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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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H4.SMR/449-41

WINTER COLLEGE ON HIGH RESOLUTION SPECTROSCOPY

(8 January - 2 February 1990)

SPECTRAL AND STATISTICAL PROPERTIES OF COLLECTIVE RESONANCE FLUORESCENCE IN A SQUEEZED VACUUM

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J. Phys. B: At. Mol. Opt. Phys. **22** (1989) 131-142. Printed in the UK

Spectral and statistical properties of collective resonance fluorescence in a squeezed vacuum

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Received 22 June 1988, in final form 2 September 1988

Abstract. We discuss the problem of collective resonance fluorescence of N driven atoms which are damped by a white squeezed noise. Analytical formulae for the spectrum of resonance fluorescence and for the degree of second-order coherence for spectral lines are calculated. The narrowing or expansion of linewidths and the asymmetry of the spectrum caused by the squeezed vacuum are shown.

1. Introduction

The fundamental property of squeezed light is the reduced quantum fluctuations in the one-quadrature phase. After early work by Caves (1983), in which the potential application of squeezed light for detection of gravitational waves was shown, a large amount of theoretical (Walls 1983, Collett and Loudon 1987, Yurke 1984, Bondurant *et al* 1984, Mandel 1982, Walls and Zoller 1981, Lakshmi and Agarwal 1984, Bogolubov *et al* 1986, 1987b, Ficek *et al* 1987, Kozierowski and Kielich 1983, Golubyev 1987, Dodonov *et al* 1980, Barnett and Knight 1987, Wodkiewicz *et al* 1987) and experimental work (Slusher *et al* 1985, Ling-An Wu *et al* 1986) has been concentrated on the generation of squeezed light and its application to overcome the shot-noise limit (Min Xiao *et al* 1987). In recent work the radiative decay (Gardiner 1986) and spectroscopic properties (Carmichael *et al* 1987a,b, Parkins and Gardiner 1988) of an atom interacting with a broad-band squeezed vacuum have been considered.

In this work we discuss the collective resonance fluorescence from N driven atoms which are damped by white squeezed noise. In the case of an intense external field the stationary solution for the density operator of the atomic system is given. In the general case (excluding the case of exact resonance) the density matrix of the atomic system is dependent on a phase difference between the driving field and squeezed vacuum. The dependence of the fluorescence spectrum and the photon statistics of spectrum components on the parameters of the squeezed vacuum are analysed.

2. Basis equations

Let N two-level atoms be concentrated in a region small compared with the wavelength of all the relevant radiation modes (Dicke model). The atoms interact with a classical driving field of frequency ω_L and with the quantised multimode radiation field. The

Hamiltonian of the system in the electric dipole, rotating-wave approximation and in the interaction picture can be written in following form:

$$H = \tfrac{1}{2}\delta(J_{22} - J_{11}) + G\,e^{i\phi_1}J_{21} + G\,e^{-i\phi_1}J_{12} + H_{\mathrm{rad}} + (J_{21}\Gamma + J_{12}\Gamma^{+}) \tag{1}$$

where $\delta = \omega_{21} - \omega_L$ is the detuning of the laser frequency from the atomic resonance frequency ω_{21}; $G\,e^{-i\phi_1} = \mu E$ where μ is the atomic dipole moment and E is the amplitude of the driving field; $J_{ij}(i, j = 1, 2)$ are the collective (angular) atomic operators which satisfy the commutation relations

$$[J_{ij}, J_{i'j'}] = J_{ij'}\delta_{i'j} - J_{i'j}\delta_{ij'} \tag{2}$$

H_{rad} is the free Hamiltonian for the quantised radiation field; Γ and Γ^{+} are operators defined in terms of the positive and negative frequency components of this field, respectively.

The normal treatment of resonance fluorescence has been considered in many works (Bogolubov *et al* 1987a, Agarwal *et al* 1979, Compagno and Persico 1982) in which the quantised radiation field is initially taken in the usual vacuum state.

In this work we assume that all the quantised radiation modes coupling to the atoms initially are squeezed (Collett and Gardiner 1985, Gardiner 1986, Carmichael *et al* 1987, Parkins and Gardiner 1988, Gardiner *et al* 1987). The bandwidth of the squeezing is assumed to be sufficiently broad that the squeezed vacuum appears as δ-correlated squeezed white noise to the atoms. Then, the correlation functions for free parts Γ_{free} and $\Gamma^{+}_{\mathrm{free}}$ (the noise operators) of the operators Γ and Γ^{+} can be written as (Collett and Gardiner 1985, Gardiner 1986, Carmichael *et al* 1987)

$$\begin{aligned}
\langle\Gamma^{+}_{\mathrm{free}}(t)\Gamma_{\mathrm{free}}(t')\rangle &= \gamma P\delta(t-t') \\
\langle\Gamma_{\mathrm{free}}(t)\Gamma^{+}_{\mathrm{free}}(t')\rangle &= \gamma(P+1)\delta(t-t') \\
\langle\Gamma_{\mathrm{free}}(t)\Gamma_{\mathrm{free}}(t')\rangle &= \gamma|Q|\,e^{i\phi_V}\delta(t-t') \\
\langle\Gamma^{+}_{\mathrm{free}}(t)\Gamma^{+}_{\mathrm{free}}(t')\rangle &= \gamma|Q|\,e^{-i\phi_V}\delta(t-t')
\end{aligned} \tag{3}$$

where γ is the spontaneous emission rate of an atom in the usual unsqueezed vacuum; P and $Q = |Q|\,e^{i\phi_V}$ are the parameters characterising the squeezing with $|Q|^2 \leqslant P(P+1)$, where the equality holds for a minimum-uncertainty squeezed state.

Using the relations (1) and (3) and after making the unitary transformation $U = \exp(-\tfrac{1}{2}i\phi_1(J_{22} - J_{11}))$, one finds the master equation for the reduced density operator ρ of the atoms in the following form (Collett and Gardiner 1985):

$$\begin{aligned}
\partial\rho/\partial t = -i[\tfrac{1}{2}\delta(J_{22} - J_{11}) + G(J_{12} + J_{21}), \rho] &+ \tfrac{1}{2}\gamma(P+1)(J_{12}\rho J_{21} - J_{21}J_{12}\rho + \mathrm{HC}) \\
&+ \tfrac{1}{2}\gamma P(J_{21}\rho J_{12} - J_{12}J_{21}\rho + \mathrm{HC}) - \tfrac{1}{2}\gamma|Q|\,e^{-i\phi}(J_{21}\rho J_{21} - J_{21}^2\rho + \mathrm{HC}) \\
&- \tfrac{1}{2}\gamma|Q|\,e^{i\phi}(J_{12}\rho J_{12} - J_{12}^2\rho + \mathrm{HC}) \equiv L\rho,
\end{aligned} \tag{4}$$

where $\phi = 2\phi_1 - \phi_V$ is the phase difference of the driving field and squeezed vacuum.

Following previous works (Bogolubov *et al* 1987a, Schwinger 1965) we introduce the Schwinger representation for the collective atomic operators J_{ij}

$$J_{ij} = a_i^{\dagger}a_j \quad (i, j = 1, 2) \tag{5}$$

where the operators a_i and $a_i^{\dagger}$ obey the boson commutation relations

$$[a_i, a_j^{\dagger}] = \delta_{ij}$$

and can be treated as the annihilation and creation operators for the atoms being populated in the level $|i\rangle$.

After performing the canonical (dressing) transformation

$$\begin{aligned}
a_1 &= C_1\cos\theta + C_2\sin\theta \\
a_2 &= -C_1\sin\theta + C_2\cos\theta
\end{aligned} \tag{6}$$

where $\tan(2\theta) = 2G/\delta$, one can split the Liouville operator L appearing in equation (4) into a slowly varying part and terms oscillating at frequencies 2Ω and 4Ω, with Ω denoting half of the Rabi frequency. We assume here that the Rabi frequency is sufficiently large and satisfies the relation

$$\Omega = (\tfrac{1}{4}\delta^2 + G^2)^{1/2} \gg N\gamma, P\gamma. \tag{7}$$

In this case the secular approximation (Bogolubov *et al* 1987a, Agarwal *et al* 1979) is justified and we retain only the slowly varying part of the Liouville operator. We then have the master equation

$$\begin{aligned}
\frac{\partial\tilde\rho}{\partial t} = -i[\Omega(R_{22} - R_{11}), \tilde\rho] &+ B(R_3\tilde\rho R_3 - R_3^2\tilde\rho + \mathrm{HC}) \\
&+ [X_1(R_{12}\tilde\rho R_{21} - R_{21}R_{12}\tilde\rho) + X_2(R_{21}\tilde\rho R_{12} - R_{12}R_{21}\tilde\rho) + \mathrm{HC}]
\end{aligned} \tag{8}$$

where $R_{ij} = C_i^{\dagger}C_j (i, j = 1, 2)$ are the collective operators of 'dressed' atoms satisfying the commutation relation

$$[R_{ij}, R_{i'j'}] = R_{ij'}\delta_{i'j} - R_{i'j}\delta_{ij'} \tag{9}$$

$$B = \gamma(P + \tfrac{1}{2} - |Q|\cos\phi)\sin^2\theta\cos^2\theta \tag{10}$$

$$X_1 = \tfrac{1}{2}\gamma[P(\cos^4\theta + \sin^4\theta) + \cos^4\theta + 2|Q|\cos\phi\sin^2\theta\cos^2\theta] \tag{11}$$

$$X_2 = \tfrac{1}{2}\gamma[P(\cos^4\theta + \sin^4\theta) + \sin^4\theta + 2|Q|\cos\phi\sin^2\theta\cos^2\theta] \tag{12}$$

$$R_3 = R_{22} - R_{11} \tag{13}$$

with $\tilde\rho = T\rho T^{-1}$ where T is the unitary operator representing the canonical transformation (6).

The master equation (8) gives the exact steady-state solution

$$\tilde\rho = Z^{-1}\sum_{n_1=0}^{N} X^{n_1}|n_1\rangle\langle n_1| \tag{14}$$

where

$$X = X_1/X_2, \tag{15}$$

$$Z = (X^{N+1} - 1)/(X - 1). \tag{16}$$

The state $|n_1\rangle$ is an eigenstate of the operators R_{11} and $R_{11} + R_{22}$. The solution (14) allows one to calculate all the stationary expectation values of the atomic observables. Some of the results that will be needed for our further considerations are given in the appendix. In the case of exact resonance $\cot^2\theta = 1$ one shows from equations (11), (12) and (15) that $X = 1$ and the solution (14) reduces to

$$\tilde\rho = (N+1)^{-1}\sum_{n_1=0}^{N}|n_1\rangle\langle n_1|. \tag{17}$$

Solution (17) is independent of the parameters P and Q of the squeezed vacuum; consequently all the one-time expectation values of the atomic observables are independent of whether the vacuum is squeezed or not. In the general off-resonance case the density operator $\tilde{\rho}$ in (14) is phase-sensitive and all the steady-state expectation values of the atomic and field observables are dependent on the parameters of squeezed vacuum. In §§ 3 and 4 we investigate the influence of the squeezed vacuum on the fluorescence spectrum and photon statistics of spectrum components.

3. Steady-state fluorescence spectrum

Following the work of Carmichael *et al* (1987) the steady-state spectrum of the fluorescent light has been calculated as the Fourier transform of the atomic correlation function

$$\langle J_{21}(\tau)J_{12}\rangle_S = \lim_{t\to\infty} \langle J_{21}(t+\tau)J_{12}(t)\rangle. \tag{18}$$

This scheme for calculating the spectrum assumes that there exists a small 'window' of unsqueezed vacuum modes through which we can view the fluorescence. For the case $N \gg P$ the intensity of the fluorescent field dominates over the squeezed noise and thus a 'window' of unsqueezed vacuum modes is unnecessary, where $\langle\cdots\rangle_S$ denotes an expectation value over the steady-state (14). According to the transformation (16) we have

$$J_{21}(t) = \sin\theta\cos\theta R_3(t) + \cos^2\theta R_{21}(t) - \sin^2\theta R_{12}(t). \tag{19}$$

The equations of motion for $\langle R_{ij}(t)\rangle$ can be derived by using the master equation (8), and have the following form

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle R_3(t)\rangle = -\gamma_0\langle R_3(t)\rangle - \gamma_c\langle R_3^2(t)\rangle + \gamma_c(N^2+2N) \tag{20}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle R_{12}(t)\rangle = -2\mathrm{i}\Omega\langle R_{12}(t)\rangle - \gamma_+\langle R_{12}(t)\rangle - \tfrac{1}{2}\gamma_c\langle\{R_3(t), R_{12}(t)\}\rangle \tag{21}$$

$$\frac{\mathrm{d}}{\mathrm{d}_t}\langle R_{21}(t)\rangle = \frac{\mathrm{d}}{\mathrm{d}t}\langle R_{12}(t)\rangle^* \tag{22}$$

where

$$\{R_3(t), R_{ij}(t)\} \equiv R_3(t)R_{ij}(t) + R_{ij}(t)R_3(t)$$

$$\gamma_0 = 2\gamma[(P+\tfrac{1}{2})(\sin^4\theta + \cos^4\theta) + 2|Q|\cos\phi\cos^2\theta\sin^2\theta] \tag{23}$$

$$\gamma_+ = \gamma[(P+\tfrac{1}{2})(1+2\sin^2\theta\cos^2\theta) - 2|Q|\cos\phi\cos^2\theta\sin^2\theta] \tag{24}$$

$$\gamma_c = \tfrac{1}{2}\gamma(\sin^2\theta - \cos^2\theta). \tag{25}$$

Equations (20)-(22) are so far exact. They contain, however, terms with the products of operators which make them unsolvable in the general case.

For the one-atom case one can use the well known operator relation

$$R_{ij}R_{i'j'} = R_{ij'}\delta_{i'j} \qquad (i, j, i', j' = 1, 2) \tag{26}$$

and equations (20)-(22) reduce to the linear differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle R_3(t)\rangle = -\gamma_0\langle R_3(t)\rangle + 2\gamma_c \tag{27}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle R_{12}(t)\rangle = -2\mathrm{i}\Omega\langle R_{12}(t)\rangle - \gamma_+\langle R_{12}(t)\rangle \tag{28}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle R_{21}(t)\rangle = \frac{\mathrm{d}}{\mathrm{d}t}\langle R_{12}(t)\rangle^*. \tag{29}$$

The linear differential equations (27)-(29) are exact solvable and their solutions are in agreement with previous work on the resonance fluorescence of an atom in a squeezed vacuum (Carmichael *et al* 1987).

For the case of exact resonance, i.e. $\cot^2\theta = 1$ we have $\gamma_c = 0$ and all equations (20)-(22) reduce to exact solvable linear differential equations.

In the general case, to deal with the product terms we apply a decorrelation scheme similar to that used by Compagno and Persico (1982). The only difference consists in the fact that we decorrelate symmetrised products of operators (anticommutator). This allows us to preserve one-atom terms unchanged and clearly separate them from the collective terms. The decorrelated operators that do not enter into the equations as 'proper' variables are replaced by their steady-state averages calculated with the density matrix (14):

$$\langle\{R_3, R_{ij}\}\rangle = 2\langle R_3R_{ij}\rangle = 2\langle R_3\rangle_S R_{ij}. \tag{30}$$

With such approximations equations (20)-(22) have simple exponential solutions with the one-atom (i.e. γ_0, γ_+) and collective (i.e. $\gamma_c\langle R_3\rangle_S$) damping constant clearly separated. Upon neglecting the collective part one immediately obtains the one-atom results. By using the density matrix (14), one can show that (Bogolubov *et al* 1987a, Compagno and Persico 1982) in the case of large N the decorrelation (30) yields a small error (of the order of $N^{-1/2}$) in the calculation of the steady-state fluorescence spectrum. The explicit expressions for the collective terms of the damping constants can be obtained with the use of the steady-state averages given in the appendix.

Using equation (19), the solutions of equations (20)-(22) and applying the quantum regression theorem (Gardiner 1986), one obtains the following expressions for the correlation function (18)

$$\begin{aligned}\langle J_{21}(\tau)J_{12}\rangle_S = {} & \sin^4\theta\langle R_{12}R_{21}\rangle_S \exp(-\tilde{\gamma}_+\tau - 2\mathrm{i}\Omega\tau) \\ & + \cos^4\theta\langle R_{21}R_{12}\rangle_S \exp(-\tilde{\gamma}_+\tau + 2\mathrm{i}\Omega\tau) \\ & + \sin^2\theta\cos^2\theta(\langle R_3^2\rangle_S - I_c)\exp(-\tilde{\gamma}_0\tau) + \sin^2\theta\cos^2\theta I_c\end{aligned} \tag{31}$$

where

$$I_c = \gamma_c(N^2+2N)\langle R_3\rangle_S/\tilde{\gamma}_0 \tag{32}$$

$$\tilde{\gamma}_0 = \gamma_0 + \gamma_c\langle R_3\rangle_S \tag{33}$$

$$\tilde{\gamma}_+ = \gamma_+ + \gamma_c\langle R_3\rangle_S. \tag{34}$$

Expressions for the weighting factors of the particular exponents are given in the appendix.

The steady-state spectrum of the fluorescent light is proportional to the Fourier transform of the correlation function (31) and has the following form:

$$\begin{aligned} S(\omega) \sim \tfrac{1}{2}\,\mathrm{Re}\left[\int_0^\infty \exp(-\mathrm{i}(\omega-\omega_L)\tau)\langle J_{21}(\tau)J_{12}\rangle_S\,\mathrm{d}\tau\right] \\ = \sin^2\theta\cos^2\theta(\langle R_3^2\rangle_S - I_e)\frac{\tilde{\gamma}_0}{(\omega-\omega_L)^2+\tilde{\gamma}_0^2} \\ +\cos^4\theta\langle R_{21}R_{12}\rangle_S\frac{\tilde{\gamma}_+}{(\omega-\omega_L-2\Omega)^2+\tilde{\gamma}_+^2} \\ +\sin^4\theta\langle R_{12}R_{21}\rangle_S\frac{\tilde{\gamma}_+}{(\omega-\omega_L+2\Omega)^2+\tilde{\gamma}_+^2} \\ +\tfrac{1}{2}\sin^2\theta\cos^2\theta\, I_e\delta(\omega-\omega_L). \end{aligned} \tag{35}$$

The fluorescence spectrum (35) contains three spectral lines centred at frequencies $\omega = \omega_L;\ \omega_L \pm 2\Omega$. In the off-resonance case, i.e. when $\cot^2\theta \neq 1$, the central line at $\omega = \omega_L$ contains an elastic component with intensity proportional to N^2 and a Lorentzian-shaped component with linewidth $\tilde{\gamma}_0$ and intensity $\sin^2\theta\cos^2\theta(\langle R_3^2\rangle_S - I_e)$. The two sidebands are Lorentzians of linewidth $\tilde{\gamma}_+$ centred at frequencies $\omega = \omega_L - 2\Omega$ and $\omega = \omega_L + 2\Omega$, and having intensities proportional to $\sin^4\theta\langle R_{12}R_{21}\rangle_S$ and $\cos^4\theta\langle R_{21}R_{12}\rangle_S$, respectively. In the case of exact resonance we have $\gamma_c = 0$ and the spectrum (34) reduces to

$$S(\omega) \sim \tfrac{1}{12}N(N+2)\left(\frac{\bar{\gamma}_0}{(\omega-\omega_L)^2+\bar{\gamma}_0^2}+\frac{3}{4}\,\frac{\bar{\gamma}_+}{(\omega-\omega_L-2\Omega)^2+\bar{\gamma}_+^2}+\frac{3}{4}\,\frac{\bar{\gamma}_+}{(\omega-\omega_L+2\Omega)^2+\bar{\gamma}_+^2}\right) \tag{36}$$

where $\bar{\gamma}_0$ and $\bar{\gamma}_+$ are the one-atom linewidths (see equations (23) and (24)) being taken for the case of exact resonance:

$$\bar{\gamma}_0 = \gamma[P+\tfrac{1}{2}+|Q|\cos\phi] \tag{37}$$

$$\bar{\gamma}_+ = \tfrac{1}{2}\gamma[3P+\tfrac{3}{2}-|Q|\cos\phi]. \tag{38}$$

It is clear from relations (36)-(38) that in the case of exact resonance the elastic component vanishes, the intensities of all three inelastic components are proportional to N^2 and are independent of the parameters of squeezed vacuum while the linewidths, and consequently the peak intensities, are dependent on the parameters of the squeezed vacuum and become the phase-sensitive quantities. For an illustration we assume the squeezed vacuum to be in a highly squeezed minimum-uncertainty state, i.e. $P \gg 1$ and $Q = (P+P^2)^{1/2}$; we have

$$\begin{cases} \bar{\gamma}_0 = 2P\gamma & \text{if } \phi = 0 \\ \bar{\gamma}_0 = \gamma/8P & \text{if } \phi = \pi \end{cases} \tag{39}$$

$$\begin{cases} \bar{\gamma}_+ = P\gamma & \text{if } \phi = 0 \\ \bar{\gamma}_+ = 2P\gamma & \text{if } \phi = \pi. \end{cases} \tag{40}$$

Thus, the sidebands are broadened while the central peak has a supernatural or subnatural linewidth by changing the phase difference ϕ by π.

In the off-resonance case, as is clear from equations (32) and (33) the linewidths contain the one-atom (γ_0; γ_+) and collective parts ($\gamma_c\langle R_3\rangle_S$) and they are, in the general case, dependent on the parameters of the squeezed vacuum. For large numbers of atoms ($N \gg 1$) from the relation for $\langle R_3\rangle_S$ given in the appendix one finds

$$\gamma_c\langle R_3\rangle_S \approx \tfrac{1}{2}\gamma N|\sin^2\theta-\cos^2\theta| \qquad \text{if } \cot^2\theta \neq 1. \tag{43}$$

Thus, for the case of $N \gg 1$, the collective part of the spectrum linewidths is independent of the parameters of the squeezed vacuum and dominates over the one-atom parts γ_0, γ_+. Consequently the spectrum linewidths are approximately independent of whether the vacuum is squeezed or not.

In contrast to the exact-resonance case the intensities of the spectrum components are strongly dependent on the parameters of the squeezed vacuum. In figures 1(*a*)-(*d*) the relative intensities of the two sidebands, i.e. the quantities $I_{-1}/N = \sin^4\theta\langle R_{12}R_{21}\rangle_S/N$ (broken curves) and $I_{+1}/N = \cos^4\theta\langle R_{21}R_{12}\rangle_S/N$ (full curves), are plotted as functions of the parameter $\cot^2\theta$ for various values of P, $|Q|$ and ϕ. In figures 2(*a*)-(*b*) the relative intensity of the central line (the sum of elastic and inelastic components), i.e. the quantity $I_0N^2 = \sin^2\theta\cos^2\theta\langle R_3^2\rangle_S/N^2$, is plotted as a function of the parameter $\cot^2\theta$ for the same values of P, $|Q|$ and ϕ. It is clear from figures 1 and 2 that, except for the point of exact resonance, the intensities of the spectrum components are strongly dependent on the parameters of the squeezed vacuum and become phase-sensitive values. In unsqueezed vacuum (see curves marked A in figures 1 and 2) the intensities of the two sidebands are equal and the spectrum is symmetric. In the squeezed vacuum (see curves B and C in figures 1(*a*)-(*d*) the intensities of the two sidebands are quite different for the off-resonance case $\cot^2\theta \neq 1$ and the spectrum becomes asymmetric. For a large number of atoms ($N \gg 1$) from the relations for $\langle R_{21}R_{12}\rangle_S$, $\langle R_{12}R_{21}\rangle_S$ and $\langle R_3^2\rangle_S$ given in the appendix, one finds

$$I_1 \approx \cos^4\theta N/|X-1| \qquad \text{if } \cot^2\theta \neq 1$$

$$I_{-1} \approx \sin^4\theta NX/|X-1| \qquad \text{if } \cot^2\theta \neq 1$$

$$I_0 \approx \begin{cases} \sin^2\theta\cos^2\theta[N^2-4N/(X-1)] & \text{if } \cot^2\theta > 1 \\ \sin^2\theta\cos^2\theta[N^2-4NX/(1-X)] & \text{if } \cot^2\theta < 1 \end{cases}$$

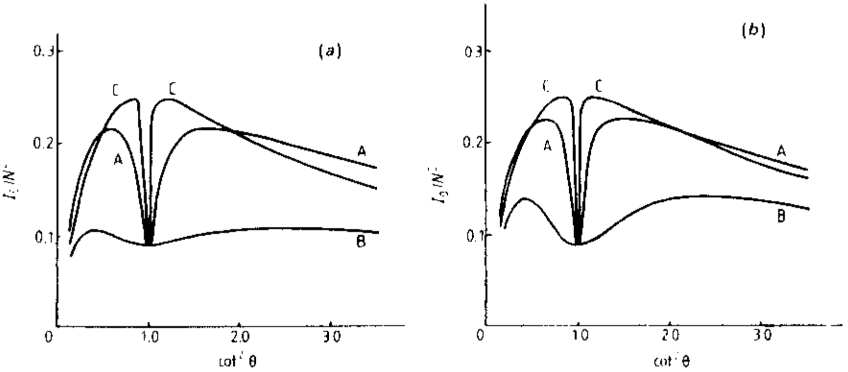

Figure 2. The relative intensities I_0/N^2 as functions of the parameter $\cot^2\theta$ for $N = 25$ (*a*) and $N = 50$ (*b*). The curves A, B and C correspond to: $P = |Q| = 0$; $P = 2$, $|Q| = (P^2+P)^{1/2}$, $\phi = 0$, and $P = 2$, $|Q| = (P^2+P)^{1/2}$, $\phi = \pi$, respectively.

In another case when $P \ll 1$ and $Q = (P + P^2)^{1/2}$ we have

$$\begin{cases} \bar{\gamma}_0 = \gamma(\tfrac{1}{2} + \sqrt{P}) > \tfrac{1}{2}\gamma & \text{if } \phi = 0 \\ \bar{\gamma}_0 = \gamma(\tfrac{1}{2} - \sqrt{P}) < \tfrac{1}{2}\gamma & \text{if } \phi = \pi \end{cases} \tag{41}$$

$$\begin{cases} \gamma^{+} = \tfrac{1}{2}\gamma(\tfrac{1}{2} - \sqrt{P}) < \tfrac{1}{4}\gamma & \text{if } \phi = 0 \\ \bar{\gamma}_{+} = \tfrac{1}{2}\gamma(\tfrac{1}{2} + \sqrt{P}) > \tfrac{1}{4}\gamma & \text{if } \phi = \pi. \end{cases} \tag{42}$$

Thus the narrowing or expansion of the linewidths of the central component and sidebands takes place depending on the phase difference ϕ. We note that the narrowing or expansion of the linewidths is sufficiently large when the squeezed vacuum is in a highly squeezed ($P \gg 1$) minimum-uncertainty state (see relations (39)–(40)).

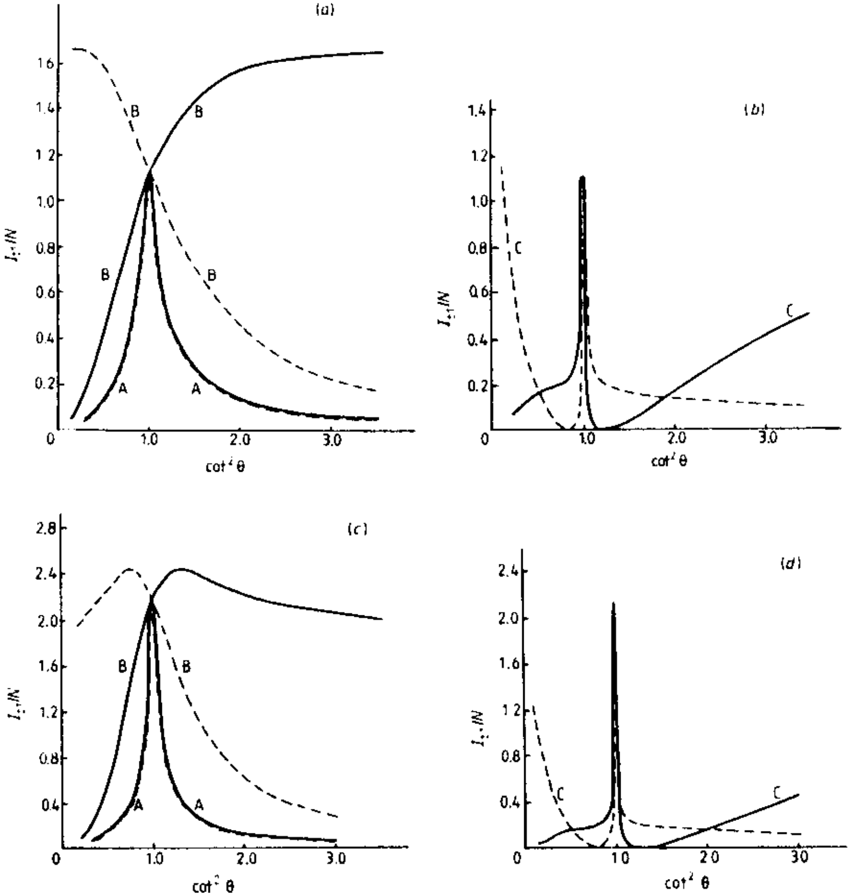

Figure 1. The relative intensities I_{+1}/N (full curves) and I_{-1}/N (broken curves) as functions of the parameter $\cot^2\theta$ for $N = 25$ (a, b) and $N = 50$ (c, d). The curves A, B and C correspond to: $P = |Q| = 0$; $P = 2$, $|Q| = (P^2 + P)^{1/2}$, $\phi = 0$; and $P = 2$, $|Q| = (P^2 + P)^{1/2}$, $\phi = \pi$, respectively.

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and thus in the off-resonance case the intensities of spectrum components are strongly dependent on the squeezed vacuum even for a large number of atoms ($N \gg 1$) when the spectrum linewidths are independent of one.

4. Photon statistics of the spectrum components

In this section we discuss the influence of the squeezed vacuum on the photon statistics of the spectrum components.

As is clear from the previous section and equation (19), the operators $\cos^2\theta R_{21}$, $\sin\theta\cos\theta R_3$ and $-\sin\theta R_{12}$ can be considered as operator-sources of the spectral lines centred at frequencies $\omega = \omega_L + 2\Omega$; ω_L and $\omega_L - 2\Omega$; for later use these operators will be denoted by S^{+}_{+1}, $S^{+}_{0} \equiv S_0$ and S^{+}_{-1}, respectively. Following Bogolubov *et al* (1987a) and Loudon (1980) we introduce the degree of second-order coherence for the spectrum component $S_l (l = 0, \pm 1)$ in the following form

$$G^{(2)}_{l,l} = \frac{\langle S^{+}_{l} S^{+}_{l} S_l S_l \rangle_S}{(\langle S^{+}_{l} S_l \rangle_S)^2} \qquad (l = 0, \pm 1). \tag{44}$$

The quantity $G^{(2)}_{l,l}$ describes the photon statistics of the spectrum component S_l.

Using equations (14) and (19) one finds

$$G^{(2)}_{0,0} = \langle R_3^4 \rangle_S / (\langle R_3^2 \rangle)_S^2 \tag{45}$$

and

$$G^{(2)}_{-1,-1} = \langle R_{12} R_{12} R_{21} R_{21} \rangle_S / (\langle R_{12} R_{21} \rangle_S)^2 = G^{(2)}_{1,1} \tag{46}$$

where the expectation values $\langle R_3^4 \rangle_S$, $\langle R_3^2 \rangle_S$, $\langle R_{12} R_{21} \rangle_S$ and $\langle R_{12} R_{12} R_{21} R_{21} \rangle_S$ are given in the appendix.

For the one-atom case, by using the operator relation (26) one finds

$$G^{(2)}_{0,0} = 1, \qquad G^{(2)}_{1,1} = G^{(2)}_{-1,-1} = 0.$$

Thus the photon statistics of the central component remains Poissonian and the sidebands have sub-Poissonian statistics as is the case in the unsqueezed vacuum.

For the collective case, the degrees of second-order coherence for the spectral lines given in equations (45) and (46) are dependent on the parameters of the squeezed vacuum and become phase-sensitive quantities. The behaviour of the degrees of coherence $G^{(1)}_{0,0}$ and $G^{(2)}_{+1,+1}$ as functions of the parameter $\cot^2\theta$ for fixed $N = 25$ and for various values of P, $|Q|$ and ϕ is as plotted in figures 3 and 4, respectively. As is clear from figures 3 and 4, except for the point of the exact resonance $\cot^2\theta = 1$, the parameters of the squeezed vacuum, including the phase difference ϕ, play an important role in determining the photon statistics of the Mollow triplet.

5. Conclusions

We have considered the problem of collective resonance fluorescence in the squeezed vacuum. For the intense external field the analytical solution for the steady-state density operator for the atomic system has been found. Analytical formulae have been derived for the spectrum of the resonance fluorescence and for the degrees of the second-order coherence for the spectrum components.

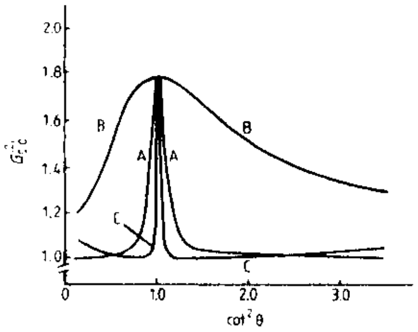

Figure 3. The degree of second-order coherence $G^{(2)}_{0,0}$ as a function of the parameter $\cot^2\theta$ for fixed $N = 25$. The curves A, B and C correspond to: $P = |Q| = 0$; $P = 2, |Q| = (P^2 + P)^{1/2}$, $\phi = 0$; and $P = 2, |Q| = (P^2 + P)^{1/2}$, $\phi = \pi$, respectively.

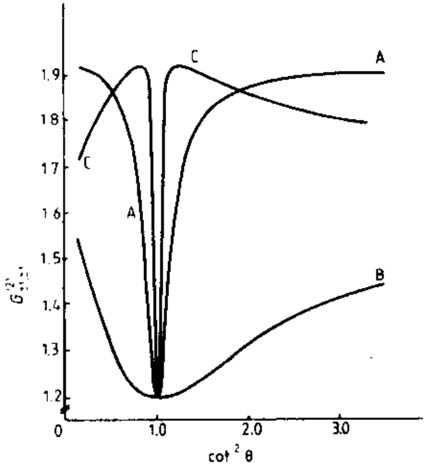

Figure 4. The degree of second-order coherence $G^{(2)}_{\pm1,\pm1}$ as a function of the parameter $\cot^2\theta$ for fixed $N = 25$. The curves A, B and C correspond to: $P = |Q| = 0$; $P = 2$, $|Q| = (P^2 + P)^{1/2}$, $\phi = 0$; and $P = 2$, $|Q| = (P^2 + P)^{1/2}$, $\phi = \pi$, respectively.

It has been shown that in the case of exact resonance the intensities of the spectrum components are independent of the squeezed vacuum while the linewidths are subnatural or supernatural in their dependence on the parameters P, $|Q|$ and phase difference ϕ of the driving field and the squeezed vacuum. In contrast to the exact resonance case, in the off-resonance case the intensities of the spectrum components are dependent on the squeezed vacuum and the spectrum becomes, in the general case, asymmetric while the linewidths are approximately independent of the squeezed vacuum for a large number of atoms ($N \gg 1$).

We have also shown that in the collective and off-resonance cases the photon statistics of spectrum components are dependent on the parameters of the squeezed vacuum and become phase-sensitive quantities.

Appendix

In this appendix we give the explicit expressions for the steady-state averages of the atomic operators that can be calculated with the use of the density matrix (14).

$$\langle R_{11}\rangle_S = Z^{-1}[NX^{N+2} - (N+1)X^{N+1} + X]/(X-1)^2 \tag{A.1}$$

$$\langle R_{11}^2\rangle_S = Z^{-1}[N^2X^{N+3} - (2N^2+2N-1)X^{N+2} + (N+1)^2X^{N+1} - X^2 - X]/(X-1)^3 \tag{A.2}$$

$$\begin{aligned}\langle R_{11}^3\rangle_S = Z^{-1}[&N^3X^{N+4} - (3N^3+3N^2-3N+1)X^{N+3} + (3N^2+6N-4)X^{N+2}\\ &-(N^3+3N^2+3N+1)X^{N+1} + X^3 + 4X^2 + X]/(X-1)^4\end{aligned} \tag{A.3}$$

$$\begin{aligned}\langle R_{11}^4\rangle_S = Z^{-1}[&N^4X^{N+5} - (4N^4+4N^3-6N^2+6N-1)X^{N+4}\\ &+(6N^4+12N^3-6N^2-12N+1)X^{N+3}\\ &-(4N^4+12N^3+6N^2-12N-11)X^{N+2}\\ &+(N^4+4N^3+4N+1)X^{N+1} - X^4 - 11X^3 - 11X^2 - X]/(X-1)^5\end{aligned} \tag{A.4}$$

$$\langle R_3\rangle_S = N - 2\langle R_{11}\rangle_S \tag{A.5}$$

$$\langle R_3^2\rangle_S = 4\langle R_{11}^2\rangle_S - 4N\langle R_{11}\rangle_S + N^2 \tag{A.6}$$

$$\langle R_{12}R_{21}\rangle_S = -\langle R_{11}^2\rangle_S + (N+1)\langle R_{11}\rangle_S \tag{A.7}$$

$$\langle R_{21}R_{12}\rangle_S = -\langle R_{11}^2\rangle_S + (N-1)\langle R_{11}\rangle_S + N \tag{A.8}$$

$$\langle R_3^4\rangle_S = 16\langle R_{11}^4\rangle_S - 32N\langle R_{11}^3\rangle_S + 24N^2\langle R_{11}^2\rangle_S - 8N^3\langle R_{11}\rangle_S + N^4 \tag{A.9}$$

$$\begin{aligned}\langle R_{12}R_{12}R_{21}R_{21}\rangle_S\\ &= \langle R_{11}^4\rangle_S - 2(N+2)\langle R_{11}^3\rangle_S + (N^2+5N+5)\langle R_{11}^2\rangle_S\\ &\quad -(N^2+3N+2)\langle R_{11}\rangle_S.\end{aligned} \tag{A.10}$$

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