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# SQUEEZING IN COLLECTIVE RESONANCE FLUORESCENCE: EFFECT OF A THERMAL FIELD

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# SQUEEZING IN COLLECTIVE RESONANCE FLUORESCENCE: EFFECT OF A THERMAL FIELD

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The influence of a thermal field on the squeezing for the mixture of two sidebands of Mollow's triplet is investigated.

Recently, the problem of a squeezed state of the light is one of the central topics of the quantum optics. A large number of theoretical papers is concentrated on the properties of the squeezed state, various schemes of the generation and applications of this quantum light.[1–3,8–10] Also the squeezed light has been generated in several laboratories.[7–10]

The squeezing in the resonance fluorescence from a single[11–14] or many atoms[15–18] has been discussed in many works. In the papers by Bogolubov *et al.*,[16,17] Lavander *et al.*[18] the large squeezing in the mixture of the two sidebands of collective resonance fluorescence has been shown for the case of the intense external field when the squeezing is absent in all Mollow's triplets taken separately. As has been shown in Refs. 16–18, the collective effects increase the degree of squeezing of two sidebands so that two sidebands may give a rather intense light (intensity is proportional to the number of atoms $N$) with large degrees of squeezing.

A more realistic calculation of the squeezing should take the black-body radiation, into consideration, especially for the experiments with Rydberg atoms.[19–22] In this letter, we investigate the effect of the thermal field on the squeezing in the mixture of two sidebands of the fluorescence field. Let $N$ two-level atoms (Dicke model) be interacting with a single-mode coherent field of the frequency $\omega_L$ and coupling to a reservoir containing all modes of the radiation field. In treating the external field classically and using the Markov and rotating wave approximation for describing the coupling of the system with the thermal reservoir, one arrives at the following master equation for the reduced atomic density operators[23]:

$$\frac{\partial\rho}{\partial t} = -i\left[\frac{\delta}{2}(J_{22}-J_{11})+G(J_{12}+J_{21}),\rho\right]$$

$$-\frac{\gamma}{2}(\bar{n}+1)(J_{21}J_{12}\rho-J_{12}\rho J_{21}+h.c.)$$

$$-\frac{\gamma}{2}\bar{n}(J_{12}J_{21}\rho-J_{21}\rho J_{12}+h.c.)\equiv L\rho, \qquad (1)$$

where $\delta = \omega_{21}-\omega_L$ is the detuning of the laser frequency $\omega_L$ from the atomic resonance frequency $\omega_{21}$ ($\omega_{21} = \omega_2-\omega_1; \hbar\equiv 1$); $G = -d_{21}E_L$ is the resonant Rabi frequency; $\gamma$ is the radiative spontaneous transition rate from the excited level $|2\rangle$ to the ground state $|1\rangle$;

$\bar{n} = \bar{n}(\omega_{21}) = [\exp(\omega_{21}/kT)-1]^{-1}$ is the photon number in the broad-hand thermal field provided by the reservoir at the atomic frequency $\omega_{21}$; $J_{ij}(i,j = 1,2)$ are the collective operators (angular momenta) describing the atomic system and having in the Schwinger representation the following form[16,17]:

$$J_{ij} = C_i^\dagger C_j \qquad (i,j = 1,2),$$

where the operators $C_i$ and $C_i^\dagger$ obey the boson commutation relations

$$[C_i, C_j^\dagger] = \delta_{ij},$$

and can be treated as annihilation and creation operators for atoms populating the level $|i\rangle$.

We note that the model master equation (1) applies to the case of many Ryberg atoms in low-$Q$ cavity provided that $\gamma$ is replaced by the appropriate cavity damping constant $(kg^2/(k^2+\delta_c^2))$ where $k$ is the decay constant of the cavity, $g$ is the atom-cavity mode coupling constant and $\delta_c$ is the cavity detuning[24] and $\bar{n}$ is taken at the frequency of the single cavity mode.

Further, we restrict our consideration to a strong laser field or to a large detuning $\delta$ so that the Rabi frequency $\Omega$ satisfies the following relation

$$\Omega = \left(\frac{1}{4}\delta^2+G^2\right)^{1/2} >> N\gamma, \bar{n}\gamma. \qquad (2)$$

After performing the canonical (dressing) transformation

$$C_1 = Q_1\cos\varphi+Q_2\sin\varphi,$$



$$C_2 = -Q_1\sin\varphi+Q_2\cos\varphi, \qquad (3)$$

where $\qquad \mathrm{tg}\, 2\varphi = 2G/\delta.$

The Liouville operator $L$ appearing in Eq. (1) is splitted into the slowly varying part and the terms oscillating at frequencies $2\Omega$ and $4\Omega$. In the case of large $\Omega$, according to the relation (2) one can use the secular approximation,[16,17] i.e., retain only the slowly varying part of the Liouville operator and find the steady-state solution of the master equation in the form

$$\tilde{\rho}_{st} = z^{-1}\sum_{n_1=0}^{N} X^{n_1}|n_1\rangle\langle n_1|, \qquad (4)$$

where $\tilde{\rho} = U_\rho U^\dagger$ with $U$ being the unitary operator representing the canonical tranformation (3)

$$X = \frac{(\bar{n}+1)\cos^4\varphi+\bar{n}\sin^4\varphi}{(\bar{n}+1)\sin^4\varphi+\bar{n}\cos^4\varphi},$$

$$Z = \frac{X^{N+1}-1}{X-1}. \qquad (5)$$

The state $|n_1\rangle$ is the eigenstate of the operator $R_{11}$ and $R_{11}+R_{22}$ where $R_{ij} = Q_i^\dagger Q_j$ $(i,j = 1,2)$. The solution (5) allows one to calculate all the stationary expectation values of the atomic observables. In particular, we find

$$\langle R_{11}\rangle = Z^{-1}(NX^{N+2}-(N+1)X^{N+1}+X)/(X-1)^2, \qquad (6)$$

$$\langle R_{11}^2\rangle = Z^{-1}[N^2X^{N+3}-(2N^2+2N+1)X^{N+2}+(N+1)^2X^{N+1}-X^2-X]/(X-1)^3, \qquad (7)$$

where $\langle B\rangle$ is the mean value of an operator $B$ over steady-state (5).

With the use of the canonical transformation (3), the atomic collective operator $J_{21}$ can be written

$$J_{21} = \sin\varphi\cos\varphi\,(R_{22}-R_{11})+\cos^2\varphi R_{21}-\sin^2\varphi R_{12}. \qquad (8)$$

The operators $-\sin^2\varphi R_{12}$, $\sin\varphi\cos\varphi\,(R_{22}-R_{11})$ and $\cos^2\varphi\, R_{21}$ can be considered as operators-sources of the spectrum components centered at frequencies

$\omega_L - 2\Omega$, $\omega_L$, $\omega_L + 2\Omega$, and for simplicity, we denote these operators by $S^{\dagger}_{-1}$, $S^{\dagger}_{0}$ and $S^{\dagger}_{1}$, respectively.[16]

It is easy to show that analogously to the case when the thermal field intensity $\bar{n} = 0$,[16] the squeezing is absent for any spectrum component of Mollow's triplet taken separately. Further, we investigate the squeezing in the operator

$$E^{(+)} = \frac{1}{\sqrt{2}}(S^{\dagger}_{-1} + S^{\dagger}_{1}), \tag{9}$$

which is connected with the squeezing of two sidebands of Mollow's triplet. The quadrature phase components of the operators $E^{(\pm)}$ are defined as

$$E_\theta = \frac{1}{2}(E^{(+)}e^{i\theta} + E^{(-)}e^{-i\theta}), \tag{10}$$

which coincide with the in-phase ($E_1$) and out-phase ($E_2$) components when $\theta = 0$ and $\pi/2$, respectively.

The normally-ordered variance of the quadrature phase component $E_\theta$ can be found using the relations (9)–(10), and takes the form:

$$\langle:(\Delta E_\theta)^2:\rangle = \frac{1}{2}(\cos^2\varphi - \sin^2\varphi\cos 2\theta)\cos^2\varphi\langle R_{21}R_{12}\rangle$$
$$+\frac{1}{2}(\sin^2\varphi - \cos^2\varphi\cos 2\theta)\sin^2\varphi\,\langle R_{12}R_{21}\rangle, \tag{11}$$

where

$$\langle R_{12}R_{21}\rangle = -\langle R_{11}^2\rangle + (N+1)\langle R_{11}\rangle, \tag{12}$$

$$\langle R_{21}R_{12}\rangle = -\langle R_{11}^2\rangle + (N-1)\langle R_{11}\rangle + N, \tag{13}$$

with $\langle R_{11}\rangle$ and $\langle R_{11}^2\rangle$ being found according to Eqs. (6), (7), respectively.

The squeezing is present in the quadrature phase component $E_\theta$ if

$$\langle:(\Delta E_\theta)^2:\rangle < 0. \tag{14}$$

For the case of exact resonance $\cos^2\varphi = \sin^2\varphi = 1/2$ we have $\langle R_{12}R_{21}\rangle = \langle R_{21}R_{12}\rangle = 1/6\,(N^2 + 2N)$ and the variance $\langle:(\Delta E_\theta)^2:\rangle$ takes the form



$$\langle:(\Delta E_\theta)^2:\rangle = \frac{1}{48}(N^2 + 2N)(1 - \cos 2\theta) \geq 0, \tag{15}$$

thus the squeezing is absent in the exact resonance case.

For the case of a large number of atoms $N \gg 1$ and $\cos^2\varphi \neq 1/2$ the variance $\langle:(\Delta E_1)^2:\rangle$ takes the form

$$\langle:(\Delta E_1)^2:\rangle = \frac{N}{2}|\cos^2\varphi - \sin^2\varphi|\,(\bar{n}\,(\cos^4\varphi + \sin^4\varphi) - \sin^2\varphi\cos^2\varphi). \tag{16}$$

It is easy to see from the relation (16) that the squeezing is present if

$$\bar{n} < \frac{\sin^2\varphi\cos^2\varphi}{\cos^4\varphi + \sin^2\varphi} \leq \frac{1}{2}. \tag{17}$$

We note that for the case $N \gg \bar{n}$ the variance $\langle:(\Delta E_1)^2:\rangle$ is a large value (proportional to $N$) and dominates over the thermal field intensity. In this case the normal-ordered variance $\langle:(\Delta E_1)^2:\rangle$ coincides with the normal-ordered variance of the two sidebands of the fluorescence field.

The detailed behavior of the normal-ordered variance $\langle:(\Delta E_1)^2:\rangle$ as a function of the parameter $\cos^2\varphi$ for $N = 50$ and for various values of the thermal field intensity is plotted in Fig. 1. As shown in Fig. 1, the thermal field strongly affects the squeezing on two sidebands of Mollow's triplet. For the case of $\omega_{21}/2\pi \approx 100$ GHz, $T = 2$ K we have $\bar{n} = 0.1$ and the large squeezing is possible.

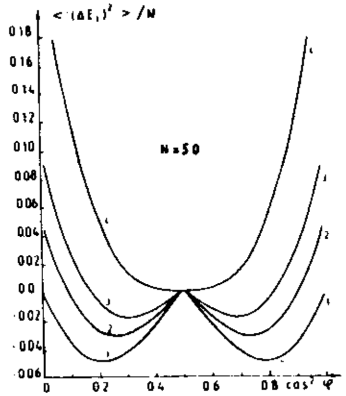


Fig. 1. The normal-ordered variance $\langle:(\Delta E_1)^2:\rangle/N$ as a function of $\cos^2\varphi$ for the case of $N = 50$. The curves 1–4 correspond to $\bar{n} = 0$, $\bar{n} = 0.1$, $\bar{n} = 0.2$, and $\bar{n} = 0.5$ respectively.

**References**

1. D. F. Walls, *Nature* **306** (1983) 141.
2. Y. Yamamoto and H. A. Hause, *Rev. Mod. Phys.* **58** (1986) 1001.
3. R. Loudon, and P. L. Knight, *J. Mod. Optics* **34** (1987) 709.
4. C. M. Cave, *Phys. Rev.* **230** (1983) 1693.
5. R. S. Bondurant and J. H. Shapiro, *Phys. Rev.* **300** (1984) 2548.
6. P. P. Drummond and S. J. Carter, *JOSA* **B4** (1987) 565.
7. R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz and J. F. Valley, *Phys. Rev. Lett.* **55** (1985) 2409.
8. Wu Ling An, H. J. Kimble, J. L. Hall and Wu Huifa, *Phys. Rev. Lett.* **57** (1986) 2520.
9. R. M. Shelby, M. D. Levenson, S. H. Perlmutter, R. G. Devoe and D. F. Walls, *Phys. Rev. Lett.* **57** (1986) 691.
10. M. G. Raizen, L. A. Orozco, Xiao Min, T. L. Boyd and H. J. Kimble, *Phys. Rev. Lett.* **59** (1987) 198.
11. L. Mandel, *Phys. Rev. Lett.* **49** (1982) 136.
12. D. F. Walls and P. Zoller, *Phys. Rev. Lett.* **47** (1981) 709.
13. R. Loudon, *Opt. Commun.* **49** (1984) 24.
14. M. J. Collett, D. F. Walls, P. Zoller, *Opt Commun.* **52** (1984) 145.
15. P. A. Lakshmi, G. S. Agarwal, *Opt. Commun.* **51** (1984) 425.
16. N. N. Bogolubov, jr, A. S. Shumovsky and Quang Tran, *Phys. Lett.* **A118** (1986) 315.
17. N. N. Bogolubov, jr, A. S. Shumovsky and Quang Tran, *Opt. Commun.* **64** (1987) 351.
18. Q. V. Lawande and S. V. Lawande, *Phys. Rev.* **36** (1988) 800.
19. S. Haroche and J. M. Raimond, *Adv. At. Mol. Phys.* **20** (1985) 347.
20. J. A. C. Gallas, G. Leuchs, H. Walther, H. Figger, *Adv. At. Mol. Phys.* **20** (1985) 413.
21. G. S. Agarwal, R. K. Bullough and G. P. Hildred, *Opt. Commun.* **59** (1986) 23.
22. A. Heidmann, J. M. Raimond, S. Reynaud and N. Zagury, *Opt. Commun.* **54** (1985) 189.
23. G. S. Agarwal, *Springer Tracts in Modern Physics* (Springer-Verlag, 1974) **70**.
24. G. P. Hildred, R. R. Puri, S. S. Hassan and R. K. Bullough, *J. Phys.* **B17** (1984) L535.