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FLUORESCENCE SPECTRUM OF A THREE-LEVEL ATOM INTERACTING WITH TWO CAVITY MODES

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1. INTRODUCTION

The Jaynes - Cummings model consisting of a single two-level atom interacting with a single quantized electromagnetic field mode is one among very few quantum optical models that are exactly soluble. Because of its being simple this model has been used for many years to explore fundamental ideas in quantum optics before applying them in more complicated situations. When the electromagnetic field is initially prepared in a coherent state the Jaynes - Commings model is found to possess a number of interesting features, an example being the collapses and revivals of the populations that have been recently studied both theoretically and experimentally for Rydberg atoms in cavities[1,2]. Much attention has been paid to the study of spectral problems. The radiation and absorption spectra have been calculated for an atom enclosed in a lossless cavity[3] and in a cavity with finite Q[4]. The effects of cooperation[5] and multiphoton transition[6] have also been studied.

In this paper we examine the emission from a three-level atom interacting with the cavity field modes. A model like that may be regarded as an extended form of the Jaynes - Commings model. We point out the spectral line splitting in comparison with the semiclassical consideration. All formulae are written for the multiphoton resonance. The one-photon transition case can easily be obtained making the multiple of resonance equal to unity.

2. SPECTRUM OF THE SCATTERED LIGHT

The three-level atom model considered here is shown in Fig.1 (for the case $m_1 = 3$, $m_2 = 2$). Let the upper level 3 be coupled with level 1 (level 2) due to the interaction with the field in mode 1 (mode 2) via m_1 - photon (m_2 - photon) transitions. In the rotating wave and electric dipole approximations, the effective Hamiltonian of the system is ($\hbar = 1$)

$$H = H_A + H_F + H_{AF}. \qquad (1)$$

$$H_A = \sum_{j=1}^{3} \Omega_j R_{jj}, \qquad H_F = \sum_{a=1}^{2} \omega_a a_a^+ a_a,$$

$$H_{AF} = \sum_{a=1}^{2} g_a (R_{3a} a_a^{m_a} + R_{a3} a_a^{+m_a}), \tag{2}$$

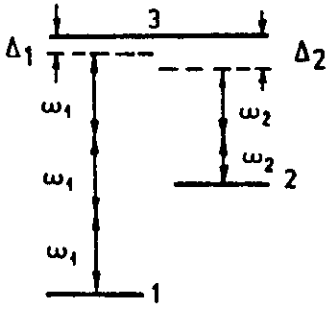

Fig. 1.

where Ω_j are the atomic level energies. The operators $R_{jj} = |j><j|$ and $R_{ij} = |i><j|$, $(i \neq j)$ describe the population of level j and the atomic transitions from level j to level i. They satisfy the relations

$$R_{ij} R_{pq} = R_{iq} \delta_{jp}. \tag{3}$$

The g_a are the coupling constants between the atom and the cavity mode a which is represented by annihilation and creation operators a_a and a_a^+. The ω_a are the mode frequencies. The detuning parameters can be defined as

$$\Delta_a = \Omega_3 - \Omega_a - m_a \omega_a, \quad (a = 1,2). \tag{4}$$

We denote by $|i; n_1 + m_1 \delta_{1i}, n_2 + m_2 \delta_{2i}>$, $i = 1,2,3$, the eigenstate vectors of the free Hamiltonian $H_A + H_F$. Solving the Schrödinger equation

$$H\Psi = E\Psi,$$

one finds the eigenvalues of the full Hamiltonian to be the roots of the third-order characteristic equation

$$X^3 - (\Delta_1 + \Delta_2)X^2 + (\Delta_1\Delta_2 - \lambda_o^2(n_1,n_2))X + \lambda_1^2(n_1)\Delta_2 + \lambda_2^2(n_2)\Delta_1 = 0, \tag{5}$$

where the notation

$$X = \Omega - E, \quad \Omega = \Omega_3 + n_1\omega_1 + n_2\omega_2,$$

$$\lambda_i(n_i) = g_i\sqrt{(n_i + m_i)!/n_i!} \equiv \lambda_i, \quad i = 1,2, \tag{6}$$

$$\lambda_o(n_1,n_2) = \sqrt{\lambda_1^2(n_1) + \lambda_2^2(n_2)} \equiv \lambda_o,$$

has been used. An analogous cubic equation has been obtained by several authors for a particular case where the one-photon transition takes place and the electromagnetic field is treated classically[7,12] or quantum-mechanically[8]. Eq.(5) in general case is irreducible, therefore it has three different real roots[12] which can be represented in the form

$$X_1 = 2\sqrt{\frac{-p}{3}}\cos\frac{a}{3} + \frac{\Delta_1 + \Delta_2}{3},$$

$$X_{2,3} = -2\sqrt{\frac{-p}{3}}\cos\left(\frac{a}{3} \pm \frac{\pi}{3}\right) + \frac{\Delta_1 + \Delta_2}{3},$$

with

$$p = -(\Delta_1^2 - \Delta_1\Delta_2 + \Delta_2^2)/3 - \lambda_o^2,$$

$$q = \frac{1}{27}[-2(\Delta_1 + \Delta_2)^3 + 9(\Delta_1 + \Delta_2)(\Delta_1\Delta_2 - \lambda_o^2) + 27(\lambda_2^2\Delta_1 + \lambda_1^2\Delta_2)], \tag{7}$$

$$a = \arccos\left(\frac{-q}{2\sqrt{(-\frac{p}{3})^3}}\right).$$

Expressions (7) show that the solutions of Eq.(5) depend in a complicated way on mode detunings and photon numbers n_1, n_2, except for two cases: (1) double resonance $\Delta_1 = \Delta_2 = \Delta$, (2) $\Delta_1/\Delta_2 = -\lambda_1^2/\lambda_2^2$; Eq.(5) becomes reducible and X_i simplify considerably. We will in detail investigate the double resonance case below.

Further, knowing the eigenvalues of the total system and consequently, the corresponding eigenstates, with the help of the dressed state formalism it is not difficult to obtain explicit expressions for the time-dependent wave function

$$|\beta; n_1 + m_1\delta_{1\beta}, n_2 + m_2\delta_{2\beta}>_t = e^{-iHt}|\beta; n_1 + m_1\delta_{1\beta}, n_2 + m_2\delta_{2\beta}> =$$

$$= \sum_{j=1}^{3} A_{\beta j}(t; n_1 + m_1\delta_{1j}, n_2 + m_2\delta_{2j})|j; n_1 + m_1\delta_{1j}, n_2 + m_2\delta_{2j}>, \tag{8}$$

$$\beta = 1,2,3.$$

Here $A_{\beta j}$ are

$$A_{\beta j}(t; n_1 + m_1\delta_{1j}, n_2 + m_2\delta_{2j}) = \sum_{m=1}^{3} a_{\beta j}^m e^{i(X_m - \Omega)t}. \tag{9}$$

with the coefficients $a^m_{\beta j}$ given in terms of X_i

$$a^m_{\beta 1}=\frac{X_m^2-\Delta_2 X_m-\lambda_2^2}{\lambda_1}C^m_\beta,\quad a^m_{\beta 2}=\lambda_2 C^m_\beta,\quad a^m_{\beta 3}=(X_m-\Delta_2)C^m_\beta,$$

$$C^m_1=\frac{\lambda_1}{(X_m-X_k)(X_m-X_l)},\quad C^m_2=\frac{\lambda_2^2+(X_k-\Delta_2)(X_l-\Delta_2)}{\lambda_2(X_m-X_k)(X_m-X_l)},\qquad (10)$$

$$C^m_3=\frac{X_m-\Delta_1}{(X_m-X_k)(X_m-X_l)},\qquad l,k,m=1,2,3,\quad l\neq k\neq m.$$

We assume that the electron was initially in an atomic level β and the cavity field was in an arbitrary state

$$\rho_F=\sum_{\substack{n_1,n_2\\ n_1',n_2'}} P_{n_1n_2}|n_1,n_2><n_1',n_2'|,$$

i.e. the initial density matrix was:

$$\rho(0)=\sum_{\substack{n_1,n_2\\ n_1',n_2'}} P_{n_1n_2}|\beta;n_1,n_2><\beta;n_1',n_2'|. \qquad (11)$$

To calculate the fluorescence spectrum from a three-level atom, we first calculate the two-time correlative function $<R_{3\alpha}(t+\tau)R_{\alpha 3}(t)>$ of the dipole-moment operators

$$D_\alpha(t,\tau)=<R_{3\alpha}(t+\tau)R_{\alpha 3}(t)>=$$

$$=\mathrm{Tr}(R_{3\alpha}(t+\tau)R_{\alpha 3}(t)\rho(0))= \qquad (12)$$

$$=\sum_{n_1,n_2}P_{n_1n_2}D_\alpha(t,\tau;\beta;n_1,n_2),$$

where

$$D_\alpha(t,\tau;\beta;n_1,n_2)=<\beta;n_1,n_2|e^{iH(t+\tau)}R_{3\alpha}e^{-iH\tau}R_{\alpha 3}e^{-iHt}|\beta;n_1,n_2>. \qquad (13)$$

One can check easily that $D_\alpha(t,\tau;\beta;n_\beta<m_\beta)=0$. Using (8) we rewrite (13) in the form

$$D_\alpha(t,\tau;\beta;n_1+m_1\delta_{1\beta},n_2+m_2\delta_{2\beta})=$$

$$=A^*_{\beta 3}(t+\tau,n_1,n_2)A_{\alpha\alpha}(\tau,n_1,n_2)A_{\beta 3}(t,n_1,n_2). \qquad (14)$$

Substitution of (9), (10) to (14) gives

$$D_\alpha(t,\tau;\beta;n_1+m_1\delta_{1\beta},n_2+m_2\delta_{2\beta})=$$

$$=\sum_{m,n,l=1}^{3}a^m_{\beta 3}a^n_{\beta 3}\tilde a^l_{\alpha\alpha}e^{i(\tilde X_n-\tilde X_m)t}e^{i\tau(\tilde X_l+X_m+m_\alpha\omega_\alpha)}, \qquad (15)$$

where $\tilde X_l$ are solutions of Eq. (5) after replacing $\lambda_\alpha(n_\alpha)$ by $\lambda_\alpha(n_\alpha-m_\alpha)=g_\alpha\sqrt{n_\alpha!/(n_\alpha-m_\alpha)!}\equiv\tilde\lambda_\alpha$, $\tilde a^l_{\alpha\alpha}$ is obtained from $a^l_{\alpha\alpha}$ after replacing X_l, λ_l by $\tilde X_l$, $\tilde\lambda_l$, respectively. Using the definition of a "physical spectrum" based on the analysis of spectral measurement[19] we define the time-dependent spectrum of the induced fluorescence as

$$S_\alpha(\nu,T)=2\Gamma\mathrm{Re}\int_0^T d\tau e^{(\Gamma-i\nu)\tau}\int_0^{T-\tau}dt e^{-2\Gamma(T-t)}D_\alpha(t,\tau). \qquad (16)$$

Here Γ is the bandwidth of the spectrometer, T is the time at which the spectrum is evaluated. Inserting Eqs. (12) and (15) into (16) we get

$$S_\alpha(\nu,T)=\sum_{n_1,n_2}P_{n_1,n_2}S_\alpha(\nu,T;\beta;n_1,n_2), \qquad (17)$$

$$S_\alpha(\nu,T;\beta;n_1+m_1\delta_{1\beta},n_2+m_2\delta_{2\beta})=$$

$$=2\Gamma\sum_{m,n,l=1}^{3}a^m_{\beta 3}a^n_{\beta 3}\tilde a^l_{\alpha\alpha}\mathrm{Re}\Big\{\frac{1}{X_m-X_n+2i\Gamma}\Big[\frac{1}{\nu-m_\alpha\omega_\alpha+X_n-\tilde X_l-i\Gamma}\times$$

$$\times\big(e^{i(X_n-X_m)T}-e^{-i(\nu-m_\alpha\omega_\alpha+X_m-\tilde X_l-i\Gamma)T}\big)+$$

$$+\frac{1}{\nu-m_\alpha\omega_\alpha+X_m-\tilde X_l+i\Gamma}\big(e^{-i(\nu-m_\alpha\omega_\alpha+X_m-\tilde X_l-i\Gamma)T}-e^{-2\Gamma T}\big)\Big]\Big\}. \qquad (18)$$

Thus, we have obtained a general expression for fluorescence spectrum $S_\alpha(\nu,T)$ which is valid for studying the spectrum of the combined atom-field system in various situations (spontaneous emission, Rayleigh and Raman scattering, etc.). In the long-time limit $\Gamma T\gg 1$ so that the terms connected with the multiplier $e^{-\Gamma T}$ are negligible and oscillating terms $e^{i(X_n-X_m)T}$ $n\neq m$ may be ignored, we get

$$S_\alpha(\nu, T\to\infty;\beta; n_1+m_1\delta_{1\beta}, n_2+m_2\delta_{2\beta}) =$$

$$= \sum_{m,\ell=1}^{3} (a^m_{\beta 3})^2 \tilde{a}^{\ell}_{\alpha\alpha} \frac{\Gamma}{(\nu - m_\alpha\omega_\alpha + X_m - \tilde{X}_\ell)^2 + \Gamma^2}. \qquad (19)$$

Let us choose two cavity modes both initially prepared in the Fock states $|n_1+m_1\delta_{1\beta}, n_2+m_2\delta_{2\beta}>$, the stationary fluorescence spectrum will have maximal nine components at frequencies

$$m_\alpha\omega_\alpha + \tilde{X}_\ell - X_m, \quad m,\ell = 1,2,3. \qquad (20)$$

This is nothing but the quantum analog of the seven components fluorescence spectrum from a three-level atom driven by two monochromatic laser fields obtained by Whitley, Stroud[10] Agarwal, Jha[11] and Kancheva et al.[12] in the situation with one-photon transition. In fact, at fixed bandwidth of the detecting mechanism when the photon number of the cavity modes increases, $\tilde{X}_\ell$ approaches a value X_ℓ and three peaks at frequencies $m_\alpha\omega_\alpha + \tilde{X}_\ell - X_\ell$, $\ell = 1,2,3$, merge to give one central peak at $m_\alpha\omega_\alpha$ with the remaining six peaks located symmetrically around it

$$m_\alpha\omega_\alpha,\ m_\alpha\omega_\alpha \pm (X_1 - X_2),\ m_\alpha\omega_\alpha \pm (X_1 - X_3),\ m_\alpha\omega_\alpha \pm (X_2 - X_3).$$

It is seen from (10), (19) that in the nine-components fluorescence spectrum the peak positions and intensive symmetry are completely destructed. In the strong field limit, as mentioned above, the symmetrical position occurs but the spectrum in general is asymmetrical because of different peak intensities. Beside the coincidence of peak the vanishing of a whatever peak amplitude also leads to reduction of the number of lines. Let us consider the two-photon resonance case $\Delta_1 = \Delta_2 = \Delta$, where the roots of Eq.(5) read as

$$X_1 = \Delta, \quad X_{2,3} = \pm\lambda_\pm. \qquad (21)$$

Here for the sake of convenience the notation

$$\lambda_\pm \equiv \lambda_\pm(n_1,n_2) = \lambda(n_1,n_2) \pm \frac{\Delta}{2},$$

$$\lambda \equiv \lambda(n_1,n_2) = \sqrt{\lambda_0^2(n_1,n_2) + \frac{\Delta^2}{4}}.$$

with $\lambda_0(n_1,n_2)$ defined in (6) has been involved

By substituting (21) into (19), after a straightforward computation one finds

$$S_\alpha(\nu;3;n_1,n_2,T\to\infty) = \frac{1}{4\lambda^2}\{\lambda_+^2 \frac{\tilde{\lambda}_\alpha^2}{2\tilde{\lambda}_0^2\tilde{\lambda}} \times$$

$$\times[\tilde{\lambda}_- \frac{\Gamma}{[\nu - m_\alpha\omega_\alpha - (\lambda-\tilde{\lambda})]^2 + \Gamma^2} + \tilde{\lambda}_+ \frac{\Gamma}{[\nu - m_\alpha\omega_\alpha - (\lambda+\tilde{\lambda})]^2 + \Gamma^2}] +$$

$$+\lambda_-^2 \frac{\tilde{\lambda}_\alpha^2}{2\tilde{\lambda}_0^2\tilde{\lambda}}[\tilde{\lambda}_+ \frac{\Gamma}{[\nu - m_\alpha\omega_\alpha + (\lambda-\tilde{\lambda})]^2 + \Gamma^2} + \tilde{\lambda}_- \frac{\Gamma}{[\nu - m_\alpha\omega_\alpha + (\lambda+\tilde{\lambda})]^2 + \Gamma^2}] + \qquad (22a)$$

$$+\frac{\lambda_\alpha^2}{\tilde{\lambda}_0^2}[\lambda_+^2 \frac{\Gamma}{(\nu - m_\alpha\omega_\alpha - \lambda_+)^2 + \Gamma^2} + \lambda_-^2 \frac{\Gamma}{(\nu - m_\alpha\omega_\alpha + \lambda_-)^2 + \Gamma^2}]\}.$$

for an atom initially excited and

$$S_\alpha(\nu;\beta;n_1+m_1\delta_{1\beta}, n_2+m_2\delta_{2\beta};T\to\infty) = \frac{\lambda_\beta^2}{4\lambda^2}\{\frac{\tilde{\lambda}_\alpha^2}{2\tilde{\lambda}_0^2\tilde{\lambda}} \times$$

$$\times[\tilde{\lambda}_- \frac{\Gamma}{[\nu - m_\alpha\omega_\alpha - (\lambda-\tilde{\lambda})]^2 + \Gamma^2} + \tilde{\lambda}_+ \frac{\Gamma}{[\nu - m_\alpha\omega_\alpha - (\lambda+\tilde{\lambda})]^2 + \Gamma^2} +$$

$$+\tilde{\lambda}_+ \frac{\Gamma}{[\nu - m_\alpha\omega_\alpha + (\lambda-\tilde{\lambda})]^2 + \Gamma^2} + \tilde{\lambda}_- \frac{\Gamma}{[\nu - m_\alpha\omega_\alpha + (\lambda+\tilde{\lambda})]^2 + \Gamma^2} + \qquad (22b)$$

$$+\frac{\lambda_\alpha^2}{\tilde{\lambda}_0^2}[\frac{\Gamma}{(\nu - m_\alpha\omega_\alpha - \lambda_+)^2 + \Gamma^2} + \frac{\Gamma}{(\nu - m_\alpha\omega_\alpha + \lambda_-)^2 + \Gamma^2}]\}.$$

for an atom initially in a lower state. In this case one can see from (10) and (21) that $a^1_{\beta 3}$ vanish and then three corresponding components disappear, the spectrum $S_\alpha(\nu;\beta;n_1+m_1\delta_{1\beta}, n_2+m_2\delta_{2\beta};T\to\infty)$ has six peaks at frequencies

$$m_\alpha\omega_\alpha \pm (\lambda-\tilde{\lambda}),\ m_\alpha\omega_\alpha \pm \lambda_\pm,\quad m_\alpha\omega_\alpha \pm (\lambda+\tilde{\lambda}).$$

Once the quantized electromagnetic fields are strong enough for $(\lambda-\tilde{\lambda}) \approx 0$ instead of two sideband peaks of frequencies

$m_a\omega_a \pm (\lambda - \tilde{\lambda})$ we have one central peak at $m_a\omega_a$. If $m_1 = m_2 = 1$, our result is in agreement with the result by Whitley, Stroud [10], Agarwal, Jha [11] showing the five-component structure of the scattered light from a three-level atom in the case when the modes are laser modes. We should like to note that in the exact resonance when both applied-field detuning are equal to zero the six-peak structure is perfectly symmetrical respectively to $m_a\omega_a$ both in position and in intensities. Next we investigate two situations of interest with various initial conditions.

(i) Spontaneous emission

Let the incoming atom be excited and two cavity modes at $t = 0$ be empty, i.e. the atom field system be in a state $|3; n_1 = 0; n_2 = 0\rangle$. Then $\tilde{\lambda}_a = 0$, $\tilde{\lambda}_a^2 = \lambda_a^2$, by the basic formula (22a) the spontaneous emission spectrum associated with transition $3\text{-}a$ is of the form

$$S_a(\nu;3;0,0;T\to\infty) = \frac{1}{4\lambda^2} \times \times \left[\lambda_+^2 \frac{\Gamma}{(\nu - m_a\omega_a - \lambda_+)^2 + \Gamma^2} + \lambda_-^2 \frac{\Gamma}{(\nu - m_a\omega_a + \lambda_-)^2 + \Gamma^2} \right], \tag{23}$$

where

$$\lambda_\pm = \lambda \pm \frac{\Delta}{2}, \qquad \lambda = \sqrt{g_1^2 m_1! + g_2^2 m_2! + \frac{\Delta^2}{4}}. \tag{24}$$

It is evident that the spectrum contains two components at frequencies

$$m_a\omega_a \pm \lambda_\pm .$$

The relation between the heights of these peaks is λ_+^2/λ_-^2. The spectrum is completely asymmetrical in the presence of nonzero detuning.

(ii) Spontaneous Raman scattering

We assume that at time $t = 0$ the system is in its state $|1; n_1 + m_1, n_2 = 0\rangle$ where the atom is in level $|1\rangle$, the field in mode 1 is in a Fock state with the photon number $n_1 + m_1$ and the field in mode 2 is empty. Then we get

$$S_2(\nu;1;n_1+m_1,0;T\to\infty) = \frac{\lambda_1^2}{4\lambda^2}\left(\frac{\Gamma}{(\nu - m_2\omega_2 - \lambda_+)^2 + \Gamma^2} + \frac{\Gamma}{(\nu - m_2\omega_2 + \lambda_-)^2 + \Gamma^2} \right). \tag{25}$$

Here

$$\lambda_\pm = \lambda \pm \frac{\Delta}{2},$$

$$\lambda = \sqrt{g_1^2 \frac{(n_1+m_1)!}{n_1!} + g_2^2 m_2! + \frac{\Delta^2}{4}}, \tag{26}$$

$$\lambda_1 = g_1 \sqrt{\frac{(n_1+m_1)!}{n_1!}}.$$

The Raman scattering spectra exhibit a doublet at frequencies $m_2\omega_2 \pm \lambda_\pm$ with the same heights.

The above presented results for the double resonance case have recently been obtained by the authors using the operator method [13].

3. FINE STRUCTURE SPLITTING OF SPECTRUM

In section 2 the fluorescence spectra of the system have been treated in detail when the field is initially in a Fock state. It should be emphasized that for a rather general state of the cavity field characterized by the photon distribution P_{n_1,n_2} one has to average $S_a(\nu; T; n_1, n_2)$, according to formula (17), and then the spectral structure may be altered drastically. In Fig.2 and Fig.3 we depict the numerical solutions for the spontaneous scattered light

$$S_a(\nu, T\to\infty) = \sum_{n_1} P_{n_1} S_a(\nu;1;n_1+m_1,0;T\to\infty). \tag{27}$$

in two usual situations where the field in mode 1 is a coherent field and a thermal field, respectively. There is no photon in mode 2 when the atom enters the cavity. The band width of the detector increases from top to bottom and the mean photon number $\bar{n}_1$ is the same for both the situations. We see if the spectra are detected by a spectrometer with a very good resolution and all the other sources of relaxation are negligible so that the only source of the line width is the width of the detector, then a fine structure splitting appears. Below we consider in detail an interesting case when

$$\Delta_1 = \Delta_2 = 0, \qquad m_1 = m_2 = 1, \tag{28}$$

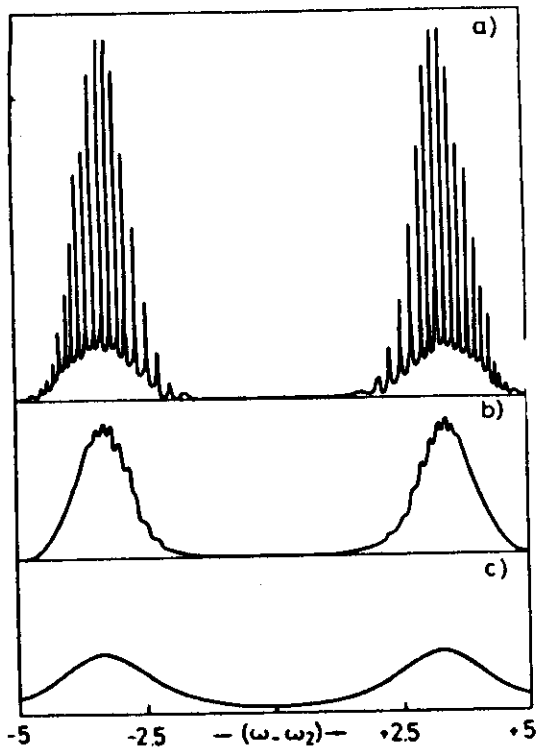

Fig.2. Spontaneous Raman scattering spectra when the field initially is in a coherent state. The parameters involved have been chosen as follows: $\Delta_1 = \Delta_2 = 0$, $g_1 = g_2 = 1$, $m_1 = m_2 = 1$ *and the average photon number* $\bar{n}_1 = 10$. *The bandwidths of the detector are, respectively, 0.02, 0.2 and 0.8 in traces (a), (b), (c).*

By substituting (28) into (25) we see that every $S_2(\nu;\ 1;\ n_1+1,\ 0;\ T\to\infty)$ has two components symmetrical in ω_2 at frequencies $\omega_2 \pm \sqrt{g_1^2(n_1+1)+g_2^2}$ with the height proportional to

$$\frac{\lambda_1^2}{4\lambda^2} = \frac{1}{4}\,\frac{g_1^2(n_1+1)}{g_1^2(n_1+1)+g_2^2}. \qquad (29)$$

The distance between two adjacent fine lines of the spectrum $S_2(\nu, T\to\infty) = \sum_{n_1} P_{n_1} S_2(\nu;1;n_1+1,0;T\to\infty)$ is

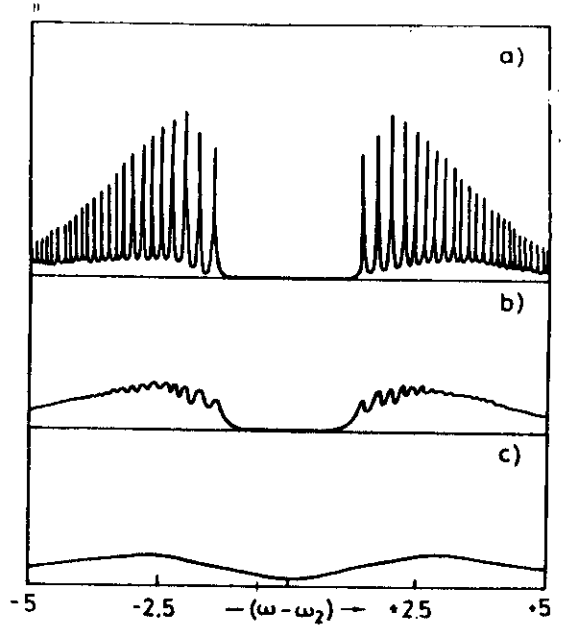

Fig.3. The same as in Fig.2 but now the field initially is in a thermal state.

$$\sqrt{g_1^2(n_1+1)+g_2^2} - \sqrt{g_1^2(n_1+2)+g_2^2}. \qquad (30)$$

The corresponding height can be obtained upon multiplying (29) by the probability P_{n_1} of finding the field in the Fock state $|n_1>$. The shape of the spectrum will essentially depend on the statistical properties of P_{n_1}. As the field becomes strong enough, the distance between two adjacent lines defined by (30) tends to zero and the spectrum $S_2(\nu, T\to\infty)$ eventually will have only two peaks near $\omega_2 \pm \sqrt{g_1^2(\bar{n}_1+1)+g_2^2}$ (if the detector has a good resolution). The width of these peaks is defined by the dispersion of the field photon distribution and the band-width of the detector. As shown in Fig.2,3, the coherent field having smaller dispersion gives a more prominent spectral envelop and the increase of Γ leads to increasing the width and decreasing the height of these envelops.

We remark that the fine splitting of the spectra is a result of the quantum nature of the field and the spread of the photon number distribution, it takes place not only in the spontaneous Raman scattering from a three-level system and can thus be considered as a pure quantum property. We hope that due to the progress of spectrometer and experimental technique with Rydberg atoms [2,14] the conditions for observing the fine splitting of spectrum will become practicable in the nearest future.

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Спектр флуоресценции трехуровневого атома, взаимодействующего с двумя модами резонаторного поля

Исследован спектр флуоресценции трехуровневого атома, взаимодействующего через многофотонный механизм с двухмодовым резонаторным полем. Предсказано наличие тонкой структуры спектра, обусловленной квантовой природой поля и конечной дисперсией распределения фотонов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

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Fluorescence Spectrum of a Three-Level Atom Interacting with Two Cavity Modes

The fluorescence spectrum of a three-level atom interacting with two-field modes in a lossless detuned cavity through multiphoton transitions is studied. The spectrum in the longtime limit is analytically treated and numerically calculated for some cases of interest. The fine structure splitting caused by the quantum nature of the field and the finite dispersion of the photon distribution is noted.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988