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**WINTER COLLEGE ON
HIGH RESOLUTION SPECTROSCOPY**

(8 January - 2 February 1990)

MULTIPHOTON IONISATION OF ATOMS

I Weak Field

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Multi-photon ionisation of atoms

1

Introduction

Typical set up

Interpretation

Orders of magnitude

Weak field effects

Perturbation theory at lowest non vanishing order

Power laws. Saturation. Coherence effects

Calculation of cross sections.

Resonances. effective hamiltonian.

Measurement of cross sections.

Strong field

Multicharged ions

Harmonic generation

Electron energy spectrum.

Calculation of cross sections

Examples

Angular distribution.

Leaving the interaction volume

Short pulses

Non perturbative calculations

Multi-photon ionisation of atoms

2

Theoretical prediction of two-photon absorption

Goppert Mayer Ann Phys 2 273 (1931)

Multi-photon ionisation invoked to explain breakdown in gases

Meyerand + Haught PRL 11 401 (63) PRL 13 7 (64)

Phenomenological model Keldysh Sov Phys JETP 20 1307 (65)

Multi-photon ionisation of noble gases

Voronov + Delone Sov. Phys JETP 23 54 (66).

Agostini Bujot Bonnal Mainfray Manne IEEE QE 4 667 (68)

Laser pulses become stronger and shorter

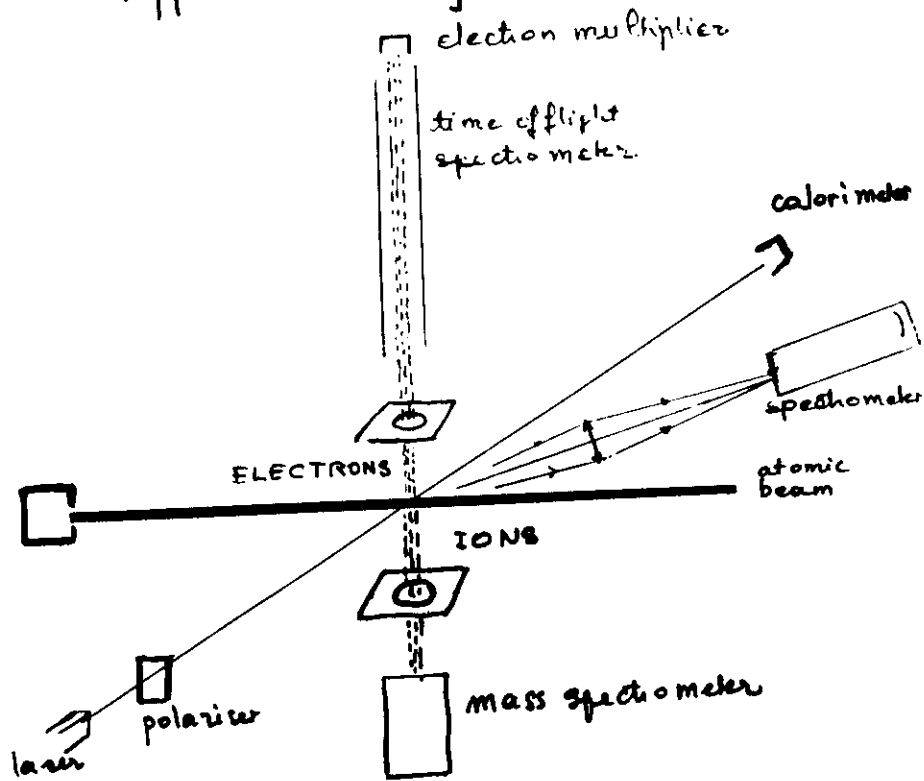
Improvement in fast electronics $\rightarrow$ pulse characteristics
in detection of electrons

Moderate fields - Yield of singly charged ions.

Strong fields - Multicharged ions.
- Harmonic generation

Short pulses - Electron yield.

Typical set up



laser pulse

frequency
bandwidth
time dependence
energy per pulse
polarisation
intensity distribution

electrons

total yield
energy spectrum
angular distribution

ions

yield
charge state
distribution

emitted light

fluorescence
harmonics

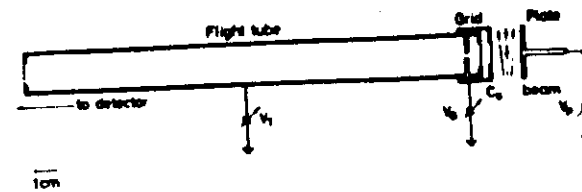


FIG. 5. Time-of-flight spectrometer.

L. Lettice, F. Fabre, F. Agostini, M. Crance, M. Aymar Phys. Rev. A29 (1984) 2677

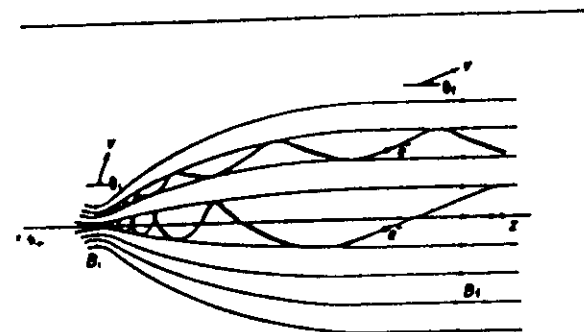
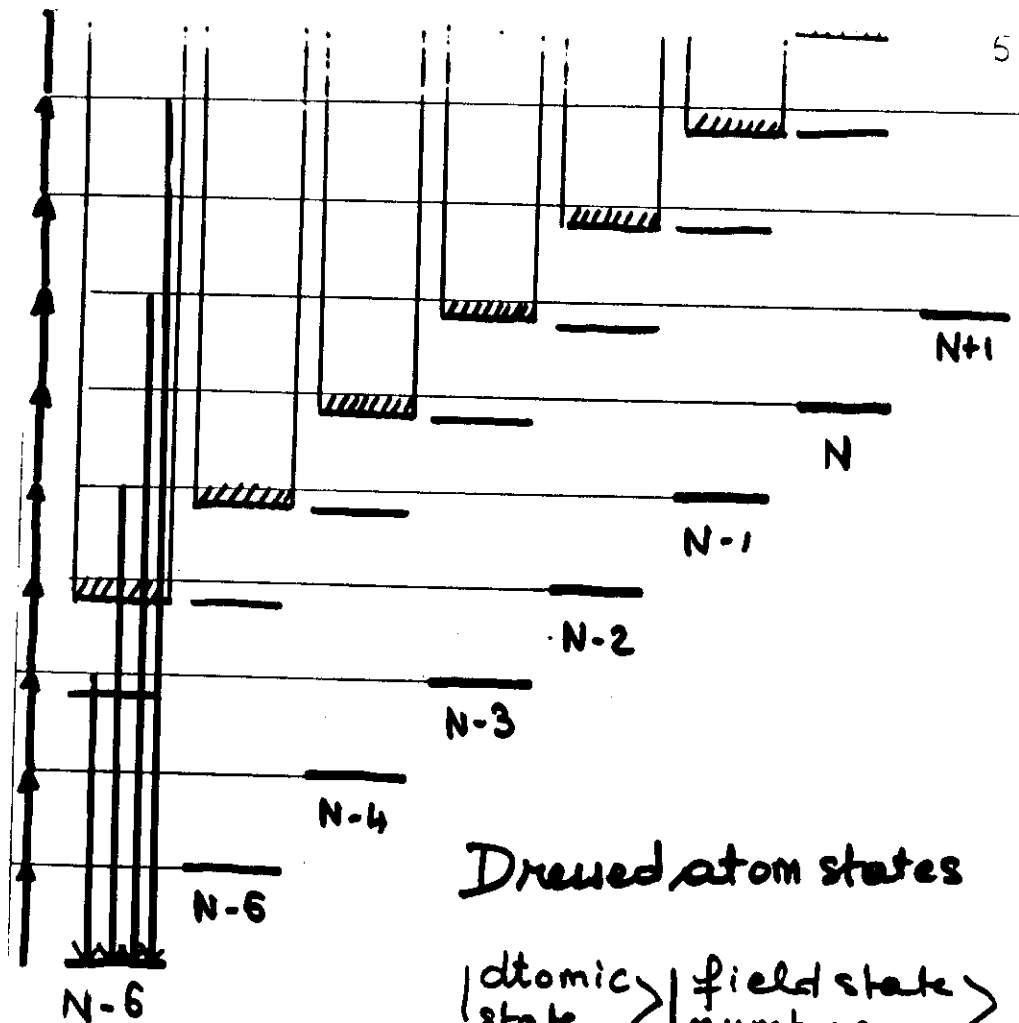


Figure 1. Schematic diagram showing the helical motion of an electron moving in a magnetic field that changes gradually from a strong field B_1 to a weaker uniform field B_2 .

P. Kruit, F. H. Read J. Phys. E16 (1983) 317



Dressed atom states

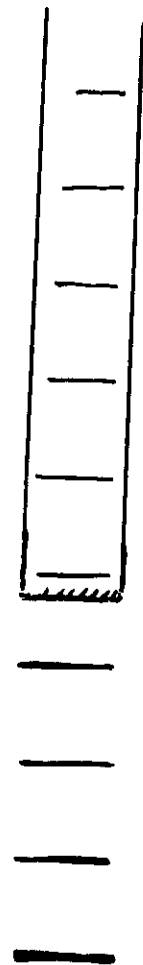
$|\text{atomic state}\rangle |\text{field state number rep.}\rangle$

$|g\rangle |N\rangle$

$|e\rangle |N\rangle$

$|e\rangle |N\rangle$

$$E = E_g + 4\hbar\omega$$

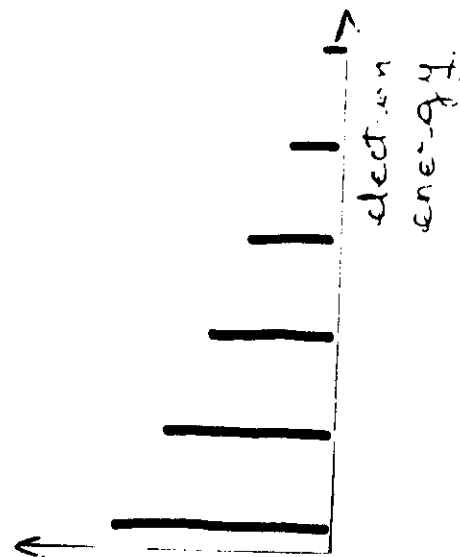


$$E_f = E_g + (n+p)\hbar\omega$$

above threshold ionisation

or Excess Photon Ionisation

absorption of $n+p$ photons.



What is a strong field?

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ground - lowly excited states $\rightarrow$ saturation mW/cm^2

bound - bound excited states $\sim \text{Wcm}^{-2}$

Resonant ionization $\sim 10^6 \text{ Wcm}^{-2}$

Non resonant ionization in alkalis or alkaline earths $\sim 10^9 \text{ Wau}^{-2}$

Non resonant ionization in noble gases $\sim 10^{12} \text{ Wcm}^{-2}$

Extreme Photon Ionisation $\sim 10^{14} \text{ Wcm}^{-2}$

Multiple ionization
harmonic generation

$\rightarrow$ STRONG SHORT PULSES

Perturbation theory at lowest non vanishing order. ⁸

Dipole approximation

time independent treatment $\rightarrow$ dressed atom picture

$$H = H_{\text{atom}} + H_{\text{field}} + \vec{E} \cdot \vec{r} (\sigma + \sigma^\dagger)$$

basis $|i\rangle \otimes |N\rangle$

time dependent treatment

$$H = H_{\text{atom}} + \vec{E} \cdot \vec{r} \cos \omega t$$

basis $|i\rangle$

$$|\Psi(t)\rangle = a_0 |0\rangle + \sum_i a_i |i\rangle + \sum_\alpha a_\alpha |\alpha\rangle$$

initial state
even states
odd states

$$i\hbar \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

$$i\dot{a}_0 = \sum_\alpha V_{0\alpha} a_\alpha (e^{i\omega t} + e^{-i\omega t})$$

$$i\dot{a}_\alpha = \omega_\alpha a_\alpha + V_{0\alpha} a_0 (e^{i\omega t} + e^{-i\omega t}) + \sum_i V_{i\alpha} a_i (e^{i\omega t} + e^{-i\omega t})$$

$$i\dot{a}_i = \omega_i a_i + \sum_\alpha V_{i\alpha} a_\alpha (e^{i\omega t} + e^{-i\omega t})$$

$$E_0 = 0 \quad \hbar V_{0\alpha} = \langle 0 | \frac{\vec{E} \cdot \vec{r}}{2} | \alpha \rangle$$

$$\hbar \omega_\alpha = E_\alpha$$

$$\hbar V_{i\alpha} = \langle i | \frac{\vec{E} \cdot \vec{r}}{2} | \alpha \rangle$$

$$\hbar \omega_i = E_i$$

$\rightarrow$ double expansion according to

- perturbation order
- rotating / counter rotating terms

Perturbative treatment:

Order 0 $\rightarrow i\dot{a}_0 = 0 \rightarrow a_0 = 1$

Order 1 $\rightarrow i\dot{a}_\alpha = \omega_\alpha a_\alpha + V_{\alpha 0} a_0 (e^{i\omega t} + e^{-i\omega t})$

$$\rightarrow a_\alpha = V_{\alpha 0} \left\{ \frac{e^{-i\omega_\alpha t} - e^{-i\omega t}}{\omega_\alpha - \omega} + \frac{e^{-i\omega_\alpha t} - e^{i\omega t}}{\omega_\alpha + \omega} \right\}$$

rotating term counter rotating term

keep $a_\alpha = V_{\alpha 0} \frac{e^{-i\omega_\alpha t} - e^{-i\omega t}}{\omega_\alpha - \omega}$

Order 2 $\rightarrow i\dot{a}_i = \omega_i a_i + \sum_\alpha V_{i\alpha} a_\alpha (e^{i\omega t} + e^{-i\omega t})$

$$\rightarrow a_i = \sum_\alpha \frac{V_{\alpha 0} V_{\alpha i}}{\omega_\alpha - \omega} \pi \left\{ \begin{aligned} & - \frac{e^{-i\omega_i t} - e^{-2i\omega t}}{\omega_i - 2\omega} - \frac{e^{-i\omega_i t} - 1}{\omega_i} \\ & + \frac{e^{-i\omega_i t} - e^{-i(\omega_\alpha + \omega)t}}{\omega_i - \omega - \omega_\alpha} + \frac{e^{-i\omega_i t} - e^{-i(\omega_\alpha - \omega)t}}{\omega_i + \omega - \omega_\alpha} \end{aligned} \right\}$$

counter rotating

↑ spurious term related to the sudden approximation

$$n_i = |a_i|^2 = \left\{ \sum_\alpha \frac{V_{\alpha 0} V_{\alpha i}}{\omega_\alpha - \omega} \times \frac{2(1 - \cos((\omega_i - 2\omega)t})}{(\omega_i - 2\omega)^2} \right\}^2$$

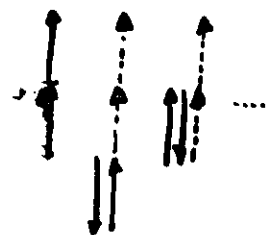
$\lim_{t \rightarrow \infty} \frac{n_i}{t}$? $\begin{cases} 0 & \text{if } \omega_i \neq 2\omega \\ \infty & \text{if } \omega_i = 2\omega \end{cases}$ $\int_{-\infty}^{+\infty} \frac{2 - 2\cos \omega t}{\omega^2 t} d\omega = 2\pi \delta(\omega)$

Finally $\lim_{t \rightarrow \infty} \frac{n_i}{t} = \left[\sum_\alpha \frac{V_{\alpha 0} V_{\alpha i}}{\omega_\alpha - \omega} \right]^2 2\pi \delta(\omega_i - 2\omega)$

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Perturbation theory at lowest non vanishing order.
The 'spurious terms'

Counter rotating terms: they contribute at orders higher than the minimum one.



Terms related to the sudden approximation:



- We have assumed

$$\xi(t) = 0 \text{ for } t < 0$$

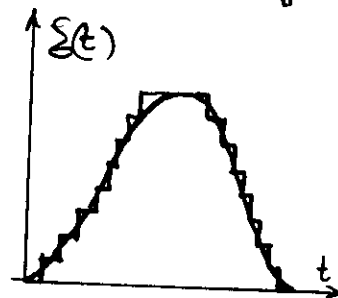
$$\xi(t) \text{ constant } \neq 0 \text{ for } t > 0$$

This is the sudden approximation

- let $\xi(t) = (1 - e^{-\lambda t}) \xi$ for $t > 0$

$\lambda \rightarrow 0$ is the adiabatic approximation

Those terms disappear in the adiabatic approximation.



- Use of perturbative result: apply the result for each step.

- Using the adiabatic approximation is a way to preserve the continuity of the phase from a step to the next one.

Time independent perturbative treatment.

$$|\psi(t)\rangle = a_0|0\rangle + \sum_i a_i|i\rangle + \sum_\alpha a_\alpha|\alpha\rangle$$

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi\rangle$$

$$i\dot{a}_0 = \sum_\alpha V_{0\alpha} a_\alpha (e^{i\omega t} + e^{-i\omega t})$$

$$i\dot{a}_\alpha = \omega_\alpha a_\alpha + V_{0\alpha} a_0 (e^{i\omega t} + e^{-i\omega t}) + \sum_i V_{i\alpha} a_i (e^{i\omega t} + e^{-i\omega t})$$

$$i\dot{a}_i = \omega_i a_i + V_{i0} a_0 (e^{i\omega t} + e^{-i\omega t})$$

$$\text{Let } a_0 = b_0^n e^{in\omega t} \quad a_i = b_i^n e^{in\omega t} \quad a_\alpha = b_\alpha^n e^{in\omega t}$$

$$\sum_n e^{in\omega t} (-ib_0^n + n\omega b_0^n + \sum_\alpha V_{0\alpha} (b_\alpha^{n-1} + b_\alpha^{n+1})) = 0$$

$$\sum_n e^{in\omega t} (-ib_i^n + (\omega_i + n\omega)b_i^n + V_{0i} (b_0^{n-1} + b_0^{n+1}) + \sum_\alpha V_{i\alpha} (b_\alpha^{n-1} + b_\alpha^{n+1})) = 0$$

$$\sum_n e^{in\omega t} (-ib_i^n + (\omega_i + n\omega)b_i^n + \sum_\alpha V_{i\alpha} (b_\alpha^{n-1} + b_\alpha^{n+1})) = 0$$

each term of $\sum_n = 0$: Floquet representation

Formally $\omega_\alpha + n\omega$ is the energy of the dressed state $|\alpha\rangle|n\rangle$

For strong intensity Floquet representation equivalent to dressed atom picture.

Perturbation theory at lowest nonvanishing order

$$P = \sum_k \left[\sum_{\alpha, \beta} \frac{V_{0\alpha} V_{\alpha i} V_{i\beta} V_{\beta k}}{(\omega - \omega_\alpha)(2\omega - \omega_i)(3\omega - \omega_\beta)} \right]^2$$

↑
summation over all possible final state -

↑
summation over all intermediate states

$$\omega_k = 4\omega$$

$$V \propto \mathcal{E} \quad P \propto \mathcal{E}^{2n} \quad P = \sigma^{(n)} I^n$$

↑
generalized cross section

↑
light intensity

↑
number of photons absorbed

$$I = I_M f(t) g(r) \quad \text{Max}(f(t)) = 1 \quad \text{Max}(g(r)) = 1$$

$$\frac{d}{dt}|a_0|^2 = -\sigma^{(n)} I_M^n f^n g^n |a_0|^2$$

$$|a_0|^2 = \exp\left[-\sigma^{(n)} I_M^n g^n \int dt f^n(t)\right] = \exp\left(-\frac{\sigma I_M^n}{I_s}\right)$$

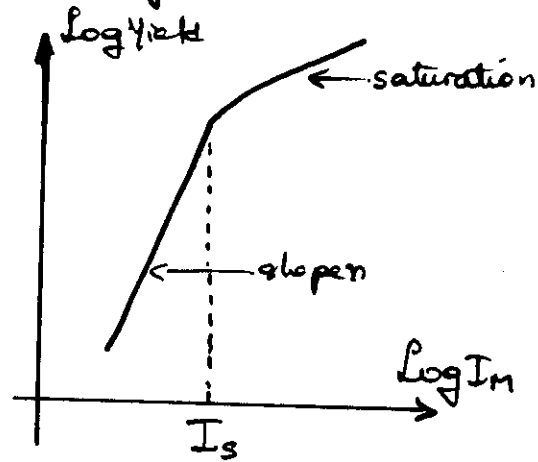
$$\text{Saturation intensity } \sigma^{(n)} I_s^n \int_0^\infty dt f^n(t) = 1$$

$$\text{Total number of ions} = \int d^3r \left[1 - \exp\left(-\frac{\sigma I_M^n}{I_s}\right)\right] \times e$$

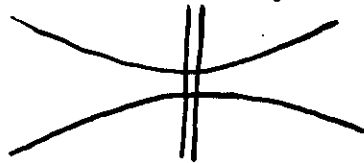
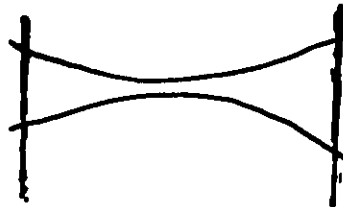
$$\text{Weak field} \rightarrow \left(\frac{I_M}{I_s}\right)^n \int d^3r e g^n$$

$$\text{Strong field} \rightarrow e \times \text{volume where } I_M \gg I_s$$

Ion yield



Saturation regime

 $N_{ion} \propto \log I$ 

spherical lens $N_{ion} \propto I^{3/2}$
 cylindrical lens $N_{ion} \propto I^2$

Measurement of ionisation cross sections

Light intensity $I_M g(r) f(t)$ $\text{Max}(g) = \text{Max}(f)$

$$\text{Number of ions } N = \rho \int d^3\vec{r} \left[1 - \exp(-\sigma^{(n)} I_M^n g^n) \right] dt f^n(t)$$

$$\text{Weak field } N = \rho \sigma^{(n)} I_M^n \int d^3\vec{r} g^n(r) \int f^n(t) dt$$

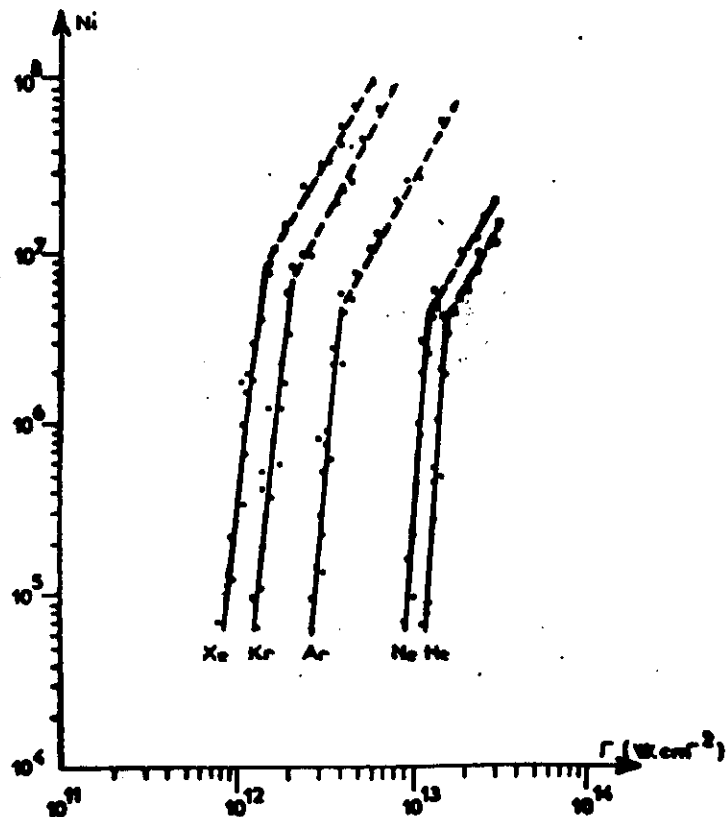
Measure ρ, I_M, g, f Strong field Measure $f(t) \rightarrow I_n = \int dt f^n(t)$

$$\text{Define } I_s \quad \sigma^{(n)} I_s^n I_n = 1$$

$$\rightarrow N = \rho \int d^3\vec{r} \left[1 - \exp\left(-\left(\frac{I_M}{I_s}\right)^n g^n\right) \right]$$

Measure $g(r)$ Calculate N

Fit $\log N$ vs $\log I$ with exp. points $\rightarrow I_s$
 avoids measurement of ρ
 absolute measurement of N .



Variation of number of multiphoton ions created as function of intensity I in focusing region; $\lambda = 1.06 \mu$; dots represent experimental points.

P. Agostini G. Barjot G. Mainfray
C. Manus J. Thebaudt.
IEEE J. Q.E. 6 782 (70)

Coherence effects

space and time - dependent intensity

$$I = I_M f(t) g(r) \quad \text{Max } f(t) = \text{Max } g(r) = 1$$

Born yield in perturbative regime

$$N = \rho \sigma^{(n)} \int d^3r g^n(r) \int dt f^n(t) I_M^n$$

$\uparrow$ atomic cross section $\uparrow$ depend on focusing $\uparrow$ depend on coherence properties

Energy per pulse $I_M \int g(r) d^3r \int f(t) dt$

In a quantum description of the field

$$\int dt f^{(n)}(t) I_M^n = \langle N^n \rangle \quad N \text{ number of photons}$$

Field state described by a density matrix $\sum_{M,N} P_{MN} |M\rangle\langle N|$

$$\langle N^n \rangle = \sum_N P_{NN} N^n$$

Two opposite cases:

Coherent (Glauber) state $P_{NN}^G = e^{-\langle N \rangle} \frac{\langle N \rangle^N}{N!}$

Chaotic (Poisson) state $P_{NN}^P = \frac{\langle N \rangle^N}{(1 + \langle N \rangle)^{N+1}}$

Coherent state $\langle N^n \rangle = \langle N \rangle^n$

Chaotic state $\langle N^n \rangle = n! \langle N \rangle^n$

Mono mode laser $\rightarrow$ coherent state

$\left\{ \begin{array}{l} \text{Multi mode laser} \rightarrow \text{chaotic state} \end{array} \right.$

Ionisation of Xenon Nd:YAG laser

17

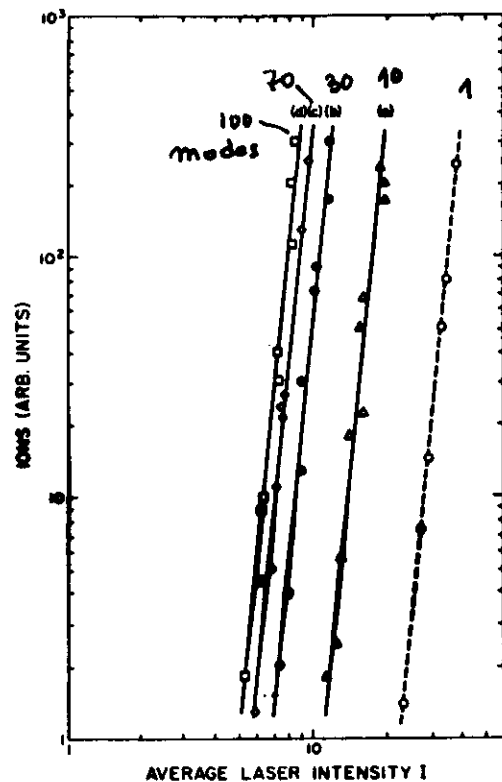


FIG. 3. Log-log plot of the variation of the number of ions in 11-photon ionization of Xe with a Nd-glass laser as a function of the average laser intensity I (arbitrary units) when the laser operates in (a) one spectral line (10 modes), (b) three lines (30 modes), (c) seven lines (70 modes), and (d) ten lines (100 modes). Dashed curve corresponds to a single-mode laser (Lecompte *et al.*, 1975).

Lecompte, Mainfray, Marcus Sanchez
PRA 11 1009 (1975)

Calculation of multiphoton ionization cross sections
Weak field

$$P_0 = \sum_{\substack{E \\ \text{final} \\ \text{states}}} M_E^2 \quad M_E = \sum_{\substack{j \\ \text{intermediate} \\ \text{states}}} \frac{V_{0\alpha} V_{\alpha j} V_{jE}}{(E_0 + \hbar\omega - E_\alpha)(E_0 + 2\hbar\omega - E_j)}$$

$$E = E_0 + 3\hbar\omega$$

Approximate methods.

- Truncated summations.
- Bebb and Gold method.
- Let us define an average energy $\langle E \rangle$ for excited states
- Replace E_α 's and E_j 's by $\langle E \rangle$
- Then $M_E = \frac{\langle 0 | V^3 | E \rangle}{(E_0 + \hbar\omega - \langle E \rangle)(E_0 + 2\hbar\omega - \langle E \rangle)}$
- The latter method for highly excited states, while low lying are introduced explicitly.

Accurate methods

Green functions

$$M_E = \langle 0 | V \sum_{\alpha} \frac{| \alpha \rangle \langle \alpha |}{E_\alpha - E_0 - \hbar\omega} V \sum_j \frac{| j \rangle \langle j |}{E_j - E_0 - 2\hbar\omega} V | E \rangle$$

$$M_E = \langle 0 | V G(E_0 + \hbar\omega) V G(E_0 + 2\hbar\omega) V | E \rangle$$

Dalgarno method

$$M_E = \langle E | V \sum_j \frac{| j \rangle \langle j |}{E_j - E_0 - 2\hbar\omega} V \sum_{\alpha} \frac{| \alpha \rangle \langle \alpha |}{E_\alpha - E_0 - \hbar\omega} V | 0 \rangle$$

$\uparrow$ $| \rho_2 \rangle$ $\uparrow$ $| \rho_1 \rangle$

$$(H - E_0 - \hbar\omega) | \rho_1 \rangle = V | 0 \rangle$$

$$(H - E_0 - 2\hbar\omega) | \rho_2 \rangle = V | \rho_1 \rangle$$

Solving step by step a hierarchy of inhomogeneous differential equations

Calculation of multiphoton ionization cross sections
any intensity

Expansion on finite basis

Slater type - Sturmian -

Tractable and accurate in the "single electron atom" approximation

alkalis and hydrogen.

noble gases and alkaline earths (M & DT).

Time dependent calculations.

Time dependent Hartree Fock.

Close coupling calculations.

splits the space in 3-ranges

$0 < r < r_1$ core effects dominate

$r_1 < r < r_2$ electron-light interaction dominant

$r_2 < r$ Coulomb interaction dominant

Solving in each range - matching on the border -

Keldysh model.

"transition probability" between
atomic ground state and Volkov state

Multiphoton ionization of hydrogen ground state
Perturbation at lowest non vanishing order

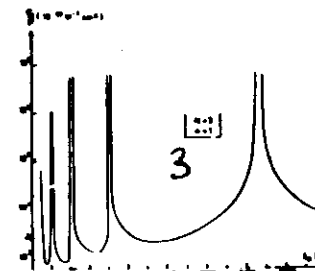


FIG. 3. Dispersion rate σ_0/I^2 for the three-photon ionization of H in ground state versus the wavelength λ_0 of the incident light.

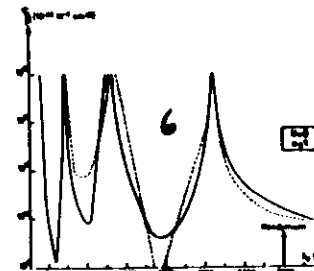


FIG. 5. Dispersion rate σ_0/I^2 for six-photon ionization of H in ground state. The value $\lambda_0 = 1300 \text{ \AA}$ corresponds to the second-harmonic of the continuous laser light. Presently calculated - solid lines; results obtained by Babb and Gold (Ref. 12) - dashed lines.

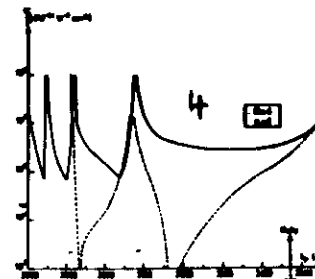


FIG. 3. Dispersion rate σ_0/I^2 for four-photon ionization of H in ground state. The value $\lambda_0 = 3471 \text{ \AA}$ corresponds to the second harmonic of the ruby laser light. Presently calculated - solid lines; results obtained by Babb and Gold (Ref. 12) - dashed lines.

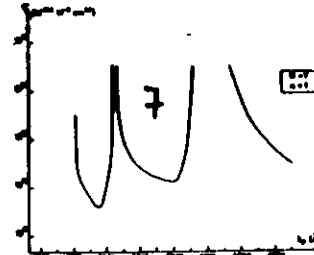


FIG. 6. Dispersion rate σ_0/I^2 for seven-photon ionization of H in ground state versus the wavelength λ_0 of the incident light.

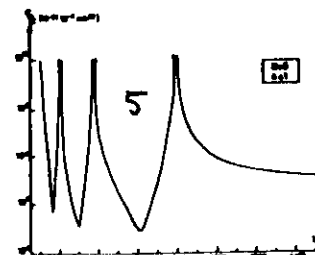


FIG. 4. Dispersion rate σ_0/I^2 for five-photon ionization of H in ground state versus the wavelength λ_0 of the incident light.

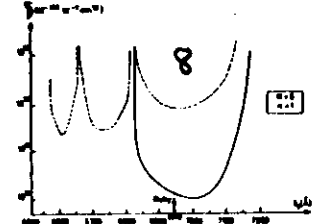
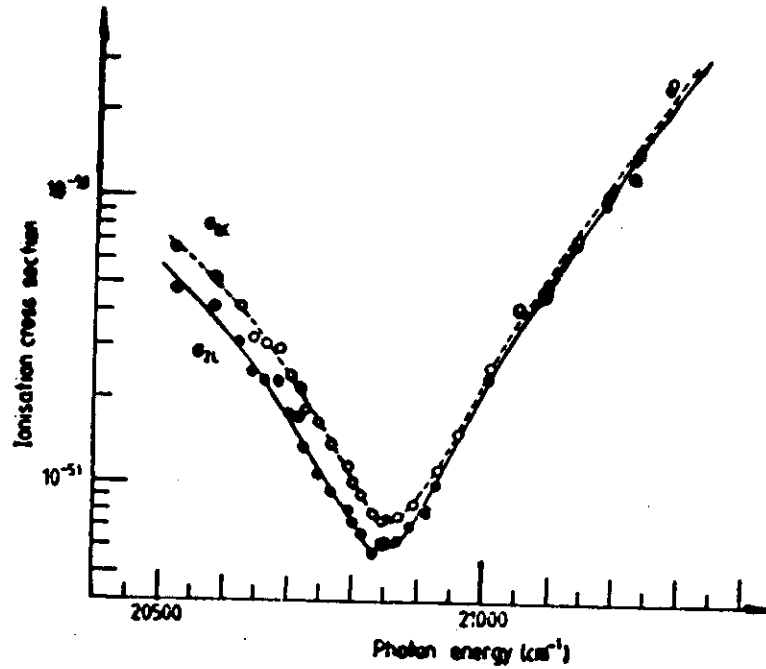


FIG. 7. Dispersion rate σ_0/I^2 for eight-photon ionization of H in ground state. The value $\lambda_0 = 8443 \text{ \AA}$ corresponds to the wavelength of the ruby laser light. Presently calculated - solid lines; results obtained by Babb and Gold (Ref. 12) - dashed lines.

Gontier Trahin (1968)



Two-photon ionisation cross sections of Cs atoms

D. Normand J. Morellec
J Phys B 14 L401 (81)

2-photon ionisation of Cs ground state

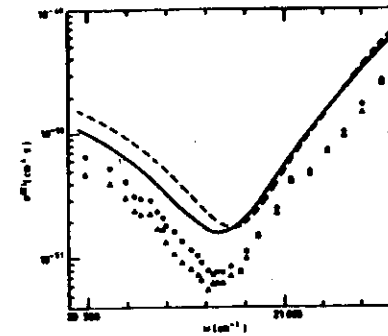


Figure 2. Two-photon ionisation cross sections of the Cs ground state as a function of the photon energy ω for linearly polarized light: —, this work; Δ , experimental data (Normand and Morellec 1981). For circularly polarized light: - - -, this work; $\square$, experimental data (Normand and Morellec 1981).

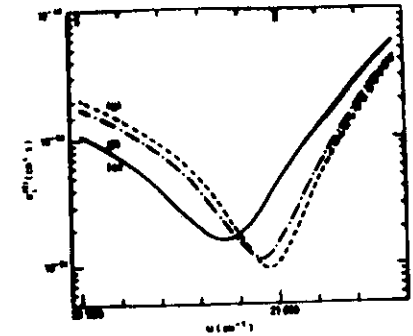


Figure 3. Two-photon ionisation cross sections of the Cs ground state by linearly polarized light as a function of the photon energy ω calculated with three levels of approximation: - - -, (1), truncated summation limited to $n=4$ levels; - - -, (2), truncated summation including all the bound intermediate levels; —, (3), infinite summation.

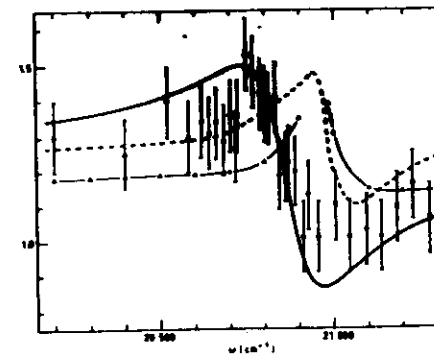


Figure 4. The ratio $R = \sigma^{(2)}_c / \sigma^{(2)}_l$ of the two-photon ionisation cross sections of the Cs ground state for circularly and linearly polarized light as a function of the photon energy ω : $\square$, experimental data (Normand and Morellec 1981). Theoretical predictions: —, this work; - - -, Geltman (1980); - · - ·, A. A. Tsevan et al (1976).

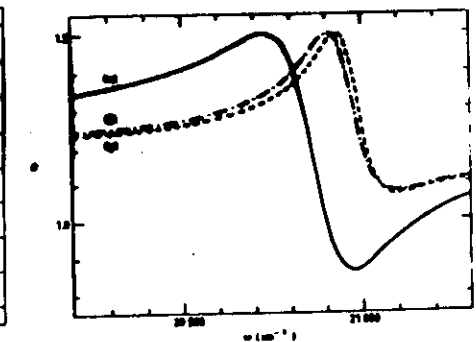


Figure 5. The ratio $R = \sigma^{(2)}_c / \sigma^{(2)}_l$ of the two-photon ionisation cross sections of the Cs ground state for circularly and linearly polarized light as a function of the photon energy ω , calculated with three levels of approximation: - - -, (1); - · - ·, (2) and —, (3).

Normand-Morellec (1981)
Geltman 1980
Symar Crance 1982

Resonant multiphoton ionisation

$$P = 2\pi \sum_k \left| \sum_{\beta} \frac{V_{0\alpha} V_{\alpha i} V_{i\beta} V_{\beta k}}{(\omega - \omega_{\alpha})(2\omega - \omega_i)(3\omega - \omega_{\beta})} \right|^2$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 energy denominators may be small

Basic assumption of the previous treatment

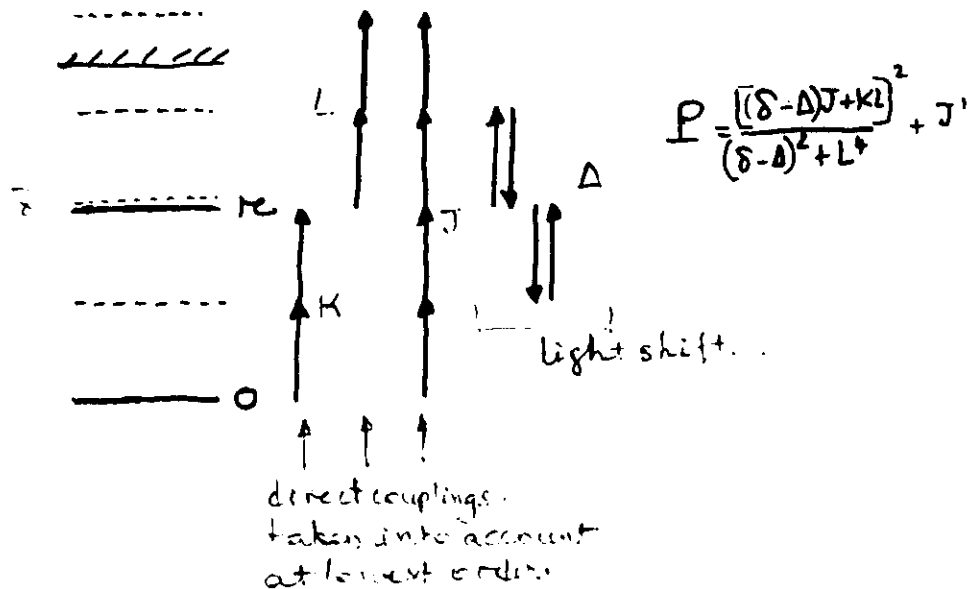
$$|\langle \psi | 0 \rangle| \gg |\langle \psi | \alpha \rangle|, |\langle \psi | i \rangle|$$

A resonance with $|i\rangle$ means $\omega_i = \omega \times \text{integer}$
 $|\langle i | \psi \rangle|$ large.

→ look for the evolution of

$$(|0\rangle\langle 0| + |i\rangle\langle i|)|\psi\rangle = P|\psi\rangle$$

→ effective operator

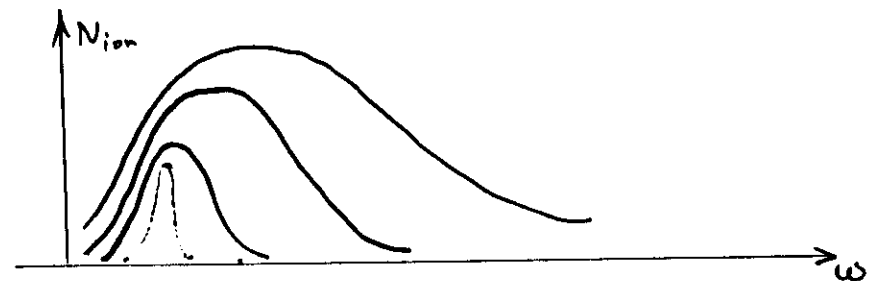


Resonant multiphoton ionisation

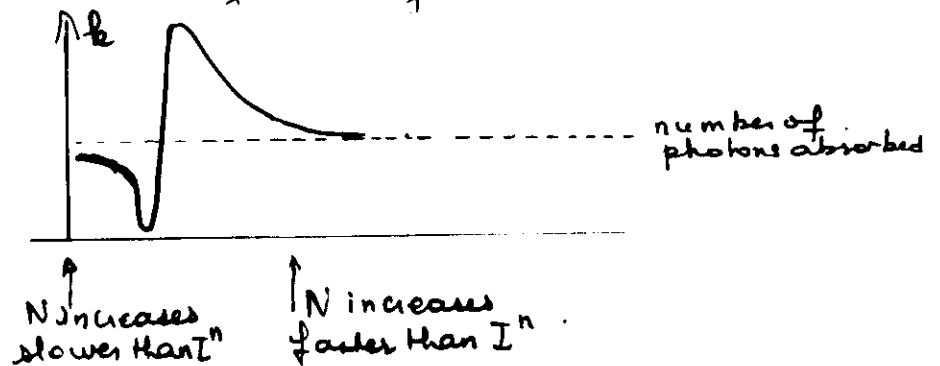
$$P = J_0 + \frac{[(\delta - \Delta)J + KL]^2}{(\delta - \Delta)^2 + L^2}$$

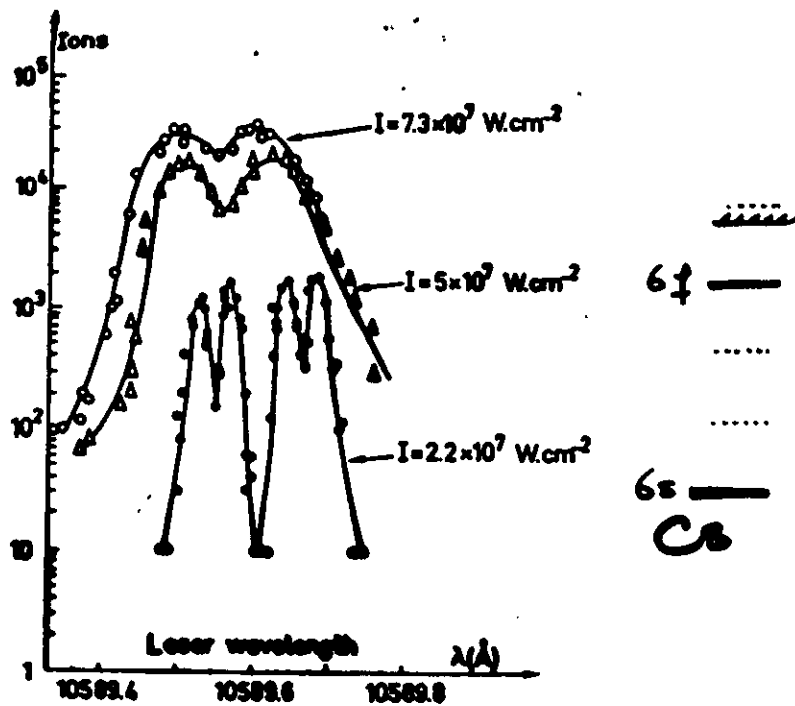
$\uparrow \quad \uparrow$
 $\propto I \quad \propto I^{2P}$

When I increases, resonances shift and broaden.



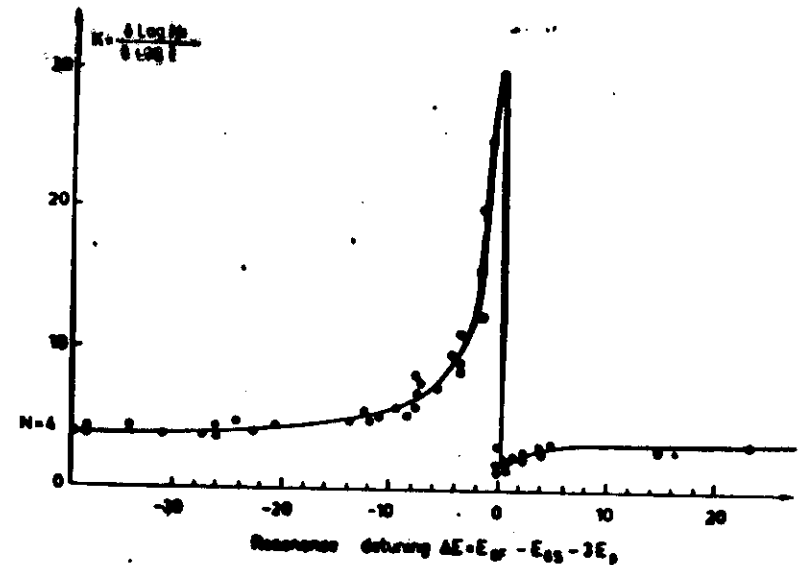
This is reflected by the effective order of non-linearity $\frac{d \log N_{\text{ion}}}{d \log I}$ = slope of the curve $\log N_{\text{ion}}$ vs $\log I$





Resonance profiles in the four-photon ionization of Cs for three values of laser intensity I . Hyperfine structure of the ground state and the fine structure of the $6F$ resonant state are well resolved for the lowest intensity value.

G. Petite J. Morelle & D. Normand
J. Phys. 40 115 (79).



Variation of the effective order of nonlinearity K as a function of the resonance detuning ΔE in the four-photon ionization of Cs.

J. Morelle D. Normand
G. Petite Phys. Rev. A 14 300 (76)

ordre effectif de non linéarité

4 photon ionisation of Cesium.

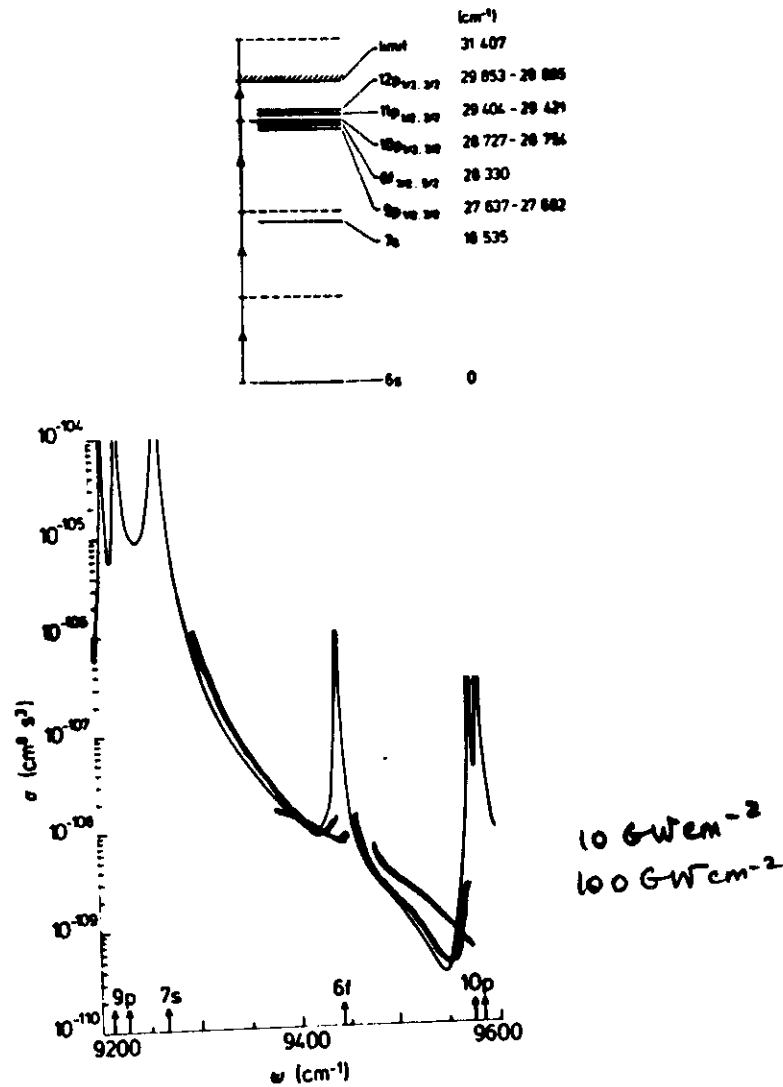
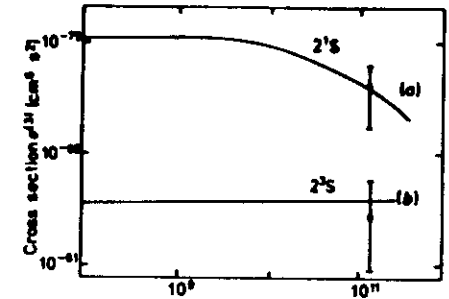
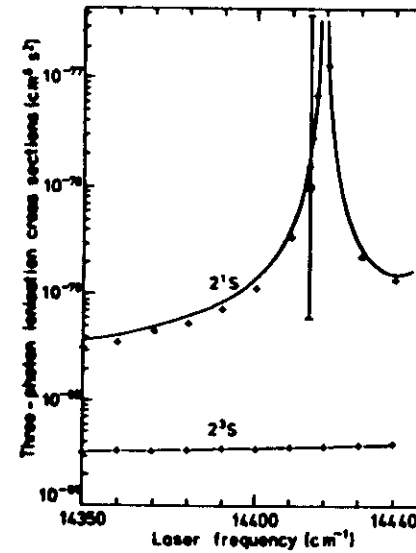


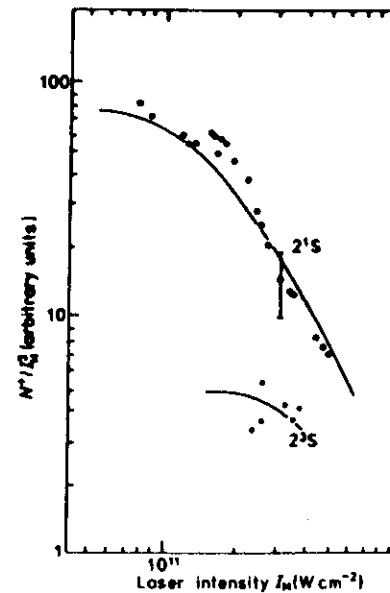
Figure 2. Four-photon ionisation cross sections for various light intensities (up to 0.1 GW cm^{-2} , —; 10 GW cm^{-2} , ---; 100 GW cm^{-2} , ···).

Crane Aymer JPhys B13 4129 (1980).



$$\omega = 14398.5 \text{ cm}^{-1}$$

3-photon ionisation of metastable Helium off resonant by 40.5 cm^{-1}



Lompre Mainfray Mathieu
Walek Aymer Crane
JPhys B13 1799 (1980)

