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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE CENTRATOM TRIESTE

SMR/402 - 19

COLLEGE ON SOIL PHYSICS
9 - 27 October 1989

"Why is Water Important ?"

K. REICHARDT
Universidad de Sao Paulo
Centro de Energia Nuclear Na Agricultura (CENA)
Departamento de Fisica del Solos
Piracicaba
Brazil

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COLLEGE IN SOIL PHYSICS
ICTP - TRIESTE

KLAUS REICHARDT
BRAZIL

WHY IS SOIL WATER IMPORTANT?

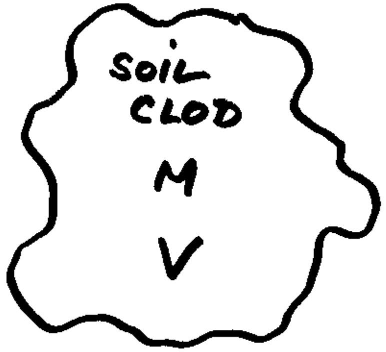
SPAC

$$[A, B \;\text{SOIL}] \rightleftarrows [B, C \;\text{PLANT}] \rightleftarrows [C, D \;\text{ATMOSPHERE}]$$

SOIL		PLANT		ATMOSPHERE
- TEXTURE - BULK DENSITY - HYDRAULIC CONDUCTIVITY - SOIL WATER CONTENT - SOIL WATER ENERGY STATUS ⋮	ROOT ACTIVITY LENGTH DEPTH ⋮	- PLANT ANATOMY, HEALTH - XILEM - STOMATA ⋮	LEAF ACTIVITY AREA SHAPE ⋮	- RELATIVE HUMIDITY - SUNSHINE - WIND ⋮

NUTRIENT UPTAKE

DIFFUSION; MASS FLOW; ROOT INTERSEPTION



$$M = m_s + m_l + m_g$$

$$V = v_s + v_l + v_g$$

$$u = \frac{m_l}{m_s} \quad g \cdot g^{-1}, \%$$

$$\theta = \frac{v_l}{V} \quad cm^3 \cdot cm^{-3}, \%$$

$$BD_d = \frac{m_s}{V} \quad g \cdot cm^{-3}$$

$$BD_w = \frac{m_s + m_l}{V} \quad g \cdot cm^{-3}$$

$$PD = \frac{m_s}{v_s} \quad g \cdot cm^{-3}$$

$$\alpha = \frac{v_l + v_g}{V} = \frac{V - v_s}{V} \quad cm^3 \cdot cm^{-3}, \%$$

$$\boxed{\theta = BD_d \times u}$$

$$\boxed{\alpha = 1 - \frac{BD_d}{PD}}$$

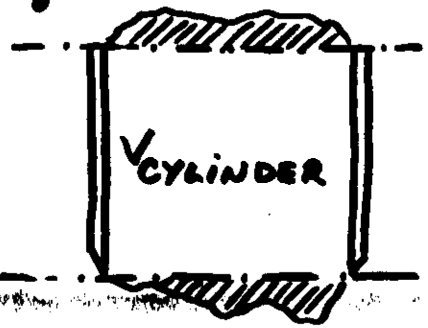
CLASSICAL METHODS

2.1. DIRECT SAMPLING

- AUGER SAMPLES

$$u = \frac{W_{ws} - W_{ds}}{W_{ds}}$$

- CYLINDER SAMPLES

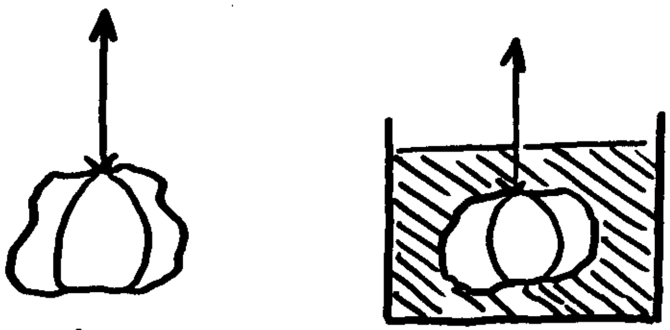


$$\theta = \frac{W_{ws} - W_{ds}}{V_{CYLINDER}}$$

$$BD_d = \frac{W_{ds}}{V_{CYLINDER}}$$

$$BD_w = \frac{W_{ws}}{V_{CYLINDER}}$$

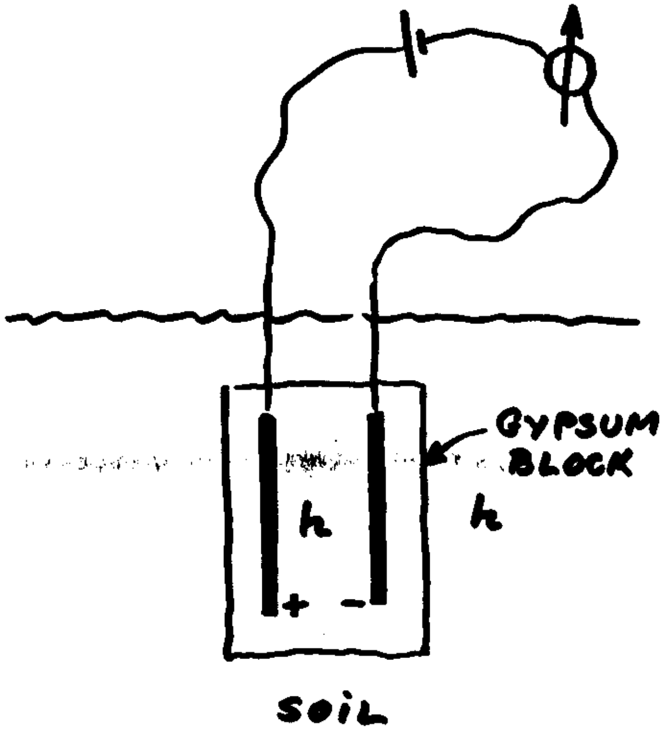
- PARAFIN SAMPLES



$W_{in\ AIR}$ $\qquad$ $W_{in\ WATER}$

2.2. SENSORS

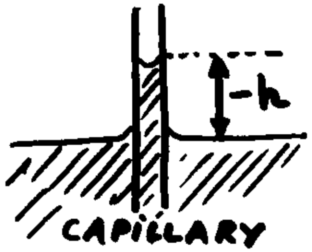
- POROUS RESISTANCE BLOCKS

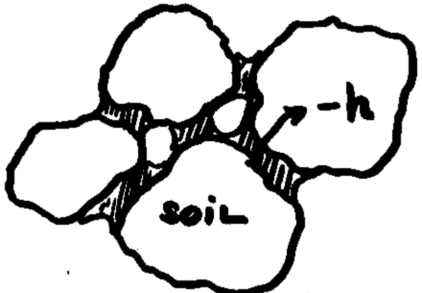


ENERGY STATUS OF WATER (HYDRAULIC HEADS)

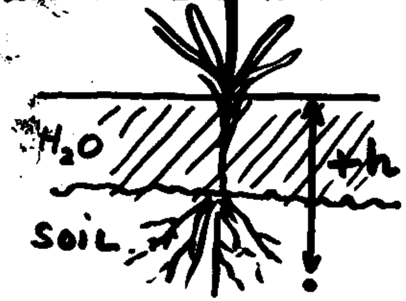
1. PRESSURE HEAD h
   a) FREE WATER $h=0$

   b) WATER UNDER TENSION, NEGATIVE PRESSURE OR SUCTION. UNSATURATED SOIL

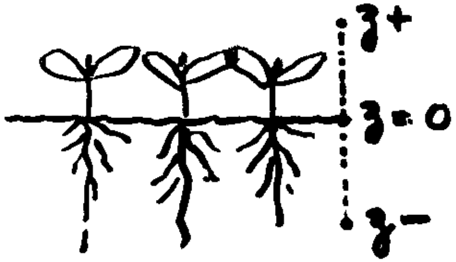




   c) WATER UNDER PRESSURE. SATURATED SOIL



2. GRAVITATIONAL HEAD z

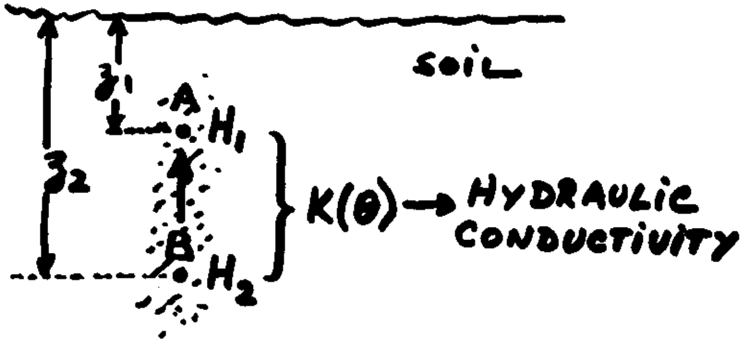


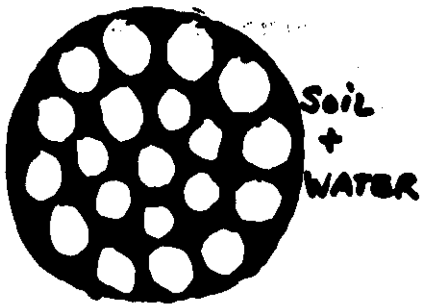
3. TOTAL HYDRAULIC HEAD

$$\boxed{H = h + z}$$

TRANSPORT OF WATER

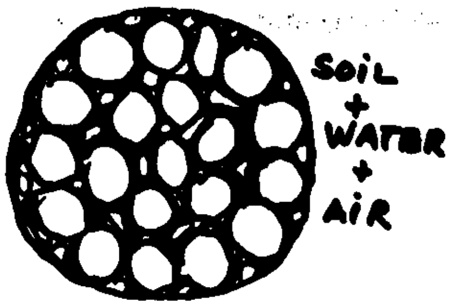
$$q = -K(\theta)\frac{\partial H}{\partial z}$$





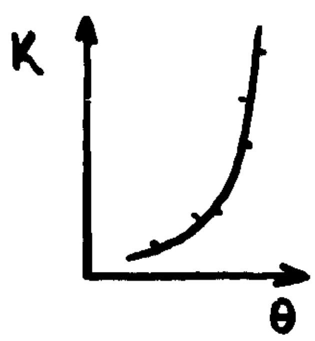
CROSS SECTION OF A SOIL WITH $\alpha = 0.5$ and $\theta = 0.5$

50% OF AREA AVAILABLE FOR FLOW

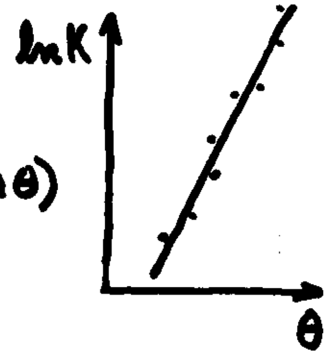


CROSS SECTION OF A SOIL WITH $\alpha = 0.5$ and $\theta = 0.25$

25% OF AREA AVAILABLE FOR FLOW



$$K = K_0 \exp(a\theta)$$

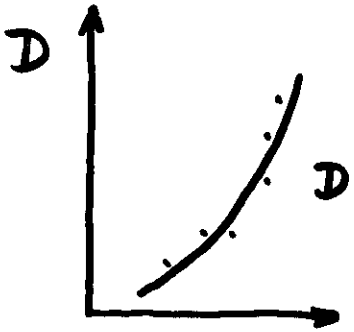


$$q = -K(\theta)\frac{\partial H}{\partial x}$$

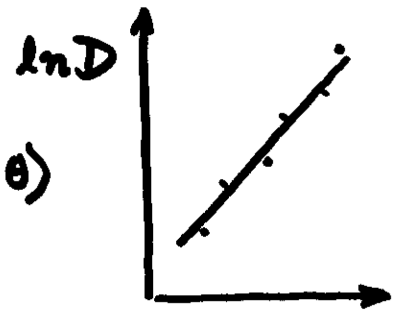
$$\Big\downarrow \quad \theta = \theta(h)$$

$$q = -D(\theta)\frac{\partial \theta}{\partial x}$$

$D(\theta)$ = SOIL WATER DIFFUSIVITY



$$D = D_0 \exp(b\theta)$$



MEASUREMENT OF WATER FLUXES

- steady-state

$$q = -K(\theta)\frac{\partial H}{\partial x}$$

- Transient-state

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial x} \quad \text{CONTINUITY}$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[K\frac{\partial H}{\partial x}\right]$$

$$\int_0^L \frac{\partial \theta}{\partial t} = K\frac{\partial H}{\partial x} = q$$

MEASUREMENT OF $\partial\theta/\partial t$ AT DIFFERENT TIMES AND POSITIONS IS OF EXTREME IMPORTANCE. WE NEED NON-DESTRUCTIVE SAMPLING $\Longrightarrow$

NUCLEAR METHODS

NUCLEAR METHODS

3.1. USE OF TRACERS

3.2 GAMMA-RADIATION ATTENUATION

3.3 GAMMA-RADIATION SCATTERING

3.4. NEUTRON MODERATION

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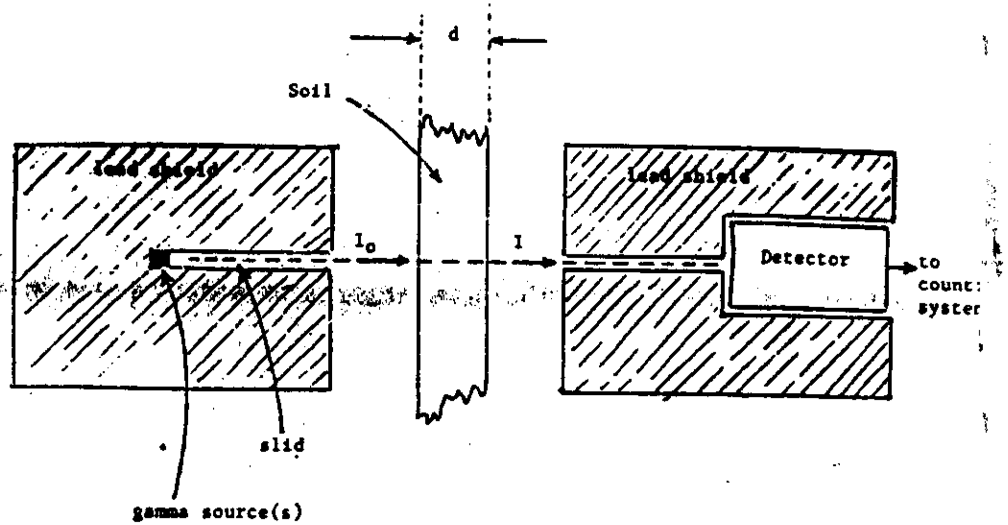
^{3}H $^{2}H/^{1}H$ $^{18}O/^{16}O$

$\underbrace{\phantom{^{2}H/^{1}H \quad ^{18}O/^{16}O}}_{\text{NATURAL AND ARTIFICIAL}}$

$^{36}Cl, \ldots$

GAMMA ATTENUATION



$$I = I_o \, e^{-d(\mu_s B D_d + \mu_w \theta)}$$

LABORATORY

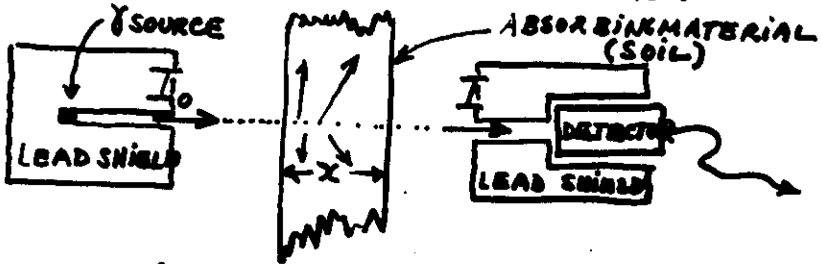
GAMMA DENSITY PROBE

GAMMA RADIATION (best range 0.1 – 1.0 MeV)

Sources Used $^{137}Cs \rightarrow 0.661$ MeV
$^{60}Co \rightarrow 1.17$ and 1.32 MeV
$^{241}Am \rightarrow 0.06$ MeV.

Interactions with Matter (PAGE 14–16 MANUAL)

1. PHOTO ELECTRIC ABSORBTION
2. COMPTON SCATTERING
3. PAIR PRODUCTION ABSORBTION



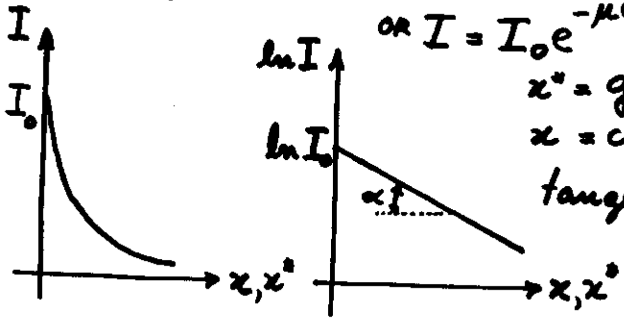
$I_0 > I$ { Because of effects 1. 2. 3. $\rightarrow I = I_0 e^{-\mu x^*}$ ..... eq. (10)

OR $I = I_0 e^{-\mu d x}$

$x^* = g \cdot cm^{-2}$

$x = cm$ $\quad d = g \cdot cm^{-3}$

tangent of $\alpha = \mu$



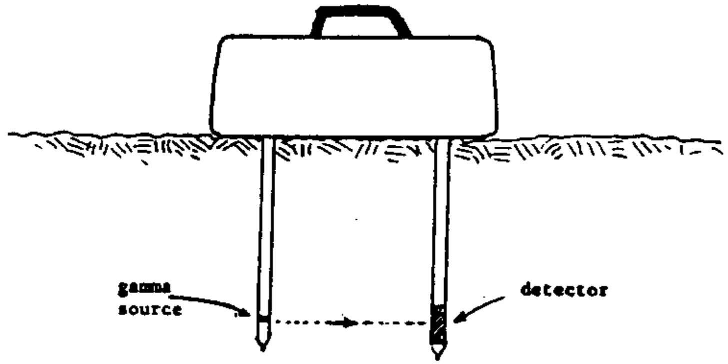
μ = average total absorbtion coefficient for soil and water (linear)

$$I = I_0 \left(\frac{1}{2}\right)^{x/x_{1/2}}$$

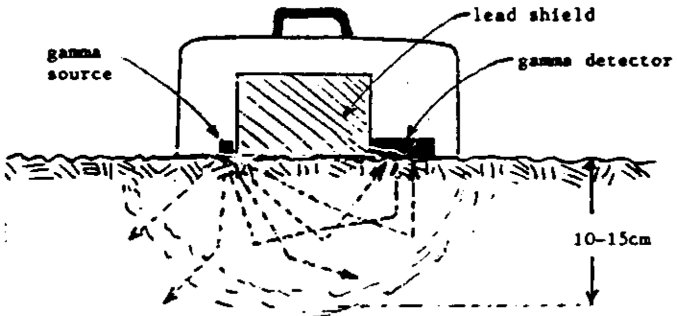
$X_{1/2}$ = thickness that reduces beam intensity by ½

GAMMA FIELD PROBES

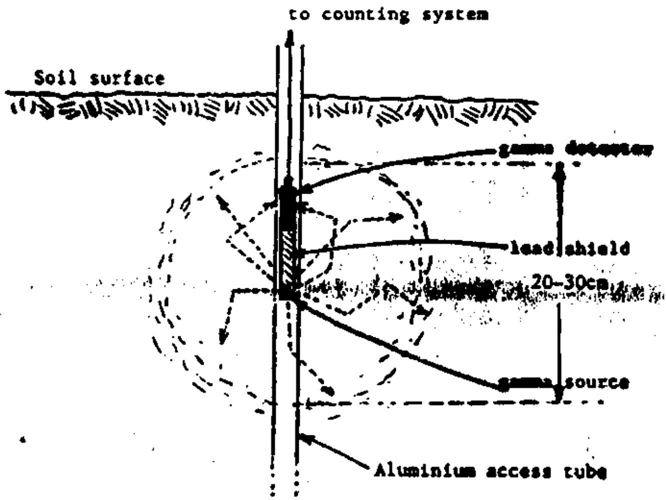
a) ATTENUATION



b) SCATTERING



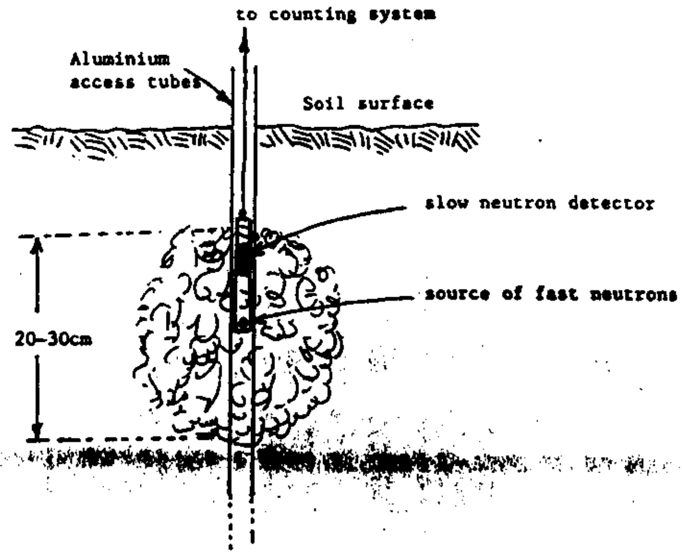
GAMMA BACKSCATTERING



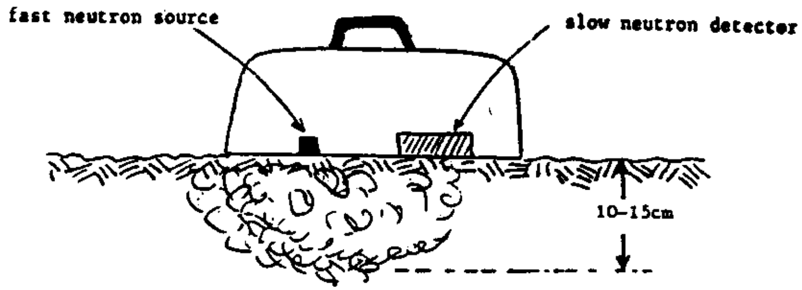
DEPTH FIELD PROBE

NEUTRON PROBES

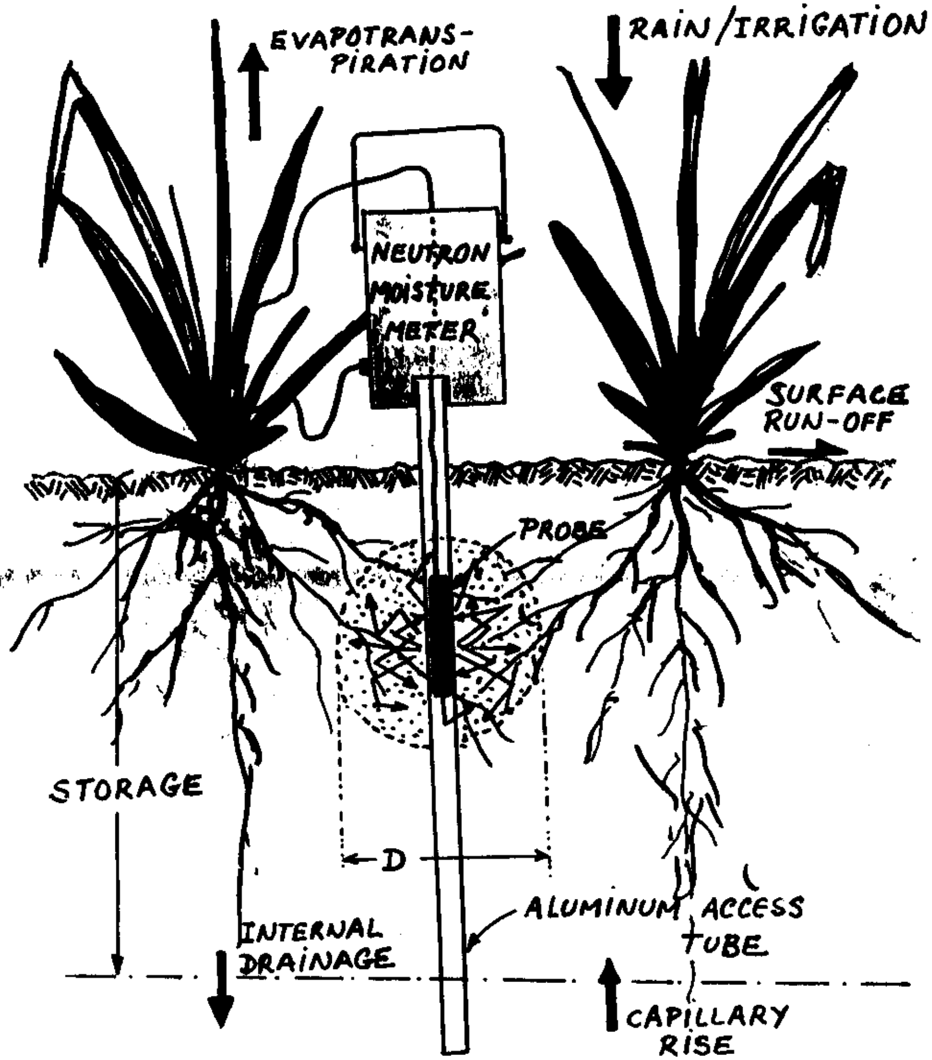
a) DEPTH



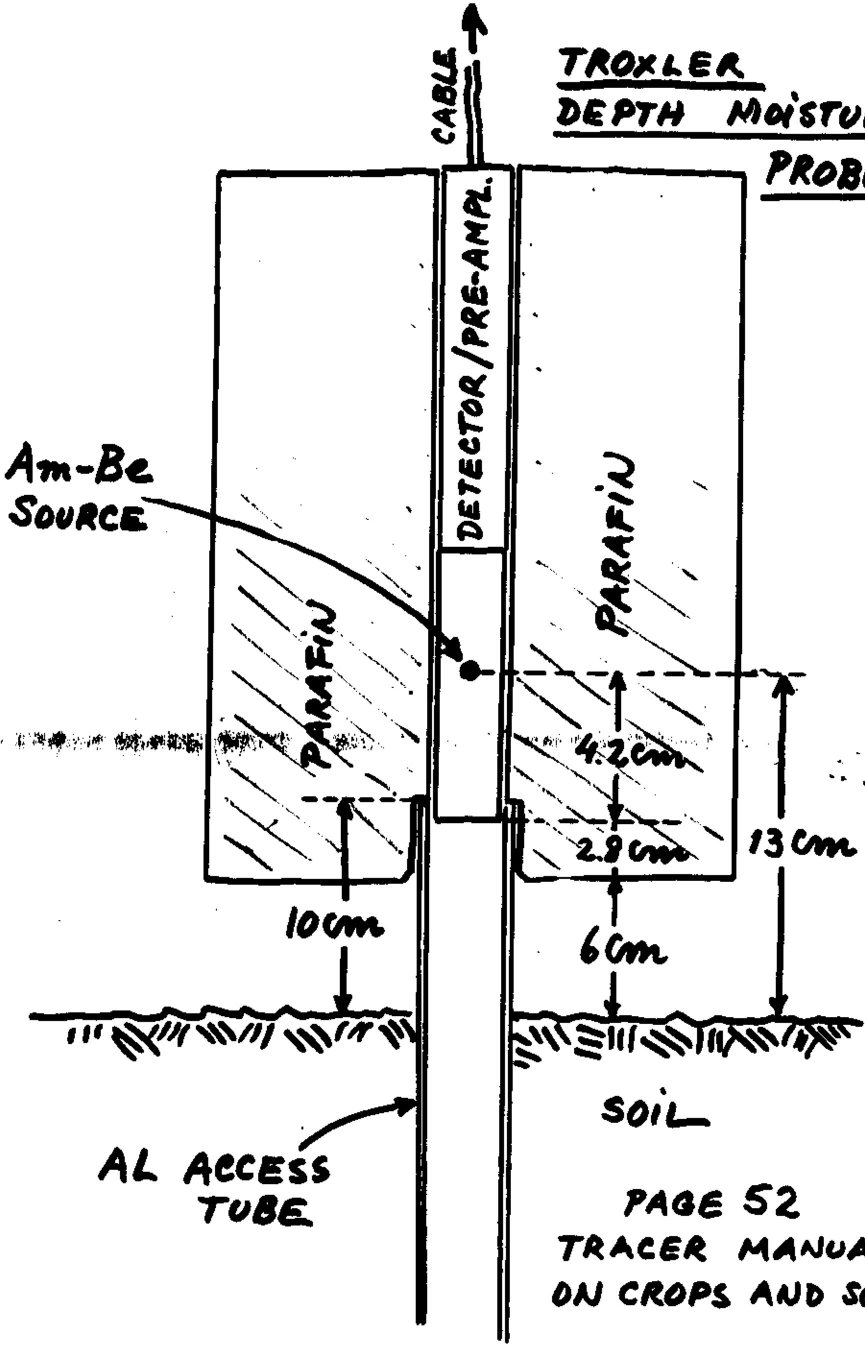
b) SURFACE



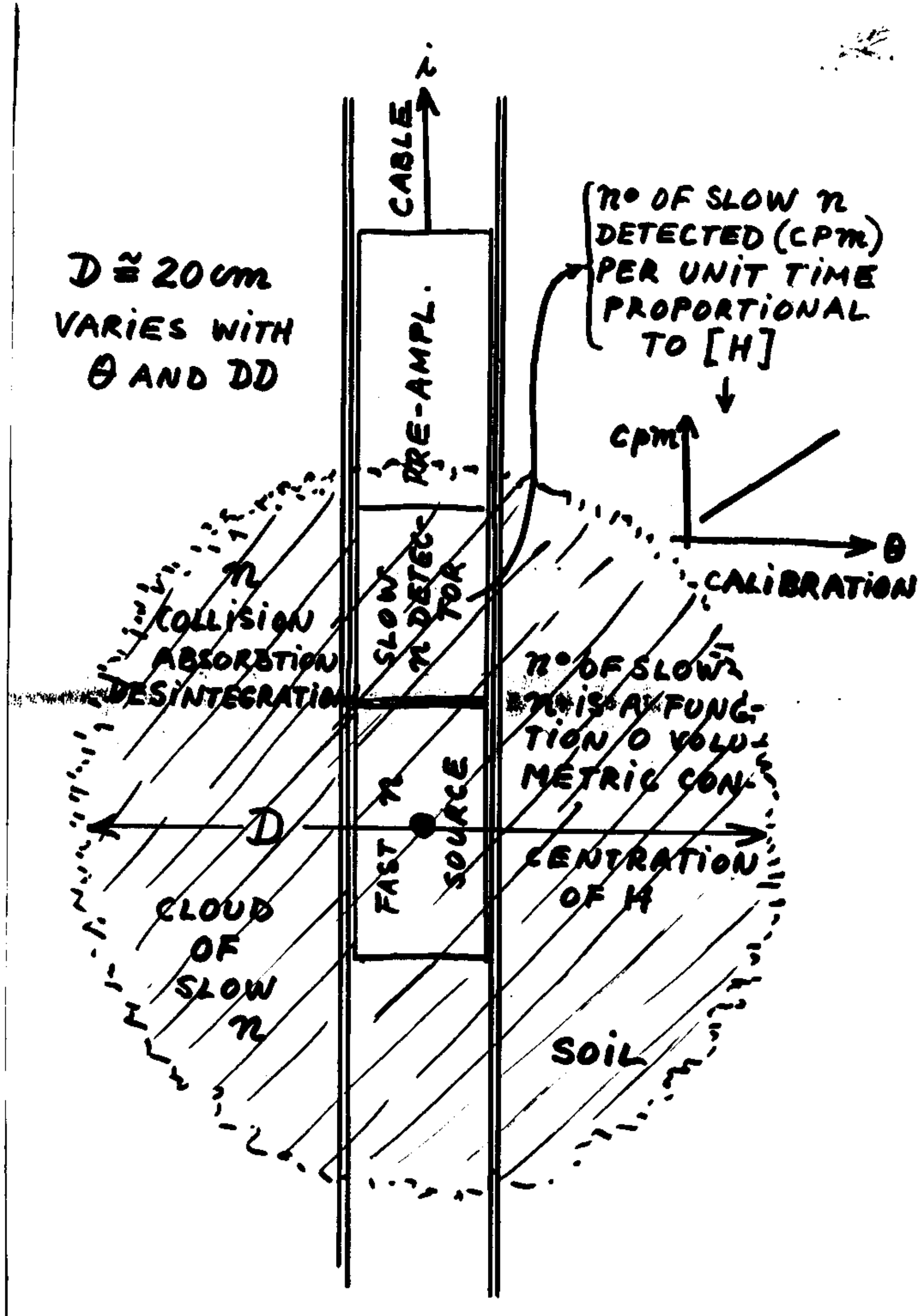
WATER BALANCE STUDIES



TROXLER DEPTH MOISTURE PROBE.



PAGE 52
TRACER MANUAL ON CROPS AND SOILS



NEUTRON SOURCES:

Ex: Am-Be (α, n) REACTION

Am is AN α EMMITTER

$${}^{4}_{2}\alpha + {}^{9}_{4}Be \rightarrow \left[{}^{13}_{6}C\right] \rightarrow {}^{12}_{6}C + \underbrace{{}^{1}_{0}n}_{\text{FAST NEUTRON}}$$

INTERACTION OF NEUTRONS WITH MATTER:

i) elastic collision
ii) absorbtion
iii) desintegration

NEUTRON THERMALIZATION
or MODERATION
or SLOWING-DOWN

$$E_{fast} \longrightarrow E_{thermal}$$

NEUTRON ENERGIES

HIGH ENERGY	$E > 10$ MeV
FAST	$10\,MeV > E > 10\,KeV$
INTERMEDIATE	$10\,KeV > E > 100\,eV$
SLOW	$100\,eV > E > 0.03\,eV$
EPITHERMAL	$E \cong 1\,eV$
THERMAL	$E = 0.025\,eV$

NEUTRON SLOWING DOWN

ELEMENT	AVERAGE N° OF COLLISIONS REQUIRED FOR THERMALIZATION OF 2 MeV NEUTRONS
H	18.2
Li	69.3
B	104.5
C	115.4
O	152
Al	251
Si	262
Ca	371
U	2169

SLOW NEUTRON DETECTORS

1. BORON TRI-FLUORIDE: SIMILAR TO A GM TUBE WITH SOME BF_3 GAS

$$^{10}_{5}B + ^{1}_{0}n \rightarrow ^{7}_{3}Li + ^{4}_{2}\alpha \qquad (n,\alpha) \text{ REACTION}$$

2. HELIUM-3 DETECTORS

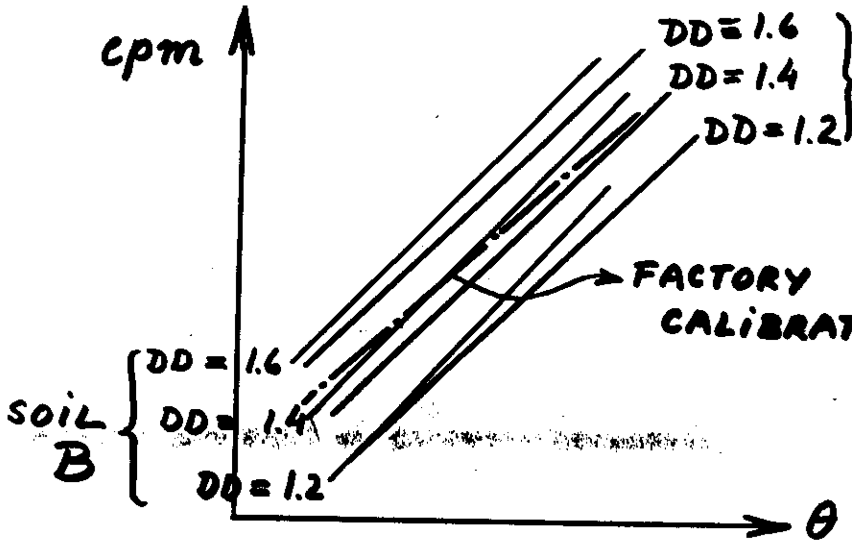
$$^{3}_{2}He + ^{1}_{0}n \rightarrow ^{3}_{1}H + ^{1}_{1}p \qquad (n,p) \text{ REACTION}$$

3. SCINTILLATION DETECTORS
   SOLID CRYSTAL OF EUROPIUM ACTIVATED WITH ^{6}Li

$$^{6}_{3}Li + ^{1}_{0}n \rightarrow ^{4}_{2}\alpha + ^{3}_{1}H \qquad (n,\alpha) \text{ REACTION}$$

CALIBRATION

MANUAL PAGES 192, 211 ... 222



ALL LINEAR FOR USEFUL RANGE

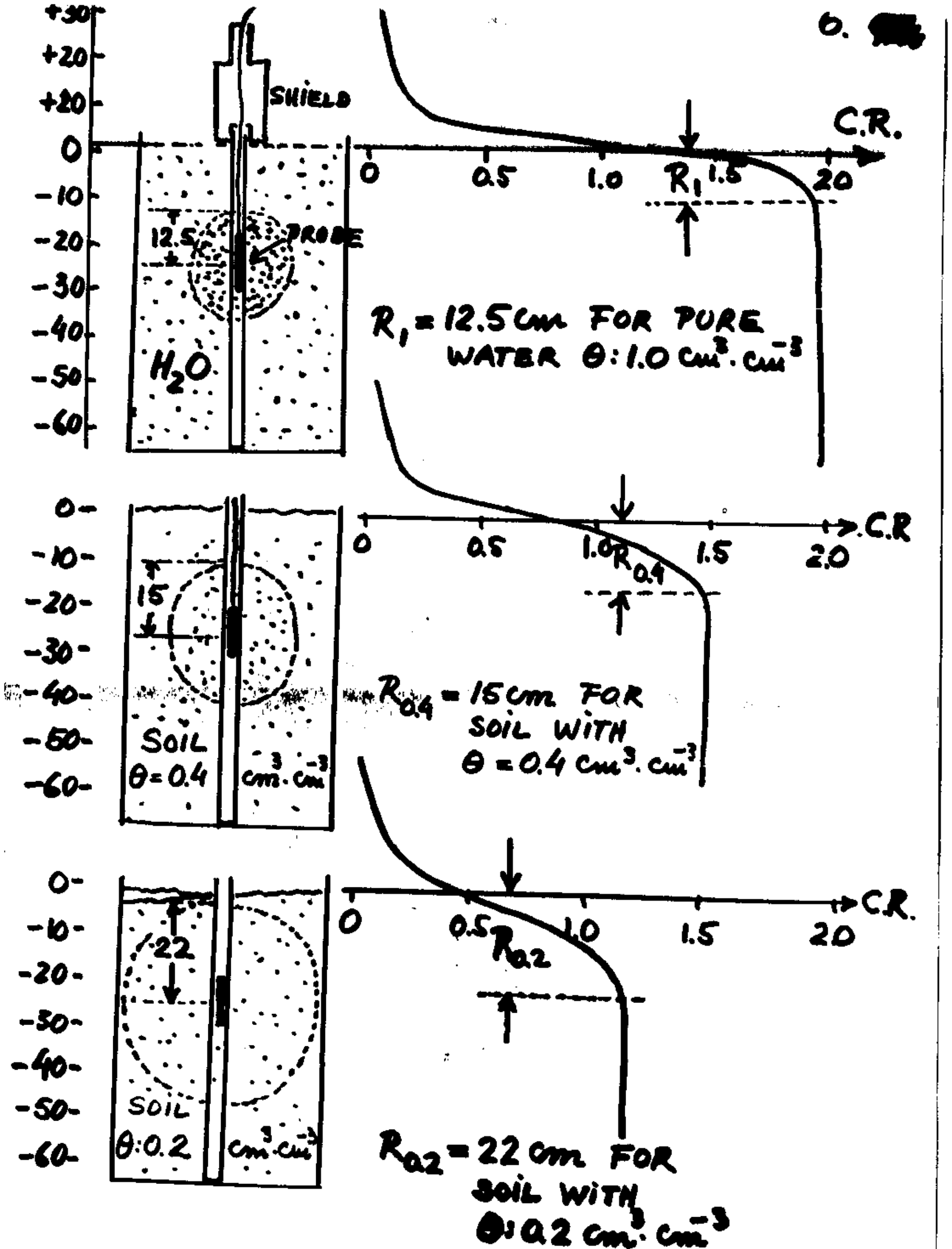
$$cpm = a\theta + b$$

a AND b ARE FED TO THE MICROPROCESSOR

- TO AVOID ELECTRONIC DRIFTS:

COUNT RATIO: $CR = \dfrac{\text{CPM SOIL}}{\text{CPM STANDARD}}$

$$CR = a'\theta + b'$$

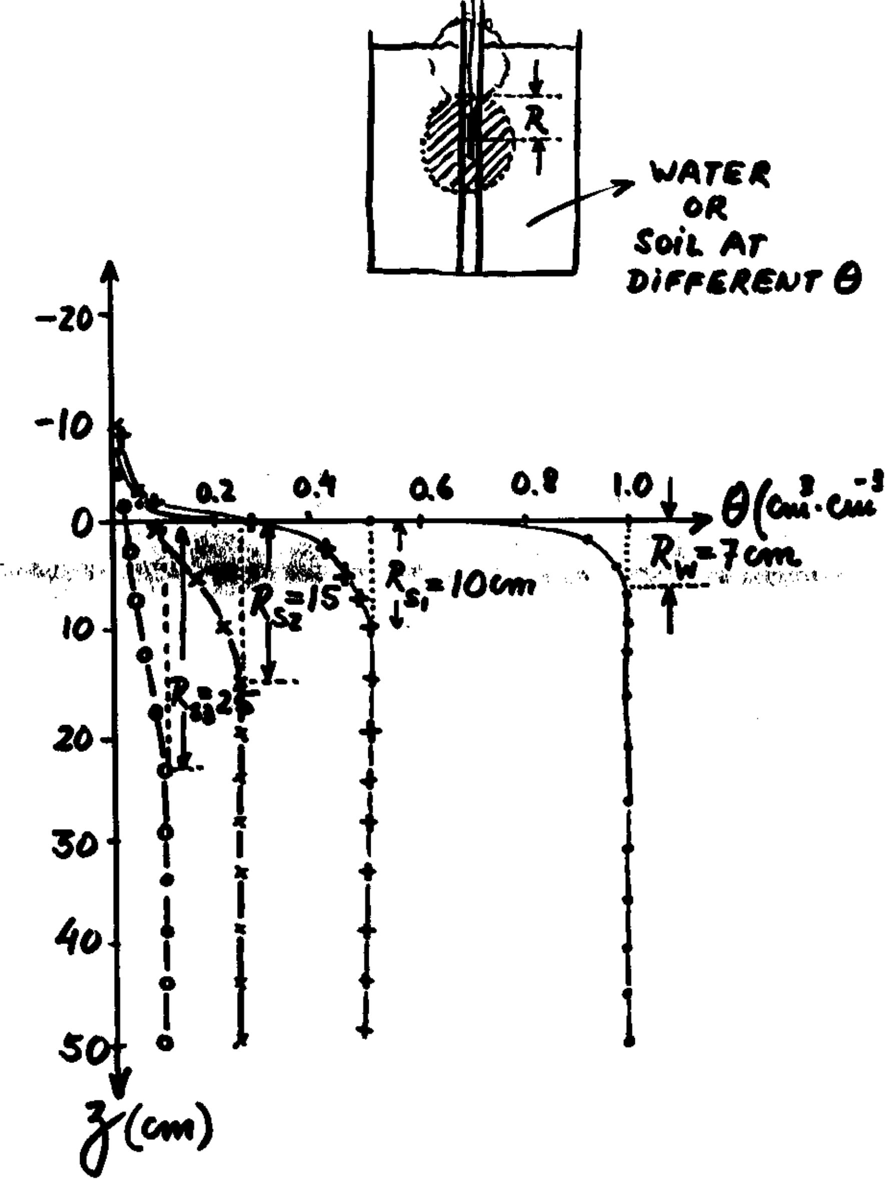


EXPERIMENT 21. PAGE 192 → MANUAL

SPATIAL RESOLUTION OF THE NEUTRON PROBE IN WATER AND IN A SOIL WITH TWO WATER CONTENTS.

z (cm)	WATER $\theta = 1\ cm^3 \cdot cm^{-3}$	SOIL $\theta = 0.4\ cm^3 \cdot cm^{-3}$	SOIL $\theta = 0.2\ cm^3 \cdot cm^{-3}$
+30	0.05	0.06	0.06
+20	0.11	0.11	0.09
+10	0.22	0.21	0.19
+5	0.40	0.38	0.31
0	1.31	0.85	0.50
−5	1.80	1.21	0.79
−10	1.92	1.41	0.98
−15	1.95	1.48	1.11
−20	1.96	1.49	1.18
−25	1.96	1.49	1.20
−30	1.96	1.50	1.20
−35	1.95	1.49	1.19
−40	1.96	1.49	1.20
−50	1.97	1.48	1.20
−60	1.96	1.49	1.20

SPATIAL RESOLUTION OF NEUTRON PROBE



DATA ANALYSIS

WE HAVE MEASUREMENTS OF θ AS A FUNCTION OF TIME AND DEPTH WITH 6 REPS.

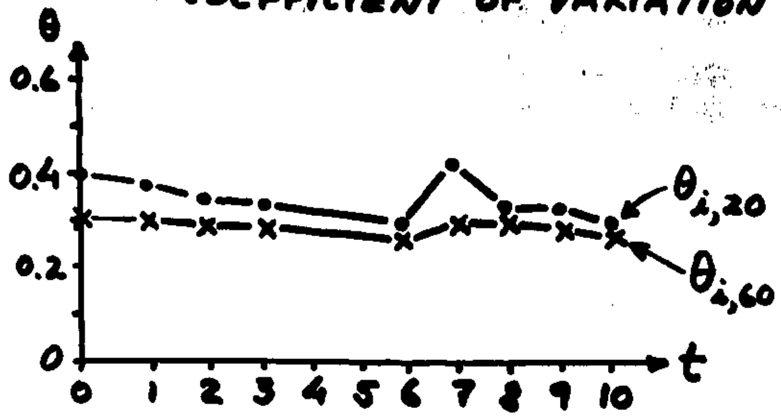
$$\theta_{i,z}\begin{cases} i = \text{time (days)} \\ z = \text{depth (cm)} \end{cases}$$

$i=0$	31/05/83	$z=0$	soil surface
$i=1$	1/06/83	$z=10$	10cm below surf.
$i=2$	2/06/83	$z=20$	⋮
⋮	⋮	⋮	

$\theta_{3,30} = ?$ ______

1. AVERAGE HOLDING i AND z CONSTANT
   - STANDARD DEVIATION
   - COEFFICIENT OF VARIATION

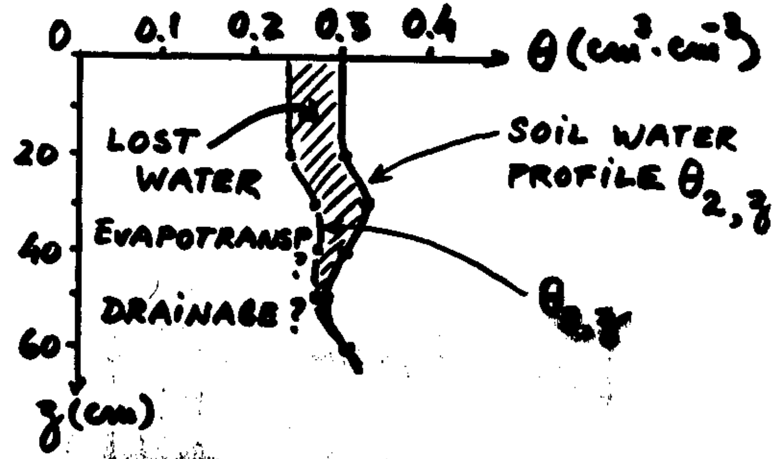
2.



$$\left[\frac{\partial\theta}{\partial t}\right]_z = \text{RATE AT WHICH } \theta \text{ OF A GIVEN DEPTH CHANGES WITH TIME}$$

$$\left[\frac{\partial\theta}{\partial t}\right]_z \cong \frac{\theta_{i,z} - \theta_{i+1,z}}{t_{i+1} - t_i} \begin{cases} - \text{ WATER LOSS} \\ + \text{ WATER GAIN} \end{cases}$$

3.

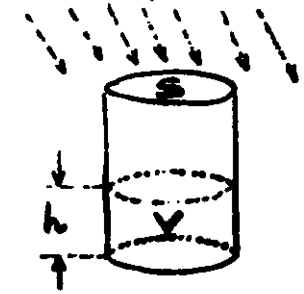


- I IRRIGATION → mm, inches,...
- R RAINFALL → mm
- ET EVAPOTRANSPIRATION → mm/day
- D DRAINAGE → mm

WATER BALANCE: $R + I - ET - D = \Delta\text{STORAGE}$

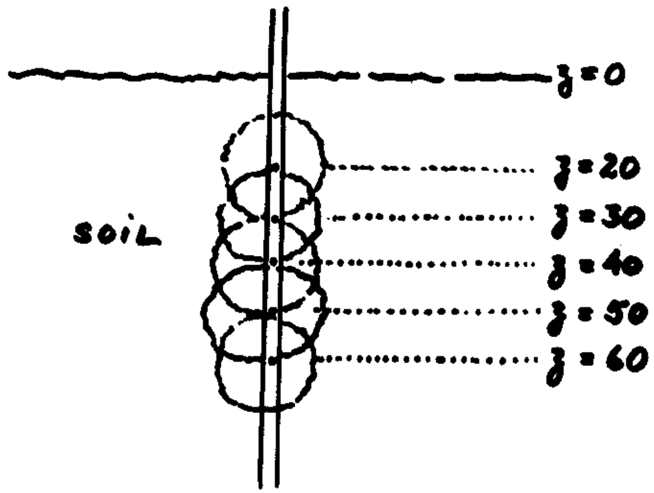
$$\theta\,(cm^3 \cdot cm^{-3}) \xrightarrow{?} mm$$

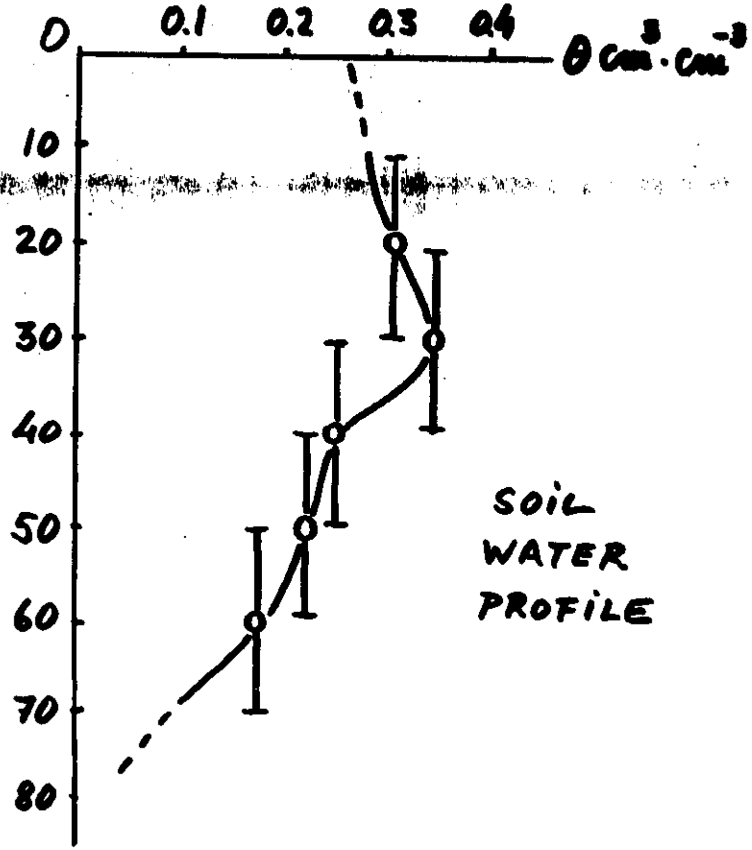
RAINFALL:



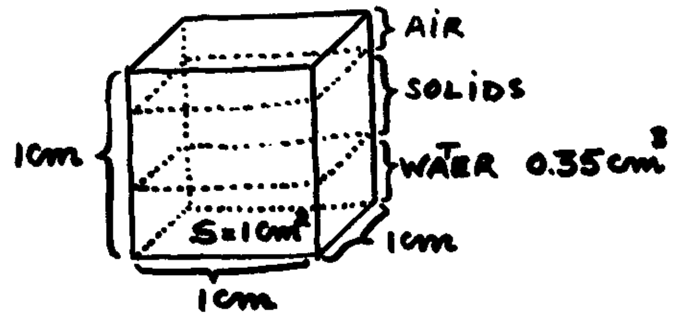
$$\frac{V}{S} = h$$

$$1\,mm = \frac{1\,\ell}{m^2}$$





ASSUME $\theta = 0.35\ cm^3 \cdot cm^{-3}$



$$h = \frac{0.35\ cm^3}{1\ cm^2} = 0.35\ cm = 3.5\ mm$$

$$\boxed{h \equiv \theta}$$

z (cm)	θ (cm³·cm⁻³)	h (cm)	Σh (cm)
0–1	0.35	0.35	0.35
1–2	0.33	0.33	0.68
2–3	0.30	0.30	0.98
3–4	0.31	0.31	1.29
4–5	0.29	0.29	1.59
5–6	0.30	0.30	1.88
6–7	0.28	0.28	2.16
7–8	0.28	0.28	2.44
8–9	0.27	0.27	2.71
9–10	0.25	0.25	2.96
⋮	⋮	⋮	⋮

$\Sigma h \rightarrow$ WATER STORAGE $= S$

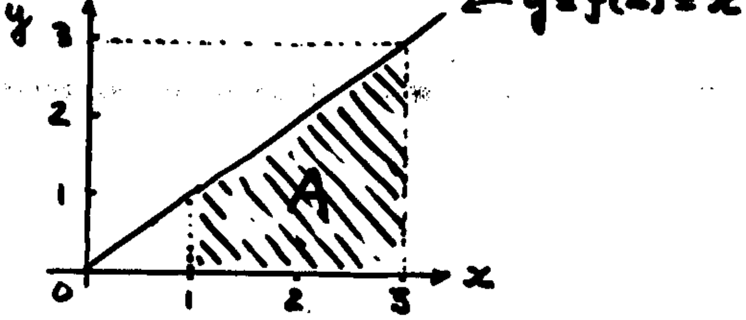
$$S_{i,L} = \int_{z_1}^{z_2} \theta \, dz \quad \text{where } L = z_2 - z_1$$

$$y = \int x \, dx \longrightarrow y = \frac{x^2}{2} + C$$

$$y = \int_1^3 x \, dx = \left[\frac{x^2}{2} + C\right]_1^3 = \left[\frac{3^2}{2} + C\right] - \left[\frac{1^2}{2} + C\right] =$$

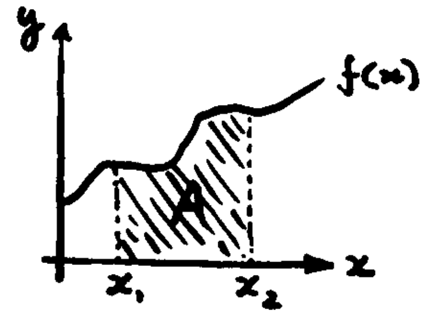
$$= 4.5 + C - 0.5 - C = 4$$

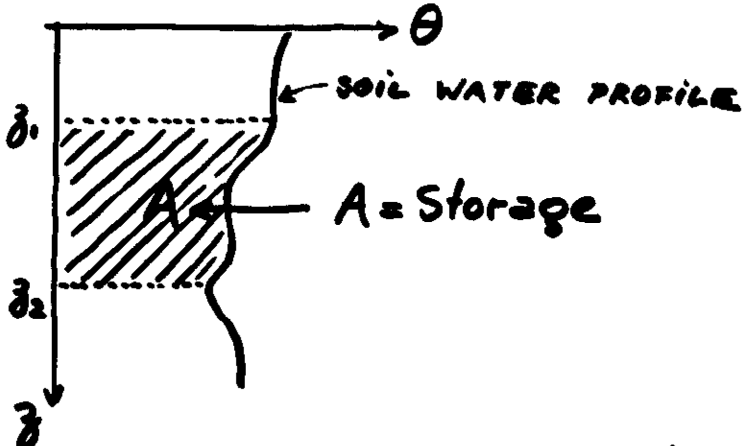
GRAPHICALLY:



$$A = \frac{1+3}{2} \cdot 2 = 4$$

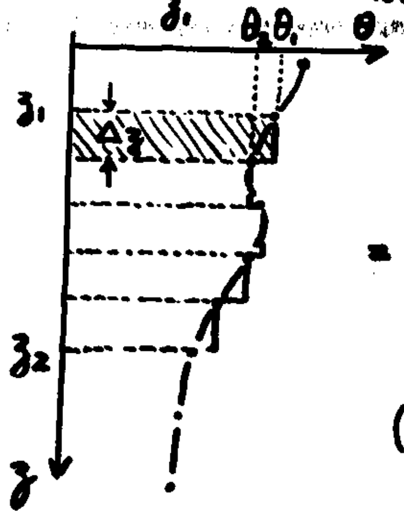
$$y = \int_{x_1}^{x_2} f(x) \, dx = A$$





BUT WE DO NOT HAVE $\theta = f(z)$!

$$S_{i,L} = \int_{z_1}^{z_2} \theta \, dz \cong \sum_{n=1}^{i} \theta \Delta z =$$



$$= \theta_1 \Delta z + \theta_2 \Delta z + \cdots + \theta_5 \Delta z =$$

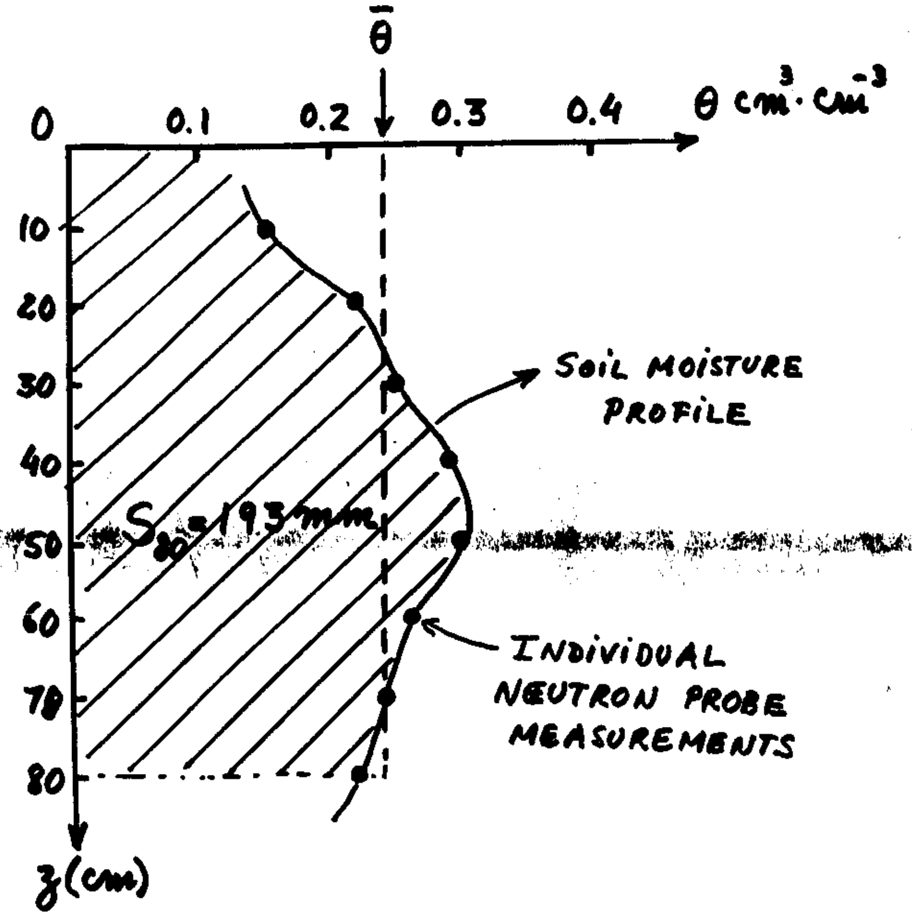
$$= \Delta z (\theta_1 + \theta_2 + \cdots + \theta_5) =$$

$$= \underbrace{5 \times \Delta z}_{(z_2 - z_1) = L} \underbrace{\frac{(\theta_1 + \theta_2 + \cdots + \theta_5)}{5}}_{\text{AVERAGE OF } \theta \text{ BETWEEN } z_1 \text{ and } z_2}$$

$$S_{i,L} = \bar{\theta}(z_2 - z_1) = \bar{\theta} \cdot L$$

3.

WATER STORAGE IN THE SOIL



$$S_{80} = \int_0^{80} \theta \, dz = \bar{\theta}(80-0) = 0.241 \times 80 = 19.3\,cm = 193\,mm$$

EXAMPLE:

z	$\theta_{4,z}$	$\theta_{7,z}$
15	0.401	0.320
30	0.396	0.328
45	0.389	0.361
60	0.392	0.385
75	0.376	0.373
	$\bar{\theta}$: 0.391	$\bar{\theta}$: 0.353

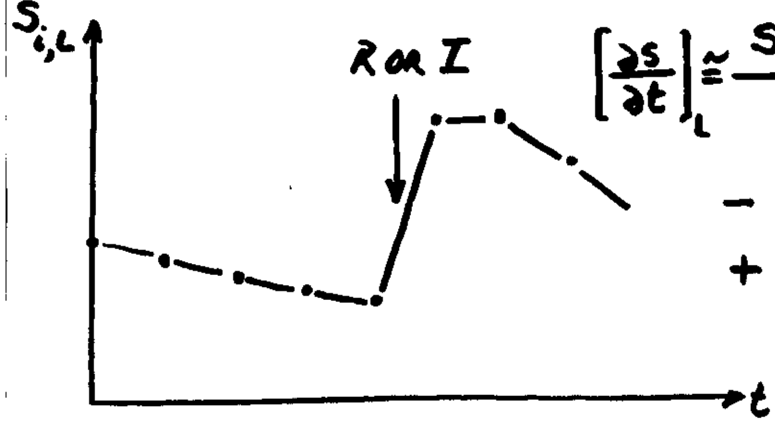
$$S_{4,75} = 0.391 \times 75 = 29.3\,cm = 293\,mm$$

$$S_{7,75} = 0.353 \times 75 = 26.5\,cm = 265\,mm$$

$$\Delta S = S_{4,75} - S_{7,75} = 28\,mm$$

$$\text{RATE OF LOSS} = \frac{\Delta S}{3} = 9.3\,mm \cdot day^{-1}$$

4.



$$\left[\frac{\partial S}{\partial t}\right]_L \cong \frac{S_{i,L} - S_{i+1,L}}{t_{i+1} - t_i}$$

− WATER LOSS

+ WATER GAIN

- AN EXAMPLE OF STORAGE CHANGES WITH TIME.
  - CROP: bean (*Phaseolus vulgaris*, L.)
  - SOIL: ALFISOL (BRAZIL)

DEPTH cm	MOISTURE CONTENT $cm^3 \cdot cm^{-3}$ 10/APRIL	18/APRIL
0	0.401	0.200
10	0.382	0.211
20	0.355	0.249
30	0.340	0.285
40	0.351	0.330
50	0.370	0.371
60	0.390	0.388

- NO RAINFALL BETWEEN DATES
- NO DEEP DRAINAGE.

$$S_{60,t_1} = 0.370 \times 60 = 22.2\ cm = 222\ mm$$

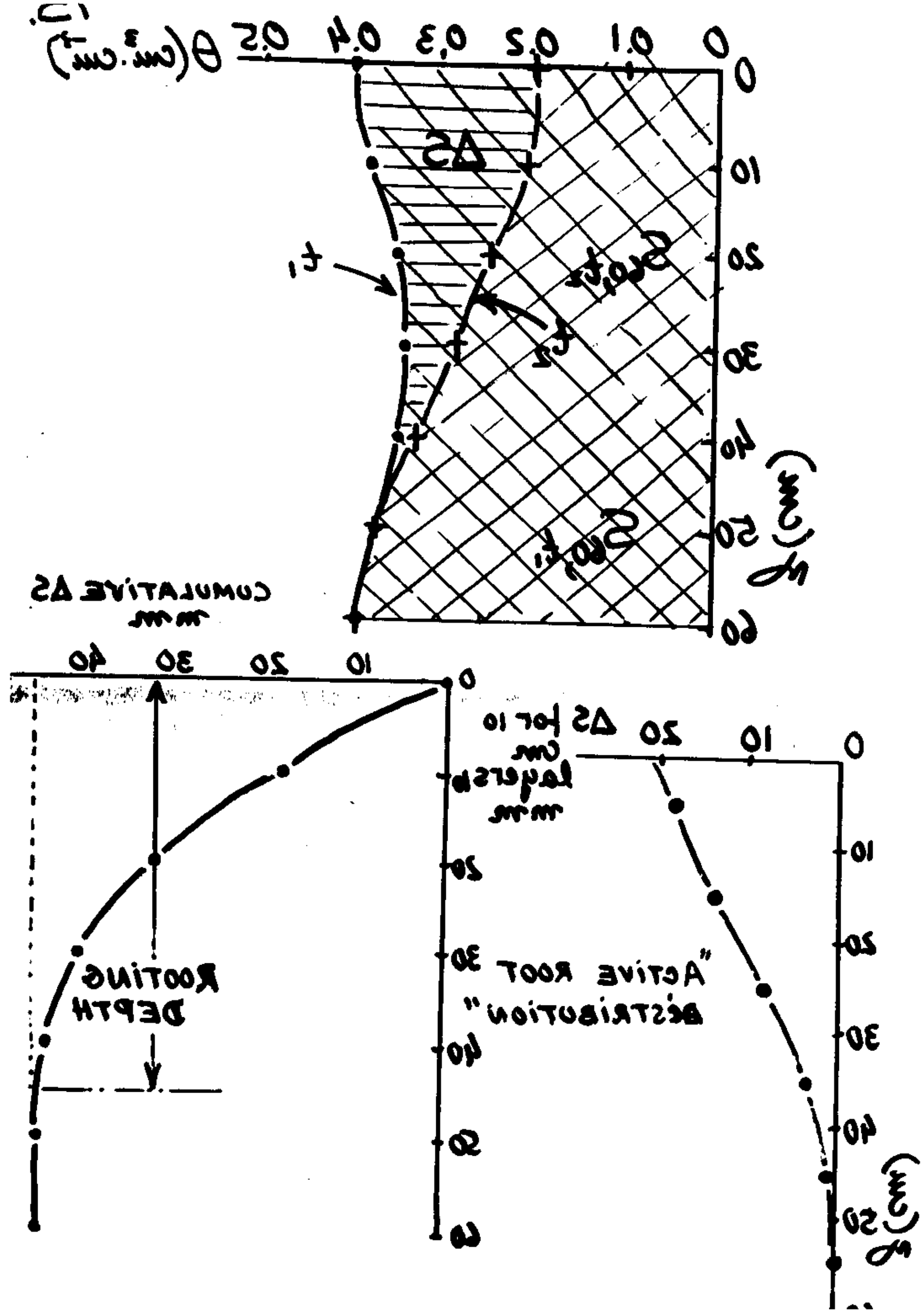
$$S_{60,t_2} = 0.291 \times 60 = 17.5\ cm = 175\ mm.$$

$$\Delta S = 47\ mm.$$

$$ET = \frac{47\ mm}{8\ days} = 5.9\ mm/day.$$

ΔS is not affected by calibration curves if they are paralell!

14.



DEPTH NEUTRON PROBE

SOURCES OF ERROR:

a) Counting time

b) Calibration

c) Instrumental performance

d) Active center position

e) Sphere of influence

f) Access tube installation

g) Relative location in the field

h) others....

Counting time:

$$s^2(\bar{\hat{N}}_s) = \frac{1}{q} \cdot \frac{\bar{\hat{N}}_s}{T_s} \quad \cdots\cdots\cdots (1)$$

$$s^2(\bar{\hat{N}}_o) = \frac{1}{p} \frac{\bar{\hat{N}}_o}{T_c} \quad \cdots\cdots\cdots (2)$$

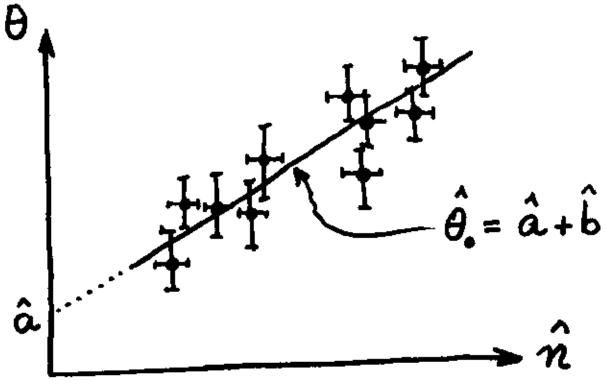
$$\hat{n}_o = \frac{\bar{\hat{N}}_o}{\bar{\hat{N}}_s} \quad \cdots\cdots\cdots (3)$$

$$s^2(\hat{n}_o) = \left(\frac{\bar{\hat{N}}_o}{\bar{\hat{N}}_s}\right)^2 \left[\frac{s^2(\bar{\hat{N}}_o)}{\bar{\hat{N}}_o^{\,2}} + \frac{s^2(\bar{\hat{N}}_s)}{\bar{\hat{N}}_s^{\,2}}\right] \cdots (4)$$

Calibration

$\hat{\theta}_o = \theta_o + \mu_i$ (classical methods)

$\hat{n}_o = n_o + v_i$ (neutron probe)



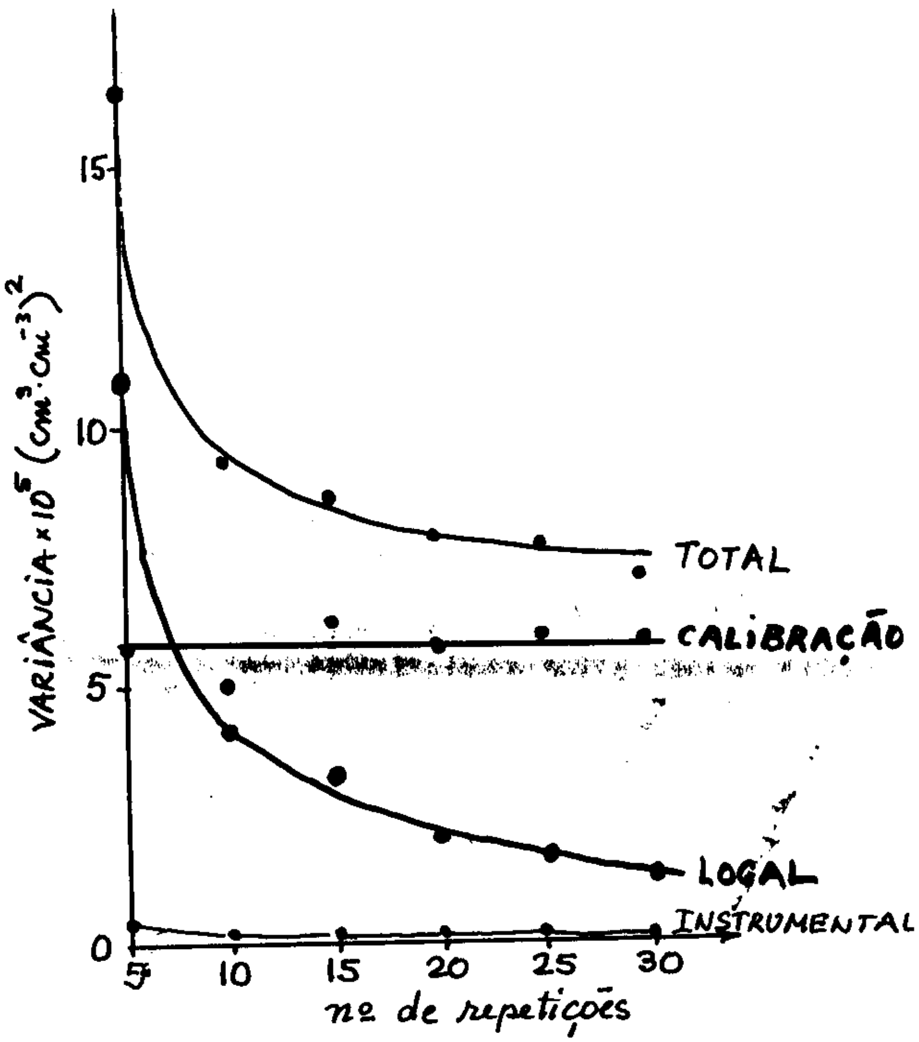
$$s^2(\hat{\theta}_o) = s^2_c(\hat{\theta}_o) + s^2_I(\hat{\theta}_o) \quad \cdots\cdots (7)$$

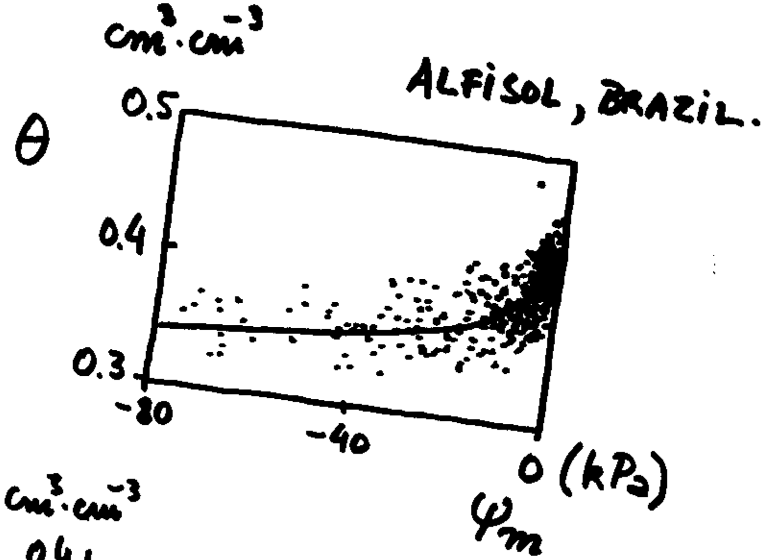
$$s^2_I(\hat{\theta}_o) = \left\{\hat{b}^2 - s^2(\hat{b})\right\}\left[\frac{\hat{n}_o}{pT_c} + \frac{\hat{n}_o^2}{qT_s}\right]\cdot\frac{1}{\hat{N}_s} \quad \cdots\cdots (7a)$$

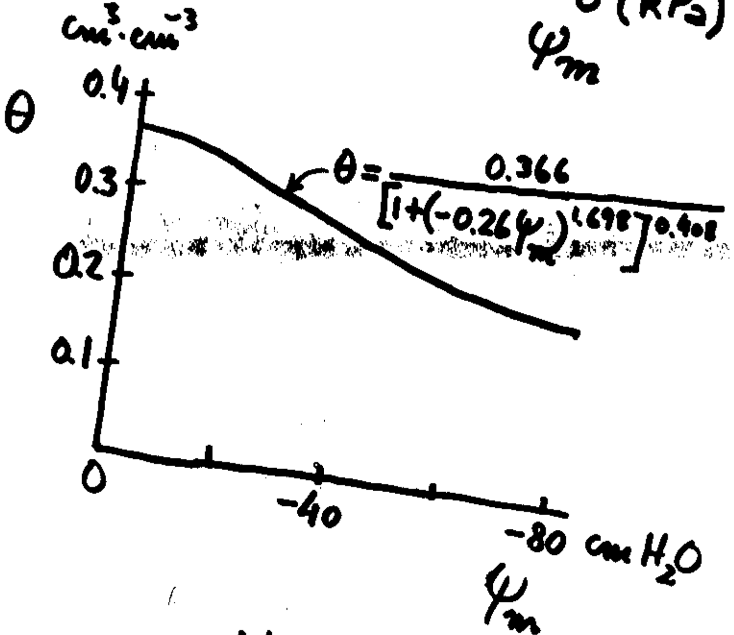
$$s^2_c(\hat{\theta}_o) = s^2(\hat{a}) + \hat{n}_o^2 s^2(\hat{b}) + 2\hat{n}_o s(\hat{a},\hat{b}) + s^2(e_a) \quad \cdots (7b)$$

$$s^2_L(\langle\hat{\theta}_o\rangle) = \left\{\hat{b}^2 - s^2(\hat{b})\right\}\cdot\frac{s^2(L_o)}{k\hat{N}_s^2} \quad \cdots\cdots (7c)$$

k different access tubes.







WORLD AVERAGE

INTERNAL DRAINAGE

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K \frac{\partial H}{\partial z}\right]$$

$$\int_0^L \frac{\partial \theta}{\partial t}\, dz = K \frac{\partial H}{\partial z}\Big|_L - \underbrace{K \frac{\partial H}{\partial z}\Big|_0}_{=0}$$

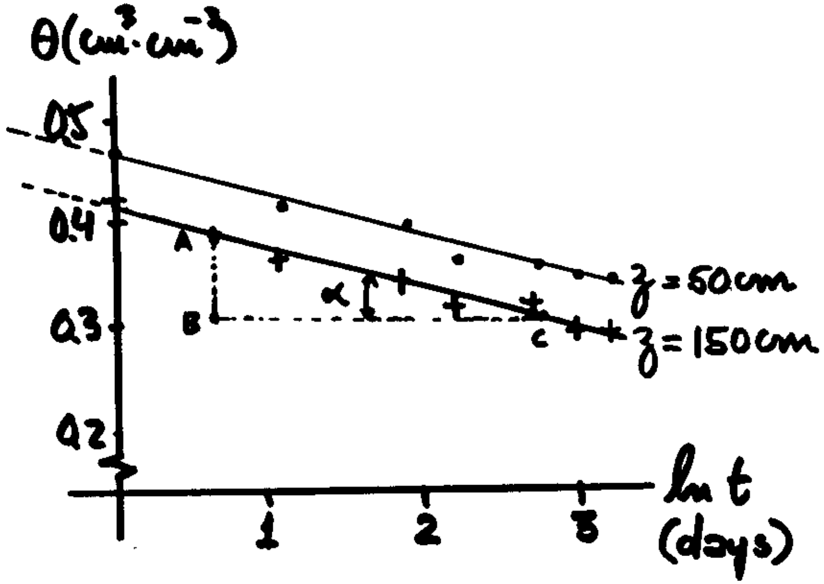
$$K = \frac{\int_0^L \frac{\partial \theta}{\partial t}\, dz}{\frac{\partial H}{\partial z}\Big|_L}$$

———— × ————

LIBARDI et al (1981)

Assuming $K = K_0 \exp \gamma[\theta - \theta_0]$

$$\theta - \theta_0 = \frac{1}{\gamma} \ln t + \frac{1}{\gamma} \ln(\gamma K_0 / L)$$



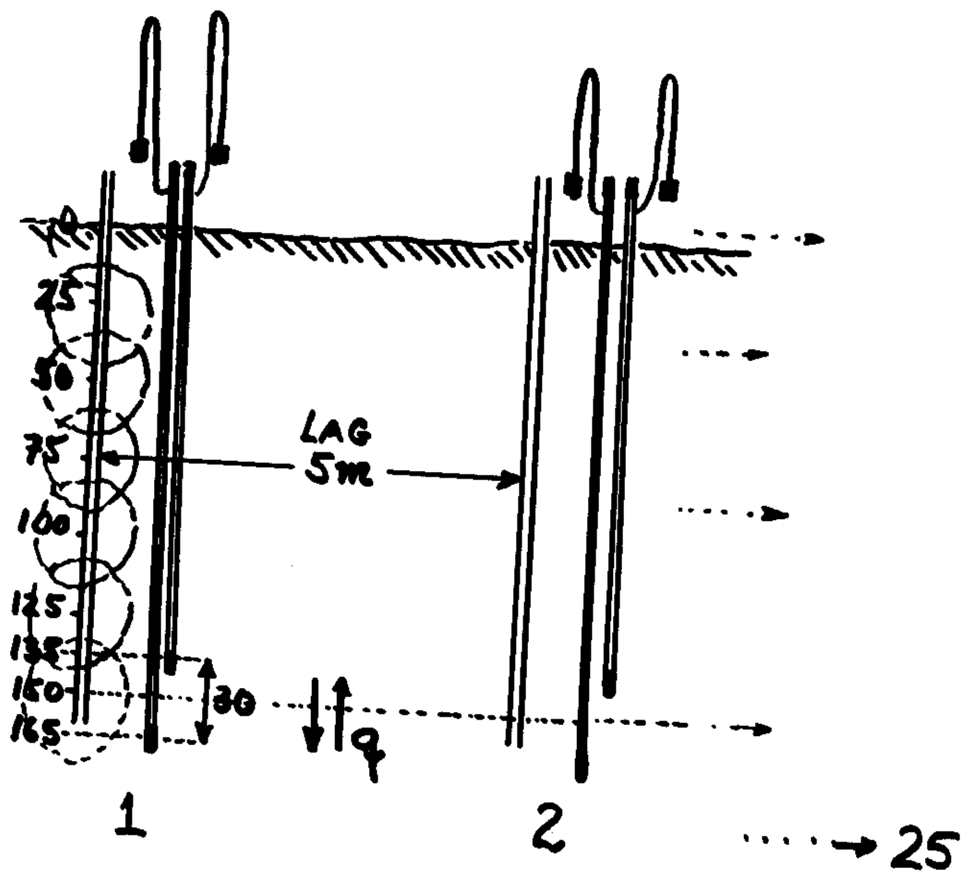
$$\frac{1}{\gamma} = \tan\alpha = \frac{AB}{BC}$$

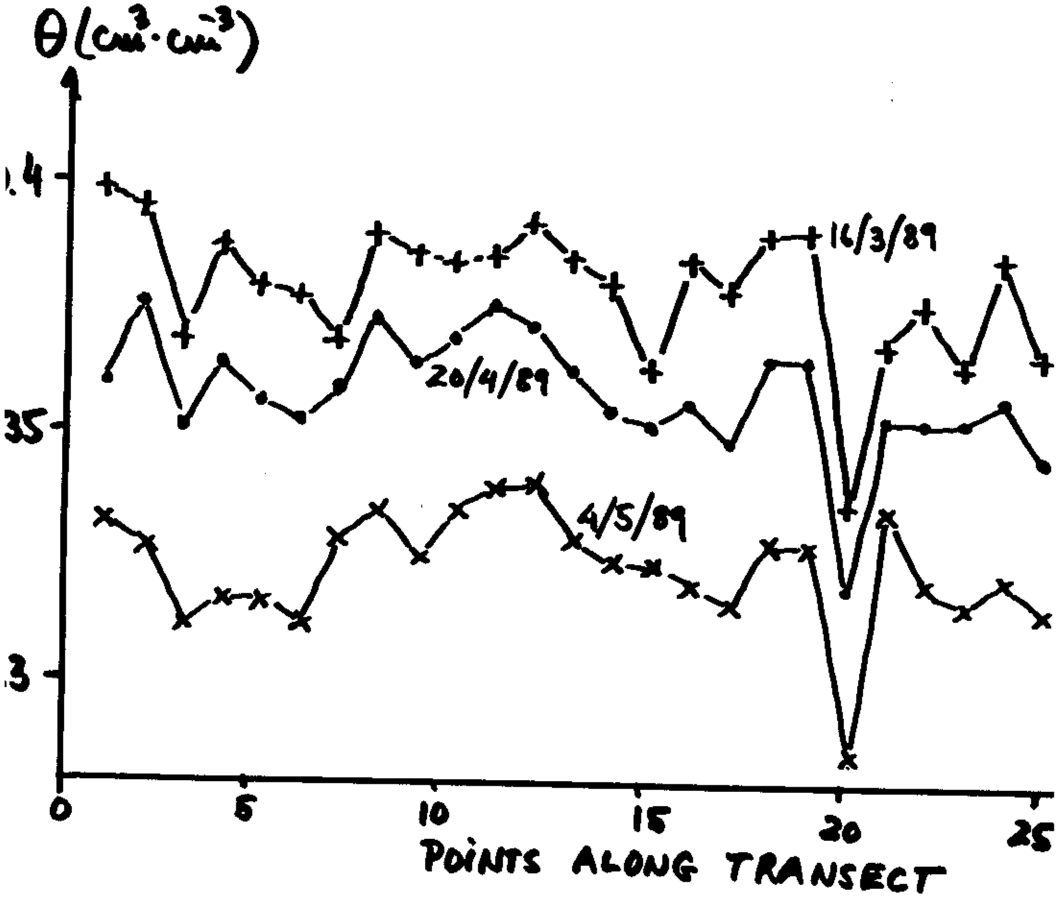
for $z = 50$ cm

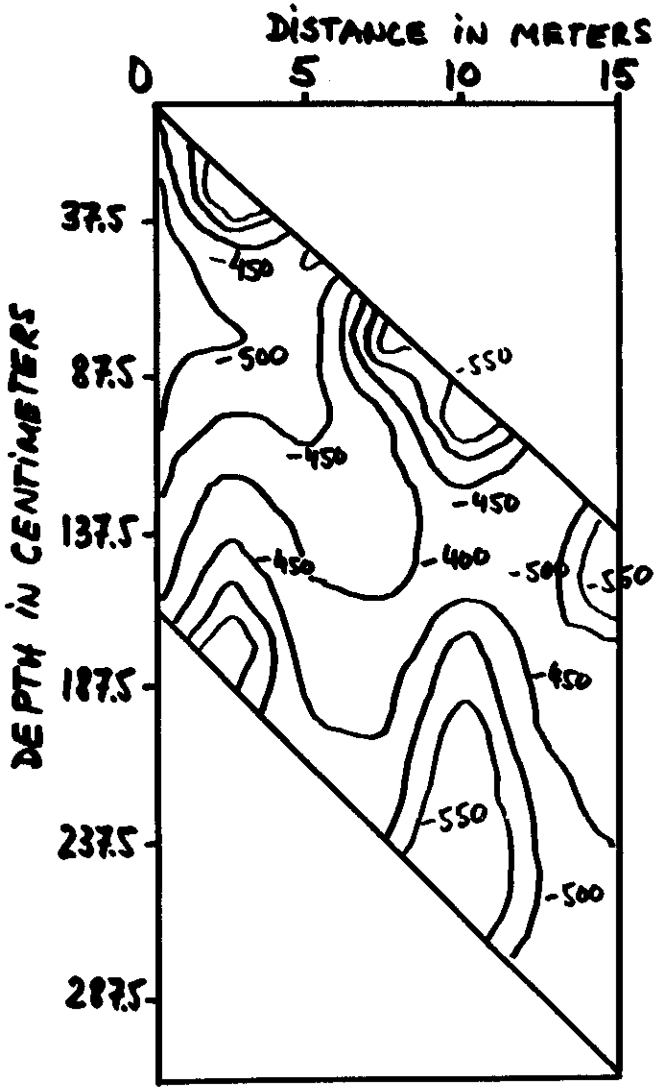
$$\frac{AB}{BC} = \frac{0.0866}{2} = 0.0433 = \frac{1}{\gamma}$$

$$\gamma = \frac{1}{0.0433} = 23.077.$$

$$K = 85.6 \exp\left[23.077(\theta - 0.450)\right]$$







SOIL WATER POTENTIAL ISOLINES ($cm\ H_2O$)
in RUBBER TREE PLANTATION