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ELEMENTARY PARTICLE PHYSICS

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Fundamental Physics: Limitations of Detectors

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FUNDAMENTAL PHYSICS

LIMITATIONS OF DETECTORS

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- "Limitations" imply an inability to achieve desireable goals.

- Prefer "CONSTRAINTS"

Perhaps the right attitude for the experimental physicist is to hold that a given end result can be achieved – but one is necessarily forced to play by natures rules: i.e. to obey the constraints of physical laws.

- Just as in sport and in law, careful study of the rules can be rewarding – often allowing one to circumvent the limitation, but, of course never the constraints.

- A given technology : constraints ⇒ limitations
  we shall study some of these.

WHAT DO WE WANT TO MEASURE?

Some
[illegible]ed
[illegible]d & Spect.)
or
or "All"
(4π detector)

of the particles produced in a high energy collision, we would like to measure or determine the following:

- The trajectories in space of the particles
- The momenta of the particles

Determines $\vec{p}$ for the particles and also the event TOPOLOGY (short lived parents, ...)

- The electric charges of the particles
- The masses of the particles

Identifies the particles

To do this we must have, in various combinations, measurements for individual particles of:

- position
- time
- Velocity
- Energy

To do this we need "PARTICLE DETECTORS"

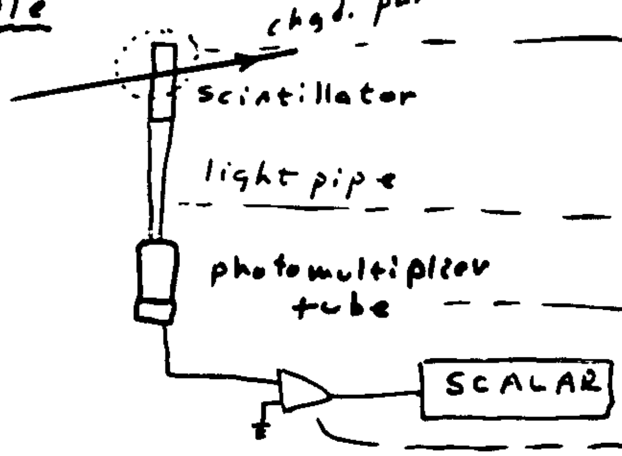
Generalized Particle Detection Process

1. Atomic and nuclear processes occur, in the interaction of a particle with the detector medium, at the "quantum level ⇒ a state which differs significantly from that which occurs naturally from thermal fluctuations in the detector medium. (e.g. ionization pairs, excited atoms with $\Delta E \gg kT$, ...

2. An "amplification" process takes place which amplifies the "differentness of the State to a level at which macroscopic physics concepts are applicable to subsequent stages.

3. Operating at the classical level, the detection is registered. (observed)

Example



molecular excitation de excitation via photon emmission

light propagation

Amplification via cascade of secondary electron emmission.

Macroscopic pulses amplified and counted.

Each stage is governed by the appropriate physical laws which impose significant constraints on the performance of the detector.

Noise - spurious outputs which confuse and degrade the detection and resolution of the desired output is a key consideration. Noise enters at each of the 3 basic stages and it enters also into systems of detectors organized to measure properties which cannot be determined by a single detector. Noise is often a "natural" phenomenon ["God loves the noise as much as the signal"]

SYSTEMS

In HEP experiments, individual detector elements are always organized into systems whose design is the result of an optimized compromise between conflicting goals of the experiment.

Many significant constraints originate in the system aspects - but to deal with them we must first understand the behaviour of the elementary detectors

Particles to be detected in high energy physics expts.

HADRONS } charged ($p, \bar{p}, K^{\pm}, \pi^{\pm}, \ldots$)
neutral ($n, K^{0}, \Lambda^{0}, \ldots$)

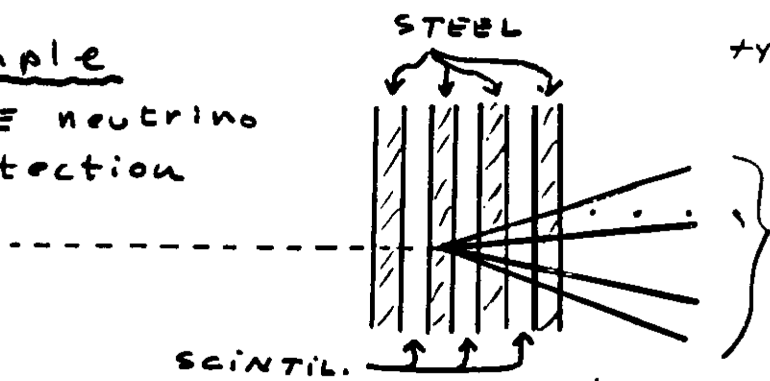
LEPTONS } charged ($e^{\pm}, \mu^{\pm}, \ldots$)
neutral ($\nu_e, \nu_\mu, \ldots$)

PHOTONS } γ

Many complex schemes to detect this variety of particles and to measure their relevant properties in experiments. HOWEVER, all detection schemes have at some stage the detection of charged particles via their electromagnetic interaction with the atoms of a detector medium.

example
high E neutrino detection

typical: $\nu_\mu + p \to \mu^- + n + \pi^+ + \pi^+$



charged particles detected by light produced in the scintillators.

Electromagnetic Interactions of Fast charged Particles with Matter

- Why Focus on EM interactions?

  Because the EM interaction is many orders of magnitude more probable than the strong or weak inter. Thus, with EM int. "thin" detectors are feasible

- Processes

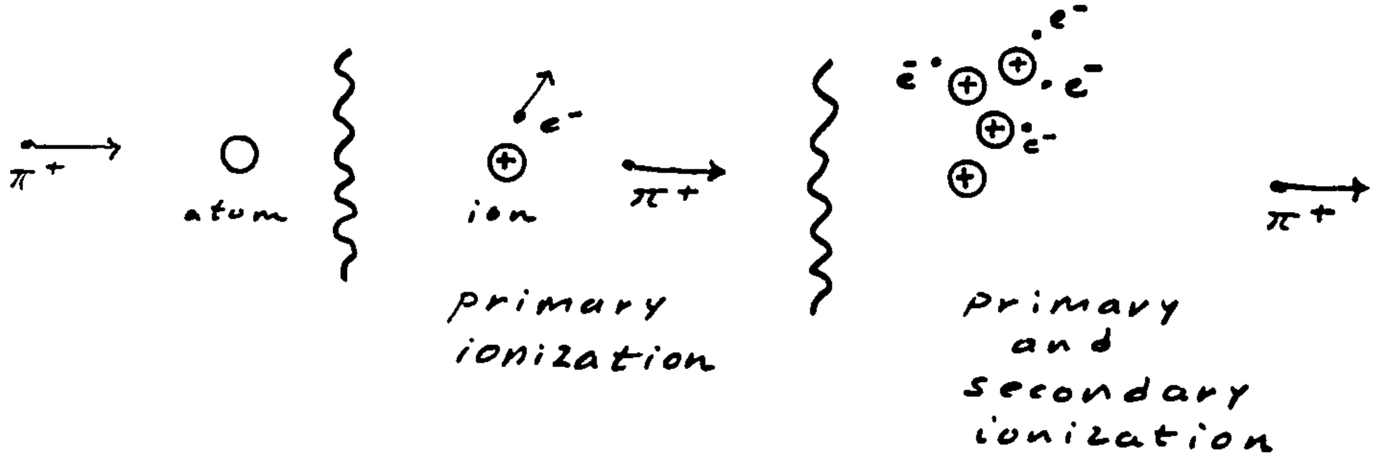
  ionization
primary
secondary
excitation
Čerenkov emmission — Energy loss mechanisms involving the atomic electrons

  multiple coulomb Scatting — From the atomic Nuclei

  elastic scattering of secondary electrons on detector atoms or molecules — important in diffusion of secondary electrons

IONIZATION (OF ATOMS + MOLECULES)

- Basic process (schematic)



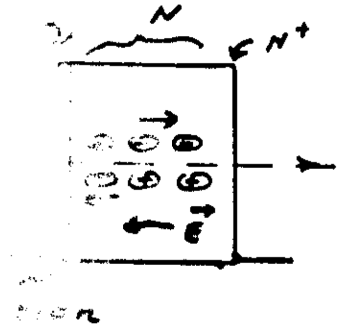
- Along the path of the fast particle there will be a trail of clusters of ion pairs. Because the primary process is described by a cross section, the number of clusters (N_c) will be Poisson distributed about a mean number $\overline{N_c}$. The total number of ion pairs will not be Poisson distributed but will reflect the probability distribution of energy transfer in the ionization process.

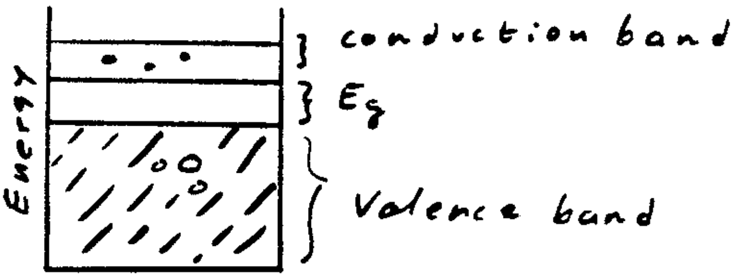
- The energy required to ionize atoms ($\gtrsim 10$ e.v.) is large compared to kT. Room temperature $kT \sim \frac{1}{40}$ e.V.

  Thermal Ion. Prob. $\sim e^{-400}$

  i.e. NEVER!

IONIZATION (Semi Conductors)





Gap between valence and conduction band is much smaller than the ionization potential for atoms. E.g. for Si $E_g = 1.12$ eV. The concentration of electrons (n) and holes (p) in thermal equilibrium at temp. T obeys

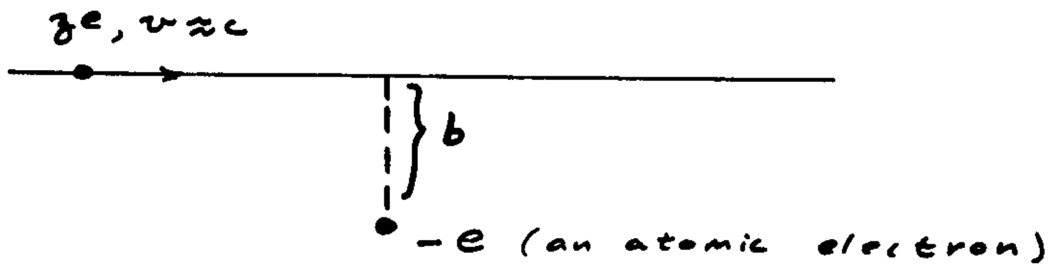
$$np \propto e^{-E_g/kT} \qquad \left(E_g/kT = 44.8 \text{ at room Temp (Si)} \right)$$

Noise considerations are complex and are strongly affected by the (external) electronics but again, it is not surprising that these devices can work at room temp.

Experimentally, it is found that for Si, 3.62 e.V of energy loss is required to make one electron-hole pair. Thus, *in principle*, the thinnest detectors could be semiconductors.

THEORY OF ENERGY LOSS – dE/dx

Simple picture – Fast particles incident, collision is over before the electron can move.



Only the transverse fields of the incident particle have a finite time integral at the atomic electron, so the momentum transfer, ΔP, to the electron is purely transverse and is (cf. J.D. Jackson, "Classical Electrodynamics")

$$\Delta P = \int_{-\infty}^{\infty} e E_t(t)\, dt = \frac{2ze^2}{v}$$

(This holds at all $\gamma = \frac{1}{\sqrt{1-\beta^2}}$)

The energy transfered to the electron (remember electron moves N.R.)

$$\Delta E(b) = \frac{(\Delta P)^2}{2m} = \frac{2z^2e^4}{mv^2}\left(\frac{1}{b^2}\right)$$

(m = electron mass)

We need to estimate the min. and max. values of b:

b_{min} determined by Quantum Mechanics b cannot be defined to better than $\hbar/p$

$$\therefore \boxed{b_{min} \approx \frac{\hbar}{p} = \frac{\hbar}{\gamma m v}}$$

(note $\gamma m v$ is the momentum of the target e^- in the rest frame of the projectile $\approx$ cm).

- b_{max} determined by the requirement that the collision time be no longer than the period of atomic oscillation of a "typical" electron. Otherwise, the atomic system will respond adiabatically and no energy will be transferred to it.

collision time $\Delta t \approx \dfrac{b}{\gamma v}$

oscillation time $\tau \approx 1/\omega$

where ω is an appropriate atomic frequency.

$$\therefore \boxed{b_{max} \cong \frac{\gamma v}{\omega}}$$

The average energy loss per unit distance traversed by the fast particle (dE/dx) is thus:

$$\frac{dE}{dx} = 2\pi N Z \int_{b_{min}}^{b_{max}} \Delta E(b)\, b\, db$$

whence $$\frac{dE}{dx} = 4\pi N Z \frac{z^2 e^4}{m v^2} \ln\left[\frac{\gamma^2 m v^2}{\hbar \langle\omega\rangle}\right]$$

where N = Number of atoms / volume

Z = atomic number of the atoms

$\langle\omega\rangle$ = a suitable mean ω for the different electrons in the atom

A less "cavalier" treatment gives (Bethe 1930)

$$\boxed{\frac{dE}{dx} = 4\pi N Z \frac{z^2 e^4}{m v^2}\left\{\ln\left[\frac{2\gamma^2 m v^2}{\hbar\langle\omega\rangle}\right] - \frac{v^2}{c^2}\right\}}$$

It gives AVERAGE energy loss rate because of the integration over b.

This formula is correct for $\gamma \lesssim 10$ for larger γ it requires modification.

Density Effect and Restricted Energy loss

t large γ the previous formula implies

$$\frac{dE}{dX} \propto \ln \gamma^2$$

(a) one power of γ in $\ln \gamma^2$ comes from b_{max} - relativistic contraction of the electric field of the incident particle shortens the collision time at a given b.

) one power of γ in $\ln \gamma^2$ comes from b_{min} because as $P_{INC.}$ increases, the wavelength decreases and we can better define b $\Rightarrow$ smaller b_{min}.

(a) is wrong (Ferr - 1940) if $b_{max} >>$ Atomic Size

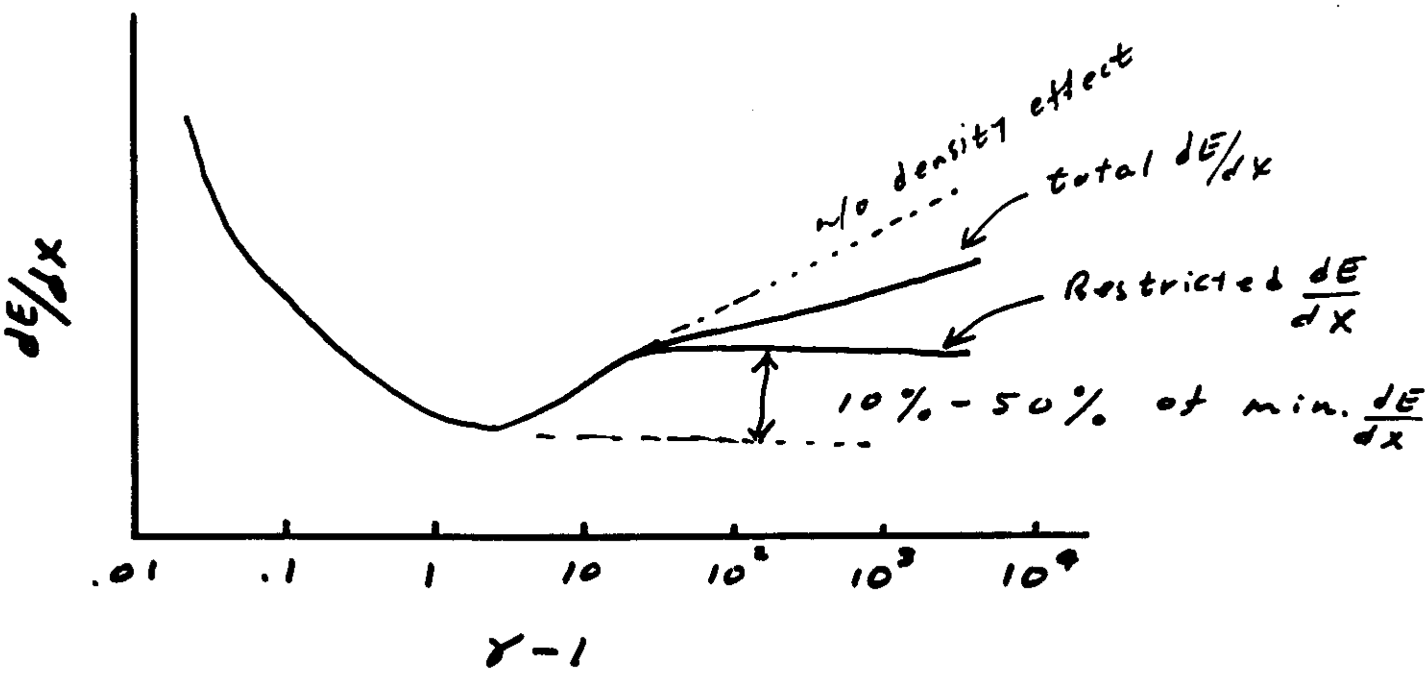
then many a s between particle and struck electron. These atoms polarize under the infuence of the fields of the fast particle and modify the field seen by the "target" electron.

The result is that the b_{max} effect saturates - removing one power of γ.

(b) If we measure dE/dx in a detector which cannot respond to energy losses greater than some maximum, b_{max} is limited to the corresponding value.

This "restricted" energy loss measured by the detector will thus reach a constant value (plateau) at very high energies.

Note: from the point of view of the particle, dE/dx continues to increase as $\ln \gamma$.



EXCITATION

Basically, excitation is a process
milar to ionization except that
e energy transfer is not
fficient to ionize the atom.

Typically, the rate of excitation and
of primary ionization are comparable.

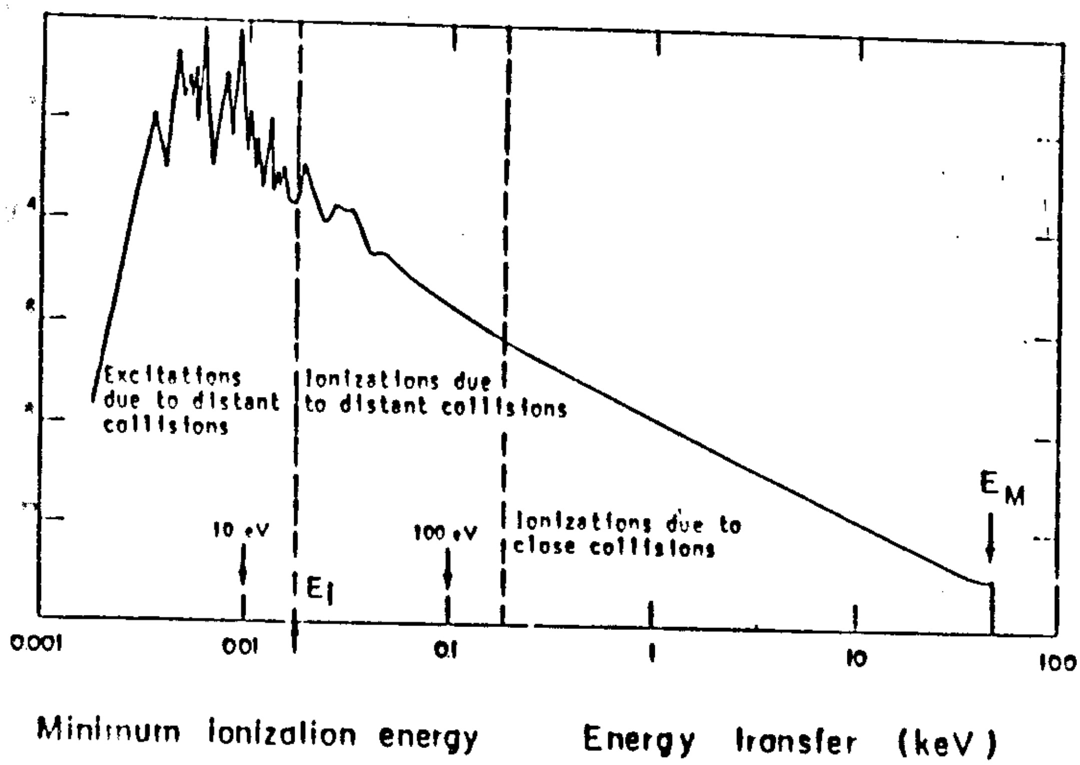


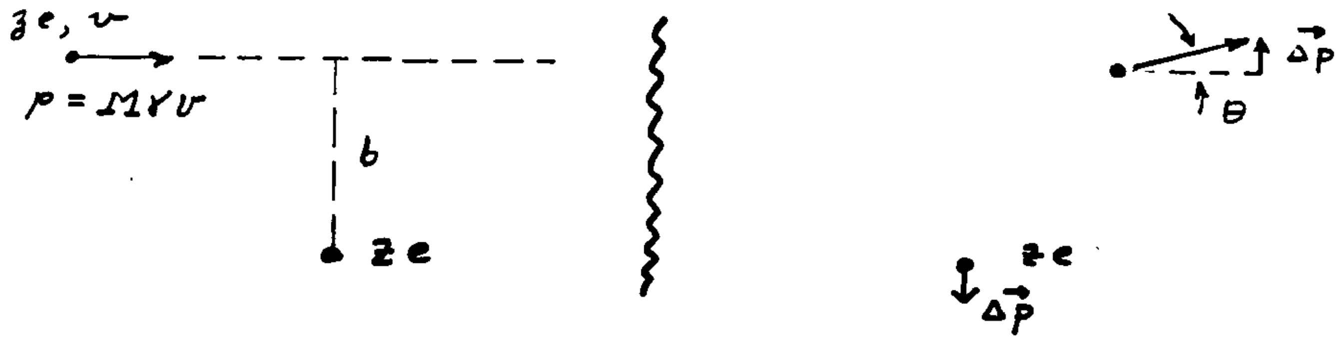
Fig. 2 Relative probability of different processes induced by fast (100 keV) electrons in water, as a function of the energy transfer in a collision[4]. The maximum kinematically allowed energy transfer, E_M = 50 keV in this case, is also shown.

(taken from Sauli 1977)

MULTIPLE COULOMB SCATTERING

In collisions with the atomic electrons the deflections of the incident particle are small even though the (cumulative) energy loss is significant.

In collisions with the atomic nuclei the (cumulative) deflection — the multiple coulomb scattering — is significant. (but the energy loss is negligible)



$$\theta = \frac{\Delta p}{p} = \frac{2 z Z e^2}{p v b}$$

with the usual definition of the differential cross section $\frac{d\sigma}{d\Omega}$

$$b\, db\, d\phi = \frac{d\sigma}{d\Omega} d\Omega = \frac{d\sigma}{d\Omega} \sin\theta\, d\theta\, d\phi$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|$$

whence (small ∡'s) $$\underline{\frac{d\sigma}{d\Omega} = \left(\frac{2 z Z e^2}{p v} \right)^2 \frac{1}{\theta^4}}$$

This (small angle) Rutherford scattering formula will apply between maximum and minimum angles :

θ_{MIN} — determined by finite size of the atom and quantum mechanics. The particle must penetrate the electron cloud to "see" the nucleus — so Δp must be large enough to define $\Delta x \lesssim a$, where a is the atomic radius.

$$\theta_{Min} \cong \frac{\Delta p_{min}}{p} = \frac{\hbar}{pa}$$

Thomas-Fermi Model of the Atom:

$$a \cong 1.4\, a_0 Z^{-1/3}$$

where a_0 = hydrogenic Bohr radius

$$= \frac{\hbar^2}{me^2} = .53 \times 10^{-8}\ \text{cm}$$

So :

$$\boxed{\theta_{Min} \approx \frac{Z^{1/3}}{192}\left(\frac{mc}{p}\right)}$$

θ_{max} — determined from Nuclear Radius

$$\theta_{Max} \approx \frac{\lambda\!\!\!^{-}}{R} = \frac{\hbar}{pR}$$

$$R \cong 1.4\, A^{1/3} \times 10^{-13}\ \text{cm}$$

$$\boxed{\theta_{MAX} = \frac{274}{A^{1/3}}\left(\frac{mc}{p}\right)}$$

The deflection in each collision is independent so after n collisions, the central limit theorem of statistics implies ($n >> 1$) that the distribution of angles will be approximately Gaussian around the forward direction with mean square angle $\langle \Theta^2 \rangle$ given by

$$\langle \Theta^2 \rangle = n \langle \theta^2 \rangle_{\text{single collision}}$$

Now:
$$\langle \theta^2 \rangle_{\text{single}} = \frac{\int \theta^2 \frac{d\sigma}{d\Omega}\, d\Omega}{\int \frac{d\sigma}{d\Omega}\, d\Omega}$$

whence
$$\langle \theta^2 \rangle_{\text{single}} = 2\, \theta_{min}^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right)$$

or if we assume $A \cong 2Z$
$$\langle \theta^2 \rangle_{\text{single}} = 4\theta_{min}^2 \ln(204 Z^{-1/3})$$

Thus for the traversal of a thickness t:

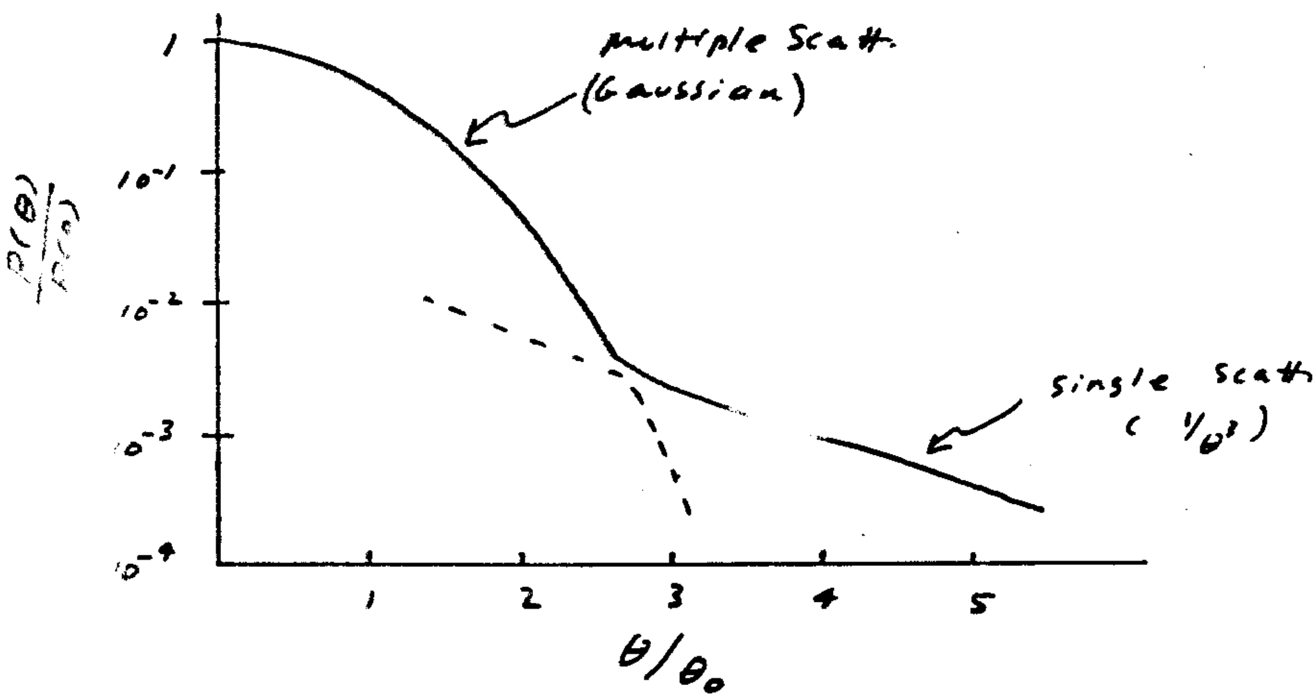
$$\langle \Theta^2 \rangle = N \sigma t \langle \theta^2 \rangle_{\text{single}}$$
$$= N \pi \left(\frac{2 z Z e^2}{p v}\right)^2 \frac{1}{\theta_{min}^2} \cdot 4\theta_{min}^2 \ln(204 Z^{-1/3})\, t$$

or
$$\boxed{\langle \Theta^2 \rangle = 4 \pi N \left(\frac{2 z Z e^2}{p v}\right)^2 \ln\left(\frac{204}{Z^{1/3}}\right) t}$$

VALIDITY OF THE GAUSSIAN APPROXIMATION

Typically, the Gaussian approximation is accurate to the $\approx$ % level or equivalently to angles of a "few" θ_0.

Since the probability of a multiply scattered angle decreases as $e^{-\theta^2/2\theta_0^2}$ but the probability of a large single scattering decreases "only" as $1/\theta^3$ (projected angle), beyond some angle single scattering will dominate. This situation is illustrated below.



Physical Data for Gases used in Drift and Proportional Chambers

(Sauli 1977)

Gases @ STP

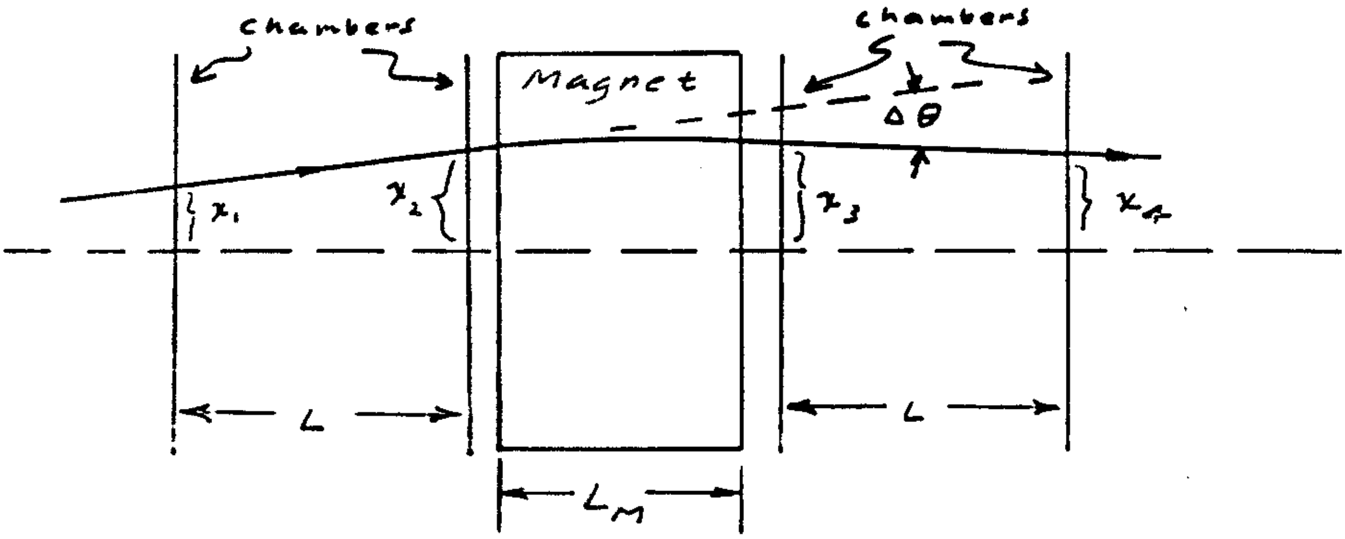
Net Energy loss/i.p.

gas	Z	A	ρ	E_{ex}	E_{ION}	I_0	W_I	dE/dx	L_R	n_P	n_T
			gm/cm^3	eV				KeV/cm	cm	i.p./cm	
H_2	2	2	8.38×10^{-5}	10.8	15.9	15.4	37	.34	7.3×10^{5}	5.2	9.2
He	2	4	1.66×10^{-4}	19.8	24.5	24.6	41	.32	5.68×10^{5}	5.9	7.8
N_2	14	28	1.17×10^{-3}	8.1	16.7	15.5	35	1.96	3.2×10^{4}	10	56
O_2	16	32	1.33×10^{-3}	7.9	12.8	12.2	31	2.26	2.57×10^{4}	22	73
Ne	10	20.2	8.39×10^{-4}	16.6	21.5	21.6	36	1.41	3.45×10^{4}	12	39
Ar	18	39.9	1.66×10^{-3}	11.6	15.7	15.8	26	2.44	1.18×10^{4}	29.4	94
Xe	54	131.3	5.49×10^{-3}	8.4	12.1	12.1	22	4.60	1.54×10^{3}	44	307
CO_2	22	44	1.86×10^{-3}	5.2	13.7	13.7	33	3.01	1.83×10^{4}	34	91
C_4H_{10}	34	58	2.42×10^{-3}		10.6	10.8	23	4.50	1.69×10^{4}	46	195

Application to a Magnetic Spectrometer Using Drift or Proportional Chamber Technology

Such devices are ubiquitous in fixed target experiments, and in somewhat different form are present in many colliding beam detectors. They provide basic charge and trajectory information for the charged particles produced in the reaction.

We shall take an oversimplified example to make the analysis transparent, but our results (within the "canonical" factor of π) will have general validity for this class of detector system.



We focus on the momentum measurement. We consider the change in direction, $\Delta\theta$, caused by the magnetic deflection.

For small angles (appropriate here for Hi E)

$$\Delta\theta = \frac{L_M}{\rho} = \frac{.3\, B(\mathrm{kg})\, L_M(\mathrm{cm})}{p\,(\mathrm{MeV/c})}$$ (Singly charged Track)

Now $\Delta\theta$ is measured by the drift ch. system:

$$\Delta\theta_{\mathrm{Meas.}} = \frac{(x_2 - x_1) - (x_4 - x_3)}{L}$$

The rms error in $\Delta\theta_{\mathrm{MEAS.}}$ due to chamber resolution is simply related to the rms error in the x_i measurements (all assumed equal for simplicity), and given by σ_x.

Thus $$\delta(\Delta\theta)\Big|_{\mathrm{Resol.}} \cong \sqrt{4}\,\frac{\sigma_x}{L} = \frac{2\,\sigma_x}{L}$$

However, there is an error in $\Delta\theta_{\mathrm{meas.}}$ due to MULTIPLE COULOMB SCATTERING in ch. 1 and ch. 3.

let θ_0 be the rms projected M.S. angle in a single chamber (all equal).

$$\delta(\Delta\theta)\Big|_{\mathrm{M.S.}} = \sqrt{2}\,\theta_0$$

The combined rms error in $\Delta\theta_{meas.}$ will therefore be (adding M.S. + Res. in Quad.):

$$\delta(\Delta\theta_{Meas.}) = \sqrt{2\theta_0^2 + 4\frac{\sigma_x^2}{L^2}}$$

This propagates to a momentum error:

$$P_{Meas} = \frac{.3BL_M}{\Delta\theta_{meas.}}$$

$$\delta(P_{meas}) = \frac{.3BL_m\,\delta(\Delta\theta_{meas.})}{(\Delta\theta_{meas})^2} = \frac{.3BL_M\,P^2}{(.3BL_M)^2}\,\delta(\Delta\theta_{meas})$$

or:

$$\left(\frac{\delta P}{P}\right)_{rms} = \frac{P}{.3BL_M}\,\delta(\Delta\theta_{meas.})_{rms}$$

whence,

$$\frac{\delta P_{meas}}{P} = \sqrt{\frac{2P^2\theta_0^2}{(.3BL_M)^2} + \frac{4P^2}{(.3BL_M)^2}\frac{\sigma_x^2}{L^2}}$$

Now we saw that $\theta_0 \cong \dfrac{14.1\,\text{MeV/c}\sqrt{\frac{t_{ch}}{L_R}}}{P}$

where $\beta \approx 1$ assumed

t_{ch} = chamber thickness

L_R = chamber gas Radiation Length.

$$\boxed{\left(\frac{\delta P}{P}\right)_{rms} = \sqrt{\frac{2(14.1)^2}{(.3BL_M)^2}\left(\frac{t_{ch}}{L_R}\right) + \frac{4P^2}{(.3BL_M)^2}\frac{\sigma_x^2}{L^2}}}$$

- Note that even if $\sigma_x = 0$, multiple coulomb scattering sets a lower limit t $\delta P/p$. Furthermore, this multiple scattering error in $\delta P/p$ is independent of momentum.

- For finite resolution ($\sigma_x \neq 0$), at high enough momentum, the measurement error will dominate. Indeed at "high" energies $\frac{\delta P}{P} \propto P$.

  As you will learn next week, "ionization calorimeters" measure particle energy with a resolution $\delta E/E$ which scales as $1/\sqrt{E}$ or, at worst, approaches a constant (due to systematic effects). Thus, we understand the increasing importance of calorimetry, relative to tracking, as particle energy increases.

How thin can we make the chambers?
(and thus reduce the m.s. "floor" to $\delta P/p$)

This is determined by the efficiency for detecting the particle which we wish the chambers to have! Let us assume that we insist on $\epsilon \gtrsim .999$.

Recalling that the number of primary clusters is Poisson distributed, we can calculate the required mean number of clusters : (N)

$$e^{-N} = 10^{-3}$$

$$N = 6.91$$

For the sake of concreteness let us choose Ar as the working gas (typical).

$n_p = 29.4$ i.p./cm (STP)

$$\therefore \quad t_{ch} = \frac{6.91}{29.4} = .235 \text{ cm}$$

also recall $L_R = 1.18 \times 10^4$ cm

$$\therefore \quad 2 \frac{(14.1)^2 \frac{t_{ch}}{L_R}}{(.3BL_m)^2} = 3.98 \times 10^{-5} \left(\frac{14.1}{.3BL_m} \right)^2$$

Typical values of the magnetic field range from 10 – [illegible] g. Let us (somewhat optimistically) [illegible] <u>30 kg.</u> Let us also assume a 2 meter long magnet. This is feasible in fixed target experiments but is probably a bit large for a collider detector magnet with 30 kg.

$$\text{so} \quad 3.98 \times 10^{-5} \left(\frac{14.1}{.3 \times 30 \times 200} \right)^2 = 2.44 \times 10^{-9} = \left(\frac{\delta P}{P} \right)^2_{M.S.}$$

$$\text{and} \quad \left(\frac{\delta P}{P} \right)_{\text{Mult. Scatt}} = 4.9 \times 10^{-5}$$

For completeness, let us estimate the contribution to $\delta P/P$ due to a state-of-the-art measuring accuracy $\sim \sigma_x = 50\,\mu m$.

$$\left(\frac{\delta P}{P}\right)_{Res.} = \frac{2P}{.3BL_M}\;\frac{\sigma_x}{L}$$

We must now fix L. Since we chose L_M to be 2 m, let us choose L to be comparable. If we make L too large, the chambers (and the downstream apparatus!) will need to be unreasonably big to have a finite acceptance. So let us choose $L = 2$ meters as well.

then, $$\left(\frac{\delta P}{P}\right)_{res.} = \frac{2}{.3\times 30\times 200}\;\frac{50\times 10^{-4}}{200}\times 10^{3}\,P(\text{GeV/c})$$

$$\left(\frac{\delta P}{P}\right)_{Res} = 2.78\times 10^{-5}\;P\,(\text{GeV/c})$$

So for this example, measurement error will dominate for P greater than a few GeV/c.

Nevertheless we have seen that even with perfect resolution, there is a minimum attainable $\delta P/P$ due to multiple Coulomb scattering. <u>This has its ultimate origin in the need for the particle to transfer a finite amount of energy >> KT, for efficient detection.</u>

Constraints on Position Measurement

Localization of the energy transferred to the detector medium:

This will have the scale of b_{Max} for the energy transfer.

Recall $b_{Max} \approx \dfrac{\gamma v}{\omega}$ $\left\{ \begin{array}{l} \text{for } \gamma \lesssim 10 \\ \text{at which point} \\ \text{polariz. of} \\ \text{medium becomes} \\ \text{important} \end{array} \right.$

where ω is a characteristic period of the electron motion in the atom.

Taking $\gamma = 10$, $v \cong c$

$$b_{max} \cong \frac{10\,c}{\omega} = 10\,\frac{\lambda}{2\pi}$$

where λ is the wavelength of a photon with energy $\hbar\omega$ (this is just for nmenonic reasons).

For gas ionization detectors

$$\hbar\omega \approx 10 - 20 \text{ e.V.}$$

Take 10 eV for concreteness:

$$\omega = \frac{10}{6.58 \times 10^{-16}} = 1.52 \times 10^{16} \text{ Hz}$$

$$\lambda = \frac{2\pi c}{\omega} = 1.24 \times 10^{-5} \text{ cm}$$

so for this case:

$$b_{max} = \frac{10}{2\pi} \times 1.24 \times 10^{-5} \text{ cm} = .2 \mu m$$

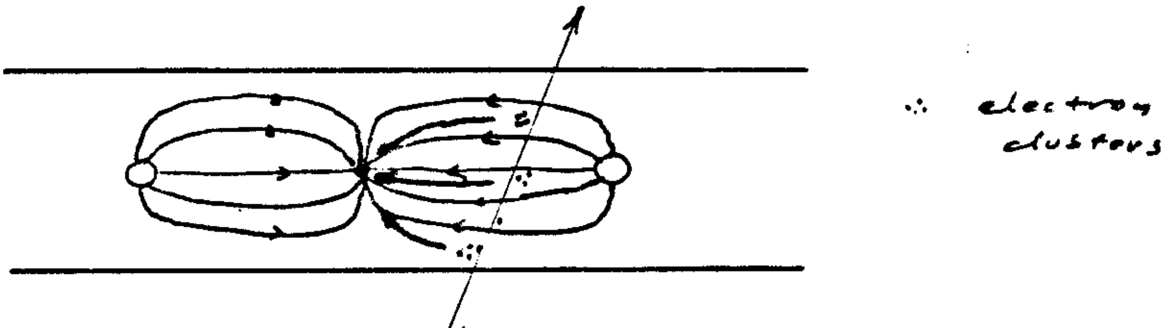
- thus the <u>initial</u> column of charge will have a radius $\sim .1 \rightarrow .2 \mu m$ just due to the lack of perfect localization of the interaction between the charged particle and the detector atoms.

- The spread of secondary ionization will of course broaden this distribution. The most serious case is that which occurs when an energetic secondary electron is created (δ-ray) and measures can be taken to correct for this.

b) Constraints on the position resolution caused by the detection of the transferred energy.

we consider an important aspect of these effects - <u>DIFFUSION</u> of the charges, as they drift to the collecting electrode, in gas ionization detectors.

Example - Simple drift Chamber



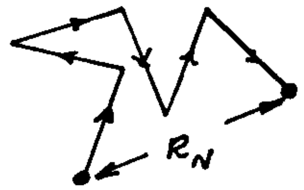
As the clusters drift to the collecting sense wire, they suffer diffusion:

This leads to a spread in arrival times and a consequent spread in measured position.

A physical insight into diffusion can be gained from simple models as follows (Rice-Evans 1974)

At the energies of typical diffusing and drifting electrons, the angular distribution of elastic scatters is nearly isotropic.

Thus, the electrons undergo nearly random walk paths and as a consequence "diffuse" <u>away</u> from regions of high concentration.



It is easy to show that (planar case)

$$\langle R_N^2 \rangle = N\ell^2$$

In a somewhat less (but still quite!) simplified model we can write an equation (Fick's law of diffusion)

$$\vec{J} = -D\,\vec{\nabla} n$$

where $\vec{J} \equiv$ Flux of particles (electrons here) per unit time per unit area in the direction $\dfrac{\vec{J}}{|\vec{J}|}$

$n(x,y,z,t) \equiv$ the concentration of particles per unit volume at time t

$D \equiv$ Diffusion coefficient

For spherically symmetric diffusion from point source at $t=0$:

$$n = \left(\frac{n_0}{4\pi D t}\right)^{3/2} e^{-r^2/4Dt}, \quad r^2 = x^2+y^2+z^2$$

" 3D case $\langle r^2 \rangle = 6Dt$

" 2D " $\langle r^2 \rangle = 4Dt \quad (= \langle \rho^2 \rangle)$

" 1D " $\langle r^2 \rangle = 2Dt \quad (= \langle x^2 \rangle)$

Kinetic Theory relates D to the mean velocity $\bar{v}$ and the mean free path ℓ, via

$$D = \frac{1}{3} \ell \bar{v}$$

(easy to see the basic idea from random walk picture

$$\langle r^2 \rangle \sim n\ell^2 = (n\ell)\ell = \bar{v} t \ell$$

but $\langle r^2 \rangle \sim Dt$ also, so $D \sim \ell \bar{v}$)

If we assume the electrons have a velocity distribution properly described by a temperature T:

$$\frac{1}{2} m \bar{v}^2 = \frac{3}{2} kT$$

For thermal (rm. Temp.) energies

$$kT \sim \frac{1}{40} \text{ e.v.}$$

$$\bar{v} = 1.16 \times 10^7 \text{ cm/s}$$

As noted in the wire chamber lectures, the electrons drifting in an electric field have an effective temperature which is greater than ambient. Thus $\bar{v}$ will depend on gas composition and E/p.

For the moment, let us ignore this and continue with the simple picture using room temp. KT, etc. We will compare with realistic case at the end but will gain insight by pursuing the simple model.

What is value of ℓ?

$$\ell = \frac{1}{n\sigma}$$

n = no. of atoms/cm³

σ = cross section for electron-atom scattering

We next consider the <u>drift</u> velocity, u, of the ensemble of electrons.

u is the velocity along $\vec{E}$. It is much less than $\bar{v}$.

Drift velocity estimate

Let t_c be the (avg.) time between collisions

In the time t_c an electron, on the average, travels a distance, x in the $\vec{E}$ direction where:

$$x = u\, t_c$$

But an "average" electron starts with zero velocity after a collision (vector average of $\vec{v}$ nearly zero) so:

$$x = \frac{1}{2} a\, t_c^2$$

$$a = \text{accel due to } \vec{E} = \frac{eE}{m}$$

$$\therefore \quad \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\ell}{\bar{v}}\right)^2 = u\,\frac{\ell}{\bar{v}}$$

where we have used $t_c = \dfrac{\ell}{\bar{v}}$

$$\therefore \quad u = \frac{1}{2}\,\frac{eE\ell}{m\bar{v}}$$

a less cavalier treatment gives

$$u = \frac{2}{3}\,\frac{eE\ell}{m\bar{v}}$$

To get a feeling for the typical numbers, let us take the case of Argon gas as detector medium.

$$\sigma = 6.5 \times 10^{-16}\ \text{cm}^2$$

at STP

$$n = \frac{6.02 \times 10^{23}}{22.4 \times 10^3} = 2.69 \times 10^{19}\ \text{atoms/cm}^3$$

$$\therefore \quad \underline{\ell = 5.72 \times 10^{-5}\ \text{cm.}}$$

To calculate u we need a value for $|\vec{E}|$. Let us take a typical drift field value of $\sim$ <u>1 KV/cm</u>

$$u = \frac{2}{3}\,\frac{e\,\ell}{m\,\bar{v}}\,E$$

$$u = \frac{2}{3}\,\frac{4.8 \times 10^{-10} \times 5.72 \times 10^{-5}}{9.11 \times 10^{-28} \times 1.16 \times 10^{7}} \times 3.33$$

(where we have used 1 STATVOLT = 300 Volts/cm)

$$\underline{u = 8.65 \times 10^{6}\ \text{cm/sec.}}$$

Let us now calculate 2D case for 1cm drift (again a "typical" value)

$$t = \frac{1\ \text{cm}}{8.65 \times 10^{6}} = 1.16 \times 10^{-7}\ \text{sec.}$$

$$\langle \rho^2 \rangle = 4Dt$$

$$D = \frac{1}{3}\,\ell\,\bar{v} = \frac{1}{3} \times 5.72 \times 10^{-5} \times 1.16 \times 10^{7} = 221\ \text{cm}^2/\text{s}$$

$$\therefore \quad \langle \rho^2 \rangle = 1.02 \times 10^{-4}\ \text{cm and } \boxed{\rho_{rms} = 101\ \mu\text{m}}$$

espite the crudeness of the model, is sets the scale. It is numerically quite wrong for pure Ar because the electric field "heats" the electrons considerably. It is not so far off for mixtures with additives which lower electron energies via inelastic scatters. An interesting summary is shown in the figure taken from an article by Sauli.

Note: the uncertainty in position measurement is really set by the uncertainty in finding the centroid of the cloud of electrons. Therefore, the measurement really depends more on the variance of the $\rho_{rms} \approx \frac{\rho_{single}}{\sqrt{n_e}}$

- The need to drift ionization to the collector electrode imposes a fundamental constraint on position measurement due to unavoidable interactions of the electrons and the detector atoms.

The effect of diffusion can be minimized but cannot be eliminated!

e.g. by increasing the pressure, by proper choice of gas, σ/p, --

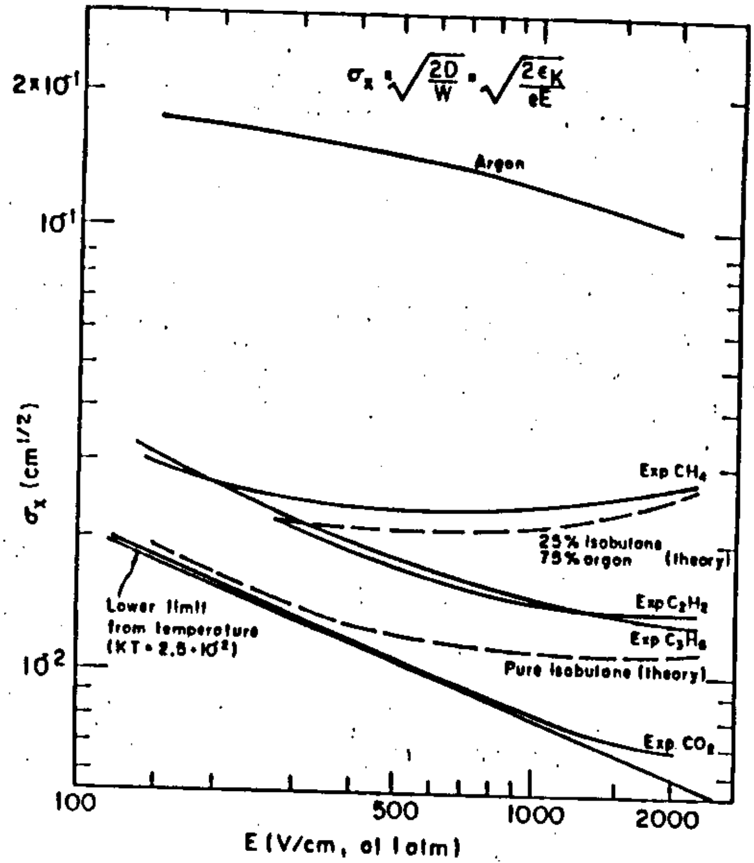


Figure 5

Taken from G. Charpak and F. Sauli, "High-Resolution Particle Detectors" Annual Rev. of Nuclear Science Vol. 34, 1984

Solid State Detectors

Similar effects occur in the drift of holes and electrons in solid state detectors. However, drift distances are typically much shorter and density is of course higher. Also there are typically more electron/holes (e.g. 300 μm Si gives ~25,000 e-h pairs).

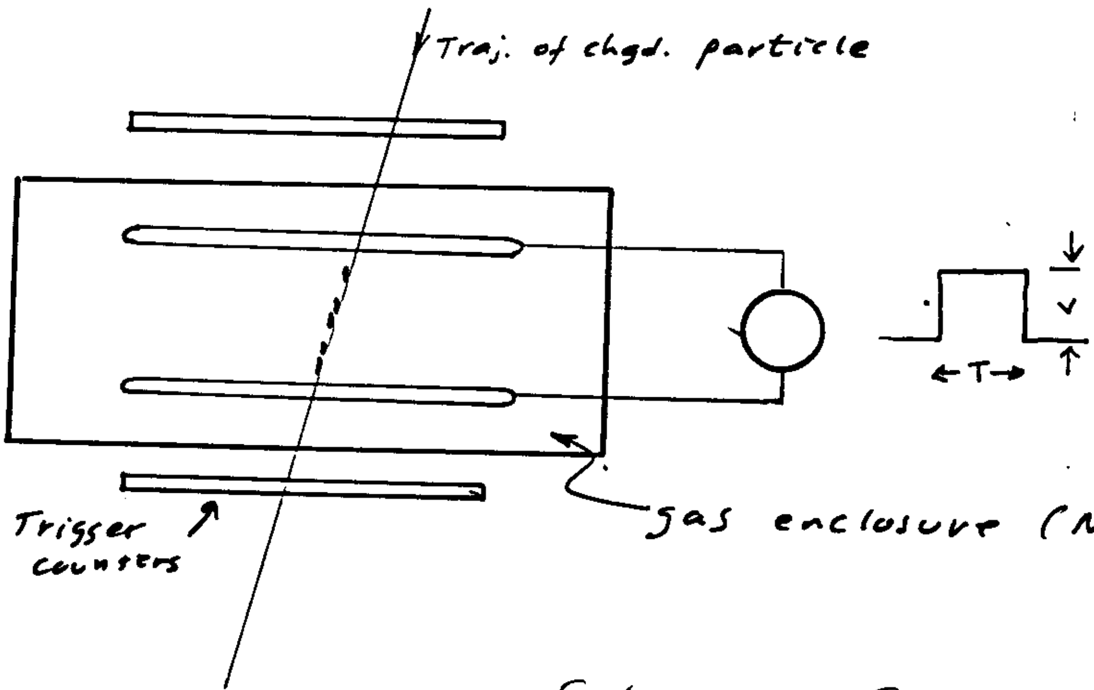
(Single e/h) diffusion widths for 1 mm of drift are ~30 μm (Charpak and Sauli – 1984)

So one would expect ~15 μm for a 300 μm thick Si detector. Taking the averaging over many e/h into account, diffusion is not a significant factor relative to those imposed by read out complexity (at present)

An Example of "Beating the Constraints"

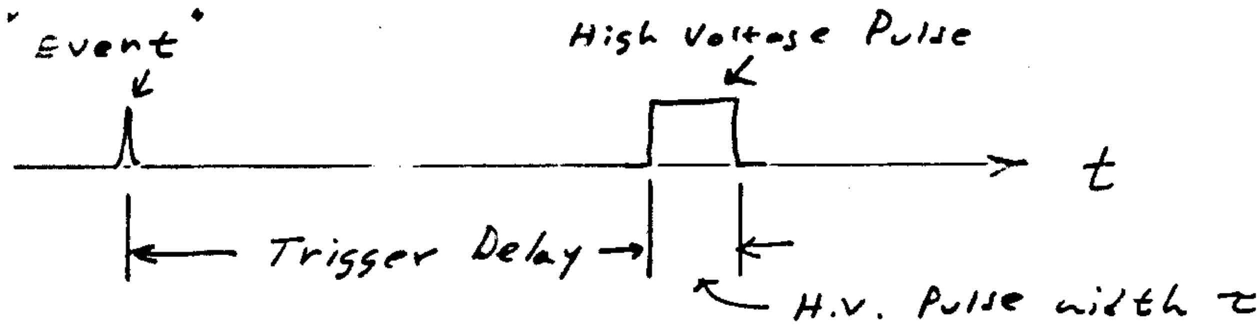
The Diffusion Suppressed Streamer Chamber

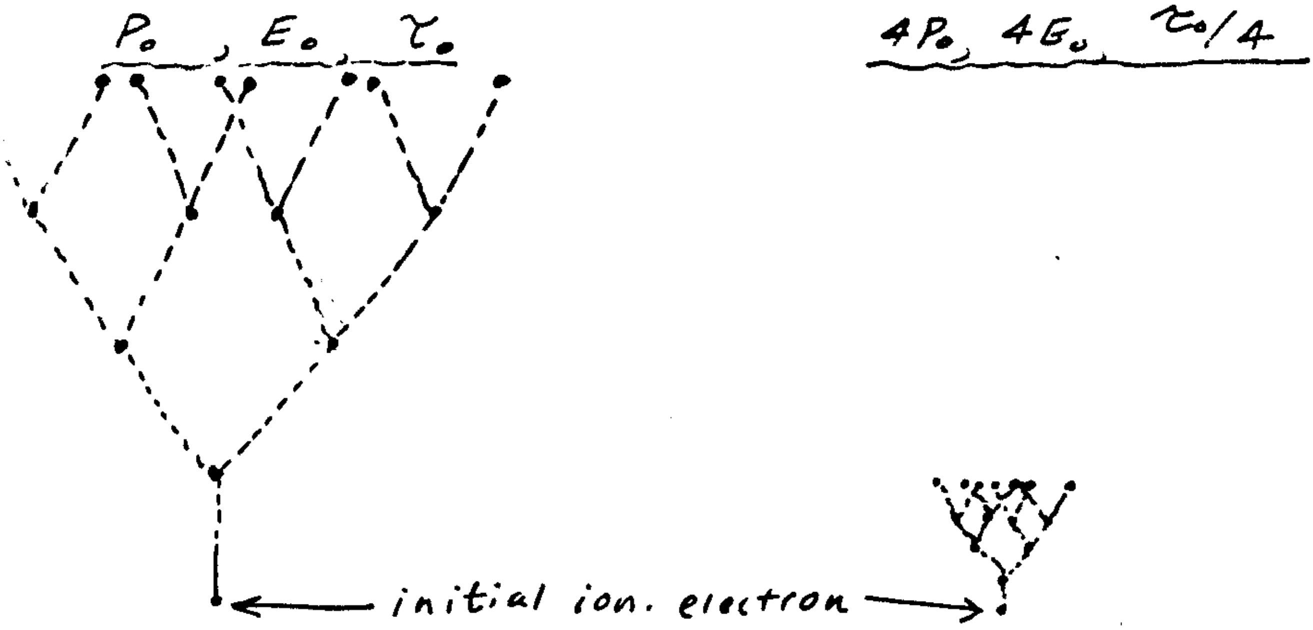
The streamer chamber is a gas detector in which a very short, intense high voltage pulse "grows" electron avalanches into small bright streamers. These are photographed, often with the use of image intensifiers and the tracks of the particles are thus observable.



Schematic Streamer Chamber

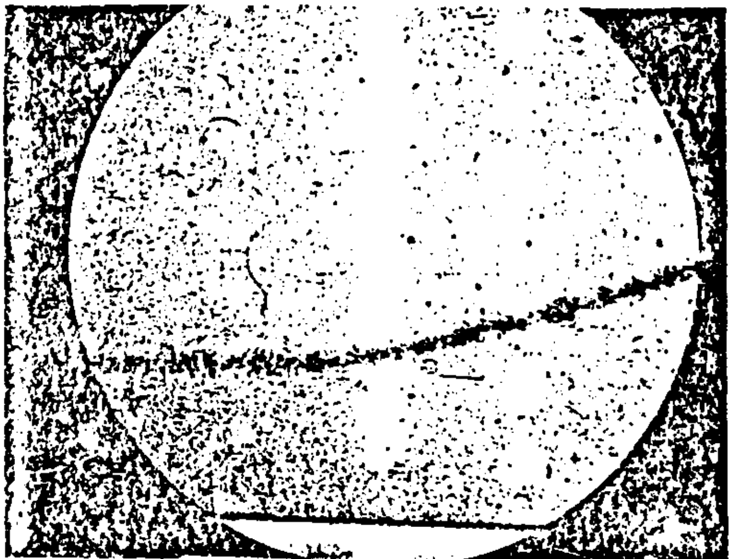
Basic Principle of "Scaling" for HRSC



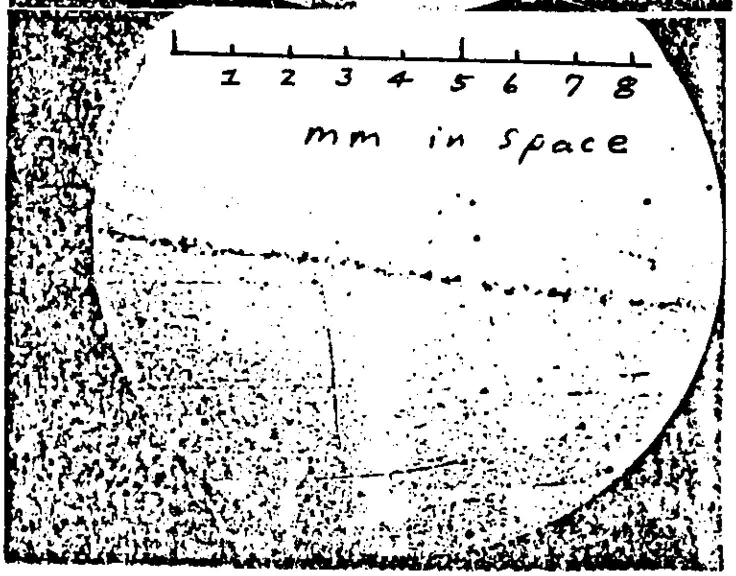


Streamer Diameter $\sim \left(\frac{1}{\sqrt{P}}\right)^2 \sim \frac{1}{P}$

<u>But</u>, Diffusion of the initial electron during trigger delay $\sim \frac{1}{\sqrt{P}}$



600 psia
Ne/He (9/1)
Track
width ~ 400 μm



600 psia
Ne/He (9/1)
+ 2% CO_2
Track width
~ 250 μm
Still Diffusion
Dominated

DIFFUSION SUPPRESSION SCHEME

A. Store ionization electrons as negative ions during trigger delay time. (Capture electrons with electronegative gas)

B. Photoionize negative ions to liberate electrons just before high voltage pulse arrives to make streamers.

C. Second species of negative ions should be formed by "stealing" electrons. This second species should have negligible photoionization cross section. (Controls chamber memory time).

Physical Chemistry of Diffusion Suppression Scheme

Add O_2 (~6 psi in 60 atm. Ne/He 80/20)

Add CO_2 ~10 μm

Initial capture : $e^- + O_2 + \left\{ \begin{matrix} O_2 \\ Ne \\ He \end{matrix} \right\} \rightarrow O_2^- + \{\ \}$

$\tau \sim 20-50$ ns {Set by H.V. + Laser timing jitter}

Conversion to O_4^- : $O_2^- + O_2 + \{\ \} \rightarrow O_4^- + \{\ \}$

$\tau \lesssim 10$ ns

Photoionization (after trigger delay ~1-3 μs) : $h\nu + O_4^- \rightarrow e + O_2 + O_2$

or $\rightarrow O_2^- + O_2$, $O_2^- \xrightarrow{h\nu} e + O_2$

{second branch not major}

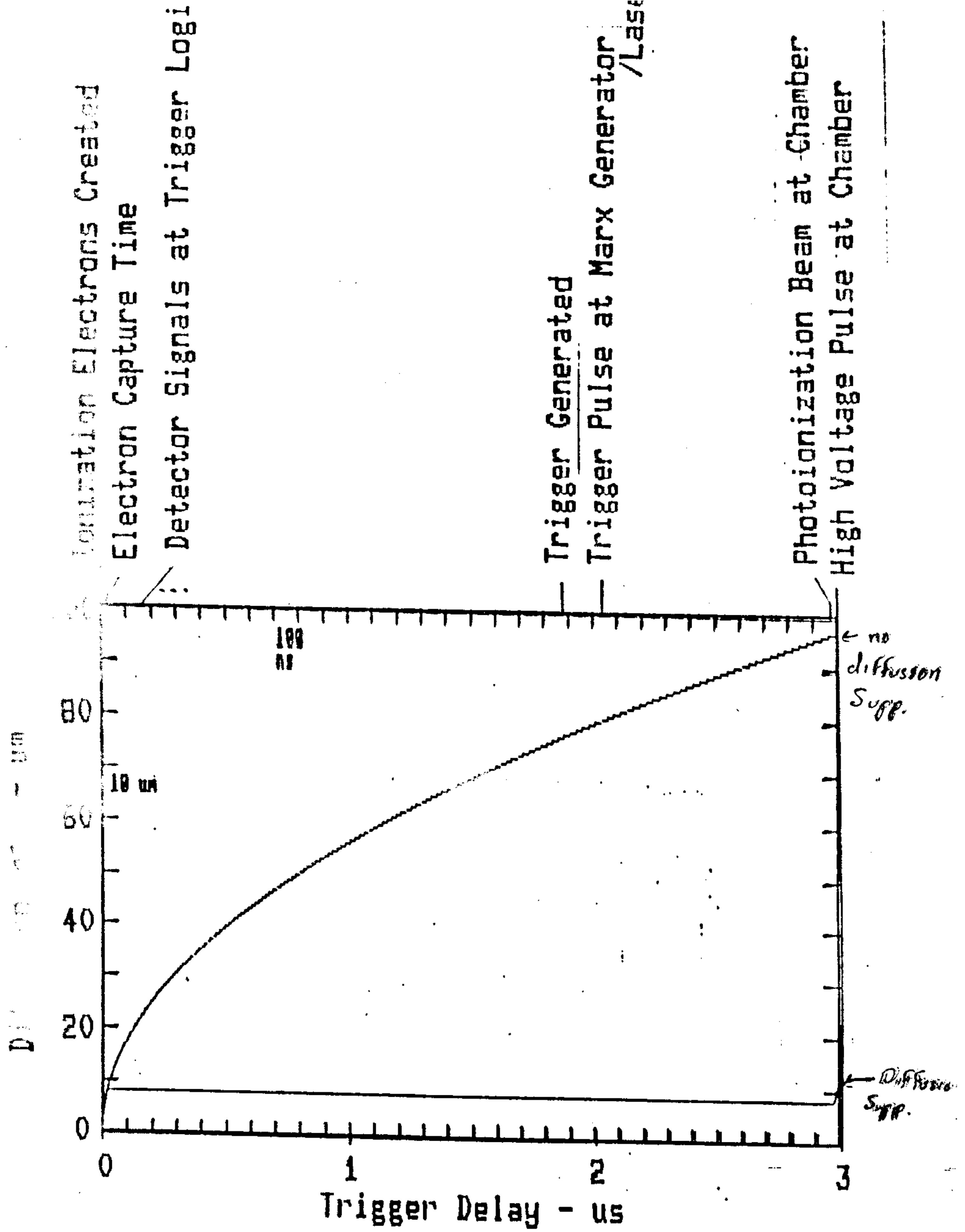
If no trigger, CO_2 provides memory time

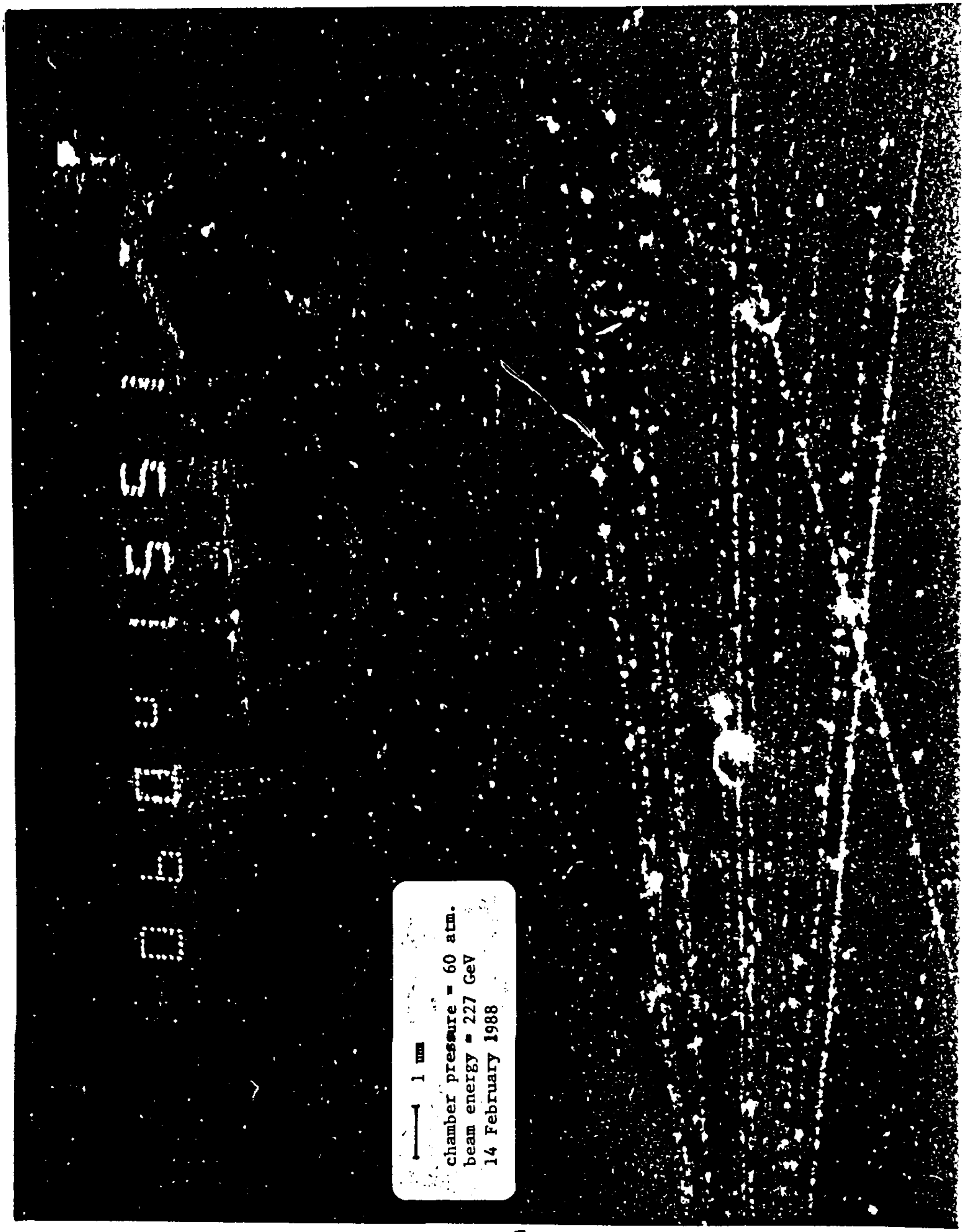
$O_4^- + CO_2 \rightarrow CO_4^- + O_2$

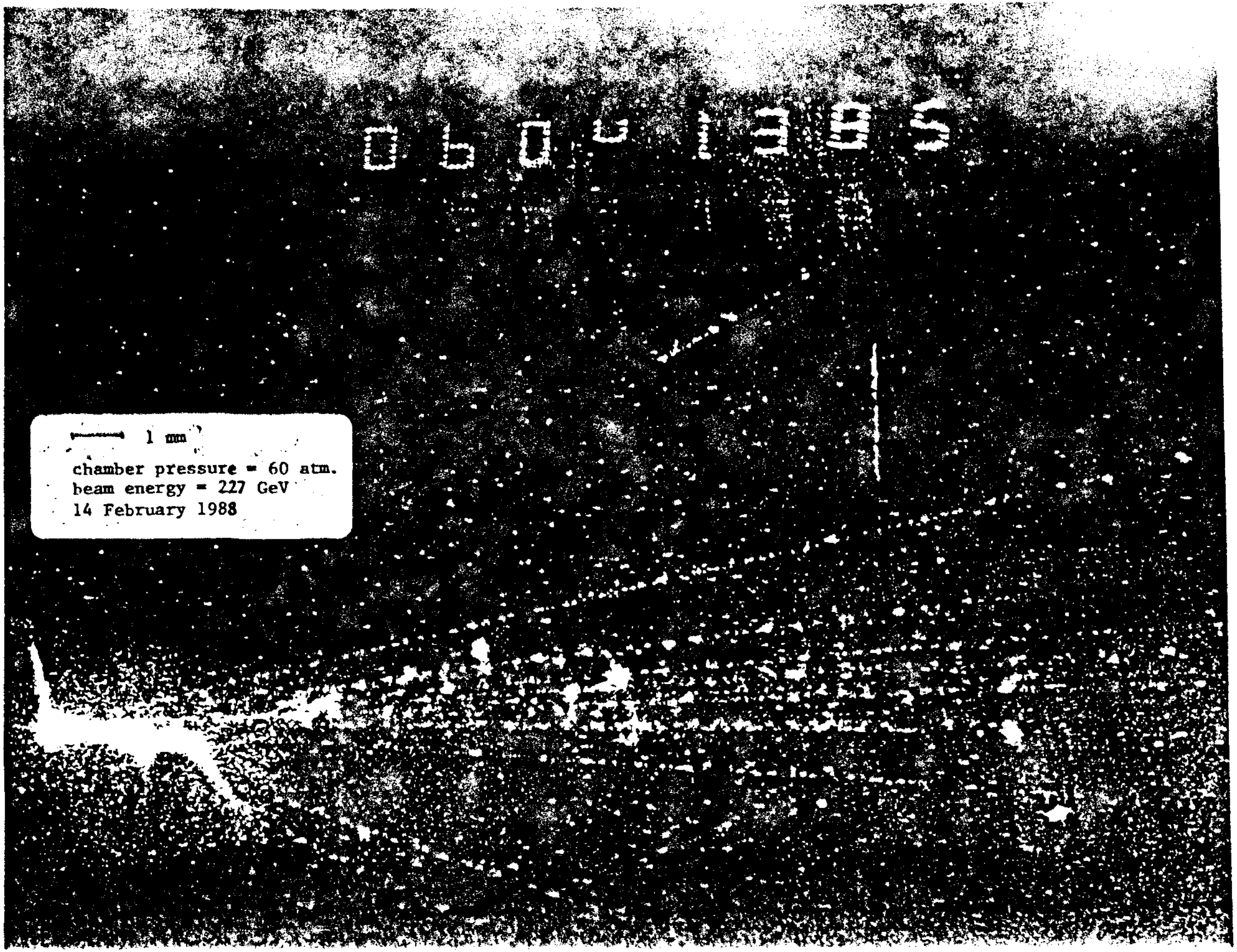
$h\nu + CO_4^- \not\rightarrow$ photoionize

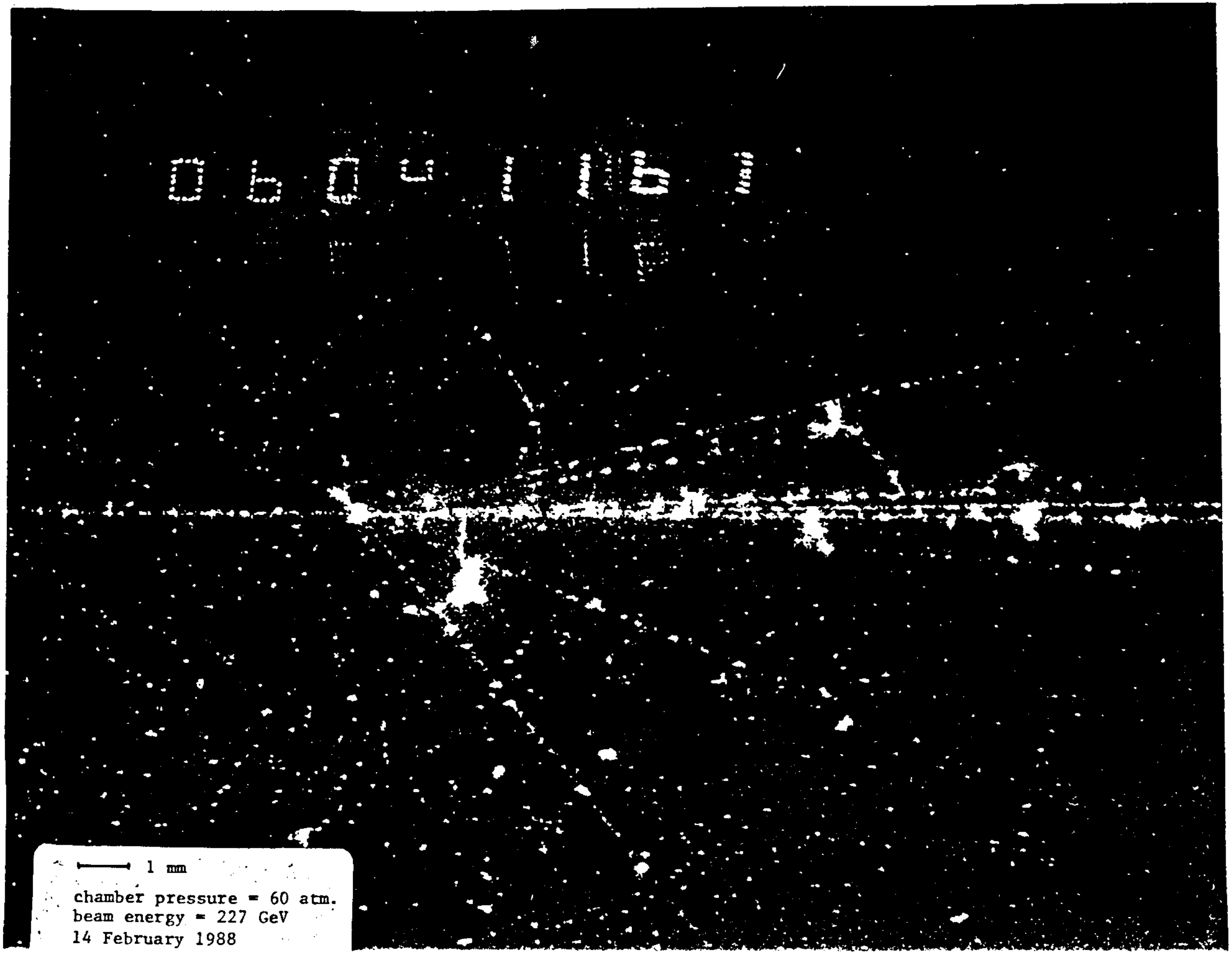
$\tau \sim 10\,\mu s$ for ~10 μm CO_2

Many important details left out
ESPECIALLY GAS PURITY!


Electron Capture Time
Detector Signals at Trigger Logi
Trigger Generated
Trigger Pulse at Marx Generator
/Lase
Photoionization Beam at Chamber
High Voltage Pulse at Chamber
100
ns
10 um
← no diffusion Supp.
← Diffusion Supp.
0
20
40
60
80
0
1
2
3
Trigger Delay - us


1 mm
chamber pressure = 60 atm.
beam energy = 227 GeV
14 February 1988


1 mm
chamber pressure = 60 atm.
beam energy = 227 GeV
14 February 1988


1 mm
chamber pressure = 60 atm.
beam energy = 227 GeV
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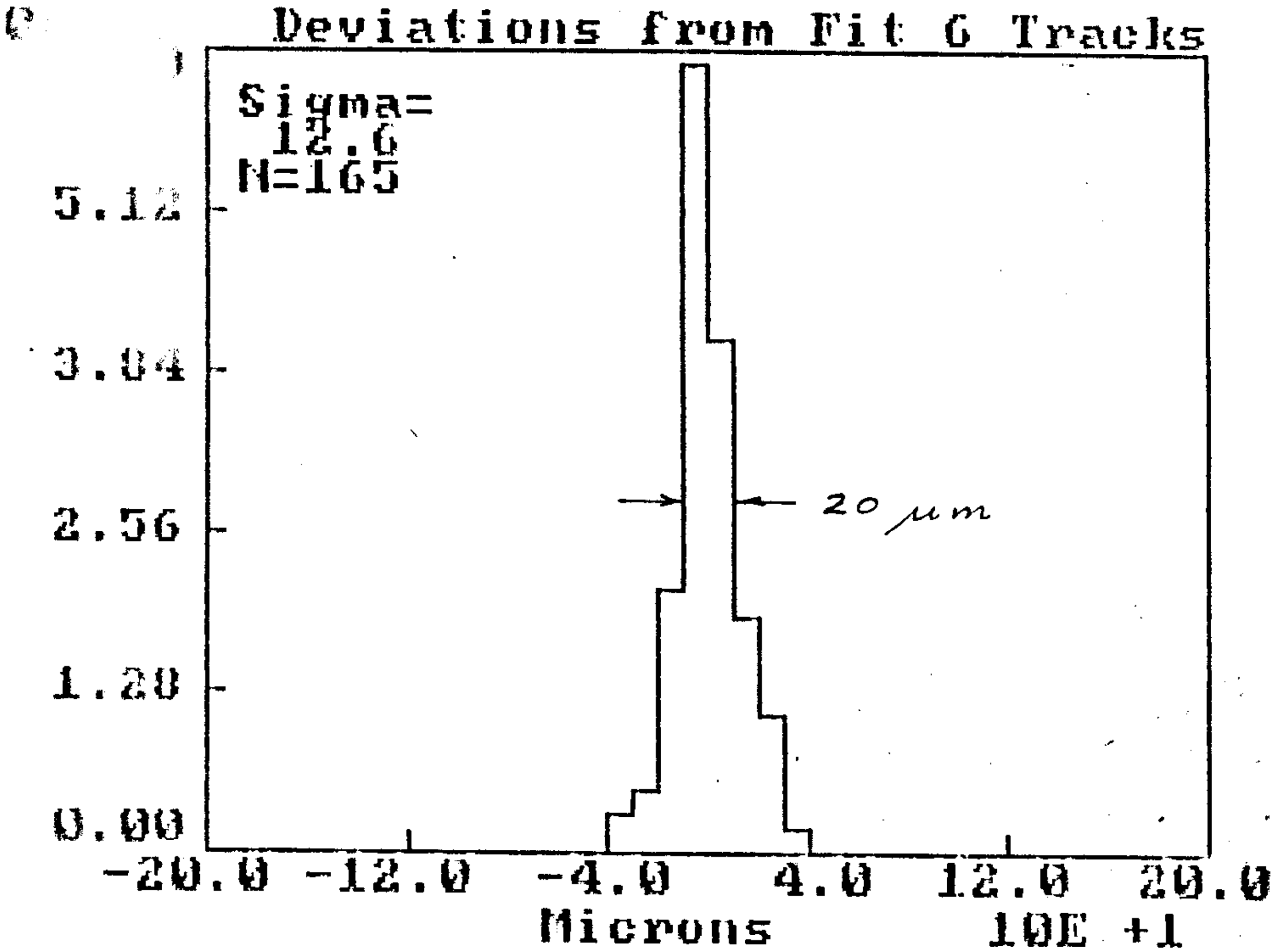

FWHM 20 μm
corresponds quite well with
expected σ ~ 12 μm from
diffusion in the time
period when the electrons
are unattached.