INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4.SMR/449-46

WINTER COLLEGE ON HIGH RESOLUTION SPECTROSCOPY

(8 January - 2 February 1990)

ONE-DIMENSIONAL HYDROGEN IN LOW-FREQUENCY RADIATION: FREQUENCY-MODULATED HYDROGEN

M.H. Nayfeh
&
D.C. Humm

University of Illinois at Urbana - Champaign
Dept. of Physics
Urbana, Illinois, U.S.A.

One-dimensional hydrogen in low-frequency radiation: Frequency-modulated hydrogen

Munir H. Nayfeh, David C. Humm, and Mark Peercy
Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801
(Received 23 March 1989)

We investigate the effect of intense, low-frequency fields on the structure of atomic hydrogen (prepared in the presence of an external dc electric field). The em field (of amplitude E_0 and frequency ω) interacts with a low-lying (n in the range of 2–7) two-level system of "one-dimensional" hydrogen [transition frequency $\omega_0=(N+1)\omega$ and transition dipole moment μ]. The field couples strongly to the permanent dipole moments of the two levels (d_1 and d_2), creating a large number of equally spaced sidebands (spacing equal to ω) that share μ among themselves. The distribution of the oscillator strength depends on the universal parameter $(d_2-d_1)E_0/\omega$. We determined the transition moment μ_N of the Nth sideband for a variety of ladder and off-ladder systems. Our results indicate that at sufficiently high intensities that are comparable to the threshold of classical chaos of the many-level hydrogen-atom system, off-ladder sidebands become as strong as ladder sidebands, thus breaking the one dimensionality of the system.

There has been considerable recent work on the excitation and ionization of atomic hydrogen in the presence of strong dc and ac fields.[1–11] In particular, one-dimensional atoms prepared by exciting high-lying states of atomic hydrogen in a strong external dc electric field have been the subject of great interest. Studies have indicated that these atoms are unconventional in their structure and spectroscopic properties.[2] Recently, attention has been drawn to the response of such atoms to intense low-frequency radiation for the investigation of chaotic dynamics in quantum systems.[4,5]

The response of this system to low-frequency radiation has been extensively analyzed in recent years with the use of a classical one-dimensional model similar to that of an electron over a liquid-helium surface.[6–9] This model predicts ionization for radiation strengths larger than some critical value, brought about by diffusive excitation due to chaotic motion of the electron. The predicted intensity thresholds for global chaos agreed well with experiments performed on three-dimensional atoms.[9] The effect of clamping dc fields on the threshold and on the number of states trapped in nonlinear resonances was also determined using this method.[7,8,10]

The quantum-mechanical one-dimensional model has also been extensively studied in recent years by numerical simulation.[11] The results indicate that, under some circumstances, quantum wave packets remain localized, in contrast to the chaotic diffusion predicted by classical calculations, pointing to a quantum limitation on classical chaos. However, the quantum-mechanical simulations still indicate the presence of an intensity threshold (lying equal to or higher than the classical one) above which diffusion takes place.[11] This has recently been attributed to multiphoton broadening sufficient enough to cause n mixing of the system.[12]

In spite of the extensive use of this one-dimensional model theoretically and experimentally, there is still an uneasy feeling lingering about its validity and applicability in the presence of intense external fields. It is known that in one-photon, near-resonance excitation, the excitation is dominated by processes where $\Delta n_1=\Delta n$, and as such the system maintains its dimensionality. However, under low-frequency interactions involving the deposition of many photon energies, it is not clear that this will continue to hold. Although there is some experimental evidence for the preservation of one dimensionality during the interaction time, this evidence is from experiments in which the field was well below the threshold for classical chaos.[4]

In this paper, we examine an effect that involves the absorption of the equivalent of many photon energies but does not involve a multiphoton process, instead coupling to the permanent dipole. In particular, we study the dimensionality under the influence of external low-frequency ac fields whose intensity is comparable to the threshold intensity for classical chaos. We take here an unusual approach, by first determining the effect of the field on the structure of atomic hydrogen. The ac fields generate atomic sidebands of spacing equal to the external field frequency in a fashion that is similar to frequency modulation of classical oscillators or laser output (frequency-modulated hydrogen). If the bandwidth of the modulation is large enough, the system can make single-photon transitions via the sidebands, resulting in strong excitation of otherwise weak transitions. The preservation or breaking of the one-dimensionality is then defined in terms of the relative strengths of such excitations within a given pair of n manifolds. Our analysis shows that at sufficiently high intensities, that are comparable to the threshold of chaos, certain off-ladder transitions become as strong as ladder transitions, thus breaking the dimensionality of the system.

We attack this problem by taking some hints from the interaction of polar molecules with intense radiation. Recently, it was noted that the existence of a permanent electric dipole in a polar molecule allows the absorption of a large number of photons in a restricted two-level system.[13] Here, we reexamine the interaction and present a view in which the absorption proceeds via single-photon absorption by sidebands that are created by the interaction of the intense field with the permanent dipole. This idea has not surfaced in the literature with regard to the recent work of interaction of polar molecules and one-dimensional hydrogen with intense radiation. We use atomic units throughout.

The interaction of a two-level system $|\phi_1\rangle$ and $|\phi_2\rangle$ of time-dependent amplitudes a_1' and a_2' with the field is described by the following two coupled equations:

$$\frac{da_1'}{dt}+iH_{11}a_1'=-iH_{12}a_2'e^{-i\omega_0 t},$$

$$\frac{da_2'}{dt}+iH_{22}a_2'=-iH_{12}^{*}a_1'e^{i\omega_0 t},$$

where $\omega_0=(E_2-E_1)$ (the field-free transition frequency) and H_{ij} are the matrix elements of the dipole operator $H_{ij}=\langle\phi_i|H|\phi_j\rangle$.

The term H_{ij} where $i\neq j$ represents the coupling of a single-photon transition between the two-level system and the external field, whereas the term H_{ii} arises from the interaction of the external field with each of the states individually, and it is nonzero if the corresponding state has a permanent dipole moment (i.e., does not have definite parity). These represent frequency shifts with H_{11} shifting the lower level and H_{22} shifting the upper level. We should mention at this point that these shifts are unconventional in atomic systems due to the absence of permanent dipoles. Their nature and origin are different from those usually encountered in multiphoton excitation of multilevel systems which are due to certain energy nonconserving transitions in the interaction. The latter energy shifts are constants (except for time dependence resulting from the envelope of the ac field) and vary slowly with the frequency of the external field. The former shifts, however, are not constants in time; they oscillate at the frequency of the external field.

To determine the effect and the nature of these shifts, we first make the following phase transformation:

$$a_i'=a_i\exp\left[-i\int_{-\infty}^{t}H_{ii}dt\right].$$

In the dipole approximation, the explicit time dependence of the H_{ij} is given as $H_{12}=\mu E_0(t)\cos\omega t$, $H_{11}=d_1E_0(t)\cos\omega t$, and $H_{22}=d_2E_0(t)\cos\omega t$, where μ is the matrix element of the transition dipole moment operator, and d_1 and d_2 are the permanent dipole moments of the lower (ϕ_1) and upper (ϕ_2) states, respectively.

We will assume that the time variation of the field amplitude is small compared to the changes in its phase, and as a result, the integrals in the phase transformation can be easily carried out for the above time dependence of H giving sine functions. Now the time-dependent exponentials of the form $e^{i\alpha\sin\omega t}$ can be expanded as an infinite series in terms of the harmonics of the frequency ω and, as a result, the two coupled equations describing the system transform into the following:

$$\frac{da_1}{dt}=-iH_{12}\left[\sum_{k=-\infty}^{\infty}J_k\left(\frac{dE_0}{\omega}\right)e^{-i\omega_0' t}\right]a_2,$$

$$\frac{da_2}{dt}=-iH_{12}\left[\sum_{k=-\infty}^{\infty}J_k\left(\frac{dE_0}{\omega}\right)e^{i\omega_0' t}\right]a_1,$$

where $\omega_0'=\omega_0-k\omega$ and k is a positive or negative integer, $d=d_1-d_2$, and J_k is the Bessel's function of order k.

The above expansion has effectively changed the two-level system of transition frequency ω into a two-level system whose transition frequency is not definite. The transition matrix element of the two-level system μ gets distributed among the sidebands according to $\mu_k=\mu J_k(dE_0/\omega)$ and the single-photon interaction coupling takes the form $H_{12}=\mu_k E_0(t)\cos\omega t$.

We now discuss the possibility of strong single-photon absorption at frequency ω. On the surface, it appears that the system cannot interact strongly with the field since $\omega\ll\omega_0$. However, from the above discussion, it is clear that the total transition probability is the sum of the individual transition probabilities to all the sidebands. In fact one can adjust or tune the external field frequency so that the field resonates strongly with one sideband ($\omega\cong\omega_0'-k_0\omega$). In this case we find that the transition frequency ω_0 is a multiple of ω: $\omega_0=(k_0+1)\omega=N\omega$, where N is an integer. Single-photon "stimulated emission" also occurs in this situation with $-\omega\cong\omega_0'-k_1\omega$, $\omega_0=(k_1-1)$, $\omega=N\omega$, and $k_1=k_0+2$. However, it is suppressed relative to absorption when $dE_0/\omega<1$, because μ_{k_1} is proportional to a higher-order Bessel function than μ_{k_0}. The other sidebands ($k\neq k_0$, $k\neq k_1$) will not interact strongly and can be neglected. However, if none of the sidebands dominates, then more than one must be retained.

In the following discussions we consider the case where one sideband dominates the interaction, and proceed to discuss specific cases of atomic hydrogen in the presence of external dc electric fields. We studied excitation from one-dimensional states originating from the $n=2$ manifold to one-dimensional states originating from the [illegible] 5, 6, and 7 manifolds. In addition, we studied excitation from the $n=2$ to 3, $n=3$ to 4, and $n=4$ to 5 manifolds.[13]

Because each n state splits in the presence of a strong dc field into a number of parabolic states (specified by the quantum numbers n_1, n_2, and m), there are several transitions within a pair of n manifolds. Ladder transitions are defined by $\Delta n_1=\Delta n$ for $n_1\geq n_2$ or by $\Delta n_2=\Delta n$ for $n_2\geq n_1$. Off-ladder transitions [illegible] defined by $\Delta n_1=\Delta n$ for $n_1\leq n_2$ or by $\Delta n_2=\Delta n$ for $n_2\leq n_1$. Finally, all other possible transitions [illegible] called "very-off-ladder" transitions. We will use radiation at 10.6 μm from CO_2 lasers in all of the following calculations. This is an interesting source of radiation since it satisfies the low-frequency condition $\omega\ll\omega_0$ and is capable of delivering high intensities.

The state of maximum n_1 for a given n corresponds to the state of that n in the one-dimensional approximation (as n increases these states become more like one-dimensional states in energy and other characteristics). Notice that if the system starts in a state of maximum n

(given n), ladder transitions will keep it in such states, maintaining the one dimensionality, whereas off-ladder transitions will take it into different states, breaking the one dimensionality. We intend to compare the strengths of ladder and off-ladder transitions to see if one dimensionality is likely to be maintained. We will also concentrate on $m=0$ states, as the "one-dimensional" states have $m=0$ and with the π polarization studied $\Delta m=0$. However, the method is much more general and we are able to study transitions between states of any n_1, n_2, and m.

Let us calculate the permanent dipole moments of the initial and final states of each of the various transitions. This is done by using $d=\langle \phi_n | (-ez) | \phi_n \rangle$. The transition matrix element is calculated using $\mu=\langle \phi_n (-ez) | \phi_m \rangle$. Although the energy spacing of the upper manifold depends on the magnitude of the dc field present, our previous studies showed that the effect of fields even as high as 20 kV/cm is negligible on the wave functions of the low-lying excited states of hydrogen.[14] Therefore we will use the zero-field parabolic wave functions for all of our calculations of d and μ.

Figure 1 gives the permanent dipole moment plotted as a function of the parabolic quantum number n_1, of the one-dimensional states of the various n manifolds. Figure 2 gives transition dipole moment $|\mu|$ for transition between the state ($n_1=1$, $n_2=0$, $|m|=0$) of the $n=2$ manifold, and the parabolic states of the $n=3$, 4, 5, 6, and 7 manifolds. Those are plotted as a function of the n_1 quantum number of the upper states.

Let us first discuss the excitation between the states of $n=2$ and 3 manifolds. Here we have a number of transitions. For any of these transitions, the energy absorbed in making a transition from $n=2$ to 3 is approximately 16.1 photon units of energy. Thus the resonant sidebands for one-photon transitions are the 15th and 17th. Since the oscillator strength of the 15th sideband becomes appreciable at a lower ac field than that of the 17th, we will concentrate on the 15th, for we are interested in the thresholds of processes. The transition from (1,0,0) to (2,0,0) is a ladder transition with an oscillator strength of 1.814 while the transition from (1,0,0) to (1,1,0) is an off-ladder transition with an oscillator strength of 1.19. The very-off-ladder transition is from (1,0,0) to (0,2,0) with an oscillator strength of 0.044. Figure 3 gives the distribution μ_N^2 for the ladder transition (1,0,0) to (2,0,0) for two field intensities. We can see that at the higher intensity, the 15th sideband appears to carry a larger fraction of the oscillator strength of the ladder transition. A similar plot for the off-ladder transition is given in Fig. 4 at the highest intensity used in Fig. 3, showing that the 15th sideband also carries a larger fraction of the oscillator strength of the off-ladder transition. The rest of the sidebands were not shown because their oscillator strengths are very small.

We will now further study the 15th sideband. Like all other sidebands, its oscillator strength will oscillate as a function of $x=(d_2-d_1)E_0/\omega$ since it is proportional to the Bessel function $J_{15}(x)$. Figure 5 gives the oscillator strength of this band as a function of intensity for the ladder, off-ladder, and very-off-ladder transitions. First,

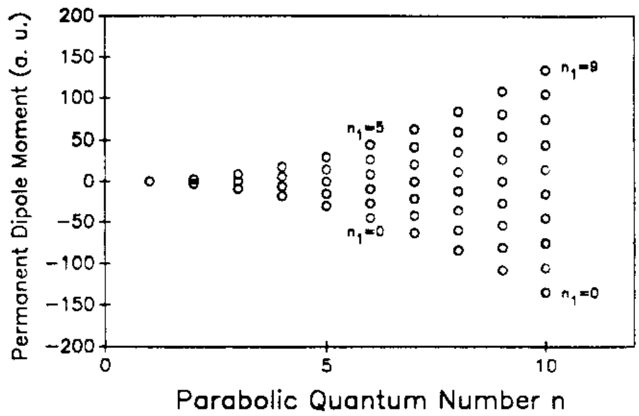

FIG. 1. The permanent dipole moment of the $m=0$ parabolic states of the $N=2-10$ manifolds as a function of the parabolic n_1 quantum numbers.

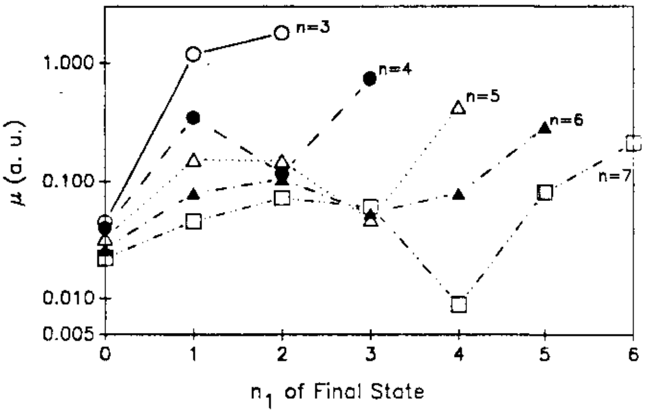

FIG. 2. The transition dipole moment from the $n_1=1$, $n_2=0$, $m=0$ parabolic state of $n=2$ to some $m=0$ parabolic states of higher-n manifolds as a function of n_1. The lines connecting the points are used to guide the eye.

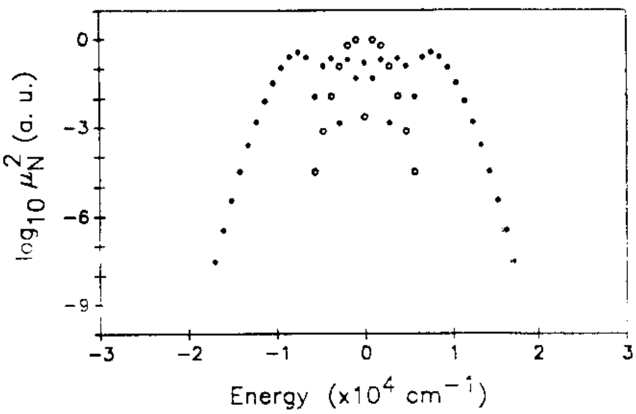

FIG. 3. The oscillator strength of the sidebands of the ladder transition between the (1,0,0) and (2,0,0) states for the two intensities 10^{11} and 1.69×10^{12} W/cm², shown as ∘ and ● respectively. Higher sidebands are weak and are not shown.

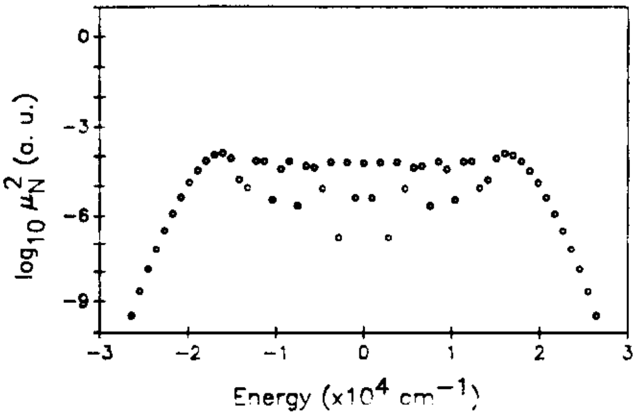

FIG. 4. The square of the oscillator strength of the sidebands of the off-ladder transition between (1,0,0) and (0,2,0) for an intensity of 1.69×10^{12} W/cm^2. Higher sidebands are weak and are not shown.

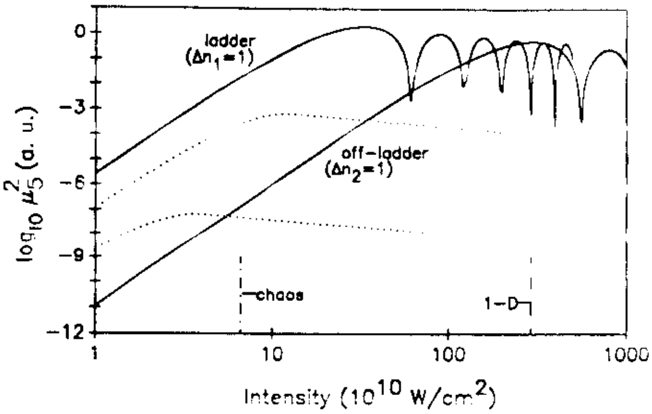

FIG. 6. The oscillator strength of the fifth sideband for ladder, off-ladder, and very-off-ladder transitions from the state (2,0,0) to the $n=4$ manifold as a function of intensity.

we observe that the contribution of the very-off-ladder sideband is very small compared to that of the ladder and off-ladder at all intensities, making it of no importance. In fact we found that this feature is generally true in all transitions studied, and as such, we will neglect all of the very-off-ladder transitions.

Another important feature of Fig. 5 is a direct result of the oscillatory nature of the Bessel function. The results show that at low intensities the strength of the ladder transition is stronger than that of the off-ladder transition. But at higher intensities, the ladder transition starts to oscillate around zero while the off-ladder continues to rise. At even higher intensities, the strength of the off-ladder transition becomes equal to that of the ladder transition and oscillations appear (of a different period and out of phase); thus both can be equally strong. The intensity at which they first become comparable (equal) defines for use the threshold for breaking of dimensionality I_b. In this case, it occurs at $I_b=2630\times10^{10}$ W/cm^2.

We next examine some transitions between $n=3$ and 4 and between $n=4$ and 5. In these cases the relevant sidebands, using $\lambda=10.6$ μm, are the fifth and second, respectively. Transitions between $n=6$ and 7 fall into the high-frequency regime ($\omega\geq\omega_0$) at 10.6 μm. There are many more transitions that can be examined here. However, we will only consider transitions from the bluest (2,0,0) and (3,0,0) states, respectively. Figures 6 and 7 give the results which show similar features to those seen in the excitation between the $n=2$ and 3 manifolds. However, as n rises, one sees a drop in the intensity I_b at which the off-ladder and ladder transitions first become comparable. These results are compiled in Fig. 8.

It is interesting to examine this behavior in relation to the intensity I_c at which one-dimensional hydrogen prepared in the initial states $n=2$, 3, 4, 5, and 6, respectively, becomes chaotic according to classical mechanics. The intensity I_c was calculated by numerical simulation using a one-dimensional classical model similar to that of an electron near a helium surface, giving the values

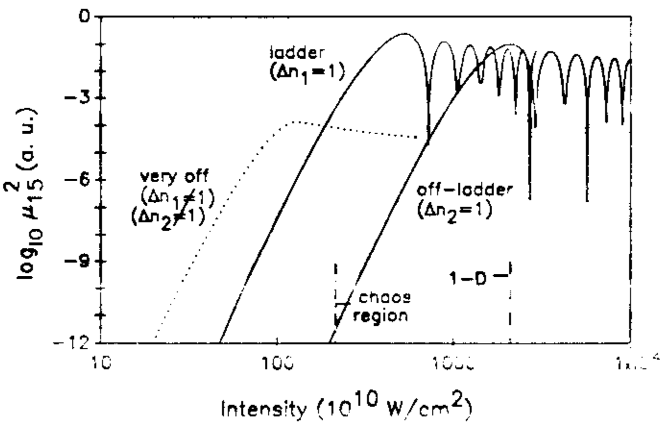

FIG. 5. The oscillator strength of the 15th sideband for ladder, off-ladder, and very-off-ladder transitions from the state (1,0,0) to the $n=3$ manifold as a function of intensity. Noted on the x axis are I_c the chaos threshold for an electron starting at $n=2$, and I_b the threshold for loss of one dimensionality. The troughs in the graph actually go to zero.

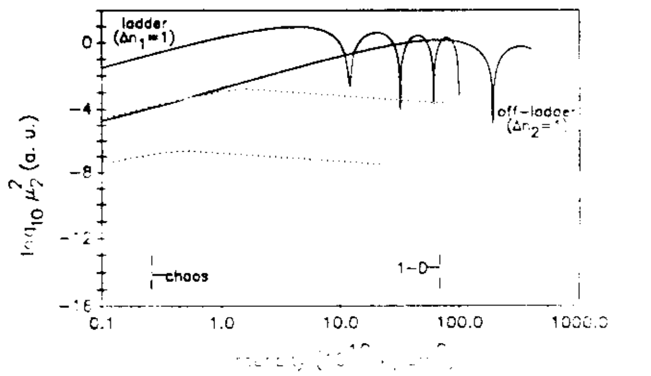

FIG. 7. The oscillator strength of the second sideband for ladder, off-ladder, and very-off-ladder transitions from the state (3,0,0) to the $n=5$ manifold as a function of intensity.

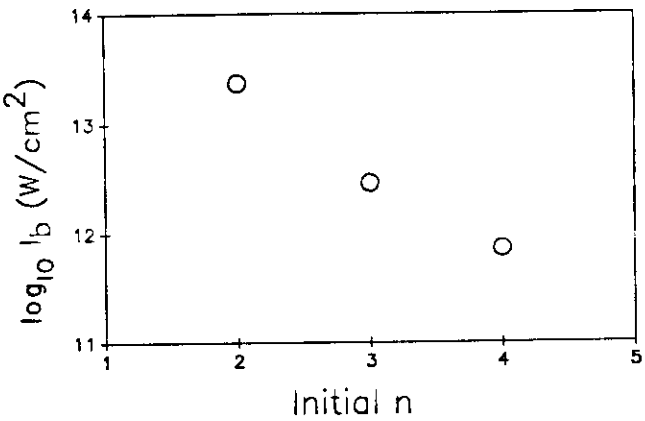

FIG. 8. The intensity at which the off-ladder and ladder transitions first become equal, I_b, is plotted for the various initial n manifolds.

214×10^{10}, 6.6×10^{10}, 0.26×10^{10}, and 0.019×10^{10} W/cm^2 for $n=2$, 3, 4, and 5 initial manifolds, respectively.[6] These values are plotted in Fig. 9 and have already been marked in Figs. 5–7. Thus we can see that $I_b>I_c$ for the initial manifolds $n=2$, 3, 4, and 5. It is therefore useful to define the condition $W=I_b/I_c\geq 1$ as an intensity window (range) such that the chaotic region may be observed in a system which is still one dimensional. The equal sign ($W=1$) is the case when the window just closes, while $W<1$ indicates that the system loses its dimensionality before it becomes chaotic. It is clear from the results that the window widens as the initial state is derived from higher n manifolds (implying improved one dimensionality).

In a single measurement in which the system is prepared in a given initial manifold, one would be interested in seeing how the dimensionality of the system changes as the excitation to higher levels proceeds. For this purpose, we take an example in which the system is initially in the (1,0,0) of the $n=2$ manifold. Dividing the intensities at which the system loses its one dimensionality by 1.7×10^{12} W/cm^2 (the threshold of chaos for $n=2$), we get Fig. 10. Notice that, although the system does not lose one dimensionality in the initial transition $n=2\rightarrow n=3$, as the excitation proceeds it would lose one dimensionality at the $n=4$ to 5 transition. This loss of one dimensionality in the subsequent excitation can begin either in the low-frequency ($\omega<\omega_0$) regime, as it does here, or in the high-frequency regime ($\omega>\omega_0$), which starts at $n=6\rightarrow n=7$ for 10.6-μm radiation).

However, we have not found any evidence of a loss of one dimensionality at or below the chaos threshold for the first transition in the excitation, starting from the state for which the chaos threshold is computed. It is only for subsequent transitions, with n greater than that for which the chaos threshold is computed, that the one dimensionality is lost.

The radiation can also interact with the system by connecting levels whose principal quantum numbers differ by two or more, with contrasting results. As an example, we studied direct excitation from the bluest state of the $n=2$ manifold, (1,0,0), to the states in the $n=3$, $n=4$, $n=5$, $n=6$, and $n=7$ manifolds. In these transitions n changes by 1, 2, 3, 4, and 5 in contrast to the above cases where $\Delta n=1$. Keeping the frequency of the radiation constant, then the 15th, 21st, 23rd, 25th, and 27th sidebands, respectively, control the interaction in these manifolds. Figures 11–14 show results for $N=4$, 5, 6, and 7, respectively.

One interesting feature which one can see from Figs. 11–14 is the drop of the intensity at which ladder and off-ladder transitions become comparable (I_b) as the quantum principal number of the final state increases, leading to a closing of the window ($W<1$). As an example we plot in Fig. 15 this intensity as a function of n for the (1,0,0) to ($n-1$,0,0) and (1,0,0) to (0,$n-1$,0) transitions. But because the absolute strength of all transitions (ladder and off-ladder) drops as n increases (see Fig. 16), these processes become weak, and thus may not be of any major consequence.

We also studied the frequency dependence of these results. Using radiation of 21.2 μm, we find that the intensity at which the off-ladder and ladder strengths become comparable does not change. This is due to a cancella-

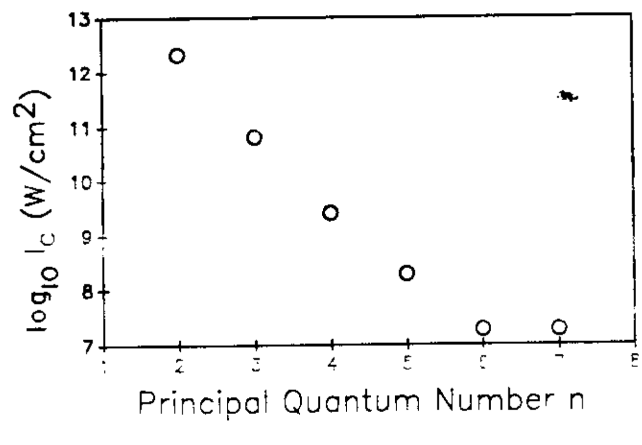

FIG. 9. The classical ionization threshold intensity calculated using a one-dimensional classical model similar to that of a surface electron for various n manifolds.

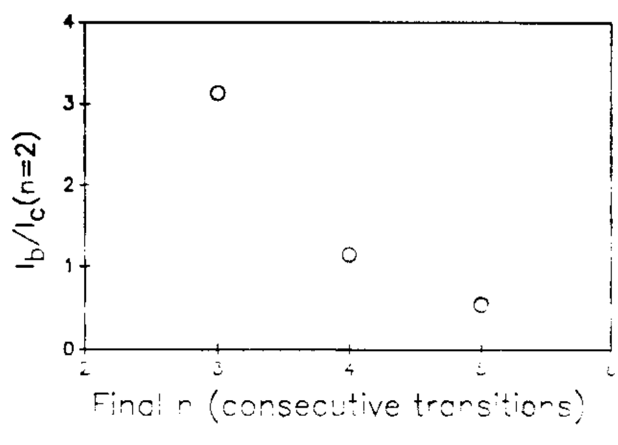

FIG. 10. The ratio of the intensity I_b of loss of one dimensionality to the $n=2$ intensity threshold of classical chaos.

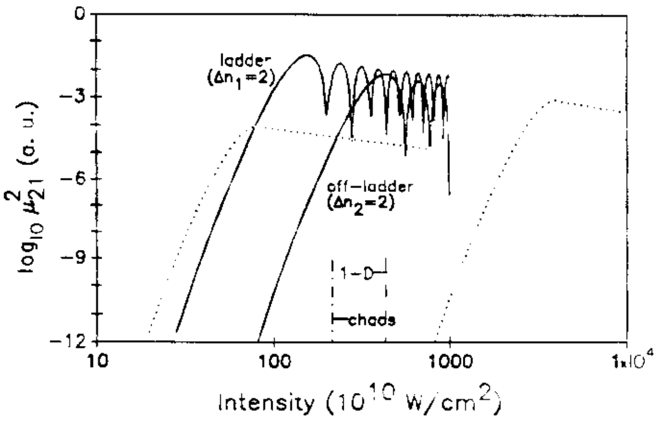

FIG. 11. The intensity dependence of the oscillator strength of the 21st sideband for the ladder, off-ladder, and very-off-ladder transitions from the state (1,0,0) to the $n=4$ manifold.

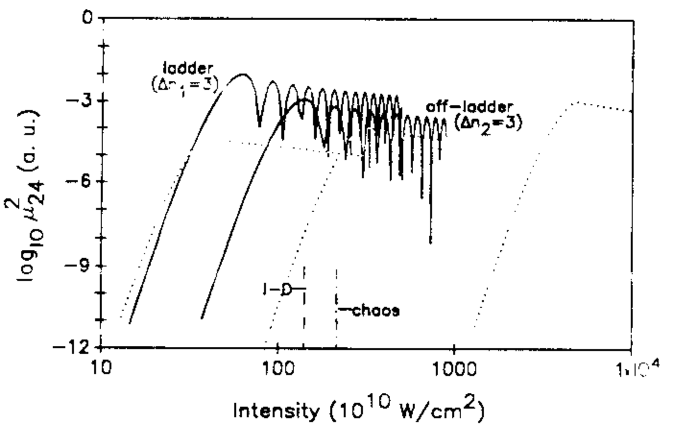

FIG. 12. Intensity dependence of the oscillator strength of the 24th sideband for the ladder, off-ladder, and very-off-ladder transitions from the state (1,0,0) to the $n=5$ manifold.

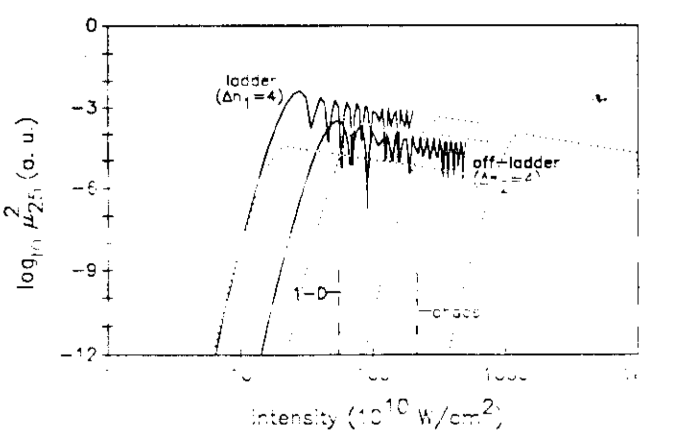

FIG. 13. Intensity dependence of the oscillator strength of the 25th sideband for the ladder, off-ladder, and very-off-ladder transitions from the state (1,0,0) to the $n=6$ manifold.

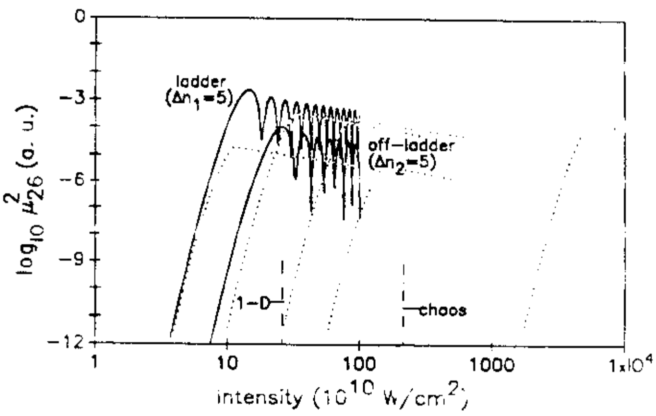

FIG. 14. Intensity dependence of the oscillator strength of the 26th sideband for the ladder, off-ladder, and very-off-ladder transitions from (1,0,0) to the $n=7$ manifold.

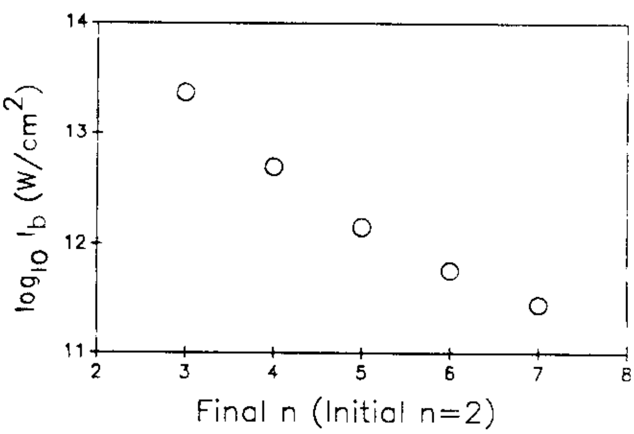

FIG. 15. The intensity at which the off-ladder and ladder transitions first become equal, I_b, for the cases given in Figs. 11–14.

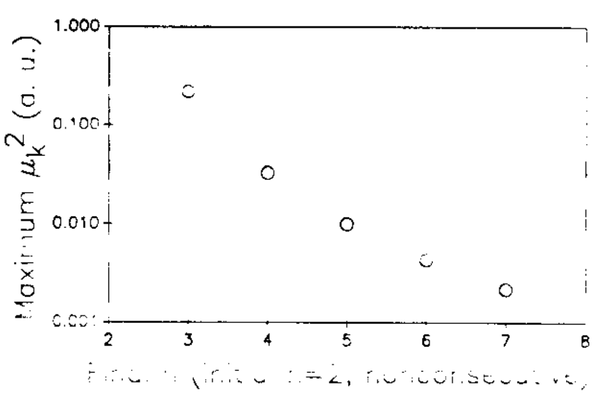

FIG. 16. The absolute oscillator strength of the ladder transitions shown in Figs. 11–14 for various n manifolds and a common initial state (1,0,0).

tion of the frequency dependence contained in the order of the Bessel function and in its argument. Given k, the number of sidebands necessary at frequency ω, $k+1$ is proportional to $1/\omega$, and the Bessel function of order k peaks at dE_{peak}/ω, which is also proportional to $k+1$. Since $(dE_{\text{peak}}/\omega) \propto (k+1)$ and $(1/\omega) \propto (k+1)$, then $(dE_{\text{peak}}/\omega) \propto (1/\omega)$. Hence, as ω changes, dE_{peak} and consequently E_{peak} (d is independent of frequency) are frequency independent. This implies that $I_{\text{peak}} \propto E_{\text{peak}}^2$, the intensity at which the off-ladder transition maximizes, is frequency independent. This maximum is simply the intensity at which the ladder and off-ladder transitions become comparable, which is thus frequency independent.

The chaos threshold, on the other hand, depends on frequency. However, at sufficiently low frequencies $\omega \ll dE/dn$, the chaos threshold is expected to be somewhat insensitive to frequency, since the conditions are close to the dc limit. Thus one expects the window W to be nearly frequency independent in this regime. Calculations for the case of 21.2 μm verified this frequency independence.

In conclusion, we find that the permanent dipole-induced sidebands are very interesting and make the one-dimensional hydrogen atom unusual among atoms in its response to external fields. The same kind of sidebands of course arise in any system which has a permanent dipole moment such as a polar molecule, and in fact their effects were first discussed in interactions of molecular systems with external fields.[13] However, the nature of these effects was explained in terms of multiphoton processes rather than sidebands.

Our results were applied to the question of whether loss of dimensionality occurs at a lower or higher laser intensity than the threshold of chaos in the analogous classical system. Such a loss of dimensionality does occur in subsequent transitions at or below the threshold of chaos for the initial state. However, it does not appear for the initial transition. The loss of dimensionality can also appear for direct transitions from the initial quantum state to a much higher one, skipping states in between, but these processes are weak and may not be significant.

The equal spacing of the sidebands indicates a linear (harmonic-oscillator-like) response of the restricted two-level system. This linear behavior, of course, does not constitute chaos, but it is a strong two-level effect and so will bear directly on the nature of any chaos effects in the full multilevel system. Nonlinearity, which is provided by the nonlinear Coulomb interaction, is necessary for classical chaos to occur in the radiation-atom system. Quantum mechanically, this translates into unequal energy-level spacings, as opposed to equal energy-level spacings for a linear system. For a two-level system, the distinction between linear and nonlinear does not apply, because there is only one energy-level spacing. However, the two-level approximation is useful by giving a simple system within which one can gauge the strength of various effects in the real system. Future work involves studying the sidebands when more levels with different spacings are included.

[1]D. A. Harmin, Phys. Rev. A **24**, 2491 (1981); Phys. Rev. Lett. **49**, 128 (1982); Phys. Rev. A **26**, 2656 (1982); E. Luc-Koenig and A. Bachelier, Phys. Rev. Lett. **43**, 921 (1979); U. Fano, Phys. Rev. A **24**, 619 (1981); V. D. Kondratovich and V. N. Ostrovsky, Zh. Eksp. Teor. Fiz. **83**, 1256 (1982) [Sov. Phys.—JETP **56**, 719 (1982)]; A. R. P. Rau and K. T. Lu, Phys. Rev. A **21**, 1057 (1980); R. J. Damburg and V. V. Kolosov, J. Phys. B 9, 3149 (1976).

[2]W. L. Glab, K. Ng, D. Yao, and M. H. Nayfeh, Phys. Rev. A **31**, 3677 (1985); W. L. Glab and M. H. Nayfeh, *ibid.* **31**, 530 (1985); M. H. Nayfeh, W. Glab, K. Ng, D. C. Humm, and J. Bowie (unpublished).

[3]H. Rottke and K. H. Welge, Phys. Rev. A **33**, 301 (1986).

[4]J. E. Bayfield and L. A. Pinnaduwage, Phys. Rev. Lett. **54**, 313 (1985).

[5]P. M. Koch, in *Fundamental Aspects of Quantum Theory*, edited by V. Gorini and A. Frigerio (Plenum, New York, 1986).

[6]R. V. Jensen, Phys. Rev. A **30**, 386 (1984).

[7]G. P. Berman, G. M. Zaslavsky, and A. R. Kolovsky, Zh. Eksp. Teor. Fiz. **88**, 1551 (1985) [Sov. Phys.—JETP **61**, 25 (1985)].

[8]D. C. Humm and M. H. Nayfeh, in *Atomic Spectra and Collisions in External Fields*, edited by C. W. Clark, M. H. Nayfeh, and K. T. Taylor (Plenum, New York, 1988).

[9]R. V. Jensen and S. M. Susskind, in *Photons and Continuum States of Atoms and Molecules*, edited by N. K. Rahman, C. Guidotti, and M. Allegrini (Springer-Verlag, Berlin, 1987), Vol. 8.

[10]M. J. Stevens and B. Sundaram, Phys. Rev. A **36** [illegible]

[11]G. Casati, B. Shirikov, D. Shepelyansky, and I. Guarneri, Phys. Rep. **154**, 79 (1987).

[12]G. Casati, in *Atoms in Strong Fields*, edited by C. Nicolaides, M. Nayfeh, and C. Clark (Plenum, New York, 1980).

[13]M. A. Kmetic and W. J. Meath, Phys. Lett. **108A**, 340 (1985).

[14]K. Ng, D. Yao, and M. H. Nayfeh, Phys. Rev. A **35**, 2508 (1987).