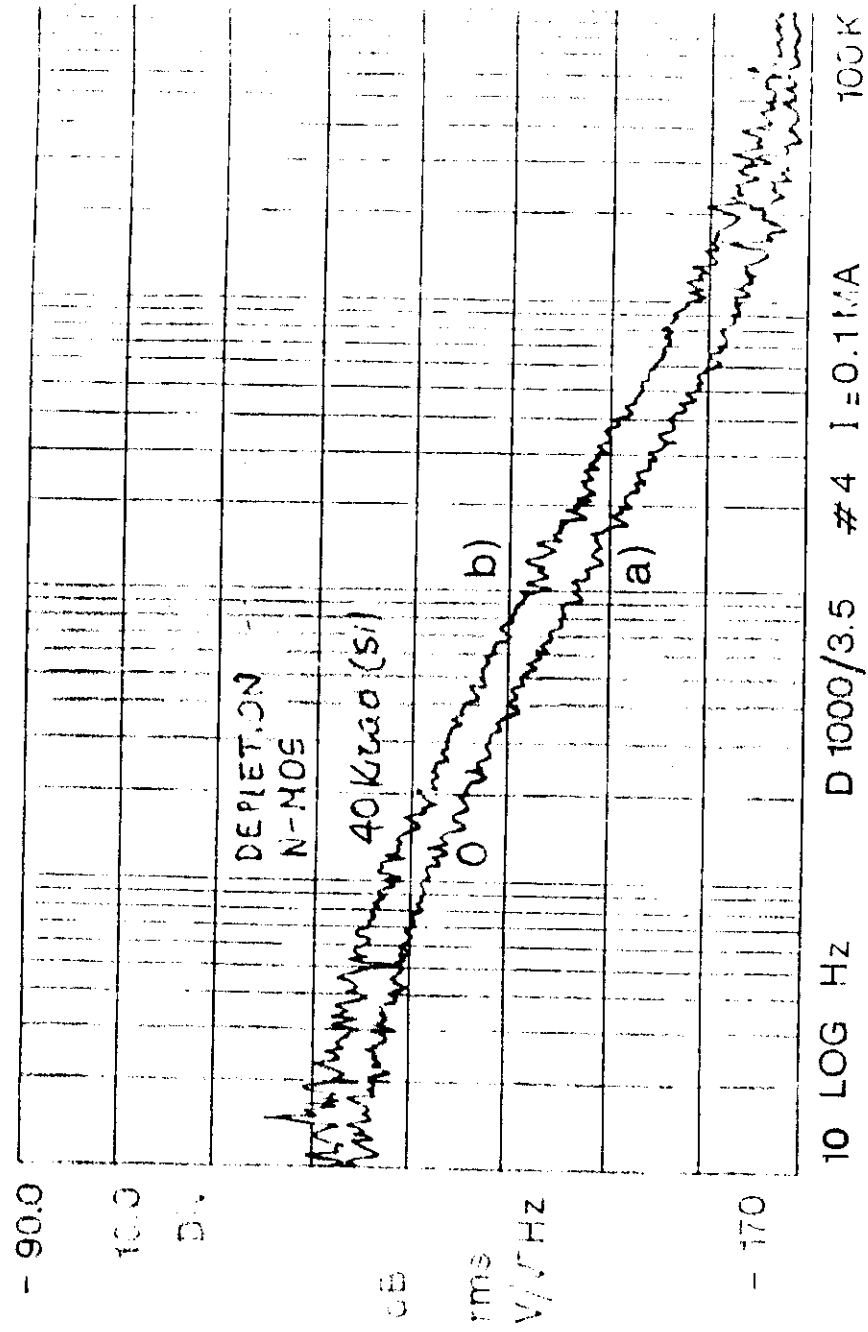
$C_D = 10 \text{ pF}$

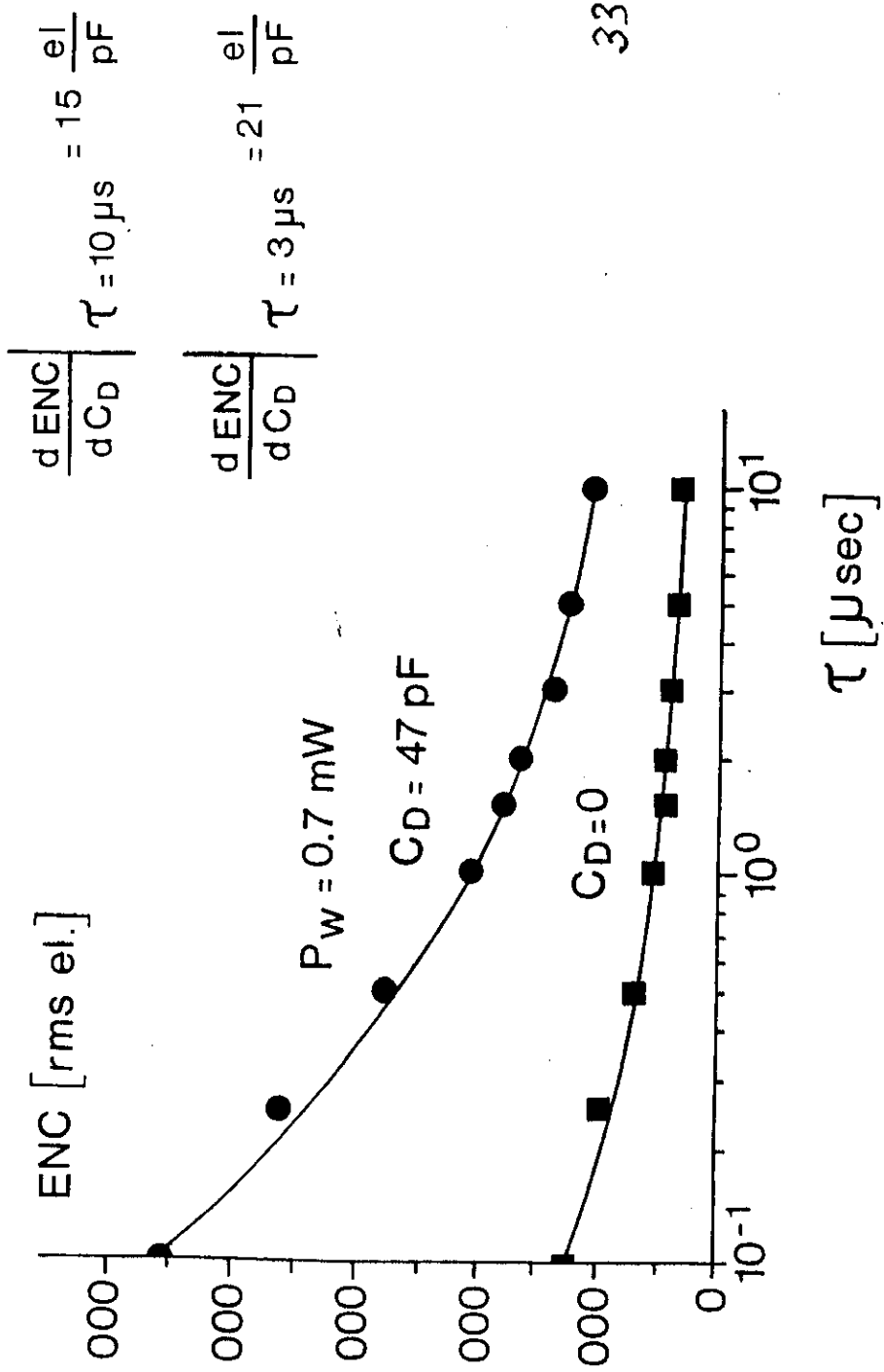
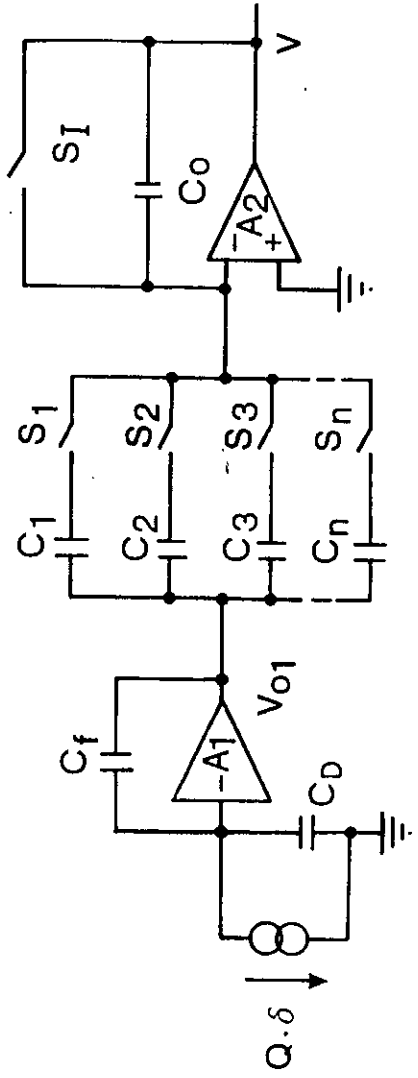
■ Dose 0  
● 40 k rad  
▲ 80 k rad

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10<sup>-1</sup> 10<sup>0</sup> 10<sup>1</sup>  
 $\tau$  [ $\mu\text{s}$ ]

b)





$$\left| \frac{dENC}{dC_D} \right| = 15 \frac{el}{pF} \quad \tau = 10 \mu s$$

$$\left| \frac{dENC}{dC_D} \right| = 21 \frac{el}{pF} \quad \tau = 3 \mu s$$

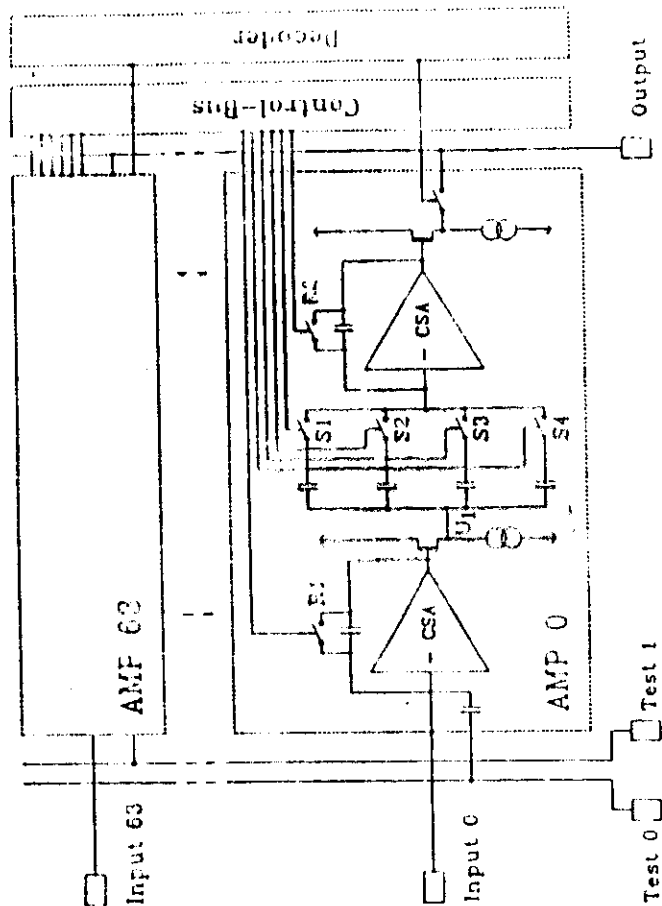
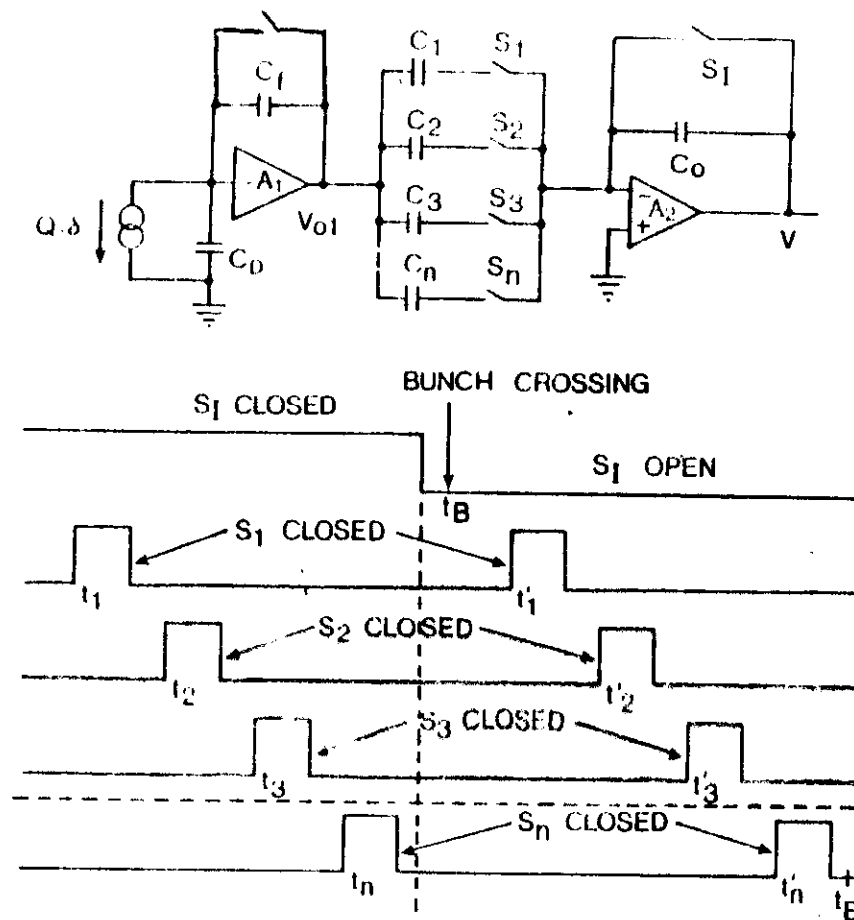
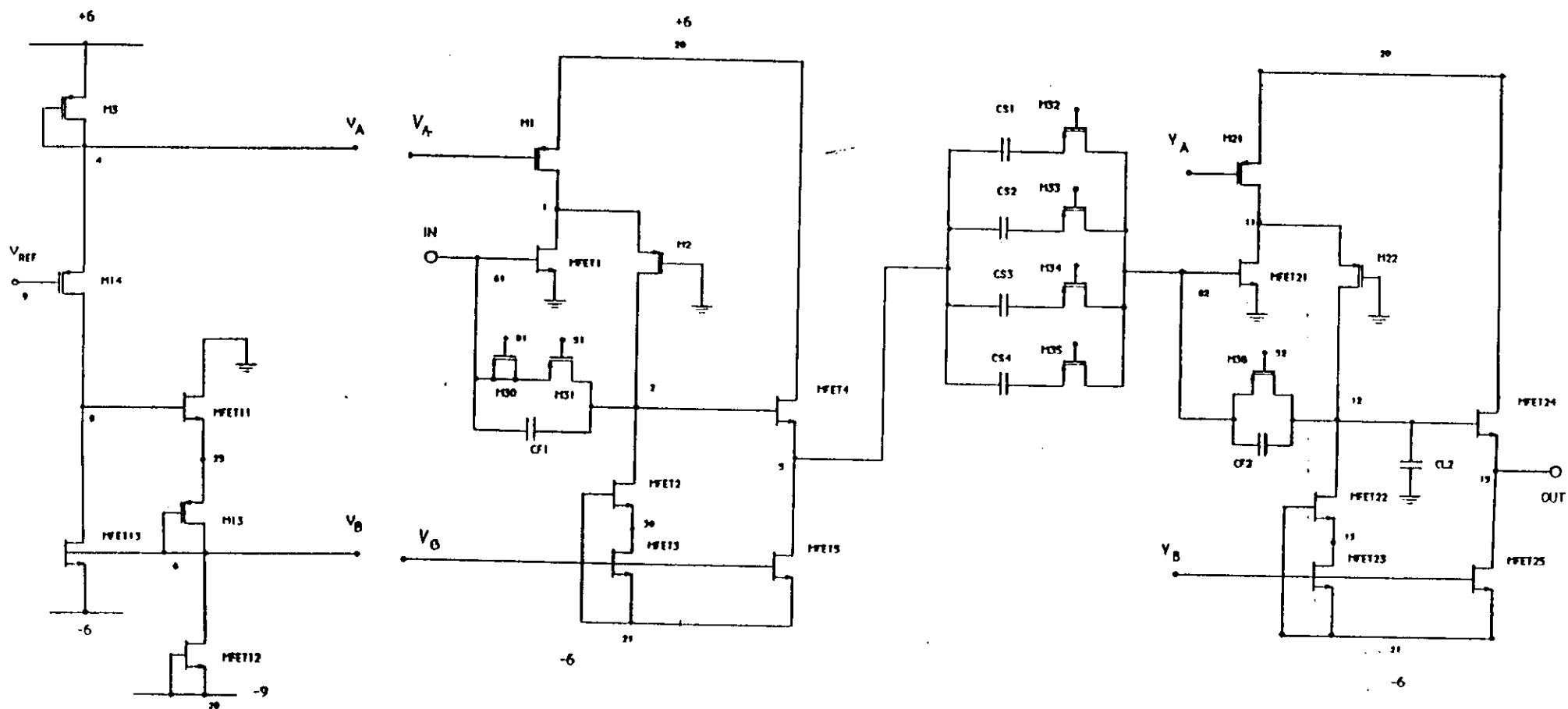


Fig. 1: Channel-amplifier

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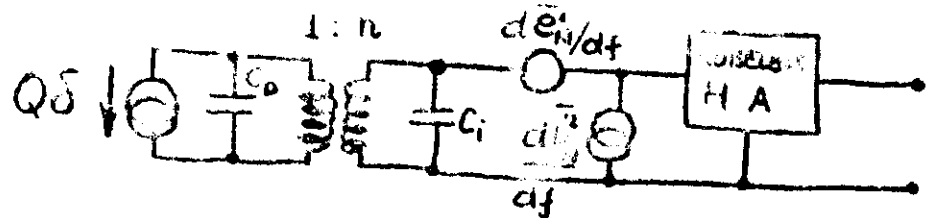


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# TRANSFORMER COUPLING FOR CAPACITIVE MATCHING

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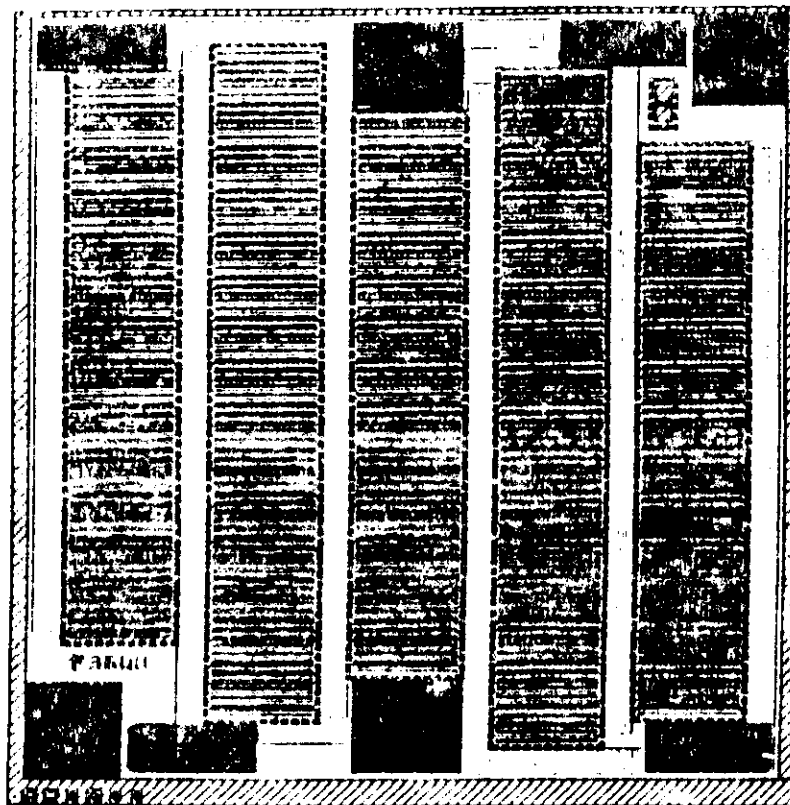
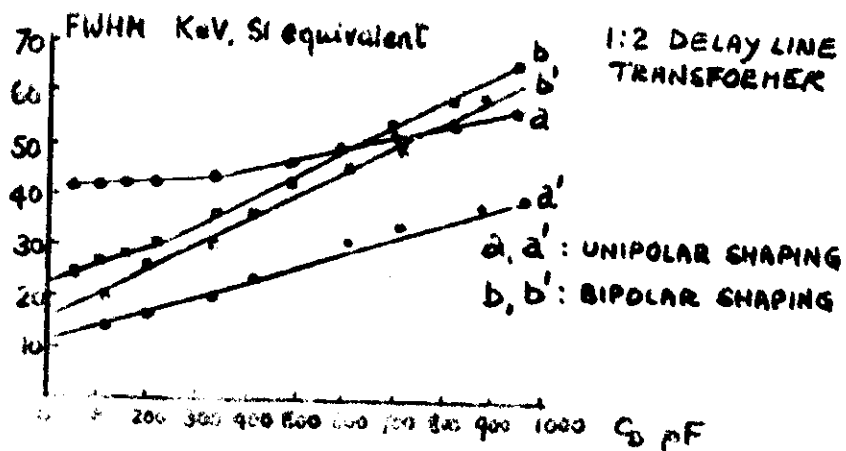


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## REAL TRANSFORMER

### UNDESIRABLE EFFECTS

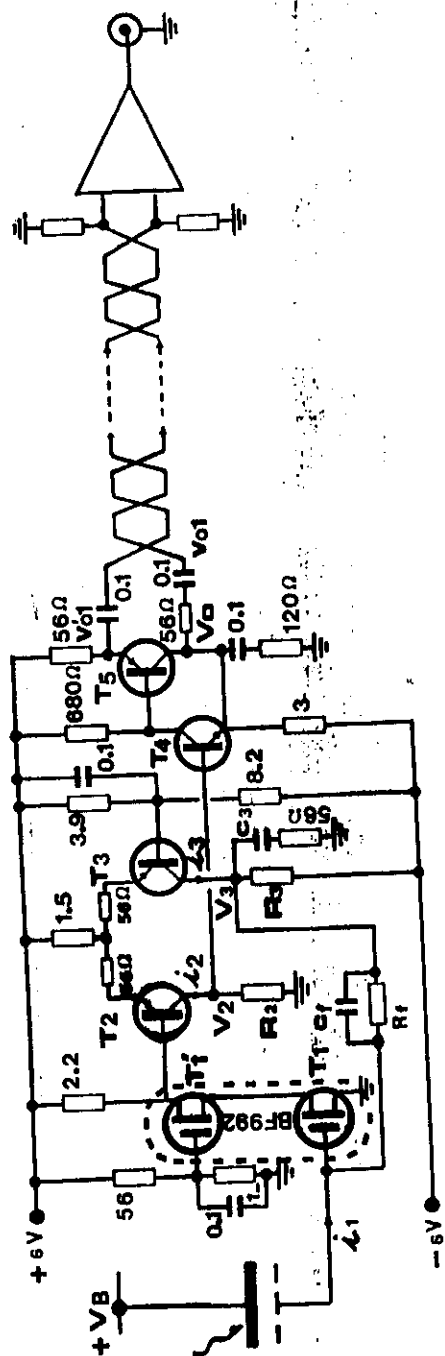
- FINITE MAGNETIZING INDUCTANCE  $L_M$
- WINDING CAPACITANCE
- NOISE ASSOCIATED WITH LOSSES IN THE CORE
- NOISE ASSOCIATED WITH LOSSES IN THE DIELECTRICS.



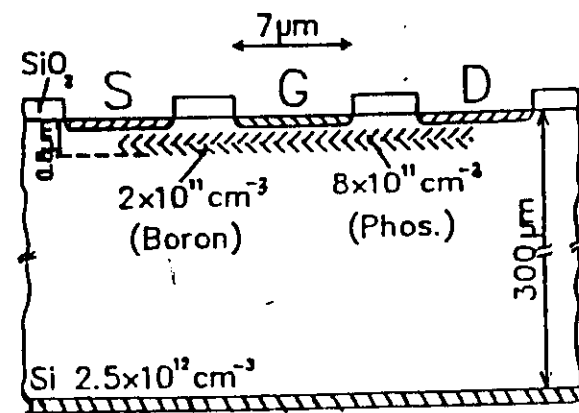
NIM A 1100 24

DELAY LINE TRANSFORMER

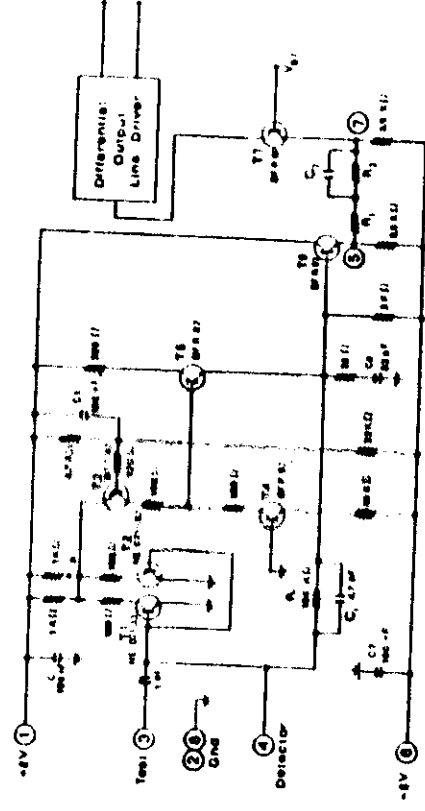
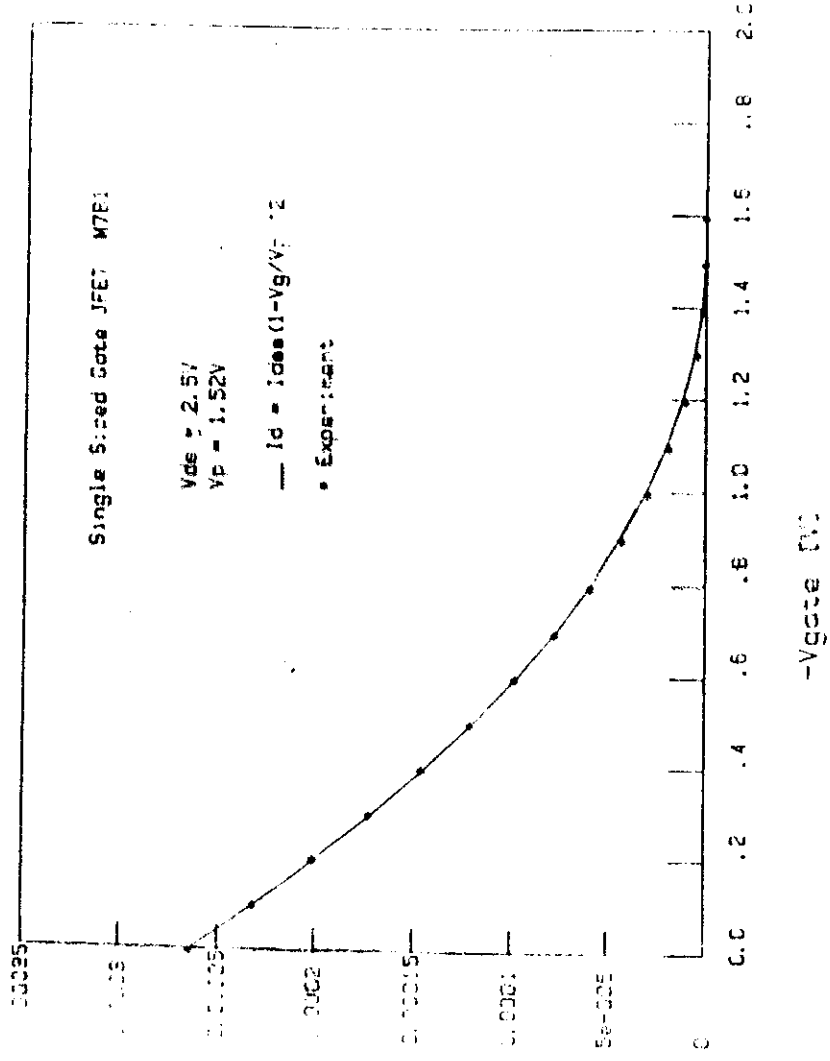
DELAY LINE TRANSFORMER



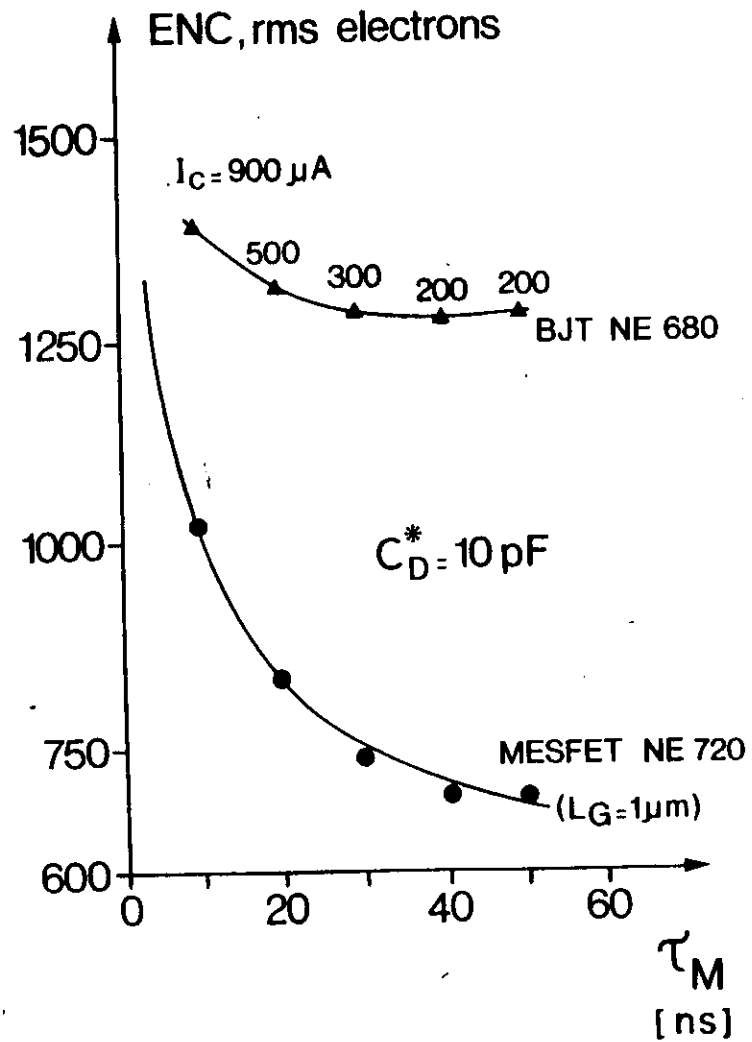
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LOOKING AHEAD AT THE TIME  
CONSTRAINTS THAT WILL BE PRESENT  
AT SSC





**SECOND ICFA SCHOOL ON INSTRUMENTATION**  
TRIESTE ICTP, June 12-23, 1989

**P.G. MANFREDI**  
**LECTURE NOTES**

**CHAPTER 2 :**  
**NOISE AND SIGNAL PROCESSING**

# Processing the Signals from Solid-State Detectors in Elementary-Particle Physics.

E. GATTI and P. F. MANFREDI

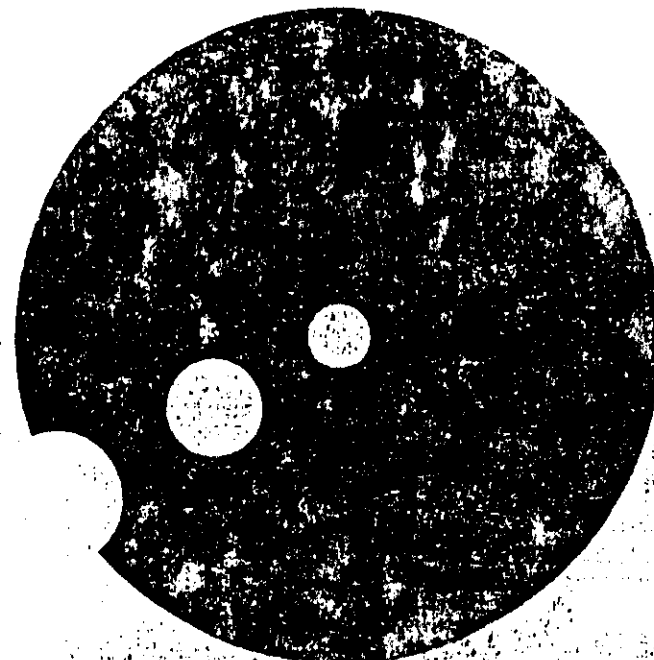
1. Review of the solid-state detectors used in elementary particle physics.
2. Shape of the signal induced in the solid-state detectors.
3. Optimum linear processing of the charge released by a solid-state detector.
  - 3.1. Statement of the problem.
  - 3.2. Time domain approach to optimum processing.
  - 3.3. Frequency domain approach to optimum processing.
  - 3.4. Partial use of the information available at the output of the noise-whitening filter.
  - 3.5. Matched filter theory in the case of finite rate of arrival of events on the detector.
  - 3.6. Optimum filter for an arbitrary pulse shape in white noise for a fixed estimation time and baseline.
4. Calculation of the signal-to-noise ratio for some practical analog processors.
  - 4.1. Introductory remarks.
  - 4.2. Time-invariant shapers.
    - 4.2.1. Frequency domain noise calculation.
      - 4.2.1.1. Triangular shaping.
      - 4.2.1.2. RC CR shaping.
    - 4.2.2. Time domain noise calculation.
      - 4.2.2.1. Triangular Shaping.
      - 4.2.2.2. RC CR shaping.
      - 4.2.2.3. Piecewise parabolic shaping.
      - 4.2.2.4. Trapezoidal shaping.
      - 4.2.2.5. Trapezoidal shaping.
  - 4.3. Contribution of the series RFI noise to the output rms noise in some particular cases.
    - 4.3.1. Triangular shaping.
    - 4.3.2. RC CR shaping.
    - 4.3.3. RC CR shaping.
  - 4.4. Time variant shapers.
5. Identification of the noise sources in preamplifier, detector and related bias network.
  - 5.1. Noise sources in the input amplifying device.
    - 5.1.1. Silicon junction field-effect transistor.
    - 5.1.2. Bipolar transistors.
    - 5.1.3. GaAs junction field-effect transistors.
    - 5.1.4. MOS transistors.
  - 5.2. Noise sources in detector and bias network.
6. General considerations about shaping.
7. Capacitive matching between detector and preamplifier.
8. Bipolar and junction field-effect transistors in elementary preamplifier configurations.
  - 8.1. Preamplifiers employing a single active device.
  - 8.2. Preamplifiers employing parallel connections of bipolar and field-effect devices.
  - 8.3. Comparison between input stages employing bipolar and junction field-effect transistors.
9. Feedback preamplifier structures.
  - 9.1. Block diagram analysis.
    - 9.1.1. Charge-sensitive configuration.
    - 9.1.2. Current sensitive configuration.
    - 9.1.3. Voltage-sensitive configuration.
  - 9.2. Basic low noise inverting amplifier.
  - 9.3. Practical preamplifier configurations.
10. Linear amplifier and shaper.
11. Conclusive remarks on some problems of analog processing.
  - 11.1. Silicon telescopes.
  - 11.2. Germanium bulk detector.
  - 11.3. Microstrip detectors.
12. Conclusions.
- Appendix.

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della Società Italiana di Fisica  
1986

E. GATTI and  
P. F. MANFREDI

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solid-state detectors in  
elementary-particle physics



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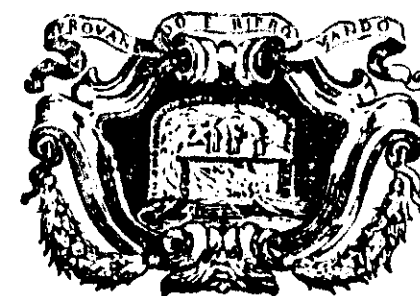
Segretario di Redazione PAOLINO PAPALI

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## Processing the Signals from Solid-State Detectors in Elementary-Particle Physics.

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## 1. - Review of the solid-state devices employed in elementary-particle physics.

More than fifteen years ago a silicon active target made of five detectors was employed on a CERN beam to investigate coherent interactions of  $\pi$ -mesons on the Si nucleus [1]. That was probably the first time that a solid-state device was used as an active target in the beam of a large accelerator. Some years afterwards the same technique was adopted in another coherent-production experiment carried out at higher energies at the accelerator of Serpukhov [2].

In both experiments the active target consisted of a telescope of surface-barrier, totally depleted silicon detectors, aligned along the beam axis at small mutual distances.

The configuration of an active target used in the coherent-production experiment is shown in fig. 1.1. The target used in the Serpukhov experiment consisted of ten 200  $\mu\text{m}$  thick detectors with an active area of 3.5  $\text{cm}^2$  each.

A device of the type shown in fig. 1.1 has a high space resolution in the beam direction. It allows the measurement of the longitudinal ionization density as a function of the position and, therefore, the pattern of the charged multiplicity



Fig. 1.1. - Si nucleus.

## PROCESSING THE SIGNAL FROM SOLID-STATE DETECTORS ETC.

can be reconstructed. The detectors where a large energy release occurs as consequence of a nucleus recoil or of a nucleus break up can be identified. An example of event reconstructed from the ionization density along the target is shown in fig. 1.2.

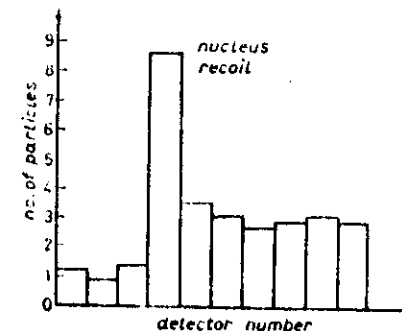


Fig. 1.2. - Coherent event reconstructed from a target of the type shown in fig. 1.1.

Signal processing associated with an active target aims at measuring the energy released along the telescope. This requires an accurate measurement of the charge made available by each detector.

A target similar to that of fig. 1.1 was employed in the first FRAMM experiment to measure the lifetime of short lived particles [3,4].

The target employed in the FRAMM 1 experiment was actually designed to measure lifetimes of charmed particles in the  $(10^{-12}, 10^{-10})$  s range.

It consisted of forty silicon detectors 300  $\mu\text{m}$  thick and with 1.5  $\text{cm}^2$  active surface. As any telescope intended for lifetime measurements it had, as compared to the configurations adopted for coherent production, the additional constraint of a very thin air gap between the detectors, 200  $\mu\text{m}$  or less. This requirement comes from the need of keeping the probability of decays occurring in the air gaps as small as possible. A FRAMM event is shown in fig. 1.3. The energies measured in the various detectors are displayed detector after detector.

The unit adopted for the energy scale is the most probable value of the Landau distribution of one minimum-ionizing particle crossing the given detector thickness,  $\sim 81$  keV in the actual case. So, the diagram of fig. 1.3 can be considered presented in units of charged multiplicity. The event corresponding to the multiplicity pattern of fig. 1.3 is shown in the same figure. A high-energy photon beam was employed. The first detectors, that the photon crosses without interacting, do not produce output signals and what appears in the charged-multiplicity axis is the energy equivalent of the electronic noise.

The production detector is identified by a large energy release due to the nucleus recoil. Two minimum ionizing particles are produced and a multiplicity  $\nu = 2$  is measured in some detectors following the one where production

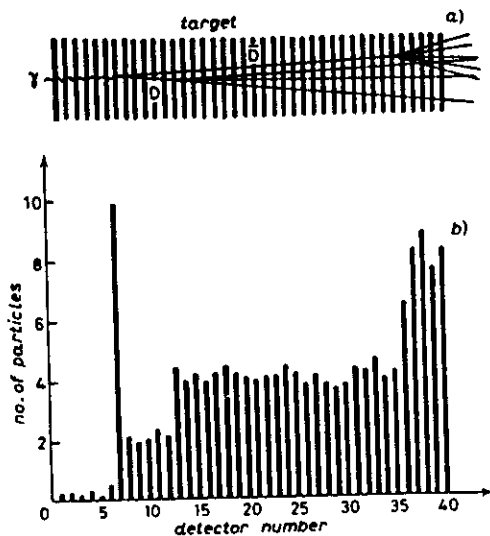


Fig. 1.3. - FRAMM event, production and decay (a), ionization pattern along the telescope (b).

took place. If, as expected, the two produced particles decay within the telescope, the decays give rise to multiplicity variations that are recorded by the silicon detectors.

The described technique of lifetime measurement was substantially improved for the FRAMM II data taking. The first target suffered from accuracy limitations in the energy measurement due to some extent to the electronics and to a larger extent to the fact that the technology employed in detector manufacturing was not adequate for the purpose. In order to meet the requirement of a close spacing between them, a substantial increase in the detector capacitance had to be accepted.

A remarkable improvement in the performances was achieved by using detectors manufactured according to the planar process introduced by KEMMER and shown schematically in fig. 1.4 [6, 7]. The detector shown in fig. 1.4 is based upon ion implantation of the junction and of the rear ohmic contact and  $\text{SiO}_2$  passivation.

As a next step, for shorter-lifetime measurements the device of fig. 1.5 was developed [8-10].

The device of fig. 1.5 is obtained from a body of high-purity  $n$ -type germanium, on the upper face of which  $p$ -type strips are implanted. A  $300\text{ }\mu\text{m}$  layer on the lower face, obtained by diffusing and drifting lithium, and the Al metallization realize the ohmic contact. When reverse bias is applied between

the strips and the lower Al plane, the body is depleted of free charge and each strip defines a detector with the section of germanium body underneath.

The device as a whole can be thought of as a sequence of equal detectors with no physical separation in between.

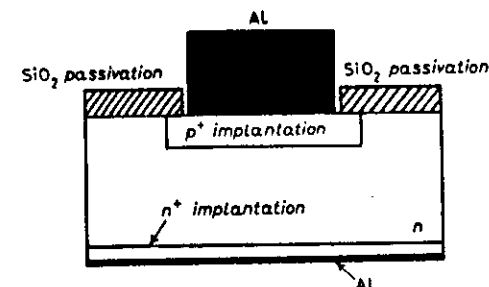


Fig. 1.4. - Silicon detector realized with the planar process.

The beam incidence is parallel to the strip plane. Therefore, the transversal dimensions must be large enough to cover the beam section. In the actual case the detector thickness is 5 mm and its width 2 cm. The longitudinal dimension is limited to a fraction of radiation length, to avoid excessive beam contamination from  $e^+$ ,  $e^-$  pairs.

The germanium bulk detector, like the silicon telescope, implements the twofold function of active target and of detector of the resulting physical event. In the sketch of fig. 1.5, a photon enters the bulk detector, a photo production occurs with nucleus recoil and creation of minimum-ionizing particles. The ionization density produced in the detector is sensed, as a function of the position along the axis, by the strips.

The germanium bulk detector of fig. 1.5 has the advantage over the silicon telescope of fig. 1.1 of a higher spatial resolution along the beam,  $(50-100)\text{ }\mu\text{m}$  vs.  $200\text{ }\mu\text{m}$ . Such a gain in granularity results from the factor-two-higher specific energy loss in germanium than in silicon, from the absence of air gap

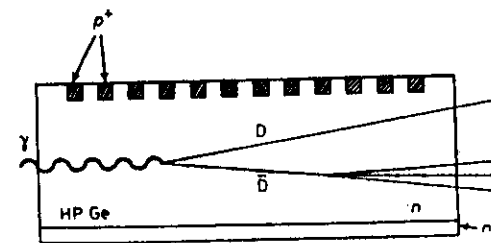


Fig. 1.5. - Ge bulk detector.

between the detection cells and from a more favourable charge transfer to the readout electronics resulting from the lower capacitance of the pick up electrodes.

The actual FRAMM target combines the properties of the Ge bulk detector and of the Si telescope, as shown in fig. 1.6a) [11]. Figure 1.6b) shows an event detected in the composite target.

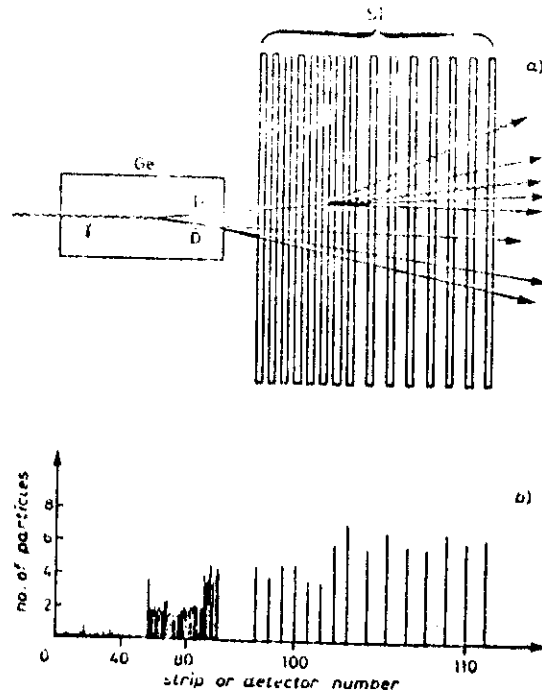


Fig. 1.6. - a) FRAMM composite active target using a Ge bulk detector followed by a silicon telescope. b) Event detected in the target.

Another application of a telescope of solid-state detectors is the evaluation of charged multiplicity of minimum-ionizing particles. The multiplicity is determined from the total energy released in the telescope. Its segmented structure allows the monitoring of the energy deposited in the individual cells, thereby reducing false multiplicity attributions that may arise from secondary interactions in the detectors. A real-time trigger based upon the difference in multiplicities sensed by two telescopes  $T_1$  and  $T_2$  has been realized as shown in fig. 1.7 [12, 13]. From the multiplicity variation between  $T_1$  and  $T_2$ , a possible decay occurring in the gap between them can be detected. Such a procedure is related to the search for  $\Lambda$  particles in the WA 71 experiment.

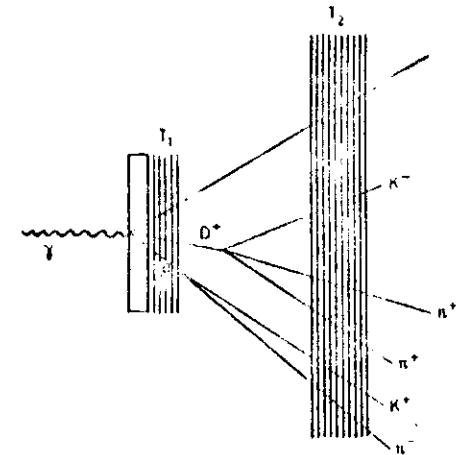


Fig. 1.7. - Real-time trigger based upon two telescopes  $T_1$  and  $T_2$ .

A silicon position-sensitive device, the microstrip detector, is already being used in several experiments and extensive use of it is being planned for future experiments at LEP, at the FNAL Tevatron and collider and elsewhere [8, 14-21].

The structure of a microstrip plane, developed with the same technology of the detector of fig. 1.4, is shown in fig. 1.8.

The structure is similar, to some extent, to the Ge bulk detector; the microstrip plane is much thinner, (200-300)  $\mu\text{m}$ , and is intended for beam incidence almost orthogonal to the strip plane. The position along the  $x$ -axis where track crosses the detector is sensed by that particular strip which collects the charge carriers.

Values of the pitch as small as 20  $\mu\text{m}$  have already been realized; the low

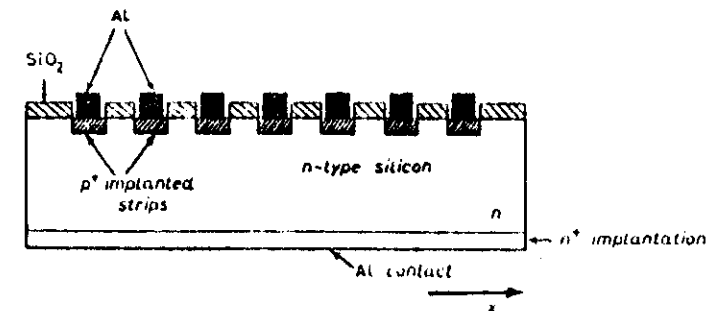


Fig. 1.8. - Microstrip plane.

limitation to the pitch rather than from the detector technology is set by other considerations.

First, considerations of physical nature, according to which, reducing the pitch below a certain limit would be meaningless, for the charge cloud being collected has its own natural width determined by charge diffusion and charge repulsion effects. The importance of these effects can, however, be attenuated by some kind of coarse interpolation.

If the microstrip detector operates in a strong magnetic field, degradation on the space resolution has also to be expected. Experimental investigation about the limits to the achievable spatial resolution with microstrip detectors or arrays of them has been recently carried on [22]. Feasibility of r.m.s. inaccuracies down to 3 to 5  $\mu\text{m}$ , depending on the readout method, has been demonstrated.

A second consideration about the lower limit to the pitch reflects the engineering difficulty in fanning out the signal lines from the detector to the readout electronics. An example of what the fanning-out printed-circuit board looks like is shown in fig. 1.9. The microstrip detector of fig. 1.9 has a 50  $\mu\text{m}$  pitch.

The present interest for microstrip detectors in elementary-particle physics



Fig. 1.9. - Printed-circuit board used to fan out the signals from the strips to the readout electronics. (Courtesy of CERN.)

is connected with the possibility of realizing a vertex detector based upon microstrip planes.

An example of this kind of application is shown in fig. 1.10, where an active target made of silicon detectors is associated with a microstrip vertex detector.

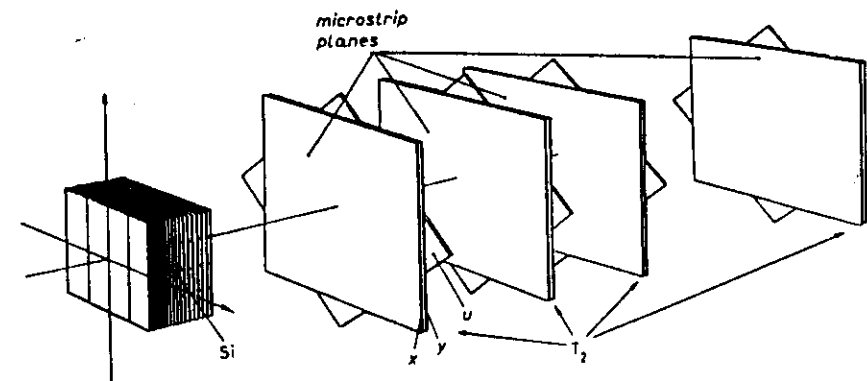


Fig. 1.10. - Silicon active target and microstrip vertex detector of FNAL E687.

The system is intended for investigation about charm and beauty decays, to be performed at the Fermilab Tevatron within the E687 experimental set-up [23, 24]. Several vertex detectors based upon microstrip planes are presently being designed for collider experiments.

An example of barrel-shaped vertex detector intended for one of LEP experiments is illustrated in fig. 1.11 [25, 26].

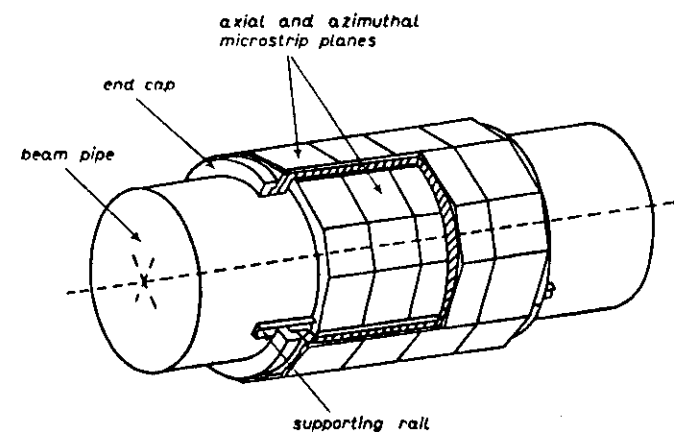


Fig. 1.11. - Barrel-shaped vertex detector employing microstrips.

Other silicon devices for position sensing and tracking applications deserve attention. One of them is the two-dimensional CCD [27, 28]. Such a device allows accurate position sensing in two co-ordinates, has an excellent two-track resolution and presents the advantage of a built-in analog readout. Using a CCD with cells of  $22 \times 22 \mu\text{m}^2$  pixel size, resolutions of 4.3 and 6.1  $\mu\text{m/rms}$  in two orthogonal directions were achieved.

The second device, the silicon drift chamber, is of very recent realization and is presently undergoing tests on the beam [29, 30]. Based upon a novel charge transport scheme, the silicon drift chamber allows accurate position sensing, yet requiring a finite number of readout channels, 1 or 2 per cm for a resolution of 4 to 8  $\mu\text{m/rms}$ .

The principle of the silicon drift chamber is shown in fig. 1.12.

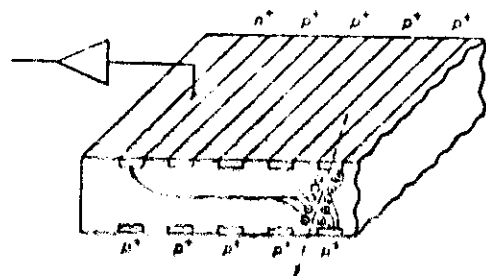


Fig. 1.12. - Silicon drift chamber.

The drift chamber is a fully depleted silicon detector with the space charge of the donors providing a potential well for the electrons. The electrons are so confined in the middle plane of the detector, parallel to its upper and lower surfaces. The confinement of the electrons is shown in fig. 1.12, where the holes created by the ionizing track drift towards the upper and lower  $p^+$  electrodes, while the electrons are confined in the middle plane. The electrons have now to drift towards a collecting electrode. Drifting is obtained with a uniform electric field parallel to the wafer surfaces. Such a field is provided for, by keeping the corresponding  $p^+$  strips of the upper and lower arrays at voltages that are negative with respect to the collecting anode and scaled down of a suitable quantity from one strip to the next one. The drift field determines a uniform motion of the primary electrons towards a low-capacitance  $n^+$  collecting electrode implanted on one side of the wafer as last electrode of the array of  $p^+$  field electrodes (fig. 1.12).

The curves of electron potential energy in the drift region of the detector for two different values of the voltage difference between two contiguous electrodes is shown in fig. 1.13.

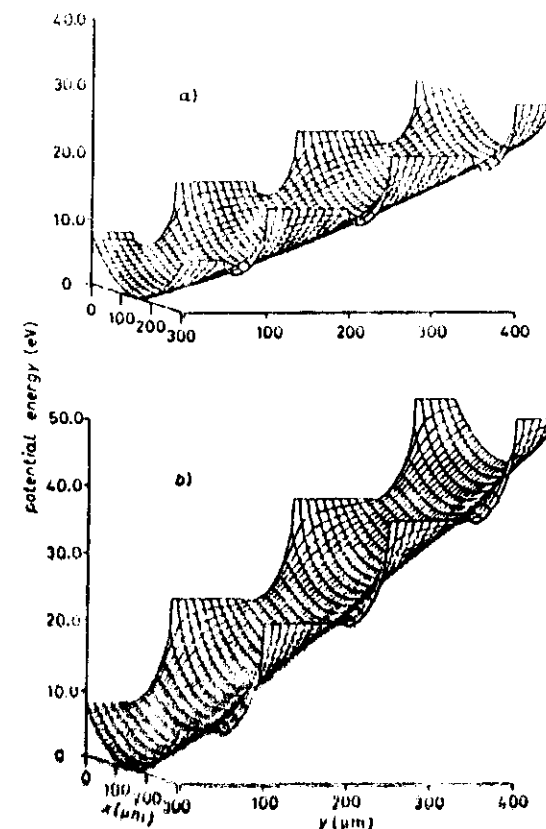


Fig. 1.13. - Potential energy of the electrons in the drift region: a) voltage difference between two contiguous  $p^+$  strips 7.5 V, b) voltage difference between two contiguous  $p^+$  strips 15 V.

The configuration of the potential surface near the collecting anode is shown in fig. 1.14.

Primary electrons due to a ionizing particle crossing the detector drift and diffuse before reaching the anode. The current pulse at the anode, which is shielded by the arrays of field electrodes, has a Gaussian shape nearly equal to the Gaussian space distributions of the electrons as they reach the anode.

Employed as a position-sensitive or tracking device the silicon drift chamber allows the measurement of the drift time, or interval elapsing between a trigger event and the time relationship with the radiation crossing the detector and the time defined on the bunch of electrons at the anode.

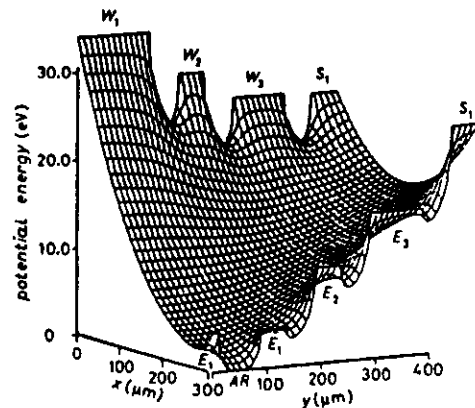


Fig. 1.14. - Potential energy of electrons in the anode region AR.

In these applications the silicon drift chambers have as a limitation a poor double-track resolution and ambiguities in the attribution of pulses to multiple triggers that may occur during a drift time interval.

The silicon drift chambers can be also employed for energy measurements and assembled into telescopes to sample the specific energy losses in the beam direction. For these applications they feature a very good energy resolution because the charge created by the ionizing radiation is made available at the output on a low-capacitance electrode.

Silicon is opening his way also in calorimetry [31-33].

Prototypes of calorimeters where the sampling cells are made of silicon detectors are presently being investigated and the results produced so far are well encouraging.

The main task of the present paper is to discuss the problems associated with the solid-state devices reviewed so far.

In some situations, accurate measurements of the energy released in the detectors are required, like in the case of the reconstruction of the ionization density along the beam, or in the charged-multiplicity evaluation, or in silicon calorimetry applications.

In some other cases it has to be distinguished whether an output signal can be safely attributed to a ionizing radiation crossing to the detector or whether it is an artifact determined by the electronic noise. This distinction has to be made in connection with microstrip detectors employed in the strip-by-strip readout mode or with CCDs, where a time slot may or may not contain the information of a particle detected. Other devices may require different kinds of measurements, like, for instance, time measurements in the case of silicon drift chambers.

Regardless of the type of signal processing required, however, the relative importance of detector signal, electronic noise and noise associated with the signal itself determines the accuracy with which the information can be extracted from the signal available at the output of the linear-amplifier system.

In order to introduce the concepts of signal and noise, it will be assumed that a beam of minimum-ionizing particles with a very narrow momentum distribution crosses a thin solid-state detector. The spectrum of the charge of the current pulses at the detector output looks like curve b) of fig. 1.15. The following effects contribute to the line broadening.

They are

- I) Landau fluctuations in the energy deposited in the detector,
- II) statistics in the process of charge creation in the detector for events corresponding to the same energy release,
- III) stochastic noise in the preamplifier.

Effect II) is of negligible importance, compared to I) and III) in high-energy physics where the energies deposited in the detectors are small, and where the Landau fluctuations dominate the statistics of the charge appearing at the detector output. The contribution of the electronic noise to the spectral line width can be disentangled from the curve b) of fig. 1.15 by injecting at the preamplifier input charge pulses and accumulating the resulting spectrum when the beam is off. The noise line is represented by the shaded peak in fig. 1.15. In this example the contribution of the electronic noise to the total width of the Landau distribution is large, as a result of a poor signal-to-noise ratio determined by the big capacitance of the detector employed.

Generally speaking, the relative importance of Landau fluctuations and electronic noise in determining the final line broadening is related to the following considerations.

At low multiplicities, the effect of noise may be dominant. It has to be pointed out, indeed, that the solid-state detectors have no built-in charge multiplication, that is, the output charge is equal to the one made available by the primary ionization process. Such a charge is determined by the specific energy loss, about 270 eV/ $\mu\text{m}$  in silicon and 550 eV/ $\mu\text{m}$  in germanium for minimum-ionizing particles, and by the depth of the depleted region in the detecting devices.

With the exception of the germanium bulk detector and of silicon detectors for some calorimetric applications, the practical depletion depths range from about 10  $\mu\text{m}$  in the case of CCDs to about 400  $\mu\text{m}$  in microstrip detectors and telescope layers. Thicker detectors are not used because they would be too slow and, in the case of the microstrip devices, they would present too much material either on the beam or on the tracks emerging from a vertex.

The charge created by one minimum-ionizing particle can be estimated to be in the range  $10^3$  to  $3 \cdot 10^4$  electrons.

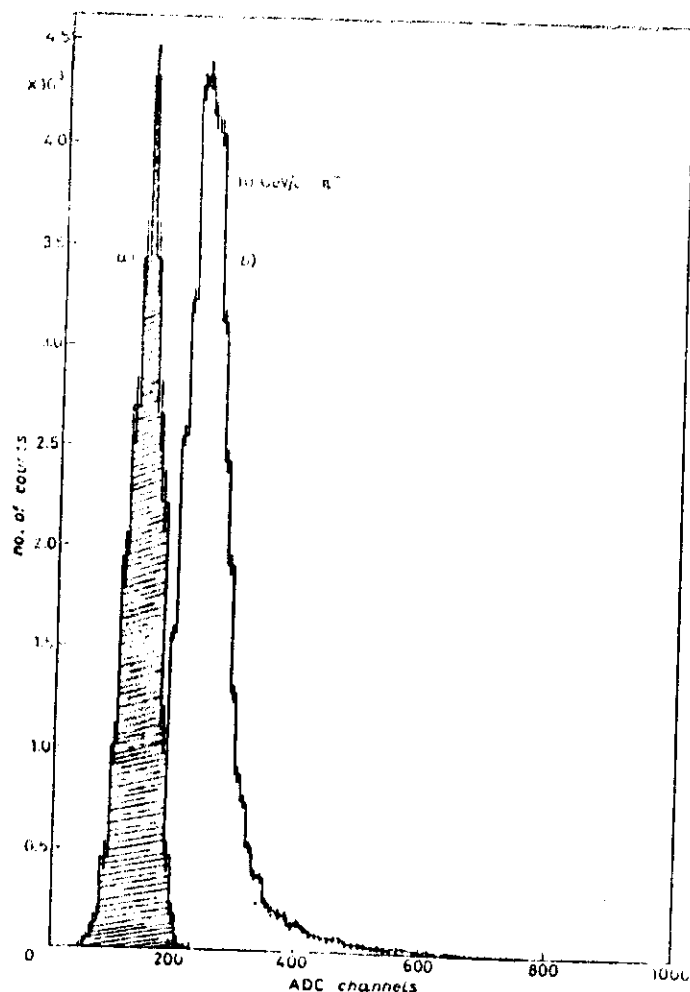


Fig. 1.15. — Landau distribution of the energies released by minimum-ionizing particles in a thin silicon detector (b) and noise line (a). Noise linewidth ENC 4600 e<sup>-</sup> FWHM, Si 40 keV.

The extent to which the electronic noise affects the accuracy in the charge measurement depends on the detector capacitance and on the time available for the measurement of the signal.

As the radiation multiplicity increases, Landau fluctuations become the dominant source of fluctuations. The relative importance of the electronic

## FONDAZIONE FRANCESCO SOMAINI PRESSO IL TEMPIO VOLTIANO, A COMO

### PREMIO PER LA FISICA E BORSA DI STUDIO PER IL 1988

Con lo scopo di premiare e incoraggiare nel nome di ALESSANDRO VOLTA gli studi di Fisica in Italia, la « Fondazione Francesco Somaini » presso il Tempio Voltiano a Como, stabilisce il Premio e indice il Concorso, come segue:

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1. — Il Premio di L. 10.000.000 (dieci milioni) nette, per il 1988, verrà direttamente assegnato senza Concorso su proposta della Commissione Scientifica della Fondazione, secondo le norme statutarie.

2. — La Commissione Scientifica della Fondazione è costituita da due Soci nazionali o corrispondenti della Categoria III, Sezione A della Classe di Scienze Fisiche, Matematiche e Naturali dell'Accademia Nazionale dei Lincei, nominati dall'Accademia stessa; da due Membri (effettivi o corrispondenti), della Sezione II della Classe delle Scienze dell'Istituto Lombardo, Accademia di Scienze e Lettere, nominati dall'Istituto stesso; dal Presidente della Società Italiana di Fisica o, in sua vece, da un Membro del Consiglio della Società nominato dal Consiglio stesso.

3. — Il Premio è indivisibile.

4. — Il conferimento del Premio avrà luogo entro il 20 Ottobre 1988. Al vincitore sarà rilasciato inoltre il diploma con la medaglia in bronzo della Fondazione.

5. — La proclamazione del vincitore, e il conferimento del Premio, del diploma e della medaglia avranno luogo, se possibile, in occasione di manifestazioni culturali di Fisica. La Commissione Amministratrice potrà chiamare il vincitore a illustrare pubblicamente, nella seduta di proclamazione e conferimento del Premio, i suoi lavori.

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1. — È aperto il Concorso alla « Borsa Francesco Somaini » per l'anno 1988, di L. 4.000.000 (quattro milioni) nette, per studi di Fisica, da assegnarsi al concorrente che, tra quelli che la Commissione Giudicatrice giudicherà in senso assoluto meritevoli della Borsa, verrà dalla Commissione stessa giudicato il più meritevole, sia per qualità e importanza scientifica, lavori già svolti e risultati già conseguiti nella Fisica, anche per il vantaggio che gli studi, per i quali è richiesta la Borsa, possono portare allo sviluppo della Fisica, in Italia.

noise is accordingly reduced. Recently, attention has been conveyed on the investigation about the specific energy losses of minimum-ionizing particles in thin silicon layers [34-37]. Corrections of the classical Landau-Vavilov relation have been suggested.

The limitations in the use of solid-state detecting devices coming from the fluctuations in the energy released will not be discussed in this paper, which is entirely oriented to the instrumental limitations [38]. Among them, as already pointed out, the electronic noise assumes a great importance. Energy measurements with any of the detecting devices presented so far imply the measurement of a charge delivered by a capacitive source with the best possible accuracy compatible with the noise in the amplifying system. The problem is schematically introduced in fig. 1.16, where  $C_D$  is the detector capacitance and  $e_N$  the total r.m.s. noise referred to the input of the amplifier system.

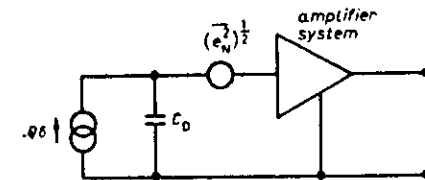


Fig. 1.16. - Basic diagram of a charge-measuring system.

The effect of the preamplifier noise can be described by saying that an infinitely narrow probability density function of detector charge would appear, at the amplifier output, as a Gaussian distribution with variance  $\overline{e_N^2} C_D^2$ . The quantity  $(\overline{e_N^2} C_D^2)^{1/2}$  is called equivalent noise charge referred to the amplifier input; it is represented with the symbol ENC and usually expressed in r.m.s. electrons. The effect of the amplifier noise on the performances of some of the devices considered in this section will be illustrated by a few examples.

In an active target of the type shown in fig. 1.1, the electronic noise may mask a signal variation occurring from one detector to the next one, thereby giving a false attribution of the point at which a multiplicity change occurs.

The electronic noise affects the accuracy in the position identification made with a detector of the type of fig. 1.5 in a similar way.

In the case of a microstrip plane, the need of achieving an adequately small electronic noise is clarified by fig. 1.17. In this example, it is assumed that the microstrip detector is read out by a system which has one channel on every strip, each channel consisting of an amplifier and a discriminator. The two spectra shown in fig. 1.17 are the noise line and the one-particle Landau distribution, whose low-energy edge is determined to a large extent again by the electronic noise. It is evident from fig. 1.17 that, if the ENC is not suitably small in comparison to the  $Q_1$ - $Q_0$  separation, the overlap between the noise line

and the Landau spectrum makes the choice of the discriminator threshold  $Q_0$  critical. In other words, it becomes impossible to find a value of  $Q_1$  which ensures in the meantime a low probability of spurious hits and a high detection efficiency. It can be shown, for instance, that, putting the requirements of less

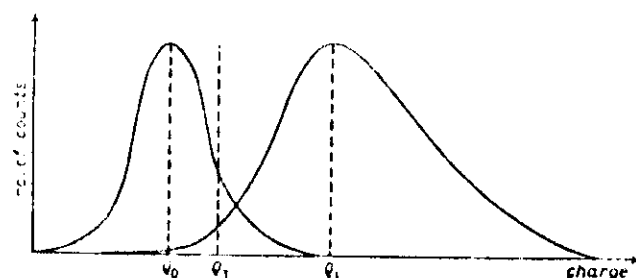


Fig. 1.17. - Effect of electric noise on the microstrip readout.

than 0.1% probability of spurious hits and less than 0.1% inefficiency and taking into account a possible charge division between adjacent strips, the ENC must satisfy the relationship

$$(1.1) \quad \text{ENC} \sim 150 d \text{ electrons r.m.s.},$$

with  $d$  detector thickness in units of 100  $\mu\text{m}$  [39]. So, for instance, a 200  $\mu\text{m}$  strip detector requires an amplifier with less than 900 electrons r.m.s.

If in a microstrip detector the position is read out by some kind of interpolation method, like, for instance, charge division, the considerations about the effects of the noise are different, though the achievement of a high resolution in the position evaluation still requires that the preamplifier noise be kept as small as possible.

Silicon drift chambers employed in energy measurements are subject to the same considerations made with reference to figure 1.16, where now  $C_b$  is meant to be the capacitance of the collecting anode, which, as already pointed out, is considerably smaller than the output capacitance of a conventional detector with the same thickness and active area.

When used as a position-sensitive or tracking device, the information processing associated with it has the purpose of measuring with the best accuracy the time taken by the electrons to drift to the collecting anode with reference to a trigger signal.

Typical drift times to achieve the best resolution are 2.5  $\mu\text{s}$  per 5 mm drift, as determined by a 135 V/cm drift field.

Silicon drift chambers have been tested with up to 2500 V/cm drift field and corresponding drift times of 140 ns per 5 mm drift with a factor 4 less in

resolution. If a pulse processor matched to the sensitive pulses were employed, the degradation in resolution would be only 1.3.

A typical pulse processor for silicon drift chambers consists of a preamplifier for low detector capacitance, of a quasi-Gaussian amplifier, 175 ns FWHM, followed by a differentiator which delivers a bipolar shape very close to the optimum  $\pi$ -crossing timing response. Again the preamplifier noise represents a dominant limitation in the achievable position resolution.

Since the solid-state detecting devices made their first appearance in elementary-particle physics, the problem of designing low-noise electronics capable of coping with the peculiar experimental situations had to be faced. If it is true that the fundamentals of signal and noise theory for capacitive sources were known since a long time and that they were extensively employed in low energy nuclear spectroscopy, it is also true that research and development work was needed to transfer that knowledge into high-energy physics.

For instance, in fixed-target experiments, silicon devices of the telescope type are employed at counting rates up to two orders of magnitude higher than the rates met in nuclear spectroscopy. The effort directed to reduce the noise at short processing times has extended the interest from the junction field-effect transistor to alternative types of amplifying devices. Problems related to very large detector capacitances and under the constraint of the high counting rates had to be solved. The need of achieving outstanding baseline stabilities at high counting rates without sacrificing the intrinsically poor signal-to-noise ratio had to be satisfied. The growing number of detecting channels in each experiment requires compact, low-noise circuitry in the front-end. Hybrid preamplifiers are already of current use, while low-noise monolithic design is being carried on, to solve the problems related to the signal extraction from microstrip detectors of small pitch [40, 41].

Moreover, degradation in signal-to-noise ratio and counting rate performances due to the long connections between beam line and counting room must be avoided.

To conclude, the instrumentation which extracts the information from solid-state devices in elementary-particle physics must be thought about with a number of constraints borne in mind.

The present paper, which aims at discussing the criteria of signal processing for solid-state detectors in elementary-particle physics, will analyse the solutions that have to be adopted to achieve the best performances from the detecting devices reviewed in this section.

## REFERENCES

- [1] G. BELLEI, M. DI CORATO, P. F. MANFREDI and G. VIGNI: *Nucl. Instrum. Methods*, **107**, 83 (1973).

- [2] M. ARTUSO, P. D'ANGELO, E. GATTI, P. F. MANFREDI and D. MARIOLI: *High rate pulse processing system for silicon active target*, in *Proceedings of the X International Symposium on Nuclear Electronics, Dresden, G.D.R., 10-16 April 1980*, Vol. 2 (1980), p. 24.
- [3] G. BELLINI, L. FOÀ and M. GIORGI: *Phys. Rep.*, **83**, 9 (1982).
- [4] G. BELLINI, P. D'ANGELO, M. DI CORATO, D. MARIOLI, C. MERONI, E. MERONI, L. MORONI, C. PALAZZI-CERRINA, F. PALOMBO, F. RAGUSA, S. SALA and G. VEGNI: *Live targets as a tool to study short range phenomena in elementary particle physics*, in *Proceedings of the Meeting on Minaturisation of High Energy Physics Detectors, Pisa, September 1980*, edited by A. STEFANINI (Plenum Press, New York, N. Y., 1983), p. 41.
- [5] G. BELLINI, P. D'ANGELO, P. F. MANFREDI, E. MERONI, L. MORONI, C. PALAZZI-CERRINA and F. RAGUSA: *Nucl. Instrum. Methods*, **196**, 351 (1982).
- [6] J. KEMMER: *Nucl. Instrum. Methods*, **169**, 499 (1980).
- [7] J. KEMMER, P. BURGER, R. HENCK and H. HEIJNE: *Performance and application in high energy physics of passive ion implanted silicon detectors*, in *Proceedings of the Conference on the Search for Charm, Beauty and Truth at High Energies, Erice, November 15-22, 1981*, edited by G. BELLINI and S. C. C. TING (Plenum Press, New York, N. Y., 1984), p. 269.
- [8] A. MENZIONE: *Latest results on silicon detector development at Pisa*, in *Proceedings of a Workshop on Silicon Detectors for High Energy Physics, Fermilab, October 15-16, 1981*, edited by T. FERBEL (1981), p. 219.
- [9] S. R. AMENDOLIA, F. BEDESCHI, E. BERTOLUCCI, D. BETTONI, L. BOSISIO, C. BRADASCHIA, M. BUDINICH, M. DELL'ORSO, F. FIDECARO, L. FOÀ, E. FOCARDI, P. GIANNETTI, A. GLAZOTTO, M. A. GIORGI, P. S. MARROCCHESI, A. MENZIONE, L. RISTORI, A. SCRIBANO, A. STEFANINI, G. TONELLI, G. TRIGGIANI, E. E. HALLER, W. L. HANSEN and P. N. LUKE: *IEEE Trans. Nucl. Sci.*, **NS-30**, 98 (1983).
- [10] S. R. AMENDOLIA, F. BEDESCHI, E. BERTOLUCCI, D. BETTONI, L. BOSISIO, U. BOTTIGLI, C. BRADASCHIA, M. DELL'ORSO, F. FIDECARO, L. FOÀ, E. FOCARDI, P. GIANNETTI, M. A. GIORGI, P. S. MARROCCHESI, A. MENZIONE, G. RASO, L. RISTORI, A. SCRIBANO, A. STEFANINI, R. TENCHINI, G. TONELLI, G. TRIGGIANI, E. HALLER, W. L. HANSEN and P. N. LUKE: *IEEE Trans. Nucl. Sci.*, **NS-31**, 945 (1984).
- [11] S. R. AMENDOLIA, G. BATIGNANI, F. BEDESCHI, G. BELLINI, E. BERTOLUCCI, D. BETTONI, G. BOLOGNA, L. BOSISIO, C. BRADASCHIA, M. BUDINICH, P. D'ANGELO, M. DELL'ORSO, B. D'ETTORRE PIAZZOLI, F. L. FABBRI, F. FIDECARO, L. FOÀ, E. FOCARDI, P. GIANNETTI, M. ENORINI, M. A. GIORGI, P. LAURELLI, F. LIELLO, P. F. MANFREDI, G. MANNOCCI, P. S. MARROCCHESI, D. MENASCE, A. MENZIONE, E. MERONI, E. MILLOTTI, L. MORONI, D. PEDRINI, L. PERASSO, P. PICCHI, F. RAGUSA, L. RISTORI, L. ROLANDI, S. SALA, A. SCRIBANO, P. SPILLANTINI, A. STEFANINI, R. TENCHINI, G. TRIGGIANI, G. TONELLI and A. ZALLO: *Nucl. Instrum. Methods Phys. Res.*, **226**, 78 (1984).
- [12] M. ADAMOVIC, Y. ALEXANDROV, G. BARONI, S. BENSO, N. CROSETTI, M. DAMERI, G. DARBO, G. DIAMBRINI-PALAZZI, P. GINOBBI, E. H. M. HEIJNE, E. LAMANNA, P. F. MANFREDI, D. MARIOLI, F. MEDDI, D. MENASCE, C. MERONI, S. MICHELETTI, B. OSCULATI, S. PETRERA, L. ROSSI, M. SANNINO, S. TENTINDO, G. TOMASINI, G. VANDERHAEGHE, G. VEGNI and S. VITALE: *Proposal for an experiment for studying beauty production and lifetime in the upgraded  $\Omega$  spectrometer, CERN-SPSC/81-18 SPSC/P159* (20 February 1981).
- [13] S. BENSO, M. DAMERI, G. DIAMBRINI PALAZZI, P. F. MANFREDI, D. MARIOLI, C. MERONI, L. ROSSI, M. SANNINO, S. TENTINDO and G. VEGNI: *Nucl. Instrum. Methods*, **201**, 329 (1982).

- [14] E. H. M. HEIJNE, L. HUBBELING, B. D. HYAMS, P. JARRON, P. LAETRAS, F. PIUZ, J. C. VERMEULEN and A. WYLIE: *Nucl. Instrum. Methods*, **178**, 331 (1980).
- [15] J. KEMMER, E. BELAU, R. KLANNER, G. LUTZ and B. HYAMS: *Development of 10  $\mu$ m resolution silicon counters for charm signature observation with the ACCMOR spectrometer*, in *Proceedings of a Workshop on Silicon Detectors for High Energy Physics, Fermilab, October 15-16, 1981*, edited by T. FERBEL (1981), p. 195.
- [16] P. BONAMY, P. BORGEAUD, J. MOVCHET and P. G. RANCOITA: *Performance of surface barrier and ion implantation silicon microstrip detectors*, in *Proceedings of a Workshop on Silicon Detectors for High Energy Physics, Fermilab, October 15-16, 1981*, edited by T. FERBEL (1981), p. 257.
- [17] B. HYAMS, U. KOETZ, E. BELAU, R. KLANNER, G. LUTZ, E. NEUGEBAUER, A. WYLIE and J. KEMMER: *Nucl. Instrum. Methods*, **205**, 99 (1983).
- [18] R. BAYLEY, E. R. BELAU, T. BOHRINGER, M. BOSMAN, V. CHABAUD, C. DAME-RELL, C. DATM, G. DE RIJK, H. DIJKSTRA, A. DWURANT, S. GILL, A. GILMANN, R. GILMORE, L. GÖRLICH, Z. HAJDUK, C. HARDWICH, W. HOOGLAND, B. D. HYAMS, J. KEMMER, R. KLANNER, U. KÖTZ, G. LUTJENS, G. LUTZ, J. MALOS, W. MÄNNER, E. NEUGEBAUER, G. POLOK, M. ROZANSKA, K. RYBICKI, H. J. SEEBRUNNER, U. STIERLIN, R. J. TAPPER, H. G. TIECKE, M. TURALA, G. WALTERMANN, P. WEILHAMMER, F. WICKENS, L. W. WIGGERS, A. WYLIE and T. ZELUDZIEWICZ: *Nucl. Instrum. Methods Phys. Res.*, **226**, 56 (1984).
- [19] S. R. AMENDOLIA, F. BEDESCHI, E. BERTOLUCCI, D. BETTONI, L. BOSISIO, U. BOTTIGLI, C. BRADASCHIA, M. DELL'ORSO, F. FIDECARO, L. FOÀ, E. FOCARDI, P. GIANNETTI, M. A. GIORGI, P. S. MARROCCHESI, A. MENZIONE, G. RASO, L. RISTORI, A. SCRIBANO, A. STEFANINI, R. TENCHINI, G. TONELLI and G. TRIGGIANI: *Nucl. Instrum. Methods Phys. Res.*, **226**, 82 (1984).
- [20] E. ENGELS jr., S. MANI, D. PLANTS, P. F. SHEPARD, R. WILKINS and S. HOSSAIN: *Nucl. Instrum. Methods Phys. Res.*, **226**, 59 (1984).
- [21] P. E. KARCHIN, D. L. HALE, R. J. MORRISON, M. S. WITHERELL, M. D. SOKOLOFF, A. KIANG, B. R. KUMAR, J. F. MARTIN and M. SARABURA: *IEEE Trans. Nucl. Sci.*, **NS-32**, 612 (1985).
- [22] E. BELAU, R. KLANNER, G. LUTZ, E. NEUGEBAUER, H. J. SEEBRUNNER, A. WYLIE, T. BÖRINGER, L. HUBBELING, P. WEILHAMMER, J. KEMMER, U. KÖTZ and M. RIEBESELL: *Nucl. Instrum. Methods*, **214**, 253 (1983).
- [23] *Fermilab Agreement E-687, High energy photoproduction of states containing heavy quarks and other rare phenomena* (21 October 1983).
- [24] G. BELLINI, M. DI CORATO, M. GIANMARCHI, P. F. MANFREDI, D. MENASCE, E. MERONI, L. MORONI, D. PEDRINI, L. PERASSO and S. SALA: *Nucl. Instrum. Methods Phys. Res.*, **225**, 019 (1984).
- [25] *Technical Report ALEPH Collaboration* (April 1983), 9.1-9.5.
- [26] *Delphi Progress Report, CERN/LEPC 84-16* (September 1984), p. 23.
- [27] P. BAILEY, C. J. S. DAMERELL, R. L. ENGLISH, A. R. GILLMAN, A. L. LINTERN, J. J. WATTS and F. J. WICKENS: *Nucl. Instrum. Methods Phys. Res.*, **213**, 210 (1983).
- [28] C. DAMERELL: *Nucl. Instrum. Methods Phys. Res.*, **226**, 20 (1984).
- [29] E. GATTI, P. REHAK and J. T. WALTON: *Nucl. Instrum. Methods Phys. Res.*, **226**, 129 (1984).
- [30] E. GATTI, A. LONGONI, P. REHAK, J. KEMMER, P. HOLL, R. KLANNER, G. LUTZ and A. WYLIE: *Nucl. Instrum. Methods Phys. Res. A*, **235**, 224 (1985).
- [31] G. BARNIELLINI, G. CECCHET, J. Y. HEMERT, F. LEMEILLIEUR, C. LENOY, G. LEV-MAN, P. G. RANCOITA and A. SEIDMAN: *Nucl. Instrum. Methods Phys. Res.*, **235**, 55 (1985).

- [32] G. BARBIERINI, K. BERNI, G. CECILINI, J. A. HENRIOT, F. LAMERZONI, P. G. RANCOTA, G. VISNARA and A. SLIDMAN: *Nucl. Instrum. Methods Phys. Res. A*, 235, 216 (1985).
- [33] A. NAKAMOTO, H. MURAKAMI, T. DOKI, J. KIKUCHI, K. MASUDA, K. KASAHARA, K. MITSUI, Y. MURAKI and T. YODA: *A Si(Li)-Pb shower calorimeter for p-p collider experiments*, report of the Institute for Cosmic Ray Research, University of Tokyo, Japan (28 November 1984).
- [34] P. G. RANCOTA and A. SLIDMAN: *Riv. Nuovo Cimento*, 5, No. 7 (1982).
- [35] P. G. RANCOTA: *J. Phys. G*, 10, 209 (1984).
- [36] S. HANCOCK, P. JAMES, J. MOVCHET, P. G. RANCOTA and L. VANROSSUM: *Nucl. Instrum. Methods Phys. Res. B*, 1, 16 (1984).
- [37] G. HALL: *Nucl. Instrum. Methods Phys. Res.*, 220, 356 (1984).
- [38] J. T. WALTON: *Nucl. Instrum. Methods Phys. Res.*, 226, 1 (1984).
- [39] E. KLEINER: *Nucl. Instrum. Methods Phys. Res. A*, 235, 209 (1985).
- [40] E. H. M. HEIJF and P. JARRON: *Nucl. Instrum. Methods Phys. Res.*, 226, 12 (1984).
- [41] V. RADEKA: *Nucl. Instrum. Methods Phys. Res.*, 226, 209 (1984).

## 2. - Shape of the signals delivered by solid-state detectors.

The knowledge of the time dependence of the signals delivered by solid-state radiation detectors is of great importance in the design of the processors that have to operate on them. For this reason, the basic method to evaluate such a time dependence will be presented here. Practical evaluation of current shapes, instead, will be limited to the case of largest application importance, that is, a  $P-N$  junction detector with planar geometry (1-4).

In the most general case, the evaluation of the output signal at a given electrode of a semiconductor detector requires (5, 6)

- a) the knowledge of the initial distributions of electrons and holes;
- b) the evaluation of the law of motion for electrons and holes in the electric field determined by the electrode biasing and by the space charge of the doping atoms; the space charge of the carriers created by the ionizing events can be neglected in the case of minimum-ionizing particles;
- c) the evaluation of the contribution of every element of charge in motion through a weighting vector  $E_w(P)$  which depends on the detector geometry, on the electrical network to which the detector is connected as well as on the velocity  $u$  of the charge element under consideration.

These three points will now be discussed in reverse order.

- a) *Weighting  $u(t)E_w(P)$* . A  $P-N$  junction detector connected to a lumped, linear, reciprocal network is schematized (fig. 2.1).

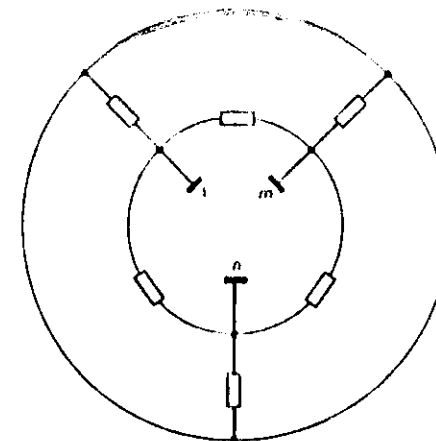


Fig. 2.1. - System of electrodes 1, ..., m, ..., n, embedded into a lumped, linear, reciprocal network.

The evaluation of the voltage signal  $v_m(t)$  determined by a charge element  $q$  on a generic electrode  $m$  can be done by applying between  $m$  and ground a current generator delivering a  $\delta$ -impulse and by calculating in the detector volume the weighting electric field  $E_{wm}(P, t)$  due to this generator alone (fig. 2.2).

The desired voltage signal  $v_m(t)$  is given, by virtue of reciprocity theorems,

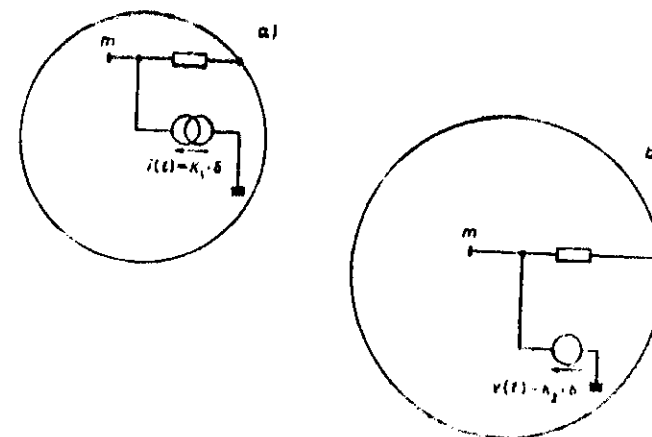


Fig. 2.2. - Evaluation of the voltage signal (a) and of the current signal (b) at the electrode  $m$ .

by

$$(2.1) \quad v_m(t) = -q \int_{-\infty}^t E_{w,m}(P, t-\tau) \times u(P(\tau)) d\tau,$$

where the symbol  $\times$  denotes the scalar product.

In eq. (2.1)  $u$  is the velocity of the charge element  $q$  at the point  $P$  at the instant  $\tau$ , as determined by the true electric field acting on that element of charge.

To calculate, instead, the current signal  $i_m(t)$  induced by the element of charge  $q$ , in a conductor shortcircuiting the electrode  $m$  to ground, a voltage generator delivering a  $\delta$ -impulse must be applied between this electrode and ground and the weighting electric field  $E_{w,m}(P, t)$  in the detector volume and due to this generator *alone* has to be evaluated.

The desired induced current  $i_m(t)$ , by virtue of reciprocity theorems, is given by

$$(2.2) \quad i_m(t) = -q \int_{-\infty}^t E_{w,m}(P, t-\tau) \times u(P(\tau)) d\tau,$$

where  $u[P(\tau)]$ , like in the previous case, is the *true* velocity of the charge element  $q$  at the point  $P$  and at the instant  $\tau$ . It has to be pointed out that delayed contributions in the voltage  $v_m$  or in the current  $i_m$  are generally present. This occurs when the  $E_w$  functions have a time dependence which differs from a  $\delta$ -impulse because of the signals induced on the  $m$  electrode by the delayed responses of all the other electrodes embedded in the network.

Only if all the other electrodes are grounded for the signals and neglecting propagation delays,  $E_w(P, t)$  can be written as

$$E_w(P, t) = -\delta(t) \cdot \text{grad } V_w = \delta(t) \cdot E_{w,static}.$$

Under this hypothesis, the induced current  $i_m(t)$ , according to (2.2), assumes the well-known expression

$$(2.3) \quad i_m(t) = -q \cdot E_{w,static}(P) \times u[P(t)],$$

where  $E_{w,static}(P)$  is the stationary electric field evaluated by connecting the electrode  $m$  to potential 1 and all the other electrodes to potential zero.

The general expressions (2.1) and (2.2) are useful in the analysis of detectors with many electrodes not connected to ground, for instance microstrip detectors with capacitive division or delay line readout.

*b) Motion law.* The law of motion for electrons and holes, which in the most adequate form is expressed by  $u[P(t)]$ , can be calculated in a relatively

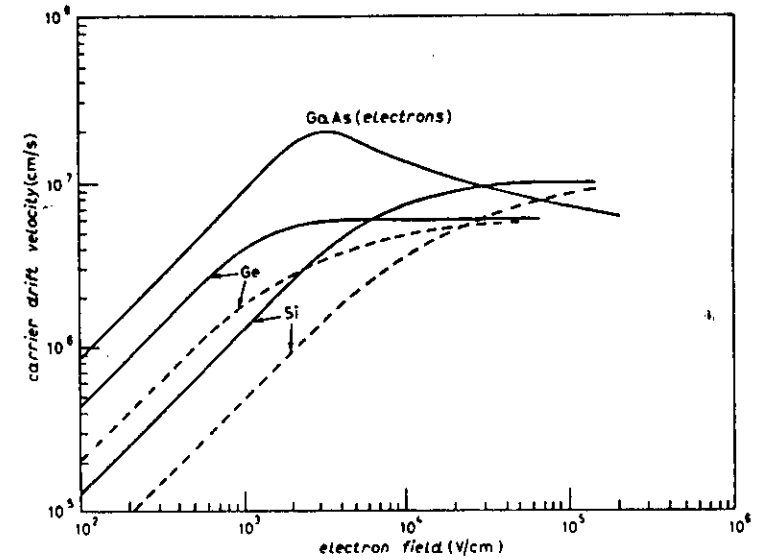


Fig. 2.3. - Carrier drift velocity as a function of the electric field for GaAs, Ge and Si at  $T = 300$  K: — electrons, --- holes.

easy way once the electron and hole mobilities,  $\mu_e$  and  $\mu_h$ , are known, as functions of the electric field. From the curves of fig. 2.3, that represent how the carrier drift velocity depends on the electric field, it can be inferred that at room temperature in silicon the mobilities of both electron and hole have to be considered constant up to an electric field of 4000 V/cm [7]. Their values are, respectively,  $\mu_e = 1200 \text{ cm}^2/\text{V}\cdot\text{s}$  and  $\mu_h = 450 \text{ cm}^2/\text{V}\cdot\text{s}$ .

*a) Initial charge distribution.* The relationships (2.1) and (2.2) give the partial contributions to the induced voltage and current at time  $t$  arising from an elementary bunch of charge  $q$  of the initial ionization track. The actual shapes of  $v_m(t)$  and  $i_m(t)$  are obtained by summing up all the elementary contributions into which the initial track can be split.

As a particular case, a  $P$ - $N$  junction with planar geometry will be considered now (fig. 2.4). The detector will be assumed to consist of a heavily doped  $P^+$  region and of a lightly doped  $N$  region and the voltage applied larger than the one required to fully deplete the  $N$  region.

Let  $N_D$  be the concentration of donor atoms supposed to be constant throughout the  $N$  region and let  $w$  be the detector thickness assumed to be equal to that of the  $N$  region. The electric field inside the detector has the linear dependence shown in fig. 2.4 starting from  $E_{min}$  at  $x = 0$  and reaching  $E_{max}$

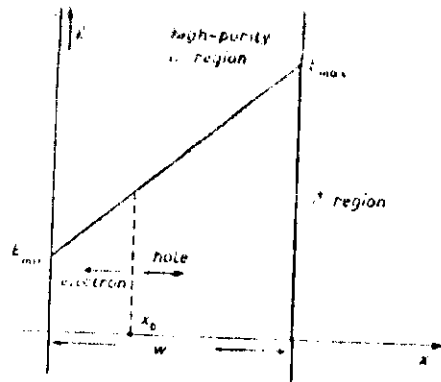


Fig. 2.4. - Electric field inside the p-n junction.

at the junction,  $x = w$ :

$$(2.4) \quad E(x) = \frac{qN_D}{\epsilon} x + E_{min},$$

where  $\epsilon$  is the dielectric constant and  $q$  the electron charge.

Considering a pair of carriers  $e, h$  at the point  $x_0$ , the following motion equations can be written:

$$(2.5) \quad \frac{dx}{dt} = \mu_h \left( E_{min} + \frac{qN_D}{\epsilon} x \right) + \mu_h E_{min} \quad \text{for the hole,}$$

$$(2.6) \quad \frac{dx}{dt} = -\mu_e \left( E_{min} + \frac{qN_D}{\epsilon} x \right) - \mu_e E_{min} \quad \text{for the electron.}$$

Integrating both equations with the initial conditions  $x = x_0$  at  $t = 0$ , the following time dependences are obtained:

$$(2.7) \quad x_h = -\frac{\epsilon}{qN_D} E_{min} + \left( x_0 + \frac{\epsilon}{qN_D} E_{min} \right) \exp \left[ \mu_h q \frac{N_D}{\epsilon} t \right] \quad \text{for the hole, } 0 < t < t_h;$$

$$(2.8) \quad x_e = -\frac{\epsilon}{qN_D} E_{min} + \left( x_0 + \frac{\epsilon}{qN_D} E_{min} \right) \exp \left[ -\mu_e q \frac{N_D}{\epsilon} t \right] \quad \text{for the electron, } 0 < t < t_e.$$

In the previous relationships  $t_h$  and  $t_e$  are the collection times of, respectively, holes and electrons.

The velocities  $u_h = dx_h/dt$  and  $u_e = dx_e/dt$  for holes and electrons are

$$(2.9) \quad u_h = \mu_h \left( E_{min} + \frac{qN_D}{\epsilon} x_h \right) \exp \left[ \mu_h q \frac{N_D}{\epsilon} t \right],$$

$$(2.10) \quad u_e = -\mu_e \left( E_{min} + \frac{qN_D}{\epsilon} x_e \right) \exp \left[ -\mu_e q \frac{N_D}{\epsilon} t \right].$$

The collection times  $t_h$  and  $t_e$  of hole and electron can be calculated from (2.7) and (2.8). They are

$$(2.11) \quad t_h = \frac{\epsilon}{\mu_h q N_D} \ln \frac{w + (\epsilon/qN_D) E_{min}}{x_0 + (\epsilon/qN_D) E_{min}},$$

$$(2.12) \quad t_e = \frac{\epsilon}{\mu_e q N_D} \ln \frac{x_0 + (\epsilon/qN_D) E_{min}}{(\epsilon/qN_D) E_{min}}.$$

According to (2.3) and observing that in the simple geometric configuration of the present detector  $|E_{x,initial}| = 1/w$  and that the velocities  $u_h$  and  $u_e$  have the same direction as  $E_{x,initial}$ , the following expressions can be written for the currents induced by the motion of holes and electrons:

$$(2.13) \quad i_h(t) = \frac{q}{w} \mu_h \left( E_{min} + \frac{qN_D}{\epsilon} x_h \right) \exp \left[ \mu_h q \frac{N_D}{\epsilon} t \right], \quad 0 < t < t_h,$$

$$(2.14) \quad i_e(t) = \frac{q}{w} \mu_e \left( E_{min} + \frac{qN_D}{\epsilon} x_e \right) \exp \left[ -\mu_e q \frac{N_D}{\epsilon} t \right], \quad 0 < t < t_e,$$

with  $i_h(t) = 0$  for  $t > t_h$  and  $i_e(t) = 0$  for  $t > t_e$ .

Time integration of (2.13) and (2.14) provides the charge induced on the cathode by both carriers:

$$(2.15) \quad Q_h = \frac{\epsilon}{wN_D} \left( E_{min} + \frac{qN_D}{\epsilon} x_0 \right) \left( \exp \left[ \mu_h q \frac{N_D}{\epsilon} t \right] - 1 \right),$$

$$(2.16) \quad Q_e = \frac{\epsilon}{wN_D} \left( E_{min} + \frac{qN_D}{\epsilon} x_0 \right) \left( 1 - \exp \left[ -\mu_e q \frac{N_D}{\epsilon} t \right] \right).$$

Introducing into (2.15) and (2.16) the collection times given by (2.11) and (2.12), the total charges induced by the two carriers on the cathode are obtained:

$$(2.17) \quad Q_{h, tot} = q \frac{w - x_0}{w},$$

$$(2.18) \quad Q_{e, tot} = q \frac{x_0}{w}.$$

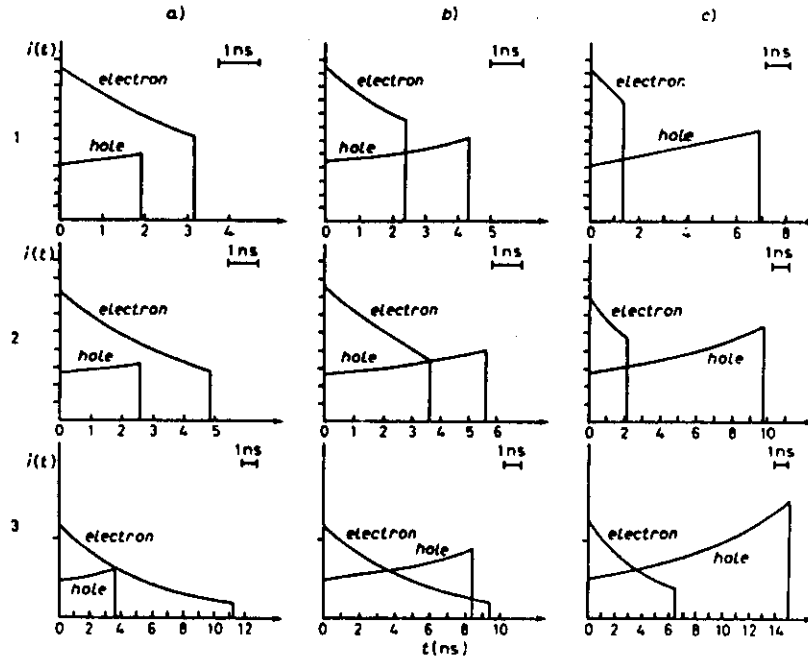


Fig. 2.5. - Shapes of current signals induced on the cathode by an electron-hole pair created at  $x_0$ . The columns a), b), c), correspond, respectively, to the values 0.75, 0.5, 0.25 of the  $x_0/w$  ratio, while the rows 1, 2, 3 correspond, respectively, to the values 0.9, 0.5, 0.1 of the  $E_{min}/E_{crit}$  ratio.

The set of curves of fig. 2.5 represents the theoretical shapes of the induced current signals for the combinations of three values of the  $x_0/w$  ratio and three values of the ratio  $E_{min}/E_{crit}$ , where  $E_{crit} = (qN_d/\epsilon)w$  is the smallest electric field at the detector junction which guarantees the total depletion, that is  $E_{min} = 0$  for  $E_{max} = E_{crit}$ . The calculations refer to a detector with  $w = 300 \mu\text{m}$  and  $N_d = 10^{11} \text{cm}^{-3}$ .

The diagrams of fig. 2.5 point out the importance of «over-depleting» the detector, that is, of using a large  $E_{min}/E_{crit}$  ratio in order to reduce the detector collection times.

In some cases, for instance for the microstrip detectors, the motion of the carriers and the resulting induced currents must be calculated taking into account the diffusion along with the drift. The importance of the diffusion can be evaluated by recalling that the average square deviation  $\overline{\Delta r^2}$  with respect to the trajectory without diffusion during a time interval  $\Delta t$  is given by

$$\overline{\Delta r^2} = 2D\Delta t,$$

where  $D$  is the diffusion coefficient of either type of carrier in silicon. The values of  $2D$  are  $60 \text{ cm}^2/\text{s}$  ( $6 \mu\text{m}^2/\text{ns}$ ) for the electrons and  $22 \text{ cm}^2/\text{s}$  ( $2.2 \mu\text{m}^2/\text{ns}$ ) for the holes. Correspondingly, the squared time spread is  $\overline{\Delta t^2} = 2D(\Delta t/u^2)$  and summing these variances, for a time-of-flight  $t_f$ ,

$$\overline{\Delta t^2} = 2D \int_0^{t_f} \frac{dt}{u^3}.$$

## REFERENCES

- [1] S. RAMO: *Proc. IRE*, **27**, 584 (1939).
- [2] C. K. JEN: *Proc. IRE*, **29**, 345 (1941).
- [3] R. E. COLLIN: *Fundamentals of Microwave Engineering* (McGraw-Hill Book Co., New York, N. Y., 1966).
- [4] G. CAVALLERI, G. FABRI, E. GATTI and V. SVELTO: *Nucl. Instrum. Methods*, **21**, 177 (1963).
- [5] A. H. WALENTA: *Nucl. Instrum. Methods*, **151**, 461 (1978).
- [6] E. GATTI, G. PADOVINI and V. RADEKA: *Nucl. Instrum. Methods*, **193**, 651 (1982).
- [7] S. M. SZE: *Physics of Semiconductor Devices*, 2nd edition (J. Wiley and Sons, New York, N. Y., 1981).

## 3. - Optimum least-square measurement of the charge released by a solid-state detector.

3.1. *Statement of the problem.* - In this section the theory of the optimum measurement of the charge delivered by a solid-state detector will be reviewed. The loss in signal-to-noise ratio due to signal processing methods different from the optimum one, but of more practical interest will be evaluated in sect. 4.

The problem of the optimum signal processing in the measurement of the charge  $Q$  delivered by a detector of capacitance  $C_D$ , briefly outlined in the introduction, can be presented as a search for a network which allows the best least-square measurement of  $Q$  in the presence of the preamplifier noise. The theory of the best least-square measurement will be introduced starting from the model of fig. 3.1.

The detector is modelled as a current source, delivering in an indefinitely short time a charge  $Q$ , proportional to the energy released, across the parallel combination of the detector capacitance  $C_D$  and of the preamplifier input capacitance  $C_i$ . Although the actual preamplifier circuits differ from the simple structure of fig. 3.1, the discussion based upon the voltage-sensitive configuration leads to a more straightforward understanding of the processes involved in the charge measurements and of the noise limitations.

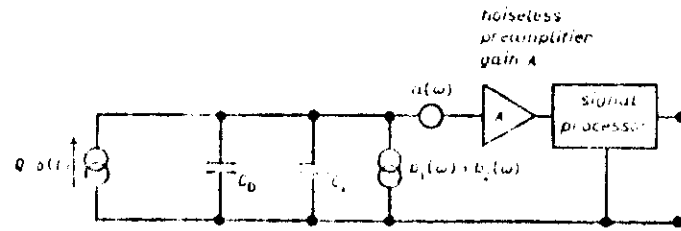


Fig. 3.1. - Analog processor for the measurement of the charge  $Q$ .

The final results, moreover, are to a large extent independent of the amplifier configurations.

The noise sources in the whole system are accounted for by a series voltage generator with spectral power density  $a(\omega)$  and by a parallel current generator with spectral power density  $b_1(\omega) + b_2(\omega)$ . While  $a(\omega)$  is a characteristic of the preamplifier alone, the parallel noise has been split into two terms,  $b_1(\omega)$  including the preamplifier contributions and  $b_2(\omega)$  describing the parallel noise associated with the detector and with the bias circuitry of detector and preamplifier. Incidentally, it has to be pointed out that, to describe a linear noisy two-port network, a series and a parallel noise generators with their power spectra  $\Sigma_{aa}(\omega)$ ,  $a(\omega)$ ,  $\Sigma_{bb}(\omega)$ ,  $b_1(\omega)$  and with the cross-correlation terms  $\Sigma_{ab}(\omega)$ ,  $\Sigma_{ba}(\omega)$  are generally required [1-3].

In the cases of interest in nuclear applications, however, the two generators  $a_1(\omega)$  and  $b_1(\omega)$  can be considered uncorrelated and, therefore, only two spectral power densities will suffice to describe the preamplifier noise. Once the noise is accounted for by the external generators, the preamplifier is represented as a noiseless two-port.

The series voltage source generally consists of a white term  $a_1$  and of a term with a spectral power density of the type  $a_2/|\omega|$ . The white noise accounts for the shot noise in the collector current in bipolar transistors or for the thermal noise in the channel of field-effect devices. The  $a_2/|\omega|$  part of the series power density is of negligible importance in bipolar transistors and silicon junction field-effect transistors as long as their use is limited to elementary-particle physics. The same might not be true for GaAs devices and is surely not true for MOS field-effect transistors, for which the contribution due to the  $a_2/|\omega|$  term turns out to be the dominant limitation [4, 5].

The spectral power density  $b_1(\omega)$  describes the shot noise associated with the base current in a bipolar transistor or with the gate leakage current in a junction field-effect transistor. In either case  $b_1$  can be considered independent of  $\omega$ .

The spectral power density  $b_2(\omega)$  includes contributions due to the shot noise in the detector leakage current and to the thermal noise in the resistors employed in the detector circuit. The noise associated with the preamplifier input is also included in  $b_2(\omega)$ .

The noise associated with the dielectric losses, which would add to  $b_1$  and  $b_2$  terms with a linear frequency dependence, is quantitatively of negligible importance in the applications covered by this paper.

The previous discussion leads to the conclusion that the  $1/|\omega|$  noise in the input spectrum is important only in one device, the MOS transistor, which cannot be considered, strictly speaking, a true low-noise device. This consideration, along with the remark that such a dependence would considerably increase the mathematical difficulties, suggests that the optimum processing be analysed in the case of white spectra for both series and parallel generators. It will be accordingly assumed  $a(\omega) = a$  independent of  $\omega$  and  $b_1(\omega) + b_2(\omega) = b$ , also independent of  $\omega$ . The effect of  $1/|\omega|$  noise will be analysed separately, by evaluating its contribution to the total noise in some particular cases.

A treatment which extends the discussion to a more general input spectrum is available in the literature [6].

In the foregoing analysis the bilateral representation,  $\omega$  ranging between  $-\infty$  and  $+\infty$ , will be adopted for the spectral power densities.

Two different methods will be followed to determine the best processing of detector signals. One, which makes use of a time domain approach to synthesize the optimum processor, is due to BALDINGER and FRANZEN [7]. The other one, which extends to the nuclear case the theory of the matched filter with a frequency domain approach, is due to RADEKA and KARLOVAC [8]. Both methods deduce the optimum filter for a single pulse of given shape under the hypothesis that the baseline to which the pulse is referred is known with infinite accuracy. Such a situation is not realistic because the time available for the baseline estimation is limited by the presence of other incoming signals. After the presentation of the methods of Baldinger and Franzen and Radeka and Karlovac, the problem related with the inaccuracies in the baseline estimation will be considered.

**3'2. Time domain approach to optimum processing.** - As a preamble to the time domain approach to optimum processing in the actual case, the following particular situation will be considered. The information about a parameter  $\mathcal{A}_1$  is carried by a signal consisting of two rectangular pulses of short width  $\Delta t$ , with amplitudes  $\mathcal{A}_1$  and  $\alpha\mathcal{A}_1$ , occurring at a mutual time distance  $\tau$ , as in fig. 3.2. The time-dependent part  $S(t)$  of the signals is represented by the two rectangles of amplitudes 1 and  $\alpha$ .

The signal is added to a noise which is assumed to be of ergodic nature and band limited white with an upper frequency limitation of the order of  $1/\Delta t$ .

If the parameter  $\mathcal{A}_1$  is determined from the first pulse of fig. 3.2, the signal-to-noise ratio is  $\mathcal{A}_1/\langle e_n^2 \rangle^{1/2}$ , where  $\langle e_n^2 \rangle^{1/2}$  is the ensemble root-mean-square value of the noise. If  $\mathcal{A}_1$  is, instead, determined from the second rectangular pulse, the signal-to-noise ratio is  $\alpha\mathcal{A}_1/\langle e_n^2 \rangle^{1/2}$ , where the ensemble root-mean-square value is the same as before owing to the hypothesis of ergodicity. In both

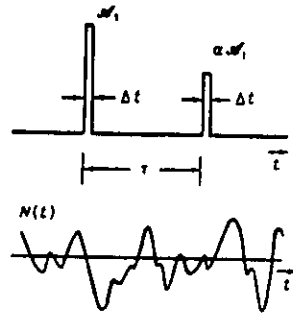


Fig. 3.2. - Best least-square measurement of the parameter  $A_1$  from a signal consisting of two rectangular pulses in the presence of band-limited white noise.

cases a signal-to-noise ratio inferior to the optimum one is achieved, because the full information carried by the signal is not utilized. To fully exploit the information available in the signal, the most general linear processing which consists in the weighted sum of the two pulses will be adopted. The time-variant network which implements it is shown in fig. 3.3.

According to the block diagram of fig. 3.3, a weighting wave form, made of two narrow rectangles of amplitudes 1 and  $\beta$  and synchronous with the two rectangles that constitute the input signal, is triggered.

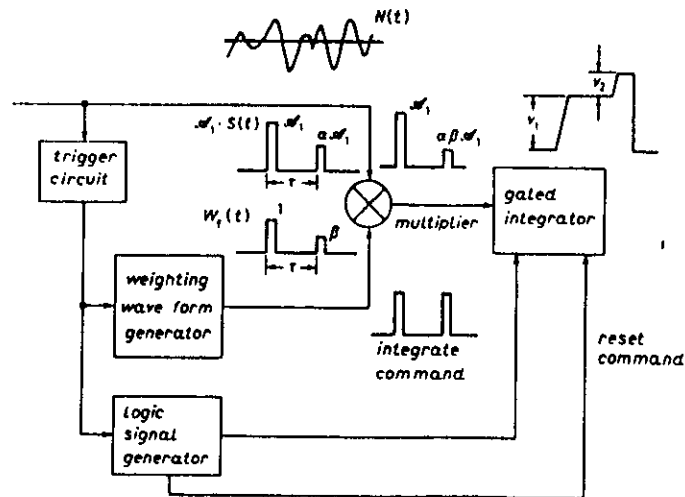


Fig. 3.3. - Time-variant realization of the weighted sum.

An analog multiplier implements the product of the input signal and of the weighting function. The resulting wave form is applied to the input of the gated integrator. At the end of the 'integrate command' signal, the voltage stored in the integrator is proportional to  $A_1(1 + \alpha\beta)$ . At the same instant the noise is proportional to  $N(t) + \beta N(t + \tau)$ . As the noise is almost white, its autocorrelation function is a very narrow function in the time domain, which means that the two values  $N(t)$ ,  $N(t + \tau)$  can be considered uncorrelated.

The ergodic hypothesis ensures that the ensemble root-mean-square values of  $N(t)$  and  $N(t + \tau)$  are equal. Therefore, the signal-to-noise ratio is

$$(3.1) \quad \frac{A_1(1 + \alpha\beta)}{(1 + \beta^2)^{1/2} \langle v_N^2 \rangle^{1/2}}.$$

Expression (3.1) considered as a function of  $\beta$  has a maximum for  $\beta = \alpha$ . The maximum signal-to-noise ratio achievable in the measurement of  $A_1$  is, therefore,

$$\frac{A_1(1 + \alpha^2)^{1/2}}{\langle v_N^2 \rangle^{1/2}},$$

which is a factor  $1 + \beta^2$  better than the larger of the two signal-to-noise ratios obtained with the information from a single pulse. The circuit of fig. 3.3 represents the time-variant version of the optimum processor, while the time-invariant one is shown in fig. 3.4.

The measurement has to be carried out, in the circuit of fig. 3.4, at  $t = \tau$  and the result obtained in this way, with  $\beta = \alpha$ , corresponds to the best least-square estimate of  $A_1$ . The linear processing discussed so far and implemented by either solution 3.3 and 3.4 can be demonstrated to be the best one with respect to any kind of processing [9].

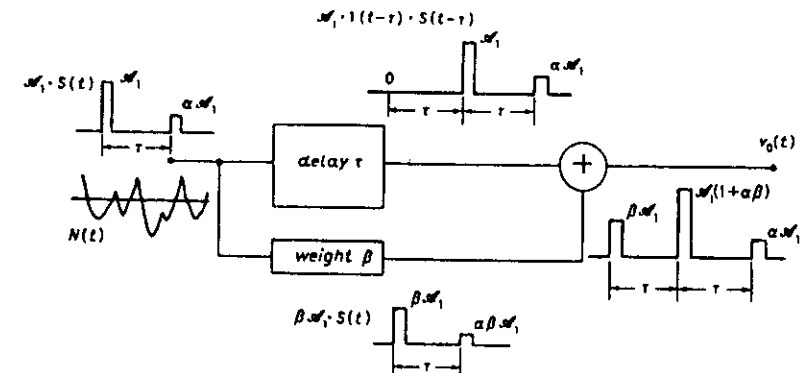


Fig. 3.4. - Time-invariant realization of the network implementing the weighted sum of two pulses.

The output of fig. 3.4 responds to a narrow rectangular pulse of unit amplitude with the wave form  $v_\delta(t)$  of fig. 3.5. The function  $v_\delta(t)$  is the mirror image of the weighting function  $w_\delta(t)$  of fig. 3.5:

$$v_\delta(t) = w_\delta(\tau - t).$$

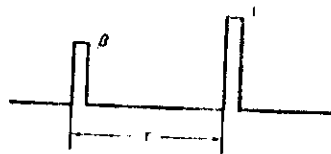


Fig. 3.5. — Response of the time invariant processor of fig. 3.4 to a narrow rectangle of unit amplitude.

In the optimum situation,  $\beta = \alpha$ ,  $v_\delta(t)$  becomes the mirror image of the time-dependent part  $S(t)$  of the input signal

$$v_\delta(t) = S(\tau - t).$$

The response  $v_\delta(t)$  of the time invariant processor of fig. 3.4 to the signal  $\mathcal{A}_1 S(t)$  can be interpreted as the convolution of  $\mathcal{A}_1 S(t)$  and  $v_\delta(t)$ .

The previous considerations can be easily extended to the case in which the information about a parameter  $\mathcal{A}$ , is carried by a continuous wave form  $\mathcal{A}_1 S(t)$  which can be considered as a sequence of samples of amplitudes  $\alpha_1 \mathcal{A}_1, \alpha_2 \mathcal{A}_1, \dots, \alpha_N \mathcal{A}_1$  and  $\mathcal{A}_1$  has to be measured with the least mean-square error in the presence of band-limited white noise.

The optimum processor implements the weighted sum

$$\mathcal{A}_1 \sum_{k=1}^N \alpha_k^2.$$

Its time-invariant implementation has, as  $\delta$ -impulse response, the mirror image of  $S(t)$

$$v_\delta(t) = S(\tau - t).$$

The hypothesis of band-limited white noise is instrumental in making the solution of the optimization problem so simple. If the noise is not white, then the synthesis of the processor which implements the best least square error measurement has to be split into two parts. One is the synthesis of a filter, called *noise-whitening*, which transforms the original noise spectrum into a white one. The second part consists in applying the weighted-sum method to the whitened signal.

The previous considerations will now be applied to the actual case of the nuclear radiation detector of fig. 3.1. The signal at the preamplifier output is a step of amplitude  $(Q/(C_D + C_i))A$  and the noise power spectral density is (see p. 21)

$$(3.2) \quad \Delta^2 \left[ a + \frac{b}{\omega^2 (C_D + C_i)^2} \right].$$

For instance, a network with transfer function

$$(3.3) \quad \mathcal{F}_s(s) = \sqrt{\frac{W}{a}} \frac{1}{A} \frac{s \sqrt{a/b} (C_D + C_i)}{1 + s \sqrt{a/b} (C_D + C_i)},$$

where  $s$  is the complex frequency, can transform the noise (3.2) into a white one with spectral power density  $W$ .

The *noise-whitening* filter is, therefore, the cascade of a gain element and of an approximate differentiator with time constant

$$(3.4) \quad \tau_1 = \sqrt{\frac{a}{b}} (C_D + C_i).$$

The time constant  $\tau_1$  is called *noise corner time constant* and is the reciprocal of that particular angular frequency  $\omega_1$  at which the contributions from series and parallel noise at the preamplifier input become equal.

As pointed out by (3.4), the noise corner time constant is determined by the preamplifier series noise, by the total parallel noise and by the total capacitance at the preamplifier input.

At the output of the whitening filter the noise has a spectral density  $W$  and the signal has the time dependence

$$(3.5) \quad \frac{Q}{C_D + C_i} \sqrt{\frac{W}{a}} \exp[-t/\tau_1] \cdot 1(t) = \frac{Q}{C_D + C_i} \sqrt{\frac{W}{a}} S_w(t) \cdot 1(t)$$

shown in fig. 3.6a). It is worth pointing out, in passing, that the transfer function  $\mathcal{F}_s(s)$  is not univocally defined by the power spectrum (3.2) and that the filter which has as output signal the mirror image of the response of fig. 3.6a) would be a noise-whitening filter as well (fig. 3.6b)). At a first glance this second filter looks to be physically unrealizable. However, if the response of fig. 3.6b) is truncated to a finite  $\tau_M$  and a delay equal to or larger than  $\tau_M$  is introduced, it becomes physically realizable.

Referring, for the sake of simplicity, to the response 3.6a) and being the noise at the filter output white, the optimum processor for the measurement of  $\mathcal{A}$  can be determined with the already known procedure.

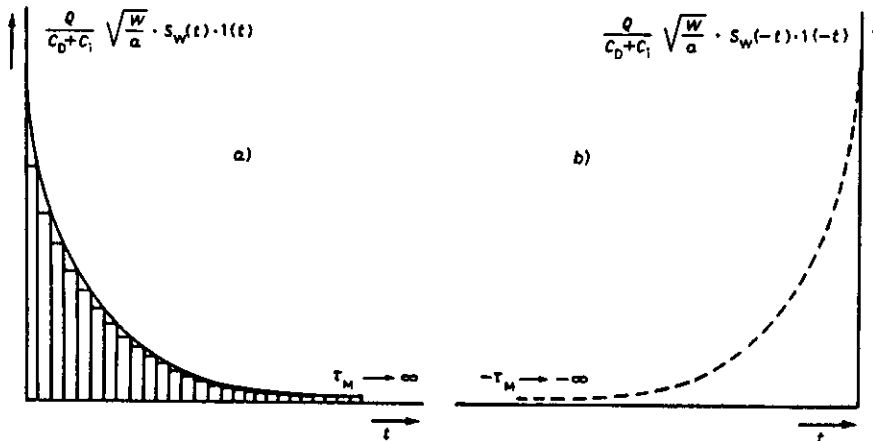


Fig. 3.6. - Signals at the output of noise-whitening filters in response to a  $\delta$ -impulse detector current of charge  $Q$ .

The signal is represented in fig. 3.6a) as a time sequence of equally spaced rectangles, each of them having an amplitude proportional to  $Q$ .

In the case of an indefinitely long measurement time  $\tau_M$  the  $\delta$ -impulse responses of the optimum processors, obtained extending to the continuum case the procedure adopted for the discrete signals of fig. 3.2, are represented in fig. 3.7a), b), respectively, for the whitening filters of fig. 3.6a), b).

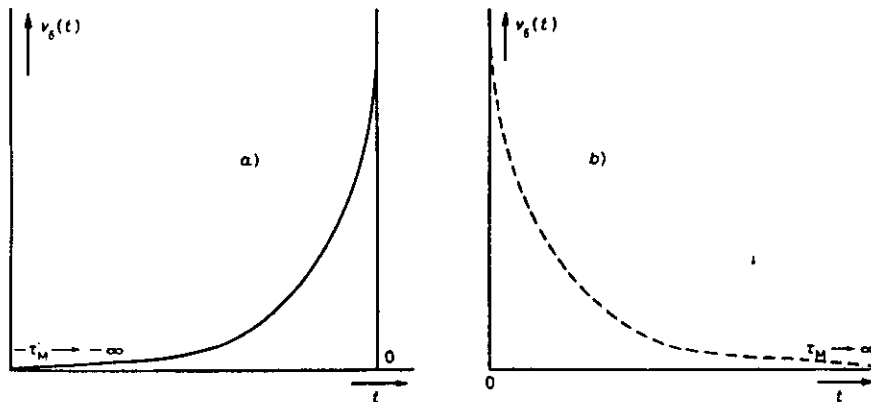


Fig. 3.7. -  $\delta$ -impulse responses of the optimum processors: a) signal at the output of the noise-whitening filter 3.6a), the  $\delta$  is supposed to be applied at  $t = -\tau_M$ ; b) signal at the output of the noise-whitening filter 3.6b).

The response of the optimum processors to their corresponding input signals, exponentially decaying or exponentially growing, is in either case the indefinite cusp of fig. 3.8.

The measurement of  $Q$  requires the evaluation of the peak amplitude of the cusp.

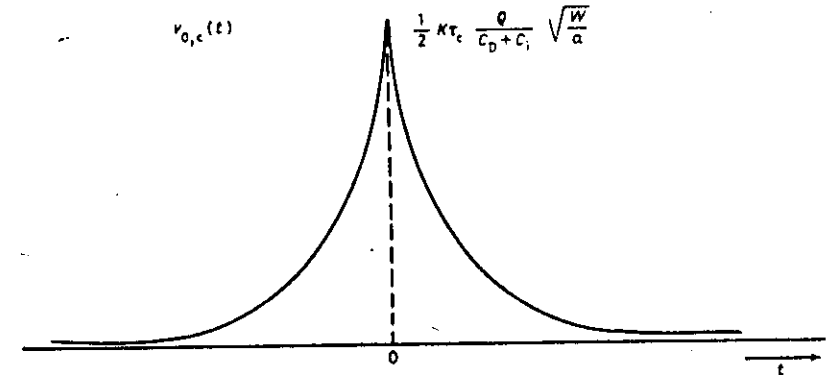


Fig. 3.8. - Response of the optimum processors to the exponentially decaying or exponentially growing pulses at the output of the corresponding noise-whitening filters.

The optimum processors are determined by the time dependence of the signals at the output of the related noise-whitening filters, that is, the processors are matched to those signals. For this reason the theory being developed is also called «matched filter» theory [10]. In order to complete it, the optimum signal-to-noise ratio has now to be evaluated.

To do this, the peak amplitude of the cusp of fig. 3.8 will be calculated at once. According to (3.5) the  $\delta$ -response of the optimum processor for the exponentially decaying signal is

$$(3.6) \quad w_r(t) = K \exp[t/\tau_c] \cdot 1(-t),$$

where  $K$  is an arbitrary constant having the dimension of a frequency. The response of the optimum processor to the exponentially decaying signal is the convolution of (3.5) and (3.6), that is,

$$v_{\delta,c}(t) = \frac{KQ}{C_D + C_I} \sqrt{\frac{W}{a}} \int_{-\infty}^{\infty} \exp\left[-\frac{t_0}{\tau_c}\right] \cdot 1(t_0) \exp\left[\frac{t-t_0}{\tau_c}\right] \cdot 1(t_0-t) dt_0,$$

which is the cusp of fig. 3.8. The function  $v_{\delta,c}(t)$  has a maximum at  $t = 0$ ,

whose value is

$$(3.7) \quad v_{0, \max} = \frac{KQ}{C_0 + C_1} \sqrt{\frac{1}{W}} \tau_c \int_0^{\infty} \exp \left[ -2 \frac{t_0}{\tau_c} \right] u \left( \frac{t}{\tau_c} \right) dt - \frac{1}{2} \frac{KQ}{C_0 + C_1} \sqrt{\frac{1}{W}} \tau_c.$$

The root-mean-square noise at the output of the optimum processor can be evaluated from the knowledge of the spectral power density  $W$  of the noise at the input and from the transfer function of the optimum processor, which is the Laplace transform of  $w_1(t)$  in eq. (3.6):

$$(3.8) \quad [v_N^2]^{-1} = \left[ \int_{-\infty}^{\infty} W |W_1(j\omega)|^2 d\omega \right]^{-1} = \left[ -W \int_0^{\infty} K^2 \exp \left[ \frac{2t}{\tau_c} \right] dt \right]^{-1} = \left[ \frac{1}{2} \tau_c W \right]^{-1} K^2.$$

In eq. (3.8)  $W_1(s)$  is the Laplace transform of  $w_1(t)$  and Parseval's theorem has been used to pass from the integral on the frequency domain to the integral in the time domain [11].

The hypothesis of ergodicity ensures that the time average of (3.8) coincides with the ensemble average previously adopted in the evaluation of the signal-to-noise ratio. The optimum signal-to-noise ratio, according to (3.7) and (3.8), is

$$(3.9) \quad \rho_{\max} = \frac{Q}{(C_0 + C_1)^{1/2} (4ab)^{1/2}}.$$

**3.3. Frequency domain approach to optimum processing.** — An alternative deduction of the matched-filter theory in the frequency domain will now be presented. Let  $Q_1 S_1(t)$  be the signal carrying the information about the parameter to be measured and let  $V(\omega)$  be the spectral power density of the noise associated with  $Q_1 S_1(t)$ .

The noise is assumed again to be ergodic. The problem of the optimum least-square measurement of the parameter  $Q_1$  can be stated in the following way. Let  $H(s)$  be the *a priori* unknown transfer function of the network which leads to the optimum measurement.

The signal at the output of this network can be written as an inverse Fourier transform

$$(3.10) \quad v_0(t) = \frac{Q_1}{2\pi} \int_{-\infty}^{\infty} H(j\omega) S_1(\omega) \exp[j\omega t] d\omega,$$

where  $S_1(\omega)$  is the Fourier transform of  $S_1(t)$ .

The r.m.s. noise at the output of the same network is

$$(3.11) \quad [v_N^2]^{-1} = \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega) |H(j\omega)|^2 d\omega \right]^{-1},$$

where the ensemble average has been evaluated as a time average thanks to the hypothesis of ergodicity. The unknown function  $H(s)$  must be determined so as to maximize the ratio between the value of the signal at the measurement time  $t_m$  and the root-mean-square noise. As the optimization procedure is more easily carried out on the square of this ratio, it can be said that the desired  $H(s)$  is the one which makes the ratio

$$(3.12) \quad \rho^2 = \frac{Q_1^2}{2\pi} \frac{\left\{ \int_{-\infty}^{\infty} H(j\omega) S_1(\omega) \exp[j\omega \tau_m] d\omega \right\}^2}{\int_{-\infty}^{\infty} V(\omega) |H(j\omega)|^2 d\omega}$$

achieve its maximum value. The existence of an upper limit for (3.12), which turns out to be the required maximum for a particular choice of  $H(s)$ , can be demonstrated by applying the Schwartz inequality. The Schwartz inequality is recalled here. Let  $u_1(\omega)$  and  $u_2(\omega)$  be two complex functions of the real variable  $\omega$ . Then

$$(3.13) \quad \left| \int_{-\infty}^{\infty} u_1(\omega) u_2(\omega) d\omega \right|^2 < \int_{-\infty}^{\infty} |u_1(\omega)|^2 d\omega \int_{-\infty}^{\infty} |u_2(\omega)|^2 d\omega.$$

In expression (3.13) the equality sign holds if

$$u_2(\omega) = K' u_1^*(\omega),$$

where  $K'$  is a constant and  $*$  denotes the complex conjugate. If the Schwartz inequality is applied to (3.12) by choosing

$$u_1(\omega) = H(j\omega) \mathcal{N}^{1/2}(\omega), \quad u_2(\omega) = \frac{S_1(\omega)}{\mathcal{N}^{1/2}(\omega)} \exp[j\omega \tau_m],$$

it can be written

$$(3.13) \quad \frac{Q_1^2}{2\pi} \frac{\left\{ \int_{-\infty}^{\infty} H(j\omega) S_1(\omega) \exp[j\omega \tau_m] d\omega \right\}^2}{\int_{-\infty}^{\infty} V(\omega) |H(j\omega)|^2 d\omega} < \frac{\left\{ \int_{-\infty}^{\infty} V(\omega) |H(j\omega)|^2 d\omega \right\} \left\{ \int_{-\infty}^{\infty} \frac{|S_1(\omega)|^2}{V(\omega)} d\omega \right\}}{\int_{-\infty}^{\infty} V(\omega) |H(j\omega)|^2 d\omega} = \frac{Q_1^2}{2\pi} \int_{-\infty}^{\infty} \frac{|S_1(\omega)|^2}{V(\omega)} d\omega.$$

Equation (3.13) is of general validity and it states that, passing a signal with Fourier transform  $S_1(\omega)$  and a noise with spectral power density  $\mathcal{N}(\omega)$  through whatever linear network, the output signal-to-noise ratio cannot exceed the integral over the whole frequency axis of the ratio between the energy density of the signal and the power density of the noise.

Out of all the possible  $H(j\omega)$ 's the only one which makes  $\rho$  reach the limit

$$(3.14) \quad \frac{Q^2}{2\pi} \int_{-\infty}^{\infty} \frac{|S_1(\omega)|^2}{\mathcal{N}(\omega)} d\omega$$

is the one which satisfies the condition

$$H(j\omega)\mathcal{N}^{1/2}(\omega) = K' \frac{S_1^*(\omega)}{\mathcal{N}^{1/2}(\omega)} \exp[-j\omega\tau_M].$$

The limit itself is, therefore, the sought  $\rho_{opt}$ . The transfer function  $H(s)$  of the optimum network must, then, satisfy the relationship

$$(3.15) \quad H(j\omega) = K' \frac{S_1^*(\omega)}{\mathcal{N}(\omega)} \exp[-j\omega\tau_M].$$

Equation (3.15) can be written by assuming for  $S_1(t)$  the signal shape  $S_-(t) = 1(t) \cdot \exp[-t/\tau_e]$ , eq. (3.5), at the output of the noise-whitening filter. A point at which the noise is white may not even exist in the actual charge-measuring system, but reference to it can be made, though in a virtual way. By putting  $S_1(t) = S_-(t)$ ,  $\mathcal{N}(\omega) = W$ , eq. (3.15) yields

$$(3.17) \quad H(j\omega) = \frac{K'}{W} S_-(\omega) \exp[-j\omega\tau_M].$$

Recalling the properties of the conjugate of the Fourier transform of a real function, it can be concluded from (3.17) that, apart from a coefficient, the  $\delta$ -impulse response of the optimum network following the noise-whitening filter is

$$(3.18) \quad S_-(\tau_M - t).$$

The same result as in the case of the time domain analysis is reached, that is, the optimum processor responds to the  $\delta$ -impulse at the output of the noise-whitening filter with the mirror image of the signal present at that point.

The optimum signal-to-noise ratio can be evaluated from (3.14) by assuming as  $Q$ ,  $S_1(t)$  the signal (3.5) at the output of the noise-whitening filter, where  $\mathcal{N}(\omega) = W$ .

By putting  $Q_1 = (Q/(C_D + C_1)) \sqrt{W/a}$ ,  $S_1(t) = S_-(t)$ , (3.14) gives

$$(3.19) \quad \rho_{opt} = \frac{Q}{C_D + C_1} \sqrt{\frac{W}{a}} \left\{ \frac{1}{W} \int_{-\infty}^{\infty} |S_-(\omega)|^2 \frac{d\omega}{2\pi} \right\}^{1/2} = \frac{Q}{C_D + C_1} \sqrt{\frac{W}{a}} \left\{ \frac{1}{W} \int_0^{\infty} \exp[-2t/\tau_e] dt \right\}^{1/2}.$$

The transformation of the integral from the frequency domain to the time domain was made possible by Parseval's theorem.

Evaluation of the integral in (3.19) gives

$$(3.20) \quad \rho_{opt} = \frac{Q}{(C_D + C_1)^{1/2}} \frac{1}{(4ab)^{1/2}},$$

which is the same result expressed by (3.9).

The optimum signal-to-noise ratio, according to (3.9) and (3.20), is proportional to the charge available  $Q$  and inversely proportional to the square root of the sum of the capacitances  $C_D$  and  $C_1$  and to the fourth root of the product of series and parallel spectral power densities.

3.4. *Partial use of the information available at the output of the noise-whitening filter.* - The result expressed by eqs. (3.9) and (3.20) refers to the case in which the information available at the output of the noise-whitening filter is fully exploited.

The need of working with a finite peaking time  $\tau_M$  suggests that only the part of the signal  $S_-(t)$  corresponding to  $0 < t < \tau_M$  is utilized, as in fig. 3.9. The shaded area in fig. 3.9a) is the part of the signal utilized for the measurement. The  $\delta$ -response of the optimum processor following the noise-whitening filter is shown in fig. 3.9b), while the response of the whole system to the detector current pulse is shown in fig. 3.9c).

The truncated cusp allows the charge measurement to be done at a finite time  $\tau_M$ ; however, no reduction in the width of the output pulse is achieved.

The reduction in signal-to-noise ratio which results from truncation can be obtained from (3.19) by limiting the integral between 0 and  $\tau_M$ :

$$(3.21) \quad \rho_{\tau_M} = \frac{Q}{C_D + C_1} \frac{1}{\sqrt{a}} \left[ \int_0^{\tau_M} \exp[-2t/\tau_e] dt \right]^{1/2} = \rho_{opt} \left[ 1 - \exp\left[-2\frac{\tau_M}{\tau_e}\right] \right]^{1/2}.$$

As pointed out by (3.21), the increase in  $\rho_{\tau_M}$  which results from extending the measurement time beyond  $\tau_e$  is very small. Increasing, for instance,  $\tau_M$  from  $\tau_e$  to infinite, only 8% is gained in signal-to-noise ratio.

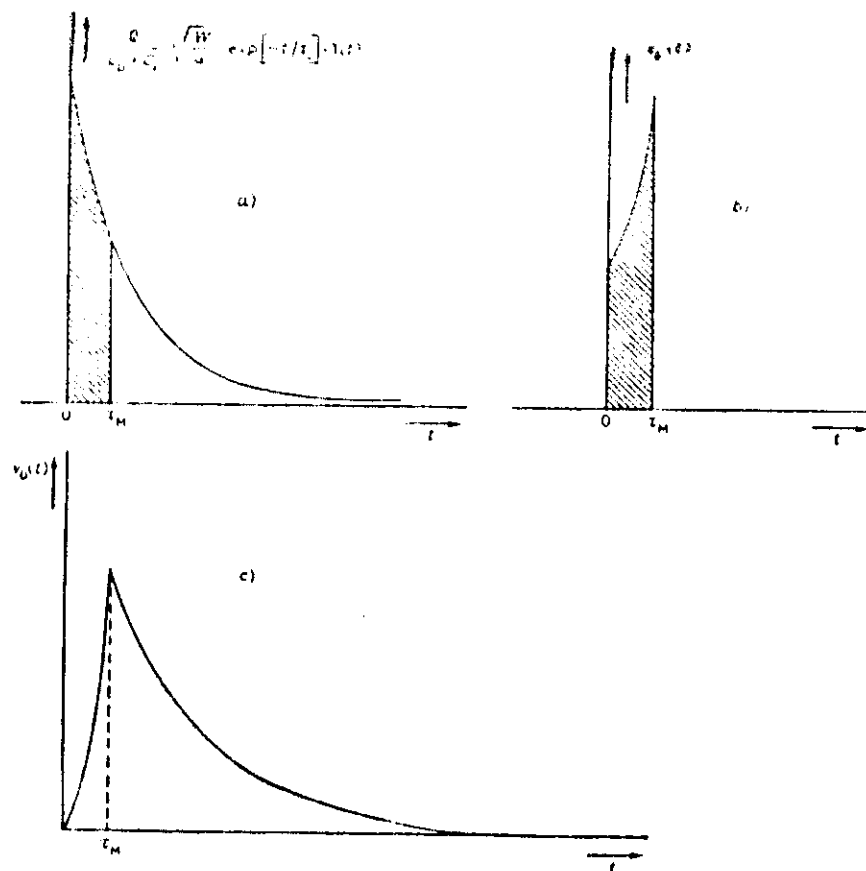


Fig. 3.9. - Use of a fraction of information contained in the signal: a) the shaded area is the part utilized in the measurement, b)  $\delta$  impulse response of the optimum network, c) truncated cusp.

**3'b. Matched-filter theory in the case of finite rate of arrival of events on the detector.** - The matched-filter theory developed so far has not been subjected to any constraint in the pulse width. The signal at the output of the matched filter has either an indefinitely long peaking time if  $\rho_{opt}$  must be achieved or a finite peaking time in the case of a truncated cusp. Both signals have infinite duration. The theory, in other words, was restricted to the consideration of a single event to be measured and according to this hypothesis the measurement of the peak amplitude of the cusp, which contains the information about  $Q$ , was referred to an indefinitely accurate measurement.

In the actual case a sequence of pulse amplitudes has to be measured and, therefore, a comparison of the maximum allowed variation of the pulse at the processor output must be added. The problem of the optimum least-square error measurement will, therefore, be restated by assuming that elementary events delivering in the detector the charge  $Q$  occur in a random sequence of finite rate.

The situation is depicted in fig. 3.10.

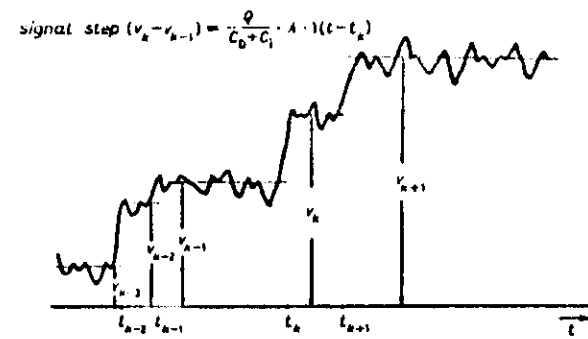


Fig. 3.10. - Stochastic staircase determined at the preamplifier output by the random occurrence of detector pulses.

The stochastic staircase of fig. 3.10 represents the effect of the random superposition of steps determined by the occurrence of the detector current pulses. The evaluation of the charge associated with the event happening at  $t = t_k$ , for instance, requires the best estimate of the difference  $v_k - v_{k-1}$ . The level  $v_{k-1}$ , existing prior to the arrival of the event at  $t = t_k$  represents, therefore, the baseline to which the event to be measured is referred. To deal with the present problem a simplifying hypothesis, which is frequently respected in the experimental situations of elementary-particle physics, will be introduced. The hypothesis is related with the property that, when the measurement time is short compared with the noise corner time constant  $\tau_c$  given by (3.4), the parallel noise becomes of minor importance. A treatment of the present problem without this limiting hypothesis is available in the literature [12]. If only the series noise is considered, the spectral noise density at the preamplifier output is white and, therefore, no additional whitening filter is needed.

This means that the problem of the best estimate of the  $v_k - v_{k-1}$  difference has to be solved in the presence of white noise, whose spectral density, according to fig. 3.1, is, at the preamplifier output,  $W = aA^2$ . As shown in fig. 3.10, a time interval  $(t_{k-1}, t_k)$  is available to estimate  $v_{k-1}$  and a time interval  $(t_k, t_{k+1})$  is available to estimate  $v_k$ . Application of the previously developed matched-filter theory to the measurement of the  $v_{k-1}$  level in the presence of white noise

shows that the best processing is a pure integration in the  $(t_{k-1}, t_k)$  interval. The optimum signal-to-noise ratio is, according to (3.14),

$$(3.22) \quad \rho_{opt} = \frac{v_{k-1}}{a^2 A} \left[ \int_{t_{k-1}}^{t_k} dt \right]^2 = \frac{v_{k-1}}{a^2 A} [t_k - t_{k-1}]^2.$$

According to (3.22) the root-mean-square noise at the output of the optimum processor is

$$(3.23) \quad \langle v_{N,k}^2 \rangle^{\frac{1}{2}} = A \left[ \frac{a}{t_k - t_{k-1}} \right]^{\frac{1}{2}}.$$

Similarly, the optimum processor to estimate  $v_k$  in the presence of white noise is a pure integrator acting in the  $(t_k, t_{k+1})$  interval. It follows again from (3.14)

$$(3.24) \quad \rho_{opt} = \frac{v_k}{a^2 A} \left[ \int_{t_k}^{t_{k+1}} dt \right]^2 = \frac{v_k}{a^2 A} [t_{k+1} - t_k]^2$$

and for the noise at the output of the optimum processor

$$(3.25) \quad \langle v_{N,k+1}^2 \rangle^{\frac{1}{2}} = A \left[ \frac{a}{t_{k+1} - t_k} \right]^{\frac{1}{2}}.$$

The signal-to-noise ratio in the measurement of the step amplitude can now be evaluated as

$$(3.26) \quad \rho_{opt}(v_k - v_{k-1}) = \frac{v_k - v_{k-1}}{[\langle v_{N,k}^2 \rangle + \langle v_{N,k+1}^2 \rangle]^{\frac{1}{2}}} = \frac{Q}{(C_D + C_i) [1/(t_{k+1} - t_k) + 1/(t_k - t_{k-1})]^{\frac{1}{2}} a^{\frac{1}{2}}}.$$

It is important to point out that the optimum signal-to-noise ratio in the measurement of the difference depends on the length of the empty intervals available for the measurement *before* and *after* the event to be processed. The network which implements the best processing for the  $v_k - v_{k-1}$  measurement is characterized by a constant distribution of weights of negative polarity between  $t_{k-1}$  and  $t_k$  and by a constant distribution of weights of positive polarity between  $t_k$  and  $t_{k+1}$ . Both distributions are shown in fig. 3.11 normalized to unit area.

Figures 3.11b) and d) show, respectively, the  $\delta$ -response of the optimum processor and the actual output signal, convolution of the wave form b) and of the step under measurement.

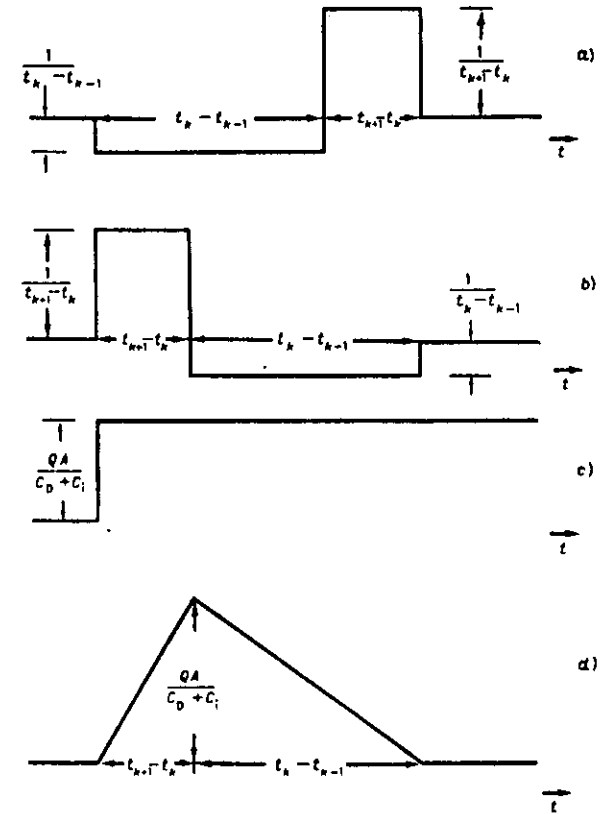


Fig. 3.11. — Optimum processing for the estimation of a voltage step in the presence of white noise: a) weight distribution, b)  $\delta$ -response of the optimum processor, c) step under measurement, d) actual response of the optimum processor to the input step.

As the detector pulses are randomly distributed in time and their instants of arrival unpredictable, the best processing based upon the exploitation of the full empty intervals before and after each event is not of practical feasibility. A good approximation consists of using equal time intervals  $\tau$  preceding and following the event [12]. The total processing time  $2\tau$  must be chosen for a given event rate short enough to keep the pile-up probability small.

In the case in which processing intervals of equal duration are employed, (3.26) simplifies into

$$(3.27) \quad \rho(v_k - v_{k-1}) = \frac{Q}{(C_D + C_i) [2a/\tau]^{\frac{1}{2}}}.$$

It is interesting now to compare the optimum processing described in fig. 3.11 with the one which would suggest Baldinger and Franzen's theory in the case of white noise and with the exploitation of a finite step duration  $t_M$ .

The corresponding wave forms are shown in fig. 3.12.

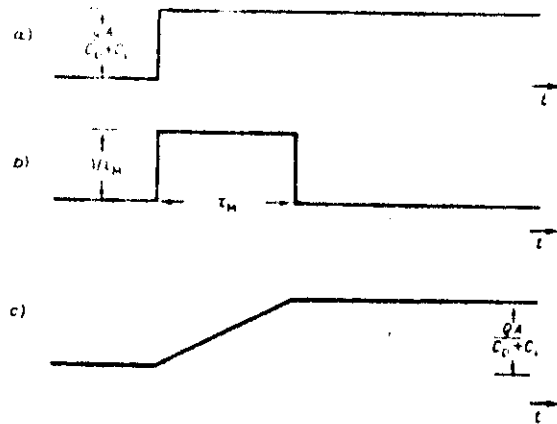


Fig. 3.12. - Optimum processing of a step in white noise and finite measurement time  $t_M$  according to the theory of Baldinger and Franzen: a) step under measurement, b)  $\delta$  response of the optimum processor, c) actual response of the optimum processor to the input step.

The processor of fig. 3.12 can be considered as a particular case of the one of fig. 3.11, in which the time interval  $t_1 - t_{M-1}$  has become indefinitely long, that is, when the free time available for baseline estimation before  $t_1$  is infinite. A time-invariant implementation of a processor measuring the amplitude of the input step with a finite resolving time  $2\tau$  is the circuit which responds to the  $\delta$ -impulse excitation with the bipolar signal of fig. 3.13.

The bipolar weighting distribution implemented by the processor of fig. 3.13 has been recognized to be the optimum one when a free time  $\tau$  is available before and after the edge of the incoming pulse [12].

The time-variant realization of the processing described in fig. 3.13 is shown in fig. 3.14 I).

The timing relationships are shown in fig. 3.14 II).

As shown in fig. 3.14 II), the  $\delta$  impulse carrying the charge information arrives at one of the multiplier inputs when the triangular wave form at the other multiplier input has its maximum amplitude. The gated integrator performs the operation,

$$(3.14) \quad \int_{-\tau}^{+\tau} \frac{0.4}{C_0 + C_i} A \cdot \delta(t) dt = \frac{0.4}{C_0 + C_i} A,$$

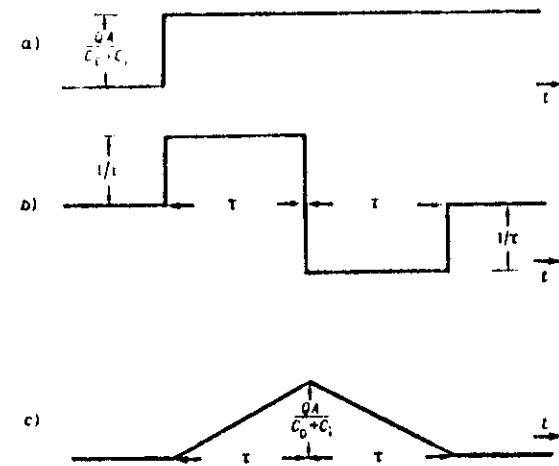


Fig. 3.13. - Processing the input step under the condition of finite resolving time  $2\tau$ : a) input step, b)  $\delta$ -impulse response of the processor, c) actual response of the processor to the input step.

having assumed as time origin the arrival instant of the radiation on the detector.

The white noise at the input of the preamplifier, which, as will be discussed exhaustively in the next section, can be represented in the time domain as a random sequence of positive and negative  $\delta$ -impulses, becomes after differentiation a random sequence of doublets. A doublet arriving at the multiplier input at a random instant  $t_0$ , with  $0 < t_0 < 2\tau$ , produces at the integrator output

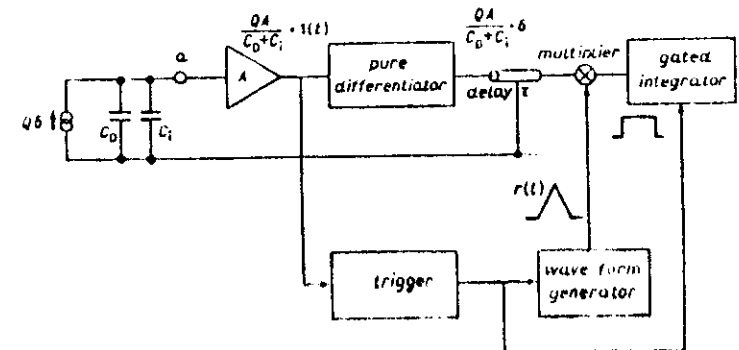


Fig. 3.14 I). - Time variant realization of the processor with bipolar weight distribution. Block diagram of the processor.

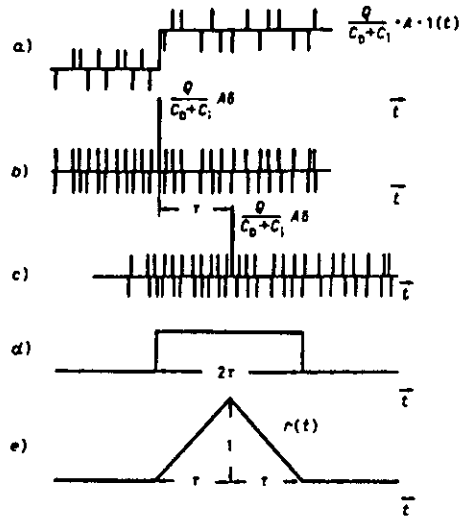


Fig. 3.14II. - Time-variant realization of the processor with bipolar weight distribution. Wave forms in the various points: a) step at the preamplifier output with  $\delta$ -noise, b)  $\delta$ -impulse at the differentiator output with doublet noise, c) same as b) after a delay  $\tau$ , d) integrator gating command, e) triangular multiplying wave form.

a step with an amplitude

$$(3.29) \quad \int_0^{2\tau} \delta'(t - t_0) r(t) dt = r'(t_0).$$

If now  $t_0$  is varied between 0 and  $2\tau$ ,  $r'(t_0)$  describes the bipolar function of fig. 3.13b).

It is, therefore, possible to conclude that the time-variant processor of fig. 3.14 I) has the same signal-to-noise ratio as the time-invariant one with bipolar  $\delta$ -response of fig. 3.13. Such a conclusion relies upon the fact that both produce the same output amplitude when the charge  $Q$  is delivered by the detector and they are characterized by the same weighting function, coinciding with the  $\delta$ -response in the time-invariant case or  $r'(t)$  in the time-variant one. Note that the output responses for the signal are different for the equivalent time-variant and time-invariant processors, while the resolving times are equal.

3'6. Optimum filter for an arbitrary pulse shape in white noise for a fixed estimation time of the baseline. - The problem of synthesizing the optimum filter for an arbitrary input pulse shape in white noise and fixed baseline estima-

tion time can be introduced. The case considered here refers, as in Baldinger and Franzen's approach, to a signal consisting of two narrow rectangles of amplitudes  $\mathcal{A}_1$  and  $\alpha\mathcal{A}_1$  in band-limited white noise (fig. 3.15a)).

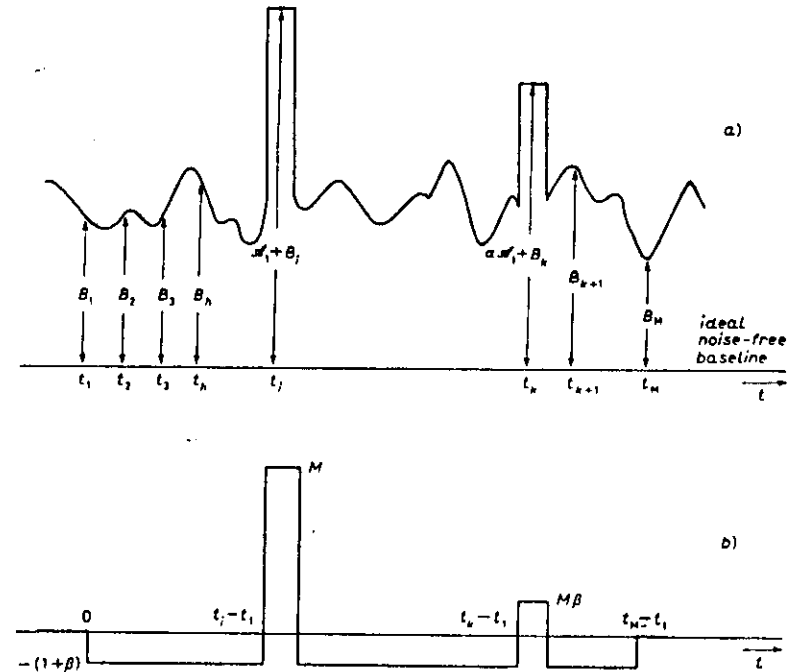


Fig. 3.15. - a) Signal consisting of two pulses in white noise, with fixed baseline estimation time. b) Weighting function of the optimum processor.

The baseline is sampled at  $M$  equally spaced instants  $t_1, t_2, t_3, \dots, t_M$ . As the noise is supposed to be band-limited white, the noise samples taken at these instants are uncorrelated. Let  $B_1, B_2, B_3, \dots, B_M$  be the baseline samples referred to an ideal, noise-free baseline. Let  $t_1, t_k$  be the instants at which the two pulses occur and suppose these instants different from those at which the baseline is sampled. It will be assumed that  $M$  is much larger than the number of wave form samplings, two in this case.

The following weighted sum can be constructed, by referring the  $\mathcal{A}_1 + B_i$ ,  $\alpha\mathcal{A}_1 + B_k$  amplitudes to all the possible baseline evaluations:

$$(3.30) \quad \sum_{i=1}^M [(\mathcal{A}_1 + B_i) - B_i] + \beta \sum_{k=1}^M [(\alpha\mathcal{A}_1 + B_k) - B_k].$$

Equation (3.40) gives for the square amplitude of the signal

$$(3.31) \quad S^2 = M^2(1 + \alpha\beta)^2 v_1^2.$$

By labelling  $v_{n,i}$  the noise amplitude at the instant  $t_i$ , from (3.30) the following expression can be written for the total noise  $v_{N,T}$ :

$$(3.32) \quad v_{N,T} = Mv_1 + \sum_{j=1}^N v_j + \beta Mv_1 + \beta \sum_{j=1}^N v_j.$$

The root mean-square noise can be evaluated from (3.32) as

$$(3.33) \quad \langle v_{N,T}^2 \rangle \approx (\beta^2 + 1)(M^2 + M) \langle v_N^2 \rangle,$$

where  $\langle v_N^2 \rangle$  is the ensemble average of the noise at the sampling instants.

The square of the signal-to-noise ratio, from (3.31) and (3.33), can be written as

$$(3.34) \quad \rho^2 = \frac{M^2(1 + \alpha\beta)^2}{\langle v_N^2 \rangle [(1 + \beta^2)(1 + 1/M)]}$$

and it achieves a maximum for

$$(3.35) \quad \beta = \alpha.$$

The weighting function of the optimum processor is shown in fig. 3.1(b).

The above outlined procedure can be extended to any input wave form.

## REFERENCES

- [1] H. ROHR and W. DÄHRKE: *Proc. IRE*, **44**, 811 (1956).
- [2] 50 I.R.E., 20 St. Committee, H. A. HACS, Chairman: *Proc. IRE*, **48**, 60 (1960).
- [3] I.R.E. Subcommittee 7.9 on Noise, H. A. HACS, Chairman: *Proc. IRE*, **48**, 69 (1960).
- [4] A. VAN DER ZIEL: *Noise Sources, Characterization, Measurements* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1970).
- [5] V. RADEKA: *Nucl. Instrum. Methods Phys. Res.*, **226**, 209 (1984).
- [6] E. GATTI and V. SVILTO: *Energ. Nucl. (Milan)*, **8**, 505 (1961).
- [7] E. BALDINGER and W. FRANZEN: *Adv. Electron. Electron Phys.*, **8**, 225 (1966).
- [8] V. RADEKA and N. KARLOVAC: *Nucl. Instrum. Methods*, **52**, 86 (1967).
- [9] P. T. ARECCHI, G. CAVALLERI, E. GATTI and V. SVILTO: *Energ. Nucl. (Milan)*, **7**, 691 (1960).
- [10] G. L. TURIN: *IRE Trans. Inf. Theory*, **IT-6**, 311 (1960).
- [11] A. PAPOLIS: *Probability, Random Variables and Stochastic Processes* (McGraw-Hill, New York, N. Y., 1965).
- [12] M. G. ACCIARI, G. BRUSOLATI, S. COVA, J. DE LOTTO and E. GATTI: *Nucl. Instrum. Methods*, **61**, 81 (1968).

## 4. Calculation of the signal-to-noise ratio for some practical analog processors.

4.1. *Introductory remarks.* — The analysis of the previous section is of fundamental nature, its purpose being that of clarifying the intrinsic limitations that occur in the measurement of the charge delivered by a capacitive source. The optimum processing realized by the indefinite-cusp response, the loss in signal-to-noise ratio introduced by the truncated cusp and the constraint set by finite empty intervals before and after the event to be processed have been reviewed.

In practical applications of nuclear radiation detectors, however, networks that respond to a  $\delta$ -impulse shaped detector current with signals different from the cusp or the triangle are sometimes employed.

Additional constraints may in fact be present, requiring, for instance, a rounded top in the  $\delta$ -response of the whole system to make the measurement of the peak amplitude more precise or a flat-topped trapezoidal response to reduce the ballistic errors [1, 2]. Cancellation of long tails following the signal may require bipolar shaping [3].

For these reasons it is useful to introduce analytical methods to evaluate the signal-to-noise ratio. The problem to be solved will be presented in the following way. In a given charge-measuring system, consisting of detector, pre-amplifier and of the network which determines the signal-to-noise ratio, the noise sources are specified through their power spectra, like in fig. 4.1. The signal-to-noise ratio has to be evaluated. The network determining the signal-to-noise ratio can be of either time-invariant or time-variant nature. In the former case the signal-to-noise ratio can be evaluated using a frequency domain approach or a time domain approach.

In the case of a network of time-variant nature, the time domain method has necessarily to be adopted.

In the nuclear-electronics literature the network which determines the signal-to-noise ratio is either referred to as a filter to emphasize its action in the frequency domain or as a shaper to emphasize its behaviour in the time domain.

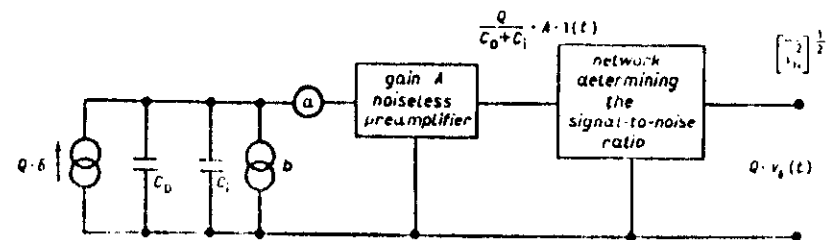


Fig. 4.1 — Evaluation of the signal-to-noise ratio in a charge-measuring system.

In the foregoing treatment the term shaper will be generally preferred, both because more importance will be attributed to the time domain analysis and because the term shaper seems to be more appropriate when referred to time-variant systems.

4'2. *Time-invariant shapers.* - When the network determining the signal-to-noise ratio in fig. 4.1 is of time-invariant nature, its behaviour is described by the transfer function  $T(s)$ . The transfer function  $T(s)$  is the Laplace transform of the response of the shaper to an input signal  $v_i$  assumed equal to a  $\delta$ -impulse. By definition of time-invariant shaper, such a response at a given instant  $t$  is a function of the difference  $t - t_i$  alone,  $t_i$  being the instant at which the input  $\delta$ -impulse is applied and is independent of the absolute time position of  $t_i$ .

4'2.1. *Frequency domain noise calculation.* Returning to the processor of fig. 4.1 and calling  $v_o(t)$  the response to a detector current signal represented by a  $\delta$ -impulse of unit coefficient,  $v_o(t)$  can be written as

$$(4.1) \quad v_o(t) = \mathcal{L}^{-1} \left[ \frac{1}{s(C_D + C_i)} A T(s) \right],$$

where  $\mathcal{L}^{-1}$  denotes the inverse Laplace-transform operator. As  $v_o(t)$  is zero for  $t < 0$  and must return to zero at either a finite time or as  $t \rightarrow \infty$ , it must have at least one maximum or minimum depending on its polarity. Let  $\text{MAX}[v_o(t)]$  be the largest absolute value of these maxima and minima. It will be assumed that the signal measurement is performed on this value. The signal amplitude corresponding to the release of a charge  $Q$  in the detector will be, therefore,

$$Q \cdot \text{MAX}[v_o(t)].$$

The output root-mean-square noise  $[\overline{v_o^2}]^{1/2}$  can be evaluated according to the frequency domain approach as [4-8]

$$(4.2) \quad [\overline{v_o^2}]^{1/2} = \left[ \int_{-\infty}^{\infty} \mathcal{N}(\omega) |T(j\omega)|^2 d\omega \right]^{1/2} = \left[ \frac{1}{2\pi} A^2 \int_{-\infty}^{\infty} \left( a + \frac{b}{\omega^2(C_D + C_i)} \right) |T(j\omega)|^2 d\omega \right]^{1/2}.$$

The signal-to-noise ratio  $\rho$  can, therefore, be expressed as

$$(4.3) \quad \rho = \frac{Q A}{C_D + C_i} \frac{\text{MAX} \mathcal{L}^{-1} \left[ \frac{T(s)}{s} \right]}{\left\{ \frac{1}{2\pi} A^2 \int_{-\infty}^{\infty} \left( a + \frac{b}{\omega^2(C_D + C_i)} \right) |T(j\omega)|^2 d\omega \right\}^{1/2}}.$$

The signal-to-noise ratio, as expressed by (4.3), does not provide a description of the intrinsic noise behaviour of the processor, for it depends on a parameter,  $Q$ , related to the incoming radiation.

The noise performances of a processor intended for capacitive sources are usually expressed in terms of equivalent noise charge referred to the input, ENC [7]. The equivalent noise charge is defined, according to (4.3), as that value of  $Q$  which makes  $\rho = 1$ . This means that the equivalent noise charge is the value of charge which injected across the detector capacitance by a  $\delta$ -like signal produces at the output of the analog processor a signal whose amplitude equals the output root-mean-square noise. By applying the definition, eq. (4.3) yields

$$(4.4) \quad \text{ENC} = \frac{\left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( a(C_D + C_i)^2 + \frac{b}{\omega^2} \right) |T(j\omega)|^2 d\omega \right\}^{1/2}}{\text{MAX} \mathcal{L}^{-1} \left[ \frac{T(s)}{s} \right]} \text{ r.m.s. coulomb}.$$

As stated by (4.3), the evaluation of  $\rho$  requires the evaluation of the peak amplitude of the  $\delta$ -response of the whole system and that of the noise integral at the denominator.

A few examples will illustrate the method.

4'2.1.1. *Triangular shaping.* In the case shown in fig. 4.2,  $T(s)$  has the following expression:

$$T(s) = \frac{1}{s\tau_M} (1 - \exp[-s\tau_M])^2.$$

The numerator of (4.3) is equal to  $Q A / (C_D + C_i)$ .

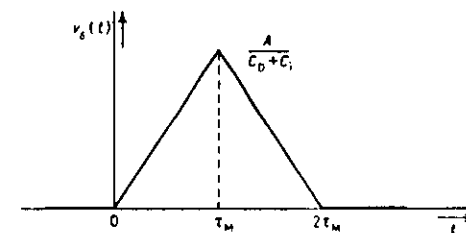


Fig. 4.2. - Triangular shaping with peaking time  $\tau_M$ .

The integral at the denominator is given by

$$(4.5) \quad 16 \int_{-\infty}^{\infty} \left[ a + \frac{b}{\omega^2(C_D + C_i)^2} \right] \frac{\sin^4(\omega \tau_M/2)}{\omega^2 \tau_M^2} d\omega = \\ = 16 \frac{a}{\tau_M} \int_0^{\infty} \frac{\sin^4 x}{x^2} dx + \frac{4b}{(C_D + C_i)^2 \tau_M} \int_0^{\infty} \frac{\sin^4 x}{x^4} dx \quad \left( x = \frac{\omega \tau_M}{2} \right).$$

If the values of the two integrals (see appendix) are introduced into (4.5), the following expression is found for the denominator of (4.3):

$$(4.6) \quad 4 \left[ 2 \frac{a}{\tau_M} + \frac{2}{3} \frac{b}{(C_D + C_i)^2 \tau_M} \right]^{\frac{1}{2}}.$$

The signal-to-noise ratio can now be evaluated from (4.3), thus obtaining

$$(4.7) \quad \rho = \frac{Q}{[2(a/\tau_M)(C_D + C_i)^2 + (2/3)b/\tau_M]^{\frac{1}{2}}}$$

and the equivalent noise charge is

$$(4.8) \quad \text{ENC} = \left[ 2 \frac{a}{\tau_M} (C_D + C_i)^2 + \frac{2}{3} b/\tau_M \right]^{\frac{1}{2}} \text{ r.m.s. coulomb.}$$

4.2.1.2. *RC-CR* shaping. A transfer function of the network determining the signal-to-noise ratio of the type

$$T(s) = \frac{s\tau_M}{(1 + s\tau_M)^2}$$

leads to the following  $v_d(t)$ :

$$v_d(t) = \frac{A}{C_D + C_i} + C_i \frac{t}{\tau_M} \exp[-t/\tau_M].$$

The time dependence of  $v_d(t)$  is shown in fig. 4.3. The peak of  $v_d(t)$  occurs at  $t = \tau_M$  and the peak value is  $(A/(C_D + C_i)) \cdot 1/e$ .

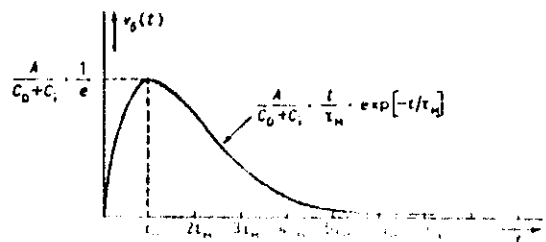


Fig. 4.3. Response of the RC-CR network to a unit current.

The integral at the denominator of (4.3) can be written as

$$2a \int_0^{\infty} \frac{\omega^2 \tau_M^2}{(1 + \omega^2 \tau_M^2)^2} d\omega + \frac{2b\tau_M^2}{(C_D + C_i)^2} \int_0^{\infty} \frac{d\omega}{(1 + \omega^2 \tau_M^2)^2} = \\ = \frac{2a}{\tau_M} \int_0^{\infty} \frac{x^2 dx}{(1 + x^2)^2} + \frac{2b}{(C_D + C_i)^2 \tau_M} \int_0^{\infty} \frac{dx}{(1 + x^2)^2} \quad (x = \omega \tau_M).$$

The value of both integrals is  $\pi/4$  (see appendix).

The signal-to-noise ratio for the *RC-CR* shaping is, therefore,

$$(4.9) \quad \rho = \frac{(2/e)Q}{[(C_D + C_i)^2(a/\tau_M) + b/\tau_M]^{\frac{1}{2}}}$$

and the equivalent noise charge is

$$(4.10) \quad \text{ENC} = \frac{e}{2} \left[ (C_D + C_i)^2 \frac{a}{\tau_M} + b/\tau_M \right]^{\frac{1}{2}} \text{ r.m.s. coulomb.}$$

4.2.2. *Time domain noise calculation.* The time domain noise calculation will now be introduced.

The time domain approach is particularly straightforward in the case of a time-invariant processor of the type shown in fig. 4.1, and for the noise generators there considered. The time domain approach is based upon two theorems. One, known as Carson's theorem, states that an ergodic noise source with bilateral power spectrum  $\mathcal{N}(\omega)$  can be considered as due to the superposition in the time domain of randomly distributed events with Fourier transform  $\Phi(\omega)$  occurring at an average rate  $\lambda$  [9, 10]. The relationship between  $\mathcal{N}(\omega)$ ,  $\Phi(\omega)$  and  $\lambda$  is

$$(4.11) \quad \mathcal{N}(\omega) = \lambda |\Phi(\omega)|^2.$$

So, for instance, white noise with bilateral spectrum  $d$  can be thought of as the result of random occurrence of  $\delta$ -impulses of unit area with an average rate  $\lambda = d$ . If the noise has a zero mean value, the  $\delta$ -impulses will be randomly positive and negative.

In the circuit of fig. 4.1 both series and parallel noise wave forms can be considered, therefore, resulting from the random occurrence of  $\delta$ -impulses with average rates  $a$  and  $b$ , respectively.

Once the noise generators are represented in this way, the output noise can be evaluated by making use of Campbell's theorem [9, 11]. This theorem states that a noise process resulting from the superposition of pulses of a fixed shape  $q(t)$ , randomly occurring in time with an average rate  $\lambda$ , has a root-

mean-square value  $(\overline{v_k^2})^{\frac{1}{2}}$  given by

$$(4.12) \quad (\overline{v_k^2})^{\frac{1}{2}} = \left[ \lambda \int_{-\infty}^{\infty} \varphi^2(t) dt \right]^{\frac{1}{2}}.$$

The evaluation of the signal-to-noise ratio on the basis of the time domain approach proceeds in the following way [12-14].

The signal amplitude is calculated, like in the previous case, as

$$Q \cdot \text{MAX}[v_s(t)] = \frac{Q}{C_D + C_i} A \cdot \text{MAX} \mathcal{L}^{-1} \left[ \frac{T(s)}{s} \right].$$

For the noise evaluation, it can be observed that a  $\delta$  in the parallel generator induces at the output the response  $v_\delta(t)$ . According to Campbell's theorem, the root-mean-square output contribution due to the parallel noise source is

$$(4.13) \quad \left[ b \int_{-\infty}^{\infty} v_\delta^2(t) dt \right]^{\frac{1}{2}}.$$

The contribution due to the series noise has to be calculated as follows. A  $\delta$ -impulse in the series generator induces at the output a signal whose Laplace transform is

$$(4.14) \quad A T(s) = s(C_D + C_i) \mathcal{L}[v_\delta(t)].$$

Equation (4.14) states that a  $\delta$ -impulse in the series generator excites at the output the response

$$(C_D + C_i) v_\delta'(t),$$

proportional to the time derivative of  $v_\delta(t)$ .

Utilizing Campbell's theorem, it can be concluded that the series generator contributes to the output root-mean-square noise with the term

$$(4.15) \quad \left[ a(C_D + C_i)^2 \int_{-\infty}^{\infty} [v_\delta'(t)]^2 dt \right]^{\frac{1}{2}}.$$

Both expressions (4.13) and (4.15) can be written in a more useful fashion remembering that

$$v_\delta(t) = \frac{A}{C_D + C_i} \mathcal{L}^{-1} \left[ \frac{T(s)}{s} \right].$$

By putting  $f(t) = \mathcal{L}^{-1}[T(s)/s]$  the previous relationship can be written as

$$v_\delta(t) = \frac{A}{C_D + C_i} f(t).$$

Let now  $u = t/\tau_n$  be a dimensionless variable, which represents the time normalized to a reference interval  $\tau_n$ . For instance,  $\tau_n$  might be the peaking time of  $v_\delta(t)$ . Observing that  $dt = \tau_n du$ ,  $dv_\delta/dt = (dv_\delta/du) \cdot 1/\tau_n$  and that  $v_\delta(u) = (A/(C_D + C_i))f(u)$ , the total root-mean-square noise resulting from (4.13) and (4.15) can be written as

$$(4.16) \quad (\overline{v_k^2})^{\frac{1}{2}} = A \left[ \frac{a}{\tau_n} \int_{-\infty}^{\infty} [f'(u)]^2 du + \frac{b\tau_n}{(C_D + C_i)^2} \int_{-\infty}^{\infty} [f(u)]^2 du \right]^{\frac{1}{2}}.$$

The two integrals in (4.16) are two numerical shape coefficients that can be easily calculated from the knowledge of  $v_\delta(t)$ . To clarify the time domain method, some examples will be discussed.

4.2.2.1. Triangular shaping. From fig. 4.2 the following expression of  $v_\delta(t)$  can be obtained, with  $\tau_n = \tau_M$ :  $v_\delta(u) = (A/(C_D + C_i))[1(u) \cdot u - 2(u-1) \cdot 1(u-1) + (u-2) \cdot 1(u-2)]$ .

The integrals in eq. (4.16) have the following values:

$$\int_{-\infty}^{\infty} f^2(u) du = \frac{2}{3}, \quad \int_{-\infty}^{\infty} [f'(u)]^2 du = 2.$$

The output root-mean-square noise is, therefore,

$$(4.17) \quad (\overline{v_k^2})^{\frac{1}{2}} = A \left[ 2 \frac{a}{\tau_n} + \frac{2}{3} \tau_n \frac{1}{(C_D + C_i)^2} \right]^{\frac{1}{2}}.$$

The signal-to-noise ratio is

$$(4.18) \quad Q = \frac{Q}{[2(a/\tau_n)(C_D + C_i)^2 + (2/3)\tau_n]^{\frac{1}{2}}}$$

and the equivalent noise charge is

$$(4.19) \quad \text{ENC} = \left[ 2 \frac{a}{\tau_n} (C_D + C_i)^2 + \frac{2}{3} \tau_n \right]^{\frac{1}{2}} \text{ r.m.s. coulomb.}$$

as already found, (4.8).

4'2.2.2. *RC CR shaping*. According to fig. 4.3, putting  $\tau_M = \tau_M$ ,  $v(u)$  becomes  $(4(C_D + C_I))u \exp[-u] \cdot \exp(-u)$ . The two integrals in eq. (4.16) become

$$\int_{-\infty}^{\infty} f(u) du = \int_0^{\infty} u^2 \exp[-2u] du = \frac{1}{4},$$

$$\int_{-\infty}^{\infty} [f'(u)]^2 du = \int_0^{\infty} (1-u)^2 \exp[-2u] du = \frac{1}{4}.$$

The root mean-square noise becomes, therefore,

$$(4.20) \quad [\bar{v}_N^2]^{\frac{1}{2}} = \frac{A}{2} \left[ \frac{a}{\tau_M} + \frac{b}{(C_D + C_I)^2 \tau_M} \right]^{\frac{1}{2}}.$$

The signal-to-noise ratio and the equivalent noise charge are given, respectively, by

$$(4.21) \quad \rho = \frac{(2/e)Q}{[(C_D + C_I)^2 (a/\tau_M) + b\tau_M]^{\frac{1}{2}}},$$

$$(4.22) \quad \text{ENC} = \frac{e}{Q} \left[ (C_D + C_I)^2 \frac{a}{\tau_M} + b\tau_M \right]^{\frac{1}{2}} \text{ r.m.s. coulomb.}$$

as already found, (4.10).

4'2.2.3. *Piecewise parabolic shaping*. A piecewise parabolic  $\delta$ -response can be obtained by multiplying the normalized function shown in fig. 4.4 by  $4/(C_D + C_I)$ . The way in which the piecewise parabolic shape is constructed from its second derivative is shown in the same figure. The signal amplitude is  $(Q/(C_D + C_I))A$ . The root mean-square output noise is, according to eq. (4.16),

$$[\bar{v}_N^2]^{\frac{1}{2}} = A \left[ a \frac{1}{\tau_M} \int_{-\infty}^{\infty} [f'(u)]^2 du + \frac{b\tau_M}{(C_D + C_I)^2} \int_{-\infty}^{\infty} f''(u) du \right]^{\frac{1}{2}}.$$

The values of the two integrals are 8/3 and 14/15, respectively. The signal-to-noise ratio is

$$(4.23) \quad \rho = \frac{Q}{[a(C_D + C_I)^2 (8/3) (1/\tau_M) + b(14/15) \tau_M]^{\frac{1}{2}}}$$

and the equivalent noise charge is

$$(4.24) \quad \text{ENC} = \left[ a(C_D + C_I)^2 \frac{8}{3} \frac{1}{\tau_M} + b \frac{14}{15} \tau_M \right]^{\frac{1}{2}} \text{ r.m.s. coulomb.}$$

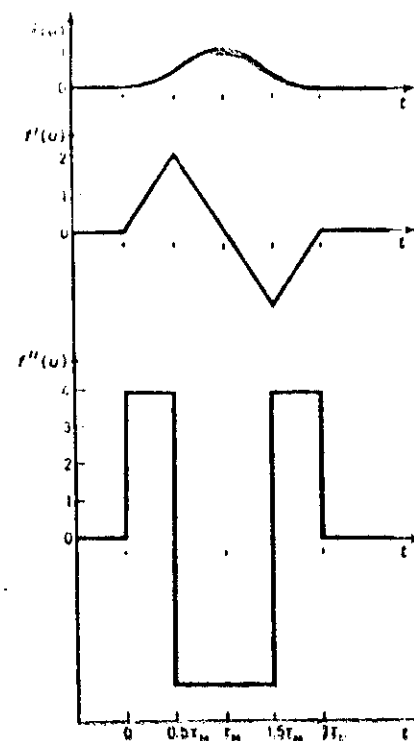


Fig. 4.4. - Normalized piecewise parabolic pulse and its first and second derivative,  $u = t/\tau_M$ .

4'2.2.4. *Trapezoidal shaping*. The trapezoidal response to a  $\delta$ -impulse detector current is shown in fig. 4.5. The contribution to the output noise due to the series noise source is equal to that of the triangular shaping with peaking

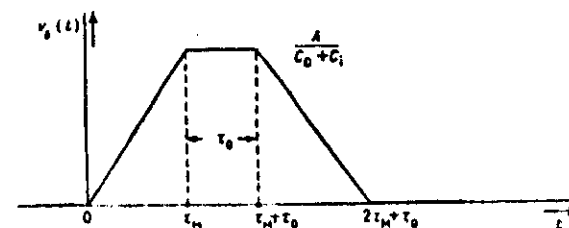


Fig. 4.5. - Trapezoidal shaping.

time  $\tau_M$ . The contribution due to the parallel noise becomes instead

$$(4.25) \quad [\overline{v_k^2}]^{\frac{1}{2}} = \frac{A}{C_D + C_i} \left[ \left( \frac{2}{3} \tau_M + \tau_0 \right) b \right]^{\frac{1}{2}}.$$

The signal-to-noise ratio  $\rho$  is given by

$$(4.26) \quad \rho = \frac{Q}{[a(C_D + C_i)^2 (2/\tau_M) + b((2/3) \tau_M + \tau_0)]^{\frac{1}{2}}}$$

and the equivalent noise charge is

$$(4.27) \quad ENC = \left[ a(C_D + C_i)^2 \frac{2}{\tau_M} + b \left( \frac{2}{3} \tau_M + \tau_0 \right) \right]^{\frac{1}{2}} \text{ r.m.s. coulomb.}$$

It can be taken advantage of the example of the trapezoidal shaping to make a statement of general validity. A processing system will be considered, which

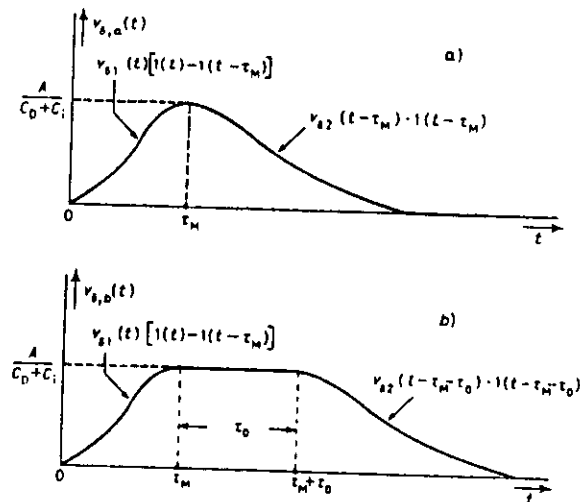


Fig. 4.6. - Responses  $v_{d,a}(t)$  and  $v_{d,b}(t)$  to the  $\delta$ -impulse detector current. The two responses have equal leading and trailing edges. The response of  $b$ ) differs from that of  $a$ ) for a flat-topped region of duration  $\tau_0$ .

is described by the  $v_{d,a}(t)$  of fig. 4.6a) along with the system described by the  $v_{d,b}(t)$  of fig. 4.6b).

As shown in fig. 4.6,  $v_{d,a}(t)$  and  $v_{d,b}(t)$  have the same leading and trailing edges; the peak region of  $v_{d,a}$  is replaced by a flat-topped region of duration

$\tau_0$  in  $v_{d,b}(t)$ . As the derivatives of  $v_{d,a}$  and  $v_{d,b}$  are equal, the contribution from the white series noise is the same in both cases. The presence of the flat-topped region in  $v_{d,b}$  enhances the effect of the parallel noise to the extent shown by eq. (4.25) which compares the equivalent noise charge in the two cases:

$$(4.28) \quad ENC_{v_{d,b}}^2 = ENC_{v_{d,a}}^2 + b\tau_0.$$

4'2.2.5. Bipolar shaping. Let  $v_d(t)$  be the response of the analog processor to a  $\delta$ -impulse detector current; it will be assumed that  $v_d(t)$  has a finite width

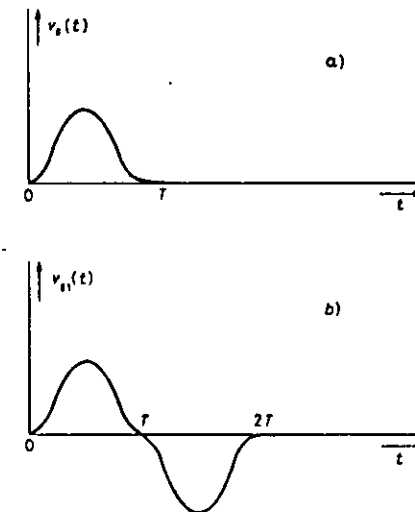


Fig. 4.7. - Unipolar response to the  $\delta$ -impulse detector current (a) and bipolar response obtained by delaying and reversing in polarity the previous one (b).

$T$  and that it is strictly unipolar, as shown in fig. 4.7a). Let  $v_{d1}(t)$  be the  $\delta$ -response obtained by applying to  $v_d(t)$  the operator  $1 - \exp[-sT]$  which adds to  $v_d(t)$  a signal equal in shape to  $v_d(t)$  but delayed of a time  $T$  and reversed in polarity. The bipolar signal  $v_{d1}(t)$  is shown in fig. 4.7b). Bipolar shaping is sometimes employed in processors intended for high counting rates owing to the following property. If the signal of fig. 4.7b) is passed through a transfer function which has a zero at  $s = 0$ , typical of an a.c. coupled processor, the tails of opposite sign determined by the positive and negative lobes cancel each other for  $t > 2T$  fairly well. As a consequence, the baseline shifts at high counting rates, that are a serious limitation in a.c. coupled processors working on unipolar signals, are almost eliminated with bipolar shaping.

As far as the signal-to-noise ratio goes, however, bipolar shaping is worse

than the unipolar one. Time domain noise evaluation of cases a) and b) of fig. 4.7 leads to the conclusion that both series and parallel contributions to the output root-mean-square noise are increased of a factor  $\sqrt{2}$  passing from unipolar to bipolar shaping.

4'3. Contribution of the series  $1/|\omega|$  noise to the output r.m.s. noise in some particular cases. — Let  $c/|\omega|$  be the bilateral power spectral density of the series noise whose contribution to the total root-mean-square output noise has to be evaluated. As already pointed out, the calculation has to be made in the frequency domain and only some cases of particular interest will be considered (13-14).

4'3.1. Triangular shaping. With reference to the shape shown in fig. 4.2 and to the procedure followed to determine the effect of series and parallel noise generators, the contribution due to the  $c/|\omega|$  term is given by

$$\begin{aligned} \bar{v}_k^2 &= \frac{16A^2}{2\pi} \int_{-\infty}^{\infty} \frac{c}{|\omega|} \frac{\sin^4(\omega\tau_M/2)}{\omega^2 \tau_M^2} d\omega = \frac{16A^2}{\pi} c \int_0^{\infty} \frac{\sin^4(\omega\tau_M/2)}{\omega^2 \tau_M^2} d(\omega\tau_M) = \\ &= \frac{4A^2}{\pi} c \int_0^{\infty} \frac{\sin^4 x}{x^2} dx \quad \left(x = \frac{\omega\tau_M}{2}\right). \end{aligned}$$

Introducing for the integral the value given in appendix, the following expression is obtained for the contribution of the  $c/|\omega|$  term to the output r.m.s. noise:

$$(4.29) \quad \bar{v}_k^2 = A^2 c \frac{\ln 2}{\pi}.$$

The contribution of the  $1/|\omega|$  series term to the ENC is accordingly

$$(4.30) \quad \text{ENC} = 2 \left( \frac{\ln 2}{\pi} c \right)^{1/2} (C_D + C_I) \approx 0.96 c^{1/2} (C_D + C_I).$$

4'3.2. RC-CR shaping. With reference to fig. 4.3 and to the procedure followed to determine the signal-to-noise ratio for the RC-CR shaping, the following expression of the contribution given by the  $c/|\omega|$  term to the output r.m.s. noise is obtained:

$$\bar{v}_k^2 = \frac{A^2}{2\pi} \int_{-\infty}^{\infty} \frac{c}{|\omega|} \frac{\omega^2 \tau_M^2}{1 + \omega^2 \tau_M^2} d\omega = \frac{cA^2}{\pi} \int_0^{\infty} \frac{x}{1 + x^2} dx \quad (x = \omega\tau_M).$$

Introducing in the integral the value given in appendix, the following expres-

sion is obtained for  $\bar{v}_k^2$ :

$$(4.31) \quad \bar{v}_k^2 = \frac{cA^2}{2\pi}.$$

The contribution brought by the  $1/|\omega|$  series noise to the total ENC is

$$(4.32) \quad \text{ENC} = c \left( \frac{c}{2\pi} \right)^{1/2} (C_D + C_I) \approx 1.08 c^{3/2} (C_D + C_I).$$

4'3.3. RC-(CR)<sup>2</sup> shaping. Although the RC-(CR)<sup>2</sup> shaper, consisting of the cascade connection of one approximate integration and two approximate differentiations, is of little interest in modern pulse processing, it is useful to evaluate its effect on the  $1/|\omega|$  noise. If in the processor of fig. 4.1 the transfer function  $T(s) = (s\tau)/(1 + s\tau)^2$  is introduced, the following  $v_k(t)$  is obtained:

$$v_k(t) = \frac{A}{C_D + C_I} \frac{t}{\tau} \exp[-t/\tau] \left[ 1 - \frac{t}{2\tau} \right],$$

which is shown in fig. 4.8.

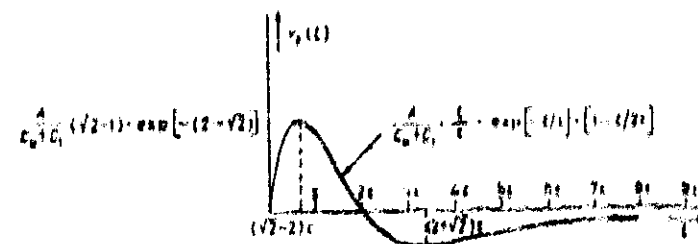


Fig. 4.8. — Bipolar shape obtained by passing the step at the preamplifier output through the cascade connection of one approximate integrator and two approximate differentiators.

The noise is evaluated from the integral

$$\bar{v}_k^2 = \frac{c}{\pi} A^2 \int_{-\infty}^{\infty} \frac{\omega^2 \tau^2}{(1 + \omega^2 \tau^2)^2} d\omega = \frac{c}{\pi} A^2 \int_0^{\infty} \frac{x^2}{(1 + x^2)^2} dx \quad (x = \omega\tau).$$

The value of the integral is  $\frac{1}{2}$ . The root-mean-square output noise due to the  $1/|\omega|$  source is, therefore,

$$(4.33) \quad (\bar{v}_k)^{1/2} = \frac{1}{2} \left( \frac{c}{\pi} \right)^{1/2} A.$$

The contribution to the total ENC is accordingly

$$(4.34) \quad \text{ENC} = \frac{1}{2} \left( \frac{e}{\pi} \right)^2 \frac{\exp[2 - \sqrt{2}]}{\sqrt{2} - 1} (C_D + C_i) \approx 1.23 e^2 (C_D + C_i).$$

The evaluations made on the ENC contribution coming from the series  $1/|\omega|$  noise show that it is independent of the shaper characteristic time  $\tau_R$  and that it is very little dependent on the wave form determined by the shaper.

**4.4. Time-variant shapers.** – Time-variant shapers were proven to be of very practical usefulness in high-energy experiments and, therefore, the more significant examples of this class of circuits will be discussed [19-27]. From a strictly conceptual standpoint, time-variant shapers do not allow better signal-to-noise ratio or counting rate performances or lower pile-up probability than the best time-invariant shapers. If the comparison between time-variant and time-invariant shapers is, however, transferred to the ground of actual behaviour, of requirements set to the instrumentation which follows and of the additional facilities they offer, the former ones have to be considered with favour in the context of elementary-particle physics. In this domain of applications, especially when fixed-target experiments are considered, the analog processors are subject to extremely high rates of events, while for very few of them the requirements of the selection logic are met. In a time-invariant shaper, all the incoming pulses are processed in the same way, while a time-variant shaper for high-energy experiment is usually a gated network which completes the processing only on those events for which the selection requirements are met. Time-variant shapers offer as advantages a fast, tail-free recovery to the baseline at the end of the shaped pulses, a built-in gating function so that at their output the counting rate is small and, in some configurations, a built-in analog store facility is easily implemented.

As pointed out by the authors [27], time-variant shapers belong to two different categories according to whether shaping occurs entirely after gating or whether a preshaping is performed before and shaping is completed after the gating. The block diagram of a time-variant shaper of the former type is shown in fig. 4.9.

The most common time-variant filter of the latter type is shown in fig. 4.10, where the second part of the shaping is done by a gated integrator.

The attractive aspect of this time-variant shaper lies in the fact that most ADCs for high-energy experiments are of the charge-sensitive type, that is, they employ a gated integrator as input analog memory. It becomes, therefore, natural to commit to this integrator the twofold function of time-variant shaper and analog store of the amplitude to be measured.

The noise analysis of time-variant shapers has to be carried out in the time domain. The approach, however, is more involved than the one adopted for

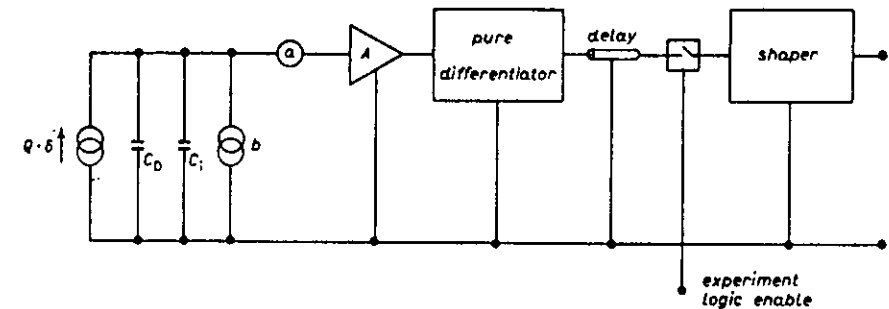


Fig. 4.9. – Time-variant shaper with the shaping entirely performed after the gate.

the time-invariant shapers. In the case of a time-invariant shaper the  $\delta$ -impulses that describe the series and parallel noise generator induce at the output signals whose form is independent of the instant at which the exciting  $\delta$  occurs. The knowledge of the response of the processor to a  $\delta$ -impulse detector signal allows, as was shown, a straightforward evaluation of the root-mean-square output noise through Campbell's theorem. In a time-variant shaper the switch opening is synchronized with the arrival of the signal to be analysed and the contribution of the  $\delta$ -impulse describing the series and parallel noise generator to the root-mean-square noise at the measuring instant depends on their time relationship with the signal.

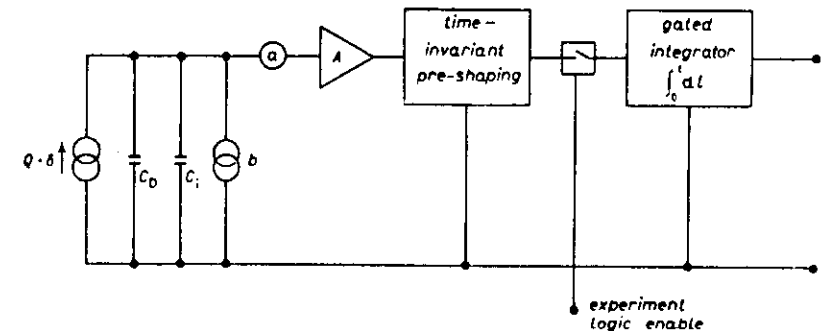


Fig. 4.10. – Time-variant shaper employing a gated integrator.

The knowledge of the processor response to the  $\delta$ -impulse-like detector current is not sufficient to evaluate the noise. This is especially true for the shapers of the type shown in fig. 4.10. The following description of the behaviour of the two types of time-invariant filters does not claim to be a comprehensive treatment of the problem, but rather an intuitive approach to their

ness. As shown in fig. 4.9, the preamplifier is followed by a differentiator which reproduces the original shape of the detector current pulses. The  $\delta$ 's describing the parallel white noise source appear again as  $\delta$ -impulses at the differentiator output, while the  $\delta$ -impulses describing the series white noise are transformed by the differentiator into doublets. An over-simplified explanation of the deterioration in signal-to-noise ratio, which may occur in this kind of shaper if a proper time relationship between switch opening and signal arrival is not respected, lies on the split-doublet effect [30].

The opening of the switch may cause a doublet splitting.

The uncompensated part of the doublet will impair the signal-to-noise ratio unless the shaper which follows the switch responds to the split doublet (a  $\delta$ -like pulse of  $\infty$  area) with a signal which goes to zero at the measurement instant.

To avoid this effect, the time relationships shown in fig. 4.11 are employed.

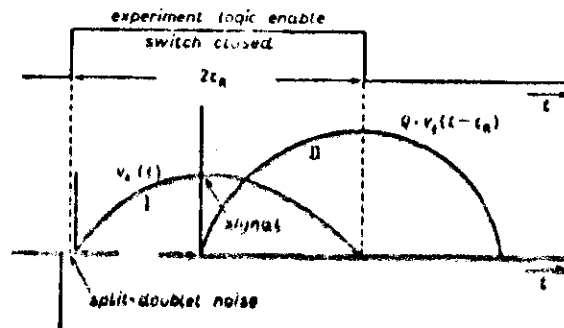


Fig. 4.11. - Timing sequence in a time-invariant shaper of the first type.

It is assumed that the shaper which follows the switch has a  $\delta$ -impulse response with a peak at  $t = \tau_R$  and a base width  $2\tau_R$  and that this response is symmetric.

Whenever the experiment logic requirements are met, a switch opening command is generated with a time duration  $2\tau_R$ . A doublet which may be split by the gate opening excites in the shaper a response which reaches zero at  $t = 2\tau_R$ . The signal to be analysed is delayed of  $\tau_R$ , so as to appear in the middle of the opening interval. The response it excites in the shaper reaches its maximum at the end of the opening interval, that is, when the response to the split doublet is zero. The amplitude measurement is performed at  $t = 2\tau_R$ .

If the effect of split doublet is avoided in the way shown, the root-mean-square noise at the measurement instant can be calculated with the same procedure adopted for the time-invariant shapers. This is a consequence of the fact that the noise is compensated after the switch

and, therefore, all the  $\delta$ -impulses and doublets that occur when the switch is open excite the same outputs, respectively  $v_s(t)$  or  $v'_s(t)$ . A time-variant filter of the type just described was employed in the coherent-production experiment of  $\pi$  mesons on the nucleus of Si [25]. The  $\delta$ -impulses transmitted by the gate excite a sinusoid in an active filter. The time relationships between switch opening and incoming detector signal are identical to those depicted in fig. 4.11. The amplitude measurement is performed on the peak of the first lobe of the sinusoid. Shortly after the peak of the first lobe, the sinusoid is nonlinearly damped. The system has a very good behaviour at high counting rates by virtue of a very neat, tail-free differentiation of the signals at the preamplifier output and of the symmetric  $\delta$ -response of the sinusoidal filter in the linear mode. The case of the sinusoidal shaper after the switch will be analysed here, just to show in a practical case an example of evaluation of the signal-to-noise ratio. As already pointed out, throughout the opening interval of the gate and as long as the split-doublet effect is neglected, the shaper is described, like a time-invariant one, by the response  $v_s(t)$  to a unit  $\delta$  detector current.

For the sinusoidal shaper

$$(4.35) \quad v_s(t) = \frac{A}{C_D + C_i} \sin \frac{\pi t}{2\tau_R} \cdot 1(t).$$

The response of the sinusoidal shaper to a unit  $\delta$  delivered by the parallel noise generator is  $v_s(t)$ , while the response of the sinusoidal shaper to a unit  $\delta$  delivered by the series voltage generator is, according to (4.14),

$$(4.36) \quad (C_D + C_i)v'_s(t) = \frac{A\pi}{2\tau_R} \cos \frac{\pi t}{2\tau_R} \cdot 1(t).$$

The root-mean-square output noise is, therefore,

$$\begin{aligned} (\overline{v_n^2})^{\frac{1}{2}} &= \left\{ a \frac{A^2 \pi^2}{4\tau_R^2} \int_0^{2\tau_R} \cos^2 \frac{\pi t}{2\tau_R} dt + b \frac{A^2}{(C_D + C_i)^2} \int_0^{2\tau_R} \sin^2 \frac{\pi t}{2\tau_R} dt \right\}^{\frac{1}{2}} = \\ &= A \left\{ a \frac{\pi^2}{4} \frac{1}{\tau_R} + \frac{b}{(C_D + C_i)^2} \tau_R \right\}^{\frac{1}{2}}. \end{aligned}$$

The signal amplitude is  $QA/(C_D + C_i)$ . The signal-to-noise ratio  $\rho$  and the equivalent noise charge ENC are, respectively,

$$(4.37) \quad \rho = \frac{Q}{[(\pi^2/4)(C_D + C_i)^2 (a/\tau_R) + b\tau_R]^{\frac{1}{2}}},$$

$$(4.38) \quad \text{ENC} = \left[ \frac{\pi^2}{4} (C_D + C_i)^2 \frac{a}{\tau_R} + b\tau_R \right]^{\frac{1}{2}} \text{ r.m.s. coulomb.}$$

A method will now be presented which allows a straightforward calculation of the signal-to-noise ratio for the time-variant shaper of fig. 4.10.

Let  $(A/(C_D + C_i))p(t) \cdot 1(t)$  be the signal at the output of the time-invariant preshaper induced by a unit  $\delta$  detector current. It will be assumed that  $p(t)$  has a finite width  $\tau_p$  (fig. 4.12a).

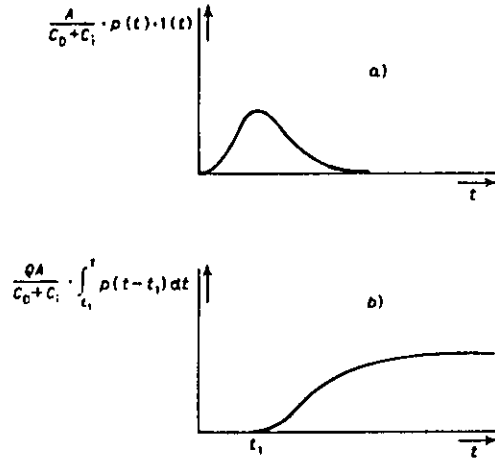


Fig. 4.12. - a) Signal at the preshaper output induced by a current  $\delta$  in the detector. b) Gated integration of the signal occurring at the preshaper output beginning at  $t = t_1$ .

A detector signal of charge  $Q$  occurring at  $t = t_1$  will produce at the preshaper output the signal

$$\frac{QA}{C_D + C_i} p(t - t_1) \cdot 1(t - t_1).$$

The switch opening is synchronized with the arrival of the detector signal and the switch remains open for a duration  $\tau_R > \tau_p$ .

The signal  $(QA/(C_D + C_i))p(t - t_1) \cdot 1(t - t_1)$  is integrated over the time interval  $(t_1, t_1 + \tau_R)$  and the amplitude measurement is performed at  $t = t_1 + \tau_R$ , after which the integrator is reset.

The noise evaluation of this time-variant filter is based upon a time domain approach which requires the knowledge of the so-called «noise weighting function» [13, 19, 20].

Contrary to what happens in a time-variant filter of the type of fig. 4.9, where no preshaping exists, in the time-variant filter of the type of fig. 4.10, the signals arriving to the gate have a finite width. This means that all the  $\delta$ -impulses delivered by the parallel and series noise generator in the time

interval  $(t_1 - \tau_p, t_1)$  as well as those occurring between  $t_1$  and  $t_1 + \tau_R$  contribute to the noise at the measuring instant.

The «noise weighting function» is defined as the contribution to the noise at the measuring time  $t = \tau_R$  given by a  $\delta$ -impulse delivered by the parallel noise generator at a time  $t_0$  and expressed as a function of  $t_0$  between  $t_1 - \tau_p$  and  $t_1 + \tau_R$ .

The way of evaluating the noise weighting function at different instants  $t_0$  is shown in fig. 4.13. The noise weighting function at a given  $t_0$  is represented by the area of the shaded region of the signal induced at the preshaper output by a  $\delta$ -impulse from the parallel noise source.

This accounts for the fact that a  $\delta$ -impulse from the parallel source, which is in the same circuit position as the signal source, induces at the prefilter output a signal whose shape is  $p(t)$ . The portion of this signal which enters the gate is integrated and stored in the integrator, therefore contributing to the noise at  $t = t_1 + \tau_R$ .

As clarified by fig. 4.13, the weighting function can be expressed in the following way:

$$W(t_0) = 0 \quad \text{for } t_0 < t_1 - \tau_p \text{ and } t_0 > t_1 + \tau_R,$$

$$W(t_0) = \frac{A}{C_D + C_i} \int_0^{t_1 + \tau_R - t_0} p(\tau_p - x) dx, \quad t_1 - \tau_p < t_0 < t_1,$$

$$W(t_0) = \frac{A}{C_D + C_i} \int_0^{\tau_p} p(\tau_p - x) dx, \quad t_1 < t_0 < t_1 + \tau_R - \tau_p,$$

$$W(t_0) = \frac{A}{C_D + C_i} \int_0^{t_1 + \tau_R - t_0} p(x) dx, \quad t_1 + \tau_R - \tau_p < t_0 < t_1 + \tau_R.$$

Once  $W(t_0)$  is determined, the noise contributions from the parallel source can be evaluated by adding quadratically all the elementary contributions appearing at the integrator output and caused by  $\delta$ -impulses occurring in the time intervals  $t_0, t_0 + dt_0$  as  $t_0$  varies. That is,

$$(4.39) \quad (\overline{v_N^2})_p^t = \frac{A}{C_D + C_i} \left[ b \int_{t_1 - \tau_p}^{t_1 + \tau_R} W^2(x) dx \right].$$

By sliding in a similar manner the derivative of  $(A/(C_D + C_i))p(t)$  through the integrator window, the noise weighting function for doublets of current injected across the  $C_D, C_i$  parallel combination can be determined. The resulting function, which is the derivative of  $W(t)$  of fig. 4.13, allows the calculation of the output noise arising from the white term in the series generator of fig. 4.1.

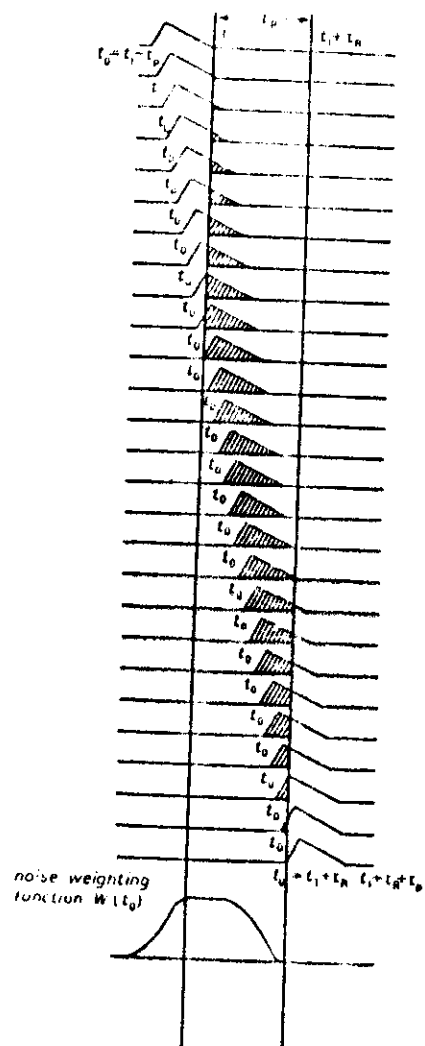


Fig. 4.13. — Graphical evaluation of the noise weighting function  $W(t)$ .

The root-mean-square output noise due to the series generator is

$$(4.10) \quad [\bar{v}_n]_0^1 = A \left[ a \int_{t_1-t_p}^{t_1+t_p} [W'(x)]^2 dx \right]^{1/2}.$$

The signal-to-noise ratio  $\rho$  and the equivalent noise charge ENC are, respectively, given by the following equations:

$$(4.41) \quad \rho = \frac{Q \int_0^1 p(x) dx}{\left\{ a(C_D + C_1) \int_{t_1-t_p}^{t_1+t_p} [W'(x)]^2 dx + b \int_{t_1-t_p}^{t_1+t_p} W^2(x) dx \right\}^{1/2}},$$

$$(4.42) \quad \text{ENC} = \frac{\left\{ a(C_D + C_1) \int_{t_1-t_p}^{t_1+t_p} [W'(x)]^2 dx + b \int_{t_1-t_p}^{t_1+t_p} W^2(x) dx \right\}^{1/2}}{\int_0^1 p(x) dx} \text{ r.m.s. coulomb}.$$

Both equations point out the fact that the knowledge of two different functions, namely  $p(x)$  for the signal and  $W(x)$  for the noise, is required in the analysis of a time-variant shaper of the type of fig. 4.10.

Two examples will clarify the behaviour of time-variant filters employing the gated integrator. A processor in which  $p(t)$  is a unit-amplitude rectangle of duration  $\tau$ , responds to a  $\delta$ -shaped signal with the function of fig. 4.14b). The noise weighting function, instead, has the symmetric shape of fig. 4.14c). The signal-to-noise ratio and the equivalent noise charge are identical to those

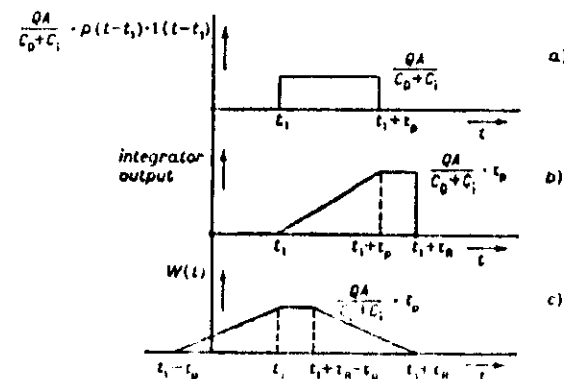


Fig. 4.14. — Wave forms at the input and at the output of the gated integrator (a), (b) and noise-weighting function (c).

calculated in the case of the time-invariant trapezoidal shaper, with  $\tau_M = \tau_r$  and  $\tau_0 = \tau_R - \tau_r$ .

If the duration of the signal at the preshaper output is made equal to  $\tau_R$ , then the flat top in the wave forms 4.14a) and b) disappears and the noise weighting function becomes a triangle of base width  $2\tau_R$ .

A processor for which  $p(t)$  is a triangle of base width  $\tau_r = \tau_R$  presents the wave forms and the noise weighting function of fig. 4.15.

The signal-to-noise ratio and the equivalent noise charge are identical to those evaluated for the piecewise parabolic shaper and their values are given by eqs. (4.23), (4.24), where  $\tau_M = \tau_R$ .

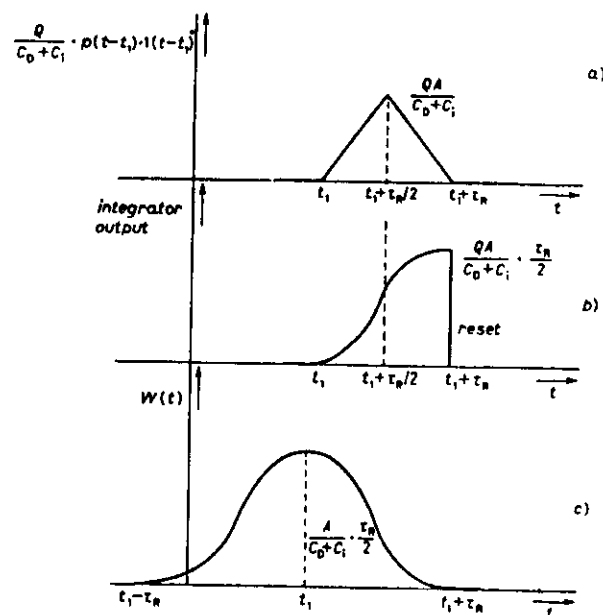


Fig. 4.15. - Wave forms at the input and output of the gated integrator (a), b) and noise weighting function (c).

As a general comment to sect. 3 and 4 it has to be pointed out that they were basically oriented to the analysis of shapers suitable for energy measurements. Shaping problems related to time measurements, for instance, were purposely neglected. It is, however, important to quote, among the shapers that have been omitted, the antisymmetric-ramp one, which is employed in the evaluation of the centroid of an arbitrary input signal [28]. The antisymmetric-ramp shaper is of particular importance in some kinds of position-sensing applications.

## REFERENCES

- [1] E. BALDINGER and W. FRANZEN: *Adv. Electron. Electron Phys.*, **8**, 225 (1956).
- [2] V. RADEKA: *IEEE Trans. Nucl. Sci.*, NS-19, 412 (1972).
- [3] E. FAIRSTEIN and J. HAHN: *Nuclear pulse amplifiers. Fundamental and design practice*, Part I: *Nucleonics*, **23**(7), 56 (1965); Part II: *Nucleonics*, **23**(9), 81 (1965); Part III: *Nucleonics*, **23**(11), 50 (1965); Part IV: *Nucleonics*, **24**(1), 54 (1966); Part V: *Nucleonics*, **24**(3), 68 (1966).
- [4] W. R. BENNETT: *Proc. IRE*, **44**, 609 (1956).
- [5] J. S. BENDAT: *Principles and Applications of Random Noise Theory* (J. Wiley and Sons, New York, N. Y., 1958).
- [6] A. B. GILLESPIE: *Signal, Noise and Resolution in Nuclear Counter Amplifiers* (Pergamon Press, London, 1953).
- [7] P. W. NICHOLSON: *Nuclear Electronics* (J. Wiley and Sons, London, 1974).
- [8] S. COVA and A. LONGONI: *An introduction to signals, noise and measurements, in Analytical Laser Spectroscopy*, edited by N. OMENETTO (J. Wiley and Sons, London, 1979).
- [9] S. O. RICE: *Bell Syst. Tech. J.*, **23**, 282 (1944).
- [10] J. R. CARSON: *Bell Syst. Tech. J.*, **10**, 374 (1931).
- [11] A. VAN DER ZIEL: *Noise-Sources, Characterisation, Measurements* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1970).
- [12] M. O. DEIGHTON: *Nucl. Instrum. Methods*, **58**, 201 (1968).
- [13] F. S. GOULDING: *Nucl. Instrum. Methods*, **100**, 493 (1972).
- [14] A. HRISOHO: *Nucl. Instrum. Methods*, **185**, 207 (1981).
- [15] V. RADEKA: *1/f noise in physical measurements, in Proceedings of the 1969 Ispra Nuclear Electronics Symposium*, Publ. EUR 4289-e (1969), p. 1.
- [16] A. VAN DER ZIEL: *Adv. Electron. Electron Phys.*, **49**, 225 (1979).
- [17] V. RADEKA: *Nucl. Instrum. Methods Phys. Res.*, **226**, 209 (1984).
- [18] P. F. MANFREDI and F. RAGUSA: *Nucl. Instrum. Methods Phys. Res. A*, **235**, 345 (1985).
- [19] M. CONRAD: *IEEE Trans. Nucl. Sci.*, NS-15, 268 (1968).
- [20] V. RADEKA: *IEEE Trans. Nucl. Sci.*, NS-15, 455 (1968).
- [21] K. KANDIAN: *High resolution spectrometry with nuclear radiation detectors*, AERE Report-R 6699-Harwell (1971).
- [22] V. RADEKA and N. KARLOVAC: *Time-variant filters for high-rate pulse amplitude spectrometry, in Proceedings of the Gallinburg Conference on Semiconductor Radiation Detectors and Circuits*, National Academy of Sciences, Publication 1593 (1969), p. 553.
- [23] K. KANDIAN, A. J. SMITH and G. WHITE: *A pulse processor for X-ray spectrometry with Si(Li) detectors, in Proceedings of the II ISpra Nuclear Electronics Symposium*, Publ. EUR 5370 e (1975), p. 153.
- [24] A. J. KALININ: private communication, Serpukhov, USSR (1977).
- [25] M. ARTUSO, P. D'ANGELO, E. GATTI, P. F. MANFREDI and D. MARIOLI: *High rate pulse processing system for silicon active targets, in Proceedings of the X International Symposium on Nuclear Electronics, Dresden, GDR, 1980*, Vol. 2 (1980), p. 24.
- [26] F. S. GOULDING and D. A. LANDIS: *IEEE Trans. Nucl. Sci.*, NS-29, 1125 (1982).
- [27] E. GATTI and P. F. MANFREDI: *Nucl. Instrum. Methods Phys. Res.*, **226**, 142 (1984).
- [28] E. GATTI, G. PADOVINI, L. QUARTEPPELLE, N. E. GREENLAW and V. RADEKA: *IEEE Trans. Nucl. Sci.*, NS-26, 2910 (1979).

### 5. Identification of the noise sources in preamplifier, detector and related bias networks.

5.1. *Noise sources, in the input amplifying device.* — In order to apply the theory developed about signal processing and signal to noise ratio, it is now necessary to discuss the explicit forms of the  $a$  and  $b$  noise generators for different kinds of preamplifying active devices, to account for the noise associated with the detector and to analyse the noise sources associated with the related bias networks. It will still be assumed that the preamplifier consists of a voltage-

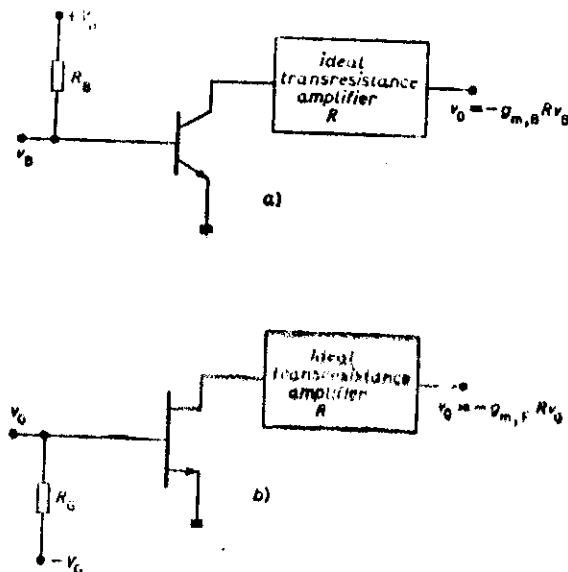


Fig. 5.1. — Voltage-sensitive preamplifiers employing as input device a grounded-emitter bipolar transistor (a) and a grounded-source field-effect transistor (b).

sensitive configuration of the type shown in fig. 5.1a, b), where the input active device is either a grounded-emitter bipolar transistor or a grounded-source field-effect transistor.

In both circuits of fig. 5.1, the output current from the first amplifying device is injected into an ideal transresistance amplifier, so that the voltage gains of the two amplifiers are, respectively,  $-g_m R$  and  $-g_{m,r} R$ , where  $g_{m,r}$  is the transconductance of the field-effect transistor, that of the field-effect transistor is  $g_m$  and  $g_{m,r}$  is the transconductance of the resistor.

### PROCESSING THE SIGNAL FROM SOLID-STATE DETECTORS ETC.

$R_B$  connected to the  $+V_B$  voltage supply its standing base current. The junction field-effect transistor is biased by connecting its gate to a negative voltage source  $-V_G$  through the resistor  $R_G$ . The bias network of fig. 5.1b) can be used, with a proper choice of value and polarity of  $V_G$ , also for depletion-type or enhancement-type MOS field-effect transistors.

The contributions determining the  $a(\omega)$  series noise spectrum and the  $b_1(\omega)$  parallel spectrum will now be discussed for the different types of active device. For this analysis the transresistance amplifier  $R$  will be considered noiseless.

5.1.1. *Silicon junction field-effect transistor.* The silicon junction field-effect transistor is still the leading device for low-noise applications at values of the processing times, either  $\tau_M$  or  $\tau_R$ , ranging from a few hundreds of nanoseconds to several tens of microseconds.

The small-signal equivalent circuit of the silicon junction field-effect transistor with the noise sources is shown in fig. 5.2.

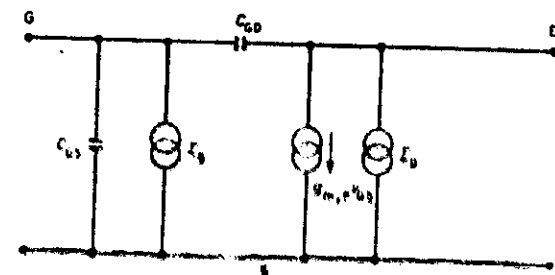


Fig. 5.2. — Small signal equivalent circuit of the junction field effect transistor including the dominant noise sources.

For a thorough analysis of the physical phenomena responsible for the noise behaviour of silicon field-effect transistors the reader can refer to the available literature [1-5].

From the point of view of the noise sources, the model of fig. 5.2 is a simplified one. Among the simplifications introduced, the effect of thermal noise in the channel coupled to the gate has been neglected.

In fig. 5.2 the two current sources with bilateral densities  $\Sigma_G$  and  $\Sigma_D$  account, respectively, for the shot noise in the gate leakage current and for the thermal noise in the channel. Their expressions are

$$(5.1) \quad \Sigma_G = qI_G,$$

$$(5.2) \quad \Sigma_D = 2kT \cdot 0.7 g_{m,r}.$$

In the previous relationships  $q$  is the electron charge,  $q = 1.6 \cdot 10^{-19}$  C,  $k$  is

Boltzmann's constant,  $k = 1.38 \cdot 10^{-23}$  J/K,  $T$  is the absolute temperature and  $I_0$  is the gate leakage current. The equivalent circuit of fig. 5.2 can be redrawn in a form closer to the representation of fig. 3.1 by referring the thermal noise in the channel to the input and by remembering that in the circuit diagram 5.2b) the drain of the JFET is connected to an ideal current input. The new equivalent circuit is shown in fig. 5.3.

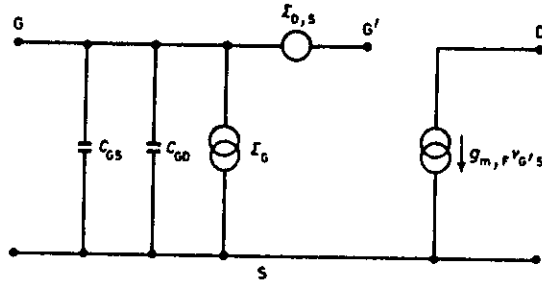


Fig. 5.3. - Equivalent circuit of the junction field-effect transistor with a series voltage source accounting for the thermal noise in the channel.

In the circuit of fig. 5.3 the thermal noise in the channel is accounted for by the series voltage source with bilateral power spectral density

$$(5.3) \quad \Sigma_{D,s} = 2kT \frac{0.7}{g_{m,r}}$$

The transconductance  $g_{m,r}$  is related to the pinch-off voltage  $V_p$  and to the drain current  $I_{Ds}$  in the saturation region by the relationship [6]

$$(5.4) \quad g_{m,r} \approx 3 \frac{I_{Ds}}{V_p},$$

where the maximum of  $I_{Ds}$  is  $I_{Dss}$ , the drain saturation current. According to (5.4) a device with a  $1V V_p$  would exhibit a  $g_{m,r}$  of 30 mA/V at a drain current of 10 mA.

Such a value of  $g_m/I_{Ds}$  is an upper limitation among silicon JFETs, whereas an average good device has typically 10 mA/V at  $I_{Ds} = 10$  mA.

The intrinsic contribution to the parallel noise in a silicon junction field-effect transistor is the shot noise in the gate leakage current  $I_0$ . The values of  $I_0$  range between  $10^{-12}$  A and  $10^{-10}$  A at  $T = 300$  K.

The junction field-effect transistor is a majority carrier device whose gain-

bandwidth product is limited by the transit time in the channel. The gain-bandwidth product, which in terms of concentrated parameters can be shown to be equal to the  $g_{m,r}/(C_{gs} + C_{gd})$  ratio, has the following upper limitation:

$$(5.5) \quad \omega_{r,r} < \frac{\mu V_p}{\pi L^2},$$

where  $L$  is the length of the channel and  $\mu$  is the mobility of the carriers.

Comparing the equivalent circuit of fig. 5.3 with the general block diagram of fig. 3.1, it can be concluded that the preamplifier input capacitance  $C_i$  is the sum of  $C_{gs}$  and  $C_{gd}$ , that the spectral power density  $\alpha$  is represented by  $\Sigma_{D,s}$  and that the spectral power density  $b_i$  is represented by  $\Sigma_0$ .

5.1.2. Bipolar transistors. The equivalent circuit which summarizes the signal action and the noise characteristics of the bipolar transistor is shown in fig. 5.4 [7, 8].

In the diagram of fig. 5.4  $R_{bb'}$  is the base spreading resistance,  $R_b$  is the

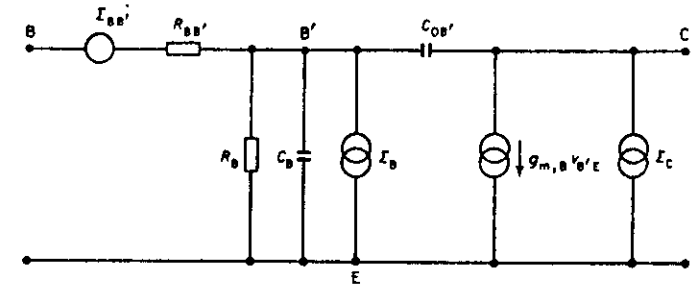


Fig. 5.4. - Small-signal equivalent circuit of the bipolar transistor including the dominant noise sources.

incremental base resistance, which depends on the base standing current  $I_b$  according to the relationship  $R_b = kT/qI_b$ ,  $C_b$  is the sum of the base-to-emitter diffusion and junction capacitances.  $C_{cb}$  is the capacitance of the base-collector junction. In ordinary operating conditions the base-to-emitter junction capacitance is small compared to the diffusion one and, therefore, it will be assumed that  $C_b$  represents the diffusion capacitance.

As to the noise sources,  $\Sigma_{bb'}$  is the bilateral spectral power density of the generator which accounts for the thermal noise in the base spreading resistance,  $\Sigma_b$  is the bilateral spectral power density of the generator describing the shot noise in the base current and  $\Sigma_c$  the one of the generator describing

the shot noise in the collector current  $I_c$ . Their expressions are

$$(5.6) \quad \Sigma_{in} = 2kTR_{bb'},$$

$$(5.7) \quad \Sigma_u = qI_u,$$

$$(5.8) \quad \Sigma_c = qI_c.$$

The circuit of fig. 5.4 can be transformed into that of fig. 5.5, where the shot

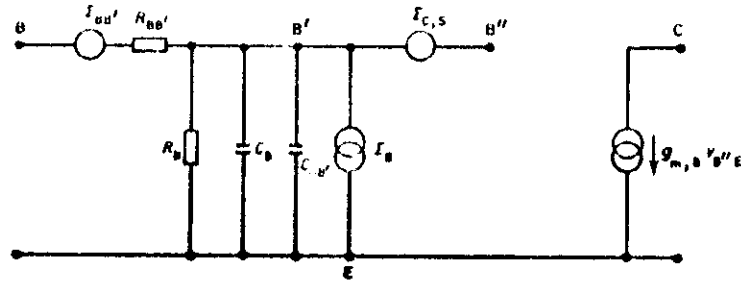


Fig. 5.5. - Equivalent circuit of the bipolar transistor with a series voltage source,  $\Sigma_{c,s}$ , accounting for the collector shot noise.

noise in the collector current is accounted for by the series voltage generator  $\Sigma_{c,s}$ . The  $C_{ob}$  capacitance has been represented in parallel to  $C_b$  as in the practical application of fig. 5.1a) the collector of the transistor terminates to a zero-impedance input. In the ordinary operating conditions  $C_{ob}$  is negligible compared to the base diffusion capacitance.

The spectral power density  $\Sigma_{c,s}$  is expressed by

$$(5.9) \quad \Sigma_{c,s} = \frac{qI_c}{g_{m,u}}.$$

Remembering that  $g_{m,u} = qI_c/kT$ ,  $\Sigma_{c,s}$  can be rewritten in a form close to the one seen for the thermal noise referred to the input in a field-effect transistor, that is,

$$(5.10) \quad \Sigma_{c,s} = 2kT \frac{0.5}{g_{m,u}}.$$

If the thermal voltage  $V_T = kT/q = 25$  mV at  $T = 300$  K is introduced into the expression of  $g_{m,u}$  and the ratios

$$\frac{g_{m,u}}{I_c} = \frac{1}{V_T} \quad \text{and} \quad \frac{g_{m,u}}{I_{cs}} = \frac{3}{V_T}$$

are compared, it can be concluded that the ratio between transconductance and standing current for the bipolar transistor is more than an order of magnitude larger than for the junction field-effect transistor.

The spectral power density  $\Sigma_{c,s}$  gives a first contribution to the series generator  $a$  in the general diagram of fig. 3.1. A second contribution, which from the circuit diagram of fig. 5.5 can be evaluated to be equal to

$$(5.11) \quad 2kTR_{bb'} \frac{C_b^2}{(C_D + C_b)^2},$$

is due to the thermal noise in the base spreading resistance. Therefore, with reference to fig. 3.1, it can be written for the white-noise part  $a_s$  of the series spectral power density

$$(5.12) \quad a_s = 2kT \frac{0.5}{g_{m,u}} + 2kTR_{bb'} \frac{C_b^2}{(C_D + C_b)^2},$$

having neglected  $C_{ob}$ . As shown by (5.12), the contribution to the series white noise introduced by  $R_{bb'}$  depends on the  $C_D/C_b$  ratio, being zero at  $C_D = 0$  and approaching  $2kTR_{bb'}$  when  $C_D \gg C_b$ . The base spreading resistance in a microwave bipolar transistor featuring typically more than 5 GHz gain-bandwidth product at a collector current of 5 mA can be in the (10-15)  $\Omega$  range.

The bipolar transistor has a large parallel noise source related to the shot noise in the base leakage current. The spectral power density of the generator  $b_1$  in fig. 3.1 is

$$(5.13) \quad b_1 = \Sigma_u = qI_u.$$

The usual values of  $I_u$  in nuclear detector applications are in the (10-100)  $\mu$ A range.

The gain-bandwidth product  $\omega_T$  in a bipolar transistor can be expressed as [6]

$$(5.14) \quad \omega_T = \frac{g_{m,u}}{C_b} = \frac{2\mu V_T}{L^2},$$

where  $\mu$  is the mobility of minority carriers in the base region and  $L$  is the base width.

Although the presence of the large parallel noise requires that the bipolar transistor be employed at a value of the collector current substantially lower than the one at which the maximum of  $\omega_T$  occurs, that is,  $I_c$  below 1 mA, still values of  $\omega_T$  of 2 $\pi$  Grad/s can be achieved.

5.1.3. GaAs junction field-effect transistors. Compared to the silicon field-effect transistor, the Schottky-barrier GaAs junction field-effect transistor has a better  $g_m/I_{DS}$  ratio and a much larger  $\omega_T$ , this feature being

a current source with bilateral power density  $2kT/R$  can be attributed to an equivalent shot noise current source with spectral power density  $qI_{L_0}$ , where  $I_{L_0} = (kT/q)(2/R)$ . Conversely, shot noise can be attributed to an equivalent thermal source. In the case of fig. 5.6b), with the detector d.c. coupled to the preamplifier, only one source of thermal noise is present, that is, the one associated with  $R_0$ . Consequently

$$(5.17) \quad b_s = qI_L + 2kT \frac{1}{R_0}.$$

Adding the  $b_s$  terms of either (5.16) or (5.17) to the  $b_i$  terms related, respectively, to the bipolar transistor, eq. (5.13), or to the field-effect transistor, eq. (5.1), the total spectral power density in the parallel noise generator of fig. 3.1 is found. Continuous monitoring of  $I_L$  during the experiment is advisable. In the spectral power density  $b$  of the parallel generator of fig. 3.1 the relative importance of  $b_i$ , contribution due to the parallel noise in the amplifying device, and  $b_s$ , contribution due to the detector and bias resistors, is subject to the following considerations. If the active device is a bipolar transistor,  $b_i$  largely dominates. If, instead, the input device is a field-effect transistor and this is connected to a silicon detector operated at room temperature, then the parallel noise is determined by the detector leakage current and by the thermal noise in the bias resistors,  $b = b_s$ .

In the case of germanium bulk detector, which operates at cryogenic temperatures, the dominant contribution to parallel noise comes from the bias resistor  $R_0$  in the preamplifier. For such an application, indeed, preamplifiers with JFET input are utilized and d.c. coupling between detector and preamplifier is adopted.

Table 5.I summarizes the expressions of the spectral power densities  $a$ ,  $b_i$ ,  $b_s$  for different combinations of detector type, bias network and active device.

TABLE 5.I.

	Type of input active device			
	Si JFET	Bipolar transistor	GaAs MESFET	MOS FET
$a(\omega)$	$2kT \frac{0.7}{g_{m,r}}$	$2kT \frac{0.5}{g_{m,u}} + R_{BB} \frac{C_D^2}{(C_D + C_u)^2}$	$2kT \frac{h}{g_{m,r}} + \frac{c_0}{ \omega }$	$2kT \frac{h}{g_m} + \frac{c_M}{ \omega }$
$b_i(\omega)$	$qI_0$	$qI_B$	$qI_0$	$\sim 0$
	Type of detector and connection to the preamplifier			
	Si d.c. coupled	Si a.c. coupled	Ge d.c. coupled	Ge a.c. coupled
$b_s(\omega)$	$qI_L + \frac{2kT}{R_0}$	$qI_L + 2kT \left( \frac{1}{R_r} + \frac{1}{R_0} \right)$	$\sim 2kT \frac{1}{R_0}$	$\sim 2kT \left[ \frac{1}{R_r} + \frac{1}{R_0} \right]$

Table 5.I enables the reader to determine the values of noise corner time constant achievable with the various possible detector-preamplifier combinations.

Recalling eq. (3.4) which can be written as

$$\tau_c = \sqrt{\frac{a_s}{b_i + b_s}} (C_D + C_i),$$

and the expressions of  $a_s$ ,  $b_i$ ,  $b_s$  in table 5.I, it can be concluded that, for a fixed value of  $C_D + C_i$ ,  $\tau_c$  is limited by the base current noise in a bipolar transistor to a much smaller value than in the other devices. For instance, neglecting the terms  $b_s$ ,  $\tau_c$  would be of the order of  $10^4(C_D + C_i)$  in a bipolar transistor and of the order of  $10^4(C_D + C_i)$  in a junction field-effect transistor. The presence of the detector leakage current and of the thermal noise in the bias resistors may actually reduce the value of  $\tau_c$  in the case of a junction field-effect transistor to much smaller values.

To understand qualitatively the effects of the noise sources in the active devices and their importance in the equivalent noise charge ENC at different processing times, the expression of ENC for the triangular shaper will be assumed as a reference [14]. Combining (4.8) and (4.27) the following result is obtained:

$$(5.18) \quad \text{ENC} = \left\{ 2 \frac{a_s}{\tau_M} (C_D + C_i)^2 + 4 \frac{\ln 2}{\pi} c(C_D + C_i)^2 + \frac{2}{3} (b_i + b_s) \tau_M \right\}^{1/2}.$$

The equivalent noise charge is expressed as the square root of the quadratic sum of three partial contributions. The first one, determined by the white component in the power spectral density of the series generator, is inversely proportional to  $\tau_M^{1/2}$ .

The second one, due to the  $1/|\omega|$  term in the power spectral density of the series generator, is independent of  $\tau_M$ .

The third one, due to the parallel noise, is proportional to  $\tau_M^{1/2}$ . According to (5.18), the ENC has a minimum at  $\tau_M = \sqrt{3}\tau_c$ . The value of  $\tau_M$  at which the minimum of ENC occurs is unaffected by the  $1/|\omega|$ -type noise.

The following interesting comments about the behaviour of the ENC as a function of  $\tau_M$  can be made with reference to fig. 5.7 [14].

As shown in fig. 5.7, the bipolar transistor has an optimum ENC at a value of  $\tau_M$  of a few tens of ns. The ENC for a junction field-effect transistor would be a continuously decreasing function of  $\tau_M$ , represented by the linear behaviour in the double-logarithmic plot of fig. 5.7 if the noise from the detector and bias resistors were not present. A 10 nA detector leakage current shifts the value of  $\tau_M$  at which the minimum of the ENC occurs to the 1  $\mu$ s range.

The MOS transistor has an initial drop of the ENC as a function of  $\tau_M$ , which reflects the behaviour of the white component  $a_s$  in the spectral power density

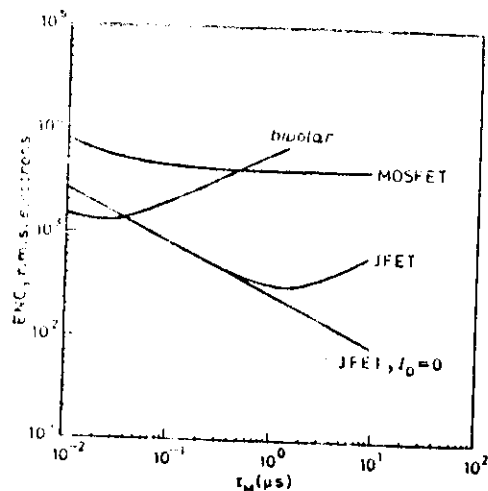


Fig. 5.7. - Equivalent noise charge as a function of  $\tau_M$  for three types of active devices and a fixed value of  $C_D$ ;  $C_D = 30$  pF,  $I_D = 10$  nA.

of the series noise source, followed by a setting to the value determined by the  $e/f[\omega]$  noise.

The curves of fig. 5.7 are theoretical curves obtained with typical values of the parameters of table 5.I.

In order to analyse the effect of series  $e/f[\omega]$  noise, the case of a MOS with  $c = 10^{-10}$  V<sup>2</sup> has been chosen.

The value of  $c$  would be considerably smaller in a Schottky-barrier field-effect transistor.

In the usual low-noise design, based upon discrete or thin- and thick-film circuit realizations, the choice of the input active device is restricted to bipolar or silicon junction field-effect transistors. GaAs field-effect transistors cover a domain of applications close to that of microwave bipolar transistors, that is, short processing times. In all these cases the contribution of the  $e/f[\omega]$  noise is either negligible or masked by the term introduced into the ENC by the series white spectral power density  $\alpha_s$ .

## REFERENCES

- [1] A. VAN DER ZIEL: *Proc. IRE*, **50**, 1808 (1962).
- [2] A. VAN DER ZIEL: *Proc. IRE*, **51**, 161 (1963).
- [3] E. GATTI and P. F. MANFREDI: *Instrumentation Techniques in Nuclear Pulse Analysis*, National Research Council, Publ. 1184 (1964), p. 70.

- [4] V. RADEKA: *Field-effect transistor in charge sensitive amplifiers*, in *Instrumentation Techniques in Nuclear Pulse Analysis*, National Academy of Sciences, National Research Council, Publ. 1184 (1964), p. 70.
- [5] V. RADEKA: *IEEE Trans. Nucl. Sci.*, **NS-20**, 182 (1973).
- [6] E. GATTI and P. F. MANFREDI: *Nucl. Instrum. Methods Phys. Res.*, **226**, 142 (1984).
- [7] A. VAN DER ZIEL: *Proc. IRE*, **46**, 1019 (1958).
- [8] B. B. FAIR: *IEEE Trans. Nucl. Sci.*, **NS-23**, 218 (1976).
- [9] V. RADEKA: *Nucl. Instrum. Methods Phys. Res.*, **226**, 175 (1984).
- [10] K. OGAWA, B. OWEN and H. J. BOLL: *Bell Syst. Tech. J.*, **62**, 1181 (1983).
- [11] G. ZIMMER: *Nucl. Instrum. Methods Phys. Res.*, **226**, 175 (1984).
- [12] B. J. HOSTICKA: *Nucl. Instrum. Methods Phys. Res.*, **226**, 185 (1984).
- [13] G. BAGGARANI: *Physics of submicron devices*, in *Proceedings of the NATO Advanced Study Institute on Large Scale Integrated Circuit Technology: State of the Art and Prospects*, edited by L. ESAKI and G. SONCINI (1982), p. 647.
- [14] P. F. MANFREDI and F. RAGUSA: *Nucl. Instrum. Methods Phys. Res. A*, **235**, 345 (1985).

## 6. - General considerations about shaping.

Some general considerations about shaping will now be made in order to outline the choice criteria that must be adopted in the design of a low-noise processor for high-energy experiments. The constraints that affect the value of the processing time will be discussed. The noise characteristics of some basic shapers are summarized in table 6.I. Each shaper is described by its «noise weighting function» which in the case of time-invariant circuits as well as in the case of time-variant ones with no preshaping before the gate coincides with the response to a detector current having the shape of a  $\delta$ -impulse. Synthesis suggestions of the shapers, based upon the time invariant or the time-variant approach, are also given. When either implementation is clearly not worthwhile compared to the other one, the corresponding space is left void.

The equivalent noise charge is given for all the shapers.

The values of  $\tau_M$  that minimize the equivalent noise charge for every processor are also presented for all the processings with the exception of the truncated cusp for which, as is obvious, the ENC is a monotonically decreasing function of  $\tau_M$ .

The last column of table 6.I gives the ratio between the equivalent noise charge evaluated for each processor at the optimum  $\tau_M$  and the equivalent noise charge achievable with the indefinite cusp.

The analysis of the optimum ENC values, summarized for each processor in the last column of table 6.I, suggests the following useful comment.  $ENC_{opt}$  can be expressed as  $\alpha(1/\alpha_s b_s (C_D + C_i)^{-1})^{1/2}$ , where  $\alpha$  is a coefficient larger than 1 which represents the «worst case factor» of the actual shaper compared with the one characterized by an indefinite-cusp weighting function. For  $\alpha_s$  and

TABLE 6.1.

Shaping	Noise weighting function	Possible time-invariant implementation	Possible time-variant implementation	Equivalent noise charge r.m.s. coulomb	$\tau_{M,opt}$	$ENC_{opt} = (4a_w b_w)^{1/2} (C_D + C_1)^{1/2}$
ideal infinite cusp		weighting network employing the principle of fig. 3.4		$(4a_w b_w)^{1/2} (C_D + C_1)^{1/2}$	$\infty$	1
truncated cusp		weighting network employing the principle of fig. 3.4		$\frac{(4a_w b_w)^{1/2} (C_D + C_1)^{1/2}}{(1 - \exp[-2(\tau_M/\tau_c)])^{1/2}}$	$\infty$	1
triangular				$\left[ 2a_w (C_D + C_1)^2 \frac{1}{\tau_M} + \frac{2}{3} b_w \tau_M \right]^{1/2}$	$\sqrt{3} \left( \frac{a_w}{b_w} \right)^{1/2} (C_D + C_1)$	1.07
trapezoidal				$\left[ 2a_w (C_D + C_1)^2 \frac{1}{\tau_M} + b_w \left( \frac{2}{3} \tau_M + \tau_0 \right) \right]^{1/2}$	$\sqrt{3} \left( \frac{a_w}{b_w} \right)^{1/2} (C_D + C_1)$	$1.07 \left[ 1 + 0.43 \frac{\tau_0}{\tau_c} \right]^{1/2}$
RC-CR $R_1 C_1 = R_2 C_2 = \tau_M$				$\frac{a}{2} \left[ a_w (C_D + C_1)^2 \frac{1}{\tau_M} + b_w \tau_M \right]^{1/2}$	$\left( \frac{a_w}{b_w} \right)^{1/2} (C_D + C_1) = \tau_c$	1.36
piecewise parabolic				$\left[ \frac{8}{3} a_w (C_D + C_1)^2 \frac{1}{\tau_M} + \frac{14}{15} b_w \tau_M \right]^{1/2}$	$2 \sqrt{\frac{5}{7}} \left( \frac{a_w}{b_w} \right)^{1/2} (C_D + C_1)$	1.25
sinusoidal lobe				$\left[ \frac{\pi^2}{4} a_w (C_D + C_1)^2 \frac{1}{\tau_M} + b_w \tau_M \right]^{1/2}$	$\frac{\pi}{2} \left( \frac{a_w}{b_w} \right)^{1/2} (C_D + C_1)$	1.25

$b_n$  the expression of table 5.1 will be introduced. Assuming, for the sake of simplicity, that the input device is a field-effect transistor and writing  $a_n$  in the approximate form  $a_n = kT/g_m$  and that the dominant contribution to  $b_n$  is the one determined by the detector leakage current, which is certainly true in the case of a room temperature Si detector,  $ENC_{opt}$  can be written, for a generic processor, as

$$ENC_{opt} = \alpha \left[ \frac{4qkT}{\omega_T} \left\{ (C_D/C_I)^4 + (C_I/C_D)^4 \right\} I_L C_D \right]^{1/2}$$

and

$$\tau_{M,opt} = \beta \left[ \frac{4T}{q} \frac{1}{\omega_T} \left\{ (C_D/C_I)^4 + (C_I/C_D)^4 \right\} (C_D/I_L) \right]^{1/2}.$$

The previous relationships show that the optimum ENC and corresponding  $\tau_n$  can be expressed, apart from a constant, by the product of three factors. One depending on the gain-bandwidth product of the preamplifier input device, which is related to the device technology, the second on a 'mismatch term' and the third on the detector characteristics.

The analysis of the various processors has been carried through by neglecting  $1/\omega$  spectral power densities in the series generator  $a(\omega)$ , whose spectrum has been assumed to be white and labelled  $a_n$ .

Spectral power densities with linear  $\omega$  dependence, typical of dielectric losses, have been neglected in the parallel generator, whose spectrum also has been assumed to be white and labelled  $b_n$  [1.3].

Both neglected kinds of noise, that is, series  $1/\omega$  spectral density and parallel  $\omega$  spectral density, would introduce into the expression of the ENC a term which is independent of  $\tau_n$  and, therefore, the value of  $\tau_{M,opt}$  is unaffected by this approximation.

As was already pointed out, the operating conditions of most experiments in high-energy physics are such that the noise due to dielectric losses is overwhelmed by the noise with  $a_n$  spectrum.

The same consideration applies very often also to the series  $1/\omega$  noise. The exception is represented by the case in which a MOS transistor is employed as input device. The corresponding ENC has the  $\tau_n$  dependence illustrated in fig. 5.7.

Table 6.1 allows the evaluation of the considered processing systems once the weighting function is chosen, the characteristics of the input device are assigned and the detector parameters, capacitance and leakage current as well as the detector and preamplifier bias networks, are known.

A further important aspect is the choice of  $\tau_n$ .

Strict considerations of signal to noise ratio would suggest for  $\tau_n$  the value which minimizes the ENC according to table 6.1. It is worth emphasizing, however, that this is not a constraint since the choice of  $\tau_n$

Considerations related to counting rates and pile-up effects put an upper limitation to the usable  $\tau_n$ .

The situation is largely different, depending on whether the detectors are employed in a fixed-target or in a collider experiment.

In several fixed-target experiments the detectors are subjected to very high counting rates, up to millions pulses per second during the accelerator spill. If a particle is detected at a given instant, the probability that other particles arrive within a time interval  $\tau_n$  from the previous one is [4.6]

$$(6.1) \quad P = 1 - \exp(-\lambda \tau_n),$$

$\lambda$  being the average counting rate. If a detector current pulse is being processed and another pulse occurs within the processing time, then pile-up occurs. Depending on the characteristics of the shaper employed and on the mutual time separation, the amplitude measurement on the second event or on both of them may lead to a wrong result.

This means that the second event or both events have to be rejected to avoid pile-up inaccuracies. Let  $\tau_n$  be the processing time, that is, the total time needed to process an event. The value of  $\tau_n$  depends on the detector pulse duration and on the shaper. Let  $P_{na}$  be the maximum tolerable pile-up probability. The processing time  $\tau_n$  must be chosen so as to satisfy the following relationship:

$$(6.2) \quad \tau_n \leq \frac{1}{\lambda} \ln \frac{1}{1 - P_{na}}.$$

For instance, a time-invariant triangular shaper with a base width  $2\tau_n$  responds to a rectangular detector current of duration  $\tau_{det}$  with a signal of total duration  $2\tau_n + \tau_{det}$ . This can be assumed to be the processing time of the system.

If a value of  $P_{na}$  lower than 5% is required at a counting rate of  $10^6$  pulses per second, then  $2\tau_n + \tau_{det}$  must not exceed 50 ns. In these circumstances, the upper limitation set to  $2\tau_n + \tau_{det}$  by the counting rate requirements becomes a mandatory one and, therefore, the signal-to-noise ratio must be optimized under this constraint on the processing time.

It has to be pointed out that in a segmented detector like a microstrip, where the counting rates on the individual strips are smaller than the counting rate  $\lambda$  on the whole detector, is still  $\lambda$  which determines the processing time. Otherwise it may occur that pulses corresponding to two different events in the detector overlap in time, thereby creating ambiguities in the assignment of the signal to the proper event [7].

In the existing colliders as well as in machines that will go into operation in a near future, the counting rate requirements are by far less severe. The time separation between each crossings is such that processing times in the  $\mu$ s

region can be employed. In collider experiments the upper limitation to the processing time is determined by other constraints. For instance, by the detector leakage current. In other cases the upper limitation may come from the need of making real-time trigger decisions.

Other effects like ballistic error, signal induction on the noncollecting electrodes of a segmented detector and cross-talk contribute to important considerations in the choice of the shaper and of the processing time.

The nature of the ballistic error is shown in fig. 6.1 [8-10].

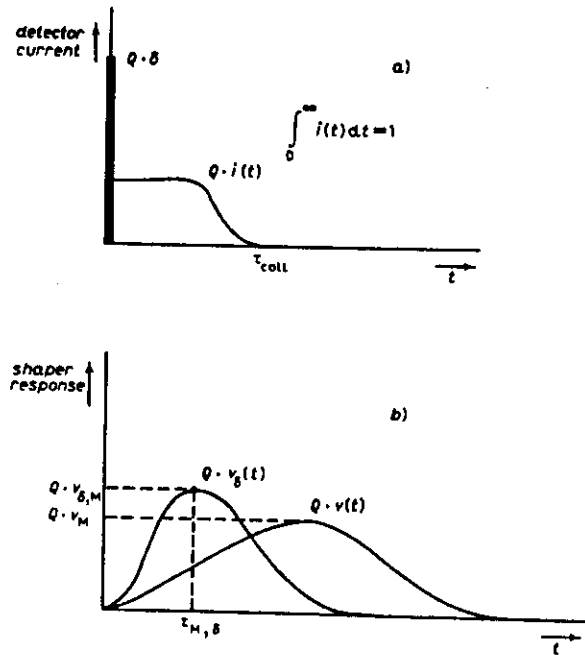


Fig. 6.1. - Response of the shaper to a  $\delta$ -impulse-like detector current and to a pulse of finite duration: a) detector current shapes, b) shaped signals.

The processor assumed as a basis of this discussion responds to a  $\delta$ -like detector current with the signal  $Qv_\delta(t)$ . A detector current of finite duration  $\tau_{coll}$ , though carrying the same charge as the  $\delta$ , excites at the processor output the response  $Qv(t)$  which reaches its peak at a later time and has a smaller peak amplitude than  $Qv_\delta(t)$ .

The ballistic error is defined, according to fig. 6.1b), as

$$(6.3) \quad v_{\delta,M} - v_M.$$

The effect of ballistic error is limited to a reduction in the equivalent noise charge, related to the decreased output amplitude in the case of detector pulses having the same shape and duration.

If, however, the duration  $\tau_{coll}$ , as frequently happens, is a random variable, then the ballistic effect contributes to increase the statistic uncertainty in the measurement of  $Q$  and, therefore, it has to be kept as small as possible [9, 10].

Analysis of the ballistic error was done by BALDINGER and FRANZEN, who showed that the term (6.3) can be expressed as

$$(6.4) \quad v_{\delta,M} - v_M \simeq -\frac{1}{2} \tau_{coll}^2 v_\delta''(\tau_{M,d}) \int_0^1 i(x) (\alpha_1 - x)^2 dx.$$

In the previous relationship  $x = t/\tau_{coll}$  and  $\alpha_1$  is the centroid of the detector current pulse normalized to unit area

$$\alpha_1 = \int_0^1 x i(x) dx.$$

As pointed out by (6.4), the ballistic error is proportional to the second derivative, that is, to the curvature of the  $\delta$ -response  $v_\delta(t)$  at the peak point. Therefore, a flat-topped  $v_\delta(t)$  is required to reduce the ballistic error. A flat-topped  $v_\delta(t)$  would eliminate the ballistic error for all the detector current pulses having a  $\tau_{coll}$  shorter than the duration  $\tau_\delta$  of the flat region [10]. If  $\tau_{coll}$  is a random variable with values ranging between 0 and  $\tau_\delta$ , no amplitude fluctuation would be noticed in  $v(t)$ . For instance, trapezoidal shaping represents a good compromise between noise performances and insensitivity to collection time variations in the detector. Conversely,  $v_\delta(t)$ 's having an angular point at the instant

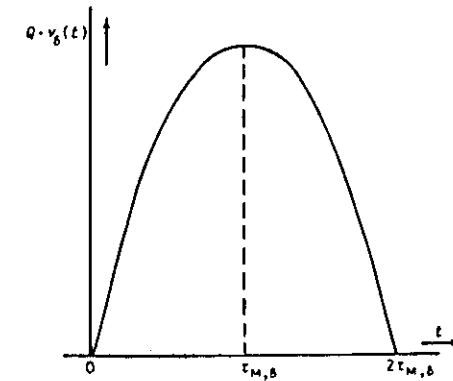


Fig. 6.2. - Parabolic  $\delta$ -response for the evaluation of the ballistic error.

$t_{M,0}$  give large ballistic errors. This is the case of the cusp and the triangle. An example of evaluation of the ballistic error is made assuming, for reference, the parabolic  $v_d(t)$  of fig. 6.2. The time dependence of  $v_d(t)$  is  $v_d(t) = 2t/t_{M,0} - t^2/t_{M,0}^2$  and has a maximum at  $t = t_{M,0}/2$ .

The relative ballistic error in the case of a rectangular current pulse of duration  $t_{on}$  is

$$(6.5) \quad \frac{v_{d,M} - v_M}{v_{d,M}} \approx -\frac{1}{12} \left[ \frac{t_{on}}{t_{M,0}} \right]^2.$$

Equation (6.5) shows that, just using a round-topped  $v_d(t)$  with a total base width equal to six times the detector pulse duration, the relative ballistic error is below 1%.

Absence of ballistic deficit can be thought of as the property of insensitivity of a measurable output parameter to the shape of the input charge pulse. In particular, insensitivity to zero-charge spurious bipolar input pulses. This kind of disturbances occurs in segmented detectors like microstrips and Ge bulk detectors [11-15].

Two origins of such disturbances are explained with reference to fig. 6.3. Induction of electron and hole pairs on output electrodes not collecting charge carriers like, for instance,  $K+1$  in fig. 6.3a). Taking into account the weighting field for electrode  $K+1$  introduced in sect. 2 and the trajectory of both carriers of a pair, it is apparent that a bipolar, zero-area pulse is induced on  $K+1$ . This pulse has an overall length equal to the transit time of the slower carrier.

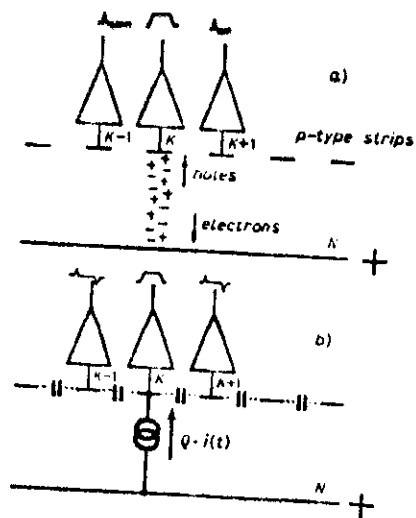


Fig. 6.3. — Induction (a) and cross talk (b) on the noncollecting electrodes. The preamplifiers are supposed to be current sensitive.

Cross-talk is illustrated in fig. 6.3b). If the input impedance of the preamplifier connected to strip  $K$ , though small, is finite, a signal appears across it. The capacitive coupling between strip  $K$  and strips  $K-1$ ,  $K+1$  is responsible for the injection of two signals, proportional to the derivative of the one appearing on the strip  $K$ , into the contiguous preamplifiers. Again, cross-talk signals have zero net area and their time separation is determined by the duration of the pulse at strip  $K$ .

Cancellation of induction and cross-talk signals requires round-topped or, better, flat-topped pulses  $v_d(t)$  of suitable duration.

The phenomena that set a lower limit to the duration of  $v_d(t)$ , namely ballistic error, induction and cross-talk on the noncollecting electrodes, combined with the pile-up effect which conversely sets an upper limitation to the processing time, may lead to incompatible requirements for the duration of  $v_d(t)$ . This incompatibility can be sometimes removed only by reducing, in fixed-target experiments, the counting rate  $\lambda$ .

## REFERENCES

- [1] A. VAN DER ZUL, *Adv. Electron. Electron Phys.*, **49**, 225 (1979).
- [2] V. RADEKA, *State of the art of low noise amplifiers for semiconductor radiation detectors*, in *Proceedings of the International Symposium on Nuclear Electronics, Fermilab 1968*, Vol. 1 (1968), p. 1.
- [3] J. HAASER, *Accurate measurements of pulse parameters in ultra low noise opto-feedback spectrometer systems*, in *Proceedings of the 11 ISPE Nuclear Electronics Symposium*, Publ. EUR 6370e (1976), p. 47.
- [4] R. L. CHASE, *Nuclear Pulse Spectrometry* (McGraw-Hill Book Co., New York, N. Y., 1963), Chapt. 9.
- [5] P. W. NICHOLSON, *Nuclear Electronics* (J. Wiley and Sons, London, 1973).
- [6] I. DE LOTTO, P. F. MANFREDI and P. PRINCIPALI: Part I, *Energ. Nucl. (Milan)*, **11**, 557 (1964); Part II, **11**, 590 (1964).
- [7] E. GATTI and P. F. MANFREDI, *Nucl. Instrum. Methods Phys. Res.*, **226**, 142 (1984).
- [8] E. BALDINGER and W. FRANZEN, *Adv. Electron. Electron Phys.*, **8**, 225 (1956).
- [9] V. RADEKA, *IEEE Trans. Nucl. Sci.*, **NS-19**, 412 (1972).
- [10] F. B. GOUVERNO, *Nucl. Instrum. Methods*, **100**, 493 (1972).
- [11] J. P. KICHER and R. L. CHASE, *IEEE Trans. Nucl. Sci.*, **NS 31**, 258 (1984).
- [12] V. RADEKA, *Position sensitive semiconductor detectors, signal formation, noise and position readout; Silicon detectors for high energy physics*, in *Proceedings of a Workshop Held at Fermilab, October 15-16, 1981*, edited by T. FERBEL (1981), p. 21.
- [13] P. F. MANFREDI and V. RAGUSA, *Nucl. Instrum. Methods Phys. Res. A*, **235**, 345 (1985).
- [14] E. GATTI and V. RADEKA, *IEEE Trans. Nucl. Sci.*, **NS-25**, 676 (1978).
- [15] D. BUCHHEITZ, R. KLANNER, G. LUTZ, H. J. SIEHRHÜNNER, A. WYTH, U. KOTZ, E. U. POSECKER and J. REMMER, *Nucl. Instrum. Methods Phys. Res. A*, **253**, 451 (1985).

### 7. - Capacitive matching between detector and preamplifier.

The forthcoming considerations apply only to the contribution to the ENO due to either thermal noise in the channel of field-effect devices or to the shot noise in the collector current of a bipolar transistor. According to table 6.I such a contribution is given by

$$(7.1) \quad \text{ENC}_s^2 = 2kT \frac{\chi}{g_m} (C_D + C_i) \frac{h_1}{\tau_M},$$

where  $h_1$  is a numerical parameter characteristic of the shaper employed (4.16), and a coefficient  $\chi$ ,  $0.5 < \chi < 0.7$ , has been introduced to describe with a single expression both bipolar and field-effect devices. As already pointed out, the  $g_m/C_i$  ratio represents the gain-bandwidth product  $\omega_T$  of the device employed. Equation (7.1) can, therefore, be written

$$(7.2) \quad \text{ENC}_s^2 = 2kT \frac{\chi}{\omega_T} \frac{(C_D + C_i)^2}{C_i} \frac{h_1}{\tau_M} = 2kT \frac{\chi}{\omega_T} \frac{h_1}{\tau_M} \left[ \left( \frac{C_D}{C_i} \right)^2 + \left( \frac{C_D}{C_i} \right) \right] C_D.$$

The gain-bandwidth product  $\omega_T$  has an important significance in the application of the device to low-noise amplification of signals from capacitive sources [1]. The device charge sensitivity can be defined as

$$(7.3) \quad \frac{dI_o}{dQ_i} = \frac{dI_o}{dV_i} \frac{dV_i}{dQ_i},$$

where  $dI_o$  is the output current change corresponding to the amount of charge  $dQ_i$  deposited on the input capacitance  $C_i$  and  $V_i$  is the voltage across  $C_i$ . As, by definition,

$$g_m = \frac{dI_o}{dV_i}, \quad C_i = \frac{dQ_i}{dV_i},$$

eq. (7.3) shows that the charge sensitivity of the employed device is expressed by its gain-bandwidth product  $\omega_T$ . It can be supposed now that both  $g_m$  and  $C_i$  can be linearly controlled in such a way that their ratio, that is, the gain-bandwidth product, remains constant [2-4].

For instance, such a control can be achieved by paralleling  $n$  subdevices with transconductance  $g_{m_s}$  and input capacitance  $C_{i_s}$ . In this case

$$g_m = n g_{m_s}, \quad C_i = n C_{i_s}$$

and the ratio  $g_m/C_i$  is equal to  $g_{m_s}/C_{i_s}$ , that is, to the gain-bandwidth product of the subdevice.

Another way of controlling  $g_m$  and  $C_i$  is based upon the control of the cross-section of the device.

In bipolar transistors, within the range of values of the collector current  $I_o$ , where the gain-bandwidth product is independent of  $I_o$  and the junction capacitance negligible, so that  $C_i$  can be assumed to be determined by the diffusion term alone,  $g_m$  and  $C_i$  are, within a good approximation, linear functions of  $I_o$ . The values of  $g_m$  and  $C_i$  can, therefore, be controlled through  $I_o$ .

Whatever is the method utilized to control  $g_m$  and  $C_i$ , it can be seen from (7.2) that the equivalent noise charge attains a minimum when  $C_i$  is made equal to  $C_D$ , that is, when the preamplifier is capacitively matched to the source [3-8]. The value of such a minimum is

$$(7.4) \quad \text{ENC}_{\text{cm}} = \left[ 8kT \frac{\chi}{\omega_T} \frac{h_1}{\tau_M} C_D \right]^{\frac{1}{2}}.$$

According to (7.4) the equivalent noise charge in the condition of capacitive matching is proportional to the square root of the detector capacitance and inversely proportional to the square root of the  $\omega_T \tau_M$  product.

The condition of capacitive matching becomes very important at large detector capacitances [8, 9]. The equivalent noise charge in the mismatched condition,  $C_i \neq C_D$ , can be written as

$$(7.5) \quad \text{ENC}_s = \text{ENC}_{\text{cm}} \frac{1+m}{2m^{\frac{1}{2}}},$$

where  $m = C_i/C_D$ . In the highly mismatched case,  $m \ll 1$ , eq. (7.5) becomes

$$(7.6) \quad \text{ENC}_s \sim \text{ENC}_{\text{cm}} \frac{1}{2m^{\frac{1}{2}}}.$$

To give an example, the case of a detector capacitance of 1000 pF and of a field-effect transistor with  $C_i = 10$  pF will be considered. The condition of capacitive matching, implemented by paralleling devices, would require 100 FET's. This, of course, would be hardly feasible. As pointed out by (7.6), however, the ENC depends on the square root of  $m$ . This means that, while a single field-effect transistor would give  $\text{ENC} \sim 5 \text{ ENC}_{\text{cm}}$ , two paralleled devices would give  $\text{ENC} \sim 3.5 \text{ ENC}_{\text{cm}}$  and four would lead to  $\text{ENC} \sim 2.5 \text{ ENC}_{\text{cm}}$ . So a considerable improvement is achieved by reducing the mismatch from  $C_i/C_D = 10^{-1}$  to  $C_i/C_D = 4 \cdot 10^{-2}$  with a reasonable number of devices in parallel. Going up with  $m$ , as pointed out by (7.5), the ENC becomes less and less sensitive to further paralleling of devices, until the feasibility of the method is impaired by the large number of devices required.

At short values of  $\tau_M$ , such that the contribution of the parallel noise to the total ENC is negligible with respect to the contribution due to the collector

shot noise, capacitive matching can be realized more easily with bipolar transistors.

To achieve capacitive matching, two parameters are available. They are the input capacitance, which can be controlled through the collector current, and the number of devices which can be connected in parallel. By fixing in each transistor a relatively large current, but not too large in order to avoid possible  $\beta$  drops due to high injection effects, the capacitive matching can be achieved with a smaller number of devices than in the case of field effect transistors.

A different approach to the capacitive matching between detector and preamplifier is based upon the use of a transformer between them (fig. 7.1).

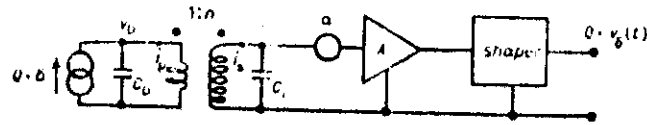


Fig 7.1. - Capacitive matching based upon transformer coupling between detector and preamplifier;  $v_s = n v_o$ ,  $i_s = -n i_o$ .

The contribution given by the white component  $a$  in the series generator to the total equivalent noise charge referred to the input of the processor 7.1 is

$$(7.7) \quad ENC_a^2 \sim 2kT \frac{1}{g_m} \frac{(C_d + n^2 C_i)^2}{n^2} \frac{h_1}{r_M} \sim 2kT \frac{1}{\omega_T r_M} \frac{h_1}{n} \left( \frac{1}{\sqrt{C_d}} + n \sqrt{\frac{C_i}{C_d}} \right)^2 C_d.$$

The contribution  $ENC_a^2$  presents a minimum, as a function of  $n^2$ , for  $n^2 = C_d/C_i$ , that is, when the condition of capacitive matching is met. Introducing the gain-bandwidth product  $\omega_T = g_m/C_d$  into (7.7), it is easily recognized that  $ENC_{a,cm}$ , that is, the equivalent noise charge in the situation of capacitive matching, has again the expression (7.4).

The transformer based approach to capacitive matching has some limitations. A real transformer would add to the total ENC a noise contribution connected with magnetic losses in the core. The finite magnetizing inductance would seriously spoil the signal-to-noise ratio unless bipolar clamping is employed. Besides, transformer coupling would be unsuitable if the preamplifiers have to be located in a strong magnetic field. Exhaustive analysis of the domains of application of transformers and of their intrinsic limitations is available in the literature [8, 10, 11].

Both ways of realizing the capacitive matching between detector and preamplifier reviewed so far aim at achieving an adequately high charge transfer from the detector to the preamplifier. It has to be pointed out, in passing, that the capacitive matching corresponds to the best transfer of the

energy available at the source to the capacitance  $C_i$  connected in parallel to  $C_d$ .

With reference to fig. 7.2, the energy stored on  $C_i$  is

$$\frac{1}{2} C_i V_i^2 = \frac{1}{2} C_i \frac{Q^2}{(C_d + C_i)^2} = \frac{Q^2}{2C_d} \frac{C_i/C_d}{(1 + C_i/C_d)^2}.$$

This expression attains a maximum for  $C_i = C_d$ .

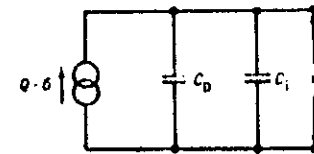


Fig. 7.2. - Energy transfer to  $C_i$ .

Capacitive matching can be employed also to reduce the contribution to the ENO arising from the  $1/|\omega|$  component of the spectral power density in the series generator. Such a feature is important in the application of MOS transistors to amplification of signals from capacitive sources [5].

The problem of improving the charge transfer between detector and preamplifiers can be tackled also in other ways that can be considered either alternative or complementary to the ones discussed so far.

For instance, charge swinging between detector and preamplifier separated by a decoupling inductor has already been proven to be feasible and to give a nonnegligible reduction in the equivalent noise charge.

All the previous considerations, though important in any situation, will become of utmost importance in the applications of silicon calorimetry, where detector capacitances of some nF are foreseen.

## REFERENCES

- [1] V. RADEKA: *IEEE Trans. Nucl. Sci.*, NS-20, 182 (1973).
- [2] P. REHAK: *Detection and signal processing in high-energy physics*, in *Proc. S.I.F., Course LXXXIV* (1981), *Data Acquisition in High Energy Physics*, edited by G. BOLOGNA and M. L. VINCELLI (North-Holland Publ. Co., Amsterdam, 1983), p. 26.
- [3] E. GATTI, A. HRIBON and P. F. MANFREDI: *IEEE Trans. Nucl. Sci.*, NS-30, 319 (1983).
- [4] V. RADEKA: *Position sensitive semiconductor detectors-Signal formation, noise and position readout*, in *Solid State Detectors for High Energy Physics, Proceedings of a Workshop held at Frascati, October 15-16, 1981*, edited by T. FINKEL (1981), p. 21.
- [5] V. RADEKA: *Nucl. Instrum. Methods Phys. Res.*, 226, 209 (1984).

- [6] P. F. MANFREDI: *Solid state telescopes for lifetime measurements*, in *Proceedings of the Conference on the Search for Charm, Beauty and Truth at High Energies*, edited by G. BELLINI and S. C. C. TING (Plenum Press, New York, N. Y., 1984), p. 257.
- [7] C. BUSSOLATI, S. COVA and E. GATTI: *Z. Angew. Math. Phys.*, 22, 14 (1971).
- [8] V. RADEKA: *Summary of noise relations for liquid argon ion chamber. Calorimeter notes*, Brookhaven National Laboratory, 9-1973 (1973).
- [9] P. F. MANFREDI and F. RAUSA: *Nucl. Instrum. Methods Phys. Res. A*, 235, 345 (1985).
- [10] E. GATTI, P. F. MANFREDI and D. MARIOLI: *Nucl. Instrum. Methods Phys. Res.*, 193, 539 (1982).
- [11] R. L. CHASE, C. GRUHN, A. HRISOBO, C. VALETTE and G. WAYSAND: *Fast electronics for a metastable superconducting detector*, in *Proceedings of the II ISPR Nuclear Electronics Symposium*, Publ. EUR 5370e (1975), p. 29.

### 8. - Bipolar and junction field-effect transistors in elementary preamplifier configurations.

8.1. *Preamplifiers employing a single active device.* - Some elementary preamplifier configurations employing bipolar transistors and junction field-effect transistors will be discussed here. They are the common-emitter connection of fig. 8.1a), the common-base connection of fig. 8.1b) and the common-source connection of fig. 8.1c) [1-5].

The three elementary preamplifiers of fig. 8.1 are followed by three different shapers, labelled, respectively, I, II, III, designed in such a way that the three circuits respond to a  $\delta$ -impulse detector current with the same output signal  $Q \cdot v_s(t)$ . To evaluate the equivalent noise charge, the active devices will be represented with the circuit models of fig. 5.2, 5.4.

The shapers of fig. 8.1 will be again considered to be noiseless and presenting to the collector of the transistor or to the drain of the JFET an ideal, current-sensitive input. The total input capacitance of the bipolar transistor will be assumed to be equal to  $C_n$  and that of the JFET to be  $C_i = C_{os} + C_{on}$ . The time domain approach of subsect. 4.2.2 will be employed for noise calculations and the measuring time  $\tau_M$  of the shaper will be adopted as a reference variable, by putting in (4.16)  $u = t/\tau_M$ . The following parameters will be introduced:

$$(8.1) \quad h_1 = \int_{-\infty}^{\infty} [v_s(u)]^2 du,$$

$$(8.2) \quad h_2 = \int_{-\infty}^{\infty} [v_d(u)]^2 du.$$

For the sake of simplicity, the index o used in the previous section to label the input capacitance and the transconductance of the individual devices will be omitted.

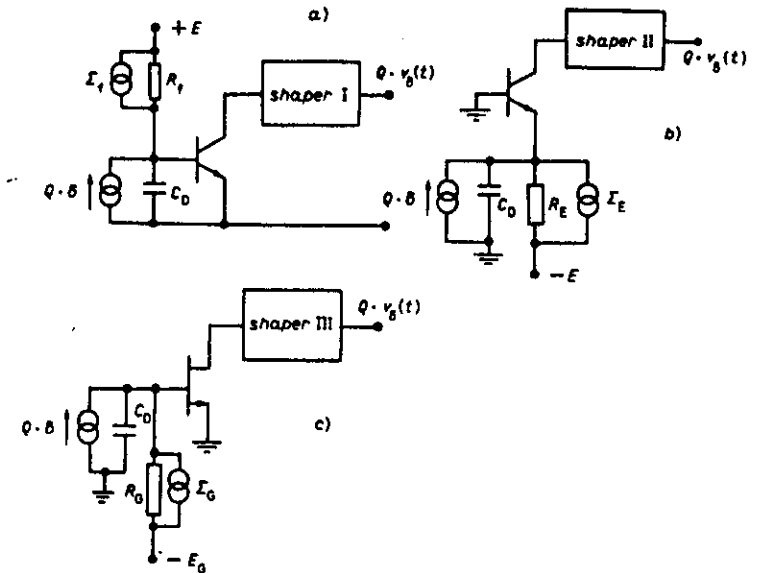


Fig. 8.1. - Elementary preamplifier configuration employing bipolar and junction field-effect transistors: a) common emitter, b) common base, c) common source.

The three bias resistors  $R_i$ ,  $R_c$ ,  $R_s$  are of negligible importance from the point of view of signal shaping, while their contribution to the noise may be of some significance.

For this reason, they will be considered to be infinitely large in the evaluation of the transfer functions, while their thermal-noise contribution will be accounted for by the relevant current sources with spectral densities

$$(8.3) \quad \Sigma_{R_i} = \frac{2kT}{R_i}, \quad \Sigma_{R_c} = \frac{2kT}{R_c}, \quad \Sigma_{R_s} = \frac{2kT}{R_s}.$$

A similar consideration allows to short circuit  $R_{nn}$  in the model of fig. 5.4, yet accounting for its thermal-noise contribution with the ideal series voltage source with spectral power density  $\Sigma_{nn}$  [6, 7].

By carrying through the calculations, the following expressions are obtained for the equivalent noise charge referred to the input of the circuits of fig. 8.1:

#### a) Common-emitter amplifier

$$(8.4) \quad \text{ENC} = \left\{ kT \left[ \left( C_n \frac{(1+\gamma)^2}{\gamma} \frac{1}{\omega_s} + 2R_{nn} C_n^2 \right) \frac{h_1}{\tau_M} + \left( \frac{\omega_s}{\beta} \frac{C_D}{\gamma} + \frac{2}{R_i} \right) h_2 \tau_M \right] \right\}^{1/2}$$

m.e.s. coulomb.

## b) Common-base amplifier

$$(8.5) \quad \text{ENC} = \left\{ kT \left[ \left( C_D \frac{(1+\gamma)^2}{\gamma} \frac{1}{\omega_a} + 2R_{bb} C_D^2 \right) \frac{h_1}{\tau_M} + \left( \frac{\omega_a}{\beta} \frac{C_D}{\gamma} + \frac{2}{R_E} \right) h_2 \tau_M \right] \right\}^{1/2}$$

r.m.s. coulomb.

In these relationships  $\beta = I_C/I_B$  is the transistor current gain, and  $\omega_a = g_{m,0}/C_D$  is the alpha-cut-off angular frequency of the device.

## c) Common-source amplifier

$$(8.6) \quad \text{ENC} = \left[ 2kT \cdot 0.7 C_D \frac{(1+\gamma)^2}{\gamma} \frac{1}{\omega_a} \frac{h_1}{\tau_M} + \left( qI_C + \frac{2kT}{R_E} \right) h_2 \tau \right]^{1/2}$$

r.m.s. coulomb.

In eqs. (8.4), (8.5),  $\gamma = C_D/C_u$  and in eq. (8.6)  $\gamma = C_D/C_i$  represent the detector capacitance normalized to the device input capacitance.

Comparison between (8.4) and (8.5) leads to the following comments. The equivalent noise charge for the common-base and the common-emitter connections differs only for the noise due to the bias resistors  $R_i$ ,  $R_E$ . Usually  $R_i$  is much larger than  $R_E$  and, therefore, the common-emitter configuration has a slightly smaller ENC.

The equivalent noise charge of a bipolar transistor has two terms in the bracket which multiplies the coefficient  $kT(h_1/\tau_M)$ .

One, determined by the shot noise in the collector current can be expressed as  $(C_D + C_u)/g_m$ . The other one, determined by the thermal noise in the base spreading resistance is  $2R_{bb}C_D^2$ . At values of  $C_D$  much larger than  $C_u$ , both terms are proportional to  $C_D^2$ . If the collector current is raised beyond 1 mA, the dominant term becomes the one related with the base spreading resistance.

Instead, at values of  $C_D$  smaller than  $C_u$ , the noise contribution due to the base spreading resistance can be neglected [5].

In the absence of noise due to the external bias network (8.4) and (8.5) become equal. The ENC as a function of  $\gamma$  presents a minimum for

$$(8.7) \quad \gamma = \left[ 1 + \frac{h_2}{h_1} \frac{\omega_a \tau_M}{\beta} \right]^{1/2}.$$

If  $\tau_M$  is so short that  $\omega_a^2 \tau_M^2 \ll \beta(h_1/h_2)$ , the minimum ENC occurs for  $\gamma = 1$ , that is, in the condition of capacitive matching between detector and transistor input capacitance [4, 8].

The behaviour of the ENC as a function of  $\gamma^{-1}$  is illustrated in fig. 8.2. Each set of curves is drawn for a fixed value of  $\omega_a \tau_M$  and with the normalized base spreading resistance  $R_{bb} \omega_a C_D$  as a parameter.

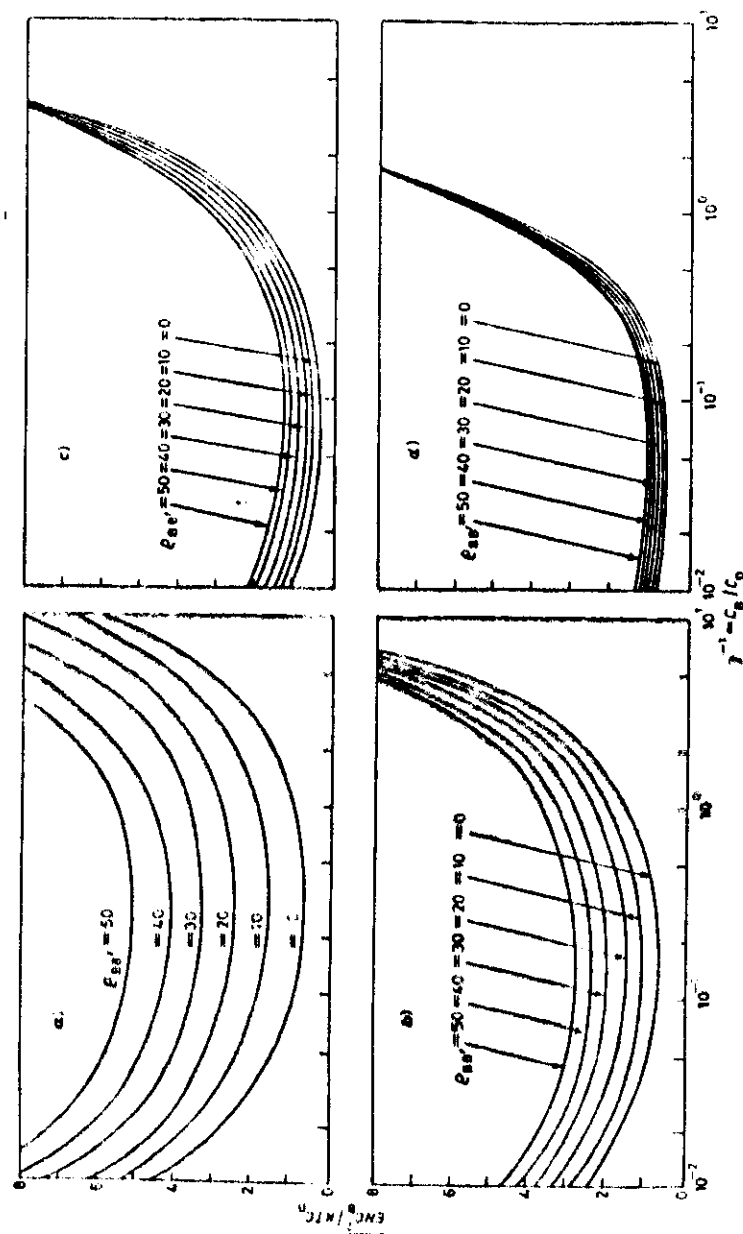


Fig. 8.2 - Normalized square of the equivalent noise charge as a function of  $\gamma^{-1}$  for a bipolar transistor,  $\beta = 100$ : a)  $\tau_M = 60/\omega_a$ , b)  $\tau_M = 120/\omega_a$ , c)  $\tau_M = 240/\omega_a$ , d)  $\tau_M = 480/\omega_a$ .

The curves of fig. 8.2 clearly show that the value of  $\gamma$  at which the minimum ENC occurs moves away considerably from  $\gamma = 1$  as  $\omega_a \tau_M$  is increased due to the effect of parallel noise.

For a given  $C_D$  and for assigned values of  $\beta$ ,  $\omega_a$ ,  $\tau_M$ , the value of  $C_B$  which gives the best ENC can be easily fixed by adjusting the collector standing current.

Comparison between the expected behaviour and some values of the ENC measured at different  $\gamma$ 's is shown in fig. 8.3.

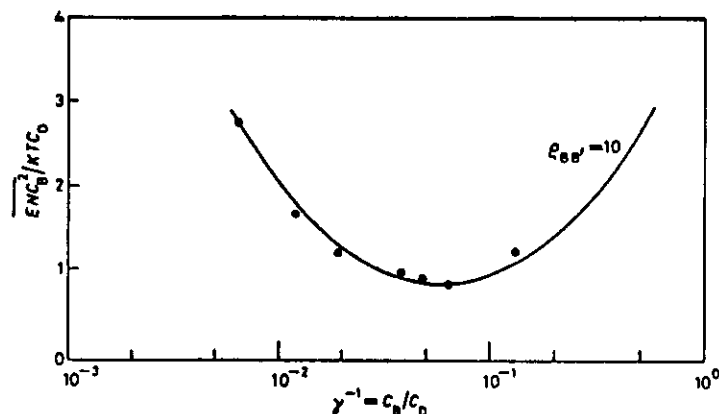


Fig. 8.3. — Comparison between the theoretical behaviour (—) of the ENC as a function of  $\gamma^{-1}$  and some experimental points (•);  $\beta = 50$ ,  $C_D = 100$  pF,  $\tau_M = 180/\omega_a$ .

In the case of a field-effect transistor, as pointed out by eq. (8.3), the value of  $\gamma$  which makes the ENC minimum is  $\gamma = 1$ . This condition of capacitive matching, however, cannot be met as  $C_D$  is varied, by using a single field-effect transistor. A way of controlling  $C$ , in a broad range of values through an electrical parameter does not exist in a field-effect transistor.

**8.2. Preamplifiers employing parallel connections of bipolar and junction field-effect devices.** — As pointed out in the previous subsection, the minimum of ENC as a function of  $\gamma$  can, in principle, be achieved in a bipolar transistor by adjusting the collector current as  $C_D$  is varied. The minimum ENC in the case of field-effect transistors occurs when the capacitive matching is satisfied and this requires, as discussed in sect. 7, the parallel connection of the devices. Also in the case of bipolar transistors, however, the parallel connection has a beneficial effect, that of reducing the noise contribution due to  $R_{BB}$ . This effect is demonstrated in fig. 8.4. The curve a) of fig. 8.4 shows the dependence of the ENC on the detector capacitance for a single bipolar transistor working at 1 mA collector current.

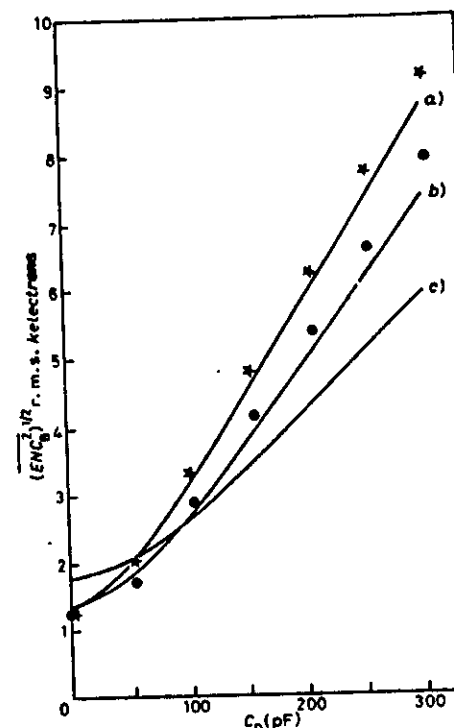


Fig. 8.4. — Reduction of the noise contribution due to  $R_{BB}$  achieved by paralleling two bipolar transistors: — experimental curves; theoretical values: \* case a), • case b),  $R_{BB} = 15 \Omega$ ,  $\beta = 10^3$ ,  $\omega_a = 2\pi \cdot 10^6$  rad/s.

The curve b) shows the same dependence for the parallel connection of two transistors, working at 0.5 mA each.

Comparison between curves a) and b) confirms that the value of the ENC at  $C_D = 0$  is unchanged, on account of the fact that the total base current and, therefore, the parallel noise are unaltered, while the sensitivity of the ENC to the detector capacitance is reduced.

Such a reduction results from the decreased noise contribution from  $R_{BB}$ , achieved by paralleling two transistors. Curve c) refers to two transistors in parallel, each working at a 1 mA collector current.

This situation brings about an increase in the parallel noise and, therefore, a larger ENC at  $C_D = 0$  and a further reduction in the sensitivity of the ENC to  $C_D$ , which accounts for the reduced contribution of the collector shot noise.

Next, the question arises of finding, for given values of  $C_D$  and  $\tau_M$  and fixed  $h_1$  and  $h_2$ , the parallel combination of bipolar transistors which gives the

minimum ENC. Two possibilities exist. One consists in fixing the number  $n_p$  of transistors to be paralleled and determining accordingly the value of  $C_D$  in the individual transistors which gives the minimum ENC.

The other one consists in fixing the standing current in the individual transistors and in finding out the number  $n$  of devices that have to be paralleled to achieve the minimum ENC. The former approach, fixed number, gives for  $\gamma_{min}$  and  $ENC_{n,min}$  the following values:

$$(8.8) \quad \gamma_{min} = n_i \left[ 1 + \frac{\omega_s^2 \tau_M^2}{\beta} \frac{h_2}{h_1} \right]^{-1/2},$$

$$(8.9) \quad ENC_{n,min} = 2kTC_D \frac{h_1}{\omega_s \tau_M} \left\{ \sqrt{1 + \frac{\omega_s^2 \tau_M^2}{\beta} \frac{h_2}{h_1}} + 1 + \frac{R_{BB'}}{n_i} \omega_s C_D \right\}.$$

The approach based upon a fixed working point in the individual transistors, that is,  $C_D$  assigned, gives for the number of devices  $n_{min}$  which minimizes the ENC and for  $ENC_{n,min}$ , respectively,

$$(8.10) \quad n_{min} = \frac{C_D}{C_{D0}} \left[ \frac{1 + 2\omega_s R_{BB'} C_D}{1 + (h_2/h_1)(\omega_s^2 \tau_M^2/\beta)} \right]^{1/2}$$

and

$$(8.11) \quad ENC_{n,min} = 2kTC_D \frac{h_1}{\omega_s \tau_M} \left\{ 1 + \sqrt{\left( 1 + \frac{\omega_s^2 \tau_M^2}{\beta} \frac{h_2}{h_1} \right) \left( 1 + 2R_{BB'} C_D \omega_s \frac{C_D}{C_{D0}} \right)} \right\}.$$

For the field-effect transistor the minimum ENC is achieved for  $\gamma = 1$ . As in most of high-energy applications  $\tau_M$  is short enough to make the parallel noise contribution negligible, the equivalent noise charge  $ENC_{FET,min}$  can be expressed as

$$(8.12) \quad ENC_{FET,min} = \left\{ 8kT \cdot 0.7 C_D \frac{h_1}{\omega_T \tau_M} \right\}^{1/2}.$$

**8.3. Comparison between input stages employing bipolar and junction field-effect transistors.** - The analysis of subject 8.1 and 8.2 opens up the possibility of comparing different configurations of bipolar and field-effect input stages, determining in the  $(C_D, \tau_M)$  plane the regions where either component gives better noise performances [4]. The present discussion will be carried on in connection to the piecewise parabolic shaper whose parameters  $h_1$  and  $h_2$  have the following values:

$$h_1 = \frac{8}{3}, \quad h_2 = \frac{14}{15}.$$

It will be assumed that the devices employed in the input stages are a bipolar transistor with a 1 GHz  $f_n$  and a junction field-effect transistor with  $g_m = 10$  mA/V and  $C_i = 10$  pF, featuring accordingly a  $10^8$  rad/s  $\omega_T$ .

As a first case, comparison will be made between a bipolar transistor whose working point is adjusted at each value of  $C_D$  so as to satisfy (8.7) and the parallel combination of JFETs which at every  $C_D$  ensures the capacitive matching. The boundary curves that in the  $(C_D, \tau_M)$ -plane limit the regions where the former input stage has smaller or larger ENC than the latter one are shown in fig. 8.5. The current gain  $\beta$  is assumed as a parameter. It is shown in fig. 8.5

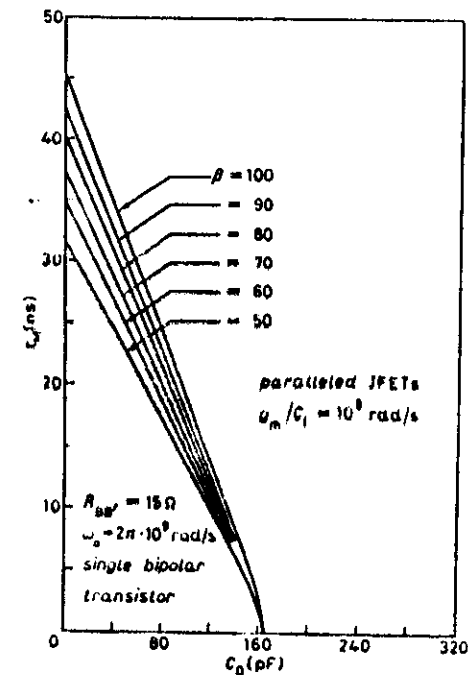


Fig. 8.5. - Boundary curves in the  $(C_D, \tau_M)$ -plane separating the regions where a single bipolar transistor gives a smaller or a larger ENC than the parallel combination of JFETs required for capacitive matching.

that, at any  $\tau_M$ , a value of  $C_D$  can be found beyond which the bipolar stage becomes more noisy than the JFET one. This can be explained by the remark that, according to the equivalent circuit of fig. 5.4, as  $C_D$  increases, the fraction of the noise generator  $\Sigma_{BB'}$  which appears at the B/E port increases.

As a second step, the parallel combination of  $n_{min}$  transistors,

is the number which, according to (8.10), minimizes  $ENC_n$  in the operation at fixed  $C_{D0}$ , can be compared to the parallel combination of JFETs required for capacitive matching. The new boundary curves in the  $(C_D, \tau_M)$ -plane are horizontal straight lines. That is, for a given  $\beta$  of the bipolar transistor, the bipolar stage is superior, regardless of the value of  $C_D$  up to a certain  $\tau_M$  beyond which the FET stage takes over (fig. 8.6). The parallel combination of JFETs required

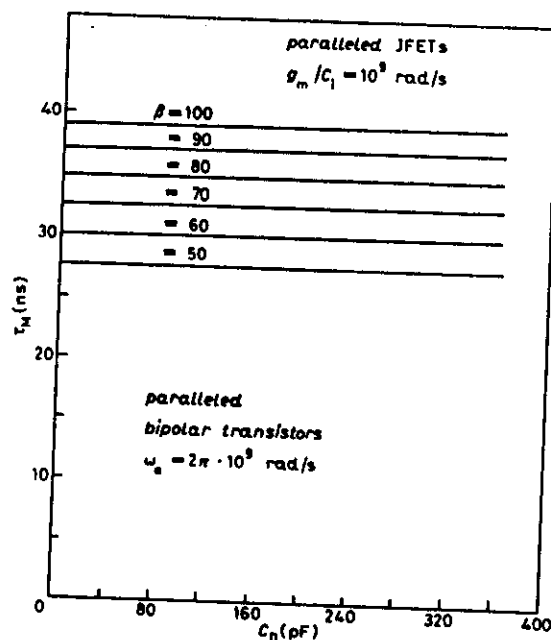


Fig. 8.6. - Boundary curves separating in the  $(C_D, \tau_M)$ -plane the regions where the parallel combination of the optimum number of bipolar transistors, each working at a fixed  $C_D$ , features a smaller or a larger ENC than the parallel combination of JFETs required for capacitive matching.

for capacitive matching may be unpractical, especially if the number of pre-amplifiers is large. It makes, therefore, sense to compare input stages consisting of a single device of either type. In this case the advantage of the bipolar transistor, which allows an optimization of the ENC based upon the simple control of its working point, emerges. This is confirmed by the boundary curves of fig. 8.7. As shown in fig. 8.7, the region where the bipolar stage features a smaller ENC than the JFET one is considerably broader than in the previous cases. In particular, the larger is  $C_D$ , the wider is the range of values of  $\tau_M$  where the

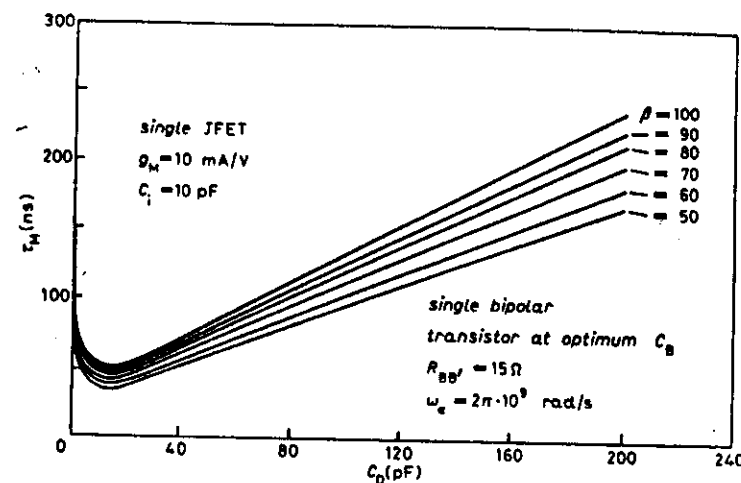


Fig. 8.7. - Boundary curves separating the regions where a single bipolar transistor put to work in the optimum condition (8.7) features a smaller or a larger ENC than a single JFET.

bipolar transistor has a better noise behaviour. This property of the bipolar transistor was successfully exploited in the early days of application of the silicon telescopes in high-energy experiments [1].

## REFERENCES

- [1] P. F. MANFREDI, D. MISLER and V. VAROLI: *Limitations in the performances of silicon detectors used as live targets in high energy experiments*, in *Proceedings of the Topical Seminar on Interactions of Elementary Particles with Nuclei*, edited by G. P. BELLINI and G. BERTOCCHI (Trieste, 1970), p. 275.
- [2] A. HRISOHO and K. THUONG: *IEEE Trans. Nucl. Sci.*, NS-27, 329 (1980).
- [3] A. HRISOHO: *Silicon detector electronics for high energy experiments*, in *Proceedings of the Conference on the Search for Charm, Beauty and Truth at High Energies, Erice, November 1981*, edited by G. BELLINI and S. C. C. TING (Plenum Press, New York, N. Y., 1984), p. 457.
- [4] E. GATTI, A. HRISOHO and P. F. MANFREDI: *IEEE Trans. Nucl. Sci.*, NS-30, 319 (1983).
- [5] E. GATTI and P. F. MANFREDI: *Nucl. Instrum. Methods Phys. Res.*, 226, 142 (1984).
- [6] H. F. KNOTT, H. SUTCLIFFE and C. ENG: *Proc. IEE*, 120, 623 (1973).
- [7] A. WILLEMSSEN and N. BELL: *IEEE J. Solid-State Circuits*, SC-15, 245 (1980).
- [8] H. SPIELER and E. E. HALLER: *IEEE Trans. Nucl. Sci.*, NS-32, 419 (1985).

## 9. - Feedback preamplifier structures.

9.1. *Block diagram analysis.* - With the only exception of the grounded-base connection, which is sometimes employed alone and has as attractive features a short risetime and a good transmission stability, all the other configurations are rather intended as input stages of feedback loops.

Under hypotheses of broad validity, the expressions of the ENC as well as the results of the comparative discussions made on stages consisting of one or more paralleled devices can be extended to the case in which the reviewed configurations are employed as input stages of feedback preamplifiers. It is sufficient for this to assume that i) the second-stage noise is negligible and ii) the feedback loop around the preamplifier does not change the signal transfer function from the detector to the shaper output as well as the noise weighting function, except for a constant factor. This statement has been implicitly already checked in subsect. 8.1, when it has been observed that the common-emitter and the common-base configurations, assuming the noise from the external bias resistors negligible, have the same ENC.

The present section is devoted to feedback preamplifier structures employing more active elements in the loop (1).

The basic configurations that will be discussed are shown in fig. 9.1 along with the input-to-output transfer functions evaluated in the simplified hypothesis of infinite gain in the amplifying modules. They are, respectively, a charge-sensitive configuration (a), a current-sensitive configuration (b) and a voltage-sensitive one (c) [2-6].

To a better approximation, the three amplifying blocks of fig. 9.1 will be represented with a single-pole transfer function

$$(9.1) \quad T(s) = \frac{g_{m,T} R}{1 + sRC},$$

where  $g_{m,T}$  is the total transconductance of the block and the parallel of  $R$  and  $C$  is the internal load across which the current  $g_{m,T}v_i$  flows, thereby producing the voltage gain. In general  $g_{m,T}$  may be greater than the transconductance of the input stage, for a current gain may exist between input stage and  $R$ ,  $C$  load. Only when such a current gain is unity,  $g_{m,T} = g_m$ . The gain-bandwidth product of the amplifying block is  $g_{m,T}/C$  and this is usually smaller than the  $\omega_T$  of the devices employed in the input stage.

The input capacitance of the amplifying blocks with the main feedback open is labelled  $C_i$  in fig. 9.1.

9.1.1. *Charge-sensitive configuration.* The charge-sensitive configuration is shown in fig. 9.1a. The input signal is a current  $i_s$  which is converted into a voltage  $v_i$  by the input capacitance  $C_i$ . The feedback resistor  $R_f$  and the feedback capacitor  $C_f$  are connected in parallel between the output and the input of the amplifier. The output voltage  $v_o$  is given by the expression  $v_o = \frac{Q}{C_f} \exp[-t/R_f C_f]$ .

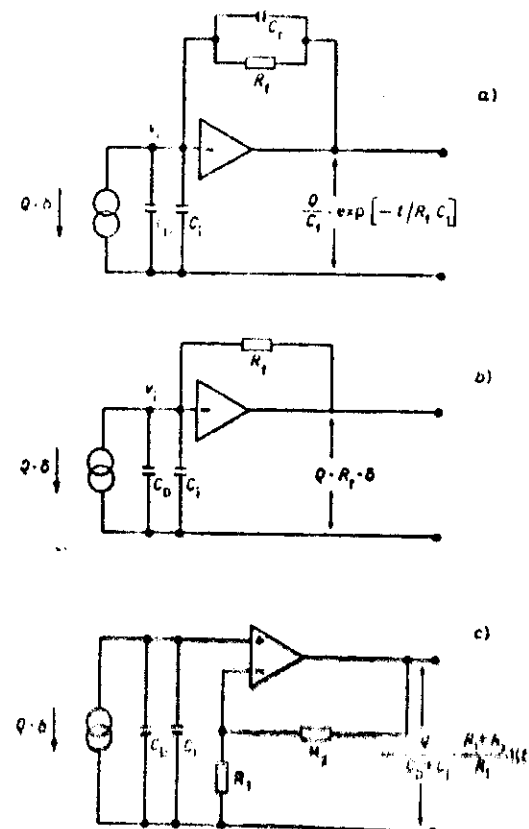


Fig. 9.1. - Feedback preamplifier configurations: a) charge sensitive, b) current sensitive, c) voltage sensitive.

contribution from the external network can be made negligible. The feedback resistor  $R_f$  serves the only purpose of providing a d.c. return for the bias currents of the input device. It can be made as large as  $10^8 \Omega$  in the case of a field-effect transistor input and of the order of  $10^4 \Omega$  or more with a bipolar input stage. Throughout the range of values of  $r_m$  either device is intended to cope with, the contribution to the ENC arising from the thermal noise in  $R_f$  is usually of minor importance.

Evaluation of the impedance  $Z_i$  presented by the charge-sensitive loop in parallel to the  $C_i + C_f$  capacitance shows the following interesting property. If the internal time constant  $RC$  is made equal to  $R_f C_f$ , the magnitude  $|Z_i|$  has the frequency dependence shown in fig. 9.2. The d.c. gain  $g_{m,T} R$  is assumed to be much greater than unity, it can be concluded that  $Z_i = C_f/g_{m,T} C_i$  from d.c. up to a

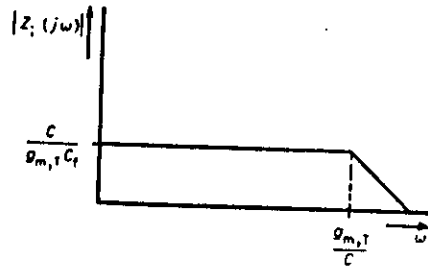


Fig. 9.2. - Frequency dependence of the magnitude of  $Z_i$  for a charge-sensitive loop.

frequency determined by the gain-bandwidth product of the amplifying block [1, 2].

Such a resistive term, which depends only on the total transconductance  $g_{m,T}$  and on the ratio of two capacitors, under certain conditions can be less noisy than a physical resistor of the same value. For this reason it deserves the denomination of «cold resistor» [2-4].

The values of the cold resistor can be varied within a broad range by a suitable choice of  $g_{m,T}$ ,  $C_i$ ,  $C$ .

Small values of the cold resistor are required in segmented detectors in order to reduce the time constants with which the spurious cross-talk signals appearing on the electrodes next to the collecting one die off.

The evaluation of the transfer functions in the charge-sensitive loop can be done on the basis of the equivalent circuit of fig. 9.3 which puts the cold resistor into evidence.

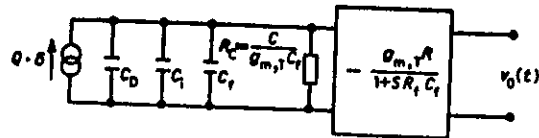


Fig. 9.3. - Equivalent circuit of the charge-sensitive loop with the cold resistance into evidence.

The amplifying block of fig. 9.3 is represented by the transfer function  $-g_{m,T}R/(1 + sR_iC_i)$ , in which the cancellation condition  $RC = R_iC_i$  has been employed.

Usually  $R_iC_i \gg (C_D + C_i + C_i)(C/g_{m,T}C_i)$  and, therefore, the risetime of the output signal  $v_0(t)$  in response to the  $\delta$ -impulse detector current is determined by the input time constant,

$$(9.2) \quad \tau_i = (C_D + C_i + C_i) \frac{C}{g_{m,T}C_i}.$$

As pointed out by (9.2), the preamplifier risetime depends on the total capacitance shunting the preamplifier input port and on the cold resistance. Equation (9.2) can be, alternatively, written as

$$(9.3) \quad \tau_i = \frac{1}{g_{m,T}/C} \frac{1}{C_i/(C_D + C_i + C_i)},$$

which expresses  $\tau_i$  as the reciprocal of the gain-bandwidth product of the amplifying block divided by the feedback return ratio. As pointed out by (9.2), in order to keep the preamplifier risetime small at large detector capacitance, the cold resistance has to be adequately reduced, which can be done by increasing  $C_i$  if the gain-bandwidth product of the amplifying block is assumed to be constant.

The signal at the output of a charge-sensitive preamplifier has usually a very large decay time constant, according to the need of keeping  $R_i$  large to reduce its thermal-noise contribution. In order to clip this signal, without introducing undesired negative tails, pole-zero cancellation is performed, as shown in fig. 9.4.

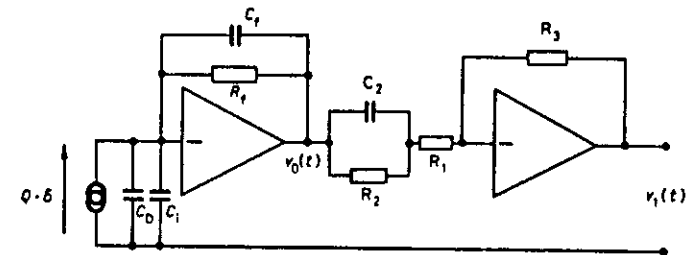


Fig. 9.4. - Signal clipping by pole-zero cancellation at the output of the charge-sensitive loop.

The foregoing analysis deals with the slow exponential decay at the output of the charge-sensitive loop and, therefore, the risetime portion of  $v_0(t)$  can be neglected, by writing

$$(9.5) \quad V_0(s) = \frac{Q}{sC_i} \frac{s}{s + 1/R_iC_i}.$$

The voltage  $v_1(t)$  can be expressed in the Laplace domain as

$$(9.6) \quad V_1(s) = -V_0(s) \frac{R_2}{R_1} \frac{s + 1/R_2C_2}{s + (R_1 + R_2)/(R_1R_2)(1/C_2)} = -\frac{R_2}{R_1} \frac{s + 1/R_2C_2}{s + (R_1 + R_2)/(R_1R_2)(1/C_2)} \frac{Q}{sC_i} \frac{s}{s + 1/R_iC_i}.$$

If the zero  $s = -1/R_2 C_2$  in eq. (9.6) is chosen so as to cancel the pole  $s = -1/R_1 C_1$ , the previous relationship becomes

$$(9.7) \quad V_1(s) = -\frac{Q R_2}{s C_1 R_1 s + ((R_1 + R_2)/R_1 R_2) (1/C_2)},$$

which shows that the original signal  $v_0(t)$  has undergone clipping to the reduced time constant  $(R_1 R_2 / (R_1 + R_2)) C_2$  and amplification determined by the  $R_2/R_1$  ratio.

If  $R_1$  is made equal to zero, eq. (9.6) yields

$$(9.8) \quad V_1(s) = -V_0(s) R_2 C_2 \left( s + \frac{1}{R_1 C_1} \right) = -Q \frac{C_2}{C_1} R_2 \frac{s + 1/R_1 C_1}{s + 1/R_2 C_2}.$$

If the condition of pole-zero cancellation  $R_2 C_2 = R_1 C_1$  is met again, (9.8) becomes

$$(9.9) \quad V_1(s) = -Q \frac{C_2}{C_1} R_2.$$

In this case the input-to-output relationship is a real transresistance  $-(C_2/C_1)R_2$ . Within the intrinsic band width limitations of the amplifying blocks, the configuration 9.1 preserves the detector current shape.

It can be concluded that a charge-sensitive loop followed by a pole-zero cancellation with  $R_1 = 0$  implements a current-sensitive configuration. Such a configuration has better noise performances than the one employing resistive feedback, shown in fig. 9.1b).

**9.1.2. Current-sensitive configuration.** The current-sensitive configuration of fig. 9.1b) presents, besides the noise sources in the preamplifier module, the thermal-noise contribution from the feedback resistor  $R_1$ . This resistor cannot be made too large, otherwise the pole determined by  $R_1$  and the stray capacitance across it would spoil the closed-loop risetime.

The frequency dependence of the magnitude of the input impedance  $Z_1$ , presented by the current-sensitive preamplifier in parallel to  $C_D + C_1$  is shown in fig. 9.5. In the frequency range of interest the input impedance has an inductive character. This may result into an oscillating response at certain values of the detector capacitance. At very small detector capacitance the circuit is not critical, but its input impedance can be not as small as desired. The advantage of the resistive feedback preamplifier over the other ones is that, in all the applications requiring that the detector current shape be preserved, no pole-zero cancellation is needed. Such an advantage becomes important in connection with detectors involving large numbers of acquisition channels, like microcalorimeters, where individual pole-zero cancellation on thousands of preamplifiers is not practicable.

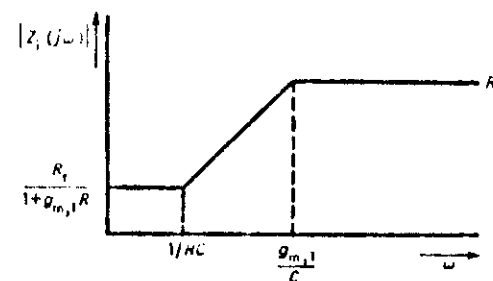


Fig. 9.5. — Frequency dependence of the magnitude of the input impedance in the current-sensitive configuration.

**9.1.3. Voltage-sensitive configuration.** In the voltage-sensitive configuration of fig. 9.1c) the external feedback network adds a contribution to the white spectral density in the series noise generator. Such a contribution, in the practical case in which  $R_2 \gg R_1$ , is equal to  $2kTR_1$ . By choosing  $R_1$  and  $R_2$  suitably small, that is,  $R_1 \sim 1 \Omega$ , the effect of the external bias network on the preamplifier noise becomes negligible.

The voltage-sensitive configuration presents to the detector an impedance of purely capacitive nature. Such a capacitance has been labelled  $C'_1$  in fig. 9.1c) to emphasize the fact that  $C'_1$ , by virtue of the series voltage feedback, is smaller than the "cold capacitance" of the active devices.

In the configuration 9.1c) the relationship between the charge delivered by the detector and the output voltage depends on the detector capacitance. If, however, the detector is totally depleted and the radiation damage effects during the experiment are negligible,  $C_D$  can be considered stable and reliable enough to guarantee an adequate accuracy in the measurement of  $Q$ .

The voltage-sensitive preamplifier has two interesting features. First, its risetime is independent of the detector capacitance and, therefore, the preamplifier can be fast enough even at very large  $C_D$ 's.

Second, contrary to what happens with the charge-sensitive and current-sensitive configurations, the voltage-sensitive preamplifier does not load the detector appreciably. It becomes, therefore, very suitable to sense the total voltage across stacks of series-connected detectors. This feature has been exploited to perform multiplicity measurements with telescopes of silicon detectors.

**9.2. Basic low-noise inverting amplifier.** — Some examples of low-noise preamplifier design, able to cope with different experimental situations in elementary particle physics, will be described in this subsection. Before going into the analysis of the specific circuits, a rather general circuit diagram of low-noise inverting amplifier will be illustrated. This circuit, shown in fig. 9.6, provides a basic building block around which charge-sensitive loops of the type shown

in fig. 9.1a) or current-sensitive configurations of fig. 9.1b) can be realized.

For reference purposes, the input device of the low-noise inverting amplifier has been represented as a field-effect transistor, but the structure of fig. 9.6 applies equally well to the case of a bipolar-transistor input stage, as well as to any parallel combinations of bipolar or field-effect devices.

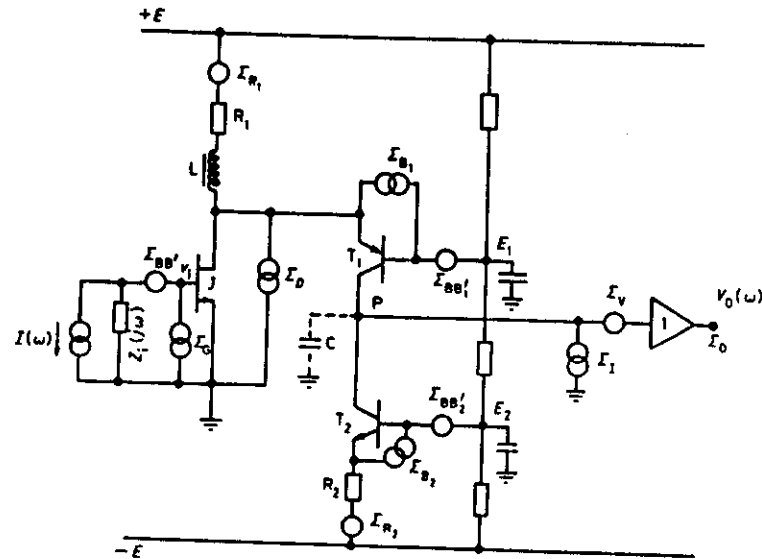


Fig. 9.6. — Basic configuration of low-noise inverting amplifier.

According to the circuit diagram of fig. 9.6 J behaves as a transconductance amplifier which injects a signal current  $g_m v_i$  into the emitter of the common-base transistor  $T_1$ . The collector of  $T_1$  is connected to that of  $T_2$ , which acts as a current source. It is reasonable to assume that the load appearing between P and ground is purely capacitive, so that the open-loop transfer function of the low-noise preamplifier is  $-g_m/sC$ , where  $C$  is the total capacitance shunting P.

The noise sources in the circuit are represented by their spectral power densities. They have the following meaning:

1)  $\Sigma_{bb'}$  is the spectral power density of the generator which accounts for the thermal noise in the base spreading resistor of a bipolar transistor if employed as input device in 9.6.

2)  $\Sigma_o$  is the spectral power density of the parallel noise related to the input bias current,  $\Sigma_o = qI_o$  or  $\Sigma_o = qI_n$  according to whether the input device is a field-effect or a bipolar transistor.

3)  $\Sigma_n$  is the spectral power density of the thermal noise in the channel of a JFET or the power density of the shot noise in the collector current of a bipolar transistor,  $\Sigma_n = 2kT \cdot 0.7 g_m$  or  $\Sigma_n = qI_c$ , respectively.

4)  $\Sigma_{R_1}$ ,  $\Sigma_{R_2}$  are the spectral power densities of the thermal noise in the resistors  $R_1$ ,  $R_2$ ,  $\Sigma_{R_1} = 2kTR_1$ ,  $\Sigma_{R_2} = 2kTR_2$ .

5)  $\Sigma_{n_1}$ ,  $\Sigma_{n_2}$  are the spectral power densities of the shot noise in the base currents of  $T_1$  and  $T_2$ ,

$$\Sigma_{n_1} = qI_{n_1}, \quad \Sigma_{n_2} = qI_{n_2}.$$

6)  $\Sigma_{bb'_1}$ ,  $\Sigma_{bb'_2}$  are the spectral power densities of the thermal noise in the base spreading resistors of  $T_1$  and  $T_2$ .

7)  $\Sigma_i$  and  $\Sigma_v$  are the parallel and series spectral power densities of the output stage.

For the sake of comparing the relative importance of the various noise sources and their relation to the signal, open-loop analysis of the circuit 9.6 is adequate.

In the circuit of fig. 9.6 the contribution of the shot noise in the collector currents of the common-base transistors  $T_1$ ,  $T_2$  to the output spectral power density  $\Sigma_o$  is small compared to the contribution due to the base current noise of the same transistors. For this reason, the generators that would represent the shot noise in the collector currents of  $T_1$  and  $T_2$  have been omitted.

The inductor  $L$  has the purpose of reducing, in the frequency range of interest, the contribution to  $\Sigma_o$  due to  $\Sigma_{n_1}$  and  $\Sigma_{bb'_1}$ . A high quality is required for this component, which must feature very low stray capacitance, must be able to work at relatively large currents without core saturation and must have an inductance between 10 and 100 mH.

The inductor  $L$  is present in most low-noise preamplifiers for nuclear spectroscopy. In high-density preamplifiers for applications in high-energy experiments it may be too bulky, expensive and even not usable if the preamplifiers are located in a strong magnetic field.

For these reasons the total output spectral density of the noise,  $\Sigma_o$ , will be evaluated in the absence of  $L$ , with the understanding that, if  $L$  is present, its effect can be accounted for by putting  $\Sigma_{n_1} = 0$ ,  $\Sigma_{bb'_1} = 0$ .

The output spectral power density is given by

$$(9.10) \quad \Sigma_o(\omega) \simeq \left\{ \Sigma_o |Z_i(\omega)|^2 g_m^2 + \Sigma_{bb'} g_m^2 + \Sigma_n + \frac{\Sigma_{R_1}}{R_1^2} + \frac{\Sigma_{bb'_1}}{R_1^2} + \right. \\ \left. + \Sigma_{n_1} + \frac{\Sigma_{R_2}}{R_2^2} + \frac{\Sigma_{bb'_2}}{R_2^2} + \Sigma_{n_2} + \Sigma_i \right\} \frac{1}{\omega^2 C^2} + \Sigma_v.$$

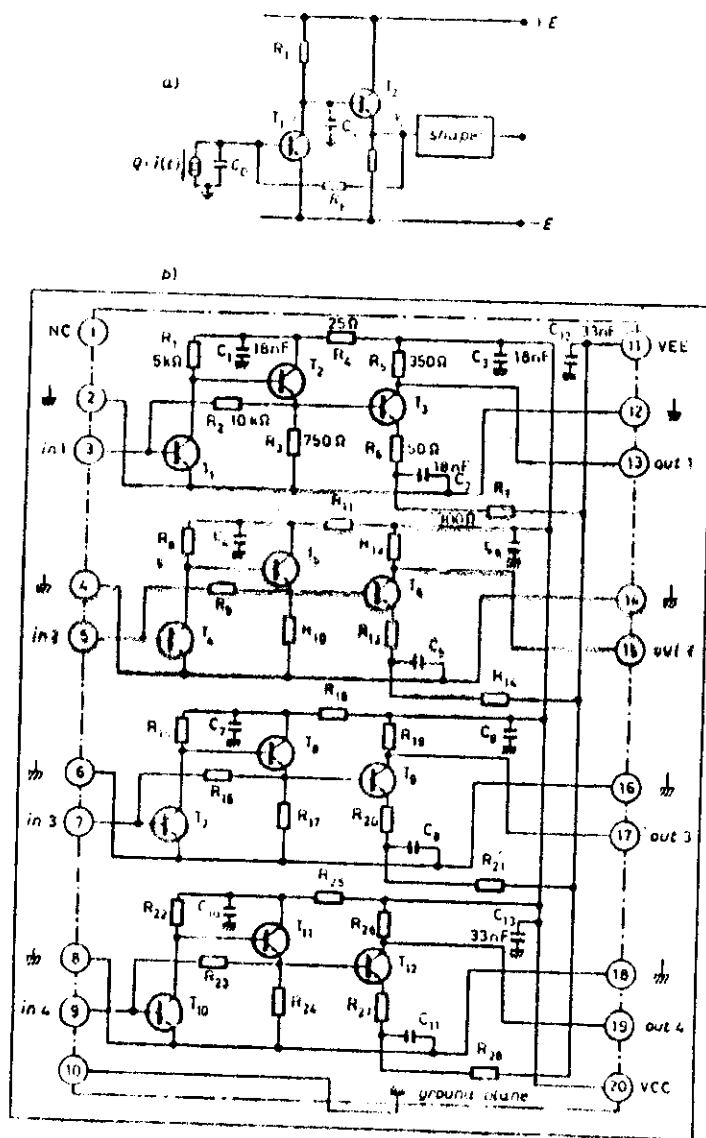


Fig. 9.7. - Current-sensitive preamplifier. a) basic preamplifier configuration followed by a shaper. b) preamplifier with 4 channels on the substrate.

The Fourier transform of the output signal is

$$(9.11) \quad V_o(\omega) = -I(\omega)Z_1(\omega)g_m \frac{1}{j\omega C}.$$

From eq. (9.10) the relative importance of the various terms can be investigated. In a well-designed preamplifier,  $\Sigma_n$  or  $\Sigma_{n=1}^{\infty} g_m^2 + \Sigma_n$  are the dominant terms. However, when  $L$  is omitted and low supply voltages are used to reduce consumption, the value of  $R_1$  may turn out to be not sufficiently large and its noise contribution may be not negligible.

To reduce the contributions due to the shot noise in the base currents  $I_{B1}$  and  $I_{B2}$ , it is advisable to fix low collector currents in  $T_1$  and  $T_2$ , that is, less than 1 mA.

The low-noise amplifying block of fig. 9.6 can be transformed into a charge-sensitive loop by connecting between input and output the parallel combination of a capacitor and a resistor. A resistor of comparatively low value,  $R_1 \sim 10 \text{ k}\Omega$ , connected between input and output implements a current-sensitive preamplifier.

**9.3. Practical preamplifier configurations.** - This subsection reviews some preamplifier configurations that have been developed during the last few years for specific applications in elementary particle physics.

The circuit of fig. 9.7 is a current-sensitive configuration designed for microstrip signal processing [7, 8].

As shown in fig. 9.7b), the transistors  $T_1, T_2$  implement the inverting preamplifier around which the current-sensitive loop is designed. The transistor  $T_3$  provides additional voltage gain. The circuit 9.7 is being extensively used at CERN in association with microstrip detectors.

The preamplifier shown in fig. 9.8 is a common-base configuration, while that of fig. 9.9 is a charge-sensitive one [9].

Comparison was made between the noise performances of the three preamplifiers of fig. 9.7-9.9 with the special aim of investigating their behaviour at short processing times [9].

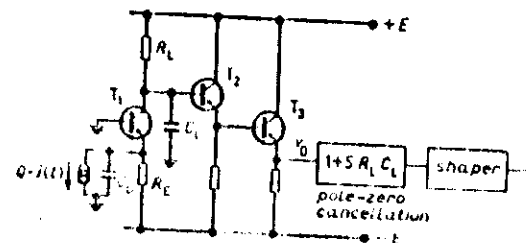


Fig. 9.8. - Common base preamplifier.

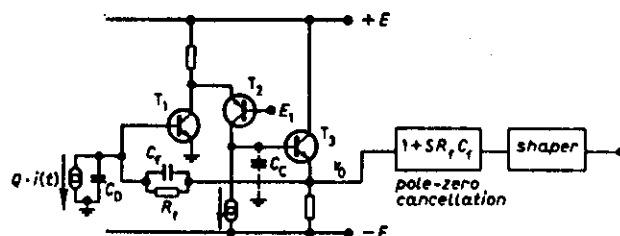
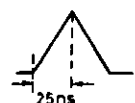


Fig. 9.9. - Charge-sensitive loop.

For this purpose, the same fast bipolar transistor, a NEC specimen featuring a  $5 \text{ GeV } f_a$  at  $I_c = 5 \text{ mA}$  and  $R_{be}$  between 10 and  $15 \Omega$ , was employed as input stage of the three preamplifiers. Its collector current was fixed around  $0.7 \text{ mA}$ .

TABLE 9.I.

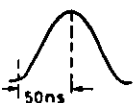
ENC <sub>p</sub> [r.m.s.e.]			
W(t)	Preamplifier		
	charge sensitive $R_i = 150 \text{ k}\Omega$	current sensitive $R_i = 10 \text{ k}\Omega$	common base $R_L = 22 \text{ k}\Omega$ $R_E = 27 \text{ k}\Omega$



1265

1434

1434



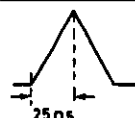
1786

2020

2020

ENC<sub>g</sub>/C<sub>D</sub> [r.m.s.e./pF]

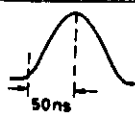
W(t)	Preamplifier		
	charge sensitive	current sensitive	common base



18

18

18



14.7

14.7

14.7

The results obtained are shown in table 9.I, which presents the values of the ENC at  $C_D = 0$  and the asymptotic slope of ENC vs.  $C_D$  for the three configurations of fig. 9.7-9.9 and for two different noise weighting functions.

The slightly better behaviour of the charge-sensitive preamplifier at  $C_D = 0$  has to be attributed to the fact that, compared to the other two circuits, it has a lower thermal-noise contribution from the bias resistors.

Preamplifiers of the types 9.7-9.9 have been realized in thick-film technology and are commercially available.

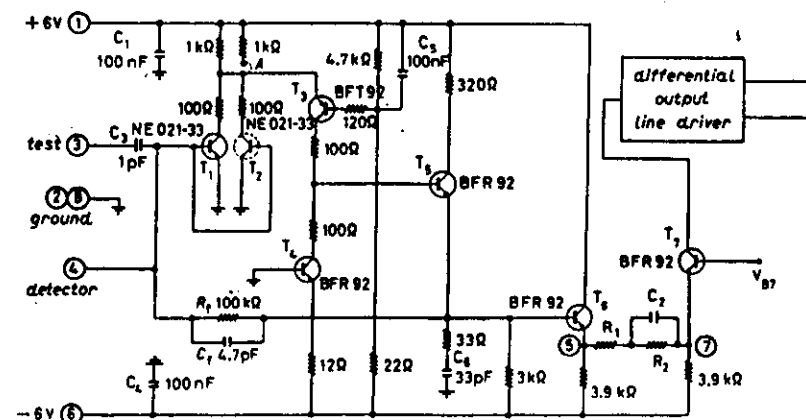


Fig. 9.10. - Preamplifier consisting of a charge-sensitive loop followed by pole-zero cancellation.

Figure 9.10 shows a complete preamplifier based upon an input charge-sensitive loop, transistors  $T_1$ ,  $T_2$ ,  $T_3$ ,  $T_4$ ,  $T_5$ , a pole-zero cancellation and amplification section consisting of transistors  $T_6$ ,  $T_7$  and passive network between their emitters and of a differential output line driver [10].

The noise behaviour of this preamplifier is close to that described in table 9.I. The circuit is commercially available in thick-film hybrid version, with either a single transistor or two paralleled transistors in the input stage. In the latter version, aiming at reducing the noise contribution from  $R_{be}$ , it was employed with the detectors of the silicon active target developed by the Milan group for the second part of the FRAMM experiment.

In that application, with a detector capacitance of about  $80 \text{ pF}$  and a shaping implemented by a semi-Gaussian prefiltering with  $25 \text{ ns}$  FWHM followed by a gated integrator with  $70 \text{ ns}$  gate width, it featured an ENC of  $2200$  electrons r.m.s.

Based upon a configuration similar to that of fig. 9.10 is the preamplifier with field-effect transistor input stage developed by the group of Milan for the



The value of the transconductance is  $1/R_1$ . If the condition  $R_1 C_i = RC$  is met, the cold resistance appearing at the preamplifier input is, accordingly,  $R_1 C/C_i$ . Two outputs are available in the circuit 9.13, the usual integrated one is taken at  $v_o$ , while a fast output, closely reproducing the detector current pulse, can be taken at  $v_i$ . The latter signal can be of some usefulness if fast trigger operations have to be performed. In the design of this preamplifier a low value of  $R_1$  must be used, as already pointed out, to avoid a noticeable increase in the series white noise.

Preamplifiers with stabilized cold resistance have been employed in data acquisition from silicon telescopes during the first part of the FRAMM experiment. They were associated with detectors of 700 pF capacitance and the counting rate during the accelerator spill was about  $10^4$  pulses per second.

An example of preamplifier of the type 9.1c) developed to sense the voltage signal across a stack of series-connected detectors is shown in fig. 9.14 [13].

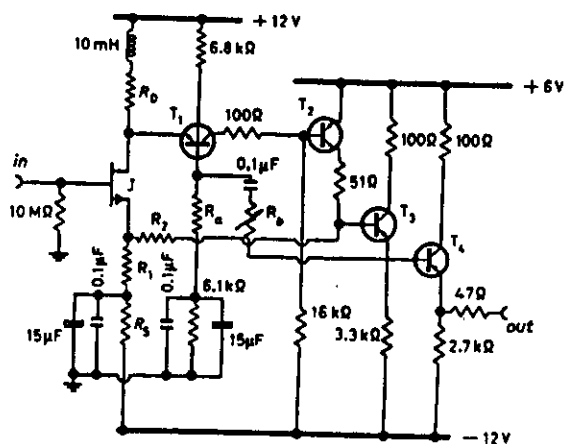


Fig. 9.14. - Voltage-sensitive preamplifier.

The feedback loop in the circuit of fig. 9.14 includes the input field-effect transistor J, the common-base transistor  $T_1$  and the common-emitter transistor  $T_2$ , whose collector current is injected into the source of J. The output is taken from the collector of  $T_2$ , through the emitter followers  $T_3$ ,  $T_4$ .

The voltage gain of the preamplifier is approximately  $(R_1 + R_2)/R_1$ .

An interesting feature of the preamplifier of fig. 9.14 is that its input capacitance can be zeroed. The gate-to-source capacitance of J appears at the input strongly attenuated by virtue of the series voltage feedback applied to the source of J. Positive feedback from the output to the base of  $T_1$  via the voltage divider  $R_1$ ,  $R_2$  has the effect of cancelling in the preamplifier input capacitance the contribution due to the gate-to-source capacitance of J.

The effectiveness in the control of the input capacitance achieved with the  $R_1$ ,  $R_2$  voltage divider is shown in fig. 9.15. As pointed out in subject. 9'1, one of the attractive features of the voltage-sensitive preamplifier is the very small dependence of risetime on detector capacitance  $C_D$ . As shown in fig. 9.16,

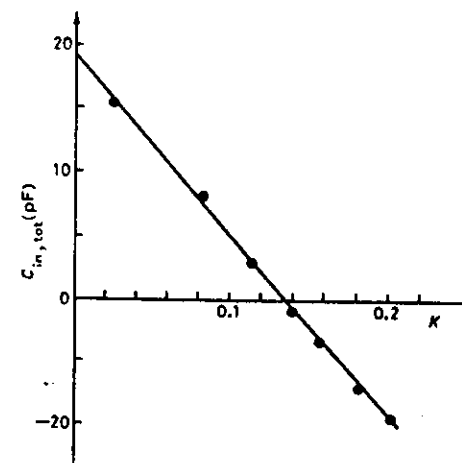


Fig. 9.15. - Input capacitance of the voltage-sensitive preamplifier as a function of the attenuation ratio  $K$  of the  $R_1$ ,  $R_2$  voltage divider.

the risetime of the preamplifier given in fig. 9.14 after an initial decrease remains constant around 6 ns up to 600 pF detector capacitance.

The noise behaviour of the voltage-sensitive preamplifier analysed with a triangular, time-invariant shaper with 100 ns base width is shown in fig. 9.17, curve a).

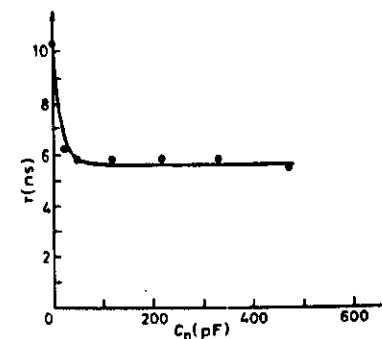


Fig. 9.16. - Risetime of the voltage-sensitive preamplifier as a function of  $C_D$ .

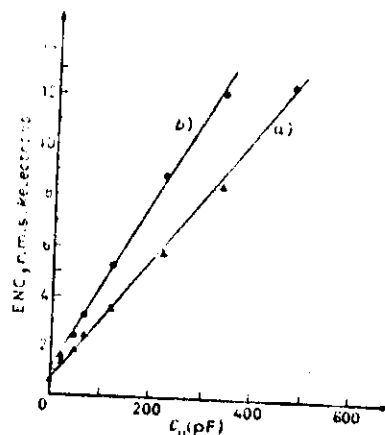


Fig. 9.17. — Equivalent noise charge as a function of the detector capacitance for the voltage sensitive preamplifier of fig. 9.14: curve a) single-ended configuration of fig. 9.14, curve b) differential configuration.

A voltage-sensitive difference configuration has been developed by coupling two circuits of the type shown in fig. 9.14. Its purpose is that of sensing the voltage across a floating detector in a stack of series-connected elements [13].

The equivalent noise charge for the differential voltage-sensitive configuration is described, as a function of  $C_D$ , by curve b) of figure 9.17.

## REFERENCES

- [1] P. F. MANFREDI: *Electronics for silicon detectors in high energy experiments*, in *Proceedings of the Meeting on Miniaturization of High Energy Physics Detectors*, Pisa, September 1980, edited by A. STEFANINI (Plenum Press, New York, N. Y., 1983), p. 77.
- [2] J. K. MILLARD, T. V. BEALOK and N. W. HILL: *IEEE Trans. Nucl. Sci.*, NS-19, 388 (1972).
- [3] V. RADEKA: *IEEE Trans. Nucl. Sci.*, NS 21, 51 (1974).
- [4] E. GATTI and P. F. MANFREDI: *Low noise current preamplifiers for large capacitance semiconductor detectors and high counting rates*, in *Proceedings II ISPR Nuclear Electronics Symposium*, Publ. EUR 5370 e (1975), p. 33.
- [5] M. GUYOT: *A hybrid, broadband, low noise charge preamplifier for simultaneous high resolution energy and time information with large capacitance semiconductor detectors*, in *Proceedings II ISPR Nuclear Electronics Symposium*, Publ. EUR 5370 e (1975), p. 99.
- [6] E. GATTI, P. F. MANFREDI and V. SPERIANI: *Nucl. Instrum. Methods Phys. Res.*, 224, 10 (1984).

- [7] E. HEIJNE and P. JARRON: *Silicon detectors, a new tool for high energy physics*, in *Silicon Detectors for High Energy Physics, Proceedings of a Workshop held at Fermilab, October 15-16, 1981*, edited by T. FIEBEL (1981), p. 1.
- [8] P. JARRON and M. GUYOT: *Nucl. Instrum. Methods Phys. Res.*, 226, 156 (1984).
- [9] P. D'ANGELO, A. BRISQHO, P. JARRON, P. F. MANFREDI and J. POINSIGNON: *Nucl. Instrum. Methods*, 193, 533 (1982).
- [10] G. BELLINI, M. CONTE, P. D'ANGELO, P. F. MANFREDI, D. MENASCE, E. MIRONI, L. MORONI, S. SALA and M. SZAWLOWSKI: *IEEE Trans. Nucl. Sci.*, NS-30, 416 (1983).
- [11] E. GATTI, P. INZANI, P. F. MANFREDI and D. MARIOLI: *Design of low noise preamplifiers for some special application in high energy experiments*, in *Proceedings of the Symposium on Nuclear Electronics, Bratislava, 6-12 September 1983*, p. 67.
- [12] E. GATTI and P. F. MANFREDI: *IEEE Trans. Nucl. Sci.*, NS-24, 66 (1978).
- [13] E. GATTI, P. F. MANFREDI and D. MARIOLI: *Nucl. Instrum. Methods Phys. Res.*, 221, 536 (1984).

## 10. — Linear amplifier and shaper.

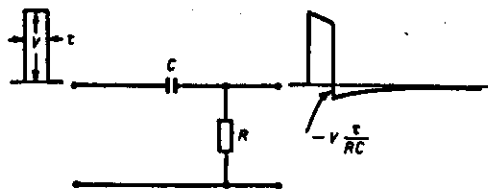
The signal delivered by the preamplifier has to go through further processing before being presented to the amplitude-measuring system. Such a processing consists of amplification and shaping.

Shaping has been exhaustively discussed in sect. 4, 5. It has been pointed out that some kinds of shaping operations—it would be better to say some kinds of noise weighting functions—can be implemented, in both time-invariant or time-variant ways.

Other weighting functions are preferably implemented in one or in the other way. The present trend in elementary-particle physics relies upon the synthesis of the weighting function based upon gated integrators. This trend finds its explanation in the fact that, as pointed out already, in most of analog signal acquisition channels a gated integrator is present as a part of the charge-sensing analog-to-digital converters.

Between preamplifier and gated integrator, a linear-amplifier system is required, whose purpose is that of matching the signal level to the amplitude range required by the gated integrator and to perform on it the time-invariant preshaping. The amplifier system may be a potential source of both inaccuracy in the amplitude measurement and of spectral-line broadening caused by baseline shifts at high counting rates. Baseline shifts occur in a.c. coupled amplifier systems because of the random superposition of the tails that follow the pulse [1-3].

The effect is explained in fig. 10.1 which shows the behaviour of a d.c. level blocking network, RC differentiator, on a rectangular signal applied at the input. The input signal is strictly unipolar and has an amplitude  $V$  and a width  $\tau$ , which is supposed to be much shorter than the differentiator time

Fig. 10.1. - Negative tail following the positive pulse and due to the  $RC$  differentiator.

constant  $RC$ . As the input signal is of finite area and the transfer function of the differentiator has a zero in the origin of the complex  $s$ -plane, the output signal has zero area. Consequently, the positive output signal is followed by a negative tail which starting from the value  $-V(\tau/RC)$  tends to zero exponentially with time constant  $RC$ . The time dependence of the tail is

$$(10.1) \quad -V \frac{\tau}{RC} \exp\{-t/RC\}.$$

Assume, for the sake of simplicity, that the input pulses have a very narrow,  $\delta$ -like amplitude distribution. The random arrival of the pulses with average rate  $\lambda$  creates a stochastic baseline. According to Campbell's theorem applied to (10.1), the mean value of the baseline is

$$(10.2) \quad -\lambda\tau V$$

independent of the time constant  $RC$  and its variance is

$$(10.3) \quad \frac{\lambda\tau^2}{2RC} V^2.$$

The original indefinitely narrow pulse amplitude distribution would undergo a relative broadening described by the standard deviation

$$(10.4) \quad \left[ \frac{\lambda\tau^2}{2RC} \right]^{1/2}.$$

It has to be pointed out that baseline shift may be present also in d.c. coupled systems employing pole-zero cancellations, if these are not correctly adjusted.

It is worth discussing now in which situations baseline shift and related statistical fluctuations are important.

The most critical condition is that of a telescope of silicon detectors in a fixed-target experiment. With reference to (10.4) it can be said that in a pulsed accelerator the time constant  $RC$  cannot be made very large, otherwise a long settling transient would be present on the baseline at the beginning of every

machine burst. For instance, in an a.c. coupled amplifier working at  $10^4$  pulses per second, each pulse lasting 100 ns and with a  $RC$  time constant of 10  $\mu$ s, the baseline shift would be 10% of the signal amplitude and the standard deviation of the output amplitude distribution would be about 2%. At low particle multiplicities, such a contribution to the line broadening is not negligible.

Three classical solutions borrowed from nuclear spectroscopy to remove baseline effects are baseline restoration, double sampling, that is, sampling the baseline before the pulse arrival and the peak of the pulse and then taking their difference, and bipolar shaping [4-9]. They are schematically illustrated in fig. 10.2. The circuit of fig. 10.2a) is the well-known time-variant

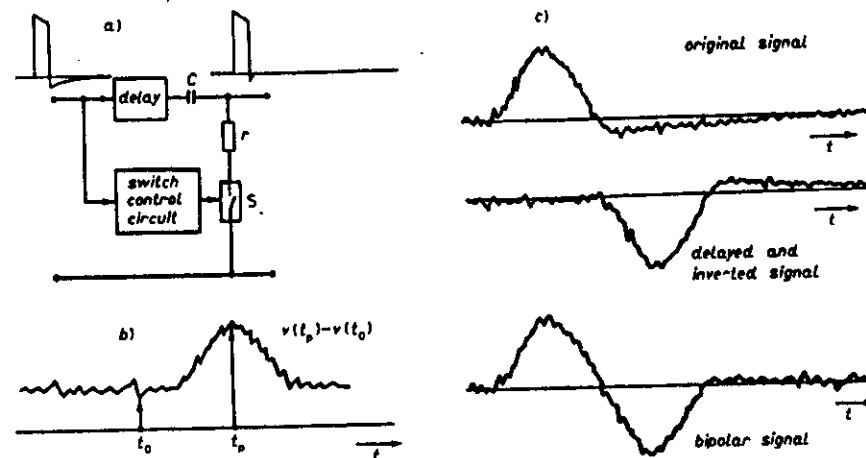


Fig. 10.2. - Three ways of avoiding baseline shift at high counting rates: a) baseline restorer, b) sampling the baseline and the peak of the pulse, c) bipolar shaping.

restorer. The incoming positive pulse through the switch control circuit opens  $S$ , thereby making the differentiation time constant associated with  $C$  infinitely larger. At the end of the positive pulse, the switch gets closed. Tails of negative sign which might exist owing to the presence of earlier differentiating stages are clipped because the time constant  $\tau C$  is chosen suitably short. In this way the zero value of the baseline is quickly recovered and the effects of baseline shift strongly reduced.

The baseline restorers, to be effective in the conditions of high-energy experiments, at counting rates of millions pulses per second, must execute a quick level clamping after the pulse and this clamping action must last until the next pulse. As a result, the faster noise components for which no correlation exists between the values immediately preceding and immediately following the positive-going edge of the pulse undergo a  $\sqrt{2}$  increase.

The method of fig. 10.2b) cancels the baseline component by measuring the peak amplitude of the signal with a double sampling of the total wave form.

As to the effect on the high-frequency noise, the double sampling behaves exactly like the baseline restorer, that is, it introduces a  $\sqrt{2}$  noise worsening. In fact, the resulting weighting function is the original one inverted in polarity and delayed of a time interval equal to its width.

Figure 10.2c) shows how a first-order tail cancellation can be achieved by using bipolar shaping, that is, by adding to the original signal the same signal inverted and delayed. As already pointed out, bipolar shaping, compared to the unipolar one based upon the same kind of signal, introduces a  $\sqrt{2}$  deterioration in the noise. For a general approach to the area-balanced wave forms and, among them, the one giving the least noise for a defined overall width, the reader can refer to paper [4].

It can, therefore, be concluded that the three methods of reducing baseline shift described in fig. 10.2 all degrade the equivalent noise charge of a comparable amount, at least if the dominant noise term is the series white one. Such a degradation is hardly tolerable in several experimental conditions of elementary-particle physics, where the signal-to-noise ratio is intrinsically poor.

Effort was accordingly concentrated on a possible way of achieving the desired counting rate performances with the slightest sacrifice in signal-to-noise ratio. A method was introduced, called periodical stabilization of the amplifier system, which was proven to meet the stated requirement. The idea it is based upon is shown in fig. 10.3 [10].

The fast signal path in the diagram of fig. 10.3 is d.c. coupled from the

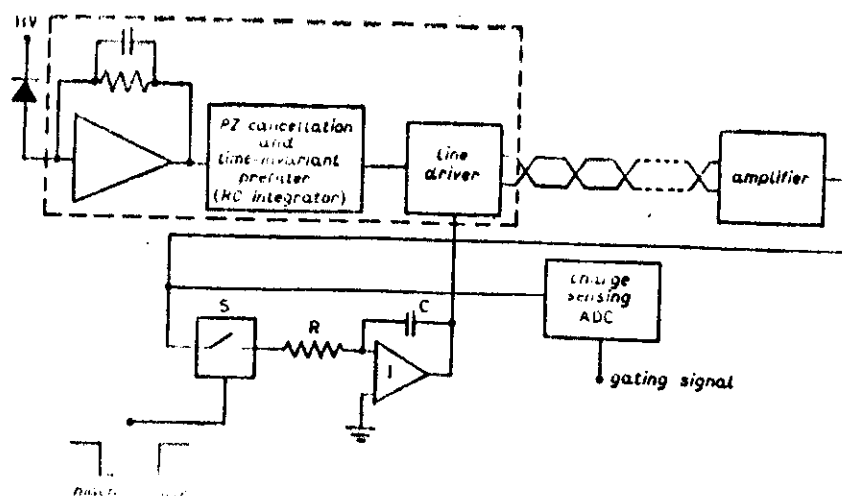


Fig. 10.3

detector to the ADC input, provided that the switch S is open. In this situation, if the pole-zero cancellation networks on the signal path are properly adjusted, the pulses along this path are strictly unipolar and no baseline shift occurs.

The linear processor as a whole can be considered as a time-variant shaper preceded by a time invariant preshaping. The time-variant shaping is implemented by the gated integrator provided in the ADC. The time-invariant preshaping results from the intrinsic band width limitations in the amplifier system for the faster realizations, and from some lumped integrations in the slower versions. The time-variant preshaper can be assumed to be Gaussian in both cases and the complete noise weighting function is almost trapezoidal with erfc-type leading and trailing edges.

The d.c. coupled fast channel cannot work alone, for the thermal and long-term drift would introduce slow instabilities in the output baseline. In order to avoid this, a correction path has been introduced, which implements the periodical stabilization. The correction path includes the switch S and a long-term integrator whose output voltage is fed back to the preamplifier. The correcting phase takes place during the interburst periods of the accelerator, when the beam is off and no radiation falls on the detector.

The switch S remains closed throughout the interburst period. The feedback loop, which encircles the preamplifier output stage, the amplifier, the switch S and the long-term integrator, forces the output of the amplifier to zero.

A correcting voltage builds up across C, which accounts for any long-term and thermal drift in the signal processor. At the beginning of the next accelerator burst, S opens again. The correcting voltage stored on C, however, keeps the working points in the amplifier stable during the accelerator burst. A new correcting phase, which has the purpose of updating the correcting voltage on C, starts at the beginning of the next interburst period. The sequence of acquisition and correction phases is shown in fig. 10.4a). Figure 10.4b) shows that no baseline shift occurs during the accelerator burst, despite the high pulse rate at which the picture was taken, more than  $10^4$  p.p.s. The effect of wrong operation of the switch S is shown in fig. 10.4c). If S is left permanently closed, then the system of fig. 10.3, with the feedback not interrupted when pulses are delivered by the detector, behaves as an a.c. coupled amplifier. Baseline shift takes place at the beginning of the accelerator burst and then the baseline settles to the negative value predicted by (10.2). The dependence of the centroid of a spectral line on the counting rate is shown in fig. 10.5 for both correctly operated and permanently closed switch S.

With the switch S correctly operated, less than 0.3% centroid shift occurs raising the counting rate up to  $\sim 3 \cdot 10^4$  pulses per second. If, instead, S is left permanently closed, a large variation is noticed in the centroid position by varying the counting rate in the same range as before.

The behaviour of the periodical stabilized amplifier system at high counting rates is also described by the spectra of fig. 10.6.

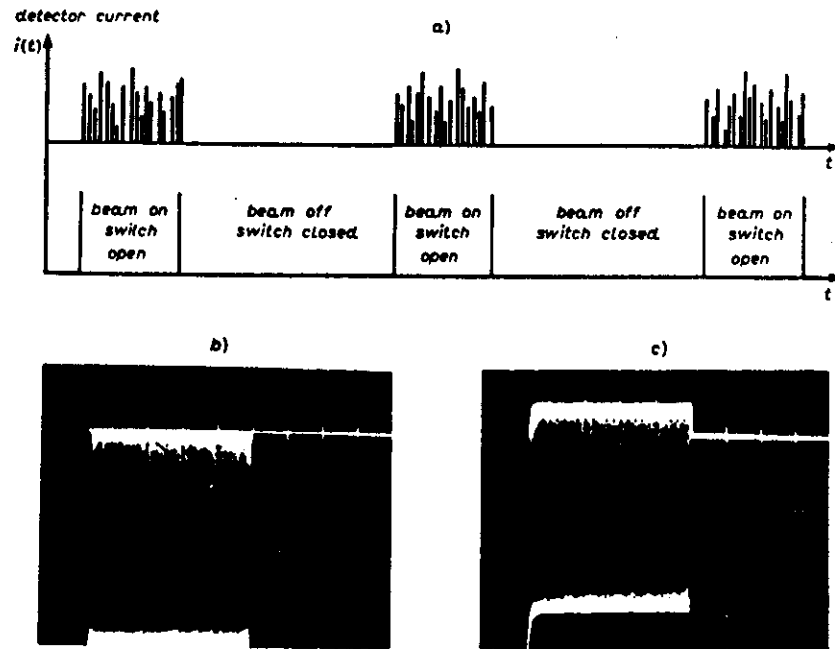


Fig. 10.4. - a) Sequence of acquisition and correction phases. b) Baseline at the amplifier output with the switch S correctly operated. c) Baseline at the amplifier output with the switch S permanently closed.

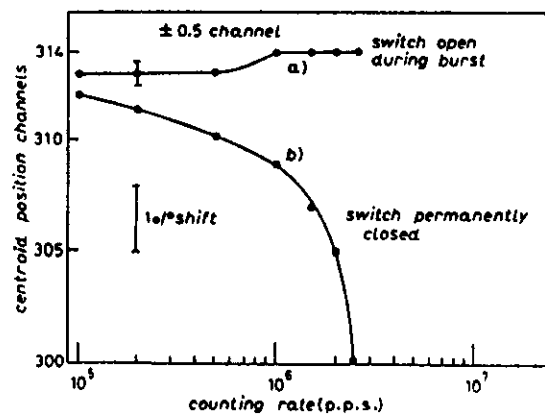


Fig. 10.5. - Dependence of the centroid of a spectral line on the counting rate: a) switch S correctly operated, b) switch S permanently closed.

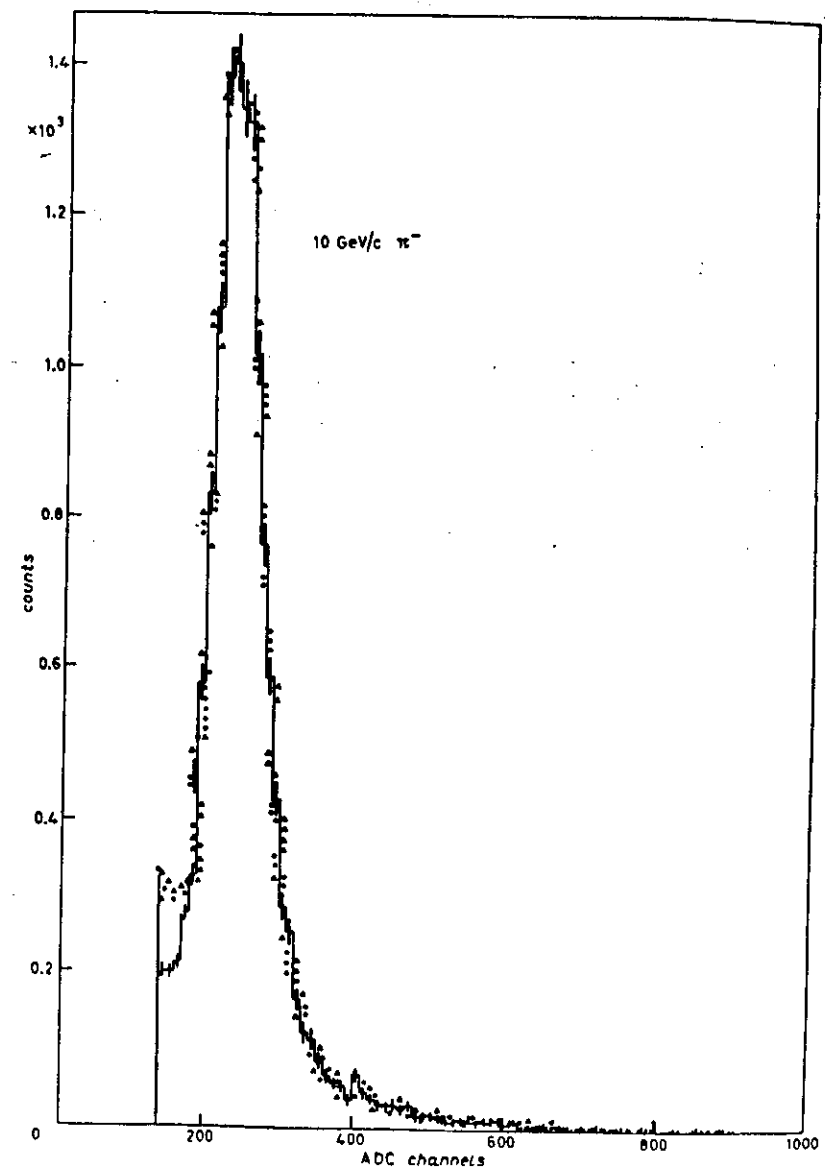


Fig. 10.6. - Landau distributions of negative  $\pi$ -mesons recorded at three different counting rates: —  $2.7 \cdot 10^5$  p.p.s.,  $\Delta\Delta\Delta$   $5.1 \cdot 10^5$  p.p.s.,  $\bullet\bullet\bullet$   $10^6$  p.p.s.

Figure 10.6 shows that no significant shape variation occurs in the Landau distribution of minimum-ionizing particles by increasing the counting rate up to  $10^4$  p.p.s.

The principle of periodical baseline stabilization is presently being employed in some fixed-target experiments, where it proved to be successful in high-duty-cycle data acquisition from both silicon and germanium active targets. It thoroughly confirmed its capability of ensuring outstanding counting rate performances without degrading the intrinsic equivalent noise charge of the detector preamplifier assembly.

Highly segmented detectors, like microstrip planes, even if employed in fixed-target experiments, do not present usually critical problems of baseline shift and baseline fluctuations because the average duty cycle in each acquisition channel is small, less than 1%. For these situations, a.c. coupled amplifier systems are suitable. The only requirement which has to be met is that the differentiating time constants in the amplifiers be adequately short to enable a quick baseline settling to the standing value at the beginning of the accelerator bursts.

In the collider applications, as long as the spacings between bunch crossings are of several microseconds, the baseline problems are negligible. This statement is reinforced by the remark that in collider experiments all the detectors are segmented and the tracks are spread over a large solid angle. Therefore, the counting rates on the individual detector segments will be definitely small. The situation might change in future large hadron colliders, where the time between bunch crossing will be a few tens of nanoseconds and the number of charged tracks per event will be large. The machine parameters for these colliders, however, are still under investigation and no conclusion can be drawn as yet.

Before concluding this section, some alternative ways of realizing the shaping function will be described. A first example is shown in fig. 10.7 [11, 12]. Accord-

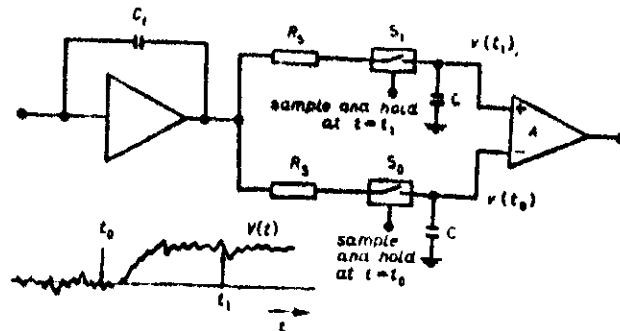


Fig. 10.7. - Shaping realized by sampling and holding the preamplifier output at different instants.

ing to the circuit diagram presented there, the output of the charge-sensitive preamplifier is split along two ways, each containing a resistor, a switch and a capacitor. A two-input amplifier takes the difference of the voltages stored on the capacitors. At  $t = t_0$ , shortly before the arrival of the pulse, the baseline is sampled by closing and reopening  $S_0$  and the corresponding voltage is held on the relevant capacitor. At  $t = t_1$ , some time after the arrival of the pulse,  $S_1$  is closed and reopened and the value  $v(t_1)$  is held. At  $t > t_1$  the voltage at the difference amplifier output provides the step amplitude to be measured.

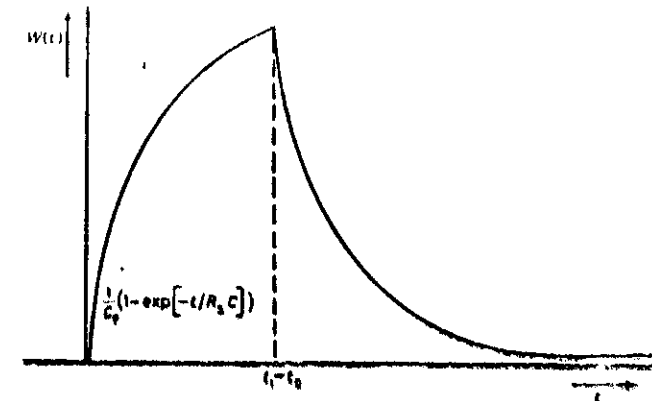


Fig. 10.8. - Weighting function realized.

The circuit of fig. 10.7 implements the weighting function shown in fig. 10.8. Such a function is identical to the one which would be obtained by applying a delay line clipping to the step response of an approximate integrator.

The sample and hold operation and the difference of the two samples, therefore, replace the delay line with two advantages. First, the delay between the two samples,  $t_1 - t_0$ , can be easily changed by acting upon the relative position of two logic pulses. Secondly, the circuit of fig. 10.7 lends itself to a possible monolithic integration, while obviously the delay line implementation of the same weighting function does not.

The present trend of realizing in monolithic form the high-density front-end circuits required by highly segmented detectors will benefit from the possibilities opened up by the principle of fig. 10.7. More generally, shaping can be based upon sampling the voltage at the preamplifier output at the instants  $t_0, t_1, \dots, t_k$ , holding the samples in  $k + 1$  capacitors and then combining the samples with suitable coefficients. For instance, a third path in the circuit of fig. 10.7 and the following linear combination of the samples  $v(t_0), v(t_1), v(2t_1 - t_0)$

$$2v(t_1) - v(t_0) - v(2t_1 - t_0)$$

would implement a bipolar weighting function [13].



## 11. - Conclusive remarks on some problems of analog processing.

Some of the experimental situations described in the introduction will be reviewed now and the related problems of noise and counting rates will be considered again, in order to outline practical solutions. The more demanding condition, that is, the one arising in fixed-target experiments, will be assumed as a reference.

**11'1. Silicon telescopes.** - By virtue of the short collection time in the detector counting rates of several million pulses per second are theoretically possible. To achieve these rates in a real system, two problems have to be solved. First, optimizing the equivalent noise charge at processing times that are constrained to be in the range of a few tens of nanoseconds. Second, designing an amplifier system capable of working at these rates with negligible spectral-line degradation [1].

The most appropriate preamplifier solution looks to be based upon microwave bipolar transistors with low  $R_{out}$ . As the detector capacitance can be not small, frequently exceeding 100 pF, capacitive matching between detector and preamplifier is advisable.

To cope with the counting rates, the use of the periodical stabilization in the amplifier system is mandatory.

Analog processors based upon bipolar-transistor preamplifiers and periodical stabilization are presently employed in association with the silicon active targets of NAI, WA71, NA14 experiments [2-5].

They all employ time-variant shaping realized with the gated integrators of charge-sensing ADCs. The analog processors of NAI experiment, for instance, has the following characteristics. The noise weighting function results from the convolution of a Gaussian pulse with 25 ns FWHM and of a rectangle 70 ns wide. The equivalent noise charge with a detector capacitance of about 80 pF is 2200 electrons r.m.s. The Landau spectra at several multiplicities of minimum-ionizing particles, taken at a counting rate of  $10^6$  p.p.s., are shown in fig. 11.1 [2].

**11'2. Germanium bulk detector.** - The germanium bulk detectors employed in NAI experiment were of two different pitches, 50  $\mu$ m and 100  $\mu$ m. Owing to the long path that the charge carriers have to travel along in their collection motion, the collection times and their fluctuations are considerably larger than in silicon telescopes. For this reason longer time constants in the time-invariant preshapers and a wider gating window had to be employed. Preshaping was achieved with standard RC integrations.

The noise weighting function results from the convolution of an almost

Gaussian pulse with 200 ns FWHM and a 500 ns wide rectangle. The capacitance presented by each electrode to the preamplifier, basically due to the connections, is of the order of 30 pF. The small detector capacitance and the long duration of the noise weighting function suggest the field-effect transistor as the most

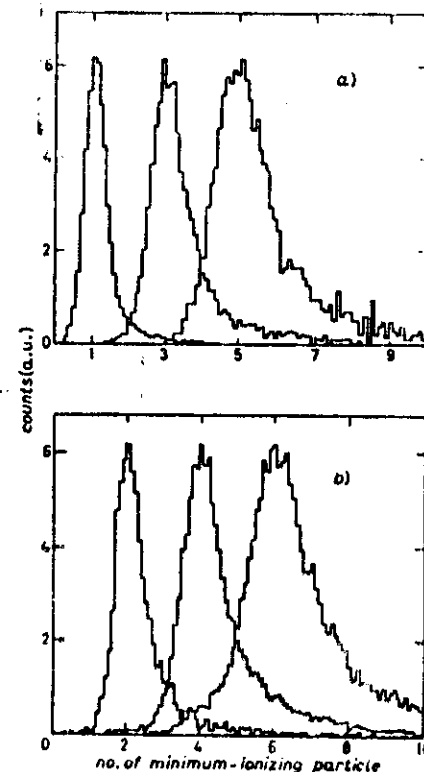


Fig. 11.1. - Landau distributions obtained with the NAI silicon active target: a) odd multiplicities, b) even multiplicities.

suitable input device in the preamplifier. The amplifier system employs periodical stabilization and, like in the processor associated with the silicon target, time-variant shaping is implemented by the gated integrator of the charge-sensing ADCs. The equivalent noise charge for the germanium analog processors is 700 electrons r.m.s.

The noise line and the Landau distributions of two and four minimum-ionizing particles are shown in fig. 11.2 [6].

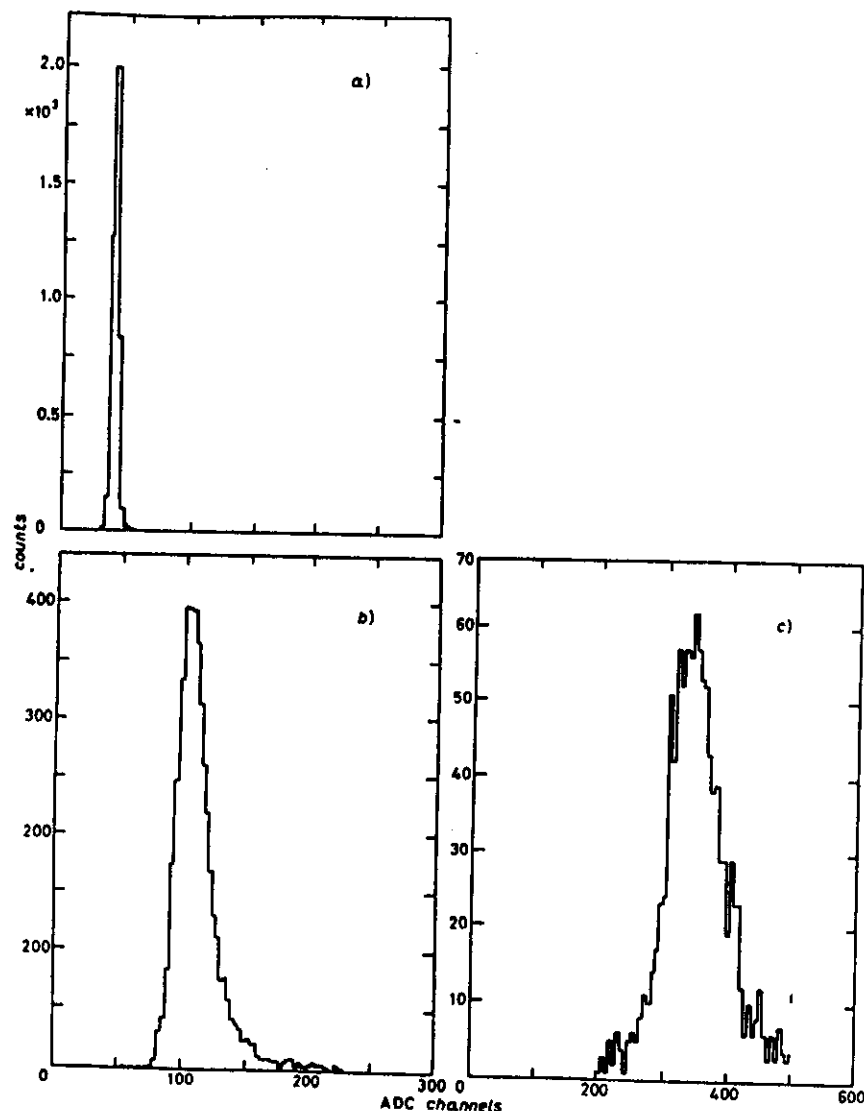


Fig. 11.2. - Noise line (a) and Landau distributions of two (b) and four (c) minimum-ionising particles.

11'3. *Microstrip detectors.* - The position information in a microstrip detector can be read out according to two different approaches [7]. One, called strip-by-strip readout, requires one analog signal processor every strip. The other is based upon an interpolation method employing charge division and allows accordingly a reduction in the number of readout channels required.

The strip-by-strip readout is shown in fig. 11.3 [8-10].

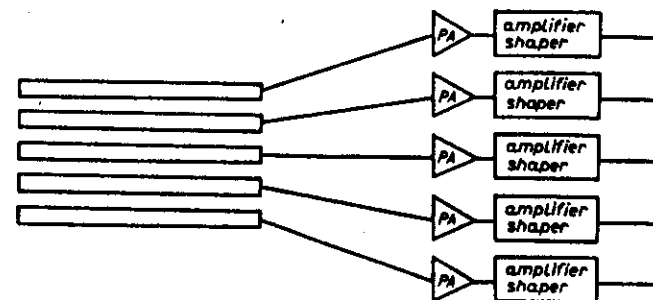


Fig. 11.3. - Strip-by-strip readout from a microstrip plane.

As the figure points out, in microstrip planes of small pitch, a serious problem is the one connected with the need of fanning out the signals from the strips to the preamplifiers. If the preamplifiers are of relatively large size, like in the case of thick-film circuits, for instance, the connections between the strips and the preamplifiers are long. The contribution of the fan-out to the total capacitance at the preamplifier input becomes dominant [7]. For instance, a printed-circuit fan-out designed to connect a microstrip detector with  $50\text{ }\mu\text{m}$  pitch to a set of hybrid, thick-film preamplifiers would present at each output more than  $30\text{ pF}$ . The capacitance presented by every strip would be of  $(5\div 10)\text{ pF}$ . The signal-to-noise ratio, therefore, is badly affected by the fan-out circuit.

Besides that, a configuration of the type shown in fig. 11.3 introduces serious problems connected with the size of the front-end electronics and with the number of cables needed to convey the signals to the remote-end electronics. These two difficulties can be overcome in a fixed-target experiment, where the number of strips hardly exceeds  $10^4$ , but would make the method unusable in the barrel-type microstrip assemblies foreseen for some collider experiments. Consequently, while present fixed-target experiments are still based upon discrete or thick-film circuits in configurations of the type of fig. 11.3, a monolithic approach to the design of the front-end electronics with a built-in demultiplexing circuit to reduce the cabling problems is pursued for collider applications. Several systems have been developed during the last few years

for low-noise data acquisition from microstrip detectors employing the strip-by-strip readout. In high-counting-rate applications, the solution based upon bipolar-transistor thick-film preamplifier of the transresistance type and described in subsect. 9.3 has been extensively used at CERN [8].

Such a preamplifier is particularly suited for processing times shorter than 50 ns and input capacitances of the order of 10 pF, that is, for applications with run-out printed-circuit boards not excessively large. Within these limitations, the preamplifier benefits from the good noise performances of the bipolar transistors at collector currents that are more than an order of magnitude lower than the usual drain currents in a field effect transistor. This results in a small power dissipation, which is an advantage in such a large front-end system as microstrip detectors require. The preamplifier can be used in association with a fast amplifier and a threshold discriminator which senses the strip hit, in which case the shaping is of time-invariant nature, implemented by the natural band width limitations in the preamplifier and in the amplifier. Alternatively, if the pulse height analysis is advisable, as it happens when microstrip detectors are employed in tracking systems, the gated integrator at the ADC input performs in the already discussed fashion time-variant shaping. In microstrip applications, the use of a gated integrator rather than of a simple discriminator, even if the purpose is just that of sensing the hit on the strip, has the advantage of a much better rejection of spurious charge induction on the noncollecting strips as well as of cross-talk signals. Cross-talk is not a trivial problem in microstrip detectors of small pitch readout with large fan out printed-circuit boards featuring a high density of conductive paths. An alternative solution of microstrip readout is presently being employed in WA75 experiment at CERN [10]. The analog channels, intended for a strip-by-strip readout, consist of a charge-sensitive preamplifier with a JFET input stage, of an  $R_C$  integration and of a track-and-hold amplifier. The outputs of the track-and-hold amplifier are digitized by analog-to-digital converters operating in a multiplexed way.

Every ADC serves 32 analog channels. The shaped analog signals are fed into the track-and-hold amplifiers, where their peak amplitude is stored upon receipt of a timing command. If the experiment trigger requirements are met, then the analog storage is confirmed to enable the multiplexed analog-to-digital conversion to be performed. Otherwise a fast clear is executed on the analog memories. The system is intended for application with a vertex detector consisting of 16 microstrip planes with 300  $\mu\text{m}$  thickness and 50  $\mu\text{m}$  pitch. The equivalent noise charge referred to the preamplifiers' input is about 1500 electrons r.m.s.

In order to meet the application requirements of microstrip detectors in collider experiments, research and development effort is presently being channelled into the monolithic realization of the front-end analog electronics. The technique chosen is based on the use of the strip-by-strip readout.

shaper and an analog multiplexer which has the purpose of reducing the number of cables leaving the front-end circuitry. The technological approach followed so far is based on VLSI MOS circuits [11, 12].

This solution, though suitable to implement a shaping employing the analog switching functions of fig. 10.7 and 10.9 as well as the analog multiplexing operations, leads to rather poor noise performances in the preamplifiers. A JFET-MOSFET compatible process, with the JFET as low-noise input stage and the MOS transistors implementing the other functions, would be certainly more suitable. Its feasibility is still to be investigated.

Analog front-end electronics employing MOS transistors only is presently undergoing the laboratory tests. The basic circuit diagram is very similar to the one shown in fig. 10.7. The outputs from several channels are multiplexed on a single analog line by a system of analog switches. This process of information serializing is controlled by a clock generated on the chip and takes place during the period between two consecutive bunch crossings.

The use of a monolithic technology does not allow the realization of resistors of large value. Therefore, an alternative solution must be found to the problem of the charge reset on the preamplifier feedback capacitance  $C_f$ , reset which in discrete or hybrid circuits is implemented by a resistor in parallel to  $C_f$ . The alternative solution adopted employs a switch in parallel to  $C_f$ , as shown in fig. 11.4. The switch  $S$  is closed during most of the time

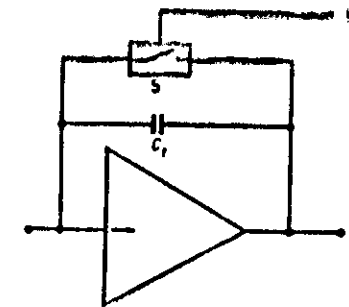


Fig. 11.4. - Switch-actuated charge reset.

between bunch crossing. A few hundreds of nanoseconds before a bunch crossing  $S$  is opened. Once the baseline at the preamplifier output has settled, the sample at  $t = t_0$  is taken (fig. 10.7). Passed the bunch crossing and after the second sampling instant,  $t = t_1$ ,  $S$  is closed again. The described solution is feasible, of course, because of the predictable time of occurrence of the events which is a characteristic of collider experiments.

Independently of the type of solution adopted for the preamplifiers, in a detector in which every strip is read out, the output noise of each preamplifier

is increased by the presence of the adjacent circuits. This statement is explained in fig. 11.5, where the three preamplifiers considered are supposed to have infinite gain-bandwidth product. The r.m.s. output noise for the  $K$ -th preamplifier can be expressed as

$$\overline{v_K^2} = \frac{\overline{e_K^2}}{C_i^2} [(2C_{i0} + C_i + C_i)^2 + 2C_{i0}^2],$$

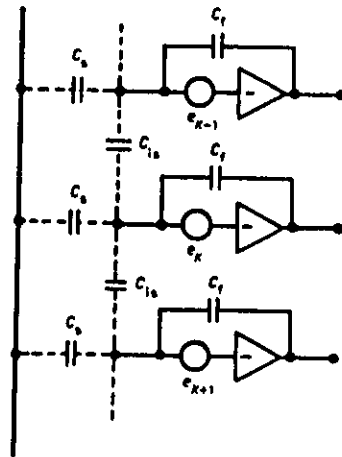


Fig. 11.5. - Effect of the adjacent preamplifiers on output noise of the  $K$ -th one.

where  $\overline{e_K^2}$  is the input root-mean-square noise assumed equal for the three preamplifiers. As usually the values of  $C_{i0}$  capacitances are larger than  $C_i$  and  $C_f$ , the presence of the adjacent preamplifiers increases the output noise of the  $K$ -th one of about 25%.

Microstrips' readout of the type analysed so far requires a large number of channels. The effect of noise increase due to the two adjacent preamplifiers must not be overlooked. As an advantage, the strip-by-strip readout has a double-track resolution which is close in value to the employed pitch.

A method which allows a reduction in the number of readout channels, more accuracy in the evaluation of the single-track position by virtue of its interpolating nature, but a lower double-track resolution, is based upon charge division and is shown in fig. 11.6 [13, 14]. As shown in fig. 11.6, there is one strip readout every  $N$ .

The hypothesis of ideal charge division between the two readout preamplifiers will be assumed at first; the parallel capacitances will be accordingly neglected for the strips that are not read out. For the strips 0 and  $N$ , to which

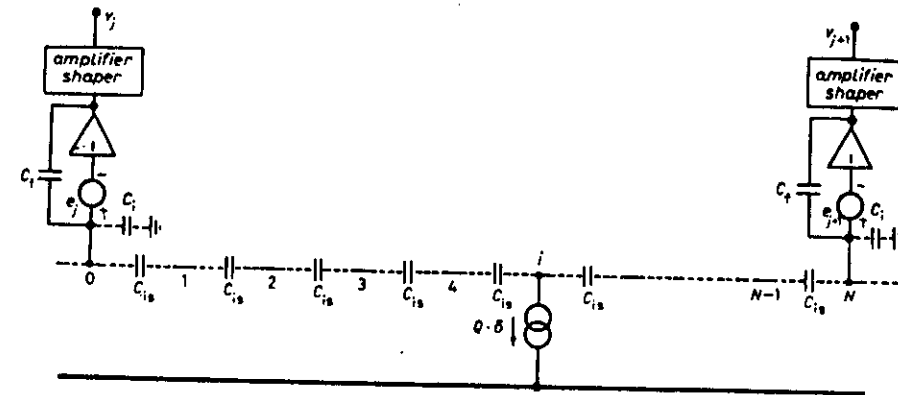


Fig. 11.6. - Position interpolation by charge division.

the preamplifiers are connected, the two capacitances  $C_i$  account for the sum of the preamplifier input capacitances and for the strays. Let  $e_i, e_{i+1}$  be the root-mean-square noise integrated on the band-pass of the shapers and referred to the input of the preamplifiers. The gain of the two analog channels will be assumed to be normalized in such a way that the peak amplitudes of the signals  $v_i, v_{i+1}$  are

$$v_{i,0} = \frac{Q}{C_i} \frac{N-i}{N}, \quad v_{i+1,0} = \frac{Q}{C_i} \frac{i}{N}.$$

The noise at the  $v_i, v_{i+1}$  outputs is given by

$$(11.1) \quad \begin{cases} v_{i,0} = e_i \left[ \frac{C_i}{C_i} + \frac{2C_{i0}}{NC_i} + 1 \right] + e_{i-1} \frac{C_{i0}}{NC_i} + e_{i+1} \frac{C_{i0}}{NC_i}, \\ v_{i+1,0} = e_{i+1} \left[ \frac{C_i}{C_i} + \frac{2C_{i0}}{NC_i} + 1 \right] + e_i \frac{C_{i0}}{NC_i} + e_{i+2} \frac{C_{i0}}{NC_i}. \end{cases}$$

To explain the basic methods that can be employed to deduce the position information, the presence of the noise will be at first disregarded.

Under this hypothesis, the sum  $v_{i,0} + v_{i+1,0} = Q/C_i$  is position independent and can be considered as a normalizing factor to obtain a position information without any dependence on the charge  $Q$ , that is, on the energy released in the detector. In the ideal case of noiseless readout amplifiers, the position is reconstructed by taking either quotient

$$(11.2) \quad \frac{v_{i+1,0}}{v_{i,0} + v_{i+1,0}} = \frac{i}{N},$$

or

$$(11.3) \quad \frac{v_{j+1,s} - v_{j,s}}{v_{j+1,s} + v_{j,s}} = 2 \left( \frac{i}{N} - \frac{1}{2} \right), \quad N \text{ even.}$$

In the former case the result is a linear function of the co-ordinate  $i$  measured from the readout preamplifier  $v_j$ ; in the latter the position is evaluated with positive and negative signals and the zero is coincident with the central strip between the two preamplifiers.

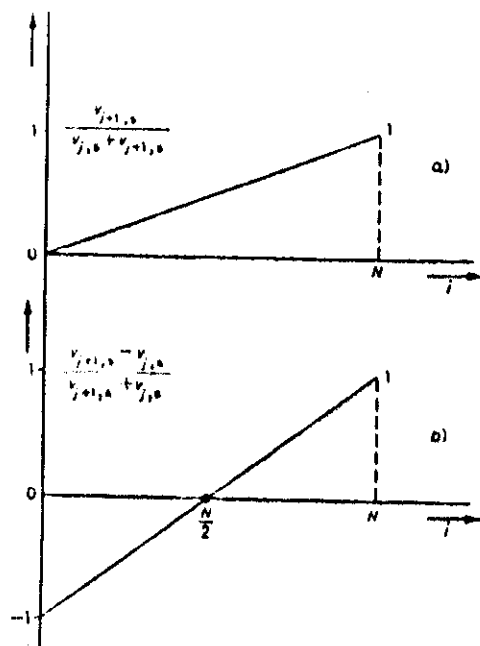


Fig. 11.7. - Signal as a function of the co-ordinate  $i$  with the two interpolation methods.

The dependence of the signal amplitude on  $i$  is shown in fig. 11.7 for the two cases.

The effect of noise in the two cases can be analysed by writing, in place of (11.2), (11.3), the actual quotients

$$(11.3') \quad \frac{v_{j+1,s} - v_{j,s}}{v_{j+1,s} + v_{j,s}} = 2 \left( \frac{i}{N} - \frac{1}{2} \right) + \frac{v_{j+1,N} - v_{j,N}}{v_{j+1,s} + v_{j,s}} + \frac{v_{j+1,N} - v_{j,N}}{v_{j+1,s} + v_{j,s}}$$

and

$$(11.3'') \quad \frac{v_{j+1,s} - v_{j,s}}{v_{j+1,s} + v_{j,s}} + \frac{v_{j+1,N} - v_{j,N}}{v_{j+1,s} + v_{j+1,N} + v_{j,s} + v_{j,N}},$$

where the suffix  $s$  stands for signal and the suffix  $N$  stands for noise.

For  $v_{j+1,N}$  and  $v_{j,N}$  expressions (11.1) have to be introduced. In the actual situation, when discrete or hybrid preamplifiers are employed, the term  $C_j/C_i$  largely dominates and  $v_{j,N}$ ,  $v_{j+1,N}$  can be accordingly written simply as

$$v_{j,N} \approx e_j \frac{C_j}{C_i}, \quad v_{j+1,N} \approx e_{j+1} \frac{C_j}{C_i}.$$

The variance of  $i/N$  is given in both cases, (11.2') and (11.3'), by

$$(11.4) \quad \frac{C_i^2}{Q^2} \left[ \frac{(N-i)^2}{N^2} + \frac{i^2}{N^2} \right] \bar{e}_N^2,$$

where  $\bar{e}_N^2$  is the square of the r.m.s. noise referred to the input and assumed to be the same for all the channels.

The function in brackets is plotted in fig. 11.8.

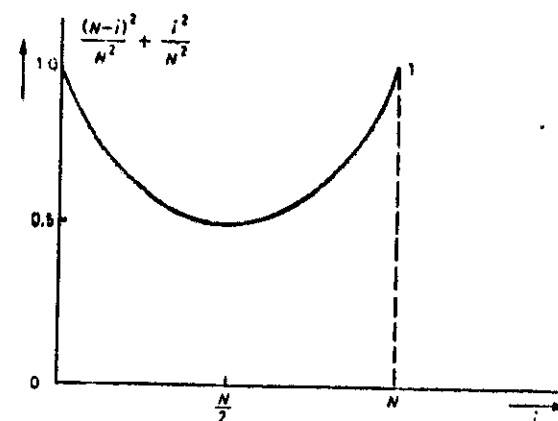


Fig. 11.8. - Dependence of the  $i/N$  variance on the  $i$  co-ordinate.

All the analysis developed so far disregards the effects of the parallel capacitances in the detector and applies, therefore, to the case in which  $C_d \gg C_p$ , where  $C_p$  is the capacitance of the diodes determined by the strips that are not read out and the depleted  $N$  region beneath them.

If the condition  $C_d \gg C_p$  is not met, the ladder-type capacitive network resulting from the  $C_d$ ,  $C_p$  cells in series introduces an attenuation in the charge

appearing at the preamplifiers' input and would affect the position measurement with a nonlinearity even in the ideal hypothesis of absence of noise.

To satisfy the condition  $C_{11} \gg C_p$ , the detector should be manufactured by making, for a given pitch, the strips as wide as possible and the gap between strips as narrow as possible. Additional interstrip capacitance might be required if the previous method does not give a suitably high  $C_{11}/C_p$  ratio.

## REFERENCES

- [1] G. BELLINI, P. D'ANGELO, P. F. MANFREDI, E. MERONI, L. MORONI, C. PALAZZI-CERRINA, F. RAGUSA and S. SALA: *Nucl. Instrum. Methods*, 196, 351 (1982).
- [2] G. BELLINI, M. CONTE, P. D'ANGELO, P. F. MANFREDI, D. MENASCE, E. MERONI, L. MORONI, S. SALA and M. SZAWLOWSKI: *IEEE Trans. Nucl. Sci.*, NS-30, 415 (1983).
- [3] S. R. AMENDOLIA, G. BATIGNANI, F. BEDESCHI, G. BELLINI, E. BERTOLUCCI, D. BETTONI, G. BOLOGNA, L. BOSISIO, C. BRADASCHIA, M. BUDINICH, P. D'ANGELO, M. DELL'ORSO, B. D'ETTORRE PIAZZOLI, F. L. FABRI, F. FIDECARO, L. FOÀ, E. FOCARDI, P. GIANNETTI, M. ENORINI, M. A. GIORGI, P. LAURELLI, F. LIELLO, P. F. MANFREDI, G. MANNOCCHI, P. S. MARROCCHESI, D. MENASCE, A. MENTIONE, E. MERONI, E. MILLOTTI, L. MORONI, D. PEDRINI, L. PERASSO, P. PICCHI, F. RAGUSA, L. RISTORI, L. ROLANDI, S. SALA, A. SCRIBANO, P. SPILLANTINI, A. STEFANTINI, R. TENCHINI, G. TRIGGIANI, G. TONELLI and A. ZALLO: *Nucl. Instrum. Methods Phys. Res.*, 226, 78 (1984).
- [4] R. BARATE, P. BONAMY, P. BERGEAUD, M. BURCHELL, M. DAVID, Y. LENOIGNE, C. MAGNEVILLE, J. MOYCHET, P. NOON, J. POINSIGNON, N. PRIMONT, P. G. RANCOITA and G. VILLET: *Nucl. Instrum. Methods Phys. Res. A*, 235, 235 (1985).
- [5] A. IOIUNI, P. INZANI, P. F. MANFREDI, D. MARIOLI, C. MERONI, N. RADAELLI, D. TORRETTA and G. VIGNI: *Nucl. Instrum. Methods Phys. Res.*, 226, 85 (1984).
- [6] P. F. MANFREDI and F. RAGUSA: *Nucl. Instrum. Methods Phys. Res. A*, 235, 345 (1985).
- [7] V. RADEKA: *Nucl. Instrum. Methods Phys. Res.*, 226, 209 (1984).
- [8] P. JARRON and M. GOYOT: *Nucl. Instrum. Methods Phys. Res.*, 226, 155 (1984).
- [9] E. H. M. HEIJNE, E. HIGON, P. JARRON, L. ROSSI, P. BURGER, W. GEENLINGS and M. RYNSANT: *Nucl. Instrum. Methods Phys. Res.*, 226, 93 (1984).
- [10] R. ALBERGANTI, E. CHESI, CH. GERKE, F. PIUZ, L. RANELLO, T. D. WILLIAMS and R. ROOSEN: *A system of 4400 silicon microstrips readout with analog multiplexed electronics*, CERN-Rep EF/0089 P (3 Novembre 1983).
- [11] R. HOFMANN, G. LUTZ, B. J. BOSTICKA, M. WREDE, G. ZIMMER and J. KEMMER: *Nucl. Instrum. Methods Phys. Res.*, 226, 195 (1984).
- [12] J. T. WALKER, S. PARKER, B. HYAMS and S. L. SHAPIRO: *Nucl. Instrum. Methods Phys. Res.*, 226, 200 (1984).
- [13] V. RADEKA: *Position sensitive semiconductor detectors signal formation, noise and position readout*, in *Proceedings of the Workshop on Silicon Detectors for High Energy Physics, Fermilab, 15-16 October 1981*, edited by T. FERBEL (1981), p. 21.
- [14] J. KEMMER, E. BELAU, R. KLANKER, G. LUTZ and B. HYAMS: *Development of 10  $\mu$ m resolution silicon counters for charm signature observation with the ACCMOR spectrometer*, in *Proceedings of the Workshop on Silicon Detectors for High Energy Physics, Fermilab, 15-16 October 1981*, edited by T. FERBEL (1981), p. 195.

## 12. - Conclusions.

During the past few years, several semiconductor detecting devices have been introduced in elementary-particle experiments performed around the large accelerators.

Examples of these devices, according to their historical order of appearance, are the arrays of silicon detectors employed as active targets or as charged-multiplicity recognizers, the germanium active targets, the silicon microstrip detectors and the CCDs used for position-sensing and for vertex reconstruction purposes. More recently, large silicon detectors have been successfully tested as sampling elements in calorimetry, while new devices, like the silicon drift chamber, are progressing along their way to the experimental set-up.

When one of these devices is introduced into a complex experimental configuration, like the one that most of the experiments require, and is exposed to a high-intensity beam, several peculiar constraints affect the strategies adopted to process the signal delivered by the detectors. These strategies are entirely different according to whether solid-state detectors are employed in a fixed-target or in a colliding-beam experiment. Besides, they depend on several other parameters of the experiment and must be versatile enough to enable the experimenters to change some of these parameters.

A review of the criteria governing the detector signal processing under the particular constraints arising in experiments around large accelerators was one of the purposes of this paper.

Its second purpose is that of disseminating among the community of high-energy physicists a number of instrumentation solutions that have been developed with the aim of making the high-energy applications of solid-state detectors as effective as possible.

## APPENDIX

The values of the integrals that more frequently appear in the frequency domain noise evaluations are given in this appendix for the readers' usefulness. A very good sourcebook of integrals is *Table of Integrals, Series and Products* by I. S. GRADSTEYN and I. M. RYZIK (Academic Press, New York, N. Y., 1965), whence some of the results quoted in this appendix were taken.

$$1) \int_0^{\infty} \frac{dx}{(1+x^2)^2} = \pi/4,$$

$$2) \int_0^{\infty} \frac{x^2 dx}{(1+x^2)^2} = \pi/4,$$

$$3) \int_0^{\infty} \frac{\sin^2 ax}{x^2} dx = a\pi/2 \quad (a > 0),$$

$$4) \int_0^{\infty} \frac{\sin^2 ax}{x^2} dx = \frac{3}{4} a \ln 3,$$

$$5) \int_0^{\infty} \frac{\sin^4 ax}{x^2} dx = \frac{a\pi}{4} \quad (a > 0),$$

$$6) \int_0^{\infty} \frac{\sin^4 ax}{x^2} dx = a^2 \ln 2,$$

$$7) \int_0^{\infty} \frac{\sin^4 ax}{x^2} dx = \frac{a^2 \pi}{3} \quad (a > 0),$$

$$8) \int_0^{\infty} \frac{\sin x}{x} dx = \pi/2.$$

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LECTURE NOTES

**CHAPTER 3 :**  
**FILTERING IN ENERGY MEASUREMENTS**

# FILTERING IN ENERGY MEASUREMENTS

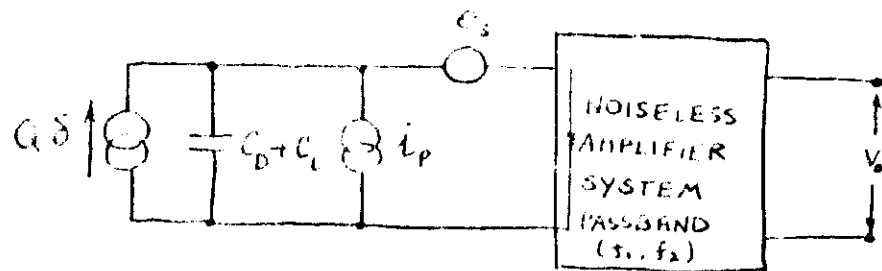


Fig. 1

$Q$  charge released by the detector

$C_D$  detector capacitance

$C_i$  preamplifier input capacitance

$i_p$  parallel noise source

$e_s$  series noise source

$i_p, e_s$  are assumed to be uncorrelated

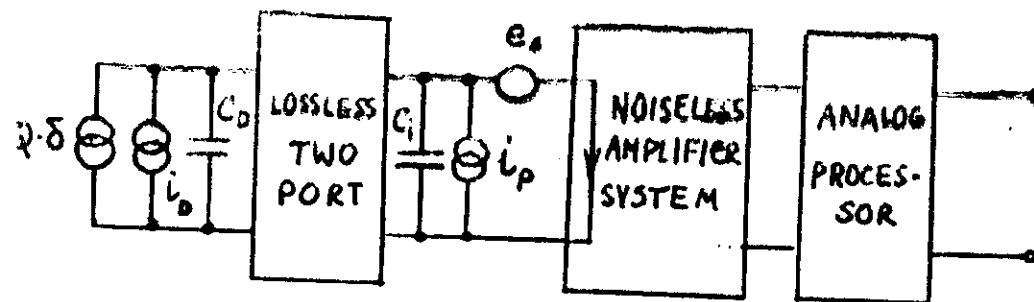
$f_1, f_2$  lower and upper frequency limitations in the amplifier system and  $f_2 \gg f_1$ , so that the peak amplitude of the signal  $V_o$  is close to

$$\frac{Q}{C_D + C_i}$$

A MORE GENERAL SITUATION SHOULD TAKE INTO ACCOUNT THE FOLLOWING POINTS:

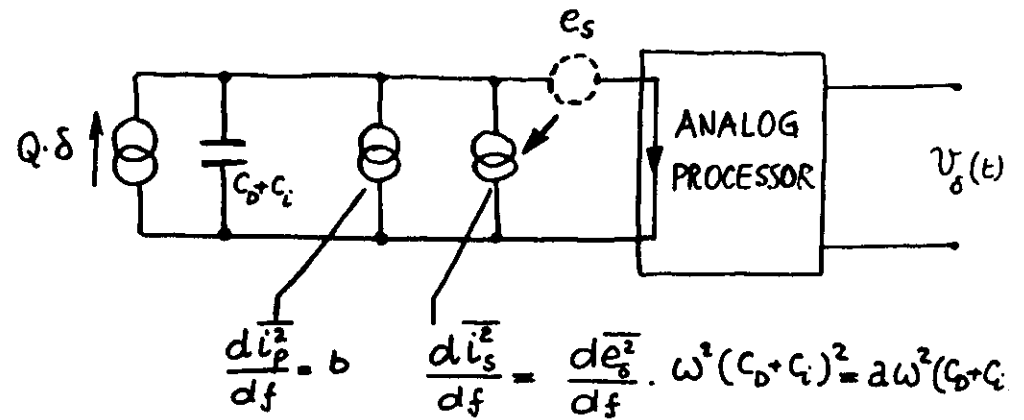
I - NOT WHITE NOISE SOURCES (AS WE SHALL CONSIDER LATER)

II - A POSSIBLY LOSSLESS TWO PORT (FOR INSTANCE A TRANSFORMER BETWEEN DETECTOR AND NOISY PREAMPLIFIER), IN WHICH CASE THE DETECTOR CURRENT NOISE CANNOT BE INCLUDED IN  $i_p$ .



AS FAR AS SIGNAL-TO-NOISE RATIO IS CONCERNED THE AMPLIFIER SYSTEM IS FULLY REPRESENTED BY  $C_i$  AND THE NOISE SOURCES WHICH ARE INVARIANT WITH RESPECT TO THE IMPEDANCE AT THE AMPLIFIER INPUT PORT, WHICH HAS BEEN CHOSEN AS A SHORT CIRCUIT IN THE FIGURE FOR THE SAKE OF CONVENIENCE. AN EQUIVALENT REPRESENTATION WOULD HAVE BEEN AN OPEN CIRCUIT. IN BOTH CASES  $C_i$  REPRESENTS THE ACTUAL CAPACITANCE OF THE INPUT DEVICE WITH THE ADDITIONAL CONTRIBUTION OF THE FEEDBACK CAPACITANCE  $C_f$  FOR A CHARGE-SENSITIVE CONFIGURATION.

fig. 3



OPTIMUM FILTER is defined as the one which leads to the best least square measurement of the charge  $Q$  in the presence of the bilateral noise spectrum  $b + 2\omega^2(C_\delta + C_i)^2$ .

FILTERING WITH SIGNAL AND EQUIVALENT NOISE SOURCES AT THE INPUT  
TIME DOMAIN APPROACH TO OPTIMUM FILTERING

Let  $V_\delta(t)$  be the response of the filter to a  $\delta$ -impulse detector current and let 1 be its peak value, so that the peak response to the actual current signal carrying the charge  $Q$  is  $Q$ .

The current noise source with white spectrum  $b$  can be represented in the time domain as a sequence of positive and negative  $\delta$ -impulses of 1 Coulomb area occurring at a rate  $\lambda_1$ .

Similarly, the generator with spectral power

density  $a\omega^2(C_0+C_i)^2$  can be thought of in the time domain as consisting of doublets, derivatives of  $\delta$ -impulses with area 1 Coulomb, occurring at an average rate  $\lambda_2$ .

In order to calculate  $\lambda_1$  and  $\lambda_2$  the noise power expressed in frequency and time domain of each of the two contributions are equalled:

$$b \int_{-\infty}^{\infty} |V_{\delta}(j\omega)|^2 df = \lambda_1 \int_{-\infty}^{\infty} v_{\delta}^2(t) dt$$

$$a(C_0+C_i)^2 \int_{-\infty}^{\infty} \omega^2 |V_{\delta}(j\omega)|^2 df = \lambda_2 \int_{-\infty}^{\infty} [v'_{\delta}(t)]^2 dt$$

According to PARSEVAL's theorems, the integrals of first and second members of the two relations are equal, whence:

$$\lambda_1 = a(C_0+C_i)^2$$

The signal-to-noise ratio can be written as:

$$(1) \quad \eta^2 = \frac{Q^2}{b \cdot \int_{-\infty}^{\infty} v_{\delta}^2(t) dt + a(C_0+C_i)^2 \cdot \int_{-\infty}^{\infty} [v'_{\delta}(t)]^2 dt}$$

The problem of identifying the optimum filter can be stated as the search for the  $v_{\delta}(t)$  which minimizes the denominator of  $\eta^2$ .

The denominator is usually called  $ENC^2$ , which is the value of  $Q^2$  in eq. (1) which makes  $\eta^2 = 1$ .  $ENC$  is the value of the charge which injected across the detector provides an output signal equal to the rms NOISE.

By applying a standard variational method under the constraints:

$$V_{\delta}(t) = 1 \quad \text{at a given instant } t_0 \\ \text{assumed for simplicity } t_0 = 0$$

$V_{\delta}(t)$  vanishing for  $t \rightarrow \pm\infty$ .

The varied function becomes:

$$V_{\delta}(t) + \Lambda \cdot f(t)$$

where  $f(t)$  vanishes for  $t = t_0 = 0$  and for  $t \rightarrow \pm\infty$ .

The varied functional is:

$$b \int_{-\infty}^{\infty} (V_{\delta} + \Lambda \cdot f(t))^2 dt + a(C_0 + C_1)^2 \int_{-\infty}^{\infty} (V'_{\delta} + \Lambda \cdot f'(t))^2 dt$$

and should be stationary for  $\Lambda \rightarrow 0$  being  $f(t)$  an arbitrary function.

By equalling to zero the derivative of (2) with respect to  $\Lambda$  and putting  $\Lambda = 0$ :

$$b \int_{-\infty}^{\infty} V_{\delta}(t) \cdot f(t) dt + a(C_0 + C_1)^2 \int_{-\infty}^{\infty} V'_{\delta}(t) \cdot f'(t) dt = 0$$

which can be written as:

$$b \int_{-\infty}^0 V_{\delta}(t) \cdot f(t) dt + a(C_0 + C_1)^2 \int_{-\infty}^0 V'_{\delta}(t) \cdot f'(t) dt + \\ + b \int_0^{\infty} V_{\delta}(t) \cdot f(t) dt + a(C_0 + C_1)^2 \int_0^{\infty} V'_{\delta}(t) \cdot f'(t) dt = 0 \quad (3)$$

The integrals of the type:

$$\int V'_{\delta}(t) \cdot f'(t) dt \quad \text{can be transformed}$$

by integrating by parts as:

$$[f(t) \cdot V'_{\delta}(t)] - \int f(t) \cdot V''_{\delta}(t) dt$$

Taking into account the boundary conditions (3)

can be written as:

$$\int_{-\infty}^0 [b \cdot V_{\delta}(t) - a(C_0 + C_1)^2 V''_{\delta}(t)] \cdot f(t) dt + \\ + \int_0^{\infty} [b \cdot V_{\delta}(t) - a(C_0 + C_1)^2 V''_{\delta}(t)] \cdot f(t) dt = 0$$

As  $f(t)$  is an arbitrary function, the only possibility to make the sum of the two integrals equal to zero is :

$$a(C_0 + C_1)^2 \cdot V_0''(t) - bV_0(t) = 0 \quad -\infty < t < 0$$

$$\text{identical expression} \quad 0 < t < +\infty$$

The common solution is:

$$(4) \quad V_0(t) = A e^{-t/\tau_c} + B e^{t/\tau_c}$$

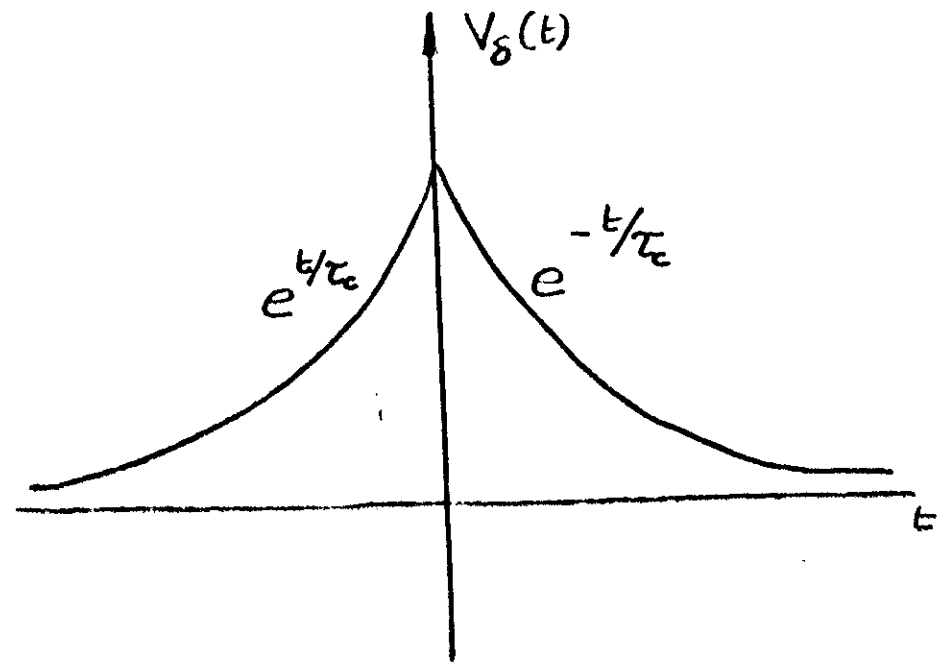
where  $\tau_c$  is the so called  
NOISE CORNER TIME CONSTANT,

$$\tau_c = \sqrt{\frac{a}{b}} (C_0 + C_1)$$

In (4)  $A$  and  $B$  can be chosen differently for  $t > 0$  and  $t < 0$ , so that to satisfy the boundary conditions,

$$\begin{aligned} & e^{t/\tau_c} & -\infty < t < 0 \\ & e^{-t/\tau_c} & 0 < t < +\infty \end{aligned}$$

The overall  $V_0(t)$  has the cusp shape shown in the figure.



Introducing the  $V_0(t)$  expressions (5) into (2), the ENC value is obtained:

$$ENC_{opt} = (4ab)^{\frac{1}{4}} \cdot (C_0 + C_1)^{\frac{1}{2}}$$

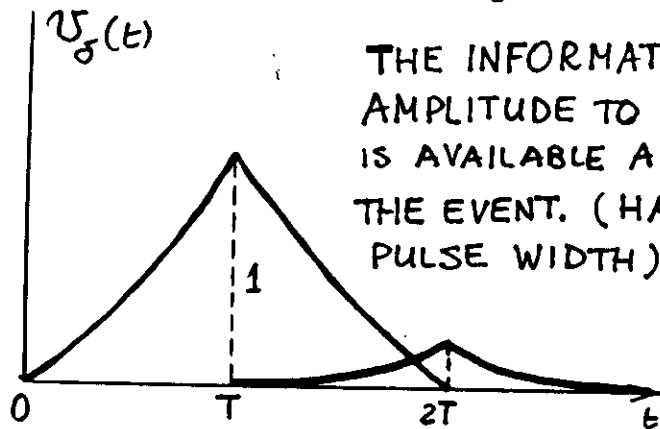
## FINITE WIDTH CONSTRAINT

$$V_{\delta}''(t) - \frac{1}{\tau_c^2} V_{\delta}(t) = 0$$

$$V_{\delta}(t) \equiv 0 \quad \text{for } t < 0 \text{ and for } t > 2T$$

$$\text{for } t = T \quad V_{\delta}(t) = 1$$

$$V_{\delta}(t) = \frac{\sinh \frac{t}{\tau_c}}{\sinh \frac{T}{\tau_c}} \quad 0 \leq t \leq T$$



THE INFORMATION ABOUT THE AMPLITUDE TO BE MEASURED IS AVAILABLE A TIME  $T$  AFTER THE EVENT. (HALF OF THE PULSE WIDTH).

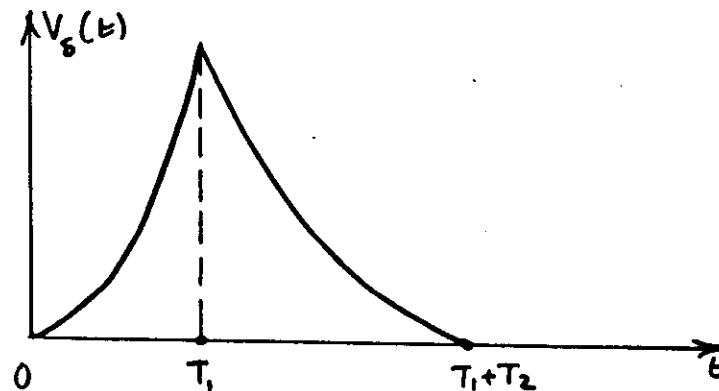
(practically a triangle when  $T < \tau_c$ )

The equivalent noise charge at finite width  $2T$  is related to  $ENC_{OPT}$  by:

$$ENC_{2T} = ENC_{OPT} \cdot \sqrt{\frac{1 + e^{-2T/\tau_c}}{1 - e^{-2T/\tau_c}}}$$

If the times available before and after the event are different, a finite cusp width can be chosen exploiting the whole available interval  $T_1 + T_2$  and the corresponding ENC is:

$$ENC_{T_1+T_2} = ENC_{OPT} \cdot \sqrt{\frac{1 - e^{-2(T_1+T_2)/\tau_c}}{(1 - e^{-2T_1/\tau_c})(1 - e^{-2T_2/\tau_c})}}$$



Let us introduce now a new concept related to the processing of a signal in any point of the system, where also one noise spectrum has to be specified:

### THE WEIGHTING FUNCTION.

This one is defined by this implicit expression.

$$(6) \quad S_o(t_M) = \int_{-\infty}^{t_M} S(t) \cdot W(t, t_M) dt$$

where

$S_o(t_M)$  is the output at the measurement time  $t_M$  due to the signal  $S(t)$  in the considered point.

$W(t, t_M)$  is the weighting function peculiar to this point.

As  $S(t)$  is an arbitrary function of  $t$ , we can choose for it a  $\delta$  function at the time  $t_0$ , with  $-\infty < t_0 < t_M$ :

$$S(t) = \delta(t - t_0)$$

It comes out from (6):

$$S_o(t_M) = W(t_0, t_M)$$

$W(t_0, t_M)$  can be so operatively defined for a given point: here a  $\delta$ -impulse is applied at time  $t_0$ . The response

$$S_o(t_M) = W(t_0, t_M)$$

is observed at  $t = t_M$ . A point of the weighting function  $W(t_0, t_M)$  is so obtained. Spanning all  $t_0$  values before  $t_M$ , the complete function is experimentally recorded.

eq. (6) represents the most general linear operation a processor can execute on the signal  $s(t)$  and on the noise  $n(t)$ . In particular the processor can be of TIME-VARIANT nature. The weighting function allows the evaluation of the contributions of signal and noise at the specified measurement time  $t_M$ . IT IS IMPORTANT TO SPECIFY THIS TIME BECAUSE A TIME-VARIANT PROCESSOR IS BASED ON A SEQUENCE OF PARAMETRIC CHANGES THAT SHOULD OCCUR AT FIXED INSTANTS WITH RESPECT TO THE MEASUREMENT TIME. THE USE OF TIME-VARIANT PROCESSORS, BECAUSE OF THESE TIME RELATIONSHIPS BETWEEN PARAMETRIC VARIATIONS AND MEASUREMENT TIME CAN BE USED

either when the time occurrence of the event is known (for instance, in COLLIDER EXPERIMENTS WITH VERY SHORT BUNCHES) or if an auxiliary channel is provided for the detection of the occurrence of an event and the starting of the processor sequence on the event of interest, which has to be delayed.

LET US FIND THE RELATIONSHIP BETWEEN THE WEIGHTING FUNCTION AND THE  $\delta$ -RESPONSE OF A TIME-INVARIANT PROCESSOR IN THE PARTICULAR CASE OF THE WEIGHTING FUNCTION REFERRED TO THE DETECTOR PORT.

# CONVOLUTION AND WEIGHTING FUNCTION INTEGRAL

## CONVOLUTION

INPUT CIRCUIT      SIGNAL  $S(t)$       NOISE  $n(t)$   
                          $\delta$ -RESPONSE  $v_{\delta}(t)$

OUTPUT

$$S_{\alpha}(t) = \int_{-\infty}^t S(\tau) \cdot v_{\delta}(t-\tau) d\tau$$

$$n_{\alpha}(t) = \int_{-\infty}^t n(\tau) \cdot v_{\delta}(t-\tau) d\tau$$

at  $t=T$   $S_{\alpha}(t)$  reaches its maximum

$$S_{\alpha}(T) = \int_{-\infty}^T S(\tau) \cdot v_{\delta}(T-\tau) d\tau \quad \text{and } n_{\alpha}(T) \text{ is}$$

the noise at the same instant

The same result can be obtained by the

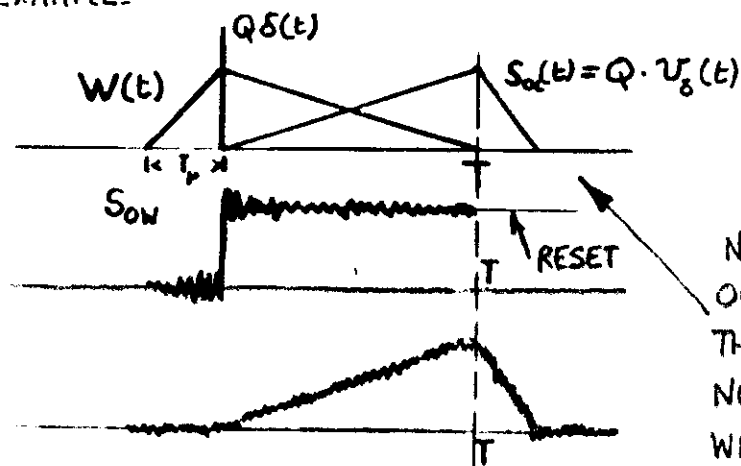
WEIGHTING FUNCTION INTEGRAL

$$S_{\alpha}(T) = \int_{-\infty}^T S(\tau) \cdot W(\tau, T) d\tau, \quad n_{\alpha}(T) = \int_{-\infty}^T n(\tau) \cdot W(\tau, T) d\tau$$

for the values at time  $T$  provided that

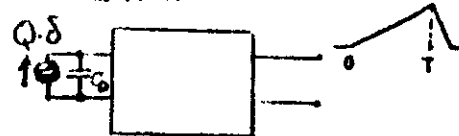
$$W(\tau, T) = v_{\delta}(T-\tau)$$

EXAMPLE.

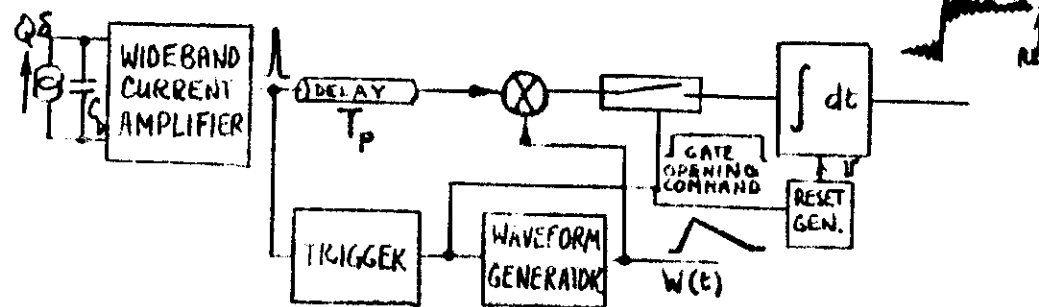


NOTE THAT THE OUTPUT SHAPE IN THE TV CASE HAS NO RELATIONSHIP WITH THE OUTPUT WAVEFORM IN THE TI ONE

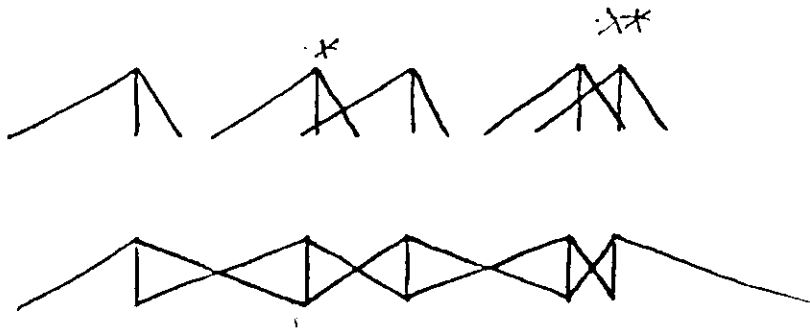
CONVOLUTION



CORRELATION



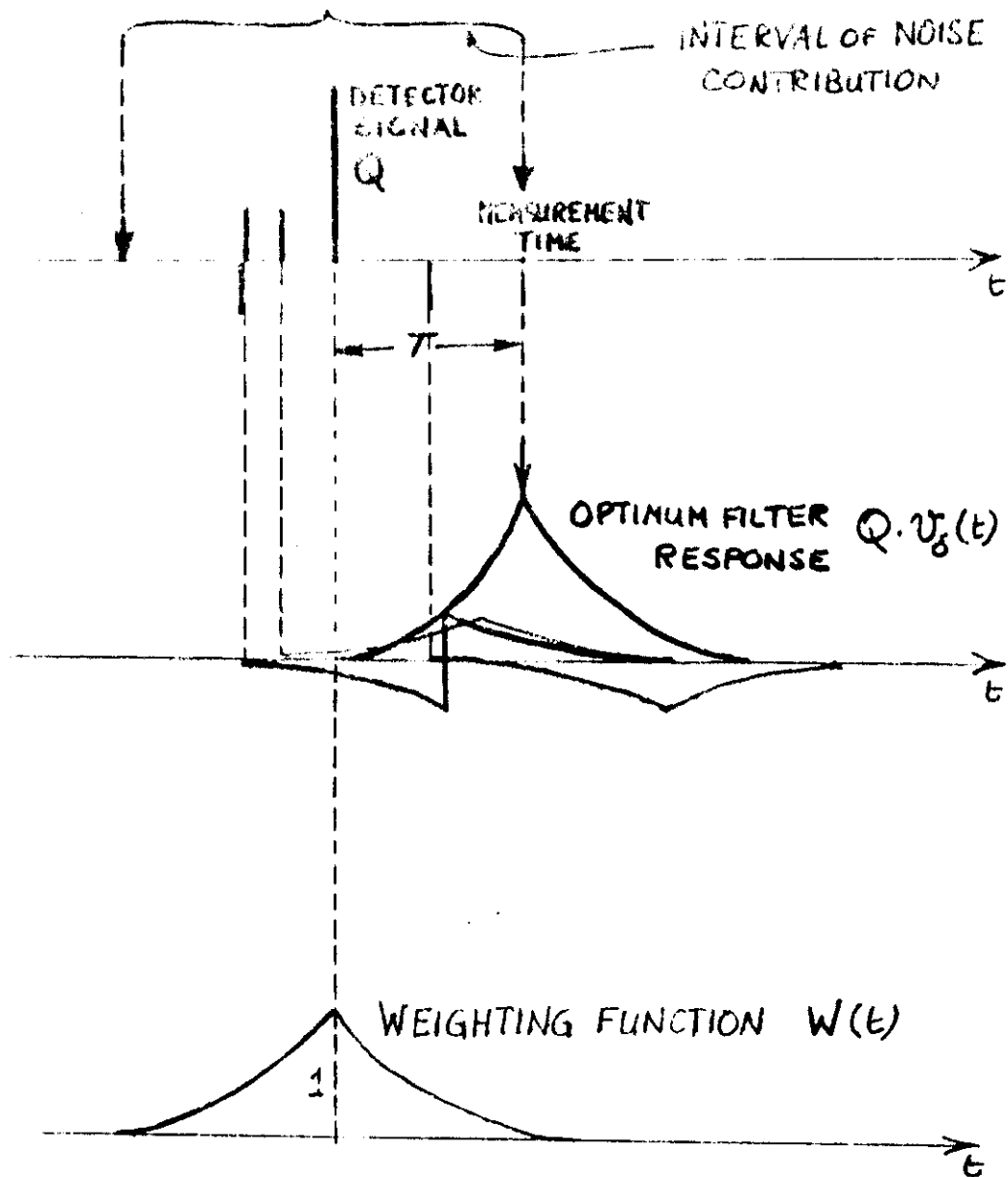
- 1) CORRELATION METHOD REQUIRES A DELAY  $T_p$  FOR PROCESSING THE SIGNAL OUTPUT BEFORE THE ARRIVAL OF THE EVENT.
- 2) THE RESULT IS OBTAINED AT A TIME  $T_p + T$ , WITH THE CORRELATION METHOD, WHILE ON THE  $T$  WITH THE CONVOLUTION.



SOME OF THE RESULTS ABOUT THE OPTIMUM PROCESSING WITH TIME-INVARIANT FILTERS WHICH LEAD TO  $V_{\delta, \text{OPT}}(t)$  CAN BE OBTAINED WITH A FREQUENCY DOMAIN ANALYSIS.

BY THE EQUIVALENCE  $V_{\delta}(t) = W(-t)$  ALSO THE OPTIMUM WEIGHTING FUNCTION CAN BE DEDUCED.

THE FREQUENCY-DOMAIN APPROACH FOR DEDUCING  $V_{\delta}(t)$  AND THE WEIGHTING FUNCTION IS USEFUL FOR DEALING WITH MORE GENERAL NOISE SPECTRA. HOWEVER IN THIS DOMAIN IT IS NOT EASY TO INTRODUCE TIME AND SHAPE CONSTRAINTS ON THE WEIGHTING FUNCTION



FILTERING WITH SIGNAL AND EQUIVALENT NOISE SOURCES AT THE INPUT  
- FREQUENCY DOMAIN APPROACH TO OPTIMUM FILTERING

Let  $f(t)$  be the response excited by the charge  $Q$ . The square of the signal-to-noise ratio can be written as

$$\eta^2 = \frac{\frac{Q^2}{2\pi} \left[ \int_{-\infty}^{\infty} f(\omega) \cos \omega t d\omega \right]_{t=0}^2}{\int_{-\infty}^{\infty} [a(c_0 + c_1)^2 \omega^2 + b] F^2(\omega) d\omega}$$

Apart from the constant  $Q^2$ , the numerator of  $\eta^2$  is the square of the peak of  $f(t)$ , with  $f(t)$  supposed to be real and expressed as Fourier transform of  $\cos \omega t$  only.

In the case of a general noise spectral power density,  $\eta^2$  can be written as

$$\begin{aligned} \eta^2 &= \frac{\frac{Q^2}{2\pi} \left[ \int_{-\infty}^{\infty} f(\omega) \cos \omega t d\omega \right]_{t=0}^2}{\int_{-\infty}^{\infty} N(\omega) f^2(\omega) d\omega} = \\ &= \frac{Q^2 \left[ \int_{-\infty}^{\infty} f(\omega) d\omega \right]^2}{2\pi \int_{-\infty}^{\infty} N(\omega) f^2(\omega) d\omega} \end{aligned}$$

The already introduced method will now be employed to determine the function  $F(\omega)$  which optimizes the signal-to-noise ratio.

For the purpose, replace

$F(\omega)$  with  $F(\omega) + \epsilon G(\omega)$ .

The relationship which defines the optimum  $F(\omega)$  is:

$$\left\{ \int_{-\infty}^{\infty} F(\omega) d\omega \right\} \cdot \int_{-\infty}^{\infty} N(\omega) F(\omega) G(\omega) d\omega = \int_{-\infty}^{\infty} G(\omega) d\omega \cdot \int_{-\infty}^{\infty} N(\omega) F^2(\omega) d\omega$$

and is satisfied for

$$(5) \quad F(\omega) = \frac{1}{N(\omega)}$$

The square of the optimum signal-to-noise ratio is

$$(6) \quad \eta_{\text{opt}}^2 = \frac{Q^2}{2\pi} \int_{-\infty}^{\infty} \frac{1}{N(\omega)} d\omega$$

In the actual case of  $N(\omega) = a(G+C)^2\omega^2 + b$ ,

$$\eta_{\text{opt}}^2 = \frac{Q^2}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{a(G+C)^2\omega^2 + b} =$$

$$= \frac{Q^2}{2\pi b} \cdot \sqrt{\frac{b}{a}} \cdot \frac{1}{G+C} \cdot \int_{-\infty}^{\infty} \frac{d\left(\omega \sqrt{\frac{a}{b}} (G+C)\right)}{1 + \left[\omega \sqrt{\frac{a}{b}} (G+C)\right]^2} =$$

$$= \frac{Q^2}{2\pi} \cdot \frac{1}{\sqrt{ab} (G+C)} \cdot \int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \frac{Q^2}{(G+C) \sqrt{4ab}}$$

The optimum response  $v_g(t)$  in this case is:

$$(7) \quad v_g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\cos \omega t}{N(\omega)} d\omega =$$

$$= \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} \frac{\cos \omega t}{a(G+C)^2\omega^2 + b} d\omega =$$

$$= 1(t) e^{-\sqrt{\frac{b}{a}} \cdot \frac{t}{G+C}} + 1(-t) e^{\sqrt{\frac{b}{a}} \cdot \frac{t}{G+C}}$$

# FILTERING IN PRESENCE OF FLICKER AND DIELECTRIC NOISE

The spectral current density of the noise reduced to the input is

$$(8) \quad N(\omega) = \omega^2 a (C_D + C_i)^2 + \omega [b_\omega + a_\omega (C_D + C_i)^2] + b$$

where  $b_\omega \left[ \frac{A^2}{Hz^2} \right]$  is the coefficient of the current spectral density  $b_\omega \cdot \omega$  of the dielectric noise and  $a_\omega [V^2]$  is the coefficient of the voltage spectral density  $a_\omega / \omega$  of the series flicker noise. Introducing (8) into (6) the achievable  $\eta_{OPT}^2$  is:

$$\eta_{OPT}^2 = \frac{Q^2}{2\pi} \cdot \frac{1}{\sqrt{ab}} \cdot \frac{1}{C_D + C_i} \cdot \int_{-\infty}^{\infty} \frac{dx}{1 + 2K|x| + x^2}$$

$$\text{where } K = \frac{1}{2\sqrt{ab}} \left[ \frac{b_\omega}{C_D + C_i} + a_\omega (C_D + C_i) \right]$$

The ratio  $\eta_{OPT} / (\eta_{OPT})_{K=0}$  is plotted as a function of  $K$  in figure 5.

$$\frac{1}{f} = \text{NOISE}$$

$$\frac{\eta_{IND. CUSP, WITH \frac{1}{f} \text{ NOISE}}^2}{\eta_{IND. CUSP, \frac{1}{f} \text{ - NOISE ABSENT}}^2} = \frac{2}{\pi} \cdot \frac{1}{\sqrt{1-K^2}} \cdot \arctg \frac{\sqrt{1-K^2}}{K}$$

$K < 1$

$$= \frac{1}{\pi} \cdot \frac{1}{\sqrt{K^2-1}} \ln \frac{K + \sqrt{K^2-1}}{K - \sqrt{K^2-1}}$$

$K > 1$

$$N(\omega) = \omega^2 a (C_0 + C_i)^2 + \omega [b\omega + a\omega (C_0 + C_i)^2] + b$$

$$K = \frac{1}{2\sqrt{ab}} \cdot \left[ \frac{b\omega}{C_0 + C_i} + a\omega (C_0 + C_i)^2 \right]$$

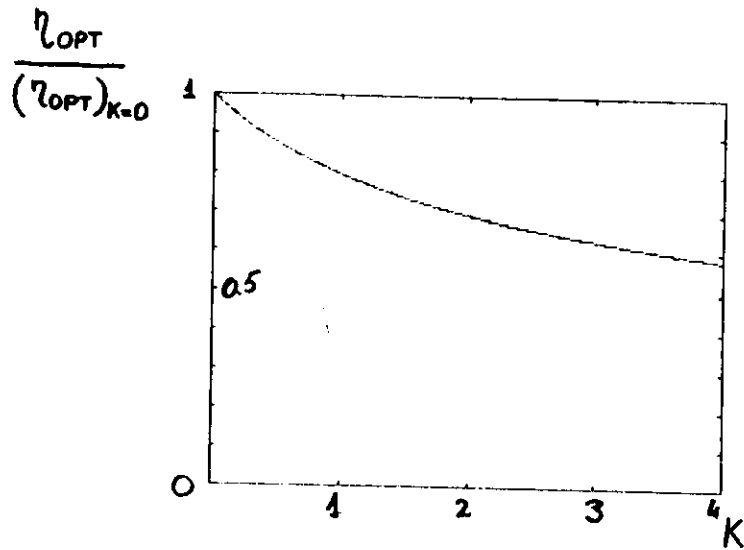


FIGURE 5

RATIO BETWEEN THE OPTIMUM  $\eta$  IN PRESENCE OF FLICKER AND DIELECTRIC NOISE AND THE OPTIMUM  $\eta$  WHEN THESE CONTRIBUTIONS ARE ABSENT

Introducing (8) into (7) we get for the optimum filter:

$$V_{\delta}\left(\frac{t}{t_c}\right) \doteq \int_0^{\infty} \frac{\cos\left(x \cdot \frac{t}{t_c}\right)}{x^2 + 2Kx + 1} dx$$

Where  $t_c = (C_0 + C_i) \cdot \sqrt{\frac{a}{b}}$  is the "noise corner time constant".

In figure 6  $V_{\delta}(t/t_c)$  is plotted for several values of  $K$ . Only the right branch is shown.

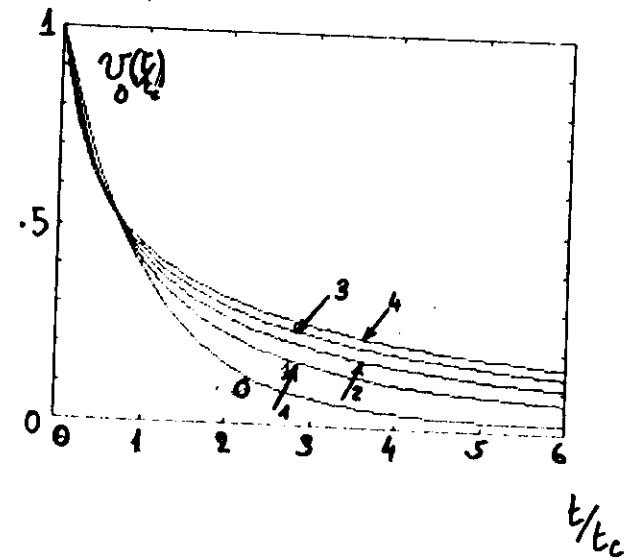


fig. 6

# NOISE - WHITENING FILTER APPROACH TO OPTIMUM MEASUREMENT

## MEASUREMENT

### TIME-DOMAIN TREATMENT

With reference to figure 3, split the filter into a NOISE WHITENING ONE and one which performs the optimum measurement on the signal at the output of the NWF and on the white noise present there.

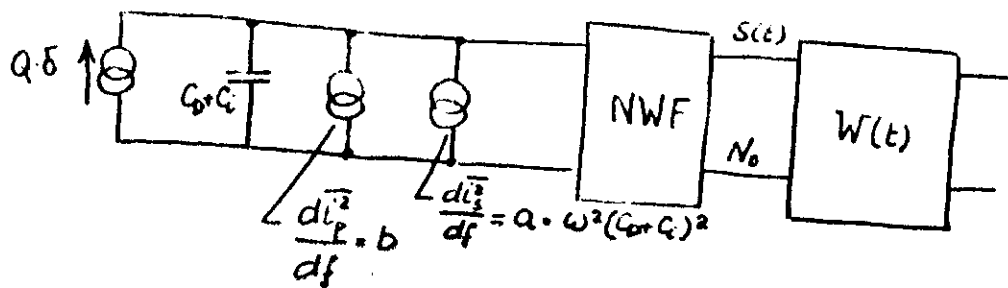
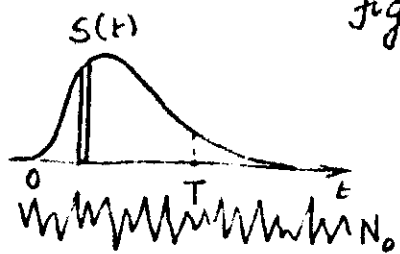


fig.7



Consider  $S(t)$  subdivided into elementary samples in the  $(0, T)$  interval and each sample weighted according to a "weighting function"  $W(t)$ .

At  $t=T$  a signal amplitude is obtained equal to:

$$\int_{-\infty}^T S(t) \cdot W(t) dt$$

The samples of the white noise taken at different times are uncorrelated and weighted

by  $W(t)$  give at  $t=T$  a contribution proportional to  $\left\{ N_0 \int_{-\infty}^T W^2(t) dt \right\}^{1/2}$

The square of the signal-to-noise ratio is therefore proportional to

$$\frac{\left\{ \int_{-\infty}^T S(t) \cdot W(t) dt \right\}^2}{N_0 \int_{-\infty}^T W^2(t) dt}$$

The identification of  $W(t)$  which optimizes the ratio above proceeds as already shown, by evaluating:

$$\begin{aligned} & \frac{\partial}{\partial \epsilon} \frac{\left\{ \int_{-\infty}^T S(t) [W(t) + \epsilon G(t)] dt \right\}^2}{\int_{-\infty}^T [W(t) + \epsilon G(t)]^2 dt} = \\ & = \frac{2 \int_0^T S(t) [W(t) + \epsilon G(t)] dt \cdot \int_0^T S(t) G(t) dt - \int_0^T [W(t) + \epsilon G(t)]^2 dt}{\left\{ \int_{-\infty}^T [W(t) + \epsilon G(t)]^2 dt \right\}^2} - \\ & - \frac{\left\{ 2 \int_{-\infty}^T W(t) G(t) dt + 2 \epsilon \int_0^T G^2(t) dt \right\} \cdot \left\{ \int_0^T S(t) [W(t) + \epsilon G(t)] dt \right\}}{\left\{ \int_{-\infty}^T [W(t) + \epsilon G(t)]^2 dt \right\}^2} \end{aligned}$$

Nulling the derivative to zero for  $\epsilon=0$  leads to a condition:

$$\int_0^T S(t) \cdot G(t) dt \cdot \int_0^T W^2(t) dt - \int_0^T W(t) \cdot G(t) dt \cdot \int_0^T S(t) \cdot W(t) dt = 0$$

which is satisfied if and only if  $W(t) = S(t)$ .

As a conclusion, the optimum least square measurement of a parameter associated with a signal  $S(t)$  in presence of white noise requires that each signal sample be weighted with a weight proportional to the sample amplitude. The network whose weighting function is  $S(t)$  is described by a  $\delta$ -impulse response, which as easy shown, is

$$S(T-t),$$

that is, the mirror image of the part of  $S(t)$  falling between the time origin and the measurement instant  $T$ .

Application of the previous considerations to the actual case gives:

Spectrum at the input of the NWF

$$b + a\omega^2(G_0 + G_i)^2$$

Transfer function of the NWF.

$$|T_w(j\omega)|^2 = \frac{N_0}{b + a\omega^2(G_0 + G_i)^2} = \frac{N_0}{a(G_0 + G_i)^2} \cdot \frac{1}{\frac{b}{a(G_0 + G_i)^2} + \omega^2}$$

$$T_w(s) = \frac{N_0^{1/2}}{a^{1/2}(G_0 + G_i)} \cdot \frac{1}{s + \left(\frac{b}{a}\right)^{1/2} \cdot \frac{1}{G_0 + G_i}}$$

$$S(t) = \frac{Q N_0^{1/2}}{a^{1/2}(G_0 + G_i)} \cdot e^{-\frac{t}{G_0 + G_i} \left(\frac{b}{a}\right)^{1/2}} \quad \text{for infinite measurement time:}$$

$$\begin{aligned} \eta_{\text{OPT}}^2 &= \frac{\int_0^\infty S^2(t) dt}{N_0} = \frac{Q^2}{a(G_0 + G_i)^2} \cdot \int_0^\infty e^{-2\frac{t}{G_0 + G_i} \left(\frac{b}{a}\right)^{1/2}} dt = \\ &= \frac{Q^2}{a(G_0 + G_i)^2} \cdot \frac{1}{2} \cdot (G_0 + G_i) \cdot \left(\frac{a}{b}\right)^{1/2} = \frac{Q^2}{(G_0 + G_i)(4ab)^{1/2}} \end{aligned}$$

that is:

$$\boxed{\eta_{\text{OPT}} = \frac{Q}{(G_0 + G_i)^{1/2} \cdot (4ab)^{1/4}}}$$

The use of the NOISE WHITENING FILTER opens up two possibilities in the realization of the optimum measurement.

One, called convolution measurement, is based upon the network which follows the NSF and has, as a  $\delta$ -response

which defines the optimum  $\hat{V}_\delta$  as the solution of the following differential equation

$$a(C_b + C_s)^2 \cdot V_\delta''(t) - b V_\delta(t) = 0$$

at is :

$$V_\delta(t) = A e^{\sqrt{\frac{b}{a}} \cdot \frac{t}{C_b + C_s}} + B e^{-\sqrt{\frac{b}{a}} \cdot \frac{t}{C_b + C_s}}$$

Introducing the boundary conditions (1 at  $t=0$ , 0 at infinity), we get :

$$V_\delta(t) = e^{-\sqrt{\frac{b}{a}} \cdot \frac{t}{C_b + C_s}}$$

The specular solution is similarly obtained for the left branch. The overall solution is given by:

$$V_\delta(t) = 1(-t) e^{\sqrt{\frac{b}{a}} \cdot \frac{t}{C_b + C_s}} + 1(t) e^{-\sqrt{\frac{b}{a}} \cdot \frac{t}{C_b + C_s}}$$

Putting (3) for  $V_\delta(t)$  into (2) yields for the optimum signal-to-noise ratio:

$$(4) \quad \eta_{\text{OPT}} = \frac{Q}{(C_b + C_s)^{1/2} (4ab)^{1/4}}$$

and  $V_\delta(t)_{\text{OPT}}$  has the shape shown in figure, that is, the indefinite cusp

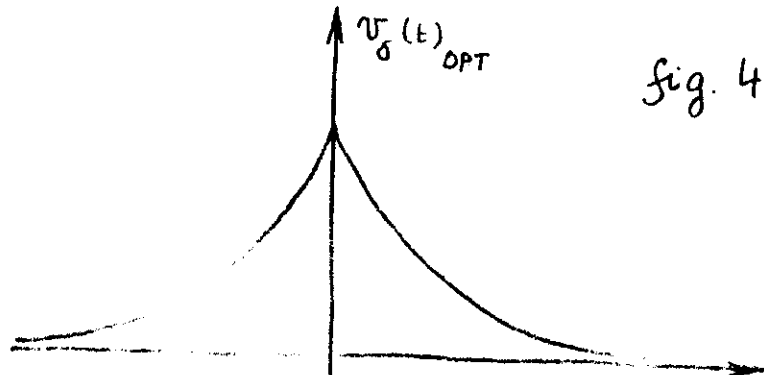
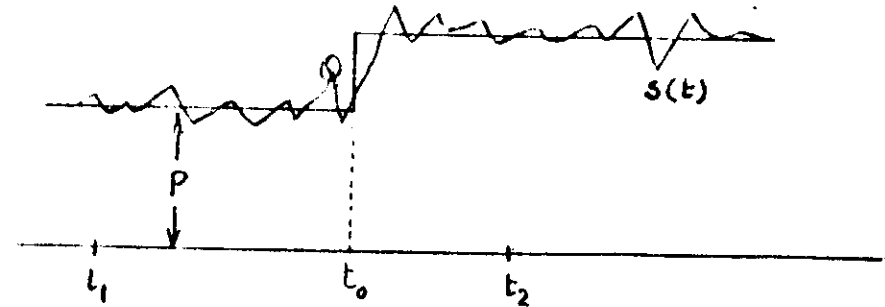


fig. 4

MEASUREMENT OF A STEP IN WHITE NOISE AND IN PRESENCE OF A dc BASELINE



the measurement of  $Q$  has to be carried out between  $t_1$  and  $t_2$ .

The expected shape of the signal is

$$P + Q \cdot 1(t - t_0)$$

$S(t)$  is the experimental signal

The least square estimate is done by minimizing the functional.

$$F = \int_{t_1}^{t_2} [(P + Q \cdot 1(t - t_0)) - S(t)]^2 dt$$

with respect to the parameters  $P$  and  $Q$

The two equations  $\frac{\partial F}{\partial P} = 0$ ,  $\frac{\partial F}{\partial Q} = 0$   
lead to the system:

$$(t_2 - t_1) \cdot P + (t_2 - t_0) \cdot Q = \int_{t_1}^{t_2} S(t) dt$$

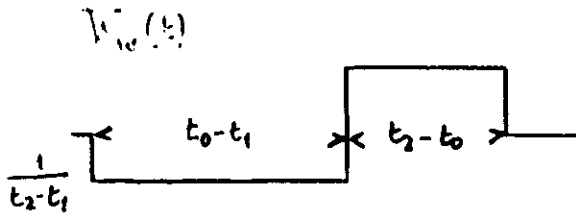
$$(t_2 - t_0) \cdot P + (t_2 - t_0) \cdot Q = \int_{t_1}^{t_2} S(t) \cdot 1(t - t_0) dt$$

by solving :

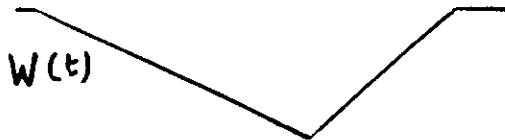
$$\text{or } Q \div \int_{t_1}^{t_2} \left[ \frac{1}{t_2 - t_1} - \frac{1}{t_2 - t_0} \cdot 1(t - t_0) \right] S(t) dt$$

whence  $W(t)$

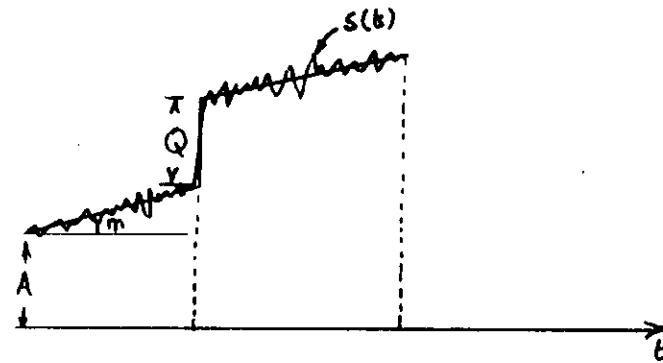
$$W_w(t) = \frac{1}{t_2 - t_1} - \frac{1}{t_2 - t_0} \cdot 1(t - t_0)$$



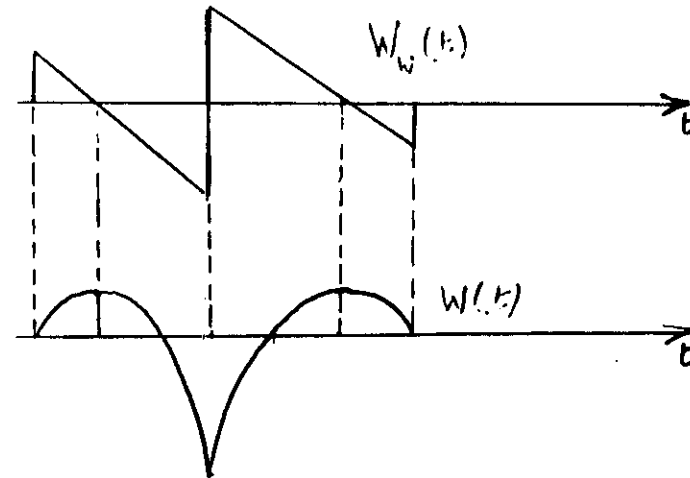
AND THE WEIGHTING FUNCTION REFERRED TO THE INPUT BECOMES



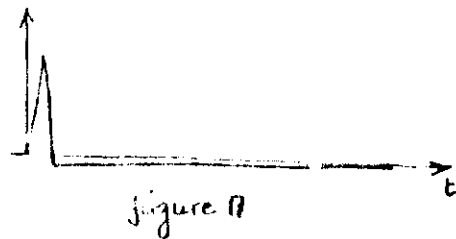
MEASUREMENT OF A STEP IN PRESENCE  
OF A LINEARLY CHANGING BASELINE



BY THE SAME LEAST SQUARE PROCEDURE, WE FIND  
THE FOLLOWING WEIGHTING FUNCTION



A unipolar signal, passed through a differentiating time constant appears at the output followed by a negative tail, as shown in figure 17, where  $RC$  is assumed to be  $\gg 2T$ .



The area of the tail is equal to the area of the positive part of the pulse. The random superposition of tails gives rise to a random fluctuation of the baseline, which is

responsible for shift in the centroid of a spectral line and for a line broadening.

The use of a bipolar signal of the type of figure 18

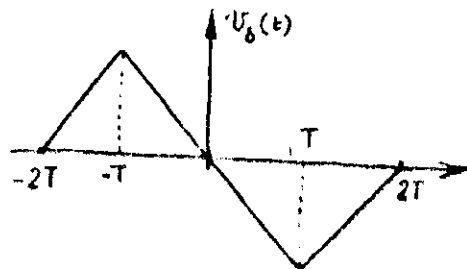


figure 18

greatly reduces the baseline shifts at high counting rates. According to (2), a symmetric bipolar signal of type 18 introduces a  $\sqrt{2}$  worsening in the signal-to-noise ratio.

A bipolar signal of unsymmetric nature, like that shown in figure 19, consisting of two lobes of equal areas but of different lengths reduces the noise worsening factor with respect to the unipolar signal, though maintaining the feature of very small baseline shift at high rates. The obvious disadvantage over the symmetric bipolar pulse is that of an increased probability of signal-mis-  
signal pick-up.

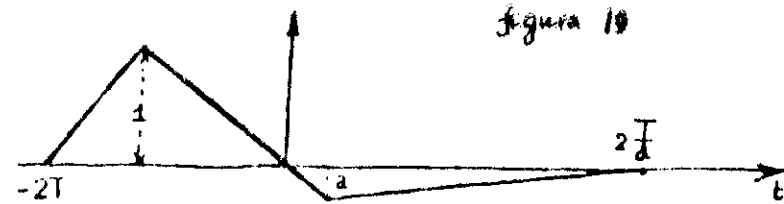
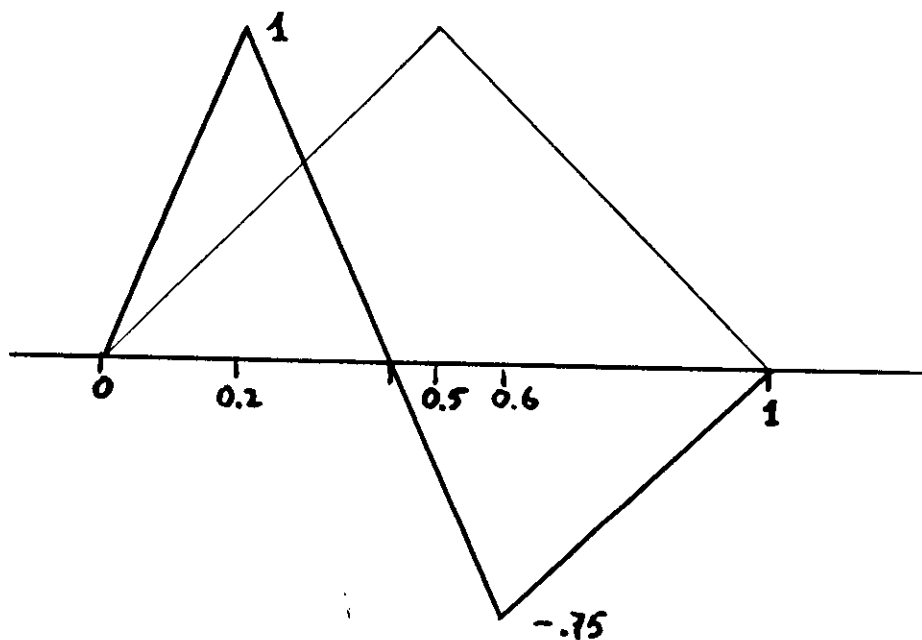


figure 19

for instance, a tail which is twice in duration the original triangle degrades the noise of the unipolar pulse of ~~the~~ about ~~the~~ 2%

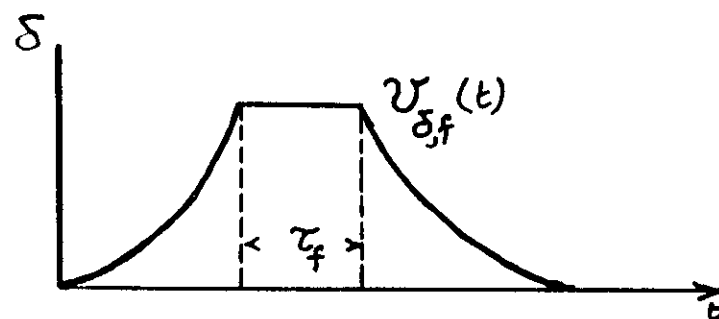
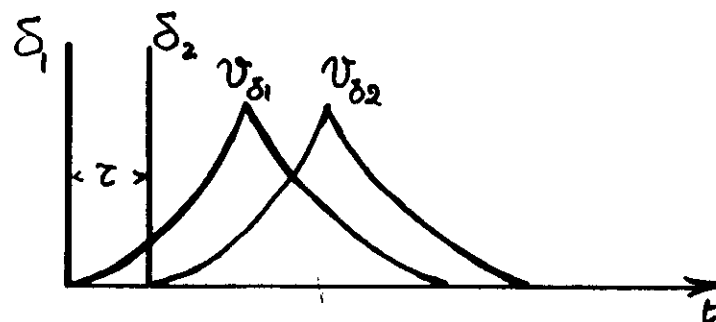
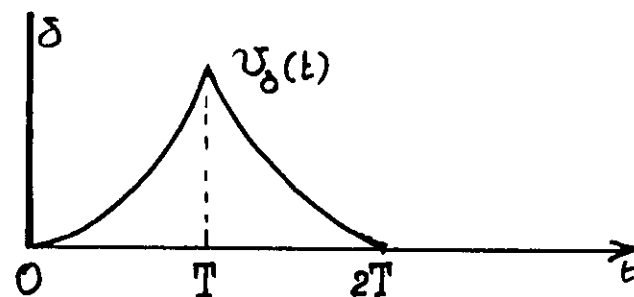


$$\frac{\eta}{\eta_0} = 0.54$$

$$\frac{\eta_{SIMH}}{\eta_0} = 0.5$$

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## BALLISTIC DEFICIT



$$\eta_f = \frac{Q}{\left\{ b \int_0^{\infty} v_{\delta f}^2(t) dt + a c^2 \int_0^{\infty} [v'_{\delta f}(t)]^2 dt \right\}^{1/2}}$$

$$= \frac{Q}{\left\{ b \int_0^{\infty} v_{\delta f}^2(t) dt + a c^2 \int_0^{\infty} [v'_{\delta f}(t)]^2 dt + b \tau_f \right\}^{1/2}}$$

## FURTHER CONSIDERATIONS ON TV PROCESSORS

Remembering the expression of the output noise:

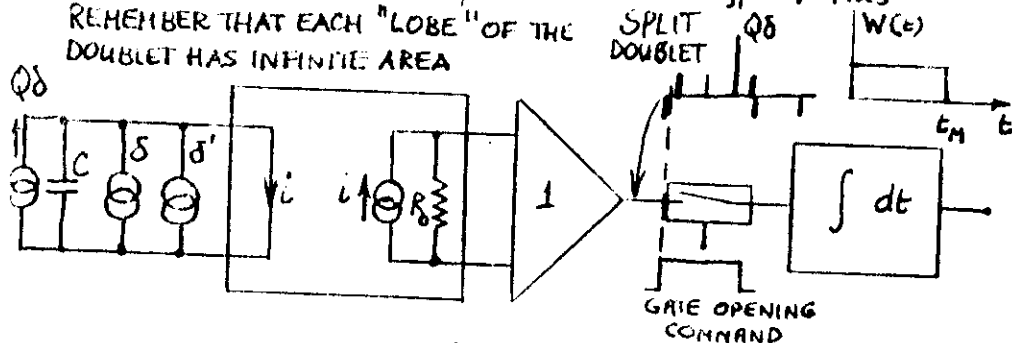
$$\overline{V_n^2} = a^2 \int_{-\infty}^{t_1} [W'(x)]^2 dx + b \int_{-\infty}^{t_M} W(x) dx$$

if the  $W(x)$  slope is infinite at the leading or trailing edge the first integral becomes infinite.

The corresponding effect occurs if  $s(t)$  is evaluated by a weighting function of rectangular shape, as it happens when the weighting integral is evaluated by switching abruptly ON and OFF a gated integrator for a signal which contains 'doublet' noise.

When using a gated integrator this effect is known as "split doublet" effect. This

REMEMBER THAT EACH "LOBE" OF THE DOUBLET HAS INFINITE AREA



effect is prevented, for instance, in processor of figure of page 18 by the triangular weighting function which attenuates to zero the gain of the integrator at the opening and closing

OTHER WAYS OF PREVENTING THIS EFFECT AND GETTING IN THE MEANTIME THE DESIRED WEIGHTING FUNCTION ARE POSSIBLE:

### 1- SHAPING BEFORE THE GATE

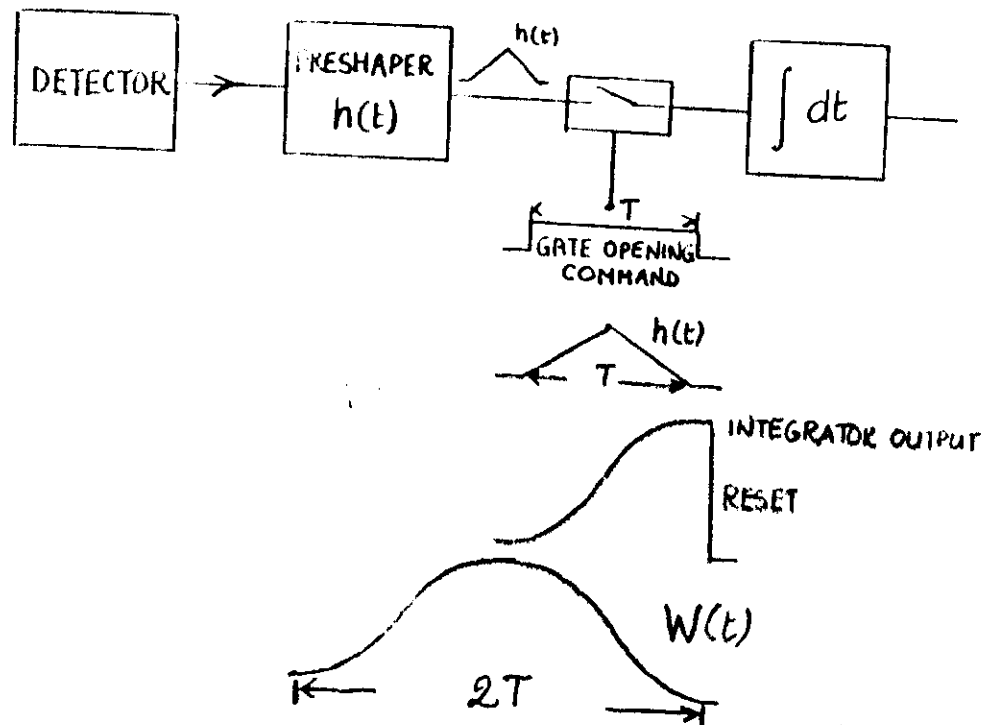
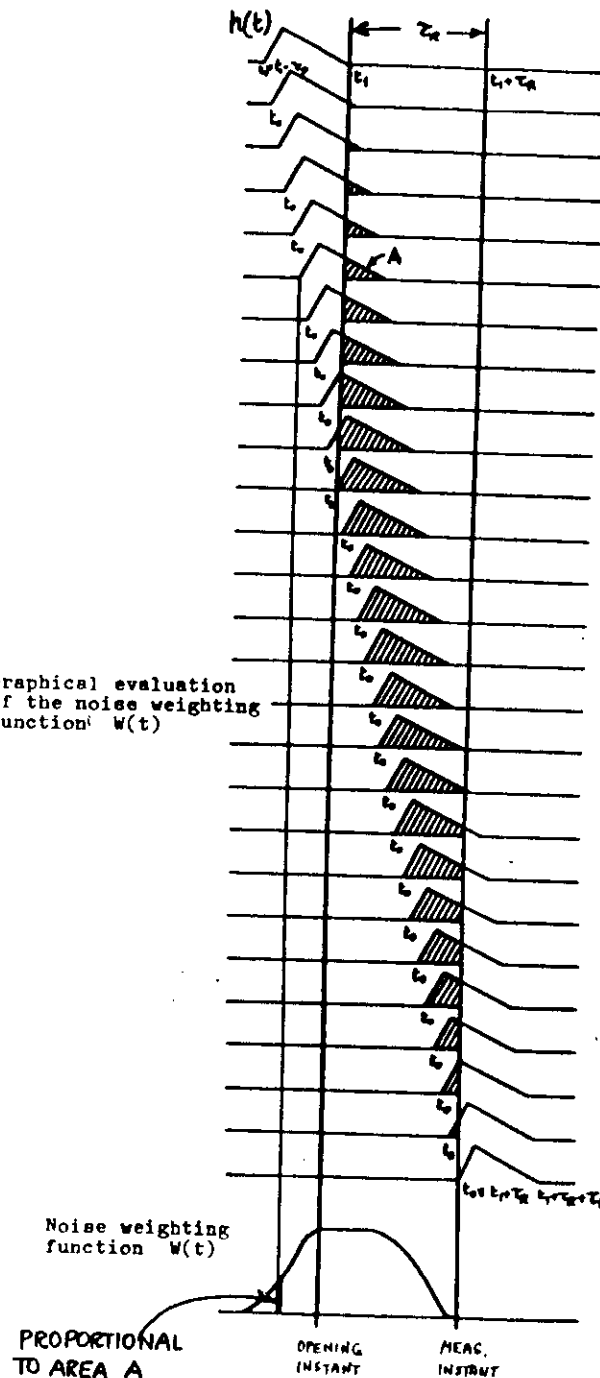


Fig. 26 - Graphical evaluation of the noise weighting function  $W(t)$



## 2 - SHAPING AFTER THE GATE

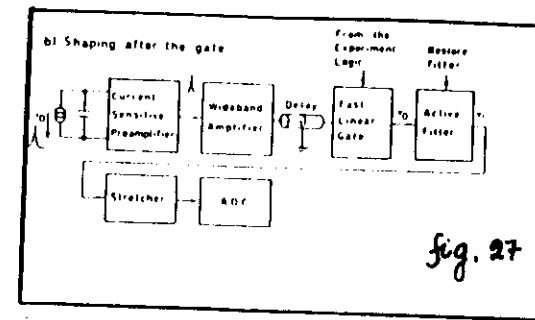
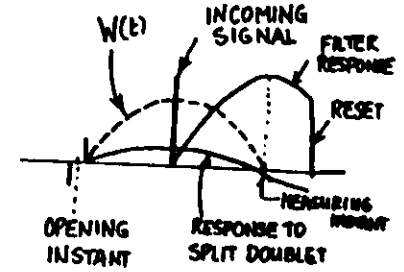


fig. 27

2) Shaping after the gate (fig. 11. 2b) The detector is followed by a fast current amplifier delivering at its output a narrow pulse similar to the  $Q(t)$  input current pulse. A linear gate is opened for the interesting events and with suitable delay, the fast pulse passes through the gate and excites the shaping network. The peak amplitude of the resulting signal is stretched and measured by a conventional ADC. The shaping network and the stretcher are reset to zero the former as soon as the stretcher has stored the peak amplitude, the latter as soon as the ADC has sensed the stretched amplitude.



In this case doublet noise is presented to the linear gate. By proper shaping, obtained by a sinewave excited by the incoming  $\delta$  and by a proper timing of the measurement with respect to the opening of the gate as well as of the opening time of the gate with respect to the signal arrival instant, the impact of the split doublet is practically removed.

## SAMPLING FILTERS

A frequent sampling of a signal in noise with suitable weights allows to process the signal with an arbitrary weighting function. Care must be taken to avoid a noise increase due to aliasing and the repetition rate of sampling should be large enough with respect to bandwidth of noise.

An effective approach to reduce the number of samplings per pulse without falling into noise aliasing problems consists in filtering the input signal, for instance to triangular shape or to an exponentially leading edge step, as shown in figure and sample it at the instants shown by dots.

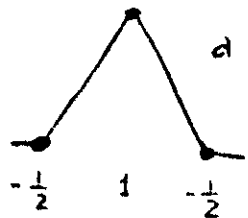
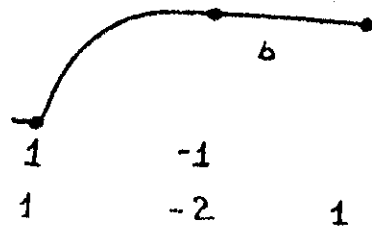


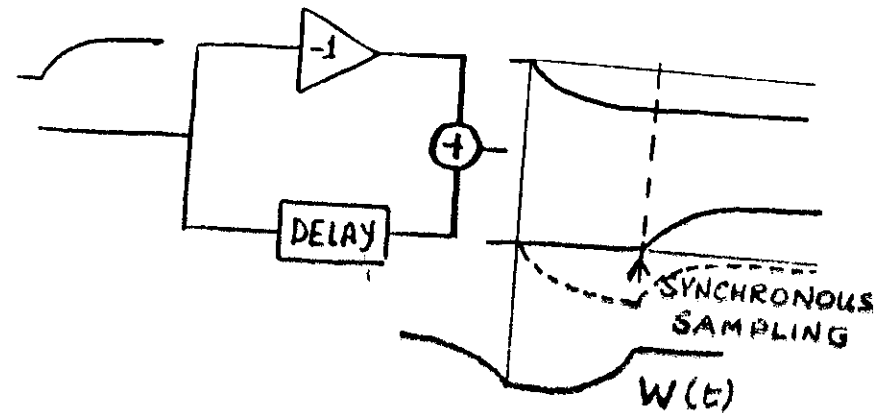
fig. 28



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SAMPLING A GIVEN WAVEFORM AT DIFFERENT INSTANTS, HOLDING THE SAMPLES AND MAKING WITH THEM A WEIGHTED SUM IS EQUIVALENT TO MAKE A SYNCHRONOUS SAMPLING ON SEVERAL REPLICAS OF THE GIVEN WAVEFORM SUITABLY DELAYED AND WEIGHTED

FOR INSTANCE, THE DOUBLE SAMPLING IN FIGURE 28 b) CORRESPONDS TO THE FOLLOWING OPERATION



SIMILARLY, THE WEIGHTING FUNCTIONS FOR THE SAMPLING ON THE TRIANGULAR WAVEFORM AND FOR THE TRIPLE SAMPLING ON THE EXPONENTIAL ARE:

