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UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
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H4.SMR/452-60

ADRIATICO CONFERENCE ON FOURIER OPTICS AND HOLOGRAPHY

6 - 9 March 1990

INTRODUCTION TO LASERS

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Lecture

INTRODUCTION TO LASERS

by A. Sona - CISE

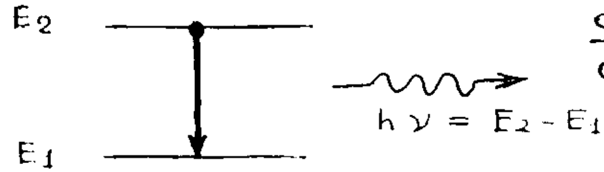
- Introduction to laser operation
  - Rate Equations Description
  - Active materials
  - Pulsed emission
- Coherence properties of the laser
  - Temporal Coherence
  - Single longitudinal Mode selection
  - Spatial coherence
  - Single transverse mode selection
- Applications Requirements
  - Interferometry
  - Holography
  - Optical Computers

Bibliography:

O. Svelto "Principles of Lasers" Plenum Press 1989

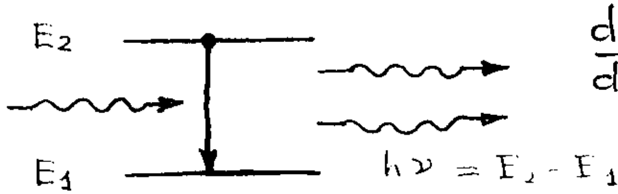
R - M INTERACTION PROCESSES AFFECTING LASER OPERATION

— SPONTANEOUS EMISSION:



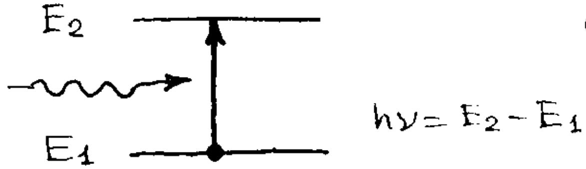
$$\frac{dn_p}{dt} = +\frac{N_2}{\tau}$$

— STIMULATED EMISSION:



$$\frac{dn_p}{dt} = \gamma N_2 n_p$$

— ABSORPTION:



$$\frac{dn_p}{dt} = -\gamma N_1 n_p$$

N_2 - Number of Atoms in the upper level [cm^{-3}]

N_1 - Number of Atoms in the Lower Level [cm^{-3}]

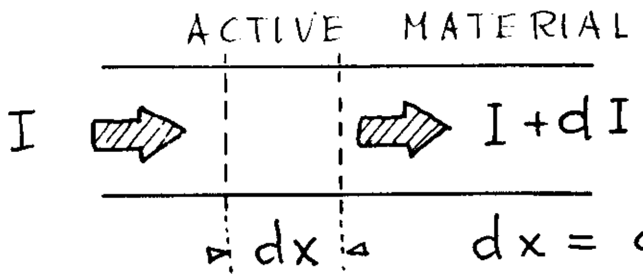
If $N_2 > N_1$ STIMULATED EMISSION IS LARGER THAN ABSORPTION

n_p - Photons density [cm^{-3}]; $\gamma = \sigma c$

$$\frac{dn_p}{dt} = \gamma n_p (N_2 - N_1) + N_2/\tau$$

- **PHOTONS BALANCE EQUATION**

$$\frac{dn_p}{dt} = \sigma c\, n_p (N_2 - N_1) + N_2/\tau$$



$$dx = c\,dt$$

$I = A\, h\nu\, c\, n_p$ A - beam cross section

$$\frac{dI}{dx} = A\, h\nu\, c \frac{dn_p}{dx} = A\, h\nu \frac{dn_p}{dt} =$$

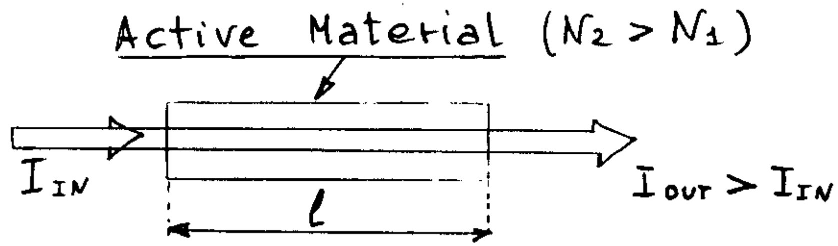
$$= A\, h\nu\, \sigma c\, (N_2 - N_1)\, n_p =$$

$$= I\, \sigma\, (N_2 - N_1) = \alpha_0\, I$$

$$\boxed{\frac{dI}{dx} = \alpha_0 I \Rightarrow I = I_0\, e^{\alpha_0 \ell}}$$

- α_0 [cm^{-1}] $= \sigma\, (N_2 - N_1)$ small signal gain
- σ [cm^2] interaction cross section

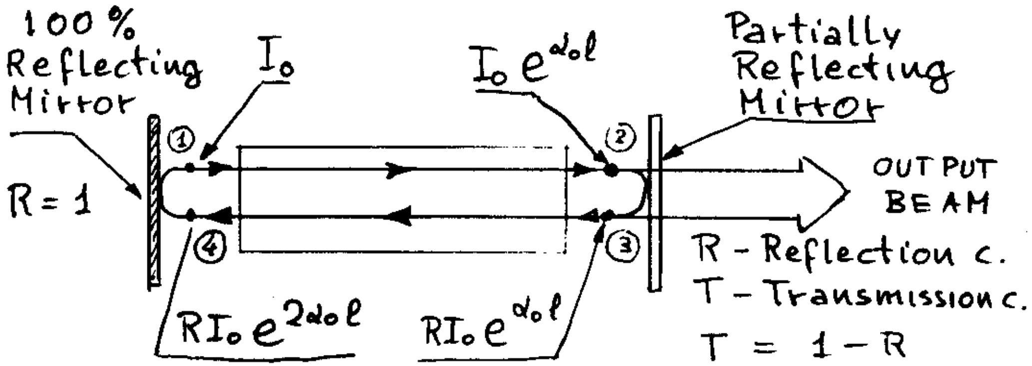
- **LASER AMPLIFIER:**



$I_{OUT} = I_{IN}\, e^{\alpha_0 \ell}$ $\alpha_0 = \sigma\, (N_2 - N_1)$

$I_{OUT} = I_{IN}\, G$ $G = e^{\alpha_0 \ell}$

- **LASER OSCILLATOR**



If $I_0\, e^{2\alpha_0 \ell} \cdot R \geq I_0$ OSCILLATION STARTS

This means $2\alpha_0 \ell \geq \ln 1/R = \ln 1/(1-T)$

and, for $T \ll 1 \Rightarrow 2\alpha_0 \ell \geq T$ or $G^2 \geq T$

The amplification in the active medium compensate for the output coupling losses in a round trip.

● THRESHOLD CONDITION (AMPLITUDE)

$$2\sigma(N_2 - N_1)\,\ell \geq T \quad \text{hence:}$$

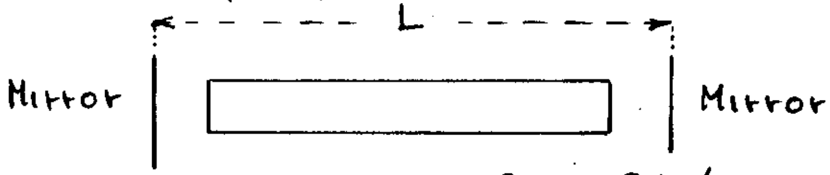
$$N_2 - N_1 \geq T/2\ell\sigma$$

(Threshold inversion)

"Pumping" is required to reach the above condition as, in equilibrium, $N_2 < N_1$ according to Boltzman eq. $N_2/N_1 = \exp-\left(\frac{E_2-E_1}{KT}\right)$

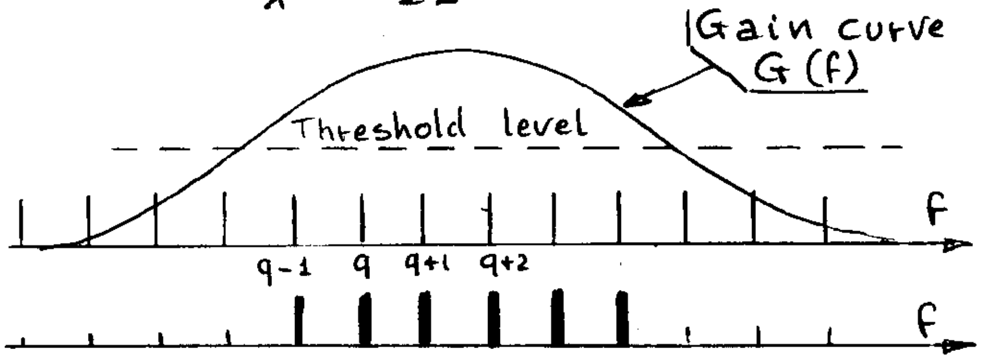
● POS. FEEDBACK CONDITION (PHASE)

The following phase condition has to be fullfilled for oscillation



$$L = q\,\lambda/2 \longrightarrow \lambda = 2L/q$$

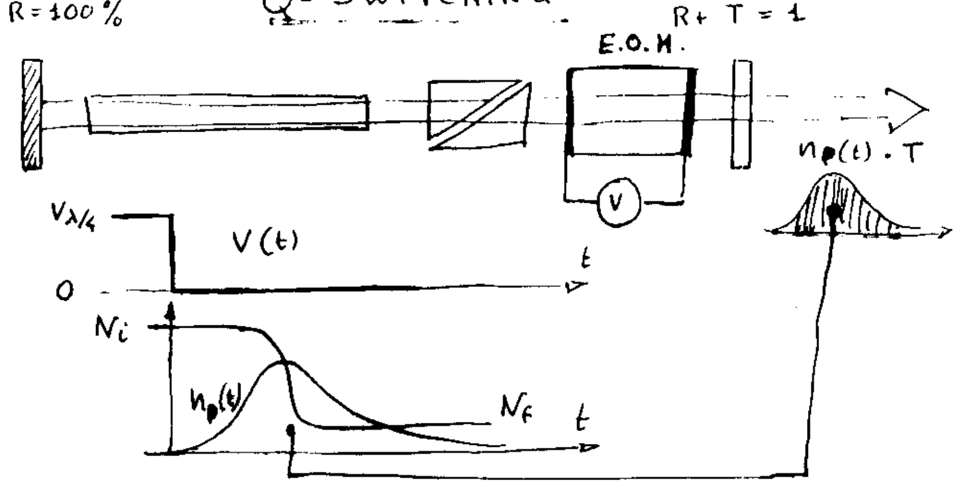
$$f = \frac{c}{\lambda} = \frac{c}{2L}\cdot q\,; \quad \Delta f_{q+1,q} = \frac{c}{2L}$$



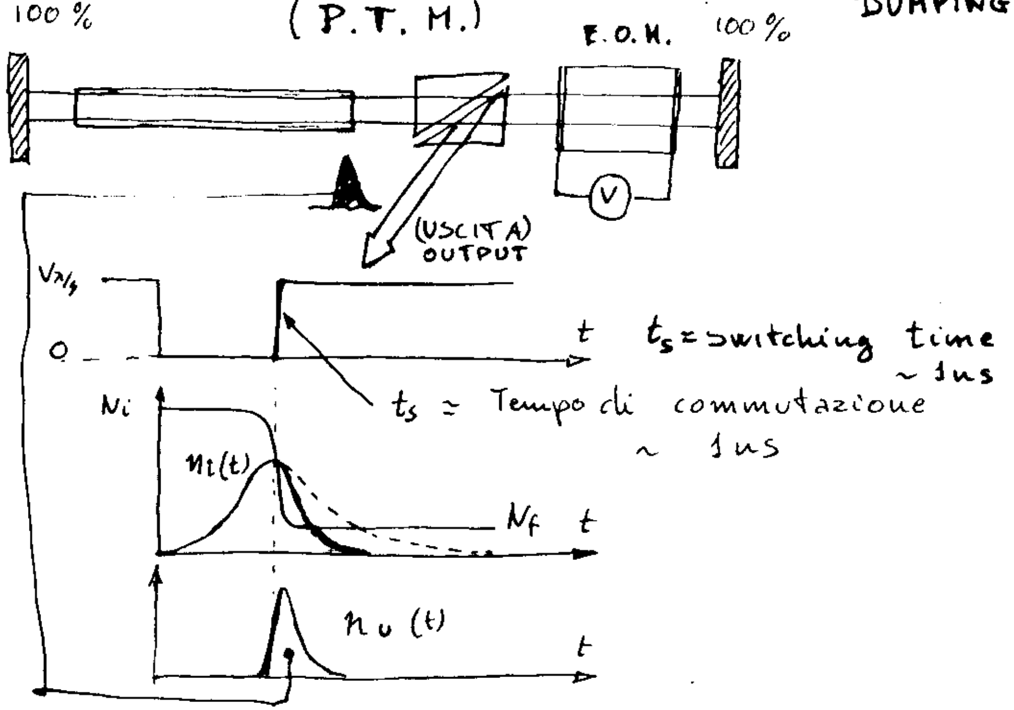
EMITTED FREQUENCIES

(Ex: for L = 0.5m Δf = 300 MHz)

Q- SWITCHING



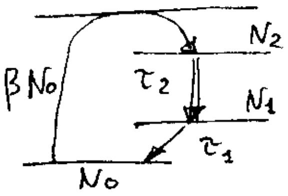
PULSE TRASMISSION MODE (CAVITY DUMPING)
(P.T.M.)



Q-switching - 4 Levels Laser

Putting: $N = N_2 - N_1$ (for $\tau_1 \ll \tau_2$ $N_1 = 0$)

- $\tau = \tau_2$ decay time for atoms
- $T = \frac{2L}{c} \frac{1}{1-R}$ decay time of photons
- βN_0 pump term



- Rate equations:

$$\begin{cases} \dfrac{dN}{dt} = \beta N_0 - \gamma n_p N - N/\tau \\ \dfrac{dn_p}{dt} = -\dfrac{n_p}{T} + \gamma n_p N \end{cases}$$

Not relevant during laser emission; important as a source term

- Before switching $\dot{N} = 0$ $n_p = 0$ $N_i = \beta N_0 \tau$

During the transient N/τ & βN_0 can be neglected

$$\begin{cases} \dfrac{dN}{dt} = -\gamma n_p N \\ \dfrac{dn_p}{dt} = -\dfrac{n_p}{T} + \gamma n_p N \end{cases}$$

for $\dot{n}_p = 0$ at $N = N_s = \frac{1}{\gamma T}$ corresponding to $\eta = 1$ n_p reaches its maximum

- The pulse energy is given by:

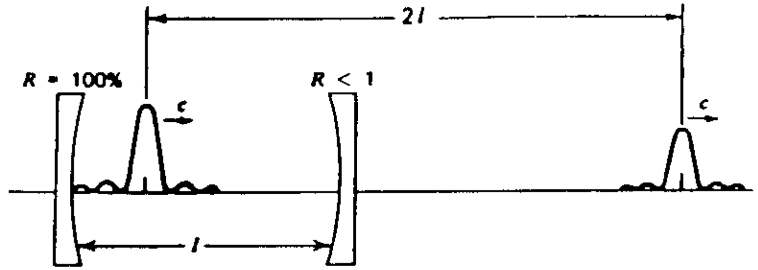
$$E = \frac{1}{T} \int_0^\infty n_p \, dt = \int_0^\infty -\frac{dn_p}{dt} dt - \int_0^\infty \frac{dN}{dt} dt$$

If $n_p = 0$ for $t = 0$ n_p final $= 0$ the first integral vanishes and the energy is

$E = N_{initial} - N_{final}$ (energy conservation)

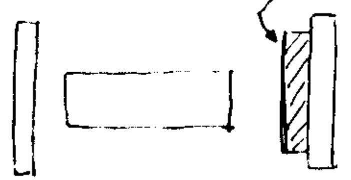
- By integrating the two equations the value of N_{final} can be derived. The energy stored in the active medium will be converted into laser energy with an efficiency
- (utilization efficiency) $K_u = \dfrac{N_{initial} - N_{final}}{N_{initial}}$

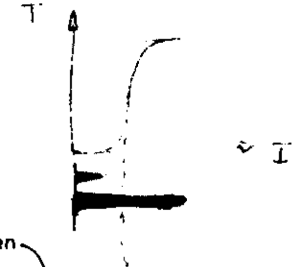
Mode Locking

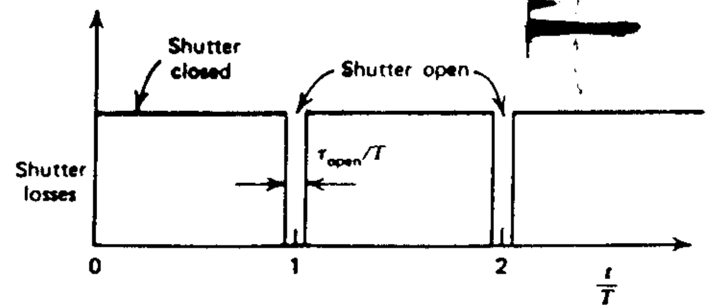


Traveling pulse of energy resulting from the mode locking of N laser modes.

Saturable absorber
Ex: DODCI in Rh 6G
< 5fs cw Ar+
1 ps Flash







Periodic losses introduced by a shutter to induce mode locking. The presence of these losses favors the choice of mode phases that results in a pulse passing through the shutter during open intervals—that is, mode locking.

La struttura dei modi si adatta all'evoluzione delle perdite nel risonatore.

Mode Locking - N modes!

The set of phase-locked modes can be described by:

- $$E(t) = \sum_n E_n e^{j[(\omega_0 + n\omega)t + \phi_n]} \qquad (1)$$

where: $\omega_0 = 2\pi\nu_0$ and ν_0 is the central mode frequency
$\omega = 2\pi\, c/2L$; $c/2L$ frequency difference between two adjacent modes; ϕ_n phase of the n^{th} mode.
If $\phi_n = 0$ (phased modes) $E_n = \text{const} = E_0$

$$E(t) = E_0 \sum_{-(N-1)/2}^{+(N-1)/2} e^{j(\omega_0 + n\omega)t} = E_0 \frac{\sin(N\omega t/2)}{\sin(\omega t/2)} e^{j\omega_0 t}$$

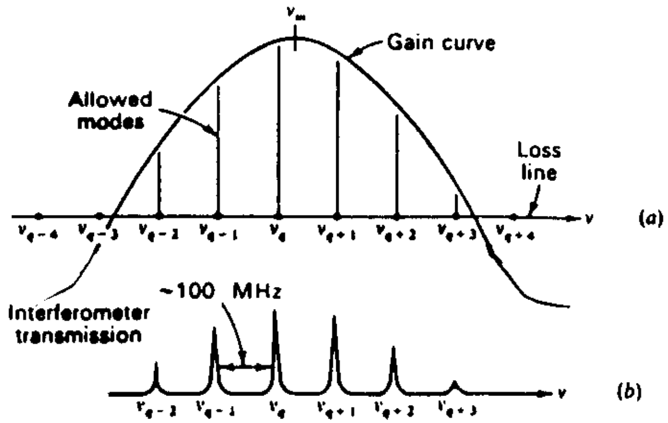
The laser output power is proportional to:

$$\boxed{P(t) \equiv E(t)E^*(t) = E_0^2 \frac{\sin^2(N\omega t/2)}{\sin^2(\omega t/2)}}$$

1. The emission consists of a periodic pulse train with period $T = 2L/c$
2. The peak power is N times the average power
   $P_{peak} = N^2 E_0^2$ $\qquad P_{ave} = N E_0^2$
3. The peak electric field is N times the field of the individual mode
4. The pulse duration (peak - first zero) is $\tau = T/N \cdot 0.886$
5. The number of modes is $N \simeq \Delta\nu / c/2L$

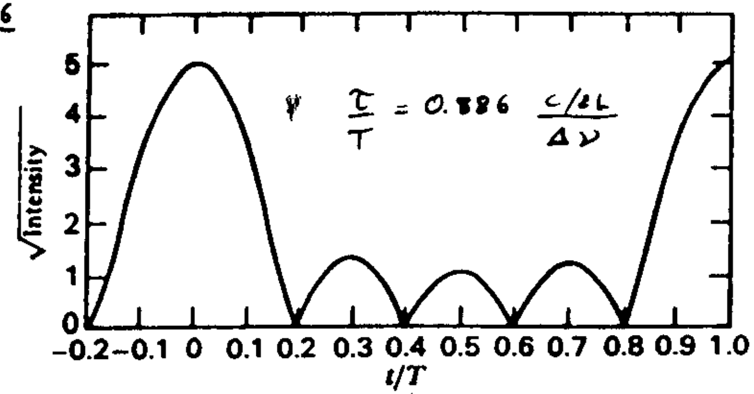
LASER	He-Ne	Nd: YAG	Ruby	Nd: Glass	Rh 6G
Wavelength	632.8 nm	1.06 μm	694.3 nm	1.06 μm	~ 550 nm
$\Delta\nu$ (Hz)	$1.5 \cdot 10^9$	$1.2 \cdot 10^{10}$	$6 \cdot 10^{10}$	$3 \cdot 10^{12}$	10^{13}
$\Delta\nu^{-1}$ (sec)	$6.6 \cdot 10^{-10}$	$8.34\ 10^{-11}$	$1.66 \cdot 10^{-11}$	$3.33\ 10^{-13}$	10^{-13}
τ (sec)	$6 \cdot 10^{-10}$	$7.6\ 10^{-11}$	$1.2\ 10^{-11}$	$4.0\ 10^{-13}$	10^{-13}

The minimum pulse duration is of $\simeq$ 8 femtosec
$\tau_{min} \simeq$ 5 cycles of the optical radiation



(*a*) Inhomogeneously broadened Doppler gain curve of the 6328 Å Ne transition and position of allowed longitudinal mode frequencies. (*b*) Intensity versus frequency profile of an oscillating He-Ne laser. Six modes have sufficient gain to oscillate. *Source:* Reference 9.

$$\tau = \frac{0.886}{\Delta\nu}$$



Theoretical plot of optical field amplitude $[\sqrt{P(t)} \propto \sin(N\omega t/2)\ \sin(\omega t/2)]$ resulting from phase locking of five ($N = 5$) equal-amplitude modes separated from each other by a frequency interval $\omega = 2\pi/T$.

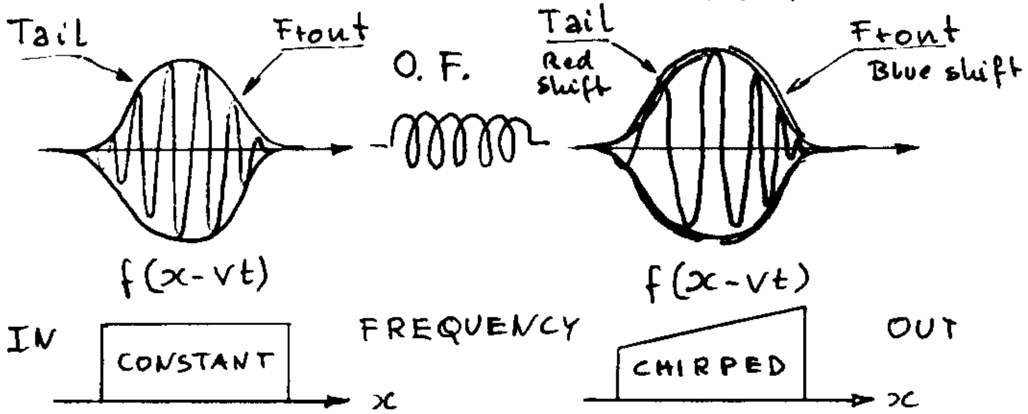
From "Principles of Lasers" O. Svelto

Pulse Compression

Optical Kerr Effect

I[st] Step - Optical Fiber - Non Linear effects in propagation induce a frequency modulation (CHIRP).

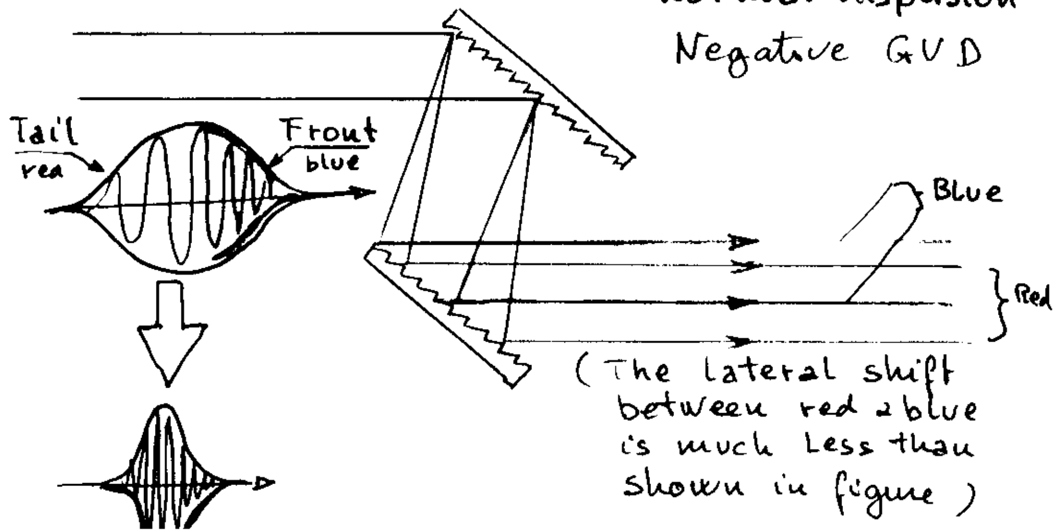
Positive GVD



II[ND] Step - Two Gratings - A frequency dependent optical path difference is introduced: short λ ⇒ longer path

normal dispersion

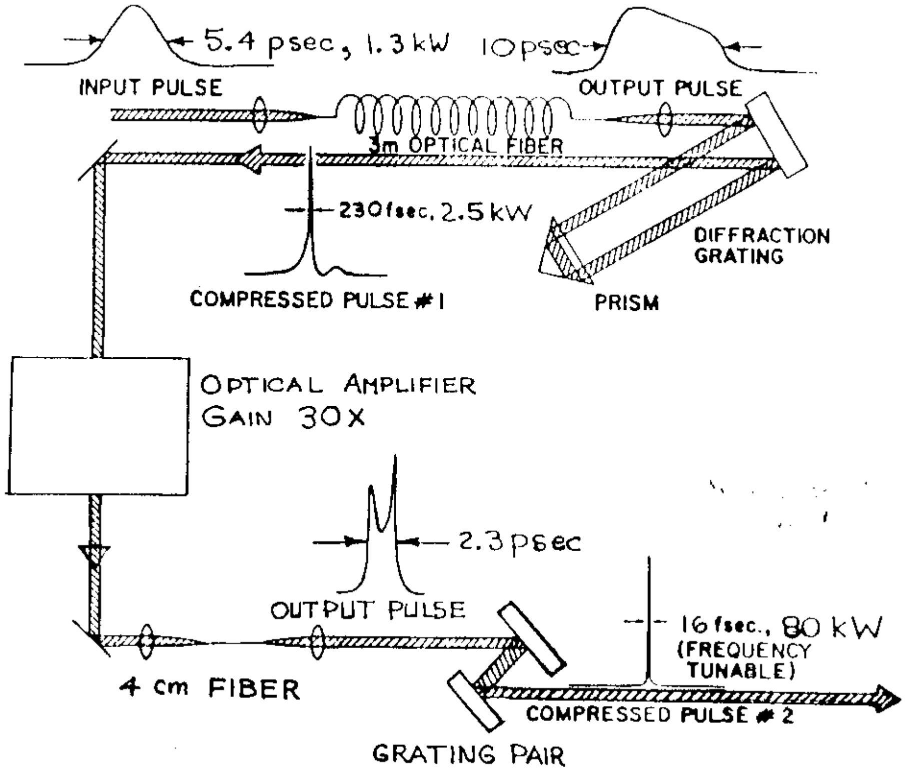
Negative GVD



(The lateral shift between red & blue is much less than shown in figure)

Concentration in Time

CLEO 85



$$\nu = \frac{c}{\lambda} = \frac{3\cdot 10^{10}\,\mathrm{cm/s}}{10^{-4}\,\mathrm{cm}} = 3\cdot 10^{14} \qquad \frac{1}{\nu} = T = 3.33\ \mathrm{fs}$$

if $\lambda = 1\,\mu m$

$$1\,\mathrm{fsec} = 10^{-15}\,\mathrm{s}$$

FIGURE 1

(from S.L. Palfrey et al. IBM)

Temporal Coherence & Monochromaticity

- First order correlation function between the field amplitudes at the same point at different times {ensemble (or time) ave.}

$$\Gamma_{11}^{(1)}(\bar{r}_1,\tau) = \langle E_1(\bar{r}_1,t)\,E_1^*(\bar{r}_1,t+\tau)\rangle$$

- The normalized function is called the "degree of temporal coherence" defined as:

$$\gamma_{11}^{(1)}(\bar{r}_1,\tau) = \Gamma_{11}^{(1)}(\bar{r}_1,\tau)\,/\,\Gamma_{11}^{(1)}(\bar{r}_1,0) \le 1$$

(in general is a complex quantity)

the value τ_{coh} for which $|\gamma_{11}^{(1)}(\bar{r}_1,\tau_{coh})| = 1/2$ can be assumed as the "coherence time" and correspondingly $\ell_{coh} = \tau_{coh} \cdot c$ as the "coherence length"

- For a pure sinusoidal wave $\tau_{coh} = \infty$
- For a conventional monochromatic source having a spectral linewidth $\Delta\nu$
  $\tau_{coh} \simeq 1/\Delta\nu$ and $|\gamma_{11}^{(1)}(\tau)| = e^{-\ln 2\,\tau/\tau_{coh}}$

In a Michelson interferometer the fringe intensity is given by

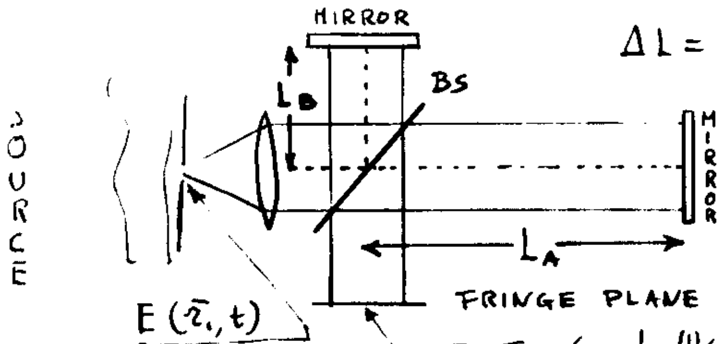
$$I = \frac{1}{2} I_1 \{1 + |\gamma_{11}^{(1)}(\tau)| \cos\omega\tau\} \quad \text{where}$$

I_1 is the beam intensity $I_1 = \Gamma_{11}^{(1)}(\bar{r}_1,0)$

$\tau = n\,\Delta L/c$ is the difference in the propagation times of the two beams due to an optical path difference

$OPD = n\,\Delta L$

Temporal Coherence & Monochromaticity

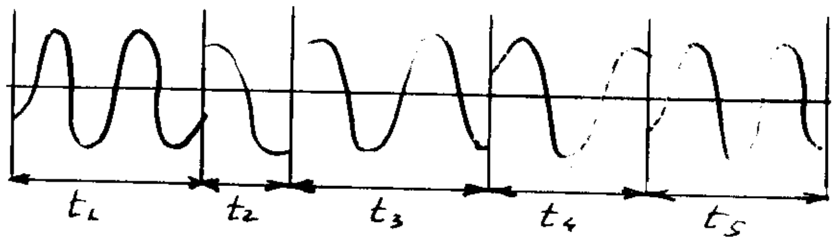


$$\Delta L = L_A - L_B \qquad \tau = \frac{n\,\Delta L}{c}$$

$$I = \frac{I_1}{2}\left(1 + |\gamma_{11}^{(1)}(\bar{r}_1,\tau)|\cos\omega\tau\right)$$

The fringe visibility $V = \dfrac{I_{MAX} - I_{MIN}}{I_{MAX} + I_{MIN}} = |\gamma_{11}^{(1)}(\bar{r}_1,\tau)|$

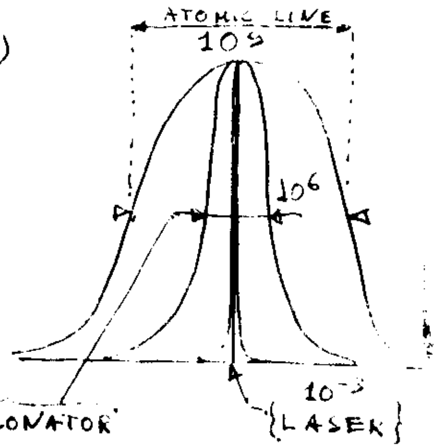
Ex. 1 Non coherent "Monochromatic" source
(Equivalent to "white noise" filtered)



τ_{coh} = Average t_i $\simeq 1/\Delta\nu$

	$\Delta\nu$ (Hz)	τ_{coh} (s)
Laser	10^{-3}	10^{3}
Resonator	10^{+6}	10^{-6}
• Atomic line	10^{+9}	10^{-9}
(Nd-YAG) XTAL	10^{10}	10^{-10}
Nd Glass	10^{12}	10^{-12}

(10^{-9} s → 30 cm)



[illegible]

Frequency selective optical elements are included in the resonator to allow single frequency oscillation such as: PRISMS - GRATINGS - SMITH CAVITY

BIREFRINGENT FILTERS - WEDGES - ETALONS

SEEDING OR INJECTION LOCKING.

LIMITS TO THE COHERENCE TIME

- Oscillation Frequency

$$\boxed{\nu_{osc} = \frac{\nu_0/\Delta\nu_0 + \nu_c/\Delta\nu_c}{1/\Delta\nu_c + 1/\Delta\nu_0}}$$

- Quantum Limit

$$\Delta\nu_{osc} = \frac{N_2}{N_2 - N_1} \frac{2\pi^2 h\nu_{osc} \Delta\nu_c^2}{P} = \frac{2\pi^2 N_2}{N_2 - N_1} \frac{\Delta\nu_c}{N_p}$$

where $N_p = \frac{P}{h\nu_{osc}} \cdot \frac{1}{\Delta\nu_c}$ is the number of photons inside the cavity.

In practice for $P = 1\,mW$, $\Delta\nu_c = 10^7\,Hz$

$\Delta\nu_{os}/\nu_{osc} = 10^{-15}$

- Thermal Fluctuations Limit

$$\Delta\nu_{osc}/\nu_{osc} = -\Delta L/L = -\Delta\lambda/\lambda = \sqrt{\frac{2KT}{VY}}$$

V Volume of the spacers; Y Youngs modulus

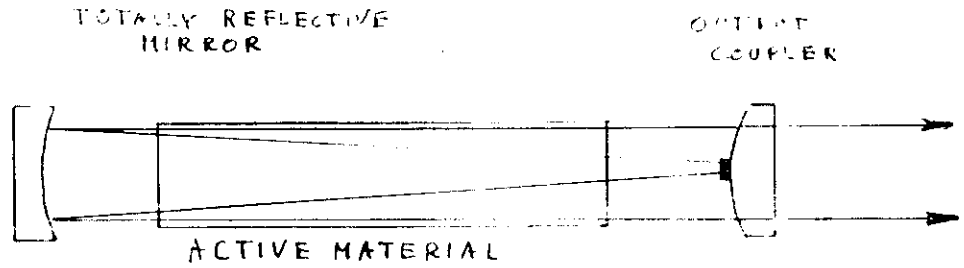
In practice $\Delta\nu_{osc}/\nu_{osc} = 10^{-11} - 10^{-10}$

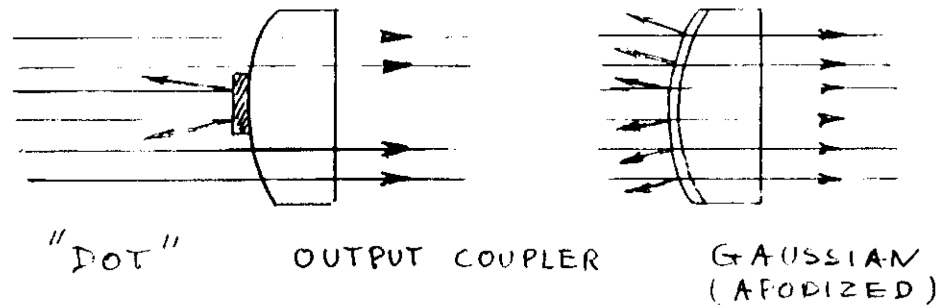
- Drifts due to change in Temperature

[illegible]

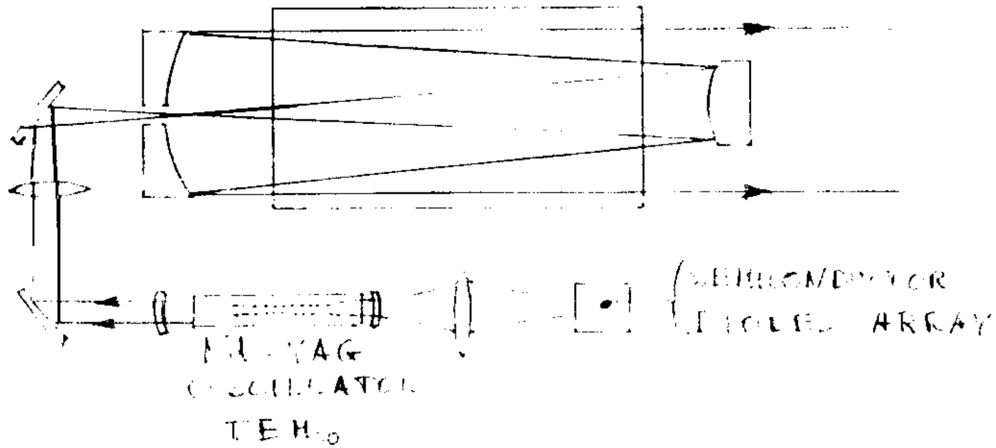
- Limits set by gain fluctuations thermal [illegible] effects

MODE SELECTION



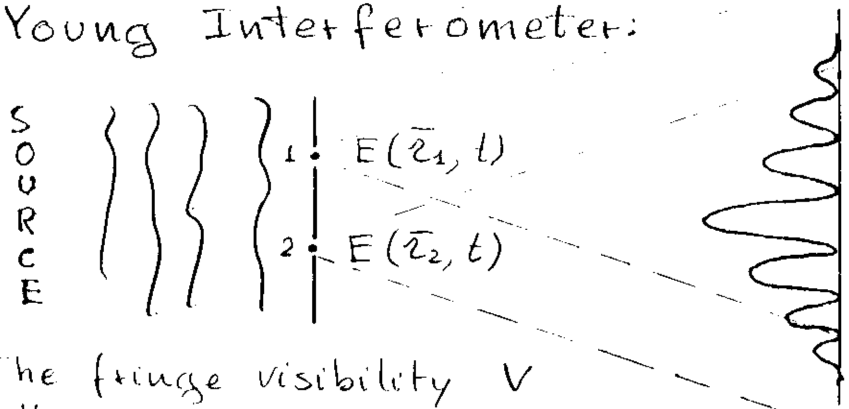


INJECTION LOCKING (SEEDING)



Spatial Coherence & Directionality

Young Interferometer:



The fringe visibility V allows the measure of the "coherence radius" and "coherence area"

Beams with partial spatial coherence have a divergence greater than for a spatially coherent beam with the same intensity distribution.

Example: Semiconductor lasers or solid state lasers with "hot spots"

The divergence is $\theta'_d = 1.22\ \lambda / d$

instead of $\theta_D = 1.22\ \lambda / D$

Multimode lasers have a divergence larger than the single mode case.

$$\Theta_{e,m} = c_{e,m} \cdot \frac{\lambda}{\pi w_0} \qquad c_{m} \to 1.16$$

$$c_{00} = 1.16$$

(if the spot size $\equiv$ 90% of beam power is considered)

The lack of spatial coherence reduces the fringe visibility and consequently the efficiency f.i. of an hologram

Spatial Coherence & Directionality

- First order correlation function between the field amplitudes at the same time in different points {ensemble (or time) ave.}

$$\underline{\Gamma_{12}^{(1)}(\bar{r}_1 - \bar{r}_2, t) = \langle E_1(\bar{r}_1, t)\, E_2^*(\bar{r}_2, t) \rangle}$$

- The normalized function is called the "degree of spatial coherence defined as:

$$\underline{\gamma_{12}^{(1)}(\bar{r}_1 - \bar{r}_2, 0)} = \Gamma_{12}^{(1)}(\bar{r}_1 - \bar{r}_2, 0) \Big/ \left\{ \Gamma_{11}^{(1)}(\bar{r}_1) \cdot \Gamma_{22}^{(1)}(\bar{r}_2) \right\}^{1/2} \leq 1$$

(in general is a complex quantity)

- The value $|\bar{r}_1 - \bar{r}_2|$ for which $|\gamma_{12}^{(1)}(\bar{r}_1 - \bar{r}_2, 0)| = 1/2$ defines the "coherence radius and the related "<u>coherence area</u>".

- For an ideal unlimited wave (f.i. coming from a single spatial mode of a laser) the coherence radius $\to \infty$.

NF Near Field
- For a wavefront limited by an aperture with a diameter D the propagation vector $\bar{k}$ is affected by a spread $\Delta k = k \cdot \Delta\theta$ where $\underline{\Delta k = 1.22 \cdot 2\pi / D}$.

FF Far Field
- If the apertured beam is focused by a lens of focal length f the spot size is defined by $\Delta s = f\,\Delta\theta = f \cdot 1.22\ \lambda/D$ and again the limitation due to diffraction holds: $\frac{1}{\lambda} \cdot \frac{D}{f} = \frac{1}{\lambda}\ \frac{\Delta k}{k} = \frac{\Delta k}{2\pi} = 1.22 / \Delta s$

$$\left(\frac{D}{f} = \frac{\Delta k}{k} = \frac{k\, \Delta\varphi^{FF}}{k} = F\text{-number} \right)$$

The <u>space concentration</u> limit is

$$\boxed{\Delta s \simeq 1.22\ \frac{f}{D}\ \lambda} \quad \text{(a few } \lambda\text{'s)}.$$

SINGLE TRANSVERSE MODE OPERATION

(Prerequisite for single frequency oscill.)

Mode selection can be accomplished by:

1. ■ Exploiting diffraction losses in stable resonators. Low Fresnel numbers ($\simeq 1$); $F = a^2/L\lambda$ (a - radius of the aperture; λ wavelength, L - cavity Length) allow the selection of the TEM_{00} mode.

2. ■ If large power or energy are requested large active volumes and, in practice, high Fresnel numbers are needed. Special mode selecting cavities must be used such as:
   - Self Filtering confocal Unstable Resonators (SFUR) negative branch
   - Unstable confocal resonators with Variable Reflectivity Mirrors (VRM):
     - Dot mirrors
     - Hard Edge phase unified couplers
     - Mirrors with Gaussian or SuperGaussian reflectivity profiles.
     - Ring cavities with expanders
     - Phase conjugated Mirrors

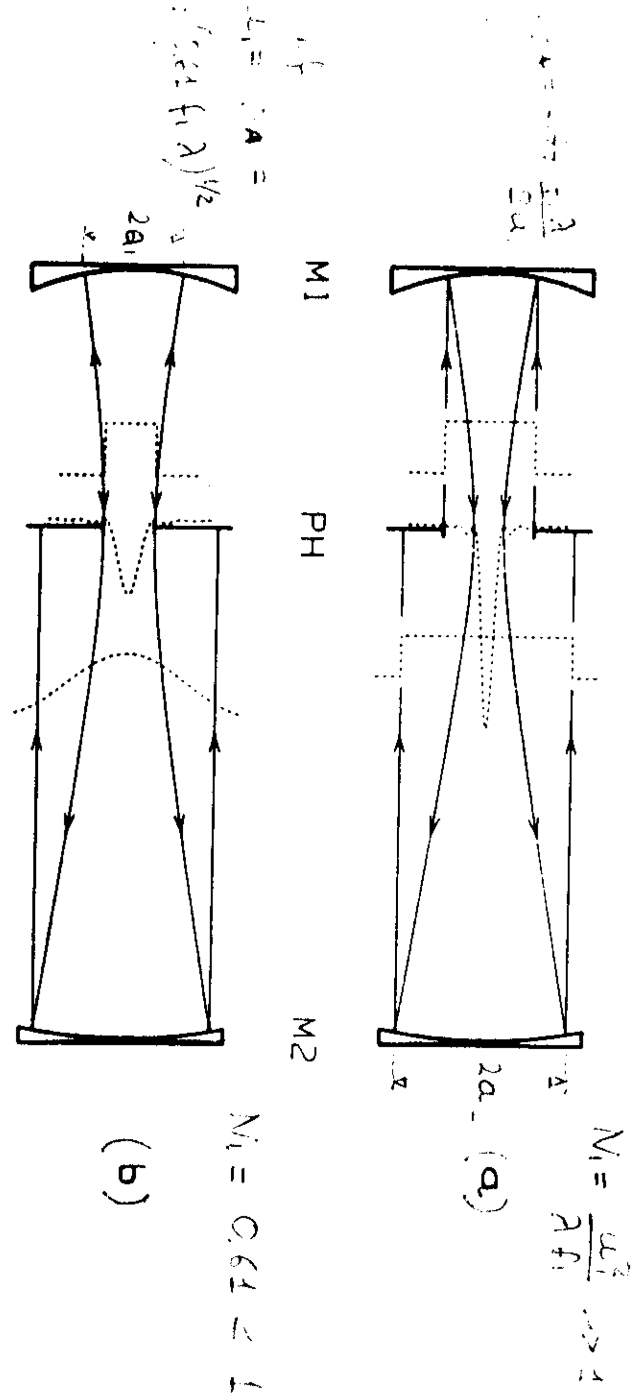
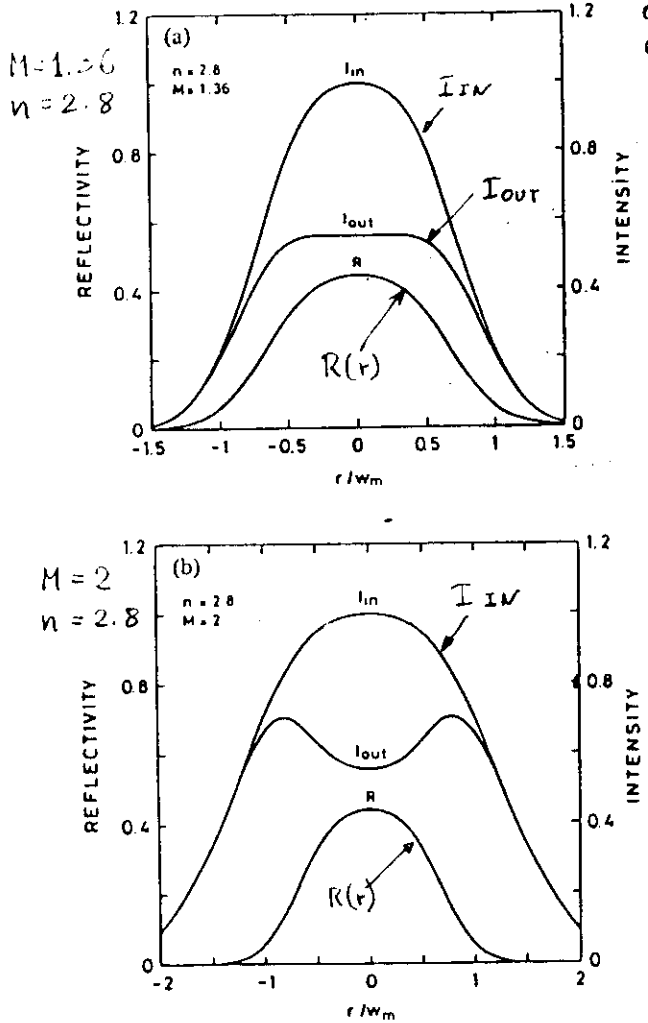


Fig. 1. Schematic of the self-imaging (a) and self-filtering (b) unstable resonators: M_1,M_2, fully reflecting concave mirrors of focal lengths f_1,f_2; PH, pinhole of diameter $2a$ placed in the common focal plane of the two mirrors.

(P. G. Gobbi et al. Appl. Opt. 24, 26 Jan 1985) [illegible] Mirror

$R(r) = R_0 \exp\left[-2\,(r/w_m)^n\right]$ (VRM)

n - ORDER OF Super-Gaussianity



Theoretical intensity profiles of the wave incident I_{in} on a super-Gaussian reflectivity mirror of order $n = 2.8$, and of the transmitted beam I_{out} as a function of radial coordinate r normalized to the mirror spot size w_m for two resonator magnifications: (a) $M = 1.36$. (b) $M = 2$. The reflectivity profile of the mirror R is also reported.

IEEE J. QE Vol 24 6 (1988)

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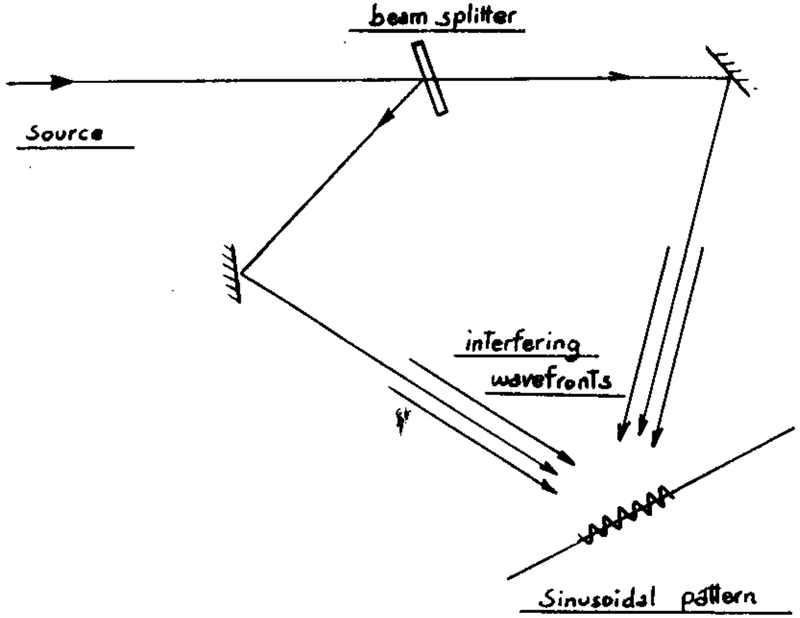
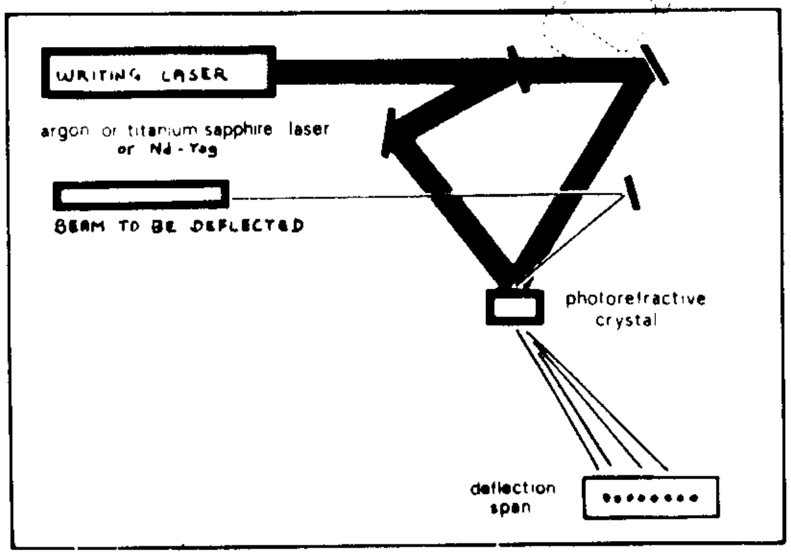
Generation of grating pattern





Scheme showing variable spacing grating recording in a photorefractive material and related deflection undergone by a signal beam.