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"The Field Water Balance"

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THE FIELD WATER BALANCE

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1. Introduction.

The various soil-water flow processes (e.g. infiltration, redistribution, drainage, evaporation and water uptake by plants) are in fact strongly interdependent, as they occur sequently and simultaneously.

To evaluate the field water cycle as a whole, and the relative magnitudes of the various processes comprising it over a period of time, it is necessary to consider the field water balance. Just as a businessman regularly summarizes the financial balance of his enterprise, including an itemized listing of all income sources, expenditures, inventory changes and net worth, so the agricultural or environmental physicists can attempt to account for all the water entering, leaving and remaining in a specified volume of soil during a specified length of time.

The water balance is merely a detailed statement of the law of conservation of matter, which states simply that matter can neither be created nor destroyed but can only change from one state or location to another. Since no significant amounts of water are normally decomposed or composed in the soil, the water content of a soil profile of finite volume cannot increase without addition from the outside (as by infiltration or capillary rise), nor can it diminish unless transported to the atmosphere by evapotranspiration or to deeper zones by drainage.

The field water balance is intimately connected with the energy balance since it involves processes that require energy. The energy balance is an expression of the chemical low of conservation of energy, which states that, in a given system, energy can be absorbed from, or released to the outside, and that along the way it can change form, but it cannot be created or destroyed.

The content of water in the soil affects the way the energy flux reaching the field is partitioned and utilized. Likewise, the energy flux affects the state and movement of water.

The water balance and energy balance interact, since they are both aspects of the same processes within the same environment.

A physical description of the soil-plant-atmosphere system, therefore, must be based on an understanding of both balances together.

In particular, the evaporation process, which is often the principal consumer of both water and energy in the field, depends, in a combined way, on the simultaneous supply of water and energy.

In the following chapters only the water balance will be described.

2. Water balance in the rootzone.

In its simplest form, the water balance merely states that, in a given volume of soil, the difference between the amount of water added, W_{in}, and the amount of water withdrawn, W_{out}, during a certain period is equal to the change in water content, Δ_W, during the same period:

$$\Delta W = W_{in} - W_{out} \quad (1)$$

When gains exceed losses, the water content change is positive and conversely, when losses exceed gains, ΔW is negative.

To itemize the accretions and depletions from the soil storage reservoir, one must consider the deposition of rain or irrigation reaching a unit area of soil surface during a given period of time. Rain or irrigation water applied to the land may in some cases infiltrate into the soil as fast as it arrives. In other cases, some of the water may pond over the surface. Depending on the slope and microrelief, a portion of this water may exit from the area as surface runoff while the remainder will be stored temporarily as puddles in surface depressions. Some of the latter evaporates and the rest eventually infiltrates into the soil after cessation of the rain. Of the water infiltrated some evaporates directly from the soil surface, some is taken up by plants for growth or transpiration, some may drain downward beyond the root zone, whereas the remainder accumulates within the root zone and adds to soil moisture storage. Additional water may reach the defined soil volume by runoff from a higher area or by upward flow from a water table or from wet layers present at some depth. The pertinent volume or depth of soil for which the water balance is computed, is determined arbitrarily.

Thus, in principle, a water balance can be computed for a small sample of soil or for an entire watershed. From an agricultural point of view it is generally most appropriate to consider the water balance of the root zone per unit area of field.

The rootzone water balance is expressed in integral form:

$$(\Delta S + \Delta V) = (P + I + U) - (R + D + E + T) \qquad (2)$$

change in storage (gains) - (losses)

wherein: ΔS: change in root zone soil moisture storage

ΔV: increment of water incorporated in the plants

P: precipitation

I: irrigation

U: upward capillary flow into the root zone

R: runoff

D: downward drainage out of the root zone

E: direct evaporation from the soil surface

T: transpiration by plants

All quantities are expressed in terms of volume of water per unit area (equivalent depth units) during the period considered.

Since the change of plant-water (ΔV) is relatively unimportant, and the upward capillary flow U and the downward drainage D can be included under the symbol D being negative if capillary rise occurs or positive when drainage is present, equation (2) can be written as follows:

$$\Delta S = P + I - R - D - E - T \qquad (3)$$

Simple and readily understandable though the field water balance may seem in principle, it is still rather difficult to measure in practice. A single equation can be solved if it has only one unknown. Often the largest component on the field water balance and the one most difficult to measure directly, is the evapotranspiration E + T, also designated ET_{cr} (crop evapotranspiration).

3. Evapotranspiration.

This term is defined as the process by which water is evaporated from wet surfaces and transpired by plants. It is sometimes referred to as comsumptive use and further defined as the quantity of water used by either cropped or natural vegetation in transpiration or in the building of plant tissues, together with the water evaporated from the adjacent soil, water bodies, or from intercepted precipitation. Since soil moisture may have an effect upon evapotranspiration, Thornthwaite (1948) introduced the term, evapotranspiration, to mean the evapotranspiration which would occur were there an adequate soil-moisture supply at all times.

It is generally recognized that climate is one of the most important factors determining the amount of water loss by evapotranspiration from the crop. Apart from the climatic factors, evapotranspiration for a given crop is also determined by the crop itself and so are growth characteristics. Local environment, soil and soil water conditions, fertilizers, insect and disease infestations, agricultural and irrigation practices and other factors may also influence growth rates and resulting evapotranspiration.

3.1. Maximum evapotranspiration ET_m.

Climate is one of the most important factors determining the crop water requirements needed for unrestricted optimum growth and yield. Crop water requirements are normally expressed by the rate of evapotranspiration (ET) in mm/day or mm/period. The level of ET is related to the evaporative demand of the air.

The approach followed was to relate magnitude and variation or evapotranspiration to one or more climatic factors (day length, temperature, humidity, wind, sunshine). For this, measured evapotranspiration data from a grass cover were used, assuming that evapotranspiration of grass occurs largely in response to climatic conditions. A reference value, ET_o, was introduced and defined as "the rate of evapotranspiration from an extended surface of 8 to 15 cm tall green grass cover of uniform height, actively growing, completely shading the ground and not short of water".

Ten methods for the calculations of the reference crop evapotranspiration or the potential evapotranspiration ET_o are mentioned briefly below.

1. Blaney-Criddle method

The relationship recommended is expressed by the following formula :

$$ET_o = c(p(0.46T + 8)) \quad (\text{mm.day}^{-1})$$

where ET_o = reference crop evapotranspiration in mm.day^{-1} for the month considered

T = mean daily temperature in ^oC over the month considered

P = mean daily percentage of total annual daytime hours for a given month and latitude

c = adjustment factor which depends on minimum relative humidity, sunshine hours and daytime wind estimates.

2. Jensen and Haise method

The Jensen and Haise formula is :

$$ET_o = (0.025T + 0.08)\frac{H_{sh}}{59} \quad (\text{mm.day}^{-1})$$

where ET_o = reference crop evapotranspiration in mm.day^{-1}

H_{sh} = incoming short-wave radiation $(\text{cal.cm}^{-2}.\text{day}^{-1})$

T = air temperature (^oC)

3 Radiation method

The relationship recommended is expressed as :

$$ET_o = c(W.Rs) \quad (\text{mm.day}^{-1})$$

where ET_o = reference crop evapotranspiration in mm.day^{-1} for the period considered

Rs = solar radiation in equivalent evaporation in mm.day^{-1}

W = weighting factor which depends on temperature and altitude

c = adjustment factor which depends on mean humidity and daytime wind conditions

4. Penman (original)

The basic formula for calculation of evapotranspiration from a free water surface (E_o) is as follows :

$$E_o = \frac{(\Delta/L)\, H_o + \gamma E_x}{\Delta + \gamma} \quad (\text{mm.day}^{-1})$$

where E_o = evaporation from a free water surface (mm.day^{-1})

H_o = net radiation $(\text{cal.cm}^{-2}.\text{day}^{-1})$

E_x = isothermal evaporation (mm.day^{-1})

Δ = slope of the temperature - vapour pressure relationship at temperature T $(\text{mm Hg}\ ^oC^{-1})$

γ = psychrometric constant $(= 0.485\ \text{mm Hg.}^oC^{-1})$

L = latent heat of evaporation of $0.1\ \text{cm}^3$ (= 59 cal)

$$ET_o = 0.8\, E_o$$

5. Uncorrected Penman

The form of the equation used in this method is :

$$ET_o = W.Rn + (1-W).\ f(u).\ (ea-ed) \quad (\text{mm.day}^{-1})$$

where ET_o = reference crop evapotranspiration in mm.day^{-1}

W = temperature related weighting factor

Rn = net radiation in equivalent evaporation in mm.day^{-1}

$f(u)$ = wind-related function

$(ea-ed)$ = difference between the saturation vapour pressure at mean air temperature and the mean actual vapour pressure of the air, both in mbar

6. Corrected Penman

The Penman equation assumes the most common conditions where radiation is medium to high, maximum relative humidity is medium to high and moderate daytime wind about double the nighttime wind. Since these conditions are not always met, correction to the Penman equation as given in 5 is necessary.

Reference crop evapotranspiration (ET_o) can be calculated using the formula :

$$ET_o = c\{(W.Rn + (1-W).f(u).(ea-ed)\}$$

The adjustment factor "c" depends on the maximum relative humidity (RH max), solar radiation (Rs), daytime wind (U day) and the value of daytime/nighttime wind.

7. Pan evaporation method

Reference crop evaporation (ET_o) can be obtained from :

$$ET_o = Kp.\ E\ pan$$

where E pan = pan evaporation in $mm.day^{-1}$

Kp = pan coefficient

Factors taken into account for the determination of the Kp factors are :

- type of pan used
- location of the pan in short green cropped or a dry fallow area
- relative mean humidity
- wind speed
- windward side distance of green crop

8. Original Turc

The original formula by Turc reads as follows :

$$ET_o = \frac{P + 80}{\sqrt{1 + \left(\frac{P+45}{L^{Tc}}\right)^2}} \quad (mm/10\ days)$$

where ET_o = reference crop evapotranspiration (mm/10 days)

P = precipitation (mm/10 days)

L^{Tc} = evaporative demand of the atmosphere, calculated according to :

$$L^{Tc} = \frac{(T+2)\sqrt{H_{sh}}}{16}$$

T = average air temperature °C at 2 m

H_{sh} = incoming short-wave radiation ($cal.cm^{-2}.day^{-1}$)

9. Simplified Turc

Turc simplified the original equation into :

a) $ET_o = 0.40 \frac{T}{T+15} (H_{sh} + 50)$ ($mm.month^{-1}$)

in case the relative humidity (h) was above 50%

b) $ET_o = 0.40 \frac{T}{T+15} (H_{sh} + 50) (1 + \frac{50-h}{70})$ ($mm.month^{-1}$)

in case h is less than 50%

10. Formula of Thornthwaite

According to Thornthwaite :

$$ET_o = 1.6b\left(\frac{10T}{I}\right)^a$$

ET_o = reference crop evapotranspiration (in $cm.month^{-1}$)

T = average air temperature (°C) calculated from daily means

I = annual heat index i.e. the sum of the 12 monthly heat indices, i, where :

$$i = \left(\frac{T}{5}\right)^{1.514}$$

a = $0.000000675I^3 - 0.0000771I^2 + 0.01792I + 0.49239$

b = depends on the month and latitude

Approximate values for ET_o in mm/day for different agroclimatic regions are given in Table 1 (Doorenbos and Kassam, 1979).

Table 1 : Reference evapotranspiration (ET_o in mm/day) for different agro-climatic regions.

Regions	Mean daily temperature, °C <10 (cool)	20 (moderate)	>30 (warm)
TROPICS			
humid	3 - 4	4 - 5	5 - 6
subhumid	3 - 5	5 - 6	7 - 8
semi-arid	4 - 5	6 - 7	8 - 9
arid	4 - 5	7 - 8	9 - 10
SUBTROPICS			
Summer rainfall:			
humid	3 - 4	4 - 5	5 - 6
subhumid	3 - 5	5 - 6	6 - 7
semi-arid	4 - 5	6 - 7	7 - 8
arid	4 - 5	7 - 8	10 - 11
Winter rainfall			
humid - subhumid	2 - 3	4 - 5	5 - 6
semi-arid	3 - 4	5 - 6	7 - 8
arid	3 - 4	6 - 7	10 - 11
TEMPERATE			
humid - subhumid	2 - 3	3 - 4	5 - 7
semi-arid - arid	3 - 4	5 - 6	8 - 9

Empirically-determined crop coefficients (kc) can be used to relate ET_o to maximum crop evapotranspiration (ET_m) when water supply fully meets the water requirements of the crop. The value of kc varies with crop, development stage of the crop, and to some extent with windspeed and humidity. For most crops, the kc value increases from a low value at time of crop emergence to a maximum value during the period when the crop reaches full development, and declines as the crop matures.

For a given climate, crop and crop development stage, the maximum evapotranspiration (ET_m) in mm/day of the periode considered is :

$$ET_m = kc. ET_o \qquad (4)$$

Maximum evapotranspiration (ET_m) refers to conditions when water is adequate for unrestricted growth and development; ET_m represents the rate of maximum evapotranspiration of a healthy crop, grown in large fields under optimum agronomic and irrigation management.

The methods presented allow prediction of ET_m within 10 to 20 percent accuracy provided the meteorological data are reliable and obtained from a representative agricultural environment, and provided total growing period and lengths of development stages are known. For details see Doorenbos and Pruitt (1977).

Meteorological data used in the calculation of ET_m should preferably be collected at stations situated within an agricultural (irrigated) area. When data are collected at stations in dry, bare areas, at airports or even on rooftops, the calculated ET_m values should be corrected since the data do not represent the different micro-climates found within the irrigation schemes. In arid and semi-arid areas with moderate wind, ET_m calculated with data obtained outside the irrigated area may need to be adjusted downward by 20 to 25 percent.

3.2. Actual evapotranspiration ET_a.

The demand for water by the crop must be met by the water in the soil, via the root system. The actual rate of water uptake by the crop from the soil in relation to its maximum evapotranspiration (ET_m) is determined by whether the available water in the soil is adequate or whether the crop will suffer from stress inducing water deficit.

In order to determine actual evapotranspiration (ET_a), the level of the available soil water must be considered. Actual evapotranspiration (ET_a) equals maximum evapotranspiration (ET_m) when available soil water to the crop is adequate, or $ET_a = ET_m$.

However, $ET_a < ET_m$ when available soil water is limited. Available soil water can be defined as the fraction (p) to which the total available soil water can be depleted without causing ET_a to become less than ET_m. The magnitude of ET_a can be quantified for periods between irrigation or heavy rain, and for monthly periods.

Total available soil water (S_a) is defined here as the depth of water in mm/m soil depth between the soil water content at field capacity (S_{fc} or at soil water tension of 0.1 to 0.2 atmosphere) and the soil water content at wilting point (S_w or at soil water tension of 15 atmosphere). Total available soil water (S_a) can vary widely for soils having a similar texture. Also, most soils are layered and integrated values of S_a over soil depth should be selected; dense layers restrict water distribution.

As a general indication, S_a mm/m for different soil textures is :

heavy textures soil	200 mm/m
medium textures soil	140 mm/m
coarse textures soil	60 mm/m

Local information on total available soil water in the root zone will be required. The need for field measurements is evident.

4. Water balance methods.

4.1. Evaluation of the parameters in the water balance equation.

The water balance of an area is given by:

$$P = R + E + \Delta S + D \qquad (5)$$

where: P = precipitation
R = runoff
E = evaporation
ΔS = change in water content of the soil
D = drainage to groundwater

Depending on the size of the area and the time scale over which the balance is determined, different methods of measuring or estimating the components can be employed.

The crop evapotranspiration also called evapotranspiration is generally a fraction of ET_0, being the reference crop evapotranspiration often called potential evapotranspiration.

The ET depends on the degree and density of plant canopy coverage of the surface, as well as on the soil moisture and root distribution. ET from a well-watered stand of a close growing crop will generally approach ET_0 during the active growing stage, but may fall below it during the early growth stage, prior to full canopy coverage and again toward the end of the growing season as the matured plants begin to dry out.

To obtain ET from the water balance one must have accurate measurements of all other terms of the equation.

It is relatively easy to measure the amount of water added to the field by rain and irrigation (P + I), though it is necessary to consider possible nonuniformities in areal distribution.

The amount of runoff generally is (or at least should be) small in agricultural fields and particularly in irrigated fields, so that it can sometimes be regarded as negligible in comparison with the major components of the water balance.

If runoff occurs it should be quantified. For short periods, the change in soil water storage ΔS can be relatively large and must be measured. This measurement can be made by sampling periodically or by use of specialised instruments viz. the neutron moisture meter.

During dry spells, without rain or irrigation $W_{in} = 0$ so that equation (5) becomes:

$$\Delta S = - D - ET \qquad (6)$$

It seems from equation (6) that the reduction in root zone water storage ΔS equals the sum of D and ET.

Common practice in irrigation is to measure the total water content of the root zone just prior to an irrigation and to supply the amount of water necessary to replenish the soil reservoir to some maximal water content, generally taken to be the field capacity. Some have tended to assume that the deficit of soil moisture which develops between rains or irrigations is due to evapotranspiration only, thus disregarding the amount of water which may flow through the bottom of the root zone, either downward or upward. This flow is not always negligible and has to be taken into account in searching of an optimal water use efficiency.

Following questions can be asked:

1. What is the lower limit of the soil water content in the profile in order to avoid a decrease in yield
2. What is the upper limit of the soil water content in the profile in order to avoid loss of water through percolation at greater depths.

It should be obvious that measurement of root zone or subsoil water content by itself cannot tell us the rate and direction of soil water movement. Even if the water content at a given depth remains constant, we cannot conclude that the water there is immobile since it might be steadily through that depth (figure 1).

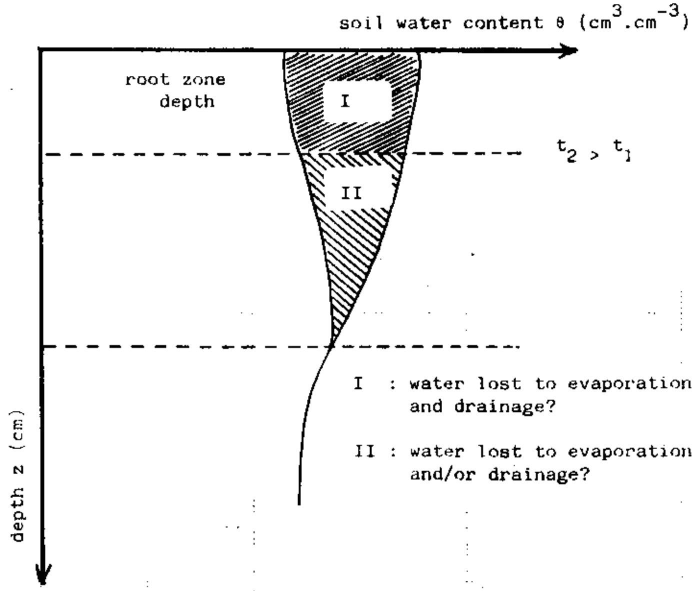


Figure 1. Soil water content profiles.

Tensiometric measurements can, however, indicate the directions and magnitudes of the hydraulic gradients through the profile and allow us to locate the zones of upward and downward water movement.

4.2. Soil water content profile - Soil water storage.

The soil water content profiles can be established at different times by lowering a neutron probe at different depths and measuring the counts per minute at some reference depths. Using the calibration curve the soil water content profile, relating the measured points as given in figure 2, can be established.

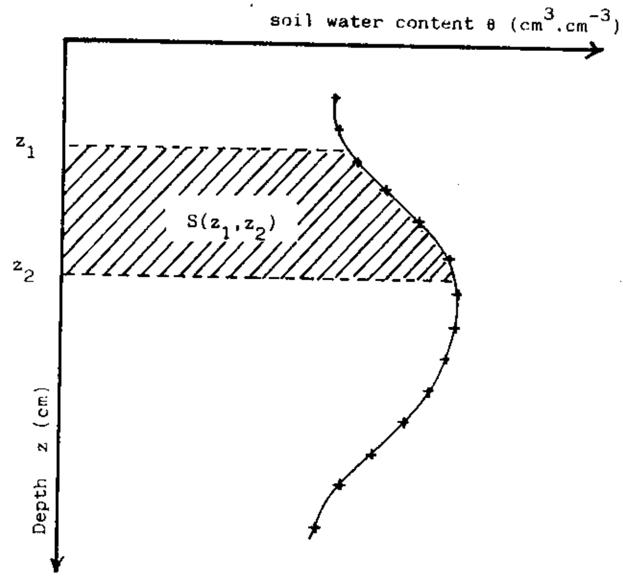


Figure 2. Soil water content profile.

When the soil water content profile is presented in one-dimension and moreover independent of the chosen vertical axis, the volume of water in the vertical soil column with a cross-section A and limited between levels Z_1 and Z_2 (figure 2) equals:

$$\int_{z_1}^{z_2} \theta \,.\, dV \quad \Lambda \quad \int_{z_1}^{z_2} \theta \,.\, dz \qquad (7)$$

or per unit area, equation (7) becomes:

$$\int_{z_1}^{z_2} \theta \,.\, dz = S\,(z_1, z_2) \qquad (8)$$

where S is called the soil water storage with dimension length allowing to compare it with precipitation or the amount of irrigation water.

The value of the cumulative storage between the soil surface and different depths is calculated by integrating the soil water content profile. This integration can be easily done by associating the water content measured at level z_k to the 10 cm soil section surrounding this level, except for the measurement done at the depth z_1 = 10 cm which is associated to the soil section extending from 0 to 15 cm. If water storage is expressed in mm water, the storage to depth z_j will equal therefore:

$$Sz_j = 1.5\,(\theta_{10} + \theta_{20} + \theta_{30} + \ldots\ldots + \theta_k + \ldots\ldots + 0.5\,\theta_j) \times 100 \qquad (9)$$

Example

Soil water storage at 50 cm soil depth (figure 3).

$$S_{50} = 150 \times \theta_{10} + 100 \times \theta_{20} + 100 \times \theta_{30} + 100 \times \theta_{40} + 50 \times \theta_{50}$$

$$= (150 \times 0.1) + (100 \times 0.15) + (100 \times 0.20) + (100 \times 0.25) + (50 \times 0.25)$$

$$= 87.5 \text{ mm water}$$

$$\text{or } S_{50} = 100\,(1.5\,\theta_{10} + 1\,(\theta_{20} + \theta_{30} + \theta_{40}) + 0.5\,\theta_{50})$$

Let us consider a soil layer with thickness Δz at depth z (axis Oz positive downwards) and with a unit cross-section area and a soil water content θ (cm^3.cm^{-3}).

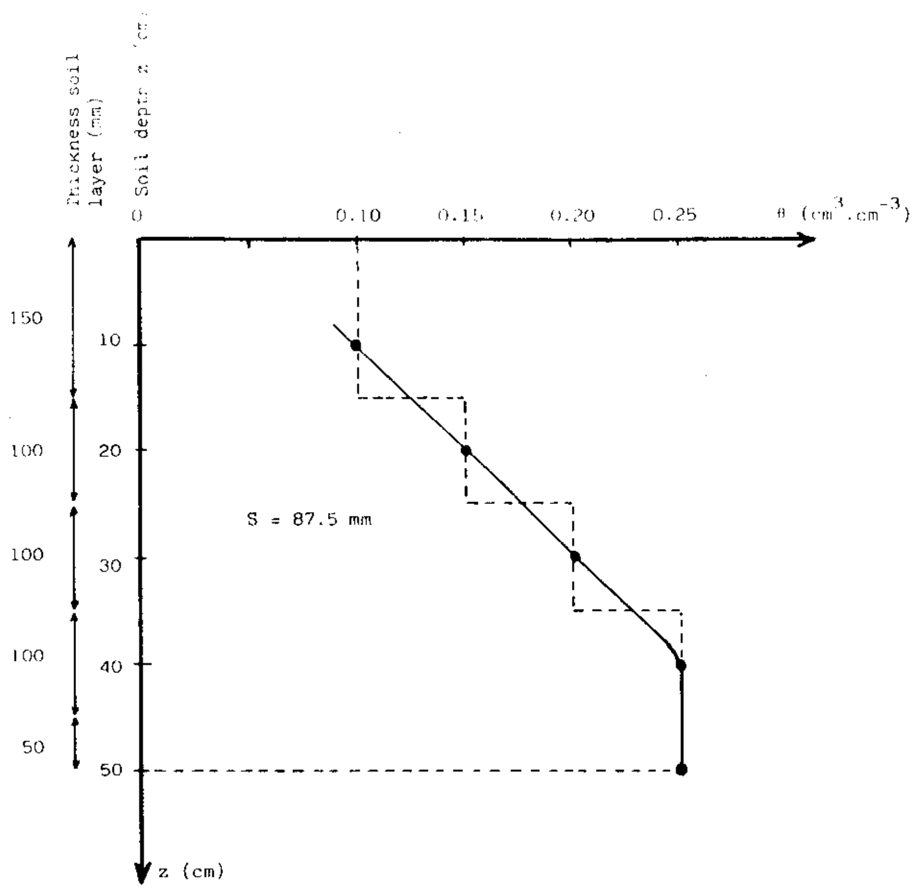


Figure 3. Calculation of soil water storage.

Transfer of water from or into that soil column can be due to:

- vapor diffusion from the soil and plant to the atmosphere in function of the energy received by the soil and the vegetation (evapo-transpiration)
- infiltration of rainfall or irrigation water.

If we consider water loss ΔS over period Δt as presented in figure 4, the following questions can be asked:

- what fraction of ΔS is due to evapo(transpi)ration
- what fraction of ΔS is due to drainage

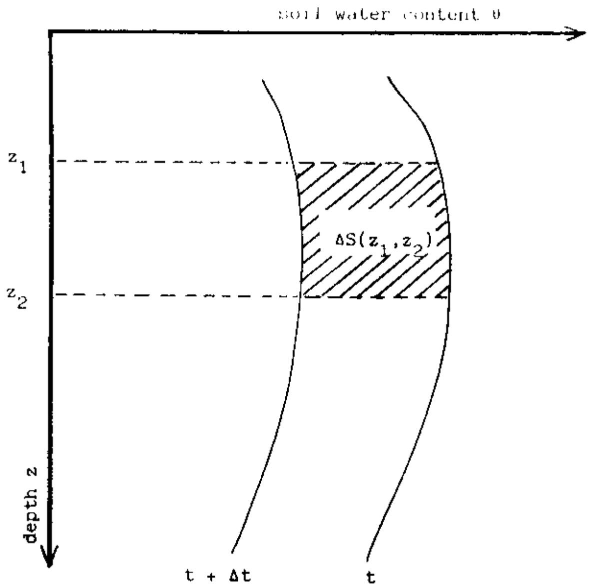


Figure 4. Soil water content profiles at two different times and soil water storage (ΔS) between z_1 and z_2.

5. Hydraulic head profile.

The basic flow equation for describing the water movement in saturated soils and conceived by Darcy was extended to unsaturated flow and if we assume that the transfer essentially occurs in the vertical direction, Darcy's law is expressed as (figure 5):

$$q = K(\theta) \frac{dH}{dz} \qquad (10)$$

where: q = the flux, which is the volume of water crossing a unit area per unit time

$K(\theta)$ = the hydraulic conductivity and function of the soil water content

dH = the difference in hydraulic potential

dz = the difference in distance over which the difference in hydraulic potential is measured

dH/dz = the hydraulic potential gradient or the driving force

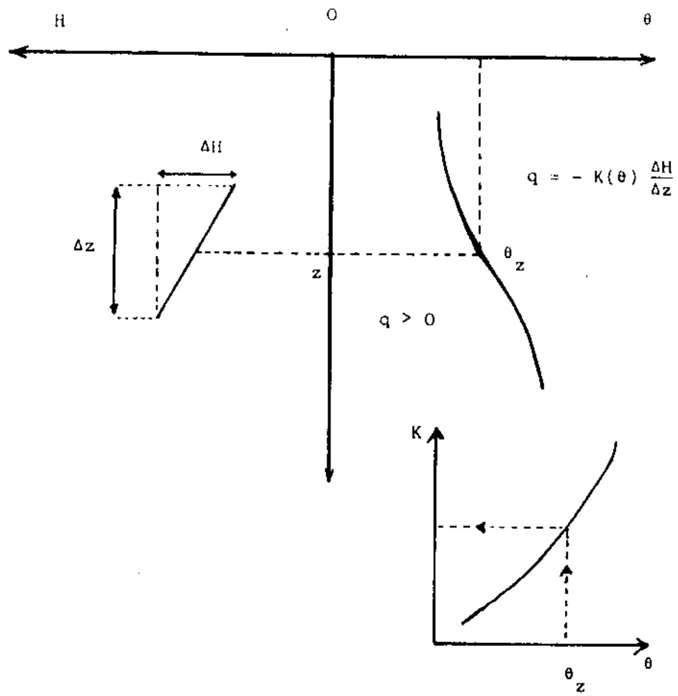


Figure 5. Darcy's law describing vertical flow in the soil profile.

and where:

$$H = z + h \qquad (11)$$

h = matric potential or soil water pressure head

z = gravitational potential or gravitational head.

The negative sign in equation (10) indicates that the driving force and the ensuing flow of water are directed towards decreasing values of the hydraulic potential.

Therefore upward or downward flux in the profile will depend on the sign of the hydraulic gradient.

In figure 6 are presented the most frequent occuring hydraulic head profiles. In this example a watertable occurs at a shallow depth. At that depth (Z_{wt}) there is atmospheric pressure, while the soil water pressure head equals zero so that equation (11) becomes:

$$H_{wt} = Z_{wt}$$

The profile in figure 6c shows that the hydraulic gradient close to the soil surface is positive ($dH/dz > 0$), so that equation (10) becomes negative ($q < 0$).

This means upward flux-supplying the demands of evapotranspiration. Under point A the water flow is in the opposite way, namely downward flux - representing drainage to the groundwater table. Indeed, $dH/dZ < 0$ and consequently $q > 0$. In A, q equals zero because the hydraulic gradient dH/dz equals 0 and that level is called "plane of zero flux", $z_{q=0}$.

Such profile occurs often during redistribution following a rainy period. Indeed, infiltration at greater dephts continues for a long period, while the upperpart of the profile is subjected to evapotranspiration. Upward flux starts at the soil surface and extends over greater depth with time.

6. Transfer of water in an unsaturated soil.

Let us consider a volume of soil of thickness Δz and a unit section where the change of water content is $\Delta\theta$ during a time interval Δt (unsteady of transient flow process), the mass conservation law, expressed in the equation of continuity states that if the rate of inflow into the volume element is greater than the rate of outflow, then the volume element must be storing the excess and increasing its water content (and conversely, if outflow exceeds inflow, storage must decrease).

For a one-dimensional flow with q_z being the flux in the direction z the continuity equation in general can be written as follows:

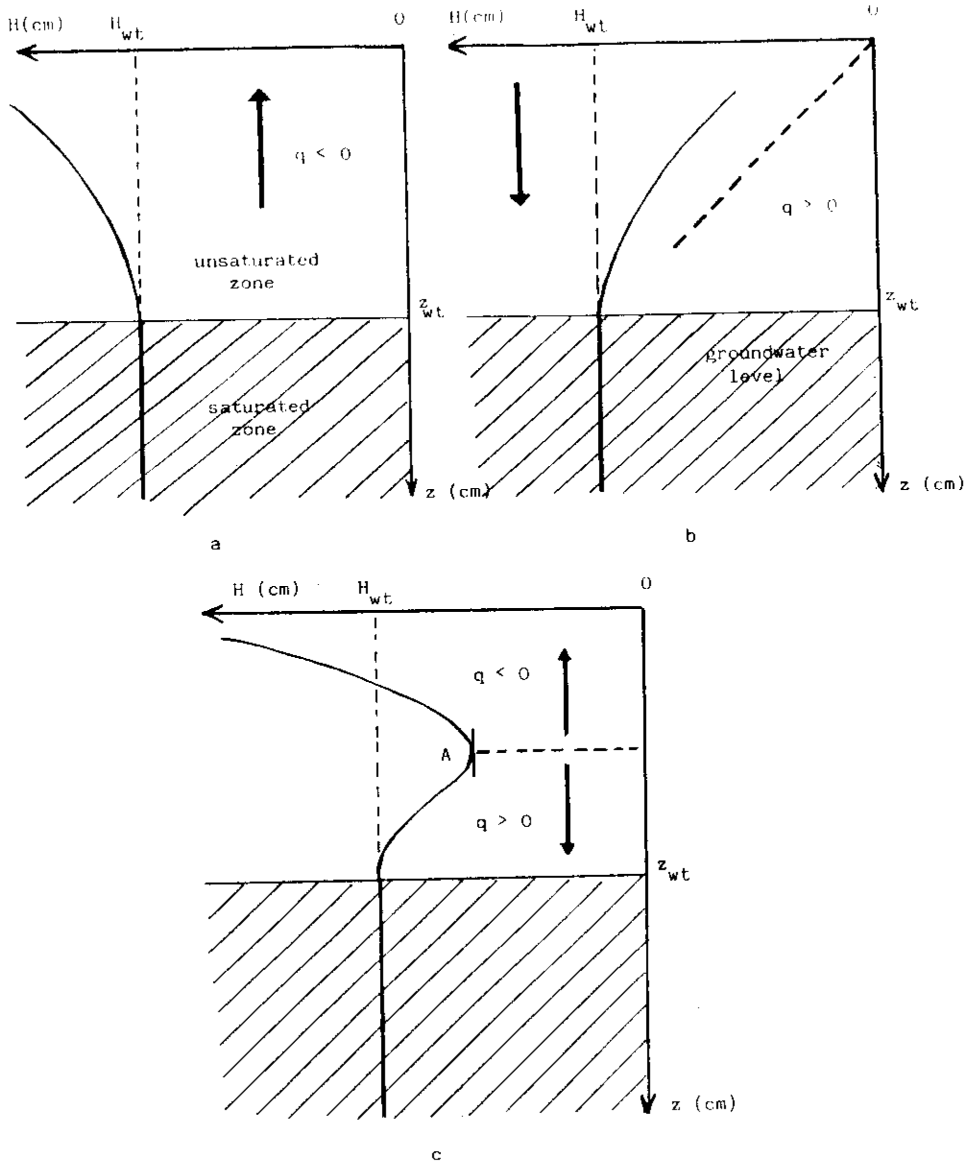


Figure 6. Hydraulic head profiles.

$$\frac{\partial \theta}{\partial t} = - \frac{\partial q_z}{\partial z} \qquad (12)$$

where: q = flux

θ = volumetric soil water content

t = time

z = distance

Equation (12) applied to a soil element as given in figure 7 can be obtained by identifying the difference between the mass of water entering by one face and that leaving the other face during the interval Δt to the variation in the mass of water contained in the volume, assuming there is no source or spring in this element. In the case of a downward flux, this is expressed as:

$$\rho_w (q_1 - q_2) \Delta t = \rho_w \Delta\theta \, \Delta z \qquad (13)$$

where: q_1 = flux entering the upper face

q_2 = flux leaving the lower face

ρ_w = density of water

or:

$$(q_2 - q_1) = - \frac{\Delta\theta \, \Delta z}{\Delta t} \qquad (14)$$

Equation (14) can also be written as:

$$q_2 - q_1 = - \frac{\Delta S}{\Delta t} \qquad (15)$$

This last formula clearly shows that it is not possible to obtain the flux through either faces or the volume of water flowing by a unit surface of soil $q.\Delta t$, from the single measurement of the variation of water storage in the element of volume taken into account.

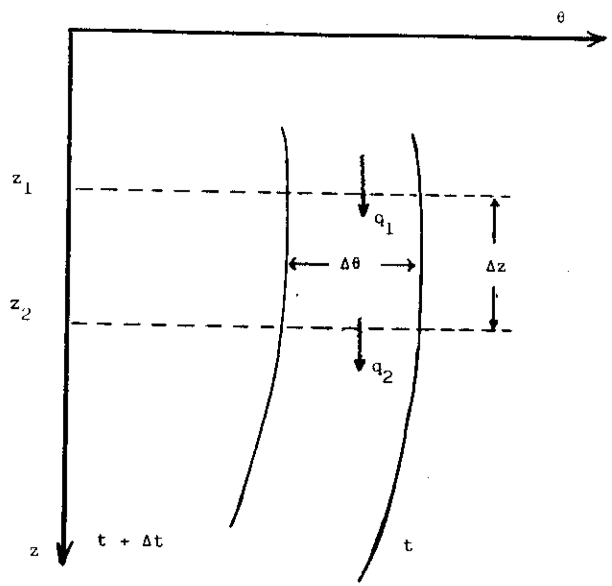


Figure 7. Equation of continuity.

It is also necessary to determine the flux flowing through the other face:

- either by applying equation (10) if the hydraulic conductivity $K(\theta)$ and the hydraulic gradient dH/dz are known in this section
- or by fixing the level of this face at a depth where the flux is known (normally z where $q = 0$ viz. where $dH/dz = 0$).

In this case, equation (15) becomes:

$$q_1 = \frac{\Delta S}{\Delta t}$$

The latter situation is presented in figure 8 and often occurs in the field. It allows to estimate the evaporation and/or drainage directly from soil water content measurements.

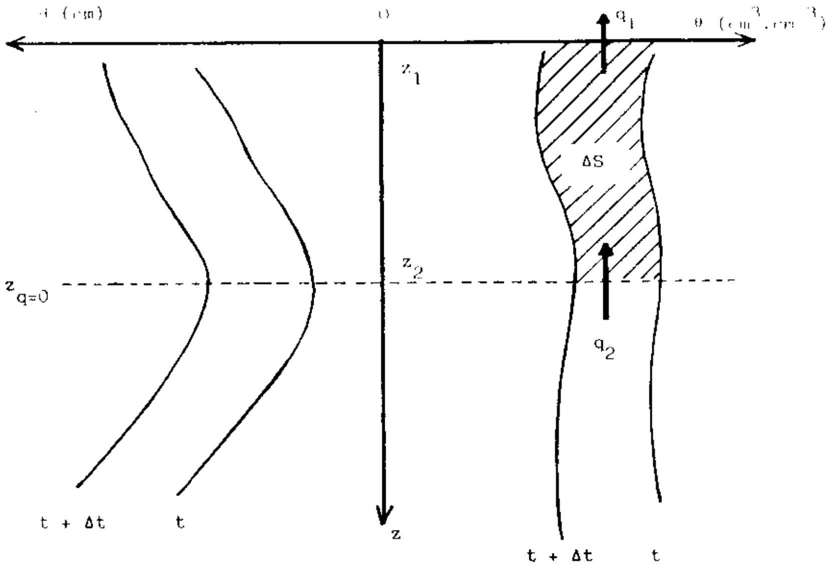


Figure 8. Calculation of evaporation using soil water content and hydraulic head profiles.

If vegetation is present equation (12) is to be written as follows:

$$\frac{\partial \theta}{\partial t} = - \frac{\partial q}{\partial z} - J(z,t) \qquad (16)$$

where J(z,t) is the soil water extraction by the roots per unit volume of soil and per unit of time

7. Water balance under bare and cropped soil.

The field water balance equation of a certain volume of soil during a time period Δt can be written as follows:

$$\left[\Delta S\right]_z^0 = P + I - ET \text{ (or E)} - q_{z_r} - R \qquad (17)$$

where: $\left[\Delta S\right]_z^0$ is the change in soil water storage between soil surface and z cm depth

P is the precipitation

I is the irrigation water applied

ET is the actual evapotranspiration in case of a cropped soil

E is the actual evapotranspiration in case of a bare soil

q_z is the soil water flux at a depth z_r

R is the surface runoff water

All variables, except Et (or E) can be measured in the field (figure 9). The main difficulty in solving the water balance equation is the estimation of the soil water flux component, which will depend on the precise knowledge of the hydraulic properties of the soil profile.

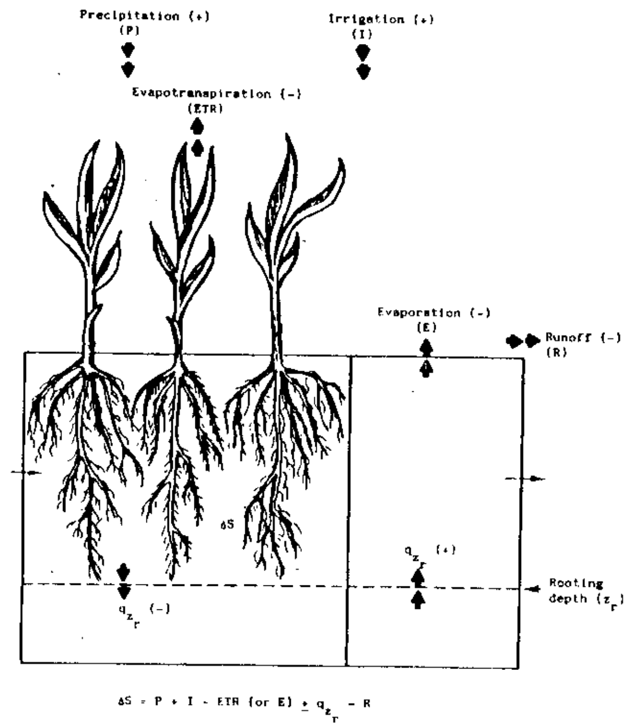


Figure 9. The field water balance equation.

If R equals zero, the measurements of ΔS, P and I are insufficient to determine accurately ET (or E).

One needs, to know the flux q_z. To solve this problem Darcy's law for vertical flow in the unsaturated zone is used:

$$q_z = - K(\theta) \left.\frac{dH}{dz}\right|_z \qquad (18)$$

Knowing the hydraulic conductivity of the soil at a certain soil water content θ and soil depth z, it is possible to estimate the evaporation or crop evapotranspiration according to following equation:

$$ET = P + I \pm R - \left[\Delta S\right]_z^0 + K(\theta) \left.\frac{dH}{dz}\right|_z \Delta t \qquad (19)$$

We shall discuss now different situations occuring under bare as well as under cropped soils.

7.1. Bare soil.

The vertical one-dimensional water flow under bare soil is controlled by following equations:

- continuity equation:

$$\frac{\partial \theta}{\partial t} = - \frac{\partial q}{\partial z} \qquad (20)$$

- Darcy's law:

$$q = - K(\theta) \frac{dH}{dz} \qquad (21)$$

Integration of equation (20) between the soil surface (z = 0) and a well defined depth z gives:

$$q(z,t) - q(0,t) = - \frac{\partial \int_0^z \theta dz}{\partial t}$$

or:

$$q(0,t) = q(z,t) + \frac{\partial \int_0^z \theta dz}{\partial t} \qquad (22)$$

where: $\int_0^z \theta dz$ represents the water storage in the soil profile from the soil surface to depth z at time t and which can be calculated from the soil water content profiles.

q(0,t) = the evaporation flux at time t

q(z,t) = the flux at depth z and time t

Two cases are possible:

7.1.1. Existence of a plane of zero flux in the soil profile.

From the hydraulic head profiles, measured by tensiometers, a plane of zero flux at time t and at depth z_0 can be observed (figure 10).

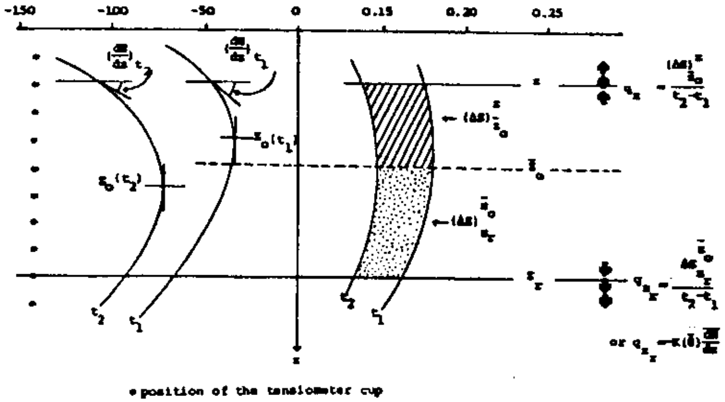


Figure 10. Diagram illustrating the calculation of soil moisture fluxes when a plane of zero flux (z_0) exists in the rooting zone.

The average plane of zero flux during a time interval Δt (being $t_2 - t_1$) is taken as the average depth of the plane of zero flux at the two different times, or

$$\bar{z}_0 = \frac{z_0(t_1) + z_0(t_2)}{2}$$

Consequently $q(\bar{z}_0,t)$ equals zero and equation (22) becomes:

$$q(0,t) = \frac{\partial \int_0^{\bar{z}_0} \theta dz}{\partial t} = \frac{\left[\Delta S\right]_{\bar{z}_0}^0}{\Delta t} \qquad (23)$$

The evaporation is than given by:

$$E = P + I + R - \left[\Delta S\right]_{\bar{z}_0}^{0}$$

The variation of the water storage between the soil surface and the mean plane of zero flux gives directly the evaporation if P, I and R are zero.

The drainage flux at a depth z_r below the plane of zero flux can be calculated in two ways:

1. by means of the plane of zero flux since the drainage will be equal to the change in storage between $\bar{z}_0$ and z_r during the time interval Δt, or:

$$q_{z_r} = \frac{\partial \int_{\bar{z}_0}^{z_r} \theta dz}{\partial t} = \frac{\left[\Delta S\right]_{z_r}^{\bar{z}_0}}{\Delta t}$$

2. by means of Darcy's equation, whereby:

$$q_{z_r} = - K(\bar{\theta}) \left.\overline{\frac{dH}{dz}}\right|_{z_r}$$

The localization of the plane of zero flux is not always very easy. The precision of the localization can be improved by increasing the number of tensiometers and the subjectivity introduced by the reader of the hydraulic head profiles by using spline functions.

The calculation of the soil water storage variation can also be inaccurate when this variation in soil water content is very low and so differs little from the statistical errors proper to the neutron probe measurements. In order to increase the precision, the time interval could be increased. To increase also the accuracy of the calculation it is desirable to measure the soil water content as near as possible to the soil surface. Therefore a surface neutron probe is necessary or a special calibration for the surface layer with the normal depth probe.

7.1.2. Absence of a plane of zero flux in the soil profile.

From the hydraulic head profiles the plane of zero flux cannot be located (figures 11 and 12). The absence of this distinct plane of zero flux in the soil profile indicates that one of the two extreme conditions i.e.continuous drainage or continuous upward water movement occurs.

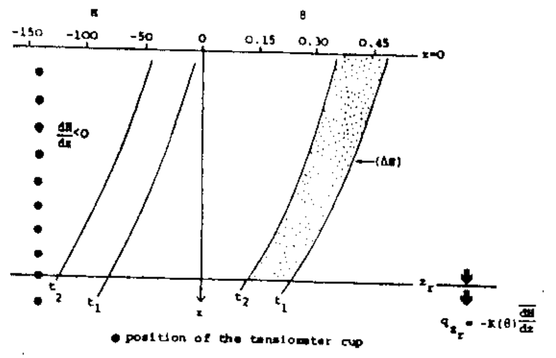


Figure 11. Diagram illustrating the calculation of soil moisture flux under continuous drainage condition.

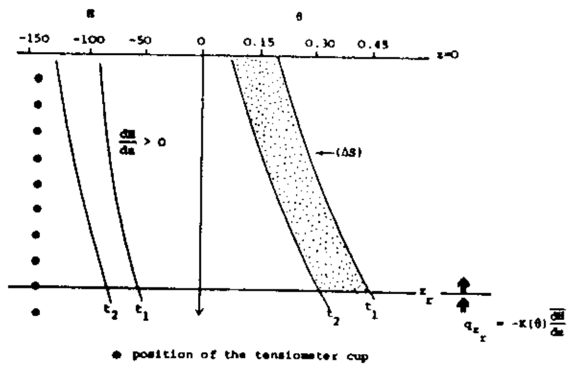


Figure 12. Diagram illustrating the calculation of soil moisture flux under continuous evapotranspiration (or evaporation).

To determine the evaporation one has to estimate, using equation (22), the flux at a depth z_r:

$$q(0,t) = q(z_r,t) + \frac{\partial \int_0^{z_r} \theta dz}{\partial t}$$

where:

$$\frac{\partial \int_0^{z_r} \theta\, dz}{\partial t} = \frac{[\Delta S]_{z_r}^{0}}{\Delta t}$$

and:

$$q(z_r,t) = - K(\bar{\theta}) \left.\overline{\frac{dH}{dz}}\right|_{z_r}$$

The latter relation assumes the knowledge of the hydraulic conductivity related to an average moisture content $\bar{\theta}$ and $\left.\overline{\frac{dH}{dz}}\right|_{z_r}$ being the average hydraulic gradient during the period considered. The hydraulic gradients are derived from hydraulic heads obtained from tensiometers situated just above and below z_r. During continuous drainage the hydraulic gradient remains negative over the whole profile, while with continuous upward flow the hydraulic gradient is positive.

Using equation (19), the evaporaton can easily be calculated:

$$E = P + I \pm R - \Delta\int_0^{z_r} \theta dz - (-K(\bar{\theta}) \left.\overline{\frac{dH}{dz}}\right|_{z_r}) \Delta t$$

7.2. Cropped soil.

In a soil with a vegetation the continuity equation is written as follows:

$$\frac{\partial \theta}{\partial t} = - \frac{\partial q}{\partial z} - J(z,t) \qquad (24)$$

where $J(z,t)$ is the soil water extraction by the roots per unit volume of soil and per unit time.

The evaporation exchanges between soil and atmosphere are due to:

- the evaporation through the soil surface:

$$q(0,t)$$

- the transpiration of the plant:

$$T(t) = - \int_0^{z_{r(t)}} J(z,t)\, dz$$

where $z_{r(t)}$ is the maximum depth of the root zone at well defined time.

Integration of equation (24) between the soil surface ($z = 0$) and a depth z gives:

$$q(0,t) = q(z,t) + \frac{\partial \int_0^{z} \theta dz}{\partial t} + \int_0^{z} J dz$$

which can also be written as:

$$q(0,t) = q(z,t) + \frac{\partial \int_0^{z} \theta dz}{\partial t} + \int_0^{z_{r(t)}} J\, dz + \int_{z_{r(t)}}^{z} J dz$$

so that:

$$q(0,t) + T(t) = q(z,t) + \frac{\partial}{\partial t} \int_0^{z} \theta dz + \int_{z_{r(t)}}^{z} J dz \qquad (25)$$

Two cases are possible:

7.2.1. Existence of a plane of zero flux in the soil profile.

a. If there is a mean plane of zero flux ($\bar{z}_0$) located under the maximal root depth (figure 13) equation (25) becomes:

$$q(0,t) + Tr(t) = \frac{\partial}{\partial t} \int_0^{\bar{z}_0} \theta dz = \frac{[\Delta S]_{\bar{z}_0}^{0}}{\Delta t} \qquad (26)$$

The crop evapotranspiration can be estimated as follows:

$$ET_{cr} = P + I \pm R - [\Delta S]_{\bar{z}_0}^{0}$$

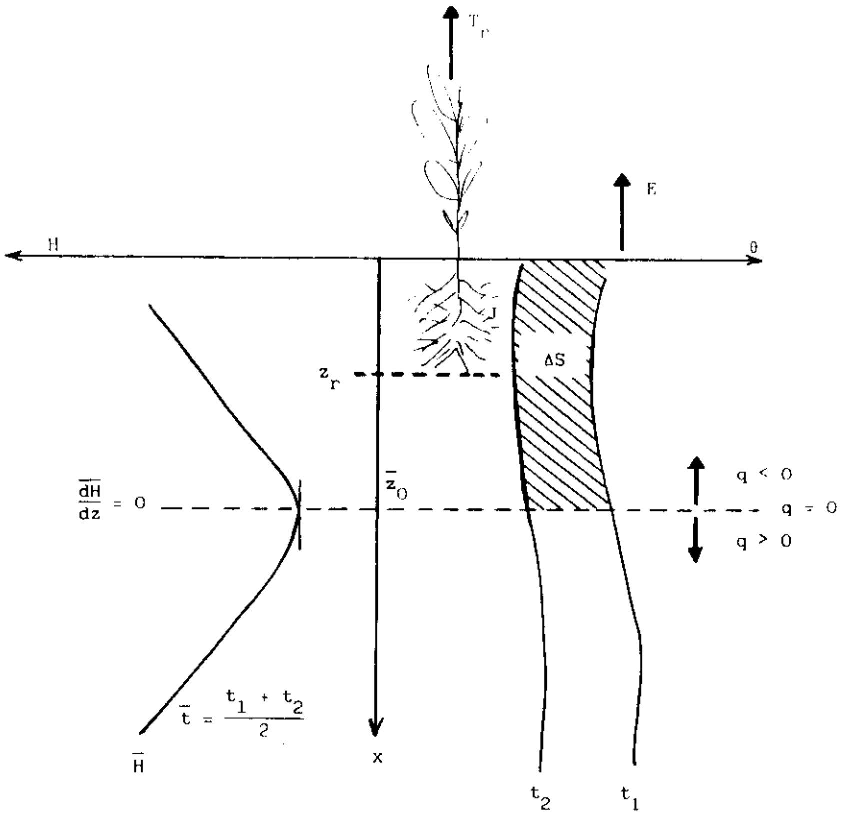


Figure 13. Diagram illustrating the calculation of crop evapotranspiration when the plane of zero flux is located under the maximal root depth ($z_0 > z_r$).

b. If the mean plane of zero flux ($\bar{z}_0$) is located <u>within</u> the root zone (figure 14) equation (25) shows that the term

$$\int_{z_{r(t)}}^{z} J(z,t)$$

is unknown and couldn't be measured by a simple method. One has to consider a plane $z_n > z_{r(t)}$ and the integration of equation (24) gives:

$$q(0,t) + Tr(t) = q(z_n,t) + \frac{\partial}{\partial t}\int_0^{z_n} \theta dz \qquad (27)$$

where:

$$q(z_n,t) = - K(\bar{\theta}) \left.\frac{d\bar{H}}{dz}\right|_{z_n}$$

and:

$$\frac{\partial}{\partial t}\int_0^{z_n} \theta dz = \frac{[\Delta S]_{z_n}^{0}}{\Delta t}$$

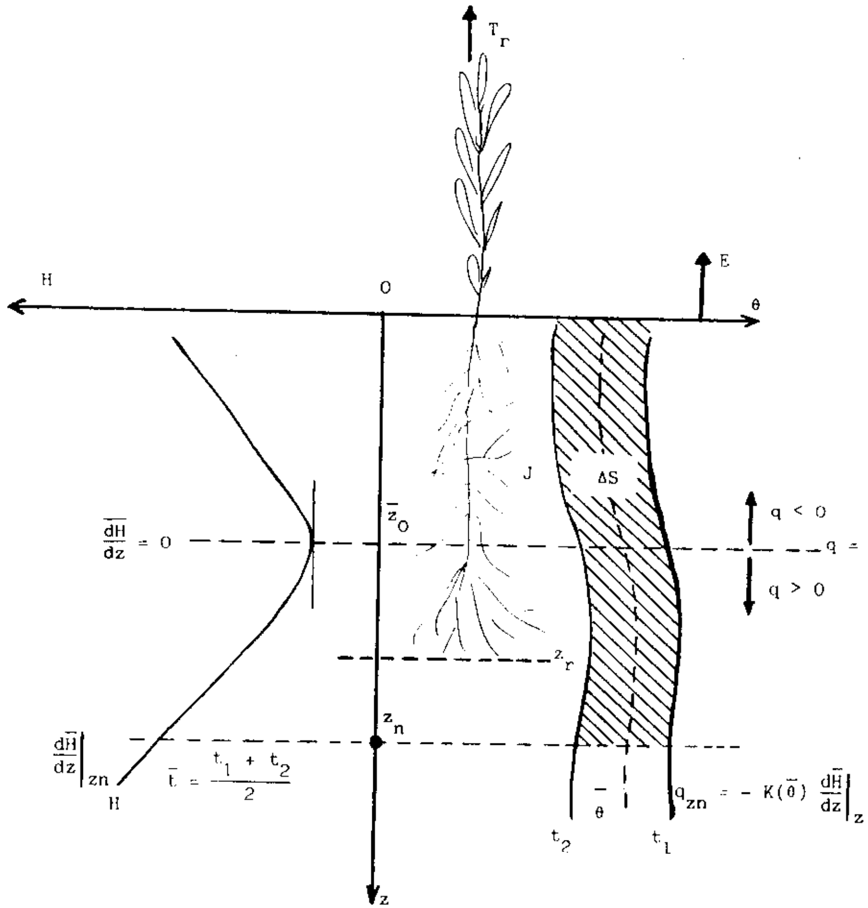


Figure 14. Diagram illustrating the calculation of the crop evapotranspiration when the plane of zero flux is located within the rootzone.

This means that in order to estimate ET_{cr} using equation (19) the hydraulic conductivity related to an average soil water content $\bar{\theta}$ and

$\left.\frac{d\bar{H}}{dz}\right|_{z_n}$, being the mean hydraulic gradient, the change of soil water storage, P, I and R during the period under investigation, have to be known

$$ET_{cr} = P + I \pm R - \Delta \int_0^{z_n} \theta dz - \left(- K(\bar{\theta}) \left.\frac{d\bar{H}}{dz}\right|_{z_n}\right) \Delta t$$

7.2.2. Absence of a plane of zero flux in the profile (figure 15).

If no plane of zero flux is present the situation is the same as when a plane exists but within the root zone. One has to consider a plane z_n located under the root zone (z_r). The ET_{cr} can be estimated in the following way:

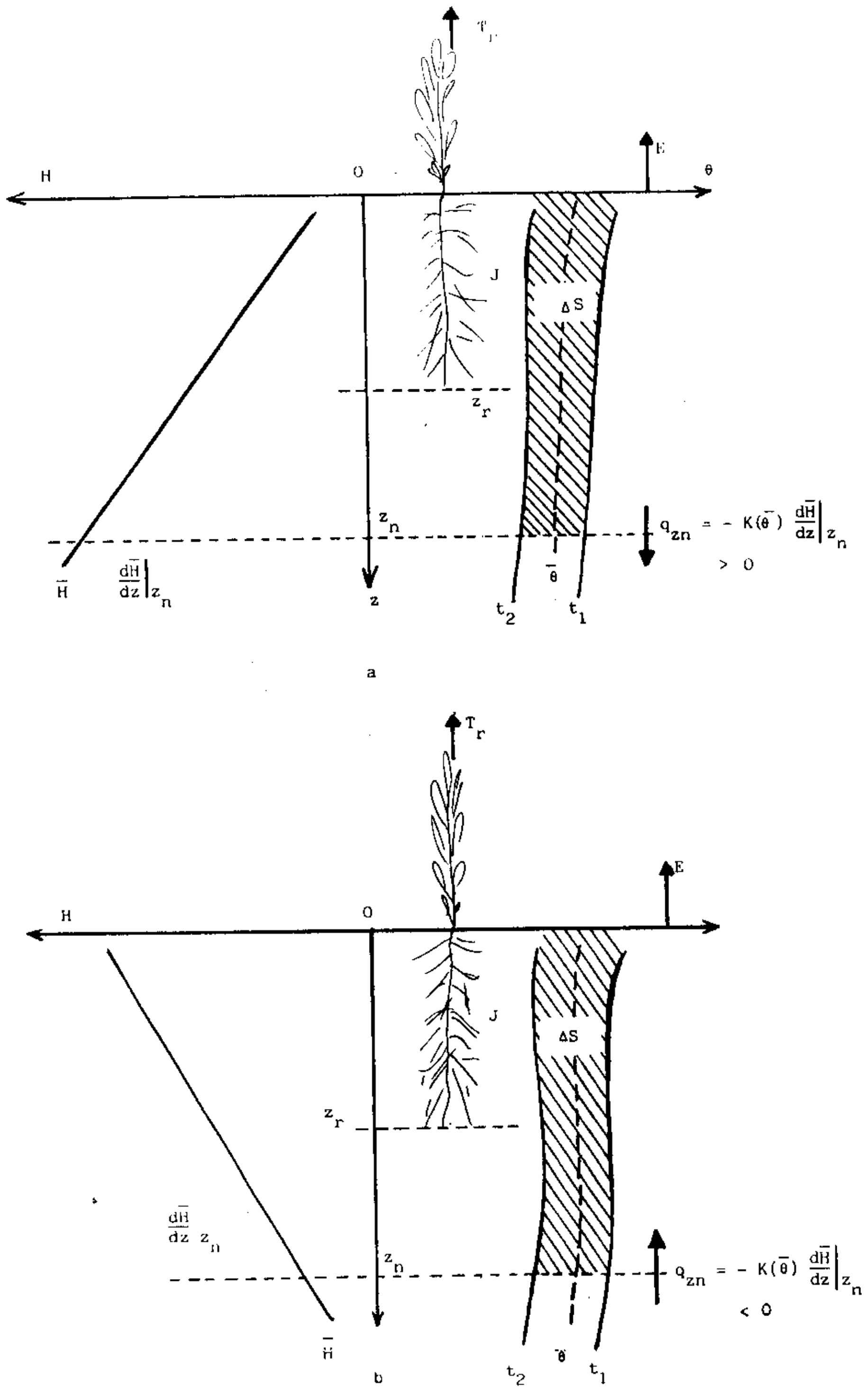
$$ET_{cr} = P + I \pm R - \Delta \int_0^{z_n} \theta dz - \left(- K(\bar{\theta}) \left.\frac{d\bar{H}}{dz}\right|_{z_n}\right) \Delta t$$

if:

$\left.\frac{d\bar{H}}{dz}\right|_{z_n} < 0$ $\quad q_{z_n} > 0$ which means downward flux through level z_n (figure 15a)

if:

$\left.\frac{d\bar{H}}{dz}\right|_{z_n} > 0$ $\quad q_{z_n} < 0$ which means upward flux through level z_n (figure 15b)



COMPUTATION OF SOIL WATER FLUX

Dr. ir. H. Verplancke

The flow of moisture through the soil can be calculated by Darcy's equation

$$q = - K \frac{\delta h}{\delta z}$$

$$h = \psi + z$$

given measurements of matric head ψ at different depths z in the soil and knowledge of the relationship between hydraulic conductivity K and ψ.

Fig. 1 shows profiles through time of moisture head measured by tensiometers located at soil depths 0.8 m and 1.8 m. The total head h is found by adding the measured matric potential head ψ to the depth z at which it was measured. These are both **negative** : z because it is taken as positive upward with 0 at the soil surface, and ψ because it is a suction force which resists flow of moisture away from the location.

Example

Calculate the soil moisture flux q (cm/day) between depths 0.8 m and 1.8 m in the soil. The data for the total head at these depths are given at weekly time intervals in columns 2 and 3 of Table 1. For this soil the relationship between the hydraulic conductivity and matric head is

$$K = 250\,(-\psi)^{-2.11}$$

where K is in centimeters per day and ψ is in centimeters.

Solution

Darcy's equation is rewritten for an average flux q_{12} between measurement points 1 and 2 as

$$q_{12} = - K \frac{h_1 - h_2}{z_1 - z_2}$$

In this case, measurement point 1 is at 0.8 m and point 2 at 1.8 m, so z_1 = -80 cm, z_2 = -180 cm, and $z_1 - z_2$ = -80 - (-180) = 100 cm.

The matric head at each depth is ψ = h - z. For example, for week 1 at 0.8 m, h_1 = -145, so
$\psi_1 = h_1 - z_1$ = -145 - (-80) = -65 cm, and ψ_2 = -230 - (-180) = -50 cm, as shown in columns 4 and 5 of the table.

The hydraulic conductivity K varies with ψ, so the value corresponding to the average of the ψ values at 0.8 m and 1.8 m is used.

For week 1, the average matric head is ψ_{av} = [(-50) + (-65)]/2 = -57.5 cm; and the corresponding hydraulic conductivity is K = $250(-\psi_{av})^{-2.11}$ = 0.0484 cm/day, as hown in column 6.

The head difference $h_1 - h_2$ = (-145)-(-230) = 85 cm.

The soil moisture flux between 0.8 and 1.8 m for week 1 is

$$q = - K \frac{h_1 - h_2}{z_1 - z_2}$$

$$= - 0.0484 \frac{85}{100}$$

$$= - 0.0412 \text{ cm/day}$$

as shown in column 8. The flux is negative because the moisture is flowing downward.

The Darcy flux has dimensions [L/T] because it is a flow per unit area of porous medium. If the flux is passing through a horizontal plane of area A = 1 m^2, then the volumetric flow rate in week 1 is

$$Q = q\,A$$

$$= - 0.0412 \text{ cm/day} \times 1 \text{ m}^2$$

$$= - 4.12 \times 10^{-4} \text{ m}^3\text{/day}$$

$$= - 0.412 \text{ liters/day}$$

Table 1 shows the flux calculated for all time periods, and the computed values of q, K and
$h_1 - h_2$ are plotted in Fig. 2. In all cases the head at 0.8 m is greater than that at 1.8 m so moisture is always being driven downward between these two depths in this example. It can be seen that the flux reaches a maximum in week 6 and diminishes thereafter, because both the head difference and the hydraulic conductivity diminish as the soil dries out. The figure shows the importance of the variability of the unsaturated hydraulic conductivity K in affecting the moisture flux q. As the soil becomes wetter, its hydraulic conductivity increases, because there are more continuous fluid-filled pathways through which the flow can move.

The complete picture of rainfall on the soil and the soil moisture head at various depths is presented in Fig.3. Rainfall during April and May flows down into the soil, reducing the soil matric head, but later the soil dries out by evapotranspiration, causing the matric head to increase again. The head profile at the shallowest depth (0.4 m) shows the greatest variability and the fact that it falls below the profile at 0.8 m from the beginning of June onwards shows that during this period, soil moisture flows upwards between these two depths to supply moisture for evapotranspiration.

Fig. 1 Profiles of total soil moisture head through time

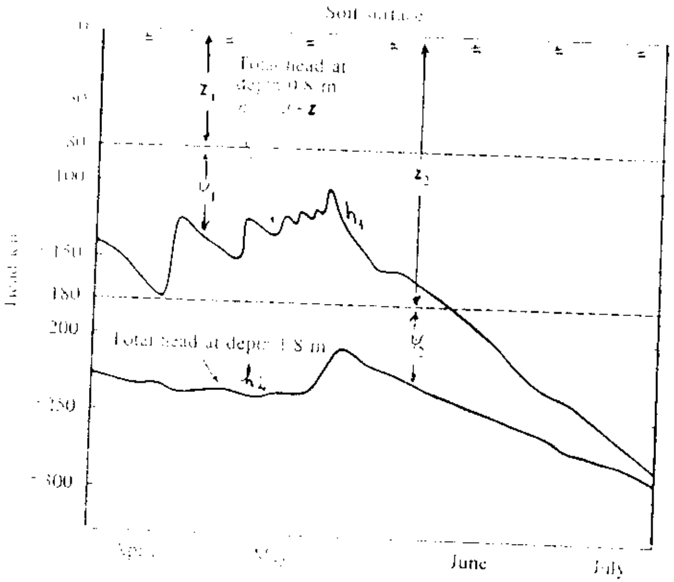
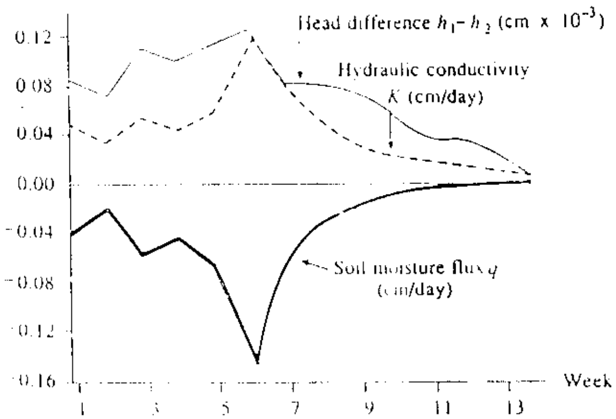


Fig. 2 Computation of the soil moisture flux



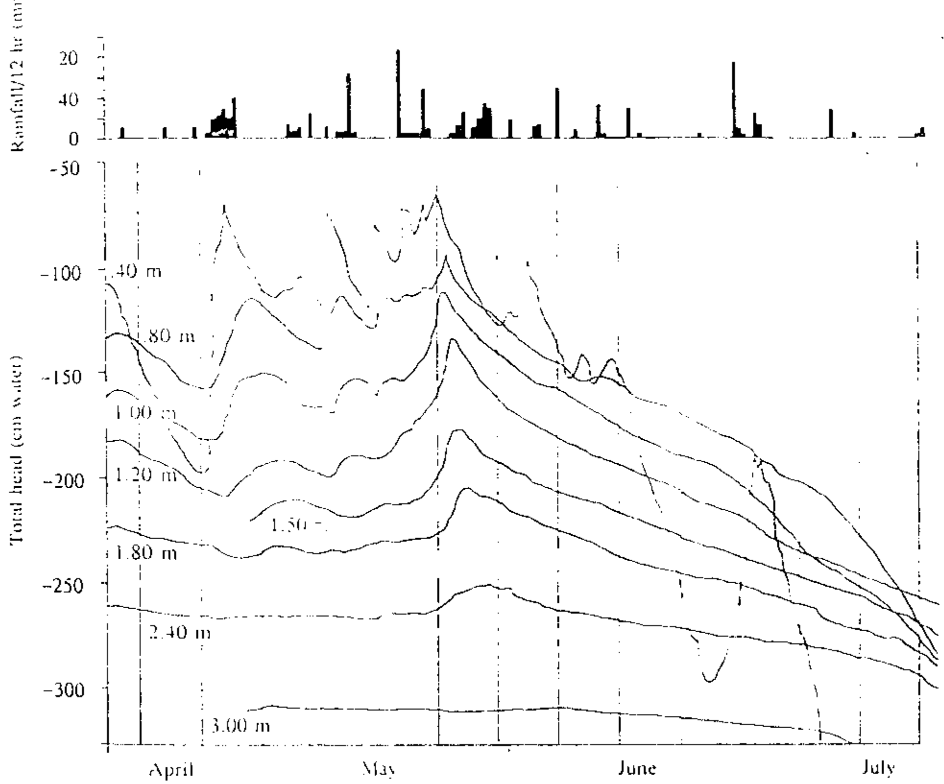


Fig. 3 Variation through time of total soil water head H at various depths in a loam soil. The infiltration of rainfall reduces the matric head which increases again and evaportranspiration dries out the soil. Matric head is the difference between the total head and the value for elevation shown on each line.

Table 1

Computation of soil moisture flux between 0.8 m and 1.8 m depth

1 Week	2 Total head h_1 at 0.8 m (cm)	3 Total head h_2 at 1.8 m (cm)	4 Suction head ψ_1 at 0.8 m (cm)	5 Suction ψ_2 at 1.8 m (cm)	6 Unsaturated hydraulic conductivity K (cm/day)	7 Head difference $h_1 - h_2$ (cm)	8 Moisture flux q (cm/day)
1	−145	−230	−65	−50	0.0484	85	−0.0412
2	−165	−235	−55	−55	0.0320	70	−0.0224
3	−130	−240	−50	−60	0.0532	110	−0.0585
4	−140	−240	−60	−60	0.0443	100	−0.0443
5	−125	−240	−45	−60	0.0587	115	−0.0675
6	−105	−230	−25	−50	0.1193	125	−0.1492
7	−135	−215	−55	−35	0.0812	80	−0.0650
8	−150	−230	−70	−50	0.0443	80	−0.0354
9	−165	−240	−85	−60	0.0297	75	−0.0223
10	−190	−245	−110	−65	0.0200	55	−0.0110
11	−220	−255	−140	−75	0.0129	35	−0.0045
12	−230	−265	−150	−85	0.0107	35	−0.0038
13	−255	−275	−175	−95	0.0080	20	−0.0015
14	−280	−285	−200	−105	0.0062	5	−0.0003