



INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
ICTP, P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE



H4.SMR/449-32

**WINTER COLLEGE ON
HIGH RESOLUTION SPECTROSCOPY**

(8 January - 2 February 1990)

**COHERENT TRAPPING IN LASER SPECTROSCOPY
EXCITATION AND LASER COOLING**

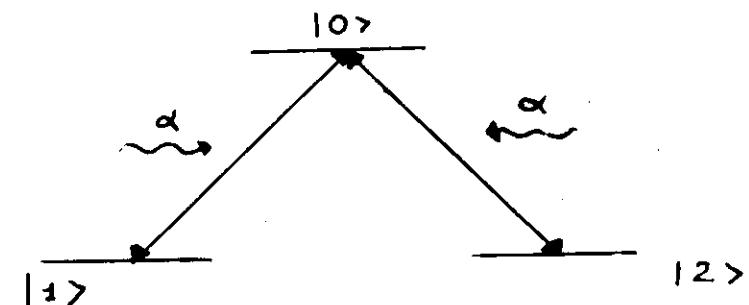
E. Arimondo

Università di Pisa
Dipartimento di Fisica
Pisa 56100
Italy

COHERENT TRAPPING IN
LASER SPECTROSCOPY, EXCITATION AND LASER COOLING

- 1) what is coherent trapping
- 2) experimental observations
- 3) new applications
- 4) constants of motion
- 5) velocity-selective coherent trapping for atom cooling

"Coherent population trapping,"



$$|\Psi_{NC}\rangle = \frac{1}{\sqrt{2}} (|1\rangle - |2\rangle)$$

"V" interaction Hamiltonian of atoms
with laser fields

$$\langle 0 | V | \Psi_{NC} \rangle = \frac{1}{\sqrt{2}} [\langle 0 | V | 1 \rangle - \langle 0 | V | 2 \rangle] = \\ = \frac{1}{\sqrt{2}} [\alpha - \alpha] = 0$$

first experimental observation in

Alzetta et al. N.C. 3CB 5 (1976)

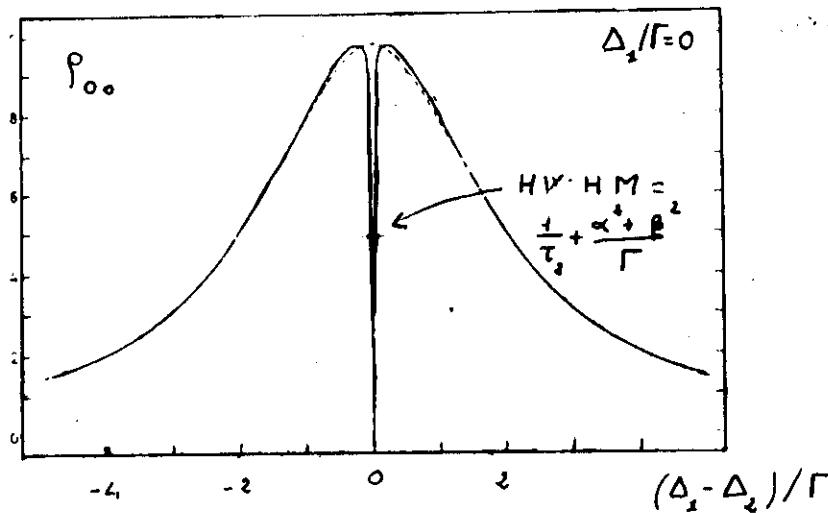
analysis in

Arimondo et al. Lett. N.C. 17 333 (1976)

Gray et al. Opt. Lett. 3 218 (1978)

Non-absorption resonances

in the D_1 transition



Non-absorption resonances in Na vapour

(Alzetta et al., 1976, 1979)

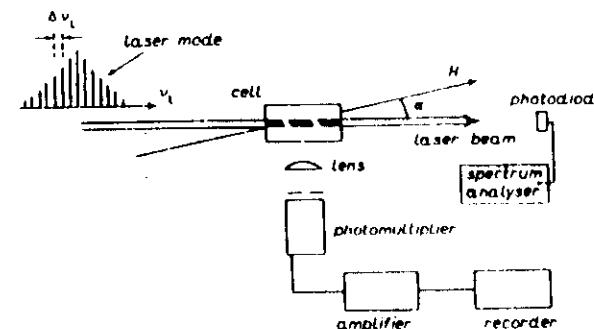


Fig. 2. Sketch of the experimental apparatus.

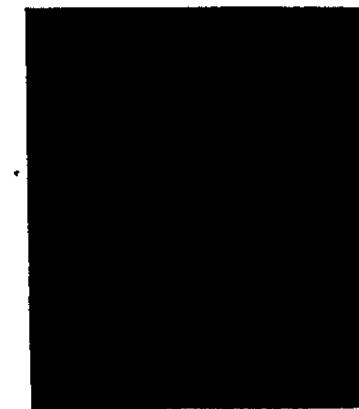
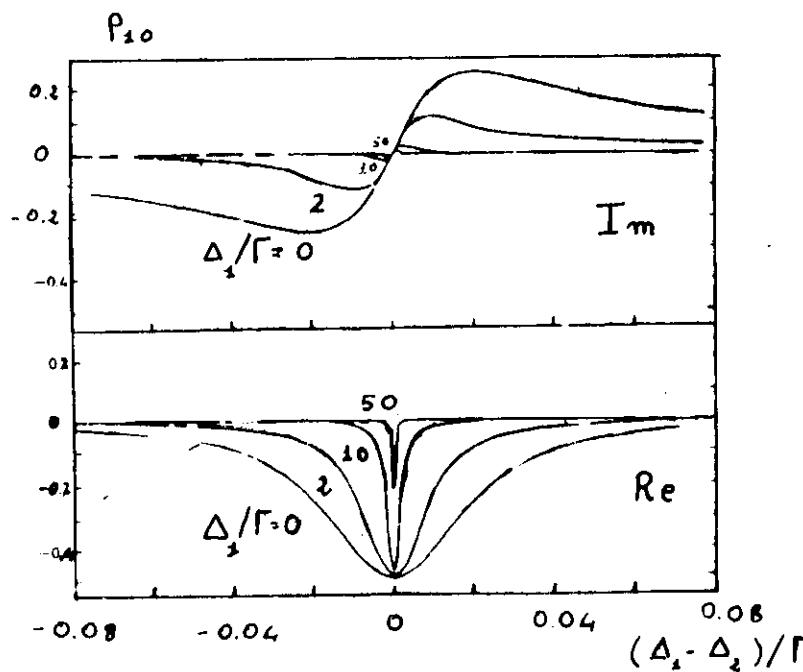


Fig. 1. – Fluorescence of sodium vapour crossed by the circularly polarized beam of a multimode laser tuned to the D_1 line.

Coherent trapping in Na experiment

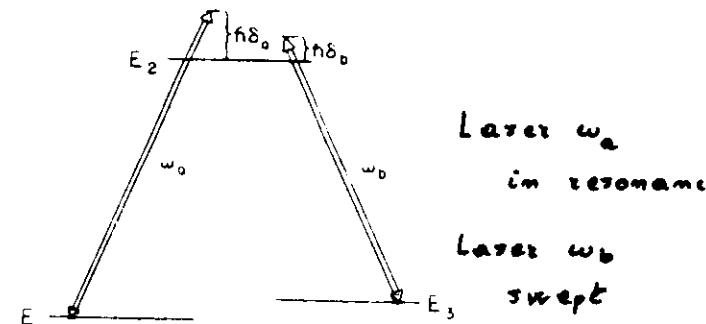
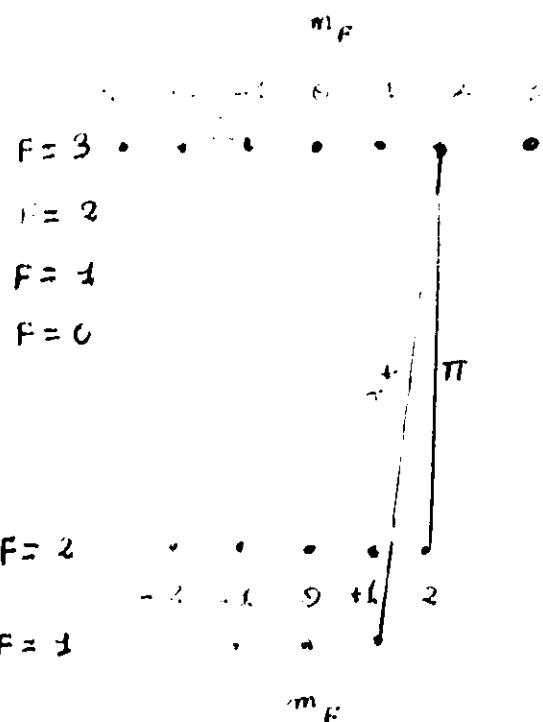


Fig. 1. Energy-level configuration. E_1 and E_3 are nondegenerate sublevels of the ground state. E_2 is a common excited state. Two monochromatic lasers of frequency w_a and w_b are tuned δ_a and δ_b off resonance.

Detection of excited state fluorescence

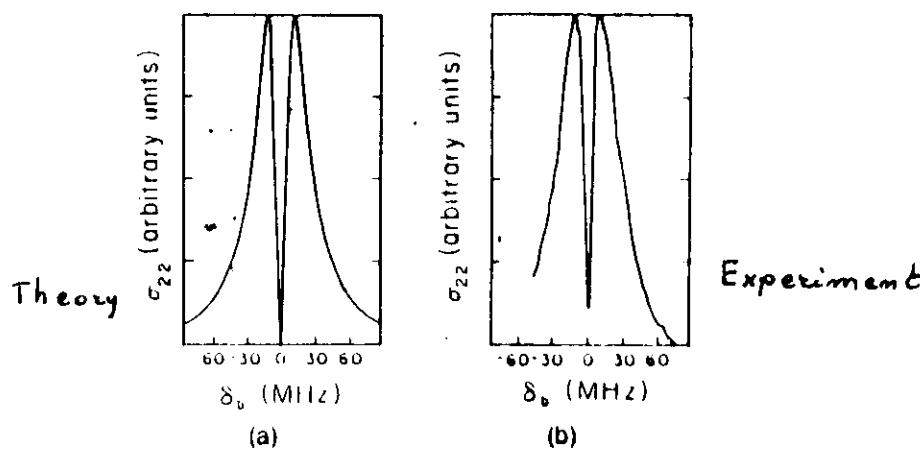
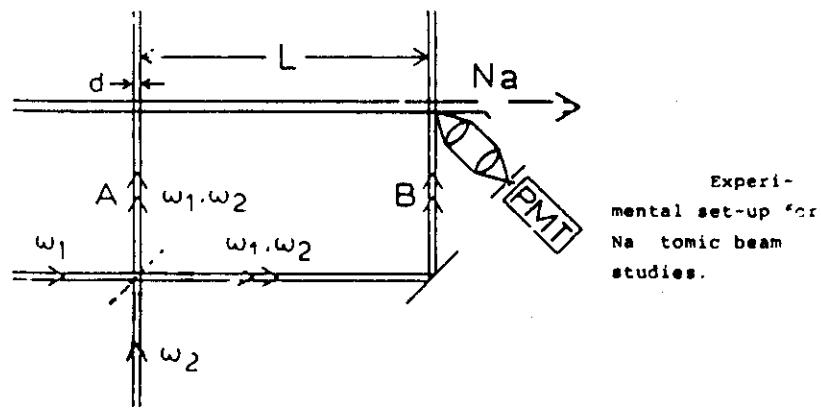


Fig. 2. (a) Experimentally observed excited-state population σ_{22} . The fixed-frequency laser is at exact resonance, $\delta_a = 0$, with an intensity of 23 mW/cm^2 . The second laser is detuned δ_b from exact resonance, with an intensity of 54 mW/cm^2 . (b) Theoretical prediction of excited-state population σ_{22} . The fixed-frequency laser is at exact resonance, $\delta_a = 0$. The second laser is detuned δ_b from exact resonance.

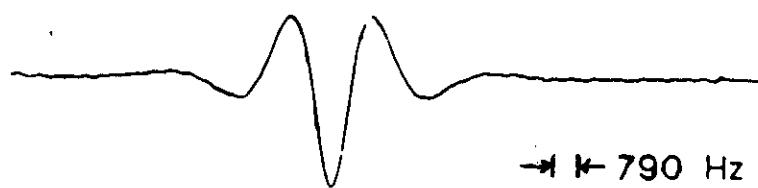
Gray, Whitley and Stroud,

Ramsey fringes in Trapping. (Eckel et al, 1982)

JOSA 86, 1519 (1989)



Experimental set-up for
Na atomic beam
studies.



$\rightarrow \Delta \approx 790 \text{ Hz}$

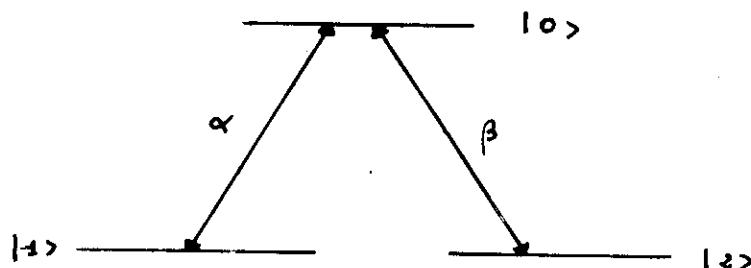
Ramsey fringes using resonance Raman excitation
in Na atomic beam.

Full width half maximum: 1.3 kHz

Constants of motion

for the optical Hamiltonian

$$\mathcal{H} = \mathcal{H}_{\text{opt}} + \mathcal{H}_{\text{spont}}$$



$$\text{Non-absorbing state } |\Psi_{nc}\rangle = \frac{1}{\sqrt{\alpha^2 + \beta^2}} [\beta |12\rangle - \alpha |11\rangle]$$

$$\langle \Psi_{nc} | \rho | \Psi_{nc} \rangle =$$

$$= \frac{1}{\alpha^2 + \beta^2} [\beta^2 \rho_{11} + \alpha^2 \rho_{22} - \alpha \beta (\rho_{12} + \rho_{21})]$$

not coupled by optical Hamiltonian:

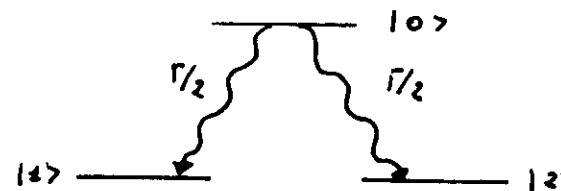
$$\partial \mathcal{H} / \partial \Psi_{nc} = 0$$

for spontaneous emission

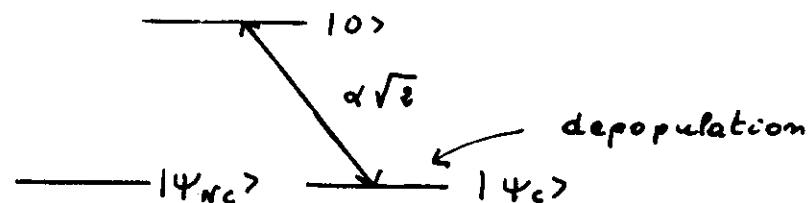
$$\frac{d}{dt} \langle \Psi_{nc} | \rho | \Psi_{nc} \rangle = \frac{\Gamma}{2} \rho_{00}$$

Preparation of coherent trapping state

through depopulation pumping



both states
are statistically
equally populated



Laser-Induced Continuum Structure in Xenon

M. H. R. Hutchinson and K. M. M. Ness^(a)

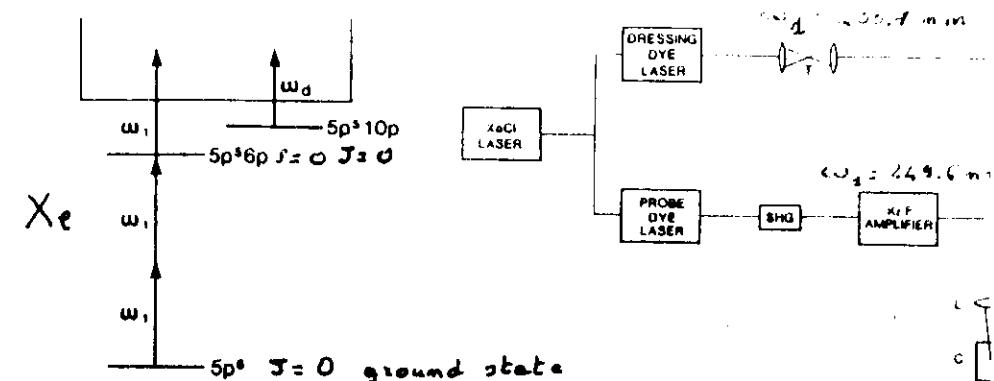
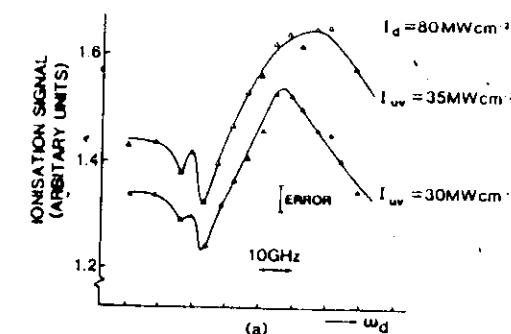
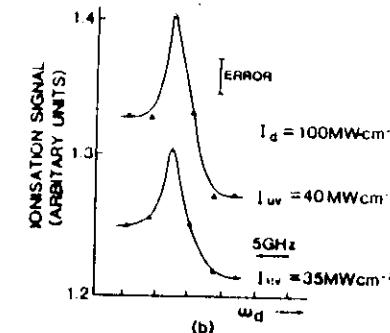


FIG. 2. Energy-level diagram for xenon, dressed by ω_d and probed by two-photon-resonant, three-photon ionization.



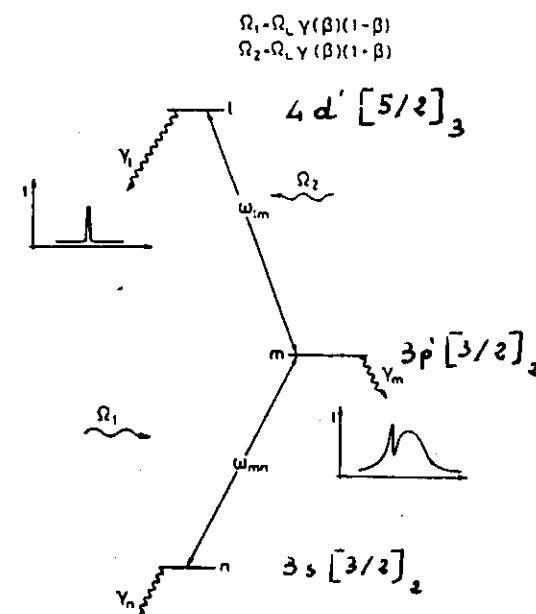
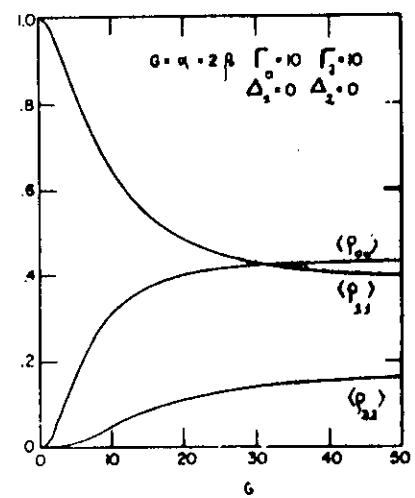
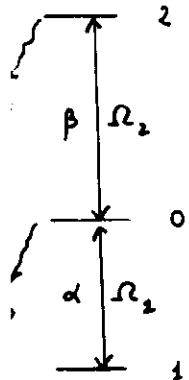
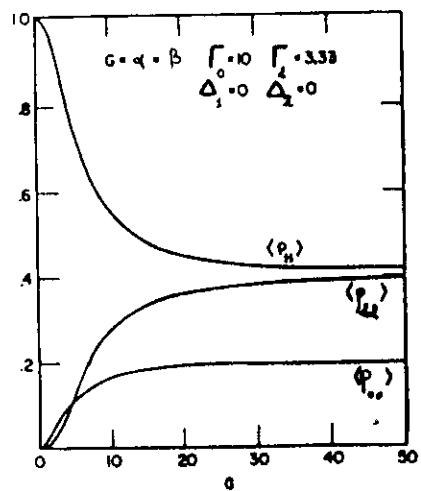
(a)



(b)

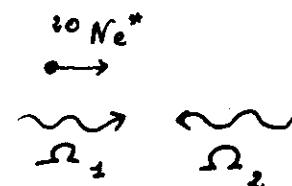
FIG. 4. Plots of the total two-photon-resonant ionization signal vs ω_d : (a) Low-resolution scan and (b) higher-resolution scan of the unknown feature.

Excited state populations
in double resonance



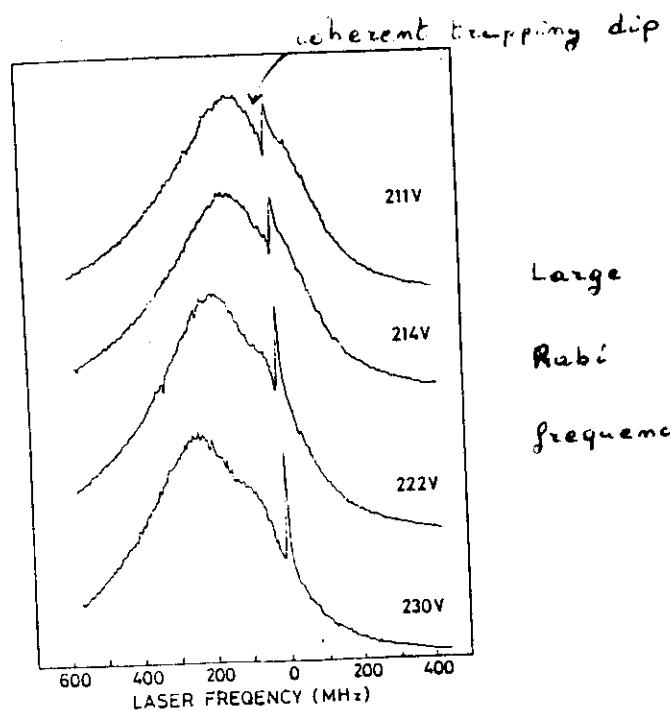
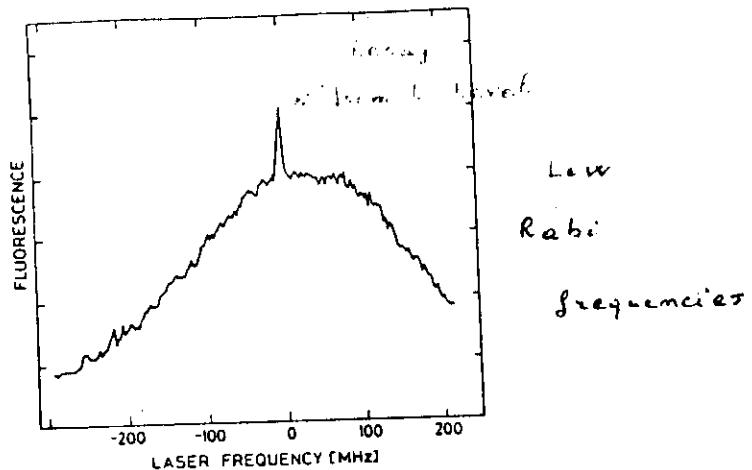
Experiment by Poulsen et al (Opt. Comm. 1984)
 on cascade configuration

accelerated $^{20}\text{Ne}^*$ metastable beam

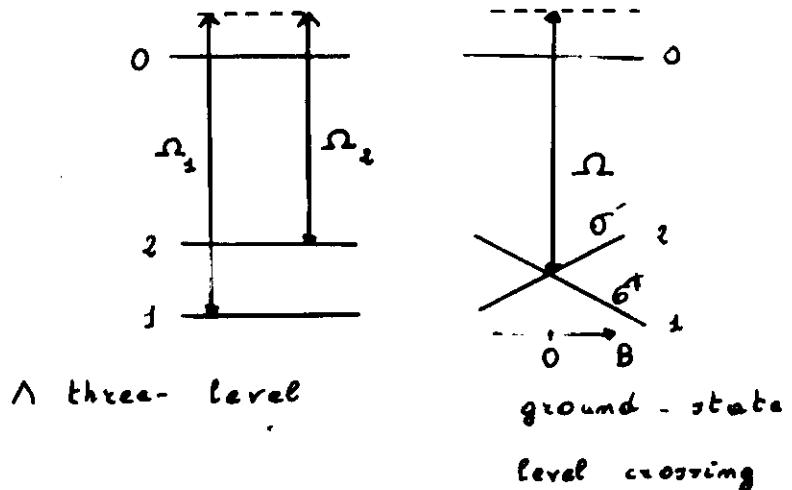


(Winetby and Streed, 1976)

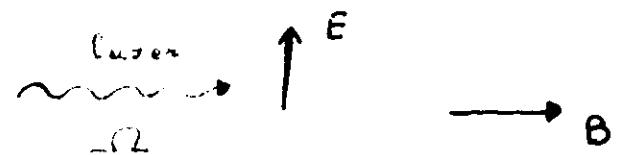
Fluorescence from m level (intermediate)



Hahn non-linear effect



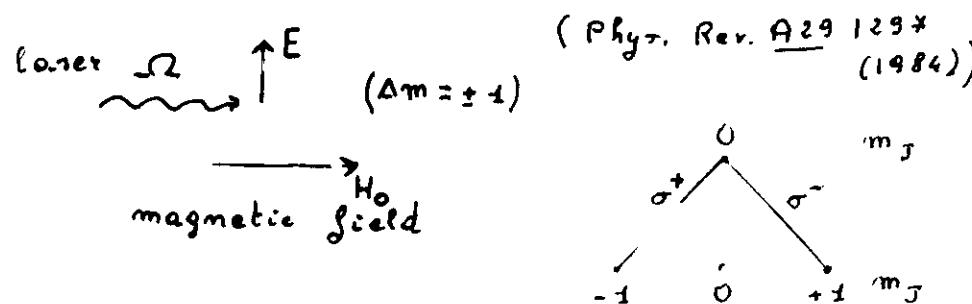
Laser configuration



Experimental configuration

for bistability based on

coherent trapping



J. MLYNEK, F. MITSCHKE, R. DESERNO, AND W. LANGE

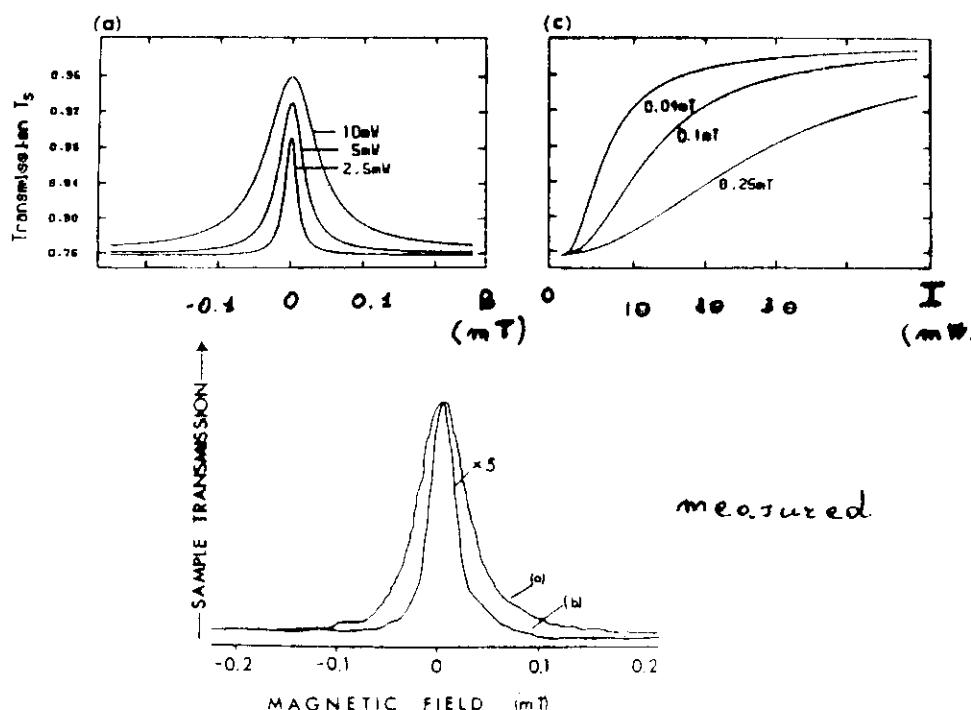


FIG. 6. Measured optical transmission of Na vapor as a function of transverse magnetic field for two input intensities (a)

Absorptive bistability

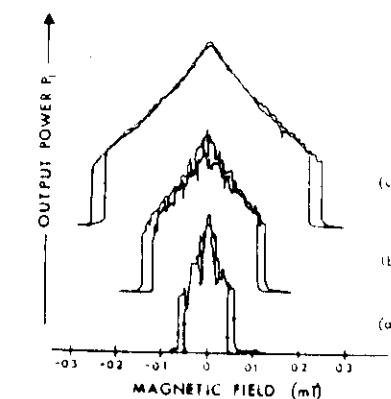


FIG. 8. Measured cavity output power in the absorptive regime as a function of the transverse magnetic field B for three different number densities: (a) $N_{Na} \approx 1.9 \times 10^{12} \text{ cm}^{-3}$, (b) $N_{Na} \approx 2.4 \times 10^{12} \text{ cm}^{-3}$, (c) $N_{Na} \approx 3.9 \times 10^{12} \text{ cm}^{-3}$. Otherwise $P_{in} = 120 \text{ mbar}$, $P_{in} = 150 \text{ mW}$, zero cavity mistuning;

Dispersive bistability

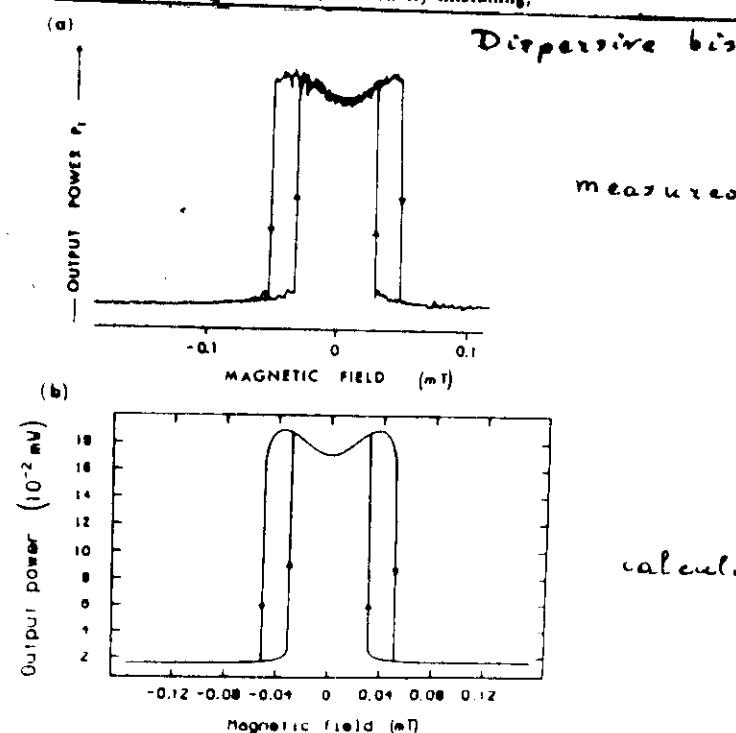
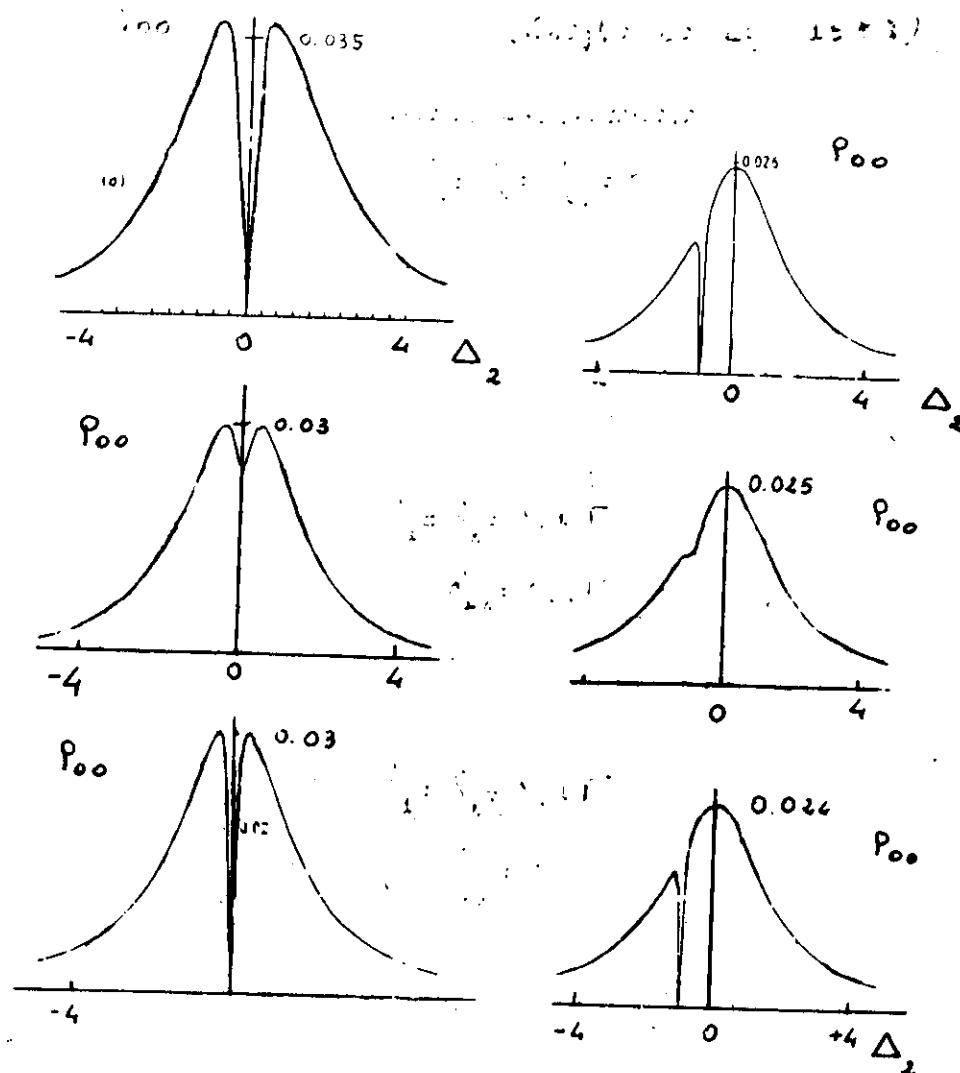


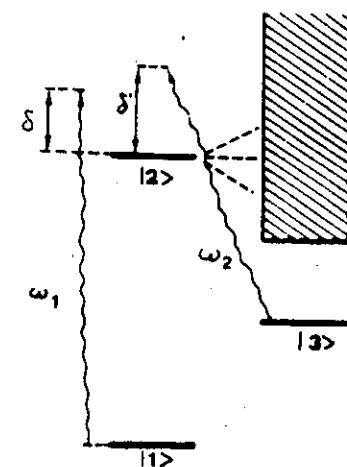
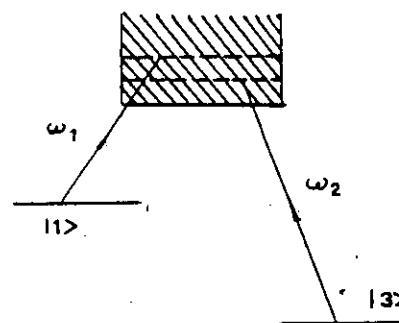
FIG. 10. Measured (a) and calculated (b) dispersive bistability and hysteresis for $\Delta_0/2\pi = 3.5 \text{ GHz}$. Parameter values are (a) $N_{Na} \approx 5 \times 10^{12} \text{ cm}^{-3}$, $p_{Ar} = 200 \text{ mbar}$, $P_{in} = 80 \text{ mW}$, $d = 240 \mu\text{m}$, cavity mistuning not experimentally determined. (b) $N = 2.7 \times 10^{12} \text{ cm}^{-3}$, $p_{Ar} = 200 \text{ mbar}$, $P_{in} = 1.7 \text{ mW}$, $d = 240 \mu\text{m}$.

Intensities of various transitions in trapping



Discrete-continuum states

investigated for
coherence trapping



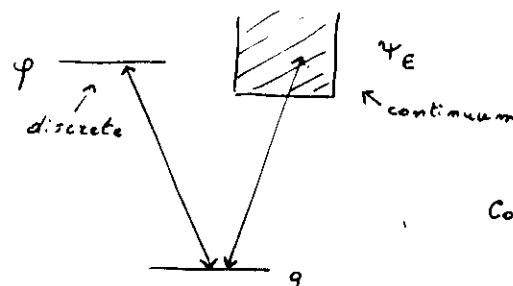
$$\Delta_2 = 0$$

$$\Delta_2 = \Gamma$$

$$\omega_2 = \omega_s = 0.4\Gamma$$

$$\omega_2 = \omega_s = 0.4\Gamma$$

Fano lineshape



Coupling continuum-autonionization
 V_E

Eigenstate

$$\phi_E = a\phi + \int dE b_E \psi_E$$

Transition probability

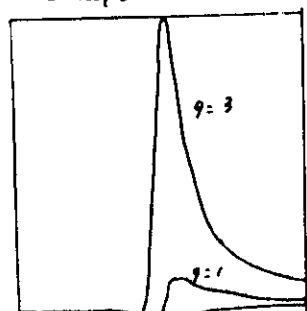
$$|\langle \phi_E | \mu \cdot E | g \rangle|^2 = |\langle \psi_E | \mu \cdot E | g \rangle|^2 \frac{[\epsilon(\omega) + q]^2}{\epsilon(\omega) + 1}$$

$$\text{where } \epsilon(\omega) = \frac{\omega - E_\phi - F(\omega)}{\pi |V_E|^2}$$

and

$$q = \frac{\langle \psi_E | \mu \cdot E | g \rangle}{\pi V_E \langle \phi | \mu \cdot E | g \rangle} \quad \begin{matrix} \text{asymmetry} \\ \text{parameter} \end{matrix}$$

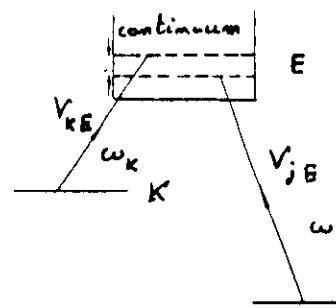
lineshape



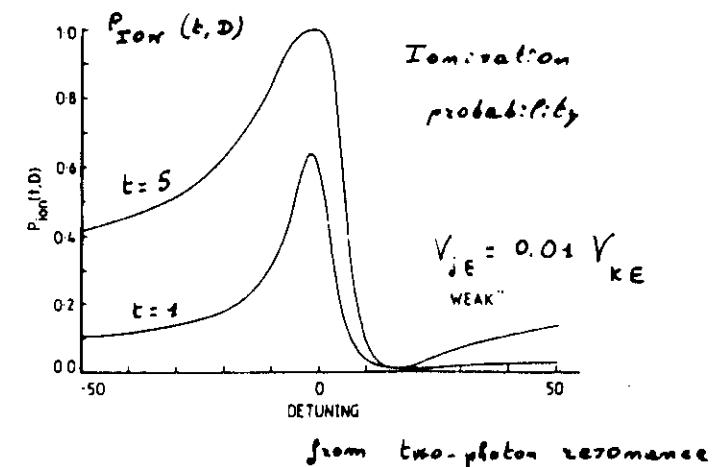
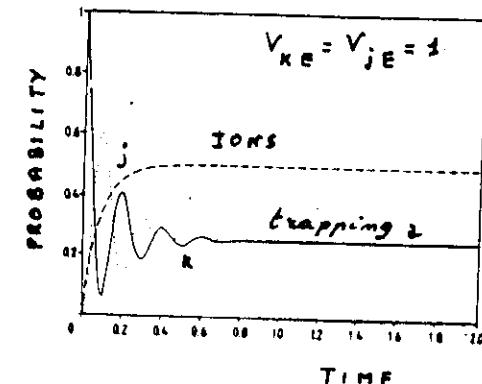
Population trapping

in photoionization

(Knight et al, 1982)



q = 5



from two-photon resonance

A diabatic following and

population trapping

(Bergmann, Hioe, (1969))

$$\mathcal{H} = \hbar \begin{pmatrix} 0 & \alpha(t) & 0 \\ \alpha(t) & \Delta(t) & \beta(t) \\ 0 & \beta(t) & 0 \end{pmatrix}$$

$$|\psi(t)\rangle = \frac{\beta(t)}{\sqrt{\alpha^2 + \beta^2}} |1\rangle - \frac{\alpha(t)}{\sqrt{\alpha^2 + \beta^2}} |2\rangle$$

$$\mathcal{H} |\psi(t)\rangle = 0 |\psi(t)\rangle$$

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \mathcal{H} |\psi(t)\rangle = 0$$

adiabatic following realized

$$\text{if } \frac{\alpha(t)}{\beta(t)} \rightarrow 0 \text{ at } t \rightarrow -\infty$$

$$\text{and } \frac{\beta(t)}{\alpha(t)} \rightarrow 0 \text{ at } t \rightarrow +\infty$$

Adiabatic transfer K. Bergmann et al.,
Kaiserslautern,
W. Germany

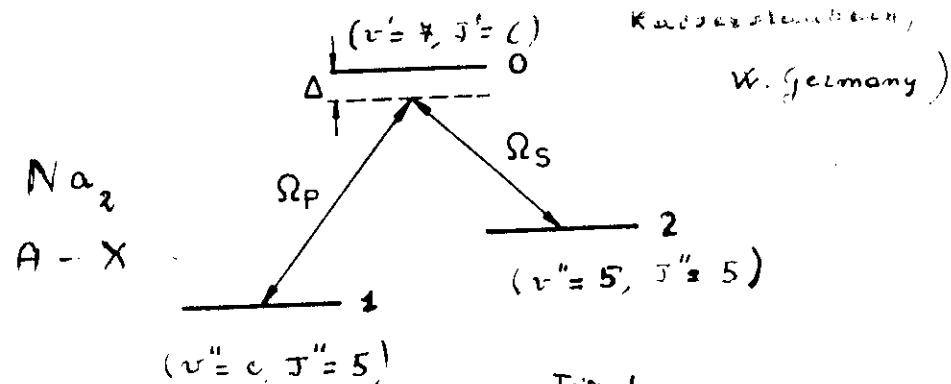


Fig. 1

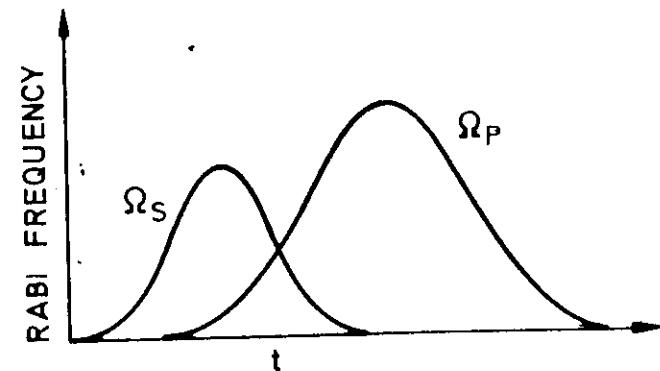


Fig. 2

Transfer to Level 3

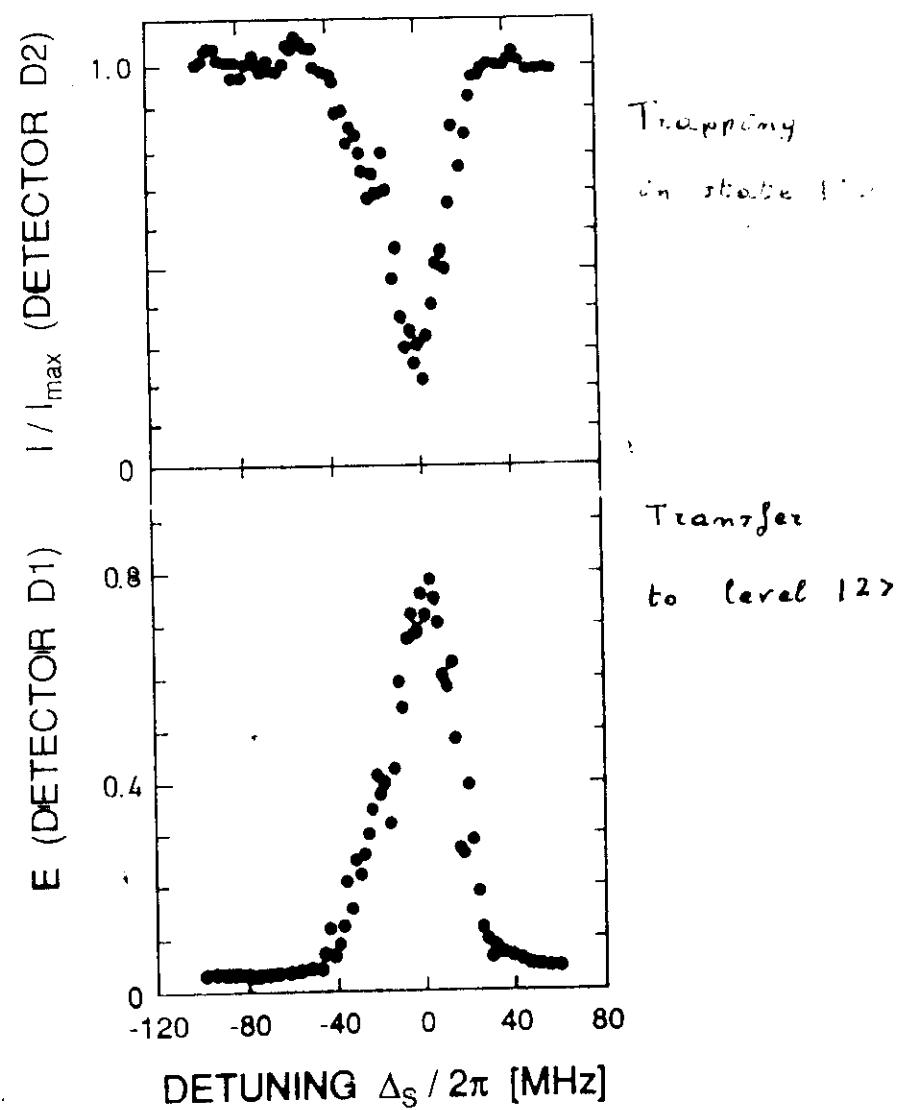
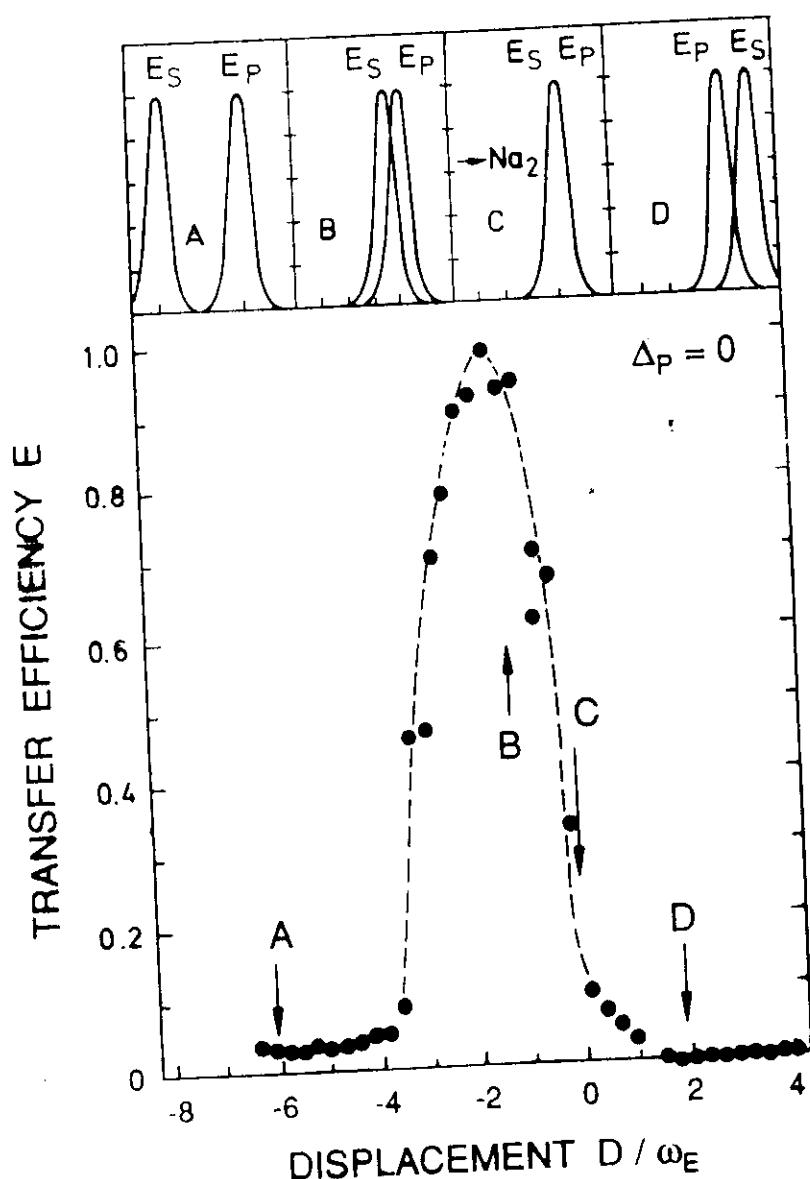
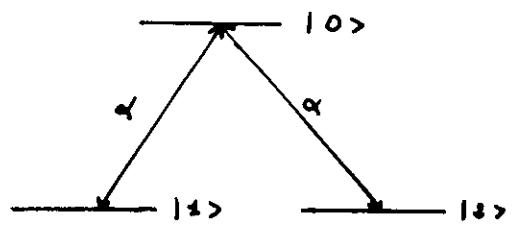


Fig. 7

Constants of motion (Hioe & Eberly
1982-89)

Bases for three-level evolutions



$$P_{00}, P_{11}, P_{22}$$

$$cx = (P_{22} + P_{00})/2$$

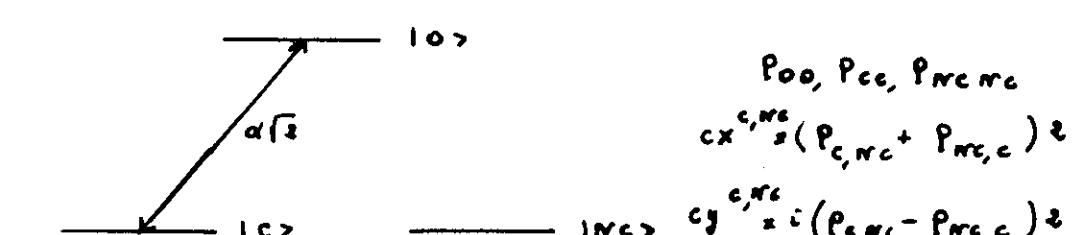
$$cy = i(P_{22} - P_{00})/2$$

$$sz_x = (P_{20} + P_{02})/2$$

$$sz_y = i(P_{20} - P_{02})/2$$

$$sz_z = (P_{20} + P_{02})/2$$

$$sy_x = i(P_{20} - P_{02})/2$$



$$P_{00}, P_{11}, P_{22,nc}$$

$$cx^{c,nc} = (P_{2,nc} + P_{0,nc})/2$$

$$cy^{c,nc} = i(P_{2,nc} - P_{0,nc})/2$$

$$sx^c = (P_{c,0} + P_{o,c})/2$$

$$sy^c = i(P_{c,0} - P_{o,c})/2$$

$$sx^{nc} = (P_{nc,0} + P_{o,nc})/2$$

$$sy^{nc} = i(P_{nc,0} - P_{o,nc})/2$$

In the 3×3 space we introduce the coherence vector \underline{S} with components:

$$\underline{S} = \begin{Bmatrix} sz^c \\ sy^c \\ P_{00} - P_{cc} \\ cx^{c,nc} \\ cy^{c,nc} \\ sz^{nc} \\ sy^{nc} \\ P_{nc,nc} \end{Bmatrix} \begin{cases} \equiv \underline{A} \\ \equiv \underline{B} \\ \equiv \underline{C} \end{cases}$$

[for $\alpha = 1$]

In absence of relaxations,

i) for two-photon resonance $\Delta_1 = \Delta_2 = \Delta$

ii) RWA

iii) same time dependence for fields and frequencies

the space may factor into 3 independent spaces of dimensions 3, 4, 1

$$\frac{d}{dt} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{bmatrix} \Delta_3 & 0 & 0 \\ 0 & \Delta_4 & 0 \\ 0 & 0 & \Delta_2 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix}$$

with $\Delta_2 \equiv 0$

following quantities are conserved:

$$|A|^2 = (sx^c)^2 + (sy^c)^2 + (p_{cc} - p_{cc})^2$$

$$|B|^2 = (sx^{nc})^2 + (sy^{nc})^2 + (cx^{nc})^2 + (cy^{nc})^2$$

$$|C|^2 = (p_{nc, nc})^2$$

furthermore following quantities are constants

$$c_j = \text{Tr}[\rho^j] \quad j = 1, 2, 3$$

$$c_1 = \text{Tr}[\rho] = p_{cc} + p_{cc, nc}$$

$$c_2 = \text{Tr}[\rho^2] = \frac{1}{2} |S|^2 + \frac{1}{3}$$

$$c_3 = \text{Tr}[\rho^3] = \dots$$

For treatment of
laser cooling phenomena

internal variables (states 10, 11, 12)

external variables ($X_{\text{at}}, P_{\text{at}}$)

treated simultaneously \rightarrow atomic momentum

Wavefunction $\psi(i, P_{\text{at}})$

Hamiltonian $\xrightarrow{\text{internal}} \kappa_{in}|i\rangle = E_i|i\rangle$
 $\xrightarrow{\text{kinetic energy}} \kappa_{KE}|\rho\rangle = \frac{p^2}{2m}|\rho\rangle$
 $\xrightarrow{\text{optical}} \kappa_{opt}$

Evolution equation for density matrix

$$P_{ij}(P, P') = \langle i, \rho | \rho | j, P' \rangle$$

$$i\hbar \frac{d}{dt} \rho = [\rho, \kappa_{in} + \kappa_{KE} + \kappa_{opt}]$$

$|1\rangle$ $\hbar\omega, \hbar k$



Statistical evolution

$|1\rangle$
atom
 $\bullet \rightarrow v$

in laser cooling.

$|1\rangle \bullet \rightarrow v$

Random walk process

with damping force

absorption $|0\rangle \bullet \rightarrow v \leftarrow \frac{\hbar k}{M}$

$$F = -\sigma v_{\text{at}}$$

and diffusion term D

stimulated } $|1\rangle \bullet \rightarrow v$
emission }

spontaneous } $|1\rangle \bullet \leftarrow \frac{-\hbar k}{M} v$
emission }

By one-photon absorption
emission

atomic energy becomes $\frac{(\hbar k)^2}{2M}$

$$\text{recoil length} T_{\text{rec}} = \frac{1}{g_B} \frac{(\hbar k)^2}{2M}$$

Equazione di Fokker-Planck

per $P(P_{at}, t)$:

$$\frac{\partial P}{\partial t} = \gamma \frac{\partial (P_{at} P)}{\partial P_{at}} + \frac{\partial}{\partial P_{at}} \left(D \frac{\partial P}{\partial P_{at}} \right)$$

dove $\gamma = - \frac{e \hbar k^2}{M} n_0 \frac{\omega - \omega_A}{(\omega - \omega_A)^2 + \Gamma^2/4}$

$$D = m_0 \frac{(\hbar k)^2}{2} (1 + \cos^2 \theta)$$

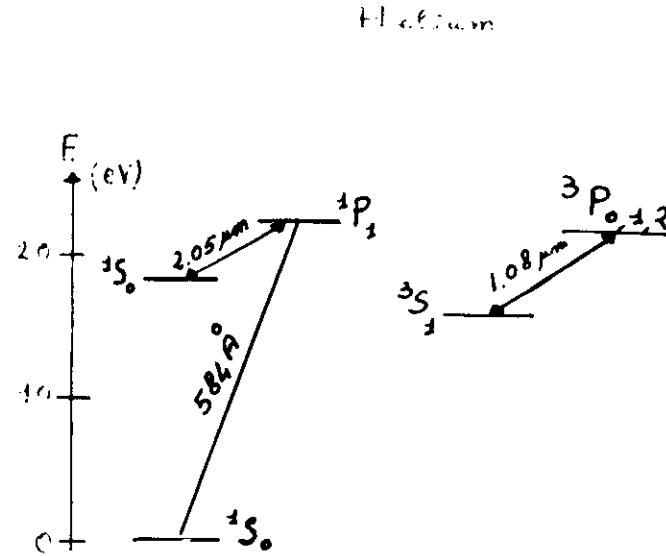
Soluzione stazionaria:

$$P_{st}(P_{at}) = C^{t_0} \exp \left(- \frac{\sigma P_{at}^2}{2D} \right) =$$

$$= C^{t_0} \exp \left(- \frac{P_{at}^2}{2m k_B T} \right)$$

per $\omega = \omega_A - \frac{\Gamma}{2}$

$$k_B T_{lim} = \frac{D}{\gamma M} = \frac{\hbar \Gamma}{2} \frac{1 + \cos^2 \theta}{4}$$



^4He metastable atoms in 3S_1 state
irradiated by $1.08 \mu\text{m}$ radiation

$$\Delta v_{\text{Recoil}} = \frac{\hbar k}{M} = 9.16 \text{ cm/sec.}$$

$$T_{\text{Recoil}} = \frac{\hbar^2 k^2}{2 M k_B} = 4 \mu\text{K}$$

$$T_{\text{Cooling, 1D}} = \frac{7 \hbar \Gamma}{20 k_B} = 26 \mu\text{K}$$

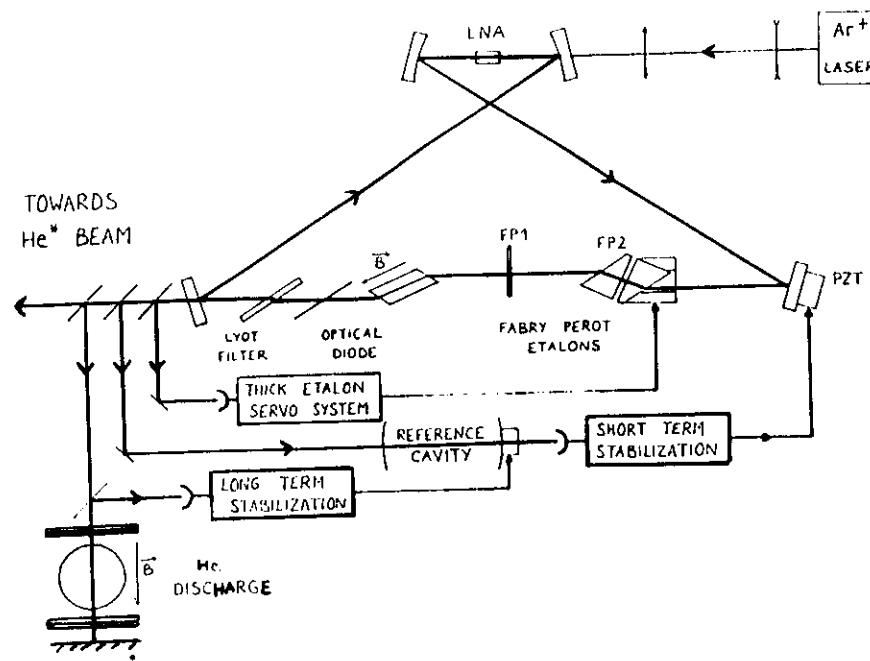
^4He experiment at ENS - Paris

(A. Aspect, E.A., R. Kaiser, N. Vansteenkiste,

C. Cohen-Tannoudji)

LNA Laser

$\text{La}_{0.9}\text{Nd}_{0.1}\text{Mg}_x\text{Al}_{3-x}\text{O}_{19}$



He metastable
beam

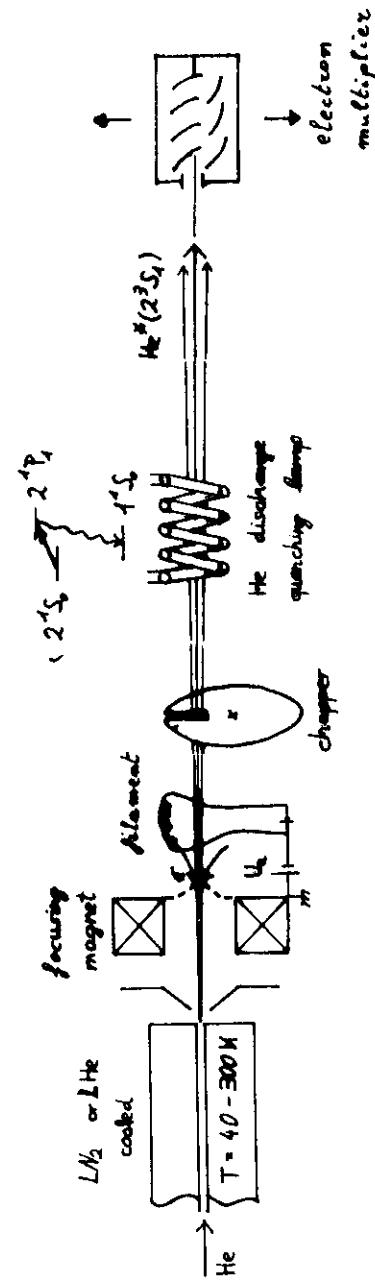
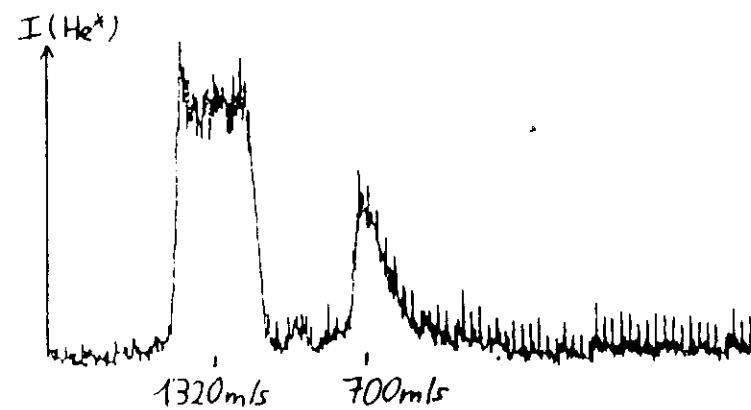


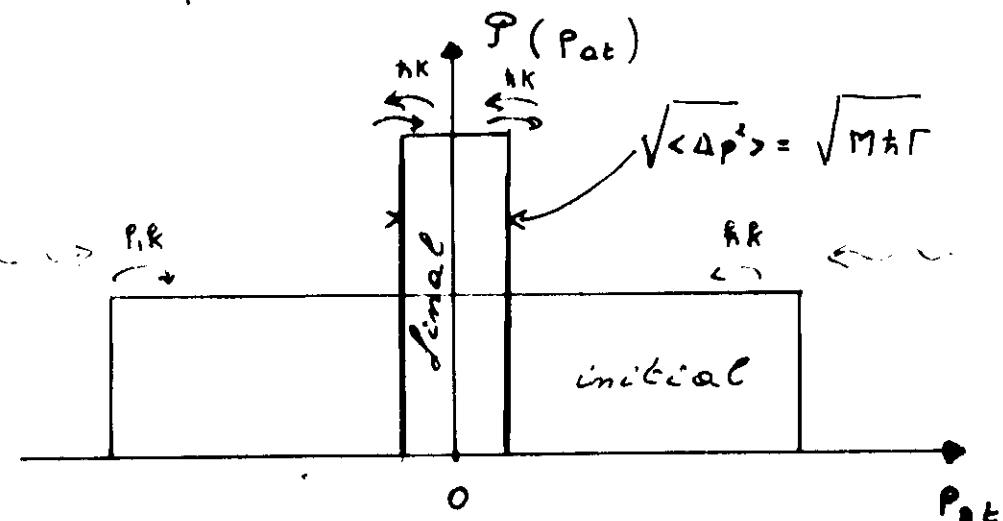
Fig F
velocity distribution



liquid nitrogen He beam

Velocity optical pumping in
spontaneous or stimulated

optical molasses:



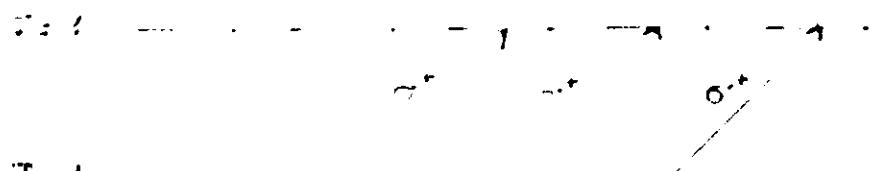
"Optical pumping in a
non-absorbing velocity selective state?

Experiments of

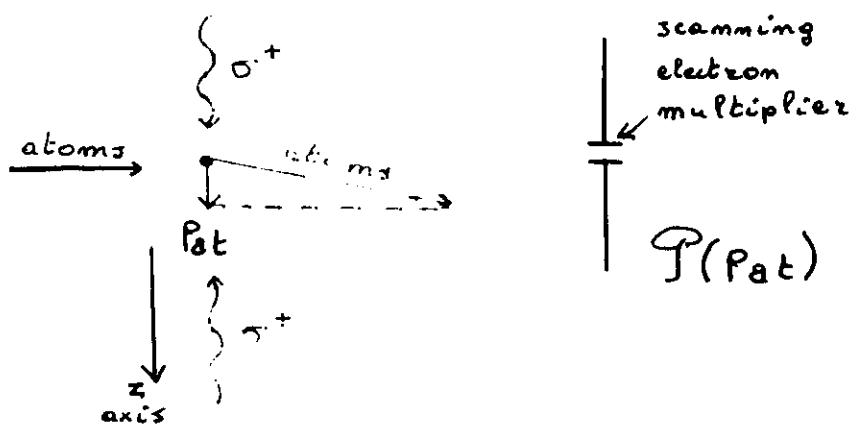
radiation pressure molasses

on $2^3S_1 \rightarrow 2^3P_2$ transition

σ^+ light.



$$m_J = -2 \quad -1 \quad 0 \quad 1 \quad 2$$

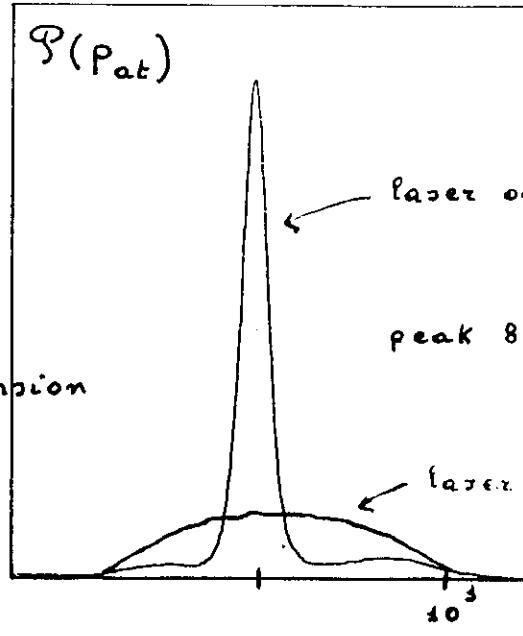


width at

$\exp(-0.5)$:

1.6 mm ≡

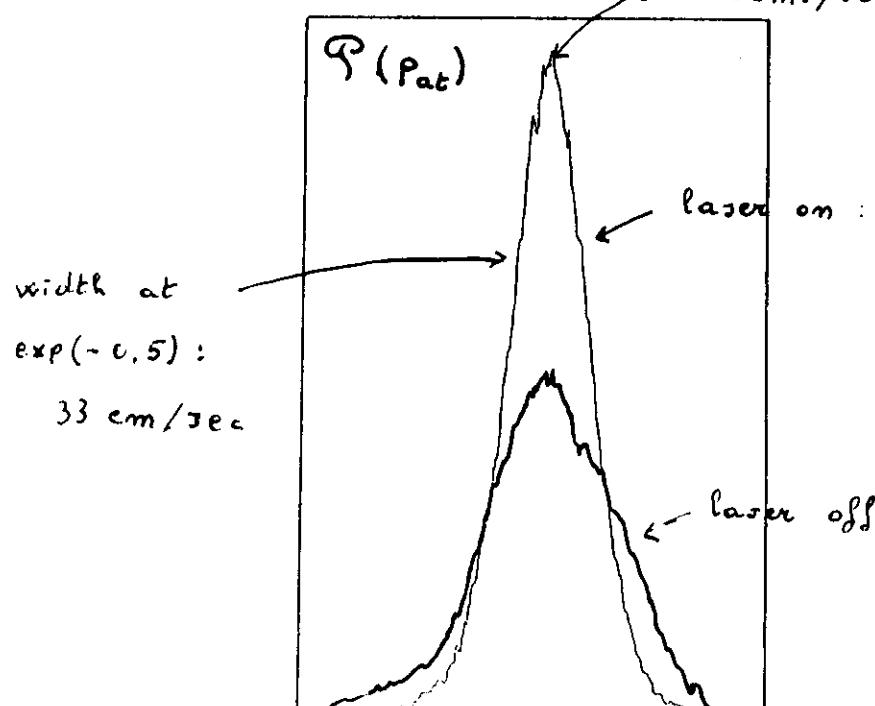
source dimension



laser on: $\Delta_L = -1.5$

$\omega_z = 2.5 \Gamma$

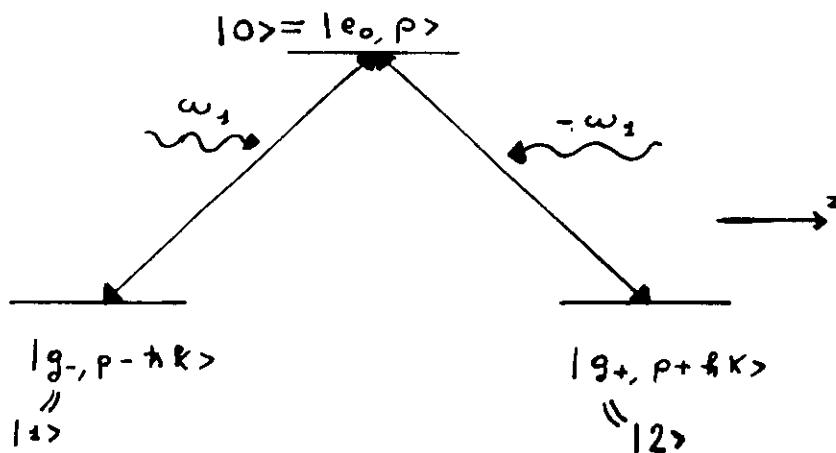
peak 8 times increas-



laser on: $\Delta_L = -0.6$

$\omega_z = \Gamma /$

"Vortex-g: selection efficient propagation trapping,"



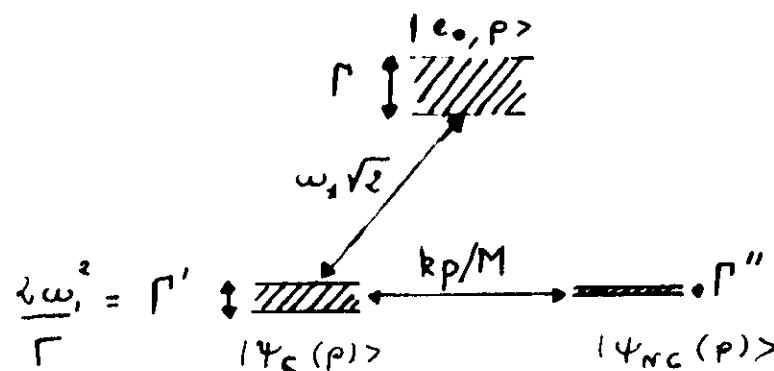
Non-absorbing state:

$$|\Psi_{NC}(p)\rangle = \frac{1}{\sqrt{2}} \left[|g-, p - \hbar k\rangle + |g+, p + \hbar k\rangle \right]$$

Absorbing state

$$|\Psi_C(p)\rangle = \frac{1}{\sqrt{2}} \left[|g-, p - \hbar k\rangle - |g+, p + \hbar k\rangle \right]$$

$$E_{\text{kinetic}} = \frac{(p - \hbar k)^2}{2M} - \frac{(p + \hbar k)^2}{2M}$$



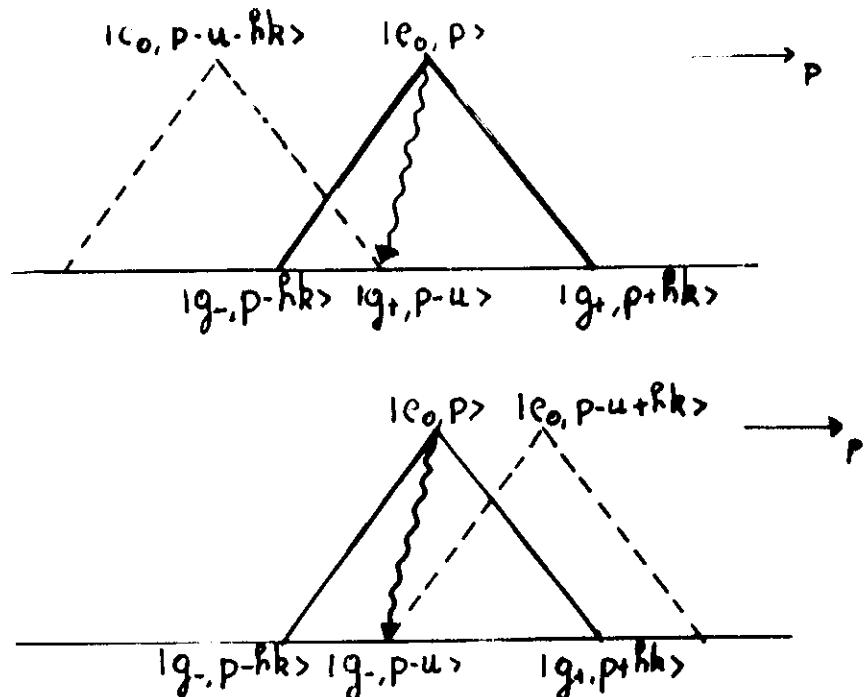
The state

$$|\Psi_{NC}(0)\rangle = \frac{1}{\sqrt{2}} \left[|g-, -\hbar k\rangle - |g+, +\hbar k\rangle \right]$$

is a perfect trap:

an atom prepared in this state will remain there indefinitely.

Accumulation into the
non-absorbing and trapping state $|\psi_{NA}(0)\rangle$
through p-diffusion



A measurement of the momentum
Pat on the $|\psi_{nc}(0)\rangle$ state

$$|\psi_{nc}(0)\rangle = \frac{1}{\sqrt{2}} [|g_-, -\hbar k\rangle + |g_+, +\hbar k\rangle]$$

yields

either $P_{at} = -\hbar k$

or $P_{at} = +\hbar k$

The atomic momentum distribution

$$\rho(P_{at})$$
 has peaks at $P_{at} = \pm \hbar k$

Density matrix equations for
coherent trapping:

$$\frac{d}{dt} \rho = \frac{i}{\hbar} [\rho(t), \mathcal{H}] + \{ \mathcal{H}_{\text{rec}}, \rho \}$$

$$\mathcal{H} = E_0 |0\rangle\langle 0| + E_z |z\rangle\langle z| + E_x |x\rangle\langle x|$$

$$+ \left\{ \alpha |0\rangle\langle z| e^{-i\omega_z t} + \beta |0\rangle\langle x| e^{-i\omega_x t} + \text{c.c.} \right\}$$

$$\left. \frac{d}{dt} \rho_{00} \right)_{\text{rel}} = -\Gamma \rho_{00} \quad (\rho)$$

$$\left. \frac{d}{dt} \rho_{0z} \right)_{\text{rec}} = -\frac{\Gamma}{2} \rho_{0z}, \quad \text{etc}$$

$$\left. \frac{d}{dt} \rho_{zz} \right)_{\text{rel}} = \frac{\Gamma}{2} \rho_{00} - \frac{1}{\tau_z} (\rho_{zz} - \rho_{xx}), \quad \text{etc} \quad (\rho)$$

$$\left. \frac{d}{dt} \rho_{xz} \right)_{\text{rec}} = -\frac{1}{\tau_z} \rho_{xz}$$

velocity-selective

$$+ \frac{\rho^2}{2M}$$

$$\frac{\Gamma}{2} \int_{-\hbar k}^{+\hbar k} du H(u) \rho_{00}(p+u)$$

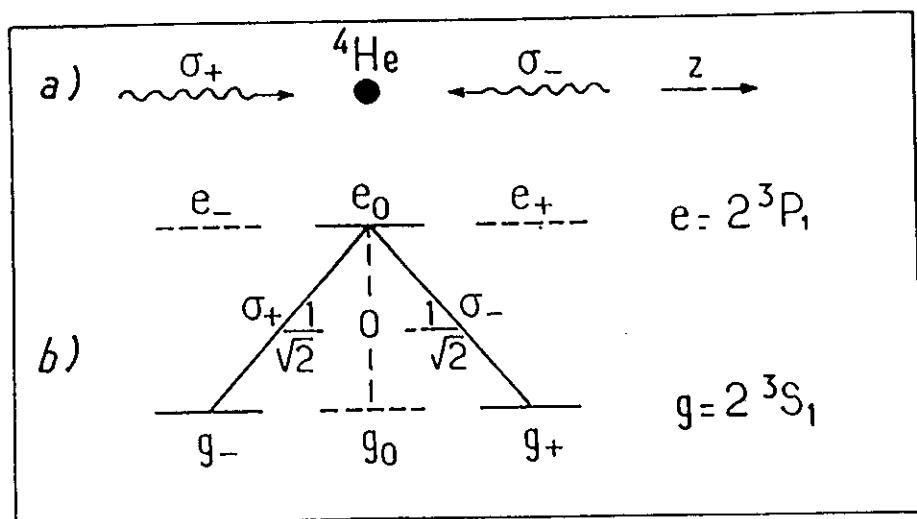
Calculated atomic momentum distribution

^4He level scheme for

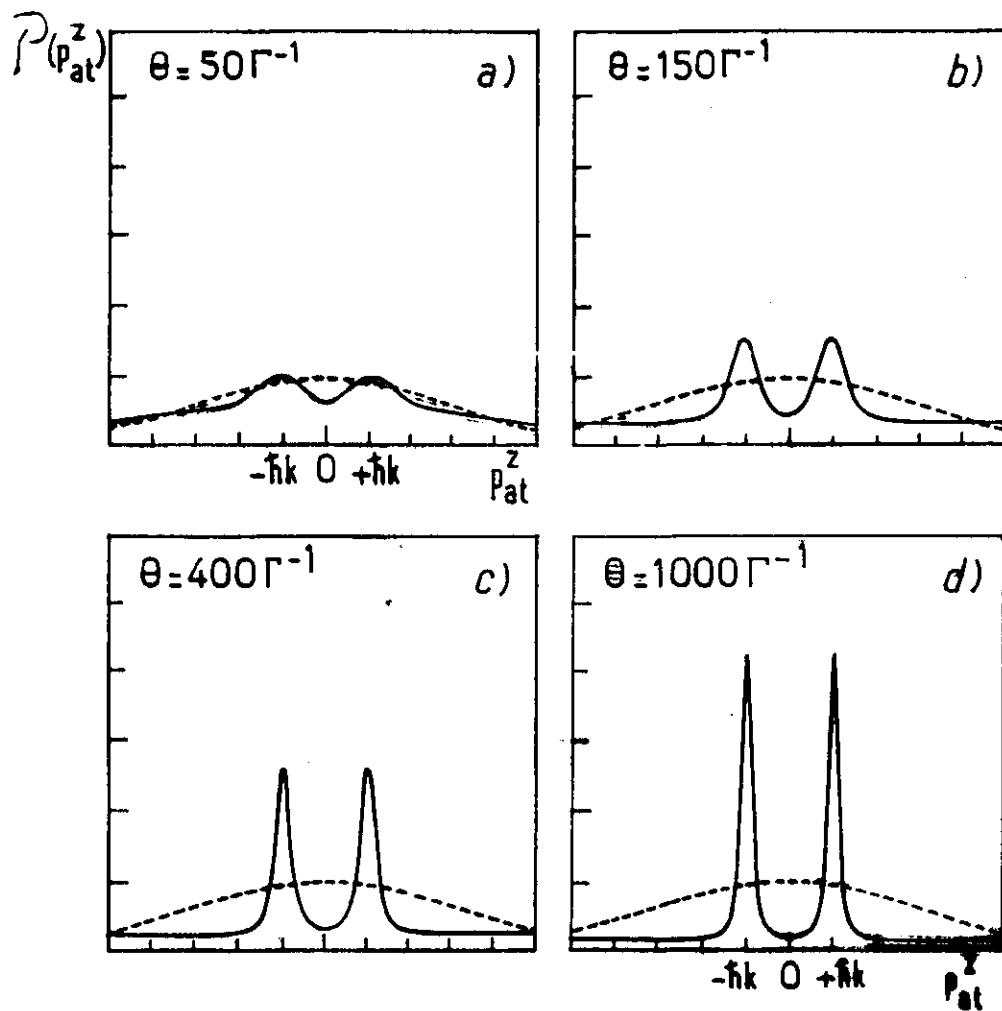
laser cooling based on

Velocity selective

Coherent population trapping



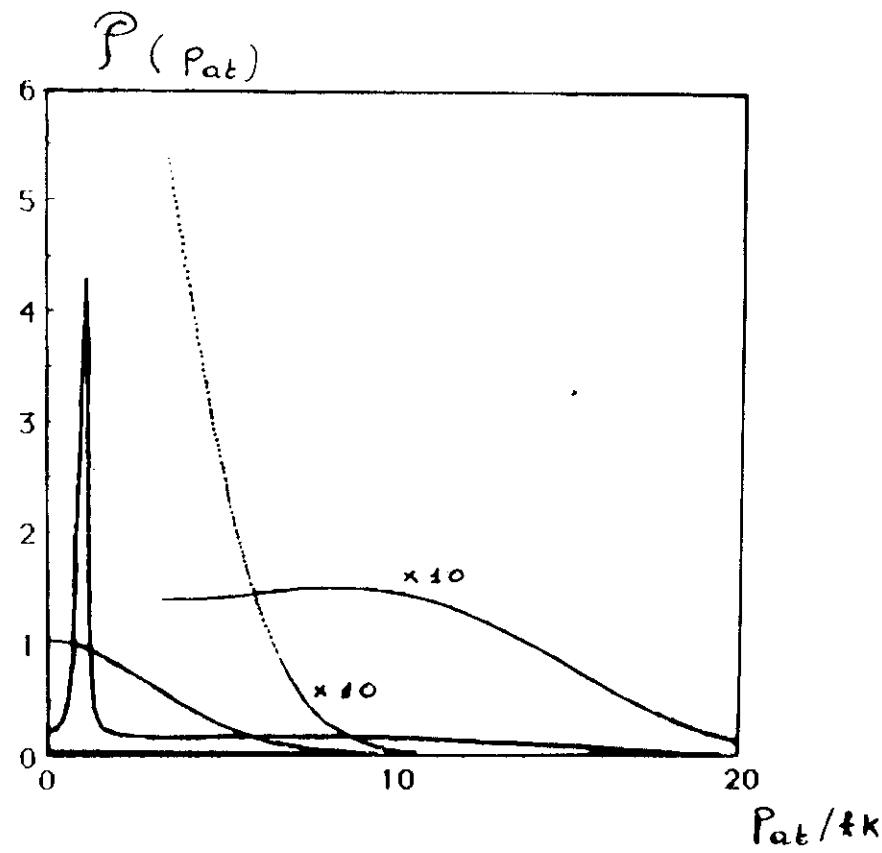
A. Aspect, E.A., R. Kaiser, N. Yastrebov, C. Cohen-Tannoudji
 Phys. Rev. Lett. 61 826 (1988)



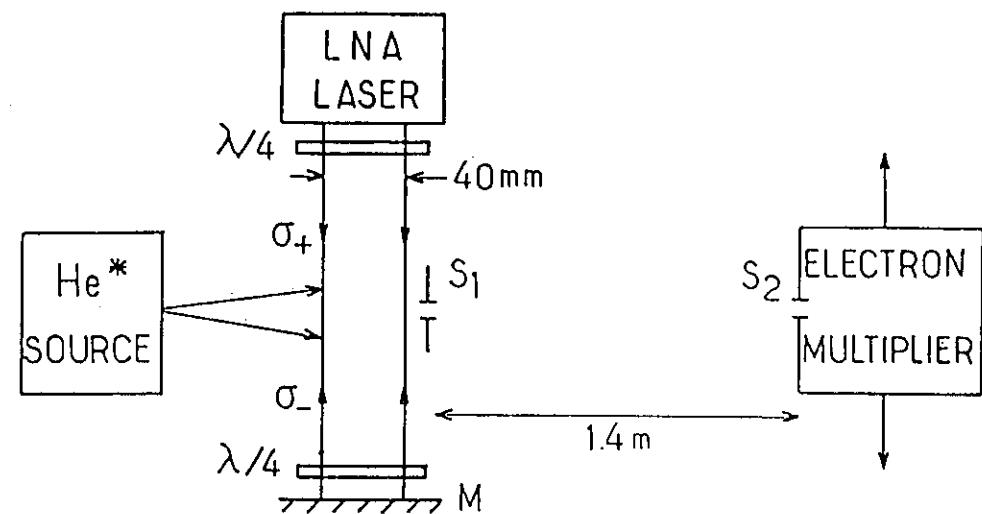
$$\Delta p_{\text{at}}(0) = 3 \hbar k$$

$$\Delta_L = 0; \quad \omega_s = 0.3 \Gamma$$

Calculated atomic momentum distribution

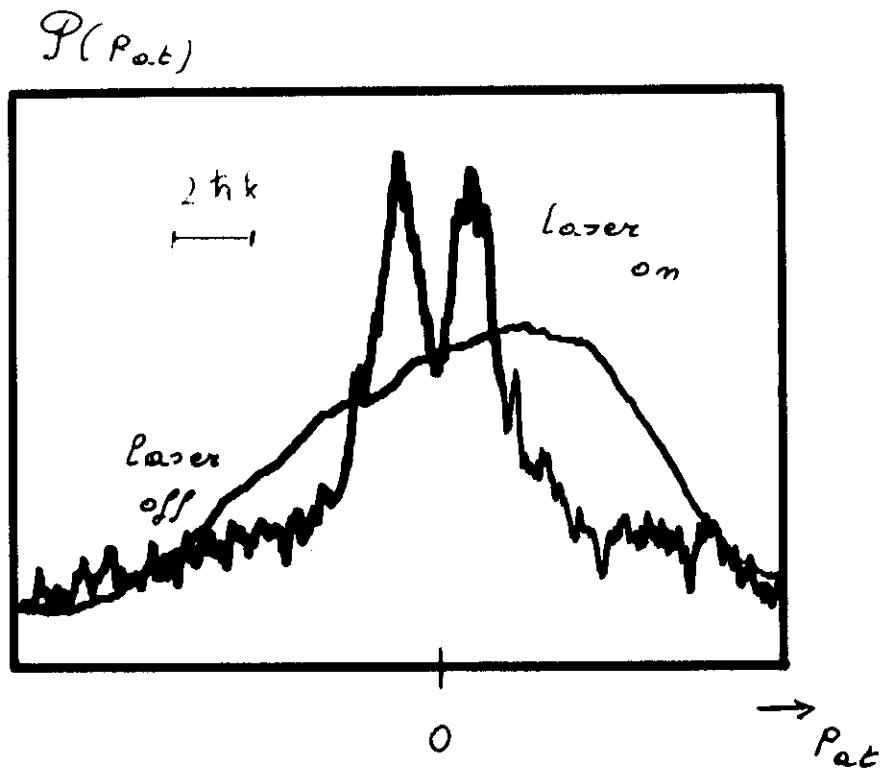


- initial $\Delta p_{ab} = 3 \text{ fk}$
- final at $\Theta \Gamma = 1000$
- $\Delta_c = 0$ $\omega_s = 0.3 \Gamma$



Schematic set-up

Transverse atomic momentum
profile



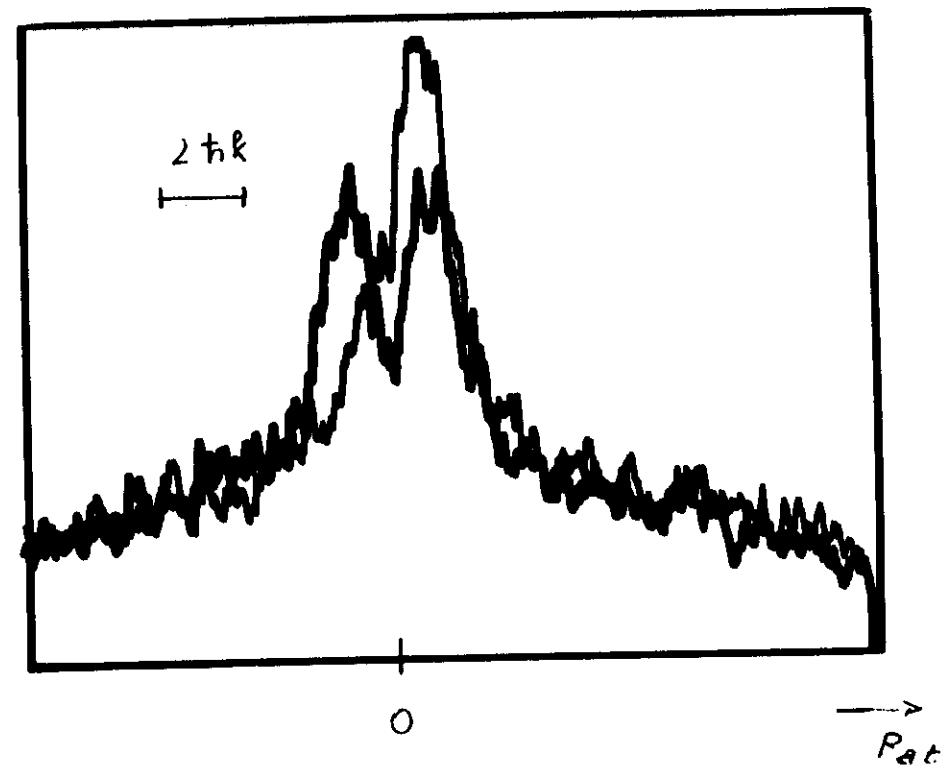
$$\Delta_L = 0$$

$$\omega_z = 0.6 \text{ } \Gamma$$

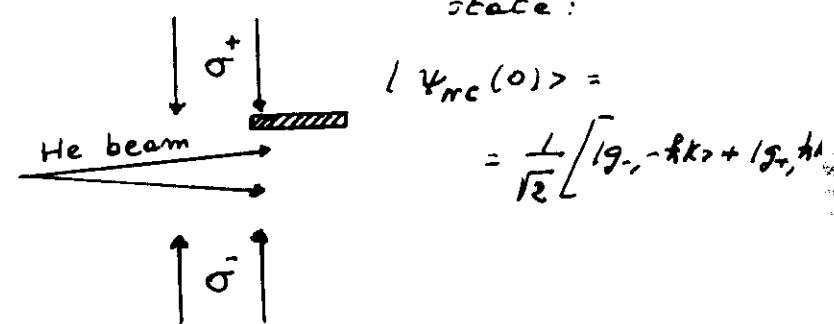
40 mm interaction region

$$T = 2 \mu\text{K}$$

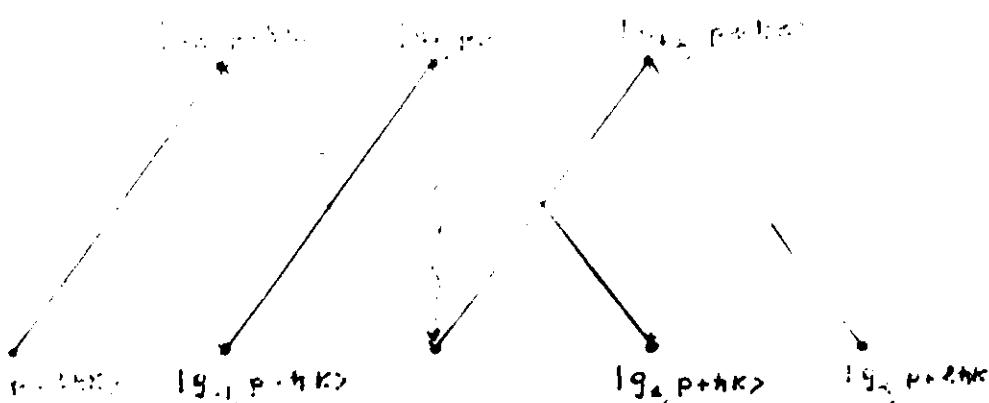
$\mathcal{P}(P_{\text{at}})$



Depopulation pumping from trapping state:



The $J=2 \rightarrow J=1$ transition is composed by following configurations



He trapping in $J=2 \rightarrow J=1$ transition

$$\omega_+ = \omega_- = 0.5 \Gamma$$

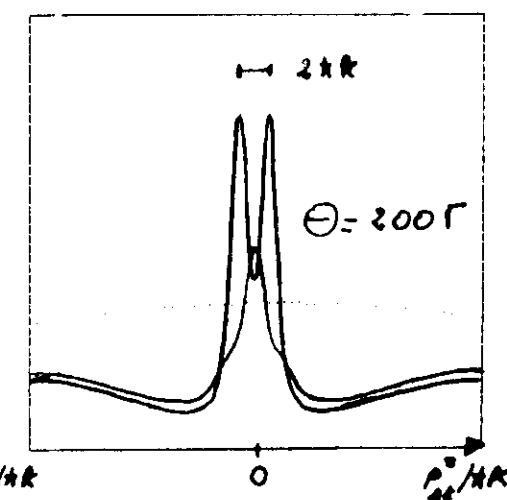
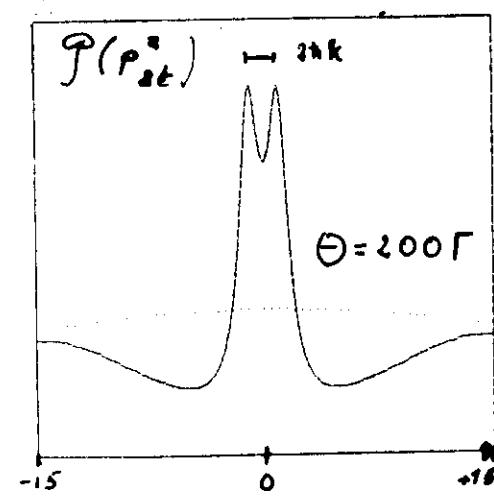
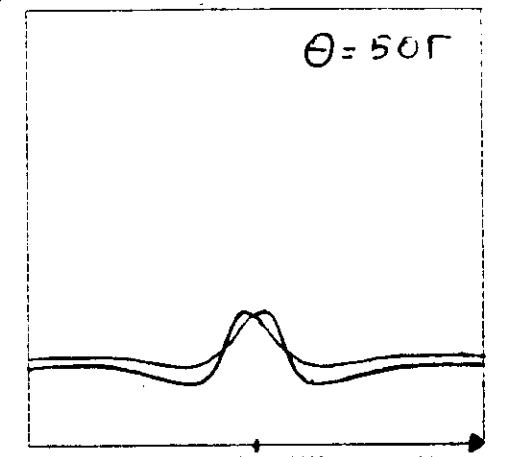
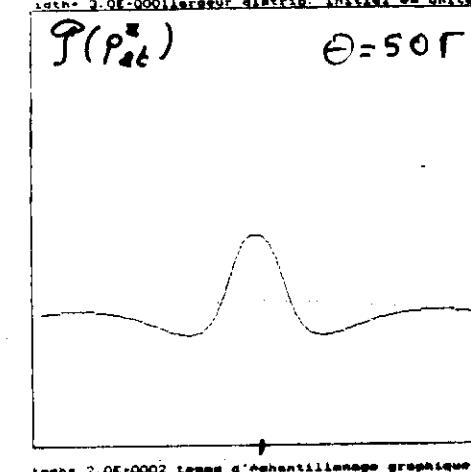
$$\delta_+ = \delta_- = 0$$

$$\Delta p_0 = 30 \text{ \AA k}$$

```

1: 3.0E-0001 fréquence de Rabi 1 en unités de gamma
2: 3.0E-0001 fréquence de Rabi 2 en unités de gamma
3: 0.0E-0000 déphasage initial entre les deux Rabi
4: 0.0E-0000 accord 2 en unités de gamma
5: 1.0E+0000 nombre total de classes de vitesses > 0
6: 1.0E+0000 nombre de préseries classes de vitesses par h/énergie
7: 0.0E+0000 pas de temps d'échantillonnage graphique
8: 0.0E+0000 temps d'échantillonnage graphique
9: 3.0E-0001 largeur distrib. initiale en unités de no.
10: 3.0E-0001

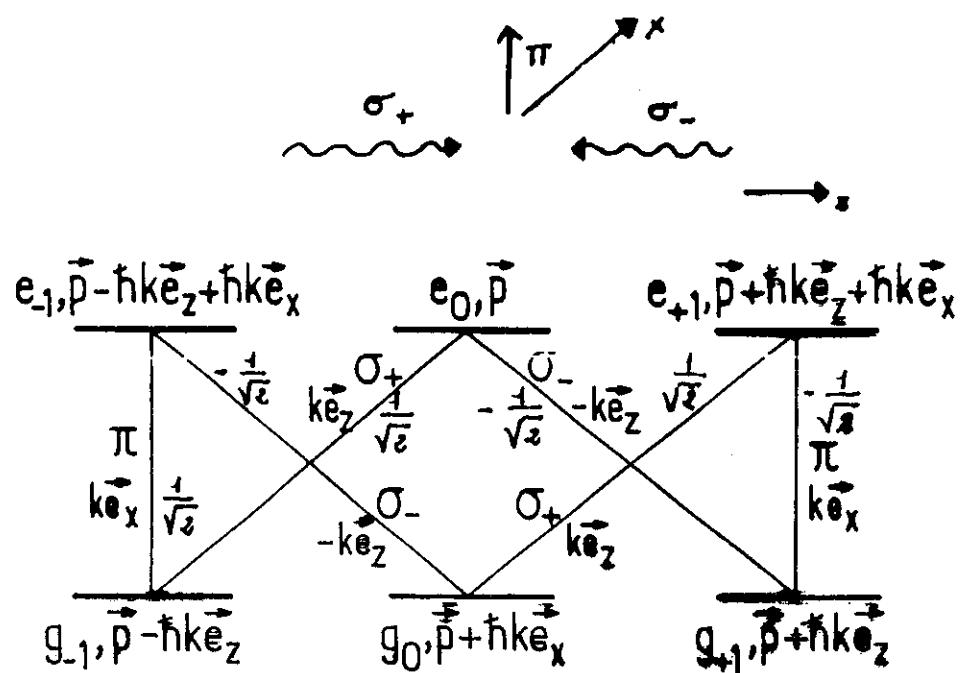
```



Contribution of each configuration:



Generalization to two dimensions

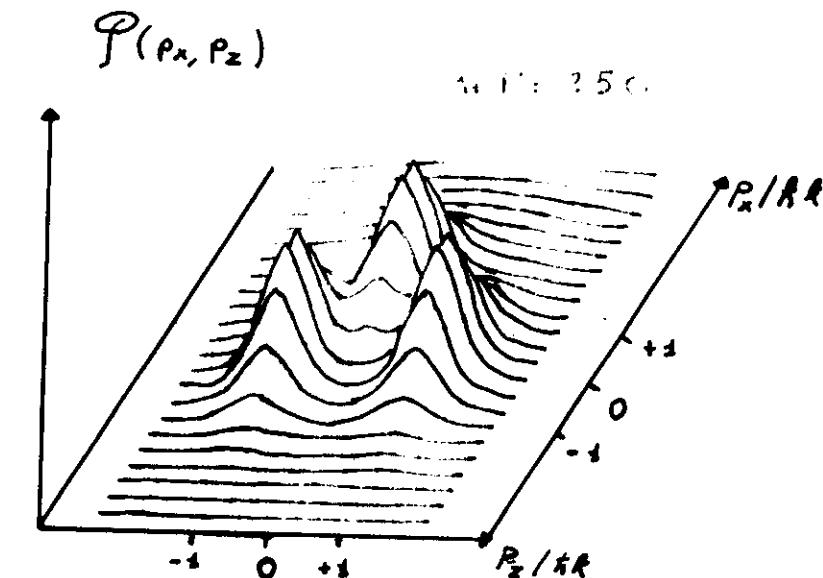


Non-compact state

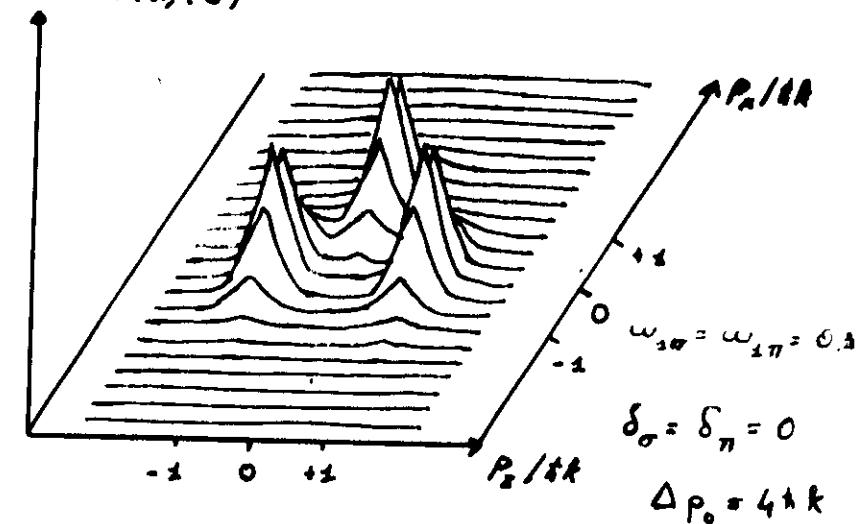
$$|\psi_{Nc}(P_x, P_z)\rangle =$$

$$\frac{1}{\sqrt{3}} \left[|g_{-1}, P_x, P_z - \hbar k\rangle + |g_0, P_x + \hbar k, P_z\rangle + |g_1, P_x, P_z + \hbar k\rangle \right]$$

Atomic distribution in
two dimension running



$$P(P_x, P_z) \quad \text{for } \Gamma = 500$$



$$\omega_{1\sigma} = \omega_{1\pi} = 0.3 \\ \delta_\sigma = \delta_\pi = 0 \\ \Delta P_0 = 4\hbar k$$

Applications of very cold atoms:

Main features of

"velocity selective coherent population trapping,"

i) cooling is not due to a friction force

but to a diffusion.

ii) for cooled atoms no interaction with

light

$$\text{iii) } |\Psi_{nc}(0)\rangle = \frac{1}{\sqrt{e}} \left\{ |g_-, -\hbar k\rangle + |g_+, +\hbar k\rangle \right\} \lambda_B = 3.4 \mu\text{m}$$

has coherence on external degrees

iv) extension to other level configurations
and geometries.

- Spectroscopy

- Metrology

- Bose-Einstein condensation

- Collisions with atoms or surfaces

In collisions of very cold atoms

it results:

in He

experiment $\rightarrow \lambda_0 = \frac{\hbar}{M v_{at}}$ \gg range of interaction
potential

\rightarrow only few partial waves
contribute to cross-section

\rightarrow dynamic of collision dominated
by weak potentials at large
distances

