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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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H4.SMR/449-35

**WINTER COLLEGE ON
HIGH RESOLUTION SPECTROSCOPY**

(8 January - 2 February 1990)

CAVITY QUANTUM ELECTRODYNAMICS

H. Walther

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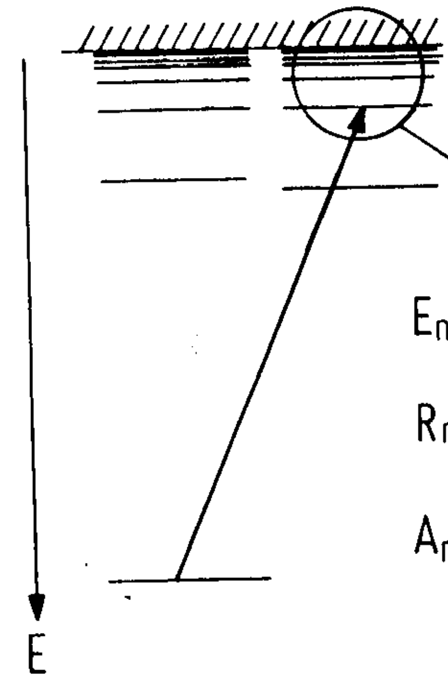
Garching D-8046
F.R. Germnay**

CAVITY QUANTUM ELECTRODYNAMICS

Properties of Rydberg Atoms



scaling laws :

$$E_n = -R/(n-\delta_l)^2 = -R/n^{*2}$$

$$R_n \propto n^{*2}$$

$$A_{nm} \propto \omega^3 |\langle n|\hat{D}|m\rangle|^2$$

Hydrogen atom

	n = 10	n = 100	units
Energy	0.14	$1.4 \cdot 10^{-3}$	eV
Radius	100	10 000	a_0
Lifetime	0.51	510	μs
Frequency	6600	6.6	GHz

Properties of Rydberg Atoms

$$E_n = R/(n-\delta_\ell)^2 = R/n^{*2}$$

n^* effective quantum number

δ_ℓ quantum defect

Radius $\propto n^{*2}$

Decay rate

$$A_{n\to m} \propto \omega^3 \, |\langle n|\hat{D}|m\rangle|^2$$

If $m \ll n \curvearrowright |\langle n|\hat{D}|m\rangle|^2 \ll 1$

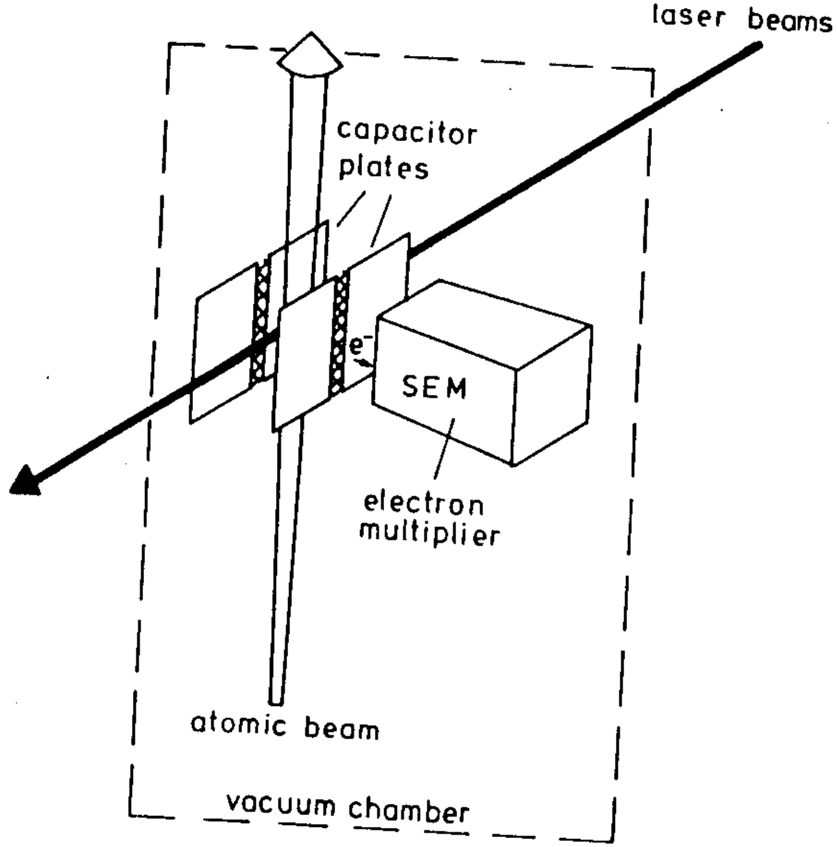
$$A_{n\to m} \propto n^{*-3}$$

If $m \approx n$ $\quad \hbar\omega = E_n - E_m \propto n^{*-3}$

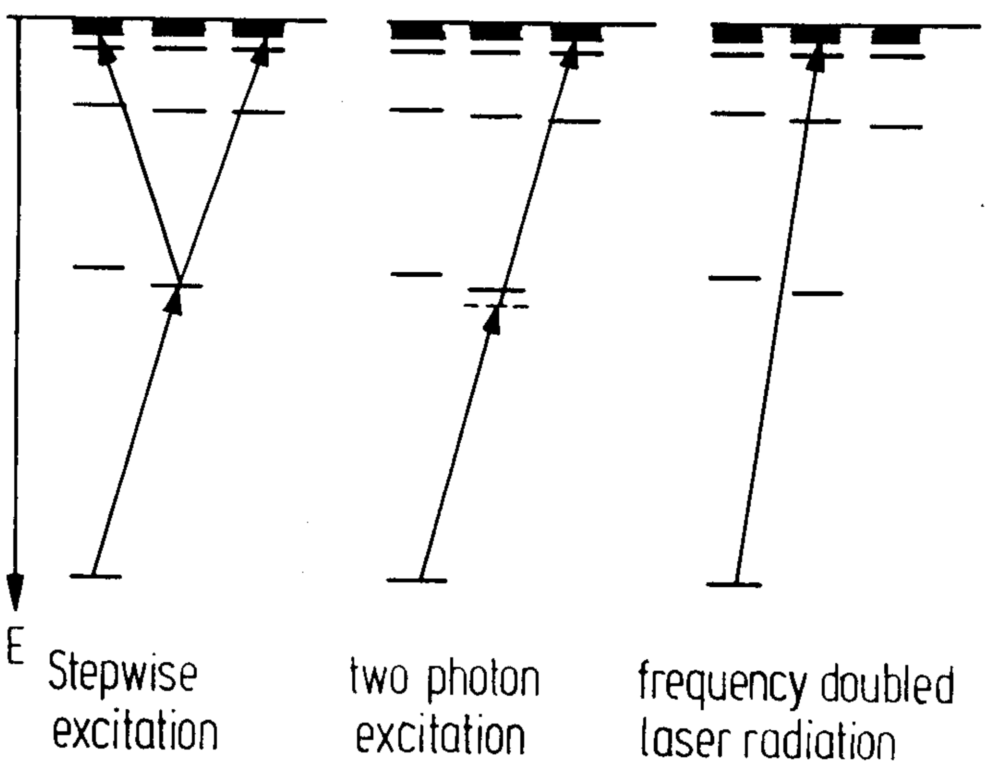
$$|\langle n|\hat{D}|m\rangle|^2 \propto n^{*4}$$

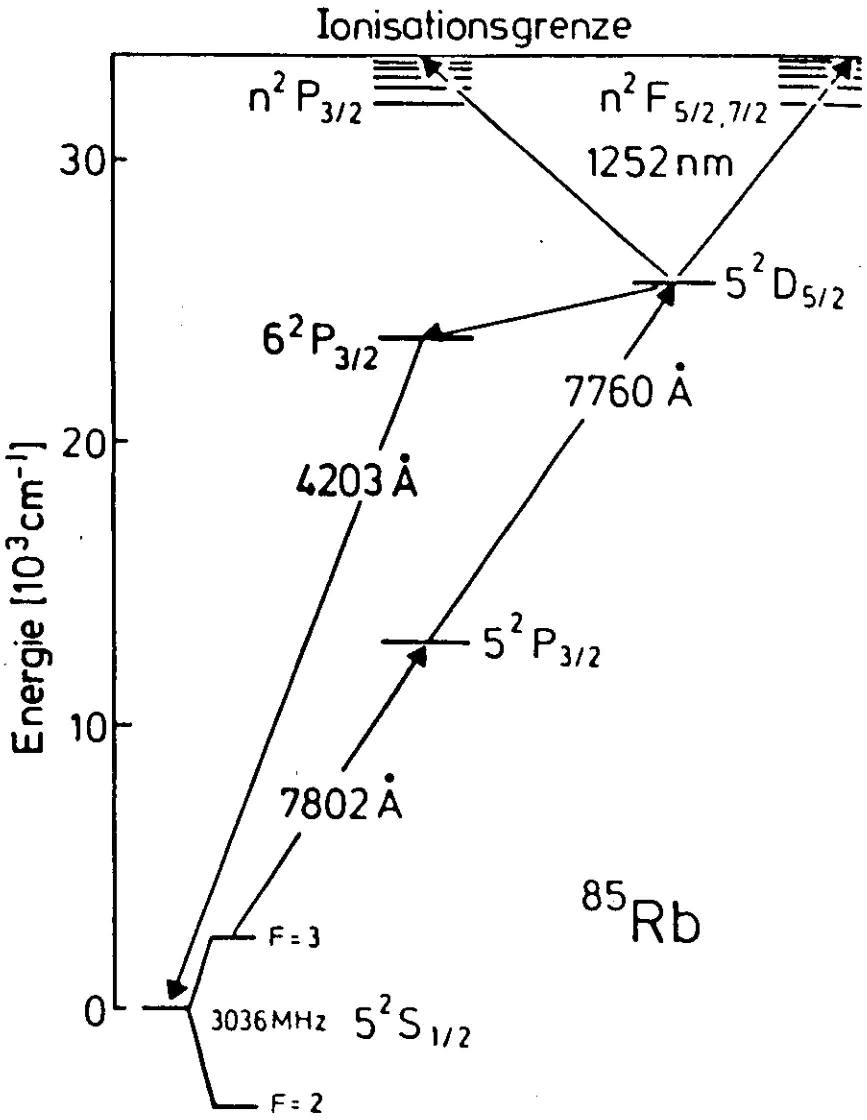
$$A_{n\to m} \propto n^{*-5}$$

Detection of Rydberg Atoms

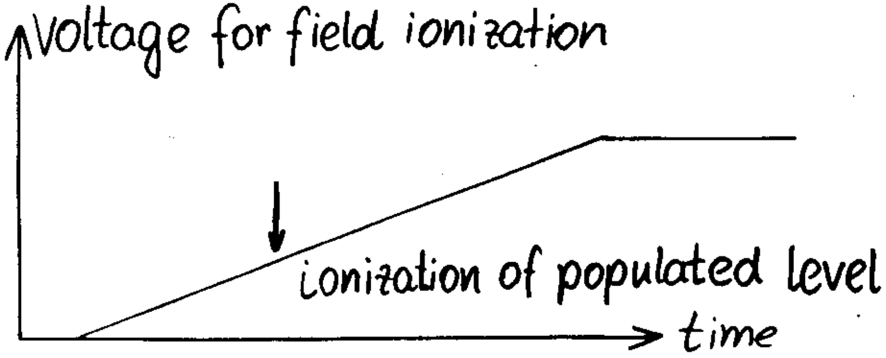


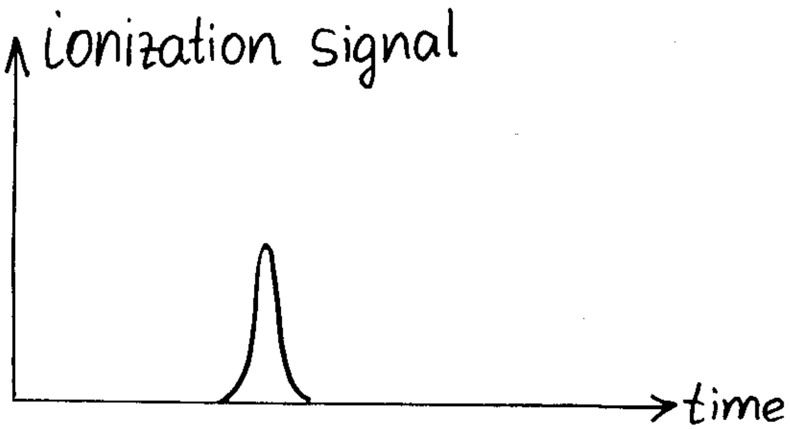
Excitation Schemes for Rydberg Atoms

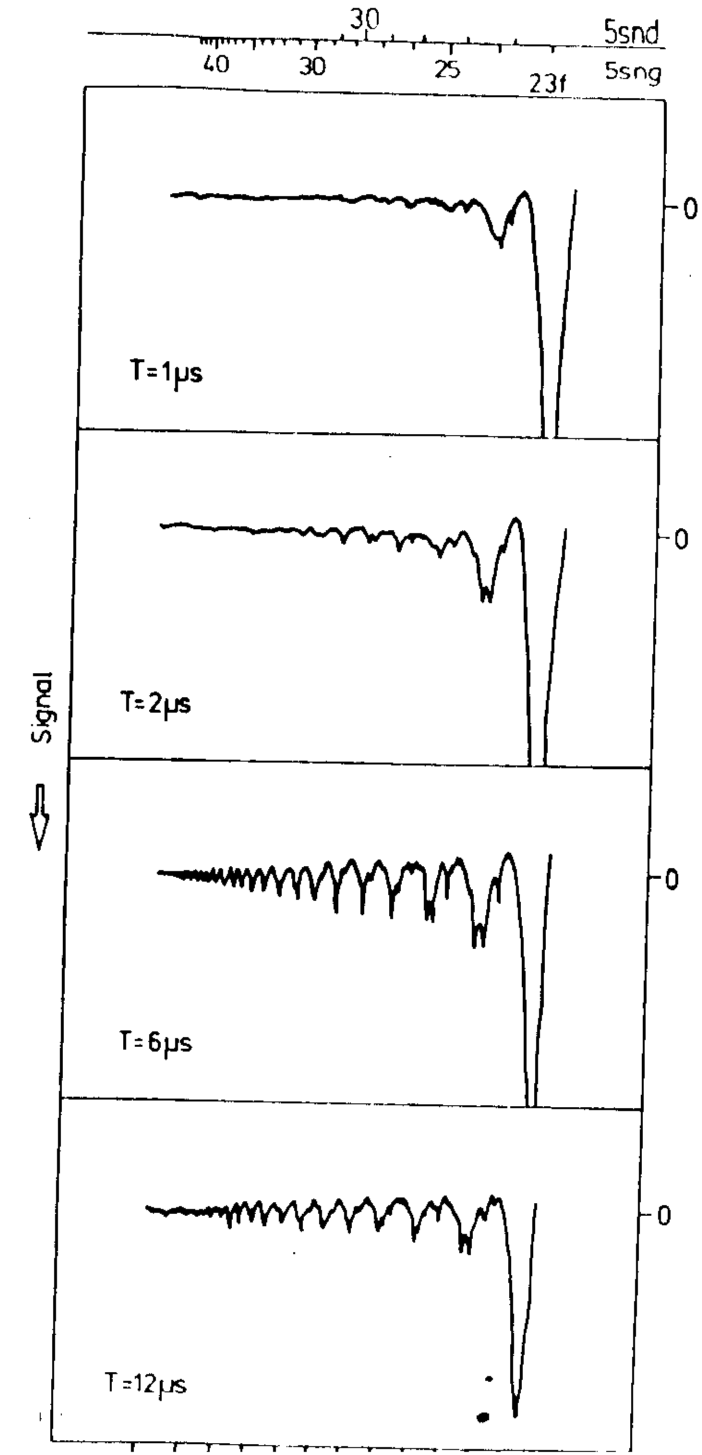




tate-selective detection of Rydberg atoms







Influence of blackbody radiation

Energy flux (Rayleigh-Jeans limit):

$$\frac{d\Phi}{d\nu} \propto \nu^2 \triangleq n^{*-6}$$

Induced transition rate:

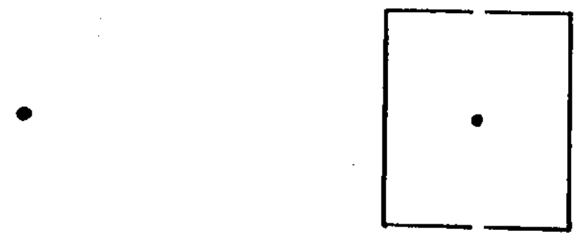
$$B_{nm} \propto \underbrace{|\langle n|\hat{D}|m\rangle|^2}_{n^{*4}} \underbrace{\frac{d\Phi}{d\nu}}_{n^{*-6}}$$

$$B_{nm} \propto n^{*-2}$$

Relative transition rate:

$$\frac{B_{nm}}{\Gamma_n} \propto \frac{n^{*-2}}{n^{*-3}} = n^*$$

Free atom versus atom in cavity



Free atom versus atom in cavity

Modification of spontaneous emission rate

Modification of Lamb shift
(also (g-2) experiments)
also level shift due to direct interaction with cavity walls

Oscillatory energy exchange
(determined by photon statistics)
(E.T. Jaynes, F.W. Cummings, Proc. IEEE 51, 89 (1963))

Modification of spontaneous emission rate

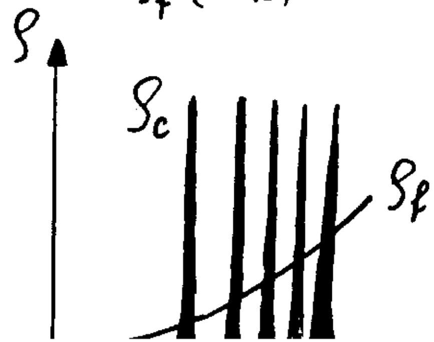
Γ: emission rate

$$\Gamma = 2\pi \int d\Omega_k \left(|V_{12}^k|^2 \rho(\omega_k) \right)_{\omega_k = \omega_0}$$

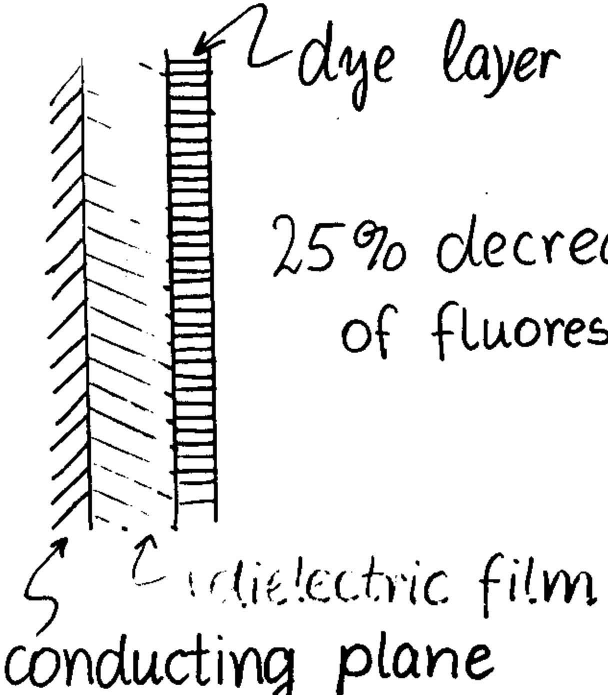
V_{12}^k dipole matrix element between states:
atom in excited state - no photon
and atom in ground state - photon in mode k

$$\rho_f(\omega_k)\,d\omega_k = \left(\frac{1}{2}\right)\left(\frac{1}{4\pi}\right)\frac{\omega_k^2\,d\omega_k}{\pi^2 c^3}$$

$$\Gamma_{cav} = \Gamma_f \frac{\rho_c(\omega_k)}{\rho_f(\omega_k)} \longrightarrow \Gamma_{cav} \approx \Gamma_f Q$$



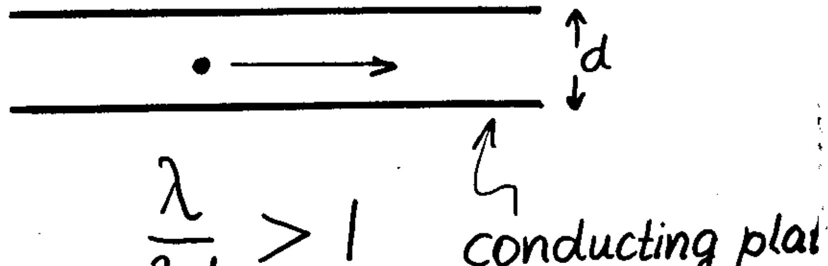
Fluorescence of dye molecules



25% decrease of fluorescence decay

Drexhage, Progress in Optics (ed. E. Wolf) Vol. 20, 163 (1974)

Inhibited spontaneous emission of Rydberg atoms



$$\frac{\lambda}{2d} > 1$$

circular states $|m| = n-1$

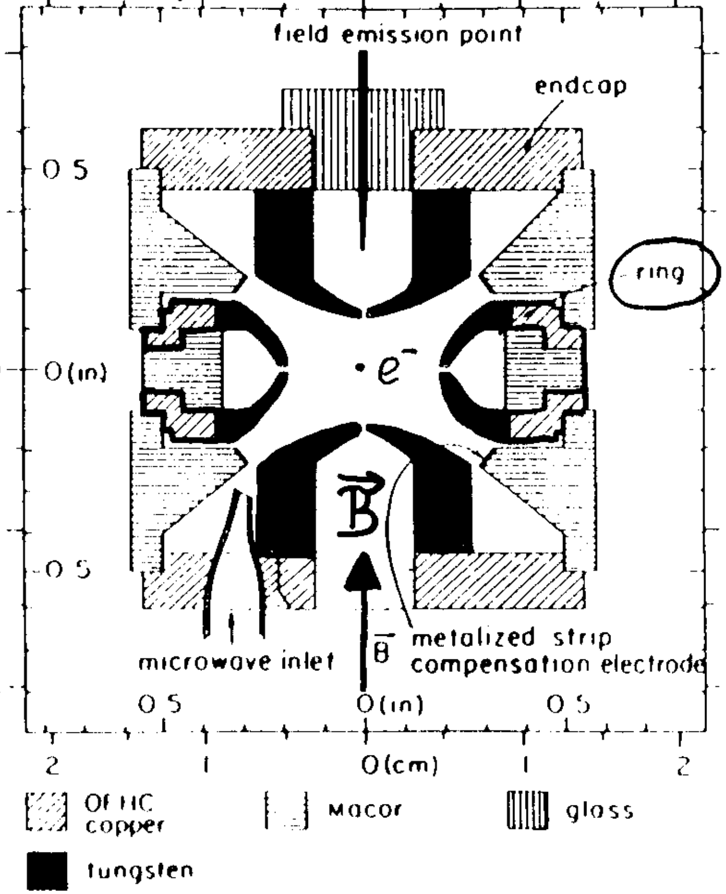
increase of lifetime observed factor 20

Hulet, Hilfer, Kleppner PRL 55, 2137 (1985)

Observation of Inhibited Spontaneous Emission

Gerald Gabrielse and Hans Dehmelt
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 16 April 1985)

radiative decay of cyclotron motion of an electron in a trap



decay rate 5 times smaller

Ihe, Anderson, Hinds, Meschede, Moi, Haroche, PRL 58, 666 (1987)

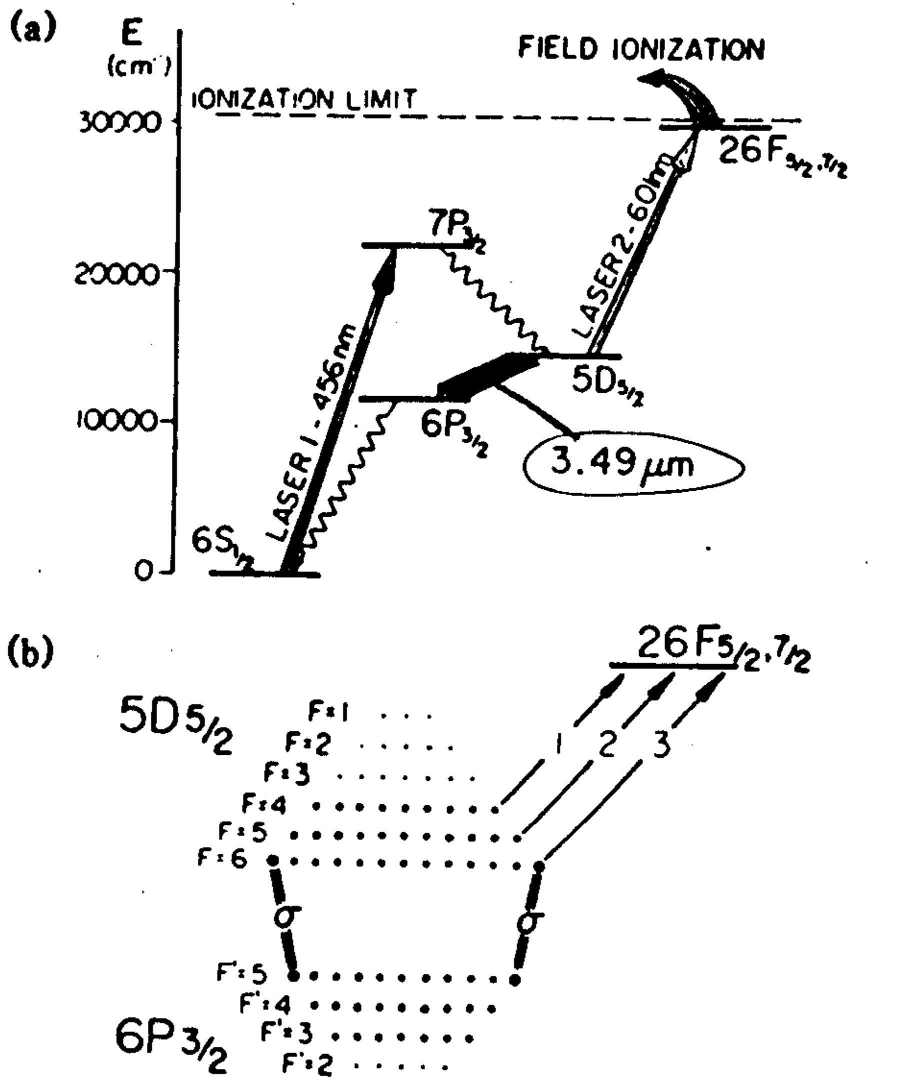
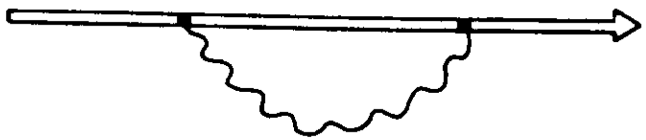
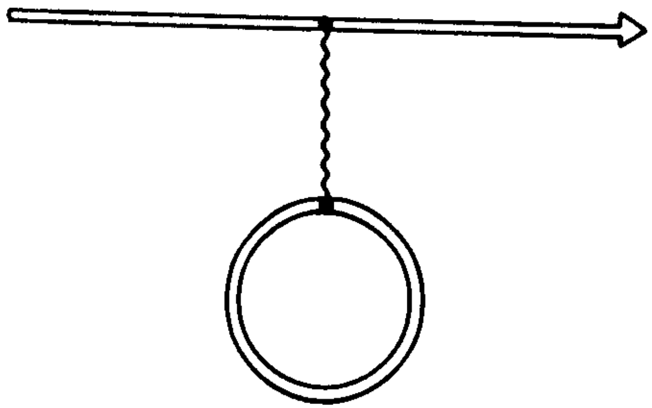


FIG. 1. (a) Cesium energy diagram showing the levels and optical transitions relevant to this experiment. (b) Closeup showing the hyperfine structure of the $5D_{5/2}$ and $6P_{3/2}$ states.

Lowest order radiative corrections to bound electron



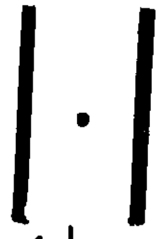
Self-energy contribution



Vacuum polarization contribution

Level shifts

Rydberg atoms between conducting planes



G. Barton
Proc. Roy. Soc.
A 410, 174 u. 175 (198

Scaling laws (hydrogen atoms)

$L < n^3 a_0/\alpha$

Interaction basically <u>electrostatic</u> (nonretarded)

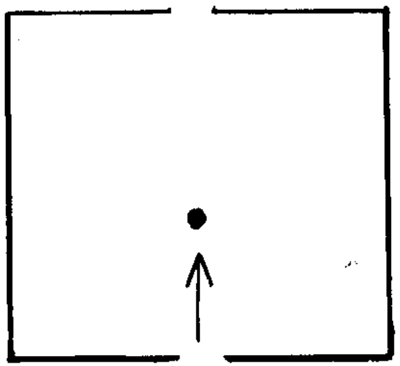
$$\Delta \approx (n^4/L^3_{(mm)}) \cdot 10^{-6}\ \mathrm{Hz}$$

$L > n^3 a_0/\alpha$

Interaction basically <u>radiative</u> (retarded)

$$\Delta \approx (2/n^2 L_{(mm)}) \cdot 10^{4}\ \mathrm{Hz}$$

nteraction of a single atom with a single cavity mode



atom enters cavity in excited state

Atom

↕ coupling constant Ω (Rabi-frequency)

Cavity field

↕ coupling constant $\frac{\omega}{Q}$

Cavity walls

$\Omega > \frac{\omega}{Q}$ Maser operation

Rabi frequency: Ω

Damping rate of field: $\frac{\omega}{Q}$

1) $\Omega < \frac{\omega}{Q}$

photon emitted in the cavity mode is absorbed much faster by cavity walls than atom decays

2) $\Omega > \frac{\omega}{Q}$

self-induced Rabi nutation

(Jaynes, Cummings, Proc. IEEE 51, 89 (1963))

One-Atom Maser

D. Meschede, H. Walther, G. Müller
Phys. Rev. Lett. 54, 551 (1985)

Superconducting cavity

$Q \approx 10^9$ at 2 K

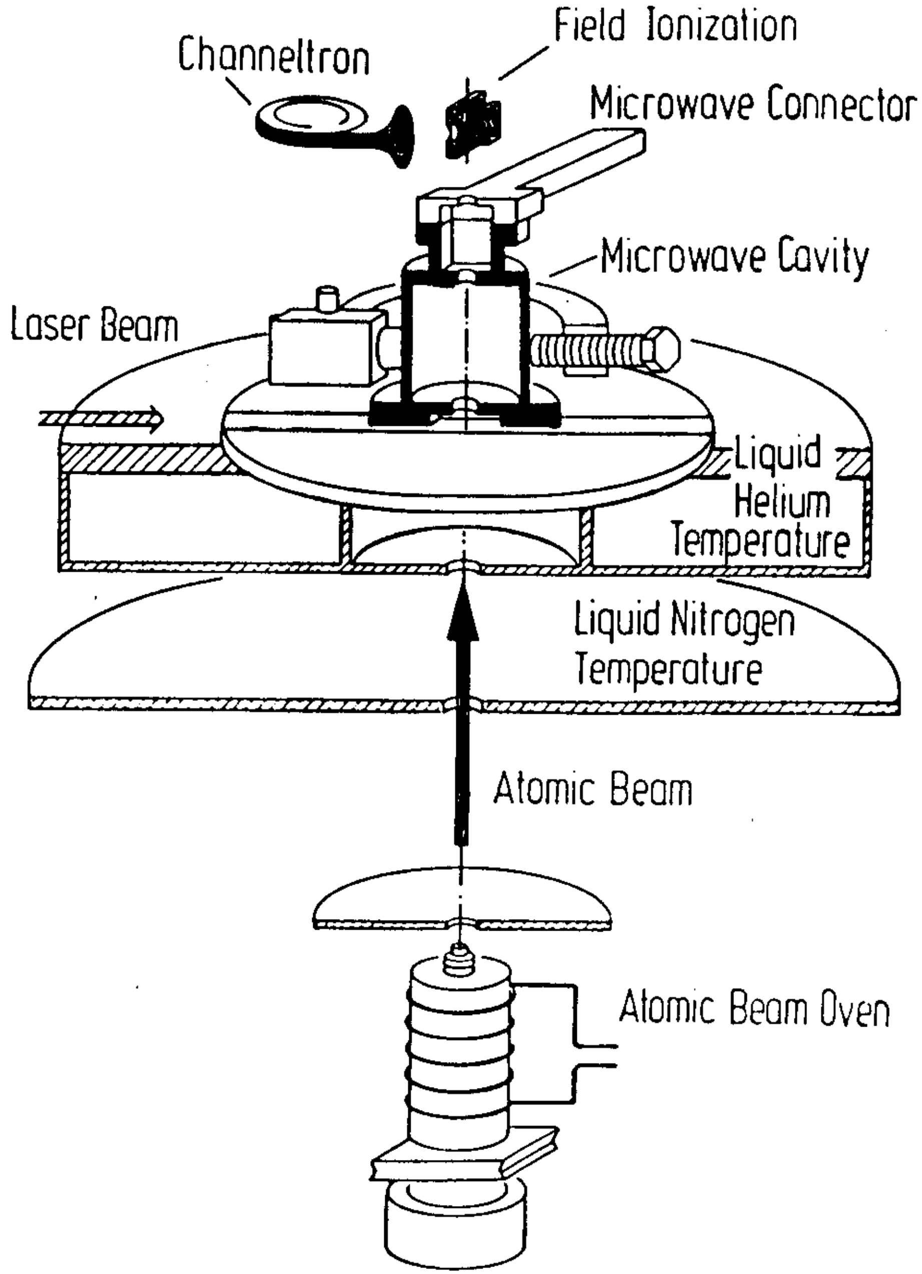
damping rate of field

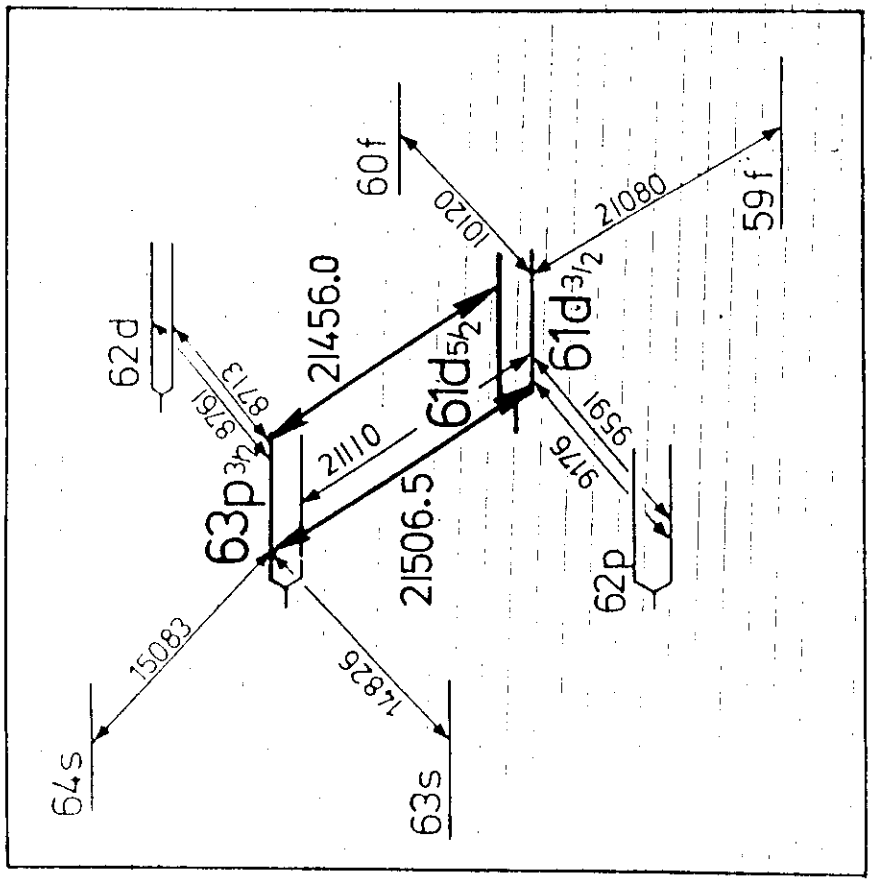
$$\frac{\omega}{Q} = 125\ s^{-1}$$

single photon Rabi-frequency

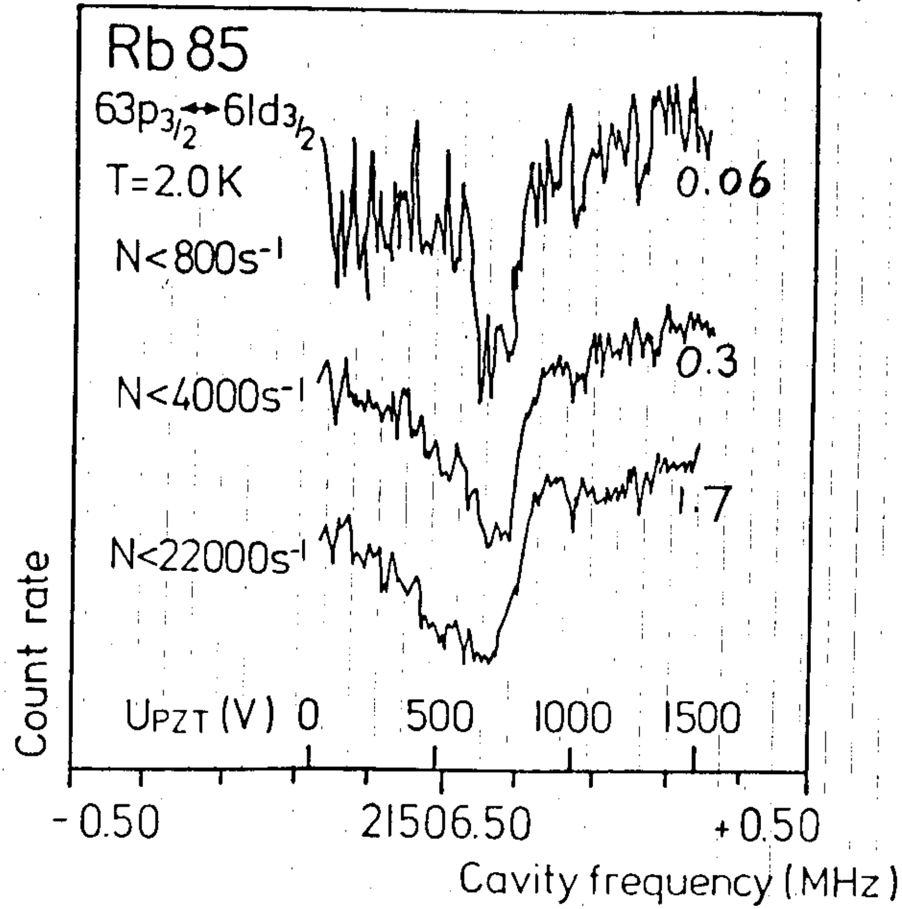
$$\Omega = 17\,000\ s^{-1}$$

D. Meschede, H. Walther, G. Müller, Phys. Rev. Lett. 54, 551 (1985)

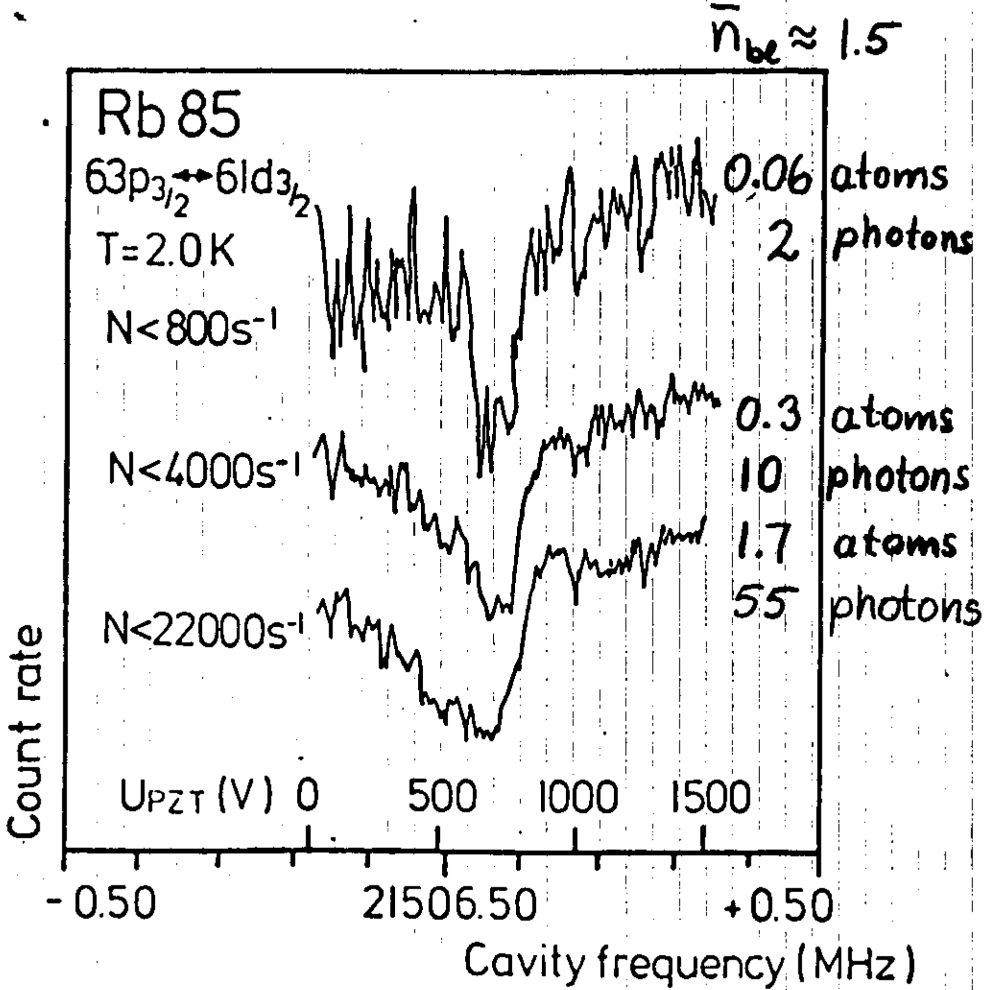




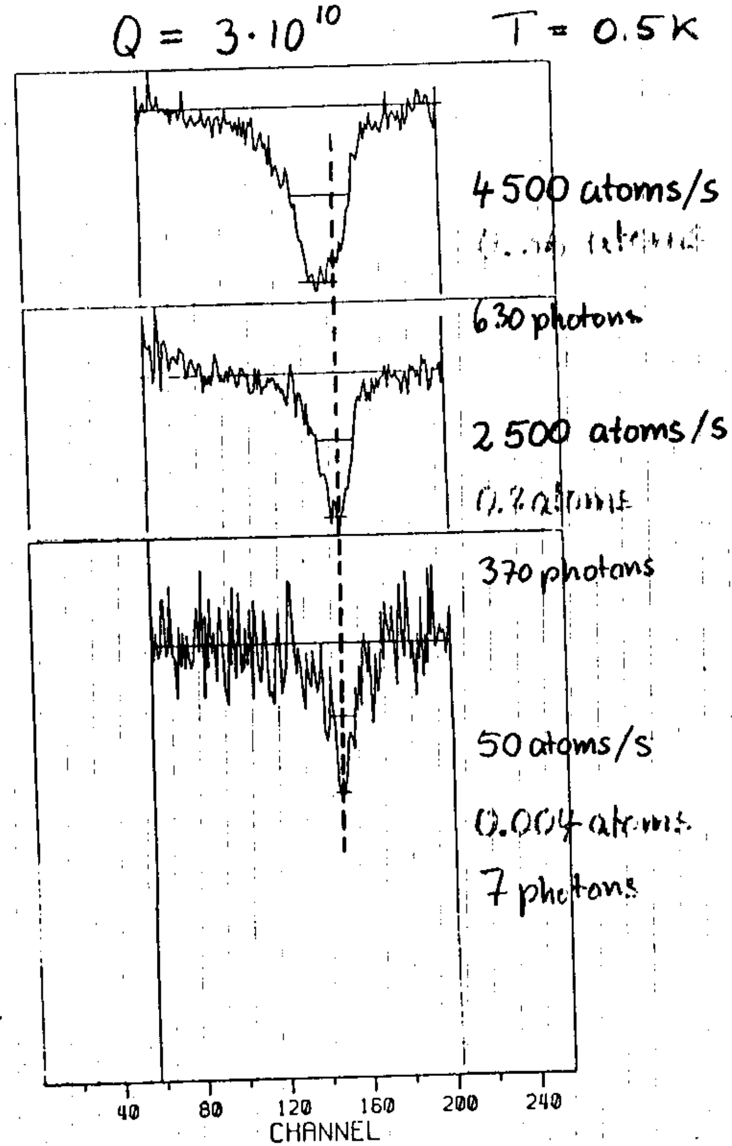
number of atoms
in the cavity



D. Meschede, H.W., G. Müller
Phys. Rev. Lett. 54, 551 (1985)



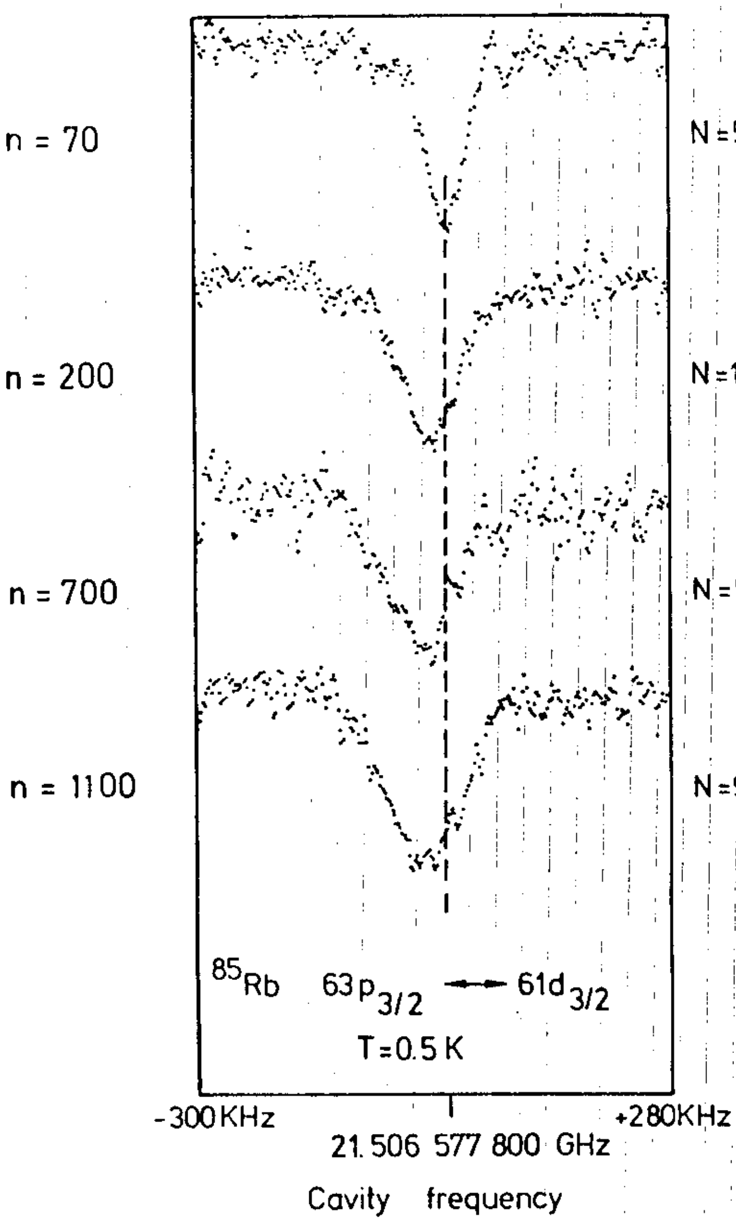
D. Meschede, H. W. G. Müller PRL 54, 551 (1985).

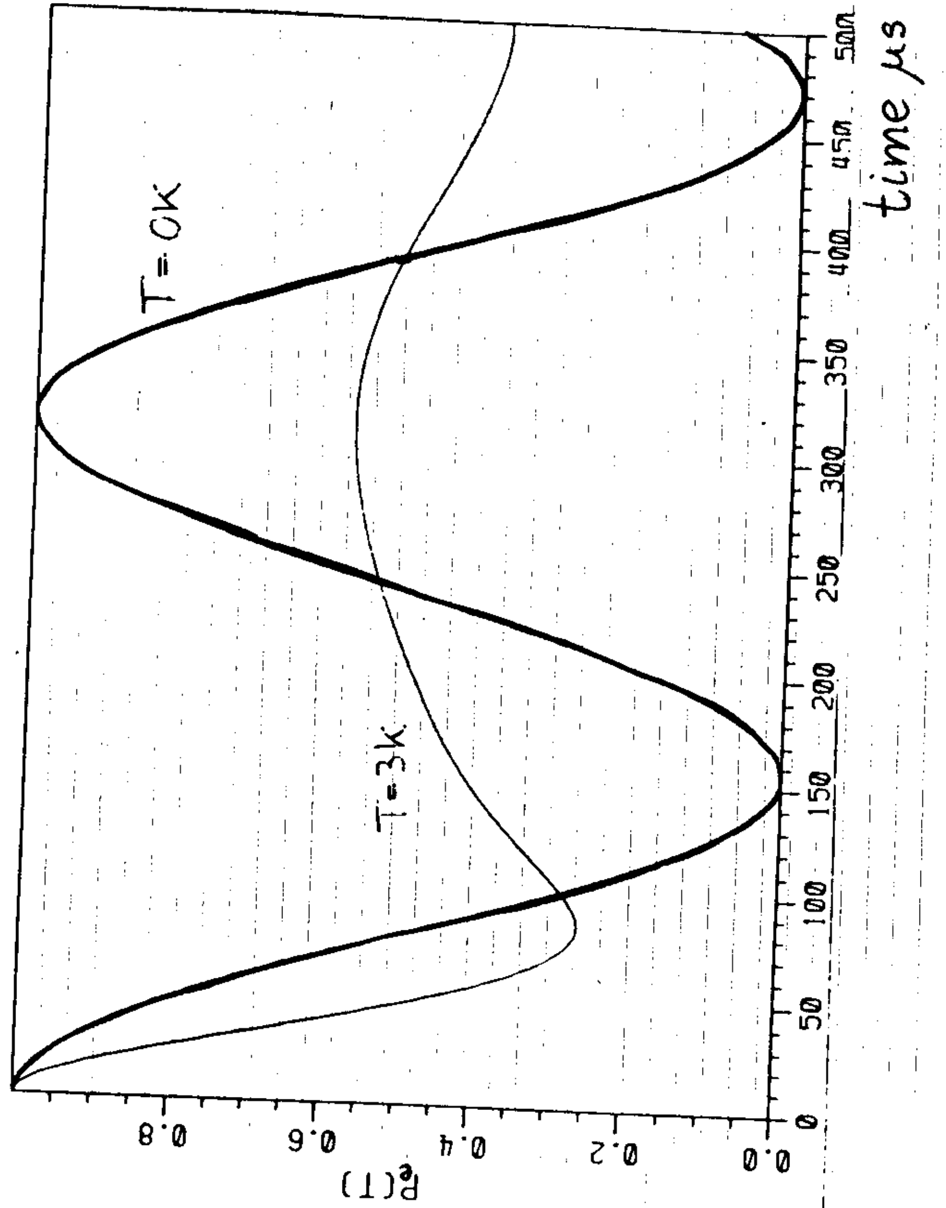


Rb^{85} $63p_{3/2} - 61p_{5/2}$

Resonance frequency : 21.456 GHz

Fig. 7





$Q = 2 \cdot 10^{10}$

G. Rempe, G. Babst, H. W.

Single atom - resonant cavity

T = 0, n photons in cavity (Fock-state)

$$P_{e,n}(t) = \frac{1}{2}\left\{1 + \cos\left(2\Omega\sqrt{n+1}\,t\right)\right\}$$

T ≠ 0; Thermal field:

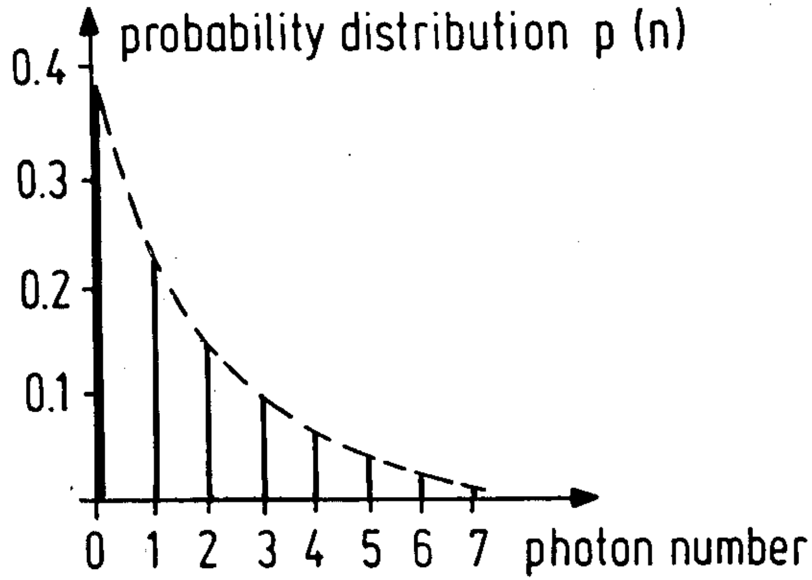
$$P_{e,n}(t) = \frac{1}{2}\left\{1 + \sum_n \underbrace{\frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}}_{p(n)} \cos\left(2\Omega\sqrt{n+1}\,t\right)\right\}$$

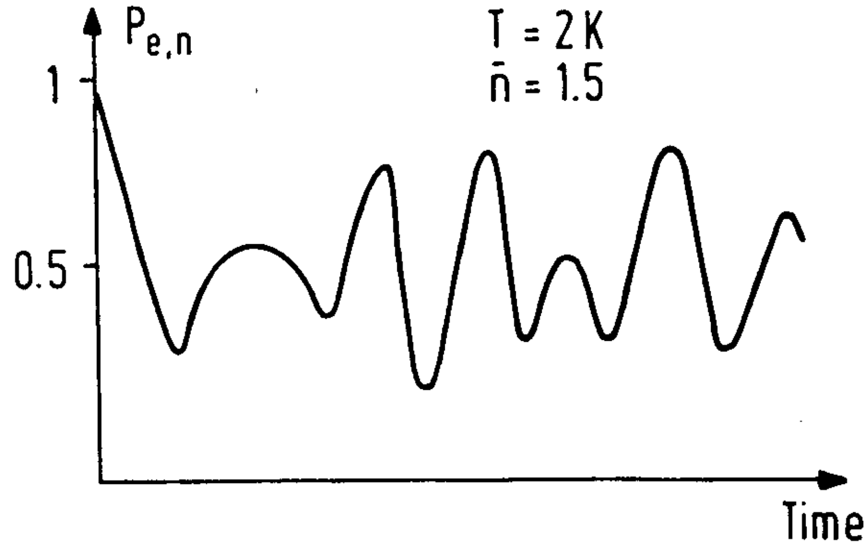
$$\bar{n} = \frac{1}{e^{\hbar\omega/kT} - 1}$$

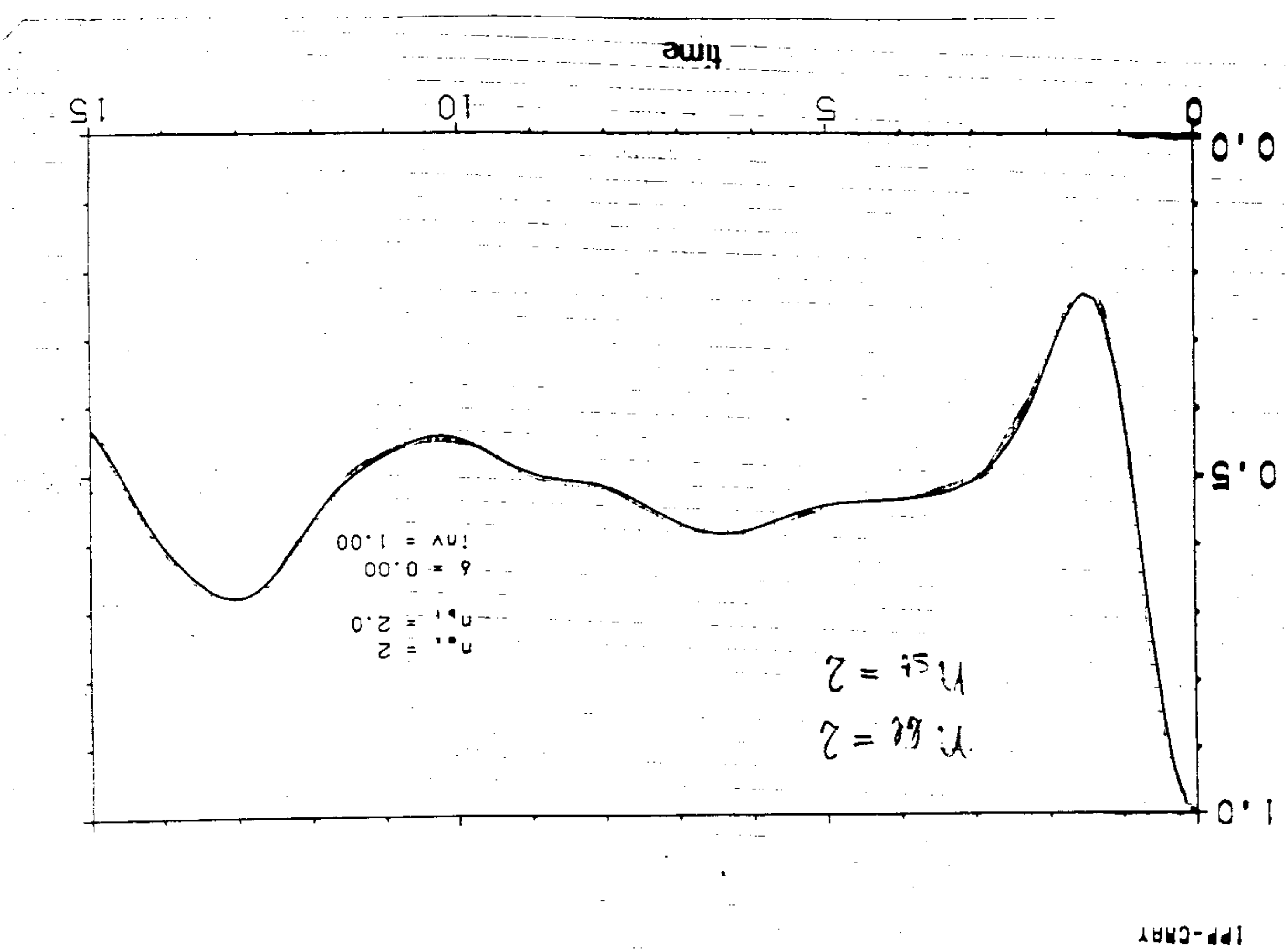
T = 0; Coherent field:

$$P_{e,n}(t) = \frac{1}{2}\left\{1 + \sum_n \underbrace{e^{-\bar{n}}\frac{\bar{n}^n}{n!}}_{p(n)} \cos\left(2\Omega\sqrt{n+1}\,t\right)\right\}$$

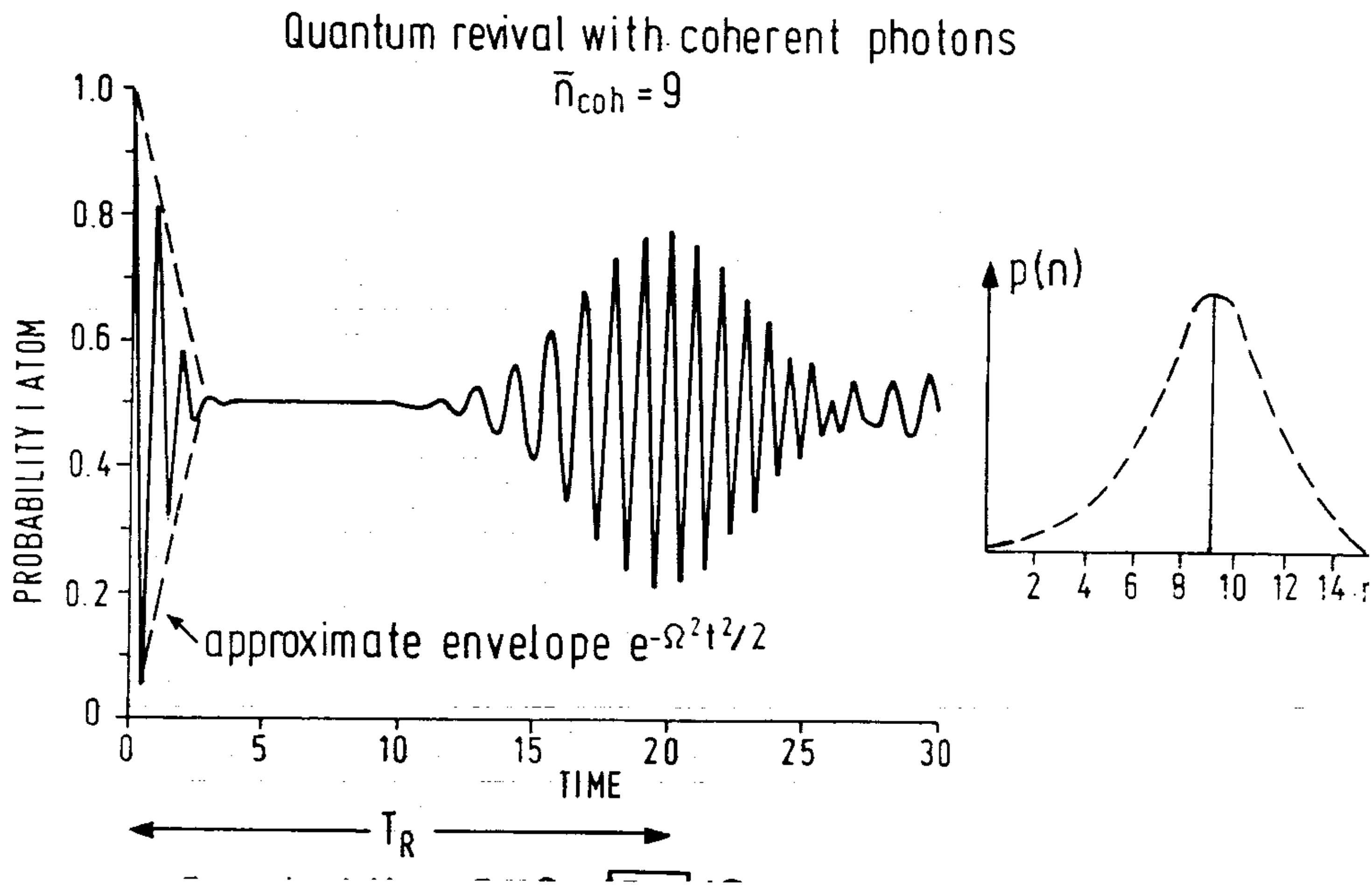
Thermal cavity field





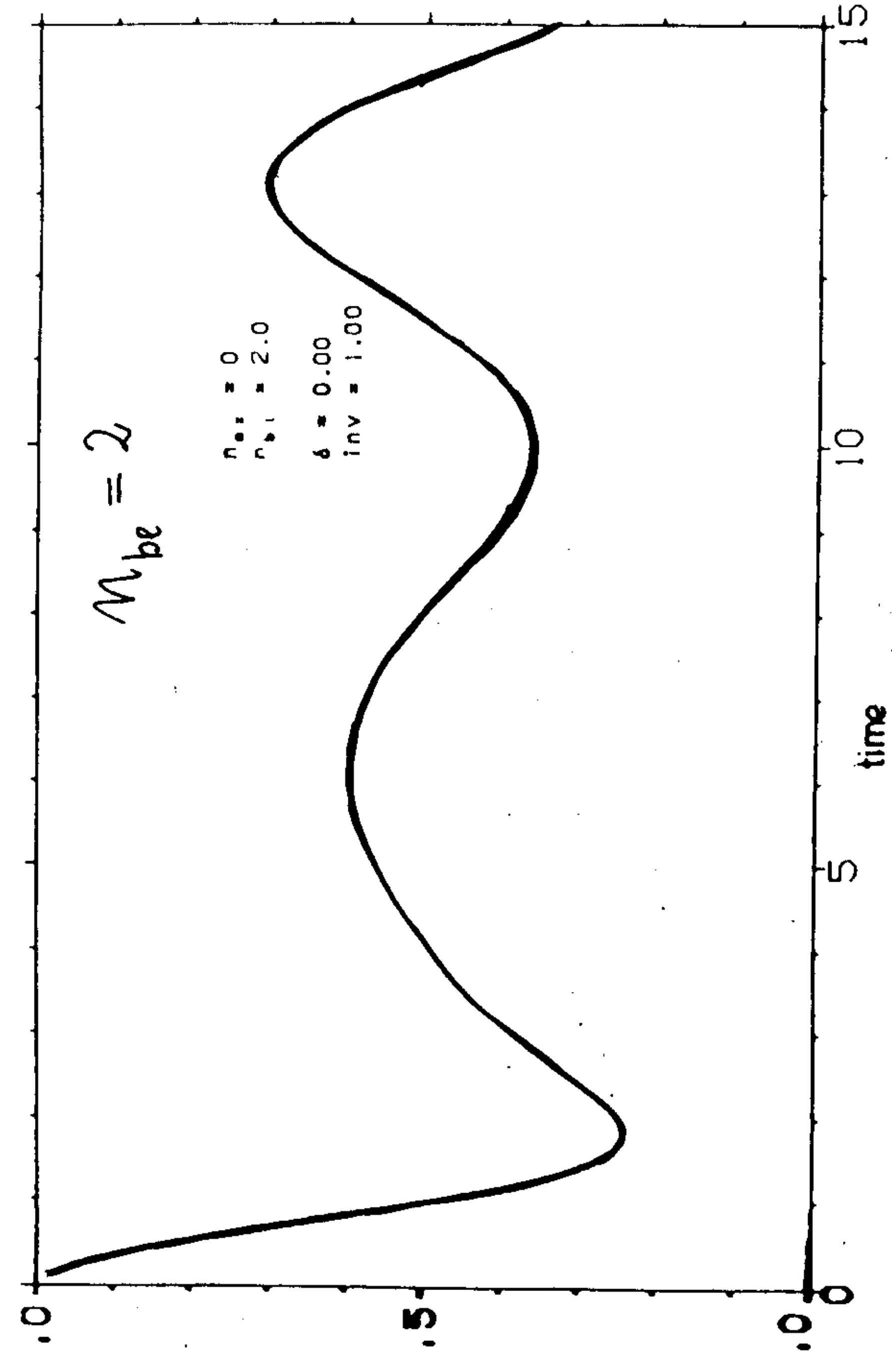


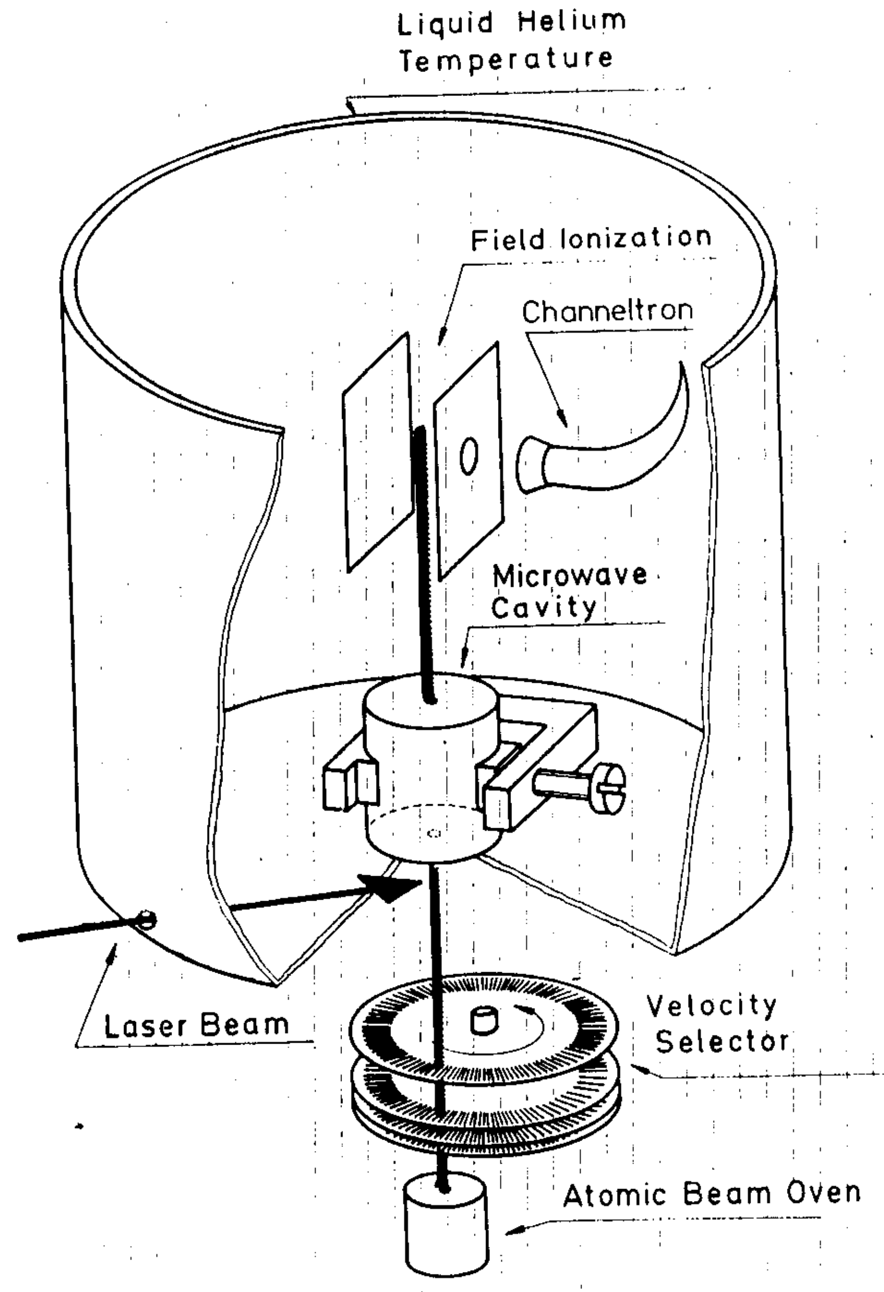
time
0
5
10
15
0.0
0.5
1.0
IPP-CRAY

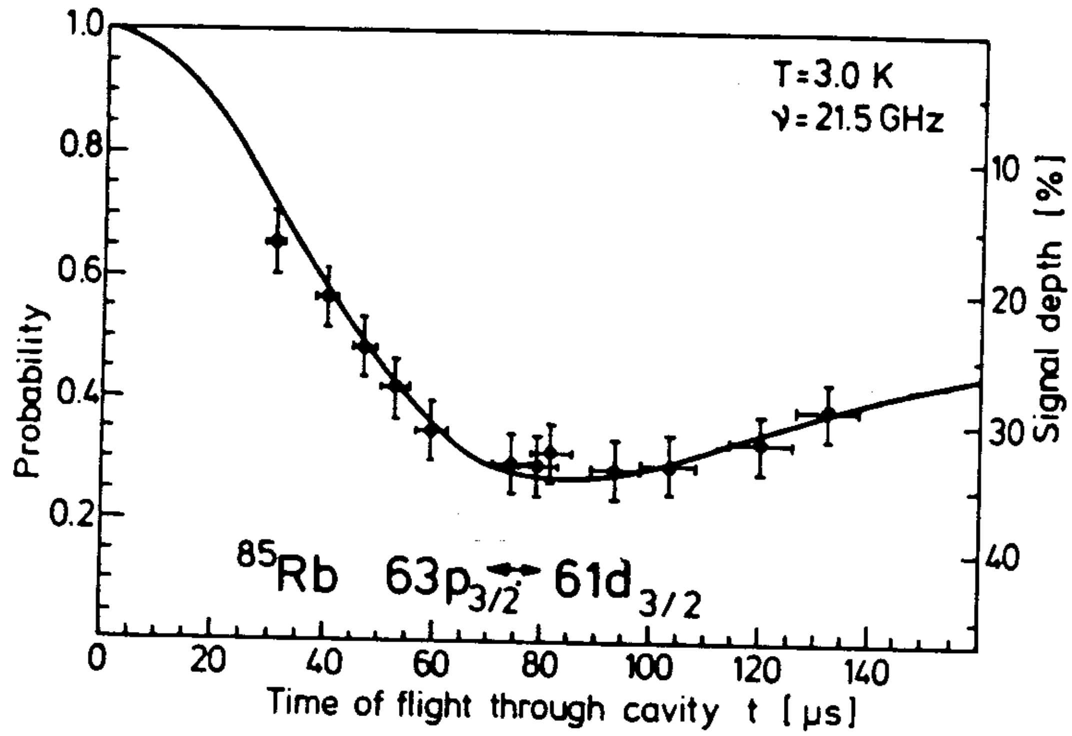


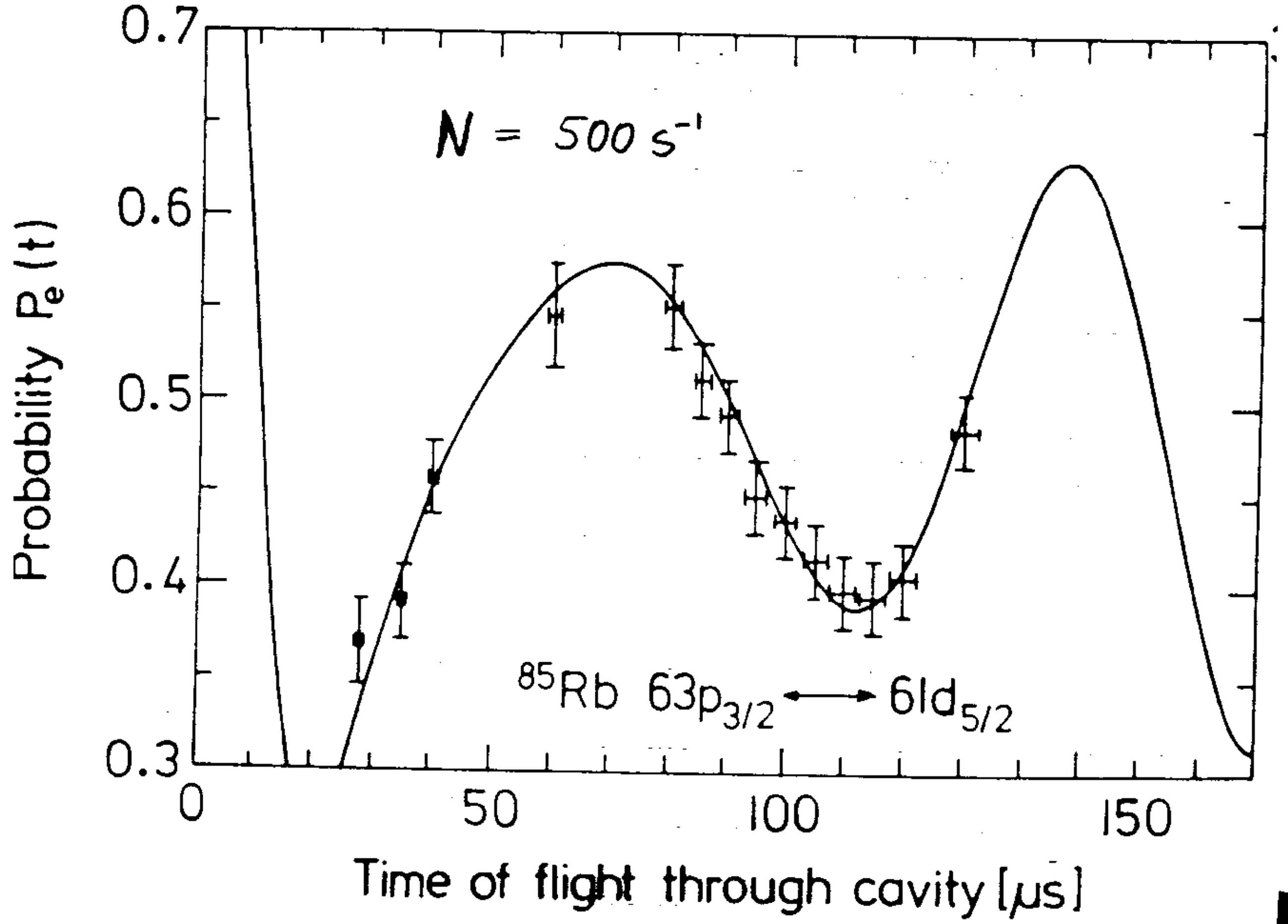
Quantum revival with coherent photons
$\bar{n}_{coh} = 9$
PROBABILITY I ATOM
approximate envelope $e^{-\Omega^2 t^2/2}$
TIME
T_R
p(n)

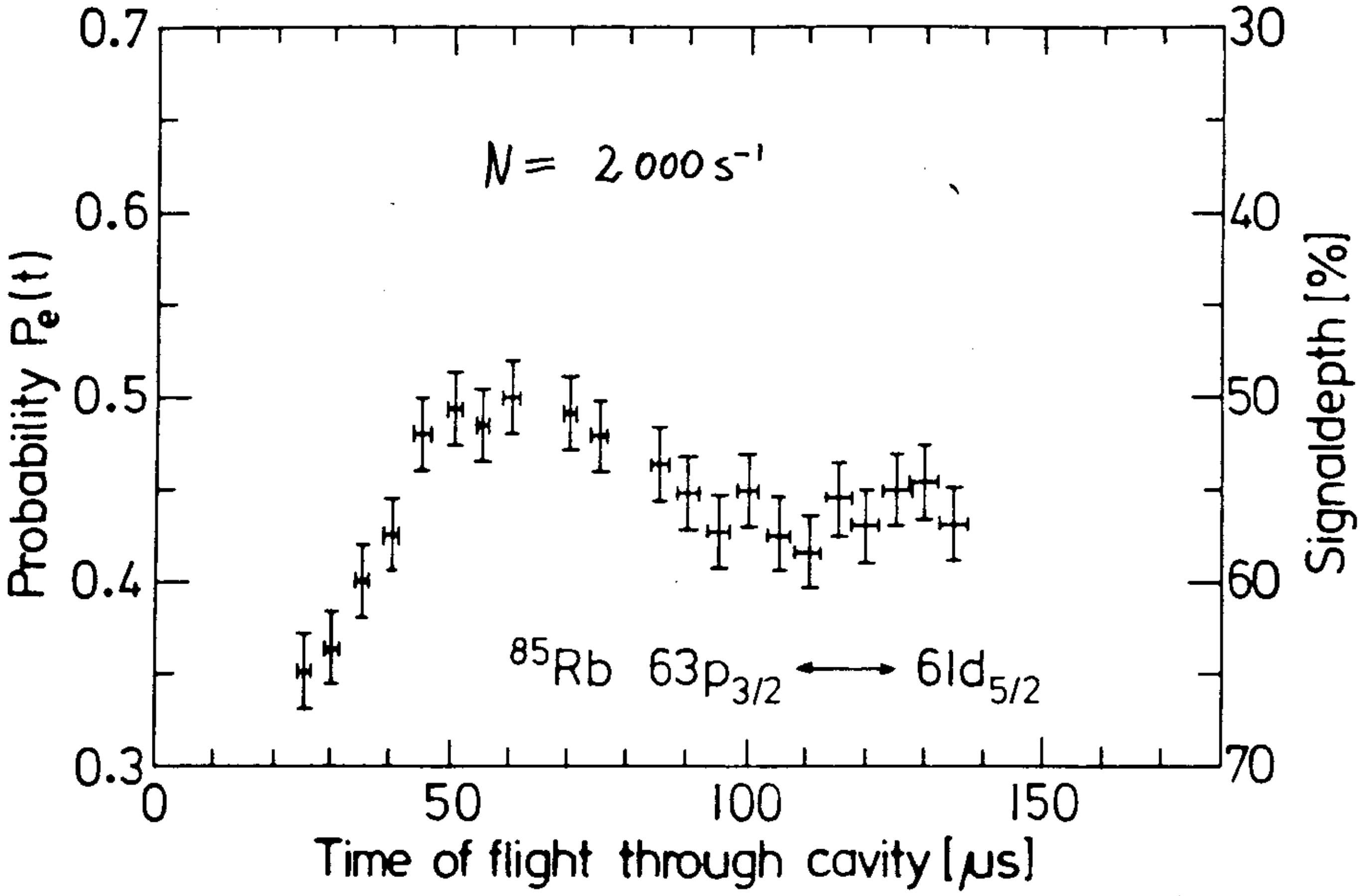
IPP-CRAY

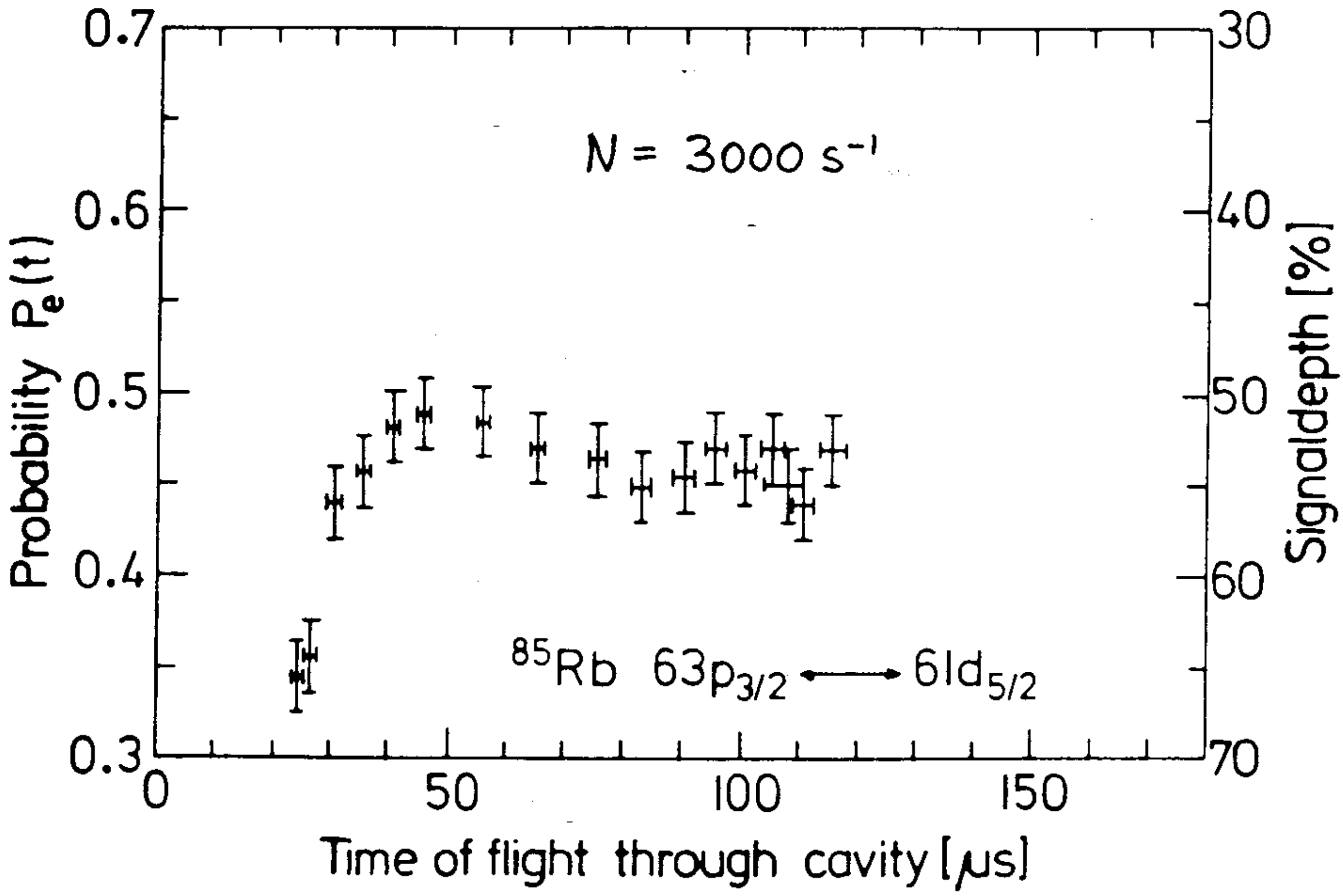

$n_{be} = 2$
δ = 0.00
inv = 1.00
time
0
5
10
15
.0
.5
0.0

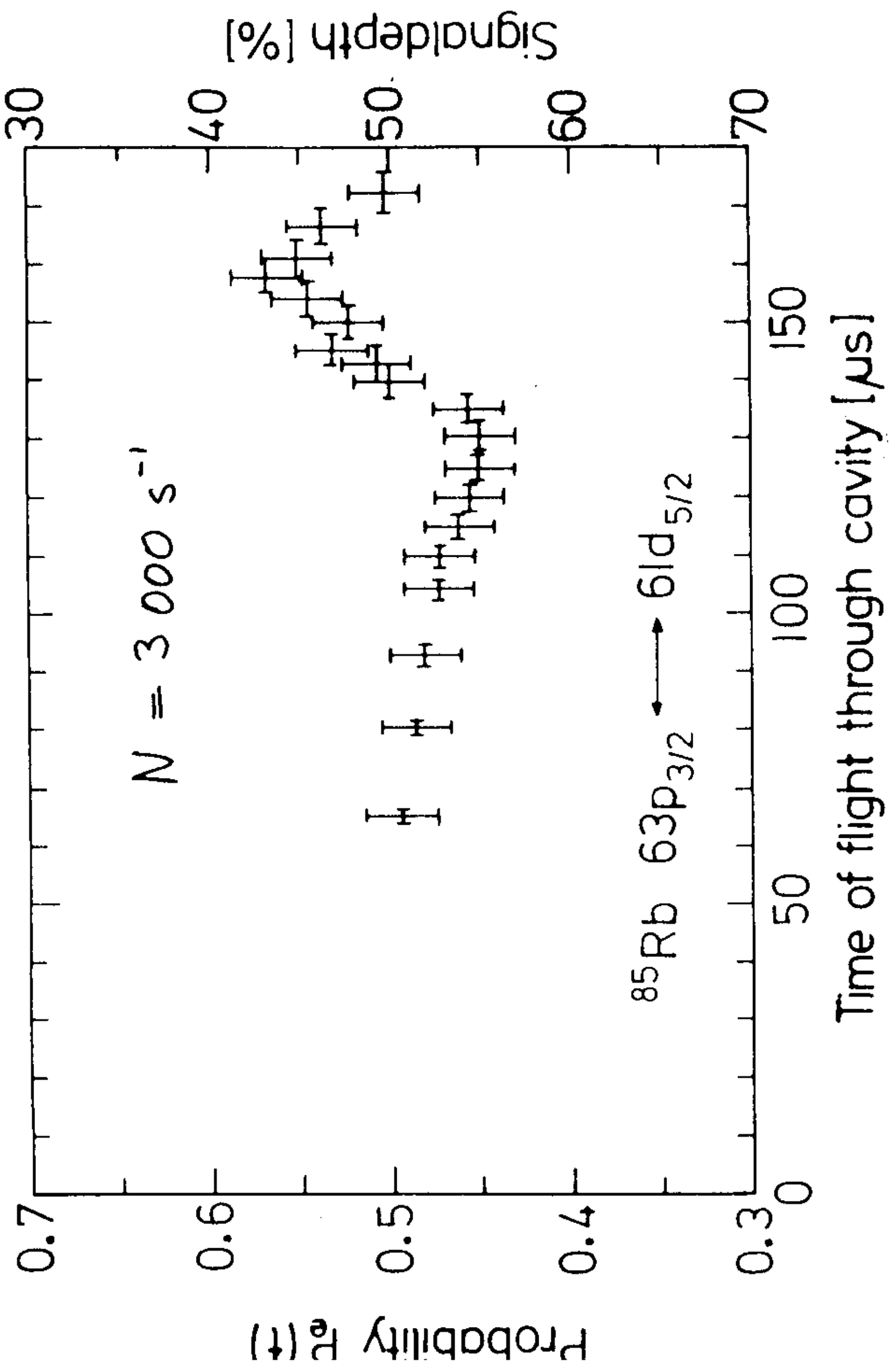

Liquid Helium
Temperature
Field Ionization
Channeltron
Microwave
Cavity
Laser Beam
Velocity
Selector
Atomic Beam Oven


T=3.0 K
ν=21.5 GHz
Probability
Signal depth [%]
^{85}Rb 63p$_{3/2}$ ↔ 61d$_{3/2}$
Time of flight through cavity t [μs]


N = 500 s^{-1}
Probability P$_e$(t)
^{85}Rb 63p$_{3/2}$ ↔ 61d$_{5/2}$
Time of flight through cavity [μs]


0.7
0.6
0.5
0.4
0.3
Probability $P_e(t)$
30
40
50
60
70
Signaldepth [%]
$N = 2000 s^{-1}$
^{85}Rb $63p_{3/2} \longleftrightarrow 61d_{5/2}$
0
50
100
150
Time of flight through cavity [μs]


0.7
0.6
0.5
0.4
0.3
Probability $P_e(t)$
30
40
50
60
70
Signaldepth [%]
$N = 3000 s^{-1}$
^{85}Rb $63p_{3/2} \longleftrightarrow 61d_{5/2}$
0
50
100
150
Time of flight through cavity [μs]

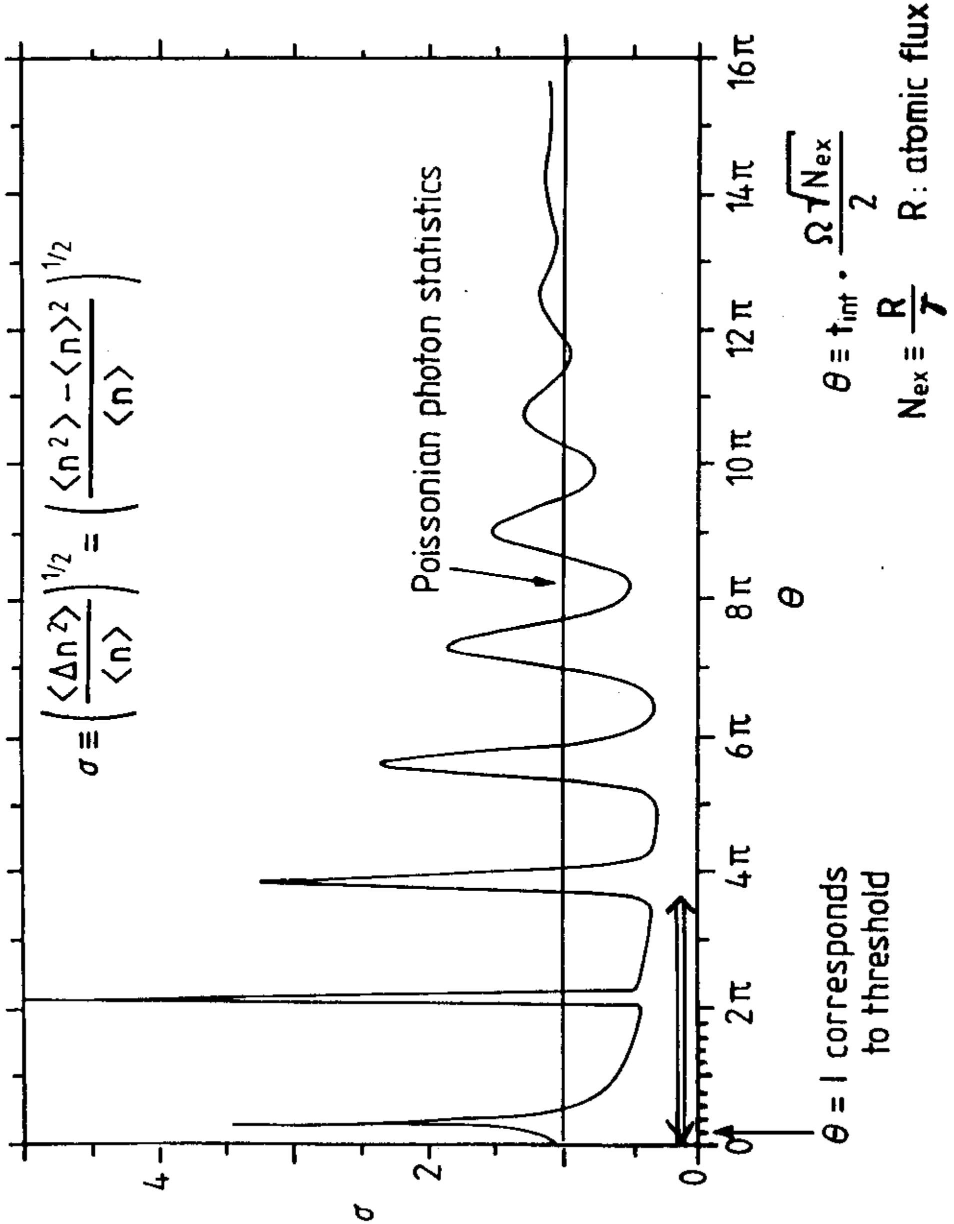


Theory of the One-Atom Maser

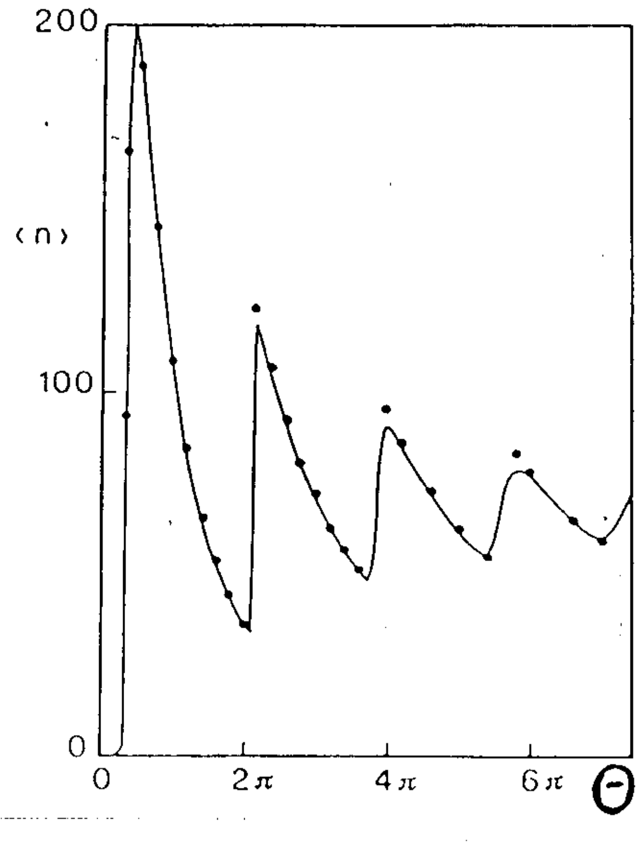
1) P. Filipowicz, J. Javanainen, P. Meystre
Phys. Rev. 34, 3077 (1986) (Oct 1986)
on the basis of Jaynes-Cummings model
and heuristic Fokker-Planck approach

2) L.A. Lugiato, M.O. Scully, H. Walther
Phys. Rev. 36, July 1987
on the basis of quantum theory of laser

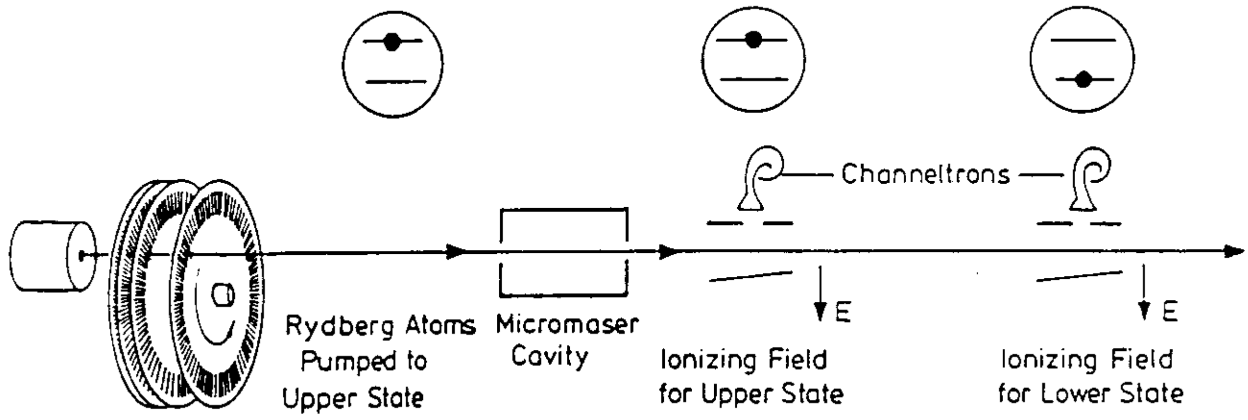
Filipowicz, Javanainen, Meystre, Optics Comm. 58, 327 (1986)



Average photon number in the micromaser

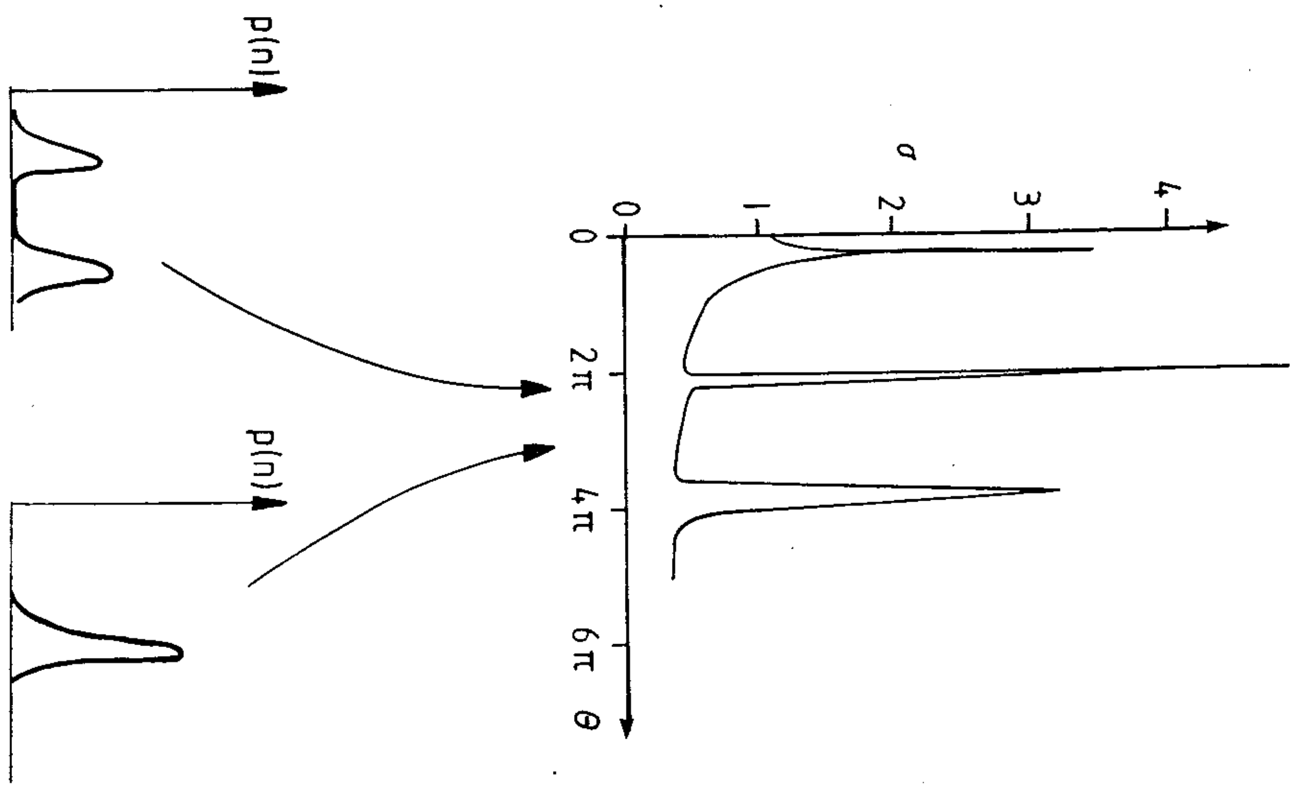


Photon statistics of the one-atom maser

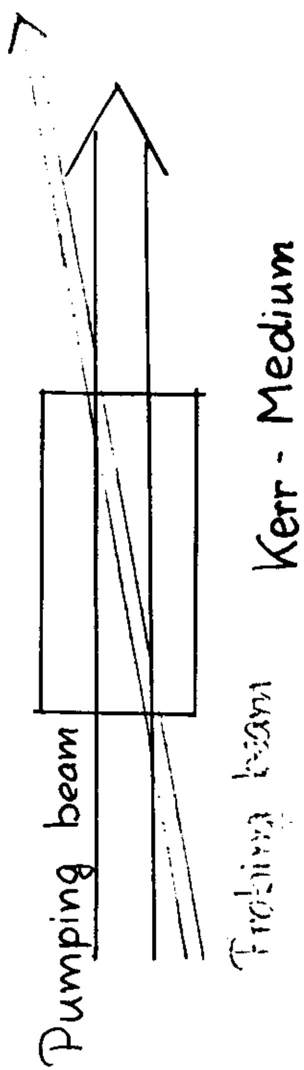


G. Rempe
F. Schmidt-Kaler
H. W. to be published

atoms in the upper and lower maser level are measured as function of the interaction time of the atoms in the cavity



Quantum-non-demolition measurement



Photon statistics ↔ Atom statistics

Photon statistics : n number of photons in the cavity

$$\sigma = \frac{\langle n^2\rangle - \langle n\rangle^2}{\langle n\rangle}$$

$$Q_{ph} = \frac{\langle n^2\rangle - \langle n\rangle^2}{\langle n\rangle} - 1$$

Atom statistics : N number of atoms in lower state measured in time interval T

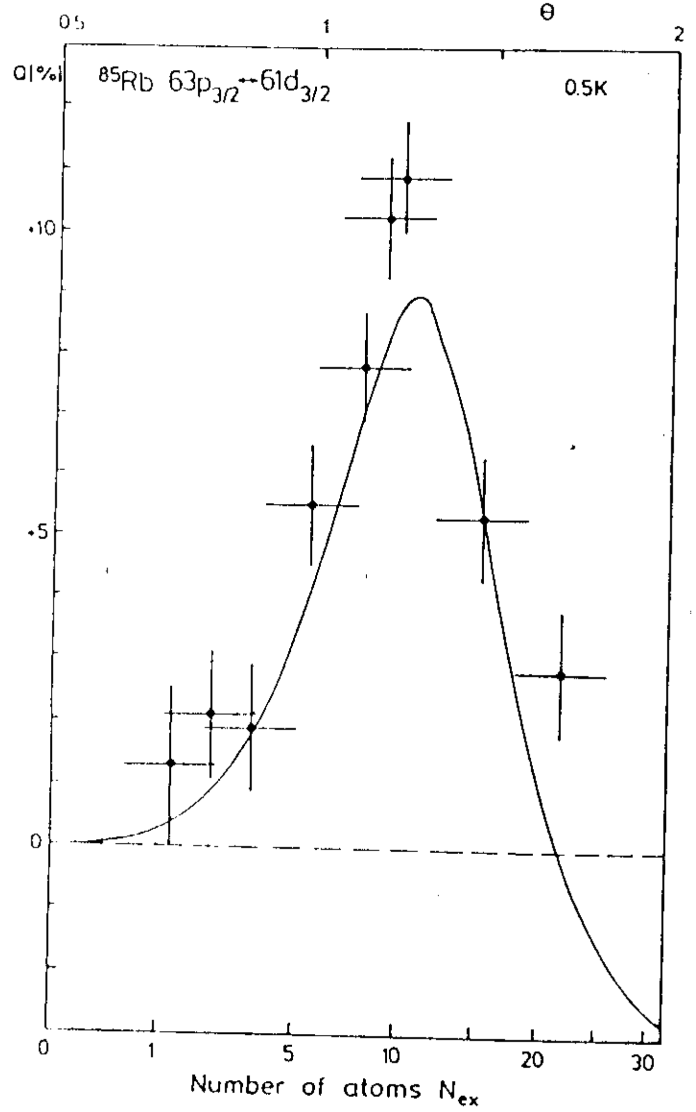
$T > T_{cav}$

$$Q_m = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle} - 1$$

$$Q_m = \eta Q$$

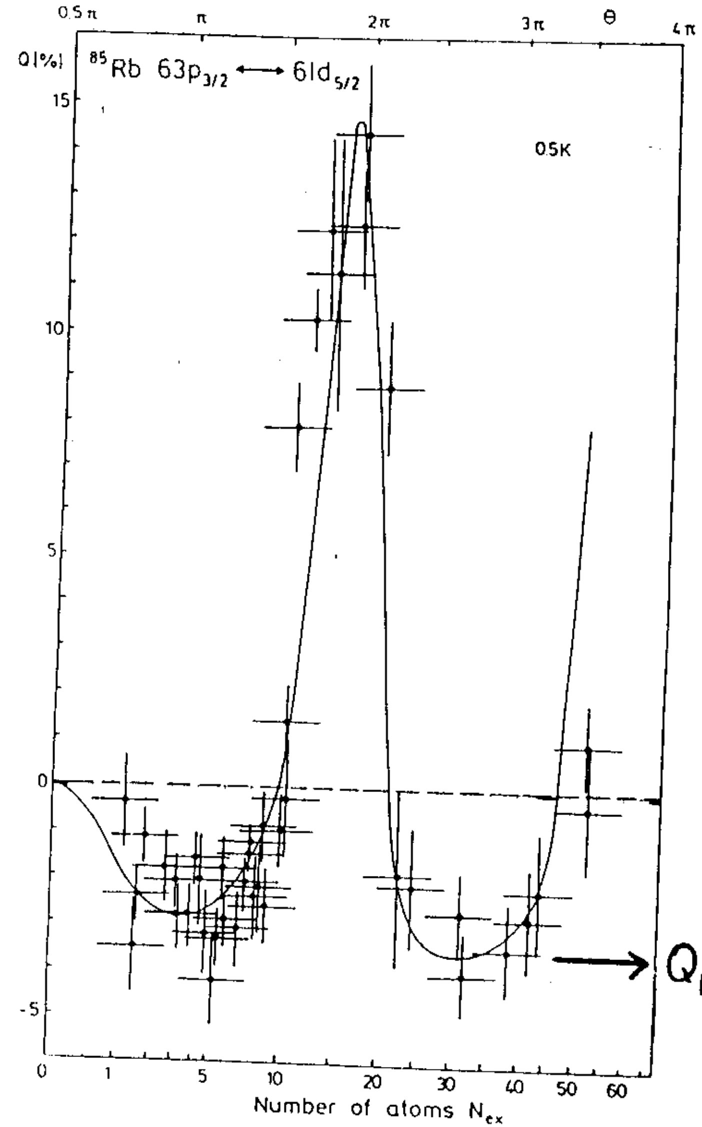
η: detection efficiency of Rydberg atom

$$Q = P(\langle N\rangle)\, Q_{ph}\,[2 + Q_{ph}]$$



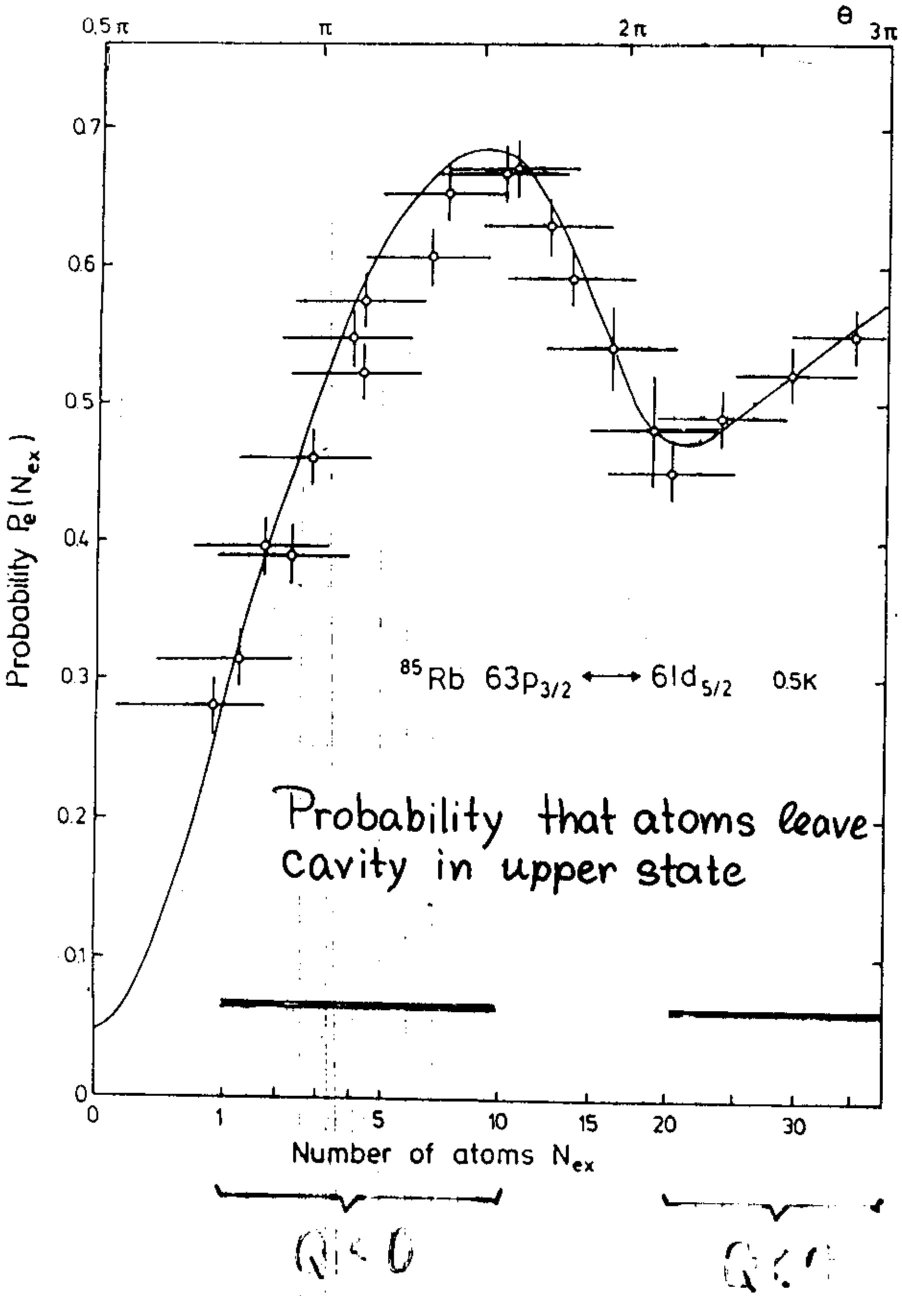
G. Remp, F. Schmidt-Kaler, H.W. to be published

Fluctuation of the atomic number in the lower state

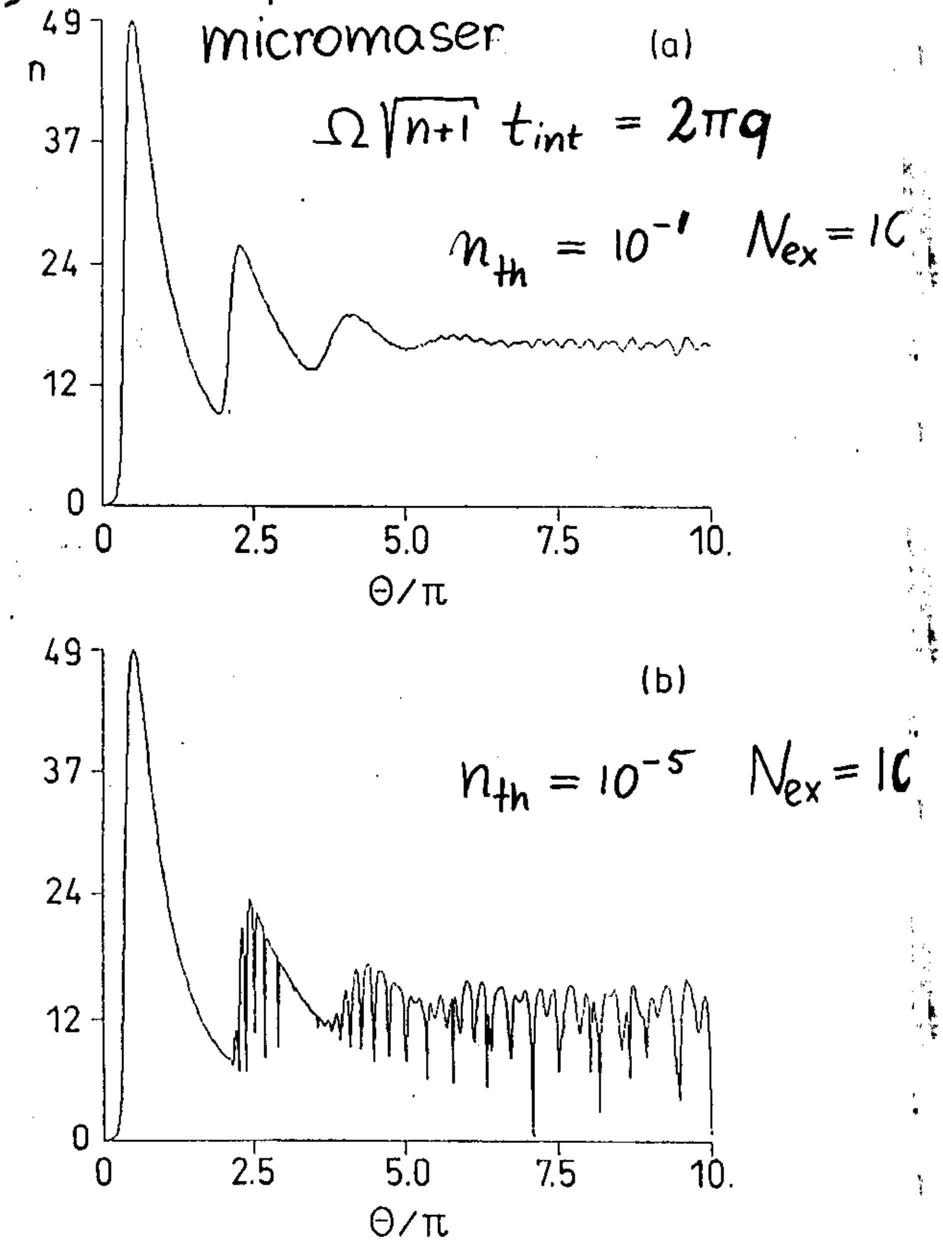


→ $Q_{ph} = -0.7$

G. Rempe, F. Schmidt-Kaler, H.W. to be published

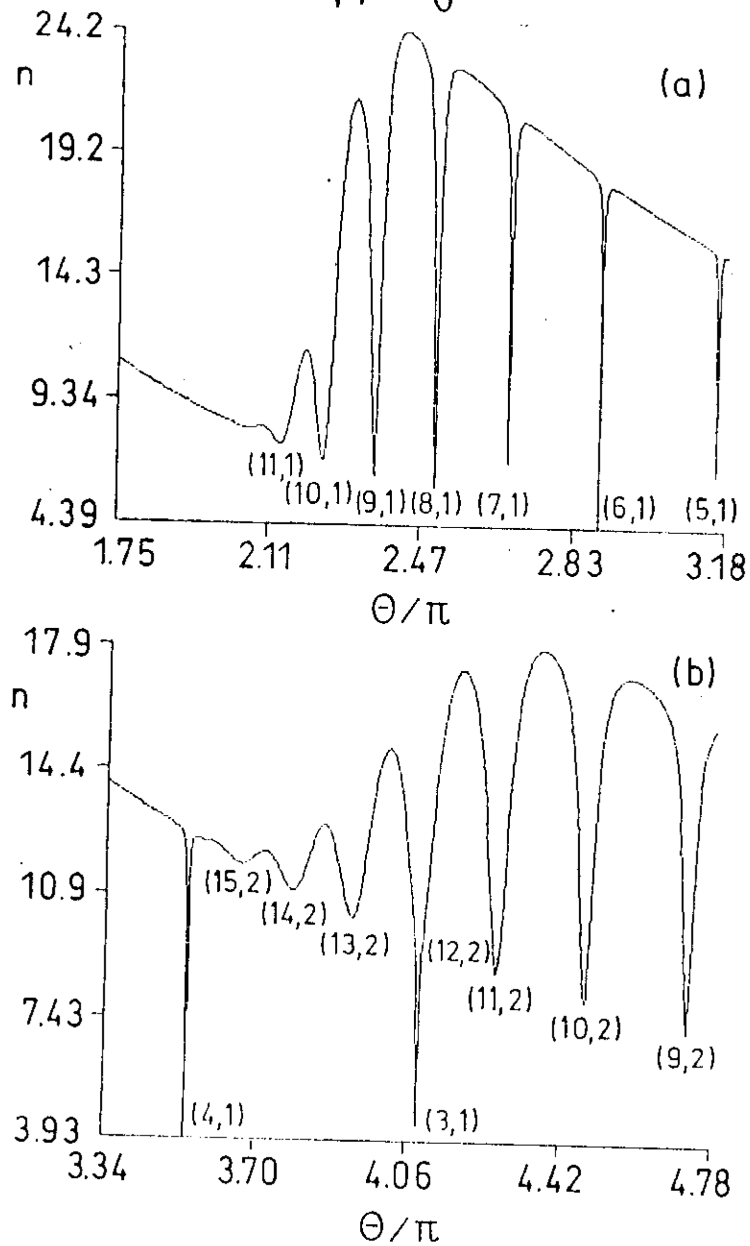


Very low temperature behaviour of the micromaser



P. Meystre, G. Rempe, H. Walther

Trapping states (n+1; q)



P. Meystre, G. Rempe, H. Walther, Opt. Lett. 13, 1078 (1988)