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FOR THE FOLLOWING:

FOURIER OPTICS AND HOLOGRAPHY

(ADRIATICO RESEARCH CONFERENCE)

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ADRIATICO CONFERENCE ON FOURIER OPTICS AND HOLOGRAPHY

OPTICAL CORRELATION

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OPTICAL CORRELATION, TRIPLE CORRELATION

AN INTRODUCTION

A. W. LOHMANN, UNIV. OF ERLANGEN

TRIESTE, MARCH 1990

PLAN

DEFINITIONS, BASIC RELATIONS

MATH OF TRIPLE CORRELATION

MOTIVATIONS

WHY? AUTO-CORRELATIONS
CROSS-CORRELATIONS
TRIPLE-CORRELATIONS

AUTOCORRELATION FOR
SHIFT-INVARIANT CHARACTER
RECOGNITION, OR SIGNAL DETECTION

COHERENT MATCHED FILTERING

THE TRIPLE CORRELATION
IN HIGH-RESOLUTION ASTRONOMY

DEFINITIONS, BASIC RELATIONS

AUTO-CORRELATION OF $u(x)$:

$$\int u(x)\,u^*(x'-x)\,dx' \equiv u_2(x)$$

IF $\int u(x)\, e^{-2\pi i \nu x}\,dx = \tilde{u}(\nu)$

THEN $\int u_2(x)\, e^{-2\pi i \nu x}\,dx = \tilde{u}_2(\nu) = |\tilde{u}(\nu)|^2$

POWER SPECTRUM

CROSS-CORRELATION OF $u(x)$ AND $v(x)$:

$$\int u(x')\,v^*(x'-x)\,dx' = C_{uv}(x)$$

$$\int C(x)\, e^{-2\pi i \nu x}\,dx = \tilde{u}(\nu)\,\tilde{v}^*(\nu) = \tilde{C}(\nu)$$

AN ALGORITHM FROM u & v TO C

$$\begin{array}{ccccccc} u(x) & & & & & & v(x) \\ \downarrow & & & & & & \downarrow \\ \tilde{u}(\nu) & \longrightarrow & \otimes & \longleftarrow & \tilde{v}^*(\nu) & \longleftarrow & \tilde{v}(x) \\ & & \downarrow & & & & \end{array}$$

$$\tilde{u}(\nu)\,\tilde{v}^*(\nu) \longrightarrow \int \tilde{u}\,\tilde{v}^*\, e^{2\pi i \nu x}\,d\nu = C(x)$$

THE (AUTO-) TRIPLE CORRELATION

(DETAILS: PROC. IEEE 72 (1984) 889 - 901)

DEF.: $\int u(x')u(x'+x)u(x'+y)\,dx' \equiv u_3(x,y)$

" BI-SPECTRUM "

$$\tilde{u}_3(\nu,\mu) \equiv \iint u_3(x,y)\, e^{-2\pi i(x\cdot\nu + y\mu)}\,dx\,dy$$

INSERT: $\tilde{u}_3(\nu,\mu) = \tilde{u}(\nu)\,\tilde{u}(\mu)\,\tilde{u}(-\nu-\mu)$

BASIC CONVOLUTION THEOREM:

IF $u(x) = f(x) * g(x) = \int f(x')\,g(x-x')\,dx'$

THEN $\tilde{u}(\nu) = \tilde{f}(\nu)\cdot\tilde{g}(\nu)$

CONVOLUTION PROPERTY OF TRIPLE CORREL.

IF $u = f * g$ THEN $u_3 = f_3 * g_3$

$$u_3(x,y) = \iint f_3(x',y')\,g_3(x-x',y-y')\,dx'dy'$$

$$\tilde{u}_3(\nu,\mu) = \underbrace{\tilde{f}_3(\nu,\mu)}_{\tilde{f}(\nu)\tilde{f}(\mu)\tilde{f}(-\nu-\mu)}\;\underbrace{\tilde{g}_3(\nu,\mu)}_{\tilde{g}(\nu)\tilde{g}(\mu)\tilde{g}(-\nu-\mu)}$$

$$= \underbrace{\tilde{f}(\nu)\tilde{g}(\nu)}_{\tilde{u}(\nu)}\;\underbrace{\tilde{f}(\mu)\tilde{g}(\mu)}\;\underbrace{\tilde{f}(-\nu-\mu)\tilde{g}(-\nu-\mu)}$$

MOTIVATIONS

WHY AUTO-CORRELATION $u_2(x) = \int u(x')u^*(x'-x)\,dx'$

INSTEAD OF $u(x)$ ITSELF ?

SOMETIMES $u(x)$ TOO NOISY, OR OVERLOADED WITH IRRELEVANT DETAILS

FOR EXAMPLE: u_2 IS SHIFT-INVARIANT

$$\int u(x')u^*(x'-x)\,dx' = \int u(x'-x_0)\,u^*(x'-x_0-x)\,dx'$$

WHY CROSS-CORRELATION ? "SIMILARITY" OF u AND v

DEVIATION:

$$\int |u(x') - v(x'-x)|^2\,dx' = \qquad (\geq 0)$$

$$= \underbrace{\int |u(x')|^2 dx'}_{(=1)} + \underbrace{\int |v(x'-x)|^2 dx'}_{(=1)} - 2\,\mathrm{Re}\left[\int u(x')v^*(x'-x)\,dx'\right] \qquad |[\;]| \leq 1$$

$(\leq) \rightarrow (=)$ " IDENTITY "

WHY AUTO-TRIPLE CORREL. u_3 INSTEAD OF u_3 ?

u_3 IS "SMARTER" THAN u_2 ; MORE NOISE IMMUNE.

$u_3(x,y) \longrightarrow u(x)$ UNIQUE, EXCEPT FOR LOCATION

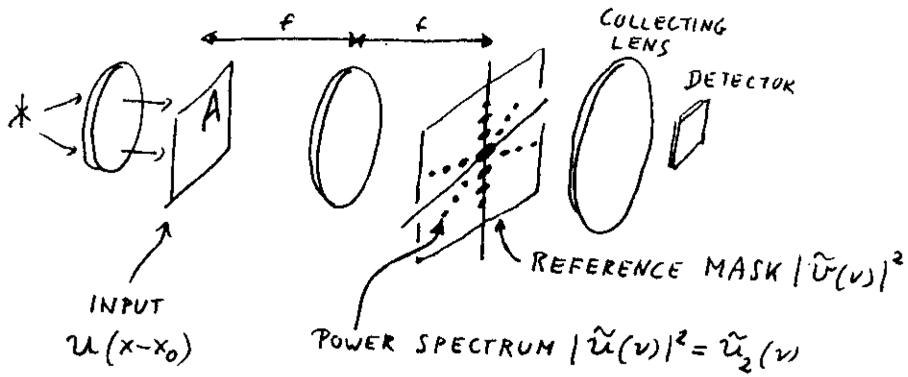
$u_2(x) \longrightarrow u(x)$ GENERALLY AMBIGUOUS

$$\tilde{u}_2(\nu) = |\tilde{u}(\nu)|^2$$

APPLICATION OF THE AUTOCORRELATION
(OR THE POWER SPECTRUM)

FOR SHIFT-INVARIANT CHARACTER RECOGNITION

APPL. OPT. 4 (1965) 461



MEASUREMENT:

$$\int |\hat{u}(\nu)|^2 \cdot |\hat{v}(\nu)|^2 d\nu$$

$$C = \int \tilde{u}_2(\nu) \cdot \tilde{v}_2^{(*)}(\nu) d\nu = \int u_2(x)\, v_2^*(x)\, dx$$

SIMILARITY OF

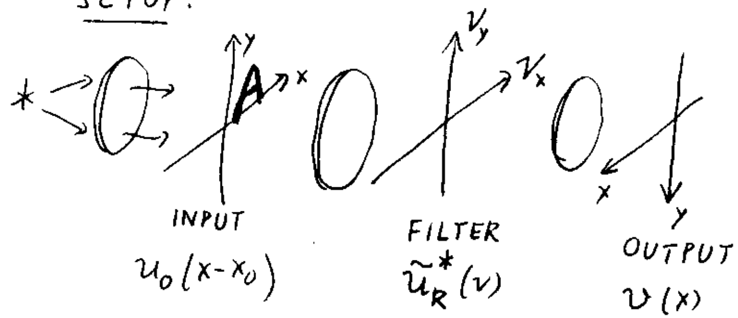
POWER SPECTRA OR AUTO CORRELATIONS

COHERENT MATCHED FILTERING

A. VANDER LUGT

(see J.W. Goodman: Intro. to Fourier Optics)

SETUP:



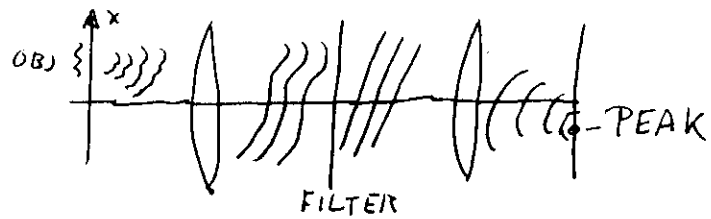
$$v(x) = \int \tilde{u}_0(\nu)\, \tilde{u}_R^*(\nu)\, e^{2\pi i \nu (x-x_0)} dx$$

"matched" if $\tilde{u}_R^* = \tilde{u}_0^*(\nu) / \underbrace{\langle |\tilde{n}(\nu)|^2 \rangle}_{\text{NOISE POWER SPECTRUM}}$

PLAUSIBILITY

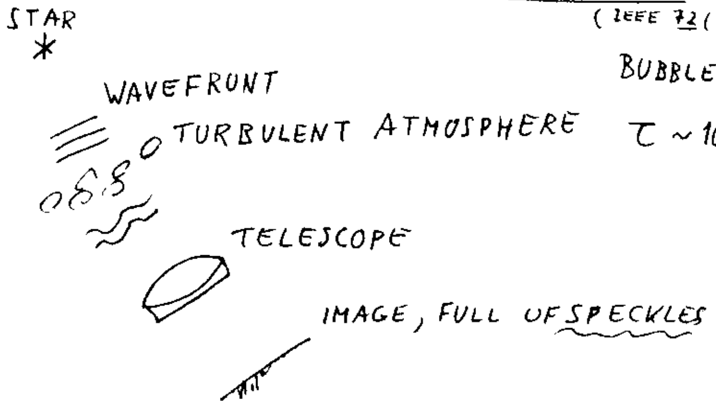
AMPLITUDE OF FILTER $|\tilde{u}_R^*|$ IS HIGH AT THOSE ν WHERE THE SIGNAL-TO-NOISE IS HIGH.

PHASE $(\tilde{u}_R^*)$ = - PHASE $(\tilde{u}_0)$



TRIPLE CORRELATION PROCESSING IN HIGH RESOLUTION ASTRONOMY (G. WEIGELT..)

(IEEE 72 (1984) 889)



BUBBLE ~ 10 cm

$\tau \sim 10 - 100$ msec

ONE STAR $*$ $\longrightarrow$

$P(x, t_m) = P_m(x)$

TWO STARS $*_*$

$\underbrace{I_0(x)}_{\text{OBJECT}} * P_m(x) = I_m(x)$

ALGORITHM

$$I_m(x) \rightarrow \tilde{I}_m(\nu) = \tilde{I}_0(\nu)\, \tilde{P}_m(\nu)$$

$$\rightarrow \tilde{I}_0(\nu) \tilde{P}_m(\nu) \tilde{I}_0(\mu) \tilde{P}_m(\nu) \tilde{I}_0(-\nu-\mu) \tilde{P}_m(-\nu-\mu) = \tilde{J}_{3m}(\nu,\mu)$$

$$\rightarrow \sum_{(m)} \tilde{J}_{3m}(\nu,\mu) = \langle \tilde{J}_{3m}(\nu,\mu) \rangle = \tilde{I}_{03}(\nu,\mu) \underbrace{\langle \tilde{P}_{m3}(\nu,\mu) \rangle}_{\text{KNOWN, POSITIVE}}$$

$$\rightarrow \tilde{I}_{03}(\nu,\mu) \rightarrow \underbrace{I_0(x)}_{\text{SHARP IMAGE}}$$