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UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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H4.SMR/449-36

WINTER COLLEGE ON HIGH RESOLUTION SPECTROSCOPY

(8 January - 2 February 1990)

SPECTROSCOPY OF TRAPPED IONS

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Garching D-8046
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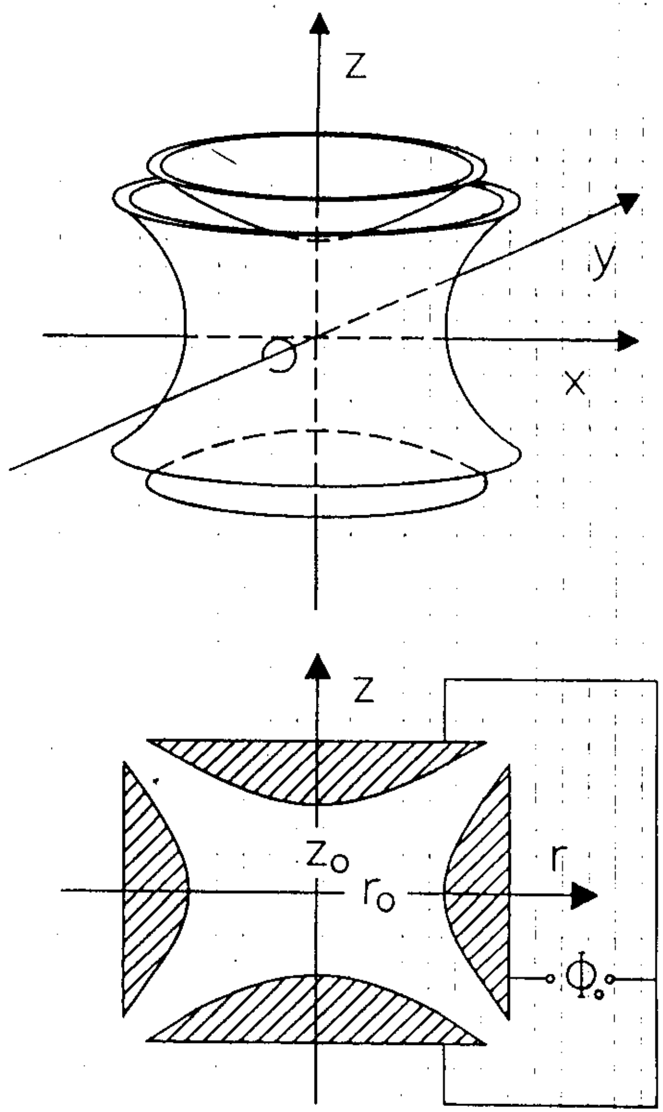
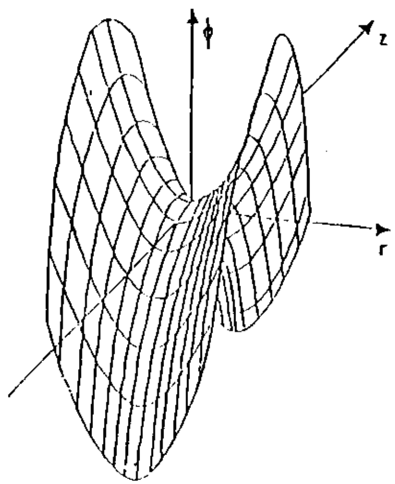
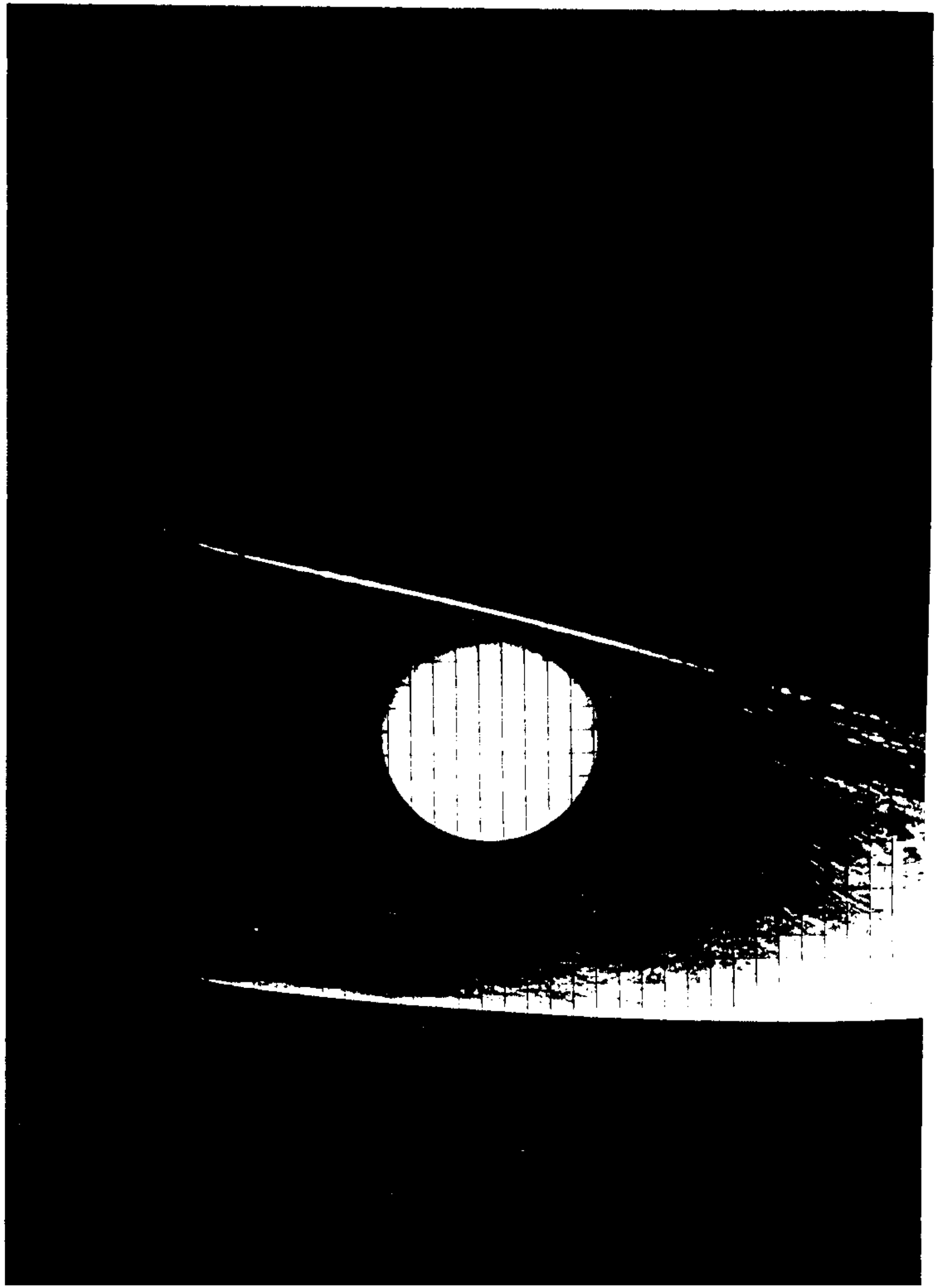
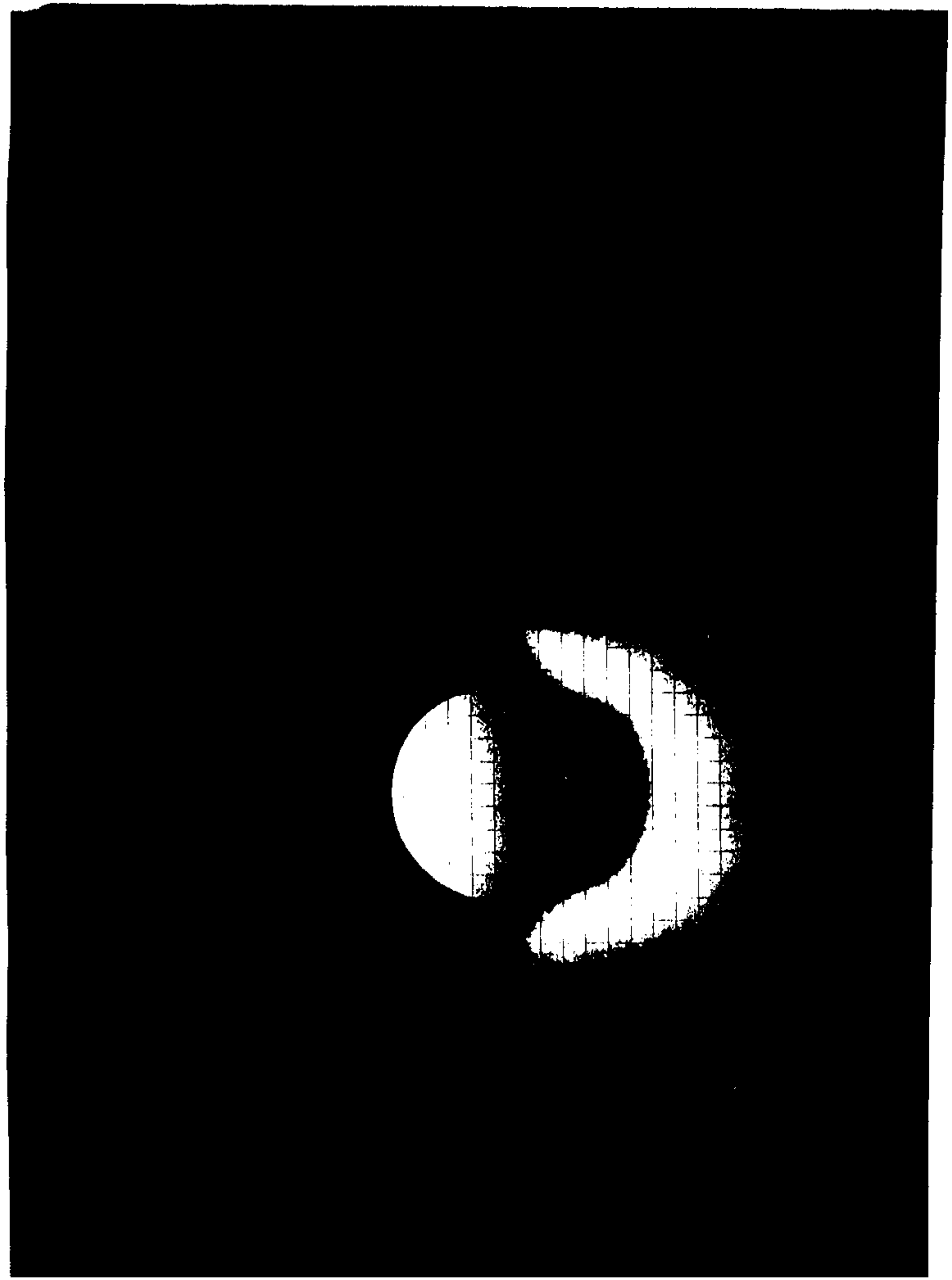
SPECTROSCOPY OF TRAPPED IONS

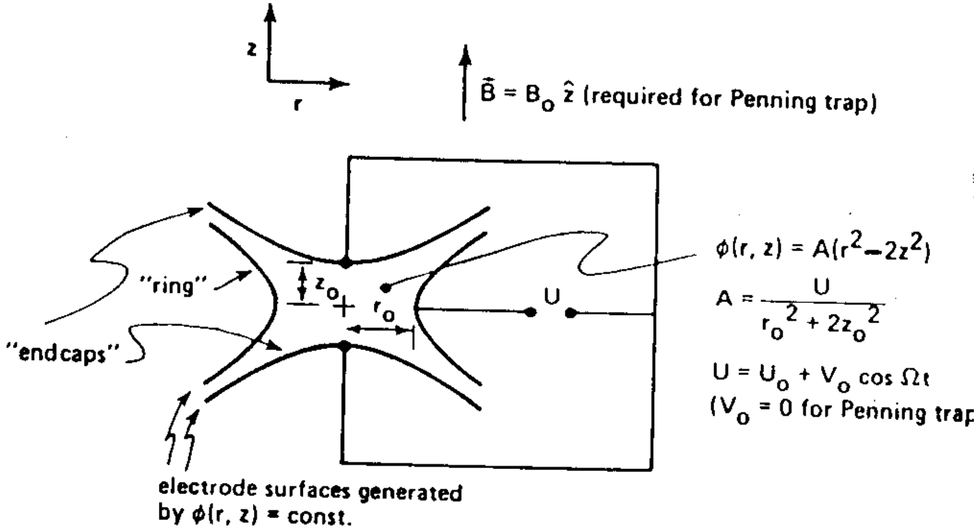



φ
z
r

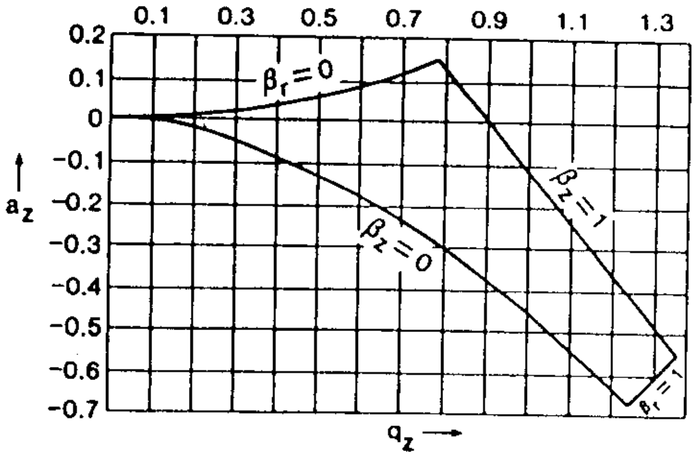




THE PAUL (rf) TRAP



Equation of motion : Mathieu-equation



$$-a_z = 2a_r = 16qU_o / m\Omega^2 (r_o^2 + 2z_o^2)$$

$$+q_z = -2q_r = 8qV_o / m\Omega^2 (r_o^2 + 2z_o^2)$$

Pseudo - Potential of the Paul-Trap

$$\phi(r, z) = \frac{U_0 + V_0 \cos \Omega t}{r_0^2 + 2z_0^2}(r^2 - 2z^2)$$

$$\frac{d^2 x_i}{d\tau^2} + (a_i - 2q_i \cos 2\tau)x_i = 0$$ Mathieu-type equation

$$\tau = \Omega t/2$$

$$-a_z = +2a_r = 16qU_0/m\Omega^2(r_0^2 + 2z_0^2)$$

$$+q_z = -2q_r = 8qV_0/m\Omega^2(r_0^2 + 2z_0^2)$$

$$\psi(r, z) = \frac{qV_0^2}{m\Omega^2(r_0^2 + 2z_0^2)^2}[(\bar{r})^2 + 4(\bar{z})^2] \qquad (U_0 = 0)$$

secular motion:

$$\bar{\omega}_z = 2\bar{\omega}_r = 2\sqrt{2}qV_0/[\Omega m(r_0^2 + 2z_0^2)].$$

micro motion: frequency Ω

Pseudo - Potential with DC-Voltage

$$\phi_{DC} = \frac{U_0}{r_0^2 + 2z_0^2}(r^2 - 2z^2)$$

$$\phi(r, z) = \psi + \phi_{DC} = \frac{k_r}{2q}(\bar{r})^2 + \frac{k_z}{2q}(\bar{z})^2$$

$$= \left[\frac{qV_0^2}{m\Omega^2(r_0^2 + 2z_0^2)^2} + \frac{U_0}{r_0^2 + 2z_0^2}\right](\bar{r})^2$$

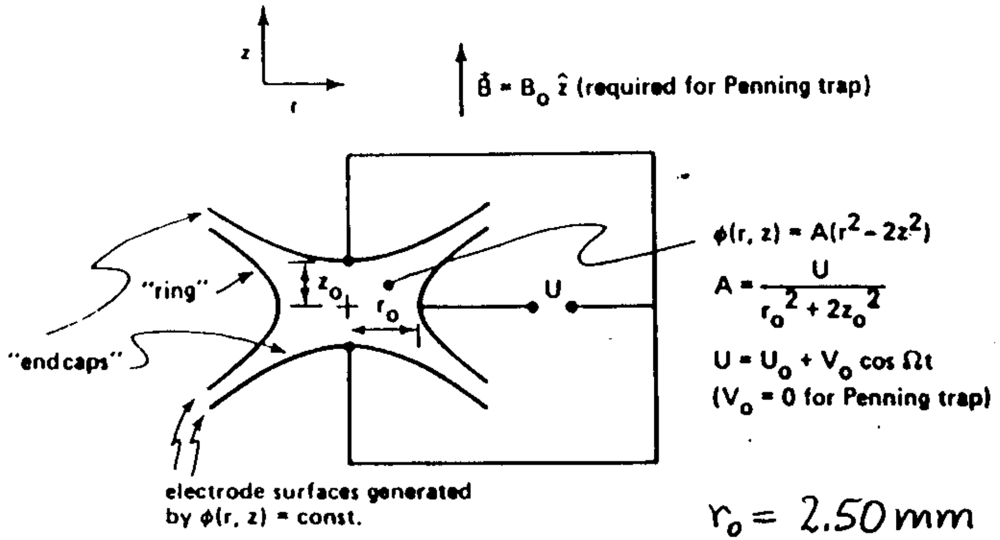
$$+ \left[\frac{4qV_0^2}{m\Omega^2(r_0^2 + 2z_0^2)^2} - \frac{2U_0}{r_0^2 + 2z_0^2}\right](\bar{z})^2$$

secular motion:

$$\bar{\omega}_r = (k_r/m)^{1/2} \text{ and } \bar{\omega}_z = (k_z/m)^{1/2}$$

The Paul (rf) Trap

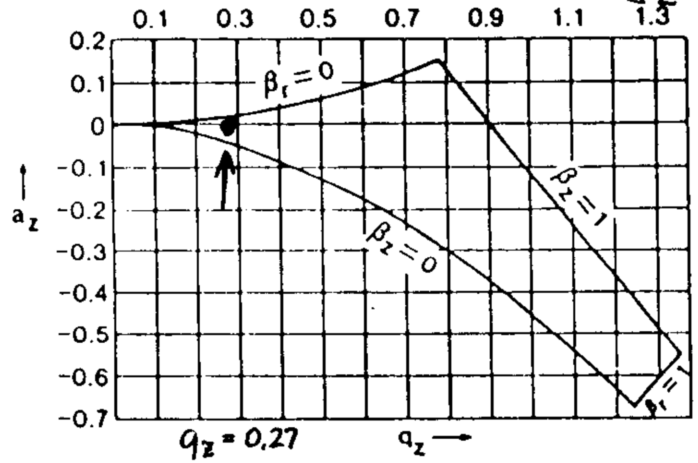
(numbers refer to Munich trap)



$r_0 = 2.50\,mm$

$z_0 = 1.77\,mm$

$\Omega = 2\pi \cdot 10.7\,MHz$



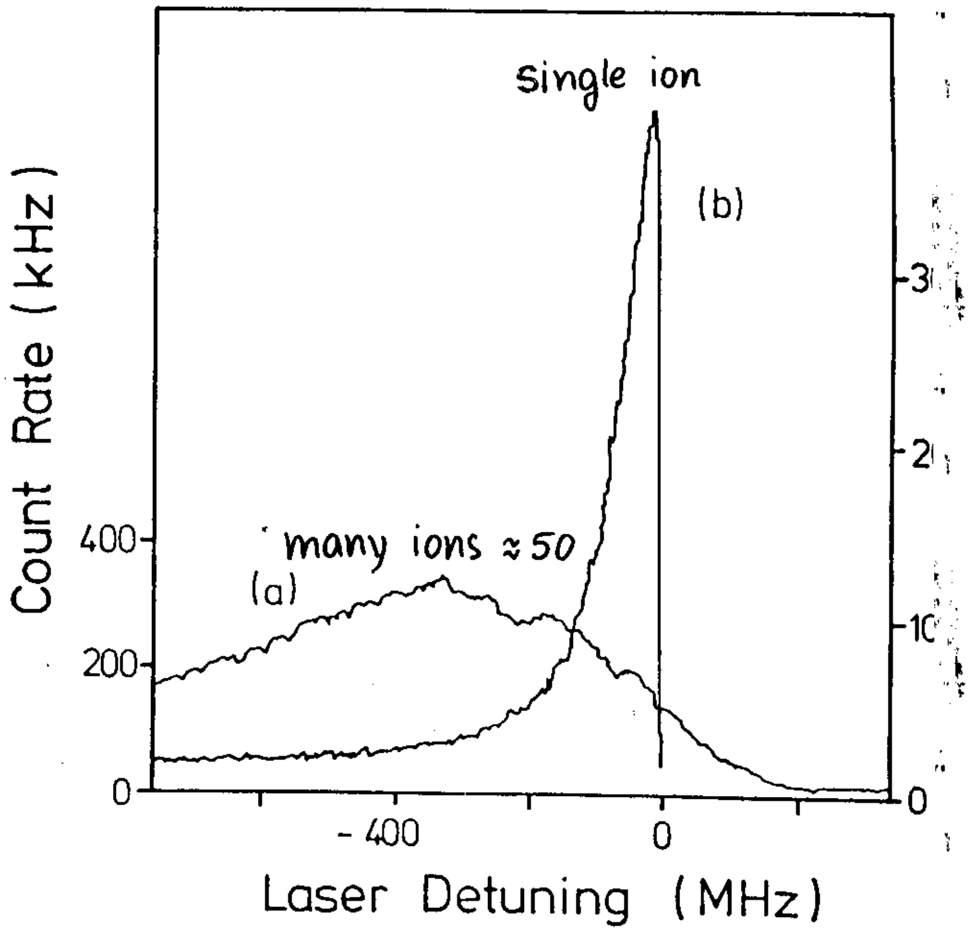
$D_r = e\,\phi(r_0, 0)$

$D_z = e\,\phi(0, r_0/\sqrt{2})$

Mg^+ : $V_0 = 500V$ $D_r = 8.4\,eV$; $D_z = 16.8\,eV$

$r_0 = 2.5\,mm$

$U_0 = 10V$ ($a_z = -0.011$) $D_r = 13.4\,eV$; $D_z = 11.8\,eV$



F. Diedrich, E. Peik, J. M. Chen, W. Quint, H. Walther

Phys. Rev. Lett. 59, 2931 (1987)

Overview on Applications

First laser spectroscopy of single ions :

W. Neuhauser, M. Hohenstatt, P. Toschek, H. Dehmelt
Phys. Rev. Lett. 41, 233 (1978)
Phys. Rev. A22, 1137 (1980)

Frequency standards and clocks :

Tl^+: H. Dehmelt, IEEE Trans. Instrum. Meas. IM-31, 83 (1982)

Hg^+: D.J. Wineland, W.M. Itano, J.C. Berquist, F.L. Walls, Proc 35th Annual. Symp. Fr. Contr. (1981)
M.D. Mc Guir, R. Retsch, G. Werth, Phys. Rev. A17, 1999 (1978)

Be^+: J.J. Bollinger, J.D. Prestage, W.M. Itano, D.J. Wineland, Phys. Rev. Lett. 54, 1000 (1985)

Yb^+: R. Blatt, H. Schmatz, G. Werth, Z. f. Physik A312, 143 (1983)

Local Lorentz Invariance

I.D. Prestage, J.J. Bollinger, W.M. Itano, D.J. Wineland, Phys. Rev. Lett. 54, 2387 (1985)

Observation of quantum jumps :

W. Nagourney, J. Sandberg, H. Dehmelt, Phys. Rev. Lett. 56, 2767 (1986)

Th. Sauter, W. Neuhauser, R. Blatt, P.E. Toschek, Phys. Rev. Lett. 57, 1696 (1986)

J.C. Bergquist, R.G. Hulet, W.M. Itano, D.J. Wineland, Phys. Rev. Lett. 57, 1699 (1986)

Crystallization of Charged Particles

Plasma Parameter Γ

$$\Gamma = \frac{\text{potential (Coulomb) energy}}{\text{kinetic (thermal) energy}}$$

Wigner (1938): quantum mechanical model
one-dimensional electron gas

Wuerker, Shelton, Langmuir (1959): crystallyzed charged particle clouds in a Paul trap

- ϕ particles 20 μm
- charge: $10^5 e$
- number: up to 100
- cooled by interaction with background gas

Condensation expected for $\Gamma \approx 150$

Ions in a Paul-trap

First observation of "ion crystals"

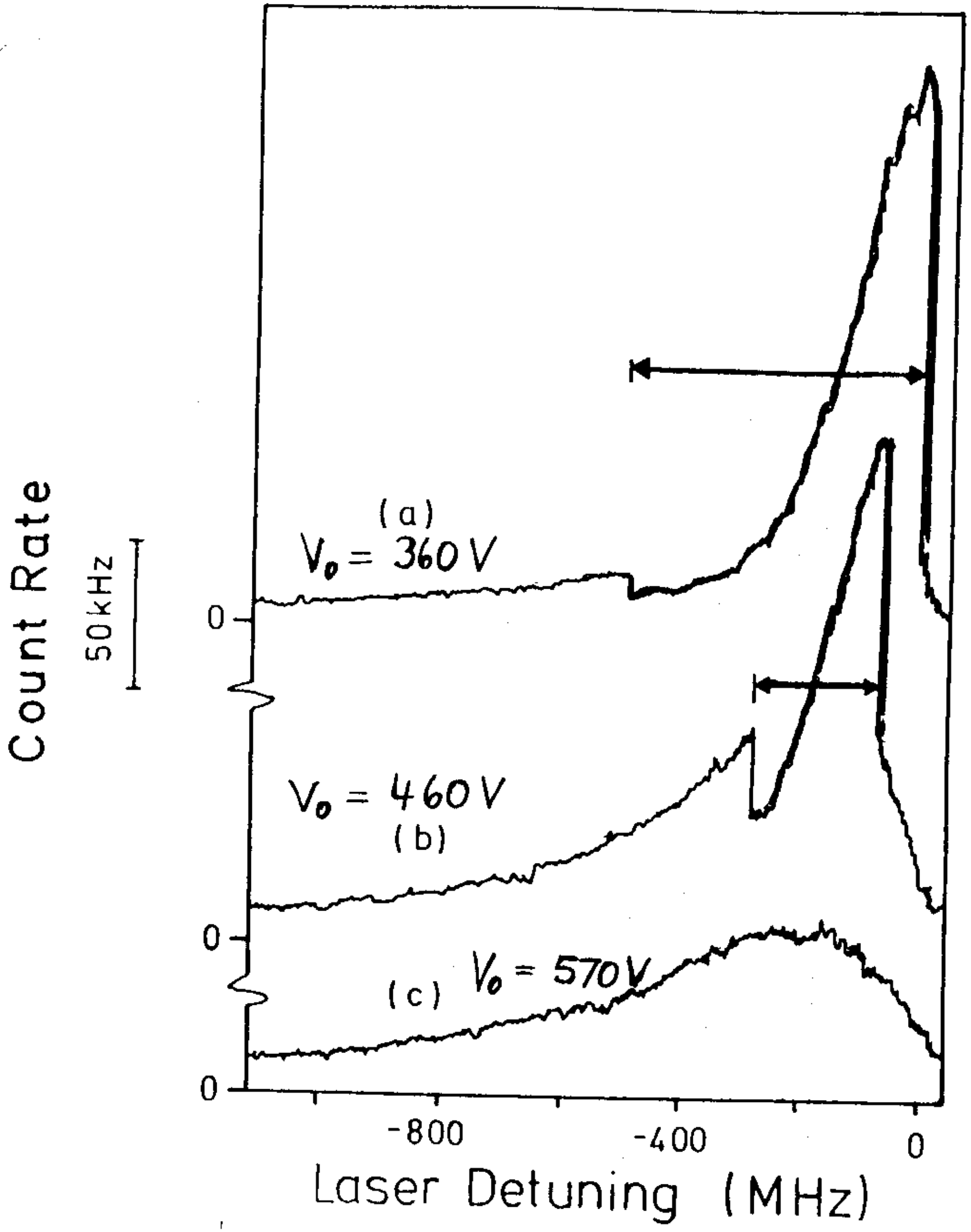
F. Diedrich, J. Krause, G. Rempe, M. Scully, H. Walther, Proceedings of the Laser Spectroscopy Conference, June, 1987, Åre, Sweden

F. Diedrich, E. Peik, J.M. Chen, W. Quint, H. Walther, P.R.L. 59, 2931 (1987)

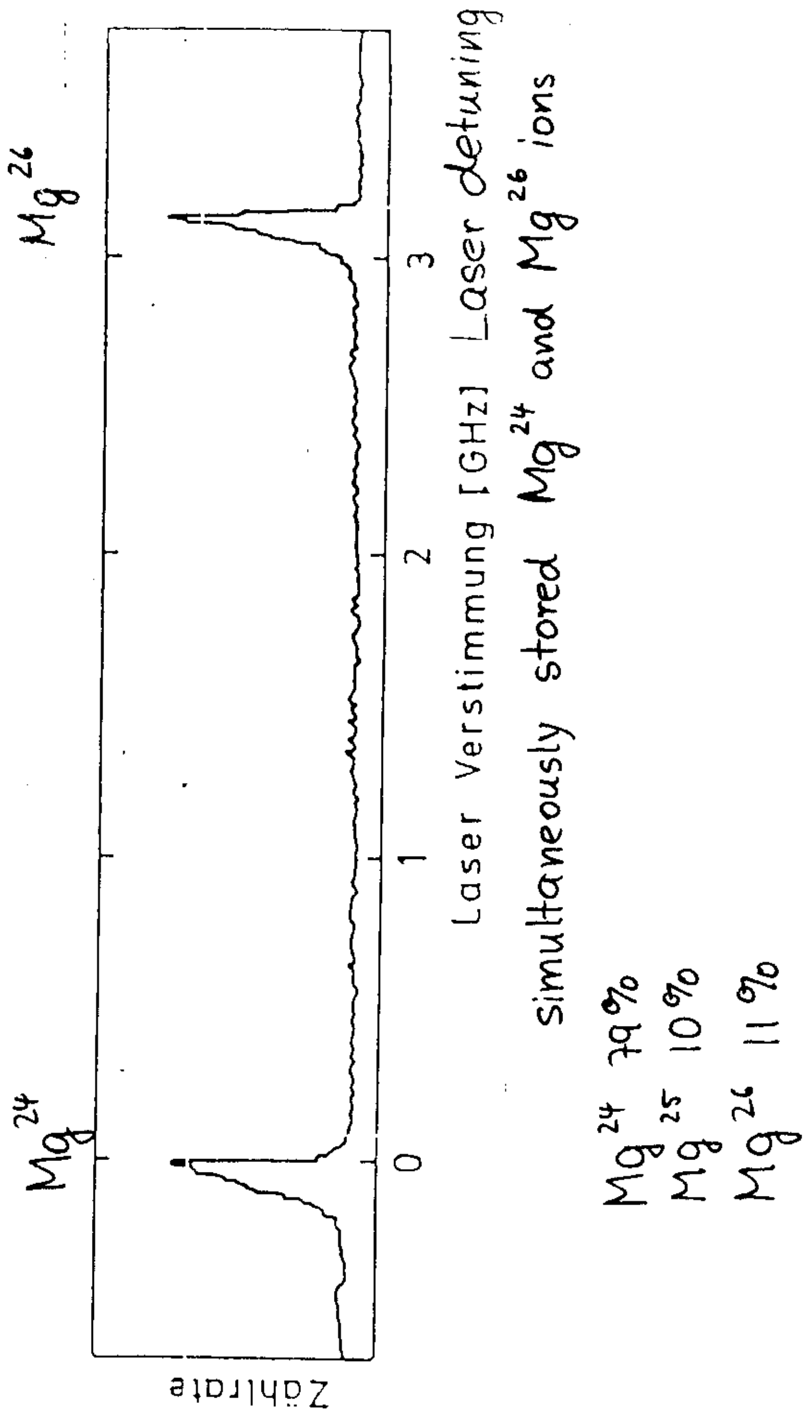
D. Wineland, J. Bergquist, W. Itano, J. Bollinger, C. Manney P.R.L. 59, 2935 (1987)

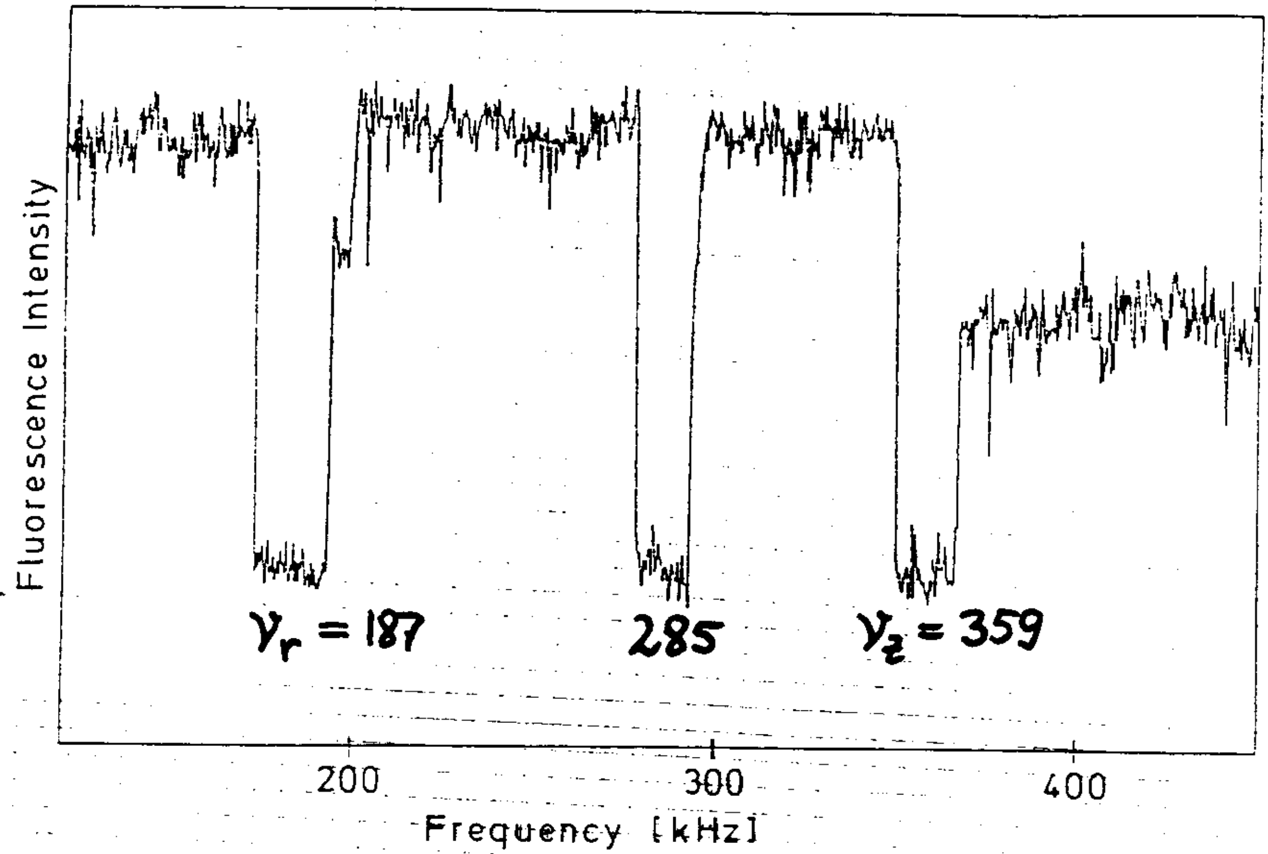
Ions in a Penning trap

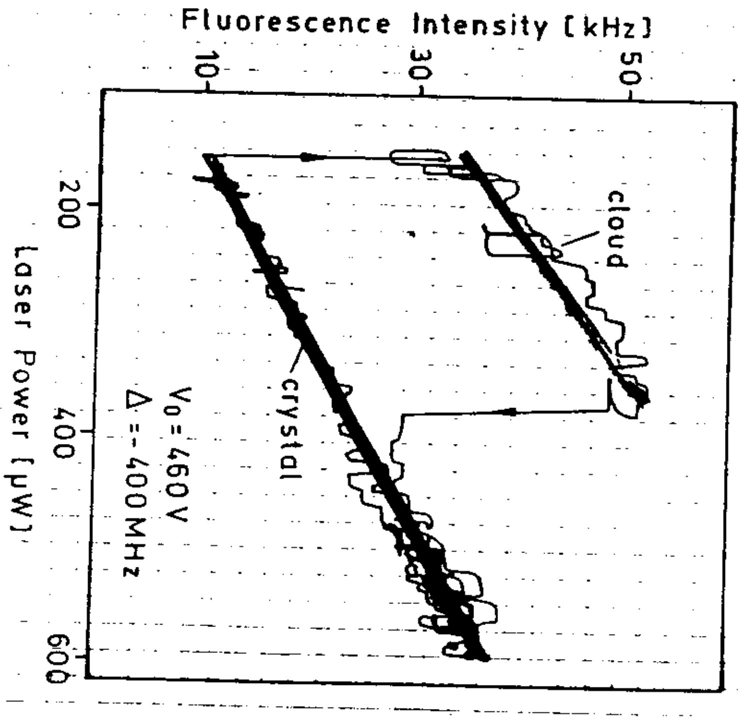
S. Gilbert, J. Bollinger, D. Wineland P.R.L. 60, 2022 (1988)

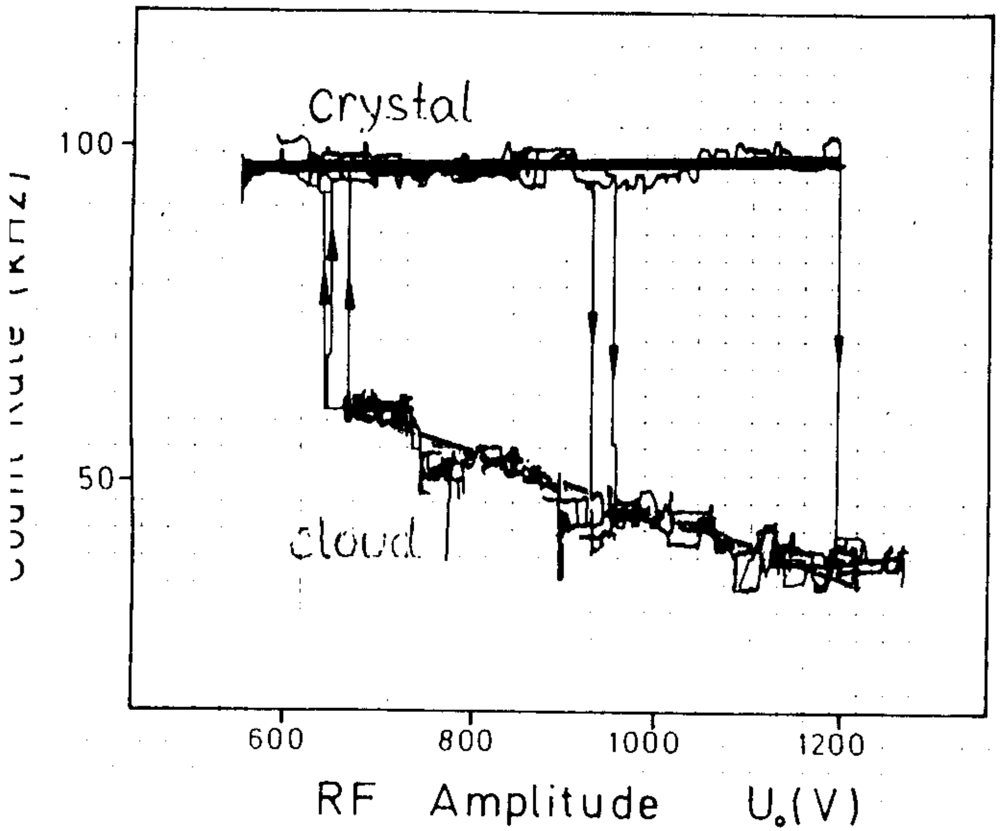


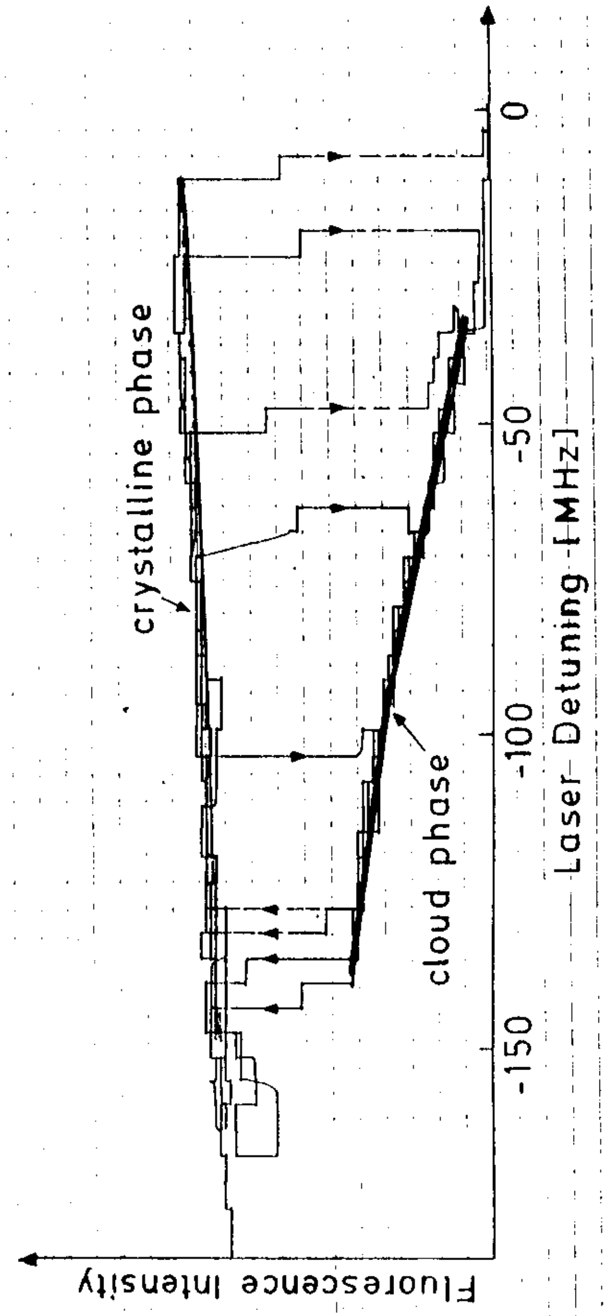
Diedrich, E. Peik, J. M. Chen, W. Quint, H. Walther,
Phys. Rev. Lett. 59, 2931 (1987)




Fluorescence Intensity
$\nu_r = 187$
285
$\nu_z = 359$
200
300
400
Frequency [kHz]


Fluorescence Intensity [kHz]
10
30
50
cloud
crystal
$V_0 = 460$ V
$\Delta = -400$ MHz
200
400
600
Laser Power [μW]


crystal
cloud
100
50
600
800
1000
1200
RF Amplitude U0(V)


crystalline phase
cloud phase
0
-50
-100
-150
Laser Detuning [MHz]
Fluorescence Intensity

Forces considered in the computer simulations

$$\vec{F}_i^{(trap)} = -2e\,g(t)\left[\vec{r}_i - 3 z_i \vec{e}_z\right]$$

$$g(t) = \frac{U_0 + V_0\cos(\Omega t)}{r_0^2 + 2 z_0^2}$$

$$\vec{F}_i^{(Coul)} = \frac{e^2}{4\pi\varepsilon_0} \sum_{m \neq i} \frac{\vec{r}_i - \vec{r}_m}{|\vec{r}_i - \vec{r}_m|^3}$$

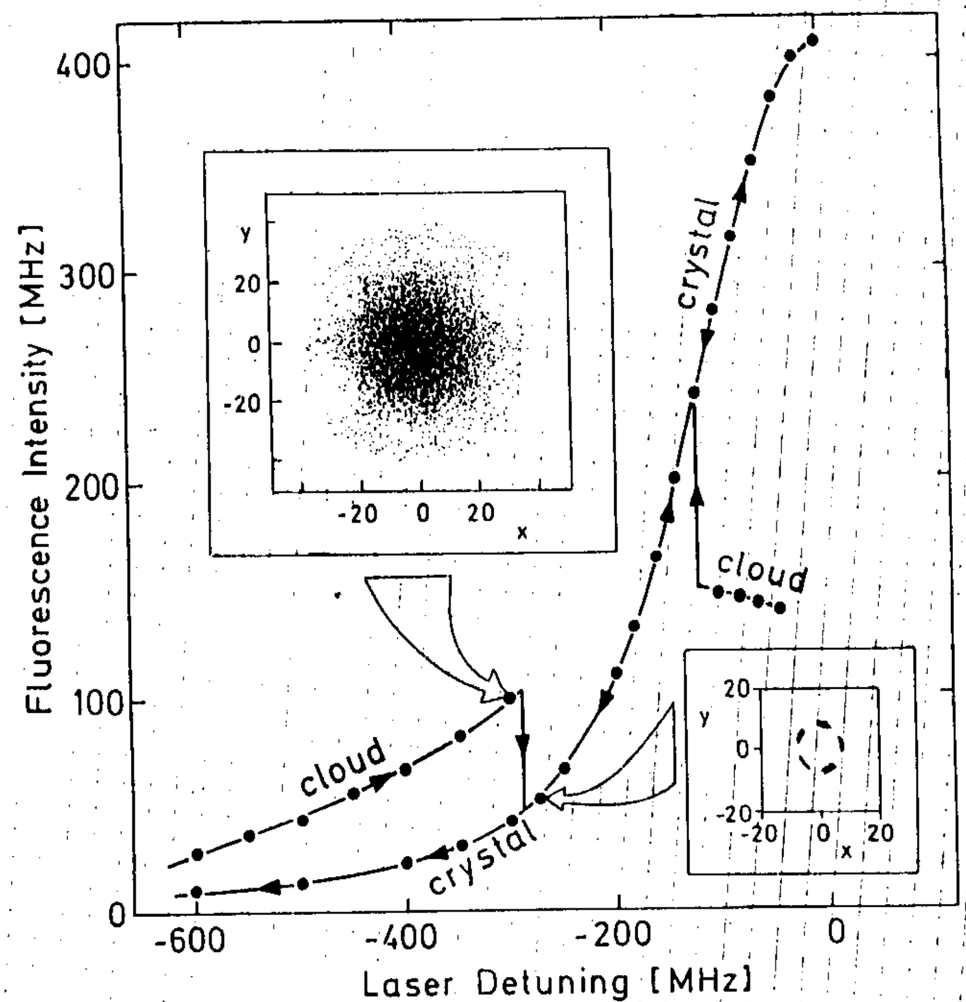
$$\vec{F}_i^{(laser)} = \hbar \vec{k}\, R\left[\vec{r}_i(t); \dot{\vec{r}}_i(t)\right]$$

R : photon scattering rate

$$\vec{F}_i^{(rand)} = \hbar k \sum_{l=-\infty}^{\infty} \delta(t - lT) \sum_{j=1}^{N_i(l)} \hat{u}_j$$

$\hat{u}_j$ is chosen at random and weighted by a $\cos^2$-distribution with respect to laser polarization

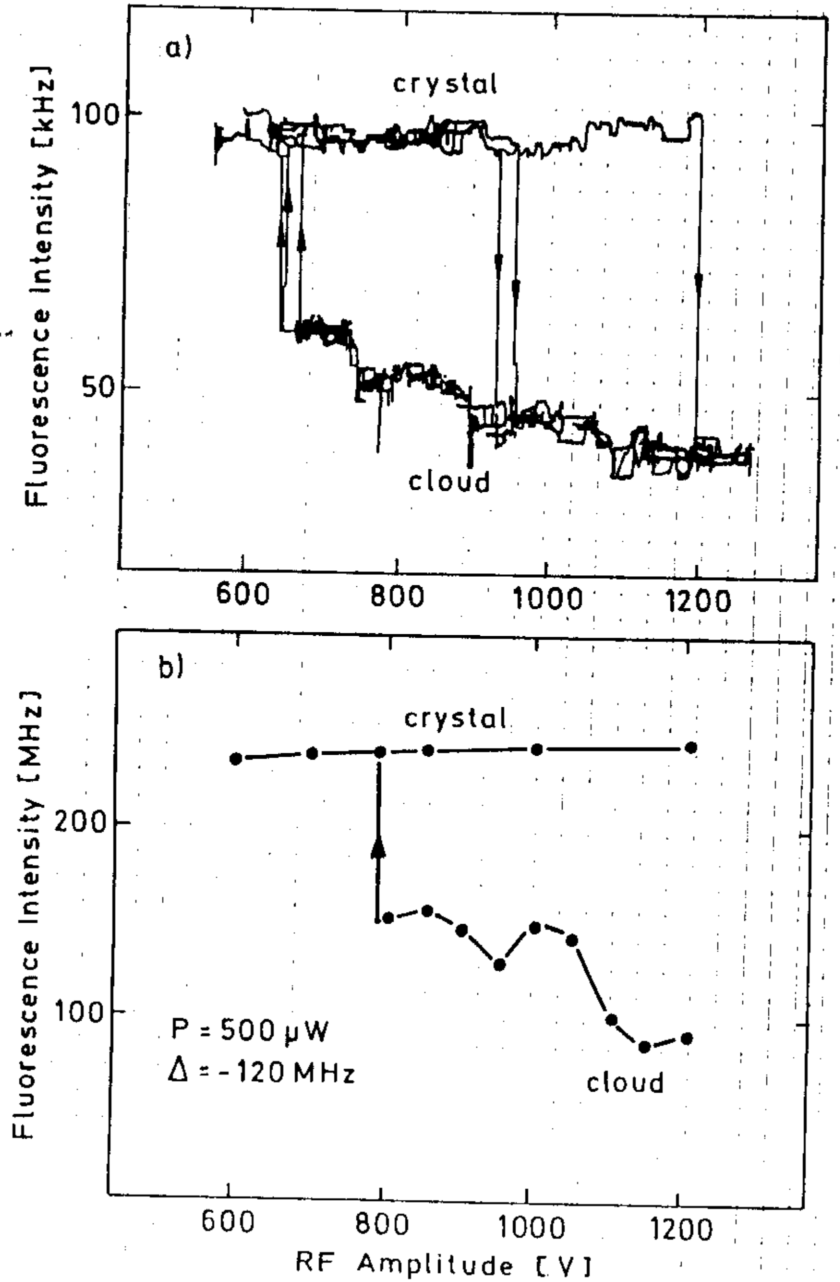
$T = 1/f \qquad \Omega = 2\pi f$

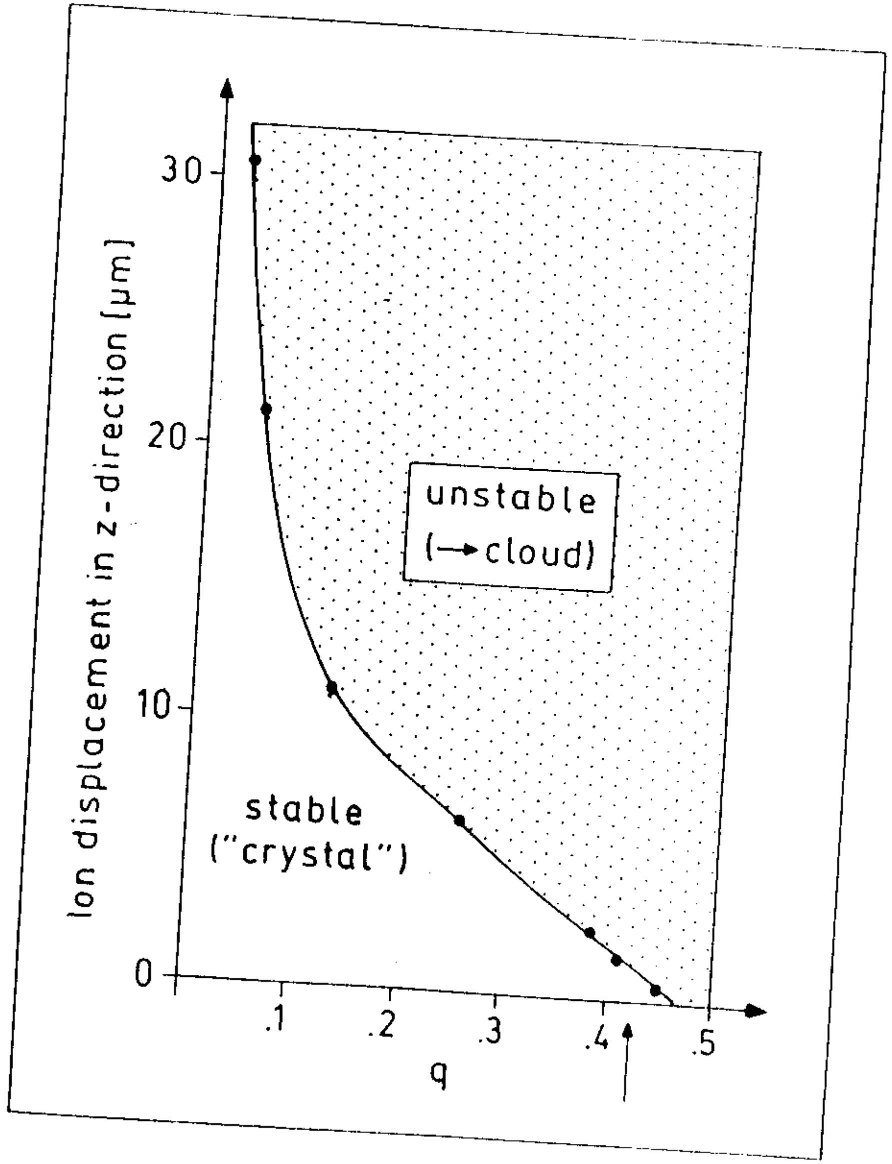


R. Blümel, J.M. Chen, E. Peik, W. Quint, W. Schleich, Y.R. Shen, H. Walther
Nature 334, 309 (1988)

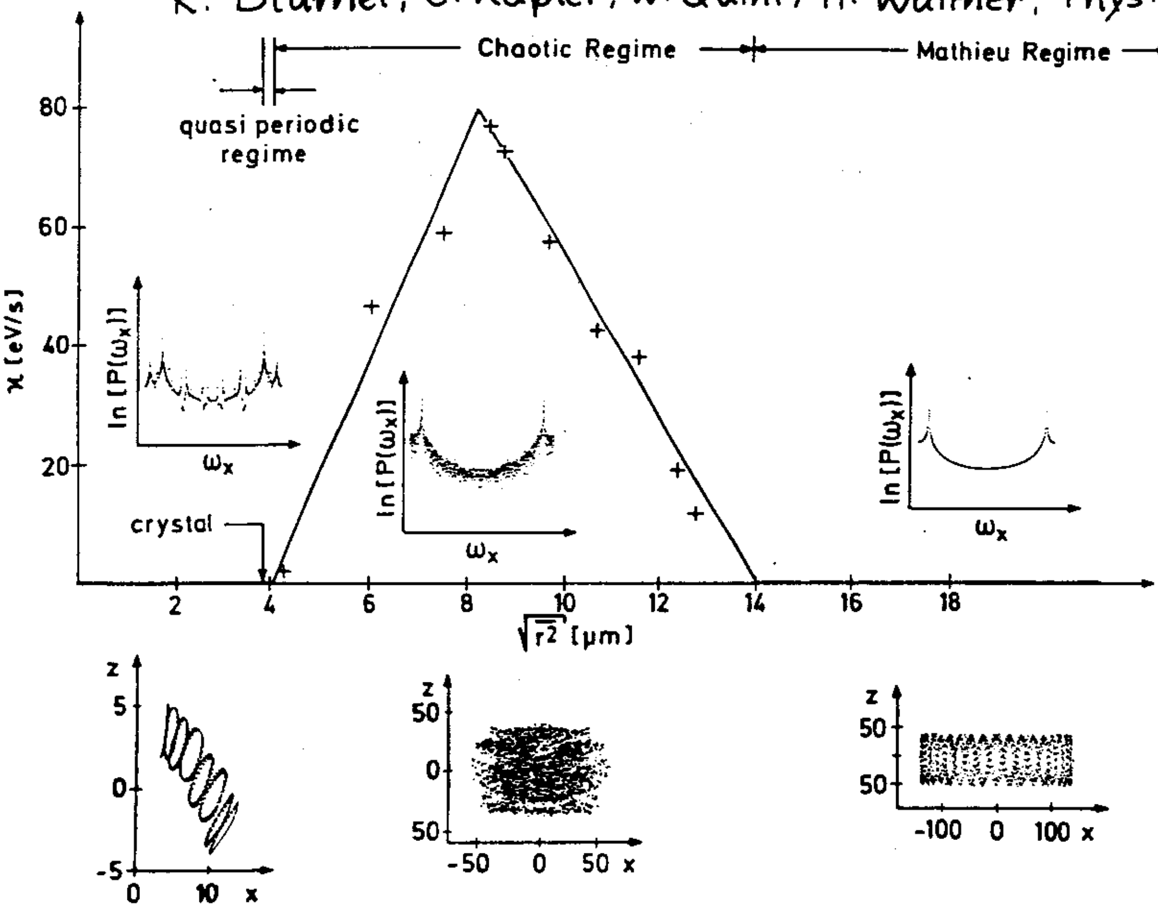
Fig. 4

Nature 334, 309 (1988)





R. Blümel, C. Kapler, W. Quint, H. Walther, Phys. Rev. July 1989



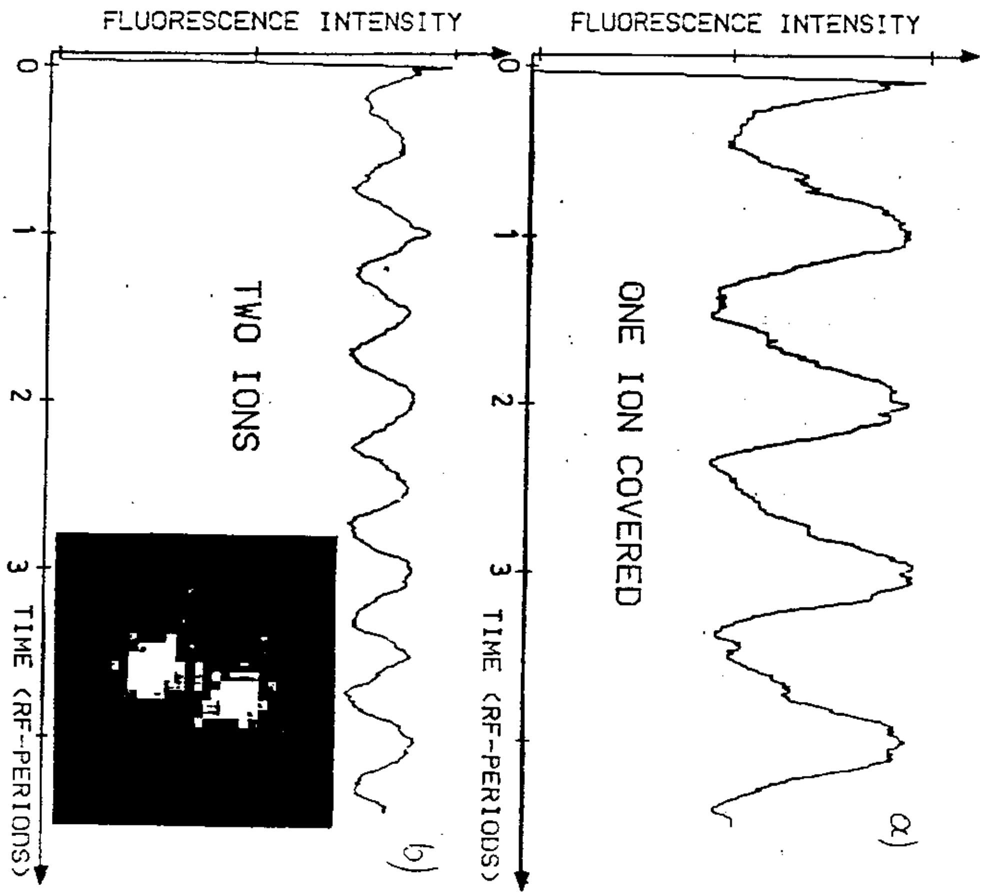
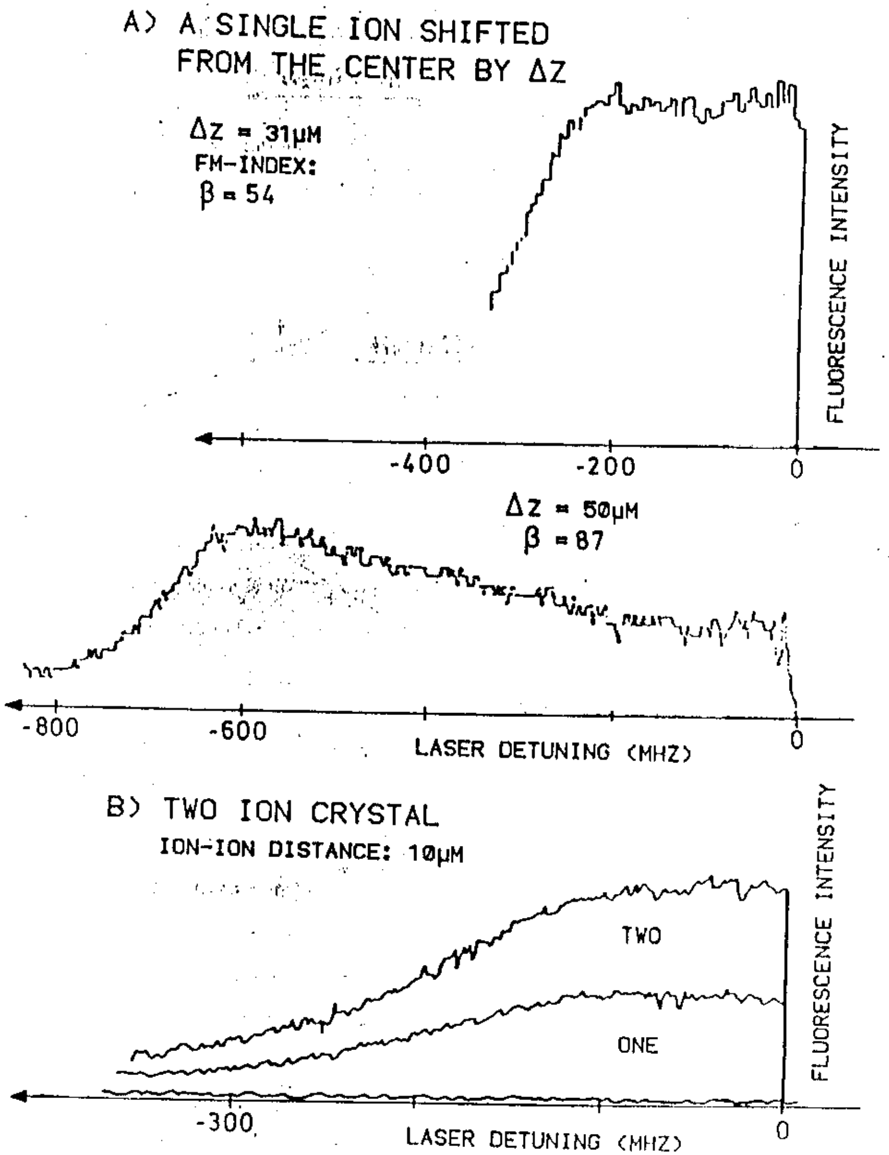


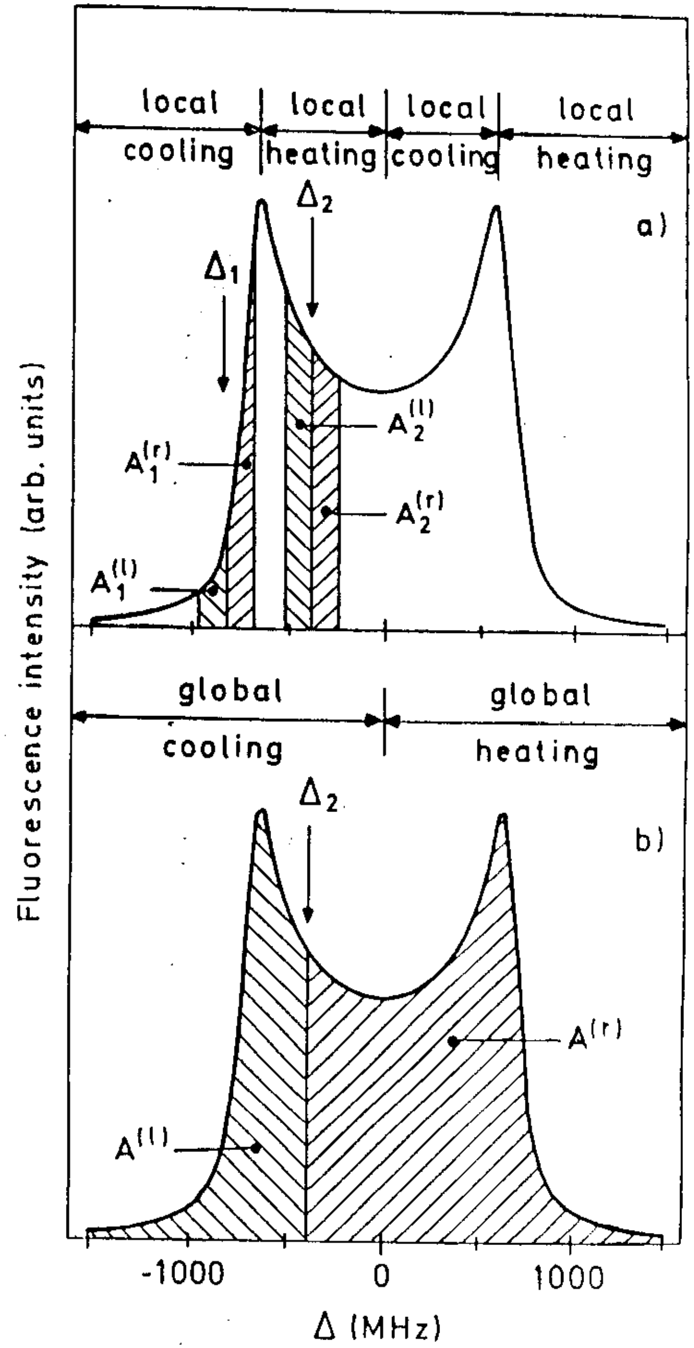
Fig. 8

Fig. 14



A) A SINGLE ION SHIFTED
FROM THE CENTER BY ΔZ
ΔZ = 31μM
FM-INDEX:
β = 54
FLUORESCENCE INTENSITY
-400
-200
0
ΔZ = 50μM
β = 87
-800
-600
0
LASER DETUNING (MHZ)
B) TWO ION CRYSTAL
ION-ION DISTANCE: 10μM
TWO
ONE
FLUORESCENCE INTENSITY
-300
0
LASER DETUNING (MHZ)

Fig. 9



local cooling
local heating
local cooling
local heating
a)
Δ1
Δ2
A1(r)
A1(l)
A2(l)
A2(r)
global cooling
global heating
Δ2
b)
A(r)
A(l)
Fluorescence intensity (arb. units)
-1000
0
1000
Δ (MHz)

Quadrupole Ring Trap