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Spatial and Temporal Instabilities in a CO_2 Laser

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We show experimentally the appearance of time-dependent and stationary complex spatial structures in a CO_2 laser. A comparison of the data with a theoretical model supports the notion that the observed phenomena result from the nonlinear interaction of transverse cavity modes with the active medium. The simultaneous appearance of spatial and temporal instabilities hints to the interesting prospect that one may be able to study turbulence in a laser system.

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Deterministic time-dependent intensity fluctuations, both of periodic and chaotic type, have been observed in a variety of free-running and driven laser systems.[1-3] Many other nonlinear systems, such as fluids and chemical oscillators, commonly display a richer phenomenology because of the appearance of spatial in addition to temporal complexity.[4,5] When dealing with a laser, one is faced with unique constraints imposed not only by the geometry and the finite transverse dimensions of the active medium, but also, and especially, by the configuration of the resonator. The laser cavity, which acts as an optical filter, imposes a significant reduction in the number of dynamical degrees of freedom. In general, this favors the development of ordered spatial structures, the so-called transverse cavity modes. Passive optical systems driven by a Gaussian field have shown recently the ability to generate complex space-time patterns.[6-8] Free-running lasers, of course, differ from passive systems because the resonant medium is active and because of the absence of an injected field.

In Ref. 9 we developed a laser model that accounts for the interaction of the transverse modes of the resonator and the active medium. Our calculations show that, of the many geometrical and physical parameters of the system, the radius of curvature of the output mirrors plays an especially critical role in affecting the strength and the nature of the modal coupling. Over a range of experimentally accessible parameter values our analysis predicts the existence of instabilities and cooperative frequency locking; the latter is a multimode regime of operation where the field modes oscillate in synchrony and the output intensity is time independent.

In this Letter we present experimental evidence for self-pulsing, cooperative frequency locking and the development of complex stationary spatial structures at the output of a CO_2 laser when the cavity is swept slowly and continuously from a quasiconfocal to a nearly planar configuration. Our experiments are carried out with a polarized CO_2 laser of the Fabry-Perot type with flat-end reflectors. The laser is designed to operate with two intracavity ZnSe lenses whose separation is controlled by a stepper motor and whose role is to produce a variable effective radius of curvature of the output couplers. The tube is designed with special care in order to maintain the cylindrical symmetry of the output beam over most of the interesting parameter range, even in the presence of the Brewster windows. Its diameter is sufficiently large to allow the operation of the second-order ($p=2$) Gauss-Laguerre mode without excessive diffraction losses.

The output beam is detected by a HgCdTe detector with a bandwidth of 100 MHz after reflection from a rotating polygonal mirror whose function is to sweep the beam across a small opening placed in front of the detector. First the detector is adjusted in such a way as to maximize the diameter of the beam cross section; the detected signal is then analyzed with a digital oscilloscope and processed by a spectrum analyzer.

The separation between the intracavity lenses controls the effective configuration of the cavity; this can be varied continuously in the experiments from nearly planar to nearly hemispherical. Varying the distance between the lenses also causes a change in the frequency spacing between the transverse cavity modes; this can be made as large as about $\frac{1}{4}$ of the longitudinal free spectral range before transverse modes characterized by a lower longitudinal index also fall under the gain curve. The longitudinal free spectral range, instead, remains constant because of the fixed length of the cavity.

For values of the effective radius of curvature of the mirrors that produce the largest separation between consecutive transverse modes, a combination of the natural diffraction losses of the cavity and the transverse profile of the atomic inversion can easily lead to single-Gaussian-mode operation. The transverse intensity profile of the beam in this case takes the form shown in Fig. 1(a). Upon increasing the effective radius of curvature of the mirrors, a small but detectable modification of the beam profile begins to develop [Fig. 1(b)]. The output field can no longer be described in terms of a single cavity mode, and even if the profile is still time independent, the contribution of the first higher-order mode ($p=1$) to the

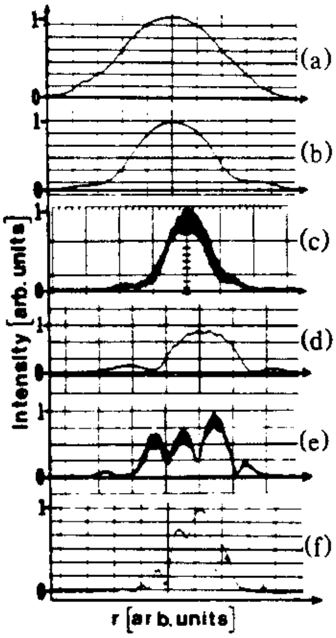

FIG. 1. Beam transverse profile for decreasing separation between transverse modes [from (a) to (f)].

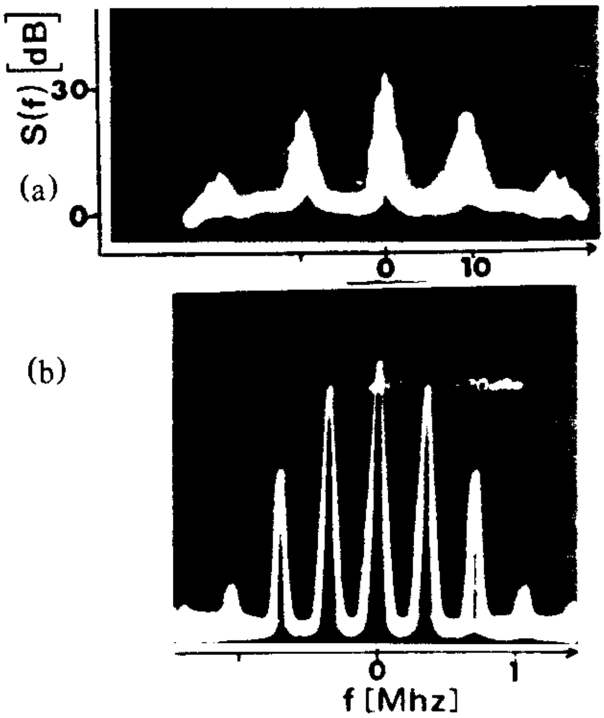

FIG. 2. Power spectrum in the quasiperiodic regime: (a) full spectrum, (b) expanded view of the central peak.

dominant Gaussian field is not negligible. When the beam profile contains contributions from two or more modes, while the laser still operates at a single frequency, we say that the system has entered a cooperative-frequency-locking state.[10]

A further reduction of the frequency spacing between cavity modes causes a break of the locking state, the contribution of the higher-order transverse mode $p=1$ grows, and the output intensity begins to pulsate at a frequency that is quite close to the mode-pulled beat note between two adjacent modes. At first sight it is tempting to attribute this result to an ordinary beating effect between transverse modes. This is certainly present. However, a detailed analysis of the modulation depth along the beam profile [see Fig. 1(c)] shows that the origin of this phenomenon is also related to the oscillating character of the modal amplitudes themselves. In fact, a simple interference of the $p=0$ and $p=1$ modes, for example, would produce no modulation of the total intensity at the radial positions where the $p=1$ mode amplitude vanishes. While a clear physical explanation for this effect is not yet available, the result is confirmed also by our theoretical analysis [see, especially, Fig. 4(b) which shows the moduli of the three lowest transverse modes]. We may also add that the simple sinusoidal nature of the oscillations seems to indicate that the interacting modes are phase locked[11] although their absolute frequencies are changing to produce the oscillations of the moduli.

A closer overlap among transverse modes, induced by a further reduction of their frequency spacing, produces first the stabilization of a structure that resembles closely the pure $p=1$ transverse pattern, as shown in Fig. 1(d) (apparently this is a consequence of the fact that the $p=1$ mode has a higher gain than the Gaussian $p=0$ mode when sufficiently close to resonance). Subsequently, a second temporal instability develops which is accompanied by a notable increase in spatial complexity as shown in Fig. 1(e). Over this range of effective radii of curvature, in addition to simple periodic oscillations, we observe also quasiperiodic behavior as shown by the power spectrum of Fig. 2. Here, frequencies of the order of MHz and kHz appear simultaneously. While the high-frequency component of these oscillations appears to match the separation between consecutive transverse modes, the low-frequency component could result from the interaction of the cavity field with the population inversion, judging at least from its proximity with the relaxation oscillation frequency.

When the cavity approaches a nearly planar configuration the temporal oscillations disappear and the output intensity becomes stationary again, with a profile that displays a high degree of spatial complexity. This configuration is no longer cylindrically symmetric and involves at least three frequency-locked modes; an example is shown to Fig. 1(f).

The regions of operation where the output intensity shows temporal oscillations are characterized by a lower average output power than neighboring stationary configurations, as shown in Fig. 3. This effect becomes especially obvious when spatial complexity and temporal oscillations both emerge.

The trends exemplified by the results shown in Figs. 1-3 are confirmed qualitatively by numerical simulations

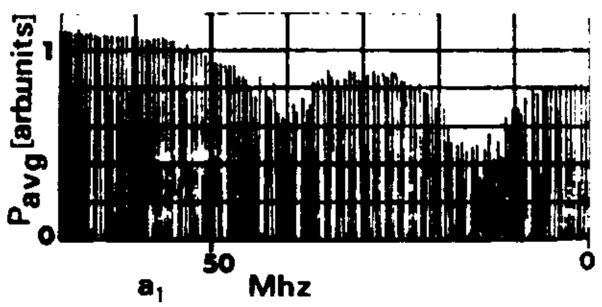

FIG. 3. Average power as a function of the distance among transverse modes. The vertical lines represent individual power measurements.

based on the model developed in Ref. 9 which we now summarize for completeness. Under the assumptions of a uniform field limit, a separation between longitudinal modes much larger than the linewidth $\gamma_\perp$ of the atomic transition, and a cavity geometry such that several higher-order transverse modes fall within the linewidth of the TEM_{00} mode, the equations of motion for the modal amplitudes Ψ_p, p_p, and d_p ($p=0,1,2\ldots$) of the field, polarization, and populations inversion are[9]

$$\frac{d\Psi_p}{d\tau}=-\kappa_p\Psi_p-i\kappa(a_p-\Delta)\Psi_p-2C\kappa p_p\,, \tag{1a}$$

$$\frac{dp_p}{d\tau}=-\sum_{qq'}\Gamma_{pqq'}\Psi_q d_{q'}-(1+i\Delta)p_p\,, \tag{1b}$$

$$\frac{dd_p}{d\tau}=-\gamma\left[-\tfrac{1}{2}\sum_{qq'}\Gamma_{pqq'}(\Psi_q^* p_{q'}+\text{c.c.})+d_p-d_p^{(0)}\right], \tag{1c}$$

where $\tau=\gamma_\perp t$, Δ is the frequency difference between the center of the gain line and the operating laser frequency, in units of $\gamma_\perp$, and γ denotes the rate of decay of the population difference (also in units of $\gamma_\perp$); the mode-dependent damping rate κ_p simulates the different diffraction losses of the transverse modes and $\kappa_0\equiv\kappa$, where κ is the cavity linewidth; the symbol a_p denotes the frequency spacing between the pth radial resonance and the reference TEM_{00} mode. The parameters $\Gamma_{pqq'}$ are the mode-mode coupling coefficients, and $d_p^{(0)}$ denotes the p-modal component of the equilibrium population difference. In our calculations we have assumed a uniform transverse profile for the pump.

For gain-to-loss ratios such that only the first three transverse modes, with indices $p=0$, 1, and 2, have a chance to play a dominant role, and upon selecting values of κ, $\gamma_\perp$, and $\gamma_\parallel$ in a range that is appropriate for CO_2 lasers, we are able to reproduce qualitatively the trends exhibited by the experimental results of Fig. 1. These are summarized in Fig. 4(a) where we plot the modulus of the modal amplitudes Ψ_0 and Ψ_1 as functions of the transverse intermode spacing. The single solid lines define steady-state operation, while the shaded regions indicate that the output is undergoing oscillations with an amplitude given by the vertical width of these domains.

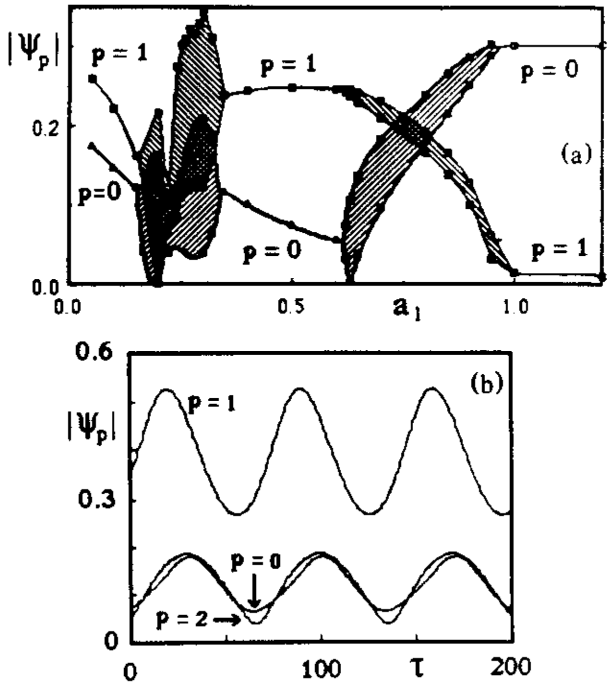

FIG. 4. (a) Behavior of the moduli of the modal amplitudes Ψ_0 and Ψ_1 as functions of the intermode spacing; (b) periodic oscillations of $|\Psi_p|^2$ for $p=0,1,2$.

For large values of a_1 (the intermode spacing is κa_1) the long-time solutions are stationary and essentially of the TEM_{00} single-mode type. For smaller values of a_1 the amplitude of the $p=1$ mode increases, as shown in Fig. 4(a), and stationarity is destroyed. At this point the moduli of the modal amplitudes begin to oscillate periodically. Over the range $0.65<a_1<1$, regular, time-dependent oscillations develop with a frequency given by the mode-pulled intermode spacing. The strength of the mode $p=1$ continues to grow for decreasing a_1 until it becomes a major contribution to the total cavity field for $a_1<0.65$. In this region the competition between the modes ceases and the system enters a regime of cooperative frequency locking.

The next region of unstable oscillations develops when a_1 becomes smaller than approximately 0.35, in response to the growth of the $p=2$ modal component which is now becoming closer in frequency to the TEM_{00} mode. Here the situation is more complicated because, in addition to the periodic oscillations [an example of which is shown in Fig. 4(b)], we find also evidence for chaotic behavior, perhaps induced by the growth of the $p=2$ modal component. This region is also characterized by small domains of quasiperiodicity in which the moduli of the dominant modes ($p=0,1,2$) move on the surface of a torus in three-dimensional space.

Below a certain well-defined threshold value (in this simulation, for a_1 less than approximately 0.2) the system enters a new locked state in which several modes oscillate synchronously with comparable strength. In this case the output field intensity becomes time independent and the transverse intensity profile develops considerable modulation.

Complex spatial structures with no radial symmetry, such as those observed experimentally in the near planar configuration, cannot be found in this model because of the assumed cylindrical symmetry. In a recent investigation, however, we have shown that the introduction of the angular degree of freedom into the dynamical equations can lead to spontaneous breaking of the cylindrical symmetry.[12] This phenomenon is currently under study.

In conclusion, we have provided experimental and theoretical evidence that the origin of certain unstable behaviors in CO_2 lasers can be traced to the interaction among the transverse degrees of freedom of the cavity field and of the atomic variables. This interaction favors the formation of stationary and oscillating spatial structures and, perhaps, under suitable conditions it may even yield turbulent motion.

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