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TRAINING COLLEGE ON
PHYSICS AND CHARACTERIZATION
OF LASERS AND OPTICAL FIBRES

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OPTICAL FIBRE THEORY

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Characteristics of single-mode optical fibres

Geometrical and Optical Characteristics

Core and cladding geometry

For single-mode fibres it was decided not to standardize the core geometrical characteristics but rather its transmission ones: this choice was made in order to permit manufacturing and use of different fibre designs which, nevertheless, could present uniformity in the transmission parameters determining joint and microbending losses. Instead of the core diameter, it is the mode field diameter which has been standardized. Concerning the cladding diameter it was decided to confirm the value already selected for multimode fibres mainly in consideration of a possible use of common parts for connectors and mechanical joints. As far as tolerances are concerned, the values are given in Rec. G.652 and as for multimode fibres they are assigned as a compromise between low jointing losses and high production yields.

Refractive index profile

It is not necessary to recommend a specific refractive index profile, in order to permit the use of different single-mode fibres (matched-cladding, depressed cladding, W-shaped and multiple cladding), with compatible values of transmission parameters. Although the use of different refractive fibres should be avoided in the same link, specifications on the mode field geometry should ensure an acceptable joint loss even in the presence of different fibres.

In general it is possible to say that the refractive index profile shape is not important from the transmission point of view, while it plays a major role in the design of dispersion shifted and dispersion flattened fibres.

Transmission characteristics

Modes in single-mode optical fibres

The simplest single-mode fibre refractive index profile involves a step change in index across the core to confine the optical power. A step-index fibre operates in the single-mode regime if the V number, or "normalized frequency", satisfies the condition

$$V = ka\ [n_0^2 - n_1^2]^{1/2} < 2.405 \qquad (1)$$

where 2.405 correspondes to the first zero of the 0th order Bessel function, a is the core radius, $k=2\pi/\lambda$ is the wavenumber (λ-the free-space wavelength), n_0 and n_1 are the refractive indexes of core and cladding respectively. From Equation (1) the cut-off wavelength, above which higher order modes cannot propagate, can be determined; in this case only the fundamental mode HE_{11} will propagate.

For the case of a graded-index profile fibre an approximate method for determining the cut-off wavelength consists in defining an "effective" number V_{eff} such that:

$$V_{eff} = \left\{2k^2 \int_o^a [n^2(r)-n_1^2]\ r\ dr\right\}^{1/2} < 2.405 \qquad (2)$$

where n(r) represents the refractive index profile (r - radial coordinate).

When the weakly-guiding condition applies ($n_0-n_1 \ll n_1$), the modes of the fibre degenerate in a way that it is possible to find linearly-polarized (LP) modes which form useful approximation to the true modes. The fundamental mode HE_{11} is however naturally linearly polarized and it is also indicated as LP_{01}. The second order mode is given by the two polarization states of the LP_{11} mode which comes from the degeneracy of the HE_{21}, TE_{01} and TM_{01} modes.

Introducing a system of cartesian coordinates (x,y,z) with the z-axis coincident with the fibre axis, the field of the LP_{01} fundamental mode can be described in terms of two components: E_x of the electric field and H_y of the magnetic field, assuming the polarization direction of the mode along the x-axis. The other transverse components of the field vanish, while the longitudinal ones are negligible. For E_x and H_y we have the following well approximated expressions:

$$E_x = \psi(r)\ \exp\ (-i\beta z)$$
$$H_y = (\varepsilon/\mu_0)^{1/2} E_x \qquad (3)$$

where ε, μ_0 are respectively the electric and magnetic permeabilities of the medium. The radial field distribution is usually assumed Gaussian as follows $\psi(r)=\exp[-1/2(r/r_0)^2]$ with r_0 depending upon index-profile and normalized frequency. Gaussian approximation althrough convenient is however no longer valid when the refractive index profile is much more complex than the step profile or when the fibre is operating at the highest wavelengths.

Attenuation

As an optical signal propagates along a fibre it is attenuated and the power level at a distance of z kilometers from the transmitter is

$$P(z) = P_T\ \exp\ (-\alpha z) \qquad (4)$$

where P_T is the power launched by the transmitter into the fibre and α the attenuation constant. It is to be noted that Equation (4), which applies for single-mode fibres, is also used for multimode ones even if in such a case α is a weighted average over the propagating modes so that its value actually changes along the fibre. Single-mode fibres have the remarkable advantage of not having such differential mode attenuation effects.

The sources of attenuation in silica fibres can be divided into three different categories: absorption, material scattering and waveguide effect.

The absorption can be intrinsic, due to UV electronic transition and IR molecular vibrations, impurity induced, due to transition metals, OH and H_2 vibration overtones and eventually, defects induced, due to oxygen vacancy, radiation, draw and H_2.

The scattering is caused by natural Rayleigh scattering which is intrinsically due to minute density and concentration fluctuations, by bulk imperfections such as bubbles, inhomogeneities and cracks or by waveguide imperfections which are due to core and cladding interfacial irregularities.

The waveguide related losses are due to macrobending (curvature induced), microbending (perturbation induced) and design.

The technology for fibre fabrication has advanced to a level that the intrinsic losses are limited by the basical causes of the Rayleigh scattering and of the infrared absorption. The OH content can be kept low and high quality fibres now in production show excess OH attenuation at 1380 nm absorption peak in the range of 0.1-2 dB/km.

Cut-off wavelength

The knowledge of refractive index profile allows the determination of the theoretical cut-off normalized frequency, V_c. For each $V<V_c$, the single-mode operation is assured. As real fibres are intended to be used in long length links the definition of an "effective" cut-off wavelength is usually assumed for design purposes. To this end the cut-off wavelength is the wavelength greater than which the ratio between the total power, including launched higher order modes and the fundamental mode power has decreased to less than a specified value, the modes being substantially uniformly excited. As effective cut-off wavelength depends upon the fibre length and curvature radius, these parameters are to be considered both in the definition and in the measurement; for the time being the length is fixed equal to 2 m and the curvature radius to 14 cm (in a single turn). In addition a cable cut-off wavelength could be defined in a similar way. In general the cable cut-off is lower than the fibre cut-off which is lower than the theoretical one (see Rec. G.652, Annex A, Paragraph A.2).

The importance of ensuring single-mode transmission at the operating wavelength is paramount. This may be approached in different ways depending on the cable and system design including the expected length, construction and curvature of repair cables and jumper cables. The dependence of the cut-off wavelength λ_c on the fibre length has been empirically determined and can be evaluated by the following expression:

$$\lambda_c(z) \approx \lambda_c(z_o) - A \log (z/z_o), \qquad z_o=2\mathrm{m} \tag{5}$$

which shows the logarithmic decreasing of the cut-off wavelength versus the fibre length z. In Equation (5) A is usually assumed in the range from 20 to 70 nm/decade, depending on matched or depressed cladding profile.

Finally it is worth noting that the second order mode attenuation coefficient $\alpha_{11}(\lambda)$ is related to the cut-off wavelength in the sense that λ_c can equivalently be defined as that wavelength at which the radiation attenuation of the LP_{11} mode has increased up to a certain prefixed value, that depends on the value, specified in the cut-off definition, of the ratio between the total and the fundamental mode power.

Mode Field Diameter

The radial electric field distribution is a fundamental property of single-mode fibres since it is related to splice-loss, bend sensitivity and dispersion. In fact it is important for single-mode fibres to have a reasonable spot-size for the fundamental mode in order to simplify connecting and splicing. Attempts have been made to characterize this distribution with a single parameter called the mode field diameter, which is a measure of the radial extent of the fundamental mode. Being the mode field diameter a single parameter derived from the electrical field which is to be used to evaluate fibre performance (bending and jointing losses, dispersion, etc.) the definition must be chosen in order to obtain meaningful values. There are two definitions of the mode field diameter with an important practical meaning. The first is given by the r.m.s. width of the near-field distribution of the fibre, according to:

$$d_n = 2 \left[2\int_o^\infty \psi^2(r)r^3dr/\int_o^\infty \psi^2(r)rdr\right]^{1/2} \tag{6}$$

and it is important for the determination of the possible microbending loss and for joint losses in presence of small tilts.

The second mode field diameter definition d_f is given by the inverse of the r.m.s. width of the far-field distribution of the fibre and it is important for the evaluation of possible joint losses in the presence of small transverse offsets and of the waveguide dispersion. This last definition of the mode field diameter has been recently accepted also if appreciable differences among the various definitions actually occur when the field distribution is far from the Gaussian shape. One of the advantage of this chosen definition is that it can be given starting from the near-field, the far-field and from the autocorrelation function of the near-field, all quantities susceptible of direct measurement.

Thus the mode field diameter d_f is given from the near-field $\psi(r)$ as follows:

$$d_f = 2 \left[2 \int_o^\infty \psi^2(r)rdr/\int_o^\infty (d\psi(r)/dr)^2 rdr\right]^{1/2} \tag{7}$$

From the far-field $\phi(p)$ ($p=k\sin\theta$, θ being the output angle with respect to the fibre axis), d_f can be expressed as:

$$d_f = 2\ [2 \int_0^\infty \phi^2(p)p\,dp / \int_0^\infty \phi^2(p)p^3dp]^{1/2} \qquad (8)$$

obtained from Equation (7), taking into account that $\phi(p)$ can be represented by the Hankel transform of $\psi(r)$ as follows:

$$\phi(p) = H\ [\psi(r)] = \int_0^\infty \psi(r)J_0(pr)r\,dr \qquad (9)$$

where J_0 is the 0th order Bessel function.

The mode field diameter d_f can be also expressed from the following near-field autoconvolution function:

$$\gamma(s) = \iint\psi(|\vec{r}|)\cdot\psi(|\vec{r}-\vec{s}|)d^2\vec{r} \qquad (10)$$

as:

$$d_f = 2\ \{-2\gamma(0)/[d^2\gamma(s)/ds^2]_{s=0}\}^{1/2} \qquad (11)$$

This Equation is derived from the preceding ones exploiting the fact that $\gamma(s)$ can be expressed as the Hankel transform of $\phi^2(p)$ as follows:

$$\gamma(s) = \psi(r)*\psi(r) = H\ [\phi^2(p)] \qquad (12)$$

This autocorelation function can be experimentally determined from transverse offset measurements.

Dispersion

The bandwidth of a single-mode fibre is determined by its structural and physical parameters as well as its material properties, mainly through chromatic dispersion. Chromatic dispersion, which for normal fibres is zero near 1300 nm, is caused by propagation delay differences among different spectral components of the signal, so that the pulse broadening can be expressed by:

$$\delta t = z\cdot(d\tau/d\lambda)_{\lambda_s}\cdot\sigma_\lambda \qquad (13)$$

where z is the fibre length, τ the group delay per unit length, λ_s the central wavelength and σ_λ the r.m.s. width of the source spectrum. The chromatic dispersion (coefficient) $D=d\tau/d\lambda$ can be evaluated knowing the composition of the fibre material and the geometrical parameters. It can be shown that D can be composed substantially by three contributions: namely the material, the waveguide and the profile dispersion. In particular the material dispersion is due to the dispersive properties of the medium that constitutes the fibre; it is generally the dominant contribution to the total dispersion, except in that wavelength region in which it vanishes, that for SiO_2 based materials is around 1300 nm. The waveguide dispersion takes into account the dispersive properties of the waveguide itself and can be expressed as function of the derivative, with respect to the wavelength, of the mode field diameter d_f. From the practical point of view, a significant property is that the waveguide dispersion can have opposite sign with respect the other ones in the wavelength range above 1300 nm. This property can be used to develop dispersion shifted and dispersion flattened fibres. For the former class the wavelength of zero dispersion is moved from the region of 1300 nm versus the 1550 nm window to coincide with the minimum loss region. For the latter, low dispersion is made to occur over an extended wavelength range, with possibly zero dispersion close to 1300 and 1550 nm, that would be very useful, for example, in wavelength multiplexed systems operating at high data rates, where a uniform performance over a wide range of wavelength is desiderable. As mentioned before the dispersion terms depend on material composition, geometrical parameters and refractive index profile so that the minimum dispersion is obtained by opportune choice of the previous parameters.

The bandwidth of a single-mode fibre at the wavelength of minimum dispersion, depends strongly on the spectral width of the source as can be seen from Table 1. An approximated relation for the fibre bandwidth BW can be derived from the assumption of a Gaussian impulse response for the optical fibre:

$$\delta t\cdot BW = 0.187 \qquad (14)$$

where δt is the r.m.s. width of the Gaussian pulse.

Another cause of dispersion in single-mode fibres is due to the difference of the group velocity of the two polarization components of the HE_{11} mode which propagate along the fibre. As low birefringence is induced by geometrical perturbation due to drawing and cabling processes, the two orthogonal polarizations can have different propagation constants. This effect is usually neglected, as the polarization dispersion coefficient is well below 1 ps/km, however it could be a limitation in future coherent systems.

Coupling loss

The problem of optical power coupling is very important in the design of optical fibre communication systems. In practice direct coupling condition can rarely be achieved and a lot of optical power can be lost in the coupling region between source and fibre and in particular in the joint region between two different fibres. Consequently the quality of launching and coupling of optical power is of vital importance for the definition of tolerance standards of sources and fibre parameters. Besides the analysis and the measurement of joint losses can lead to an adequate design of splices and connectors which can be used in an optical system.

In the analysis of the problem of source-fibre coupling the launching efficiency Λ, which evaluates the quality of coupling is defined as:

$$\Lambda = W_0/W' \tag{15}$$

where W_0 is the optical power collected and guided by the fibre and W' the power emitted by the source. In single-mode fibres the quality of the coupling depends on the matching between the field distribution of the radiation emitted by the source and the distribution of the field that the receiving fibre can propagate. A quantitative evaluation of such a matching is given by the so-called overlap integral that represents the degree of correlation of the two fields. The launching efficiency Λ is just given from the square modulus of this overlap integral. It can be also convenient to introduce in the following the coupling loss $L=1-\Lambda$. It is of interest to consider, among the other causes of coupling loss, those due to the various coupling errors. In the source-fibre coupling three fundamental kinds of coupling errors are usually considered:

- axial separation, if source and fibre input surfaces have the same axis but are separated by a gap s;
- lateral displacement: if the axes of the two surfaces are parallel but separated by a distance d;
- angular misalignement: if these two axes form a certain angle α.

These three error configurations may be simultaneously present; in such a case two possible displacements are taken into account: u along the axis around which the source surface is rotated by α and v along the axis perpendicular both to the u-axis and to the fibre axis; it follows that $u^2+v^2=d^2$, d being the distance between the two axes.

Coupling losses due to the joint between two fibres may be divided into two groups: intrinsic and extrinsic losses. The former are due to mismatch in the characteristics of both optical fibres at the jointing point while the latter originate from inaccuracies in the execution of a joint. The most significant parameters contibuting to intrinsic losses include the difference between core diameters, the difference between the refractive index profile of the two fibres: possible joint losses are in this case proportional to the squared relative difference in mode field diameters, between the fibres to be coupled, and therefore it is very modest in practice. Extrinsic losses mainly arise from:

- lateral displacement and angular misalignement between the ends of the coupled optical fibres;
- quality of the two surfaces with reference to smoothness, flatness, orthogonality to longitudinal axes, roughness, concavity and convexity.

As far as small lateral displacements are concerned, the coupling loss can be written in terms of the transverse offset d and the mode field diameter d_f as follows:

$$L = (2d/d_f)^2 \tag{16}$$

Concerning small angular misalignments, the coupling loss is given through the angular tilt α and the mode field diameter d_n according to:

$$L = (k\cdot\alpha\cdot d_n/4)^2 \tag{17}$$

Macrobending

Practical use of the optical fibres makes bending unavoidable so that an additional form of loss is present when the fibre axis is curved. The phenomenon may be interpreted observing that in a curved fibre the external part of the wavefronts should be constrained to travel at velocities greater then the light speed in that medium: this causes a rearrangement of the wavefronts with an unavoidable radiation of optical power. Theoretically it has been demonstrated that the attenuation coefficient depends exponentially on the radius of curvature. This means that an increase of a factor of two of the radius around a critical value can change the loss from negligible to prohibitive values.

However the bending loss coefficient is dependent on the particular refractive-index profile design of the single-mode fibre; and the depressed cladding fibres appear to suffer lower macrobending losses.

Microbending

Another important feature of single-mode fibres concerns their sensitivity to microbending. This phenomenon could arise particularly when the fibre is cabled and could lead to serious excess losses. Microbending can be caused by strains induced by plastic jacket surrounding the fibre or by the process of incorporating individual fibres in a fibre cable. Small imperfections in the mechanical structure can result in contact forces between the fibres and the supporting surface: random lateral deviations of the fibre axis will arise which may produce significant losses. The microbending loss can be approximately assumed to depend on the mode field diameter d_n as: ${d_n}^{4p+2}$; when p is an exponent characterizing the spatial spectrum of the perturbation assuming values typically around 2.

Table 1 - Bandwidth results for silica based fibres

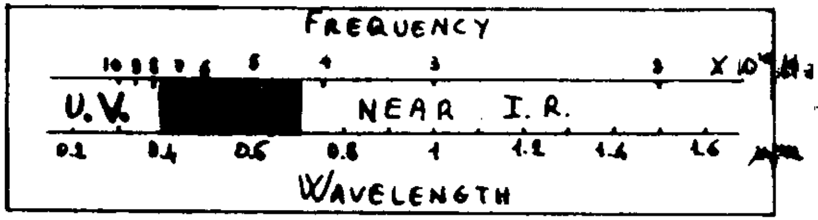
wave-length (nm)	source type	source σ_λ (nm)	single-mode fibres: Theoretical bandwidth (GHz km)	single-mode fibres: Meas. (max) bandwidth (GHz km)	multimode fibres: Theoretical bandwidth (GHz km)	multimode fibres: Meas. (max) bandwidth (GHz km)
850	laser	0.6	3.2	3.3	3.1	3.1
1300	laser	1.5	120(*)	92	13	6.7

(*) Bandwidth greater than 1000 GHz km are possible if the laser operates at the zero dispersion wavelength.

WHY FIBRE OPTICS FOR TELECOMMUNICATIONS?

[illegible] (VIS, Near IR) HAS AN EXCEEDINGLY HIGH OSCILLATION FREQUENCY ($\sim 3 \cdot 10^{14}$ Hz)

→ ENORMOUS MODULATION BANDWIDTH POTENTIALLY AVAILABLE

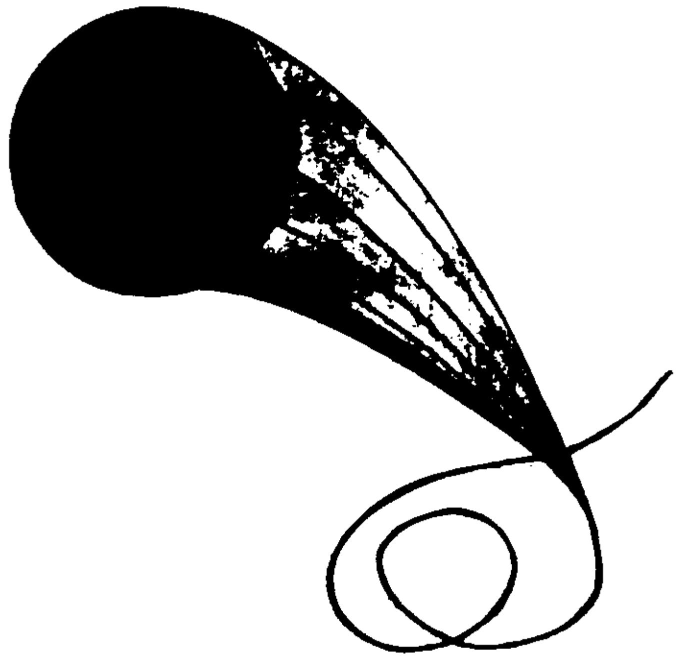


- LASERS: POWERFUL OPTICAL SOURCES

- GLASSES: VERY LOW ATTENUATION (IN THE NEAR IR. REGION)

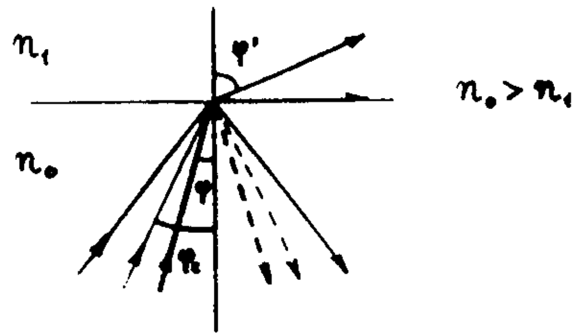
⇒ SEVERAL GHz OVER HUNDREDS Km REPEATERLESS

THE OPTICAL FIBRE

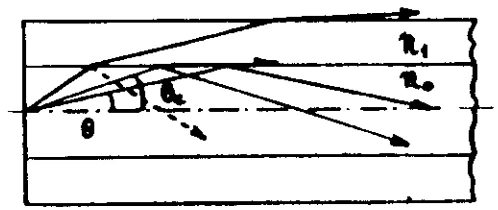


	REFRACTIVE INDEX	DIAMETER
CLADDING	$n_1 \simeq 1.46$	$\sim 125\ \mu m$
CORE	$n_1 + 5\text{-}10‰$	$\sim 5\text{-}10;\ 50\ \mu m$

TOTAL INTERNAL REFLECTION



- SNELL'S LAW : $n_0 \sin\varphi = n_1 \sin\varphi'$
- LIMITING CONDITION: $\varphi' = 90° \Rightarrow n_0 \sin\varphi_c = n_1$
- TOTAL REFLECTION FOR $\varphi > \varphi_c$



$\theta = 90° - \varphi$

LIMITING ANGLE FOR GUIDED RAYS: $\theta_c = 90° - \varphi_c \Rightarrow \cos\theta_c = \frac{n_1}{n_0}$

GUIDED RAY (TOT. INT. REFLECTION) FOR $\theta < \theta_c$

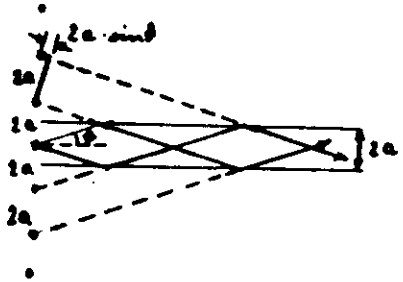
NUMERICAL APERTURE: $A = n_0 \sin\theta_c$

$$A = \sqrt{n_0^2 - n_1^2} \simeq \sqrt{2n_1(n_0 - n_1)}$$

($A \sim 0.1\text{-}0.2$ typic.)

GEOMETRICAL APPROACH TO E.M. MODES

A SIMPLE MODEL: THE PLANAR (MIRROR) GUIDE



THE CORE/CLADDING REFLECTION INTERFACE GENERATES MULTIPLE VIRTUAL IMAGES OF THE SOURCE, WITH 2a SPACING

$\Longrightarrow$ GRATING WITH 2a PERIOD

ONLY FEW PROPAGATION ANGLES ϑ ARE ALLOWED, SATISFYING A GRATING-TYPE EQUATION:

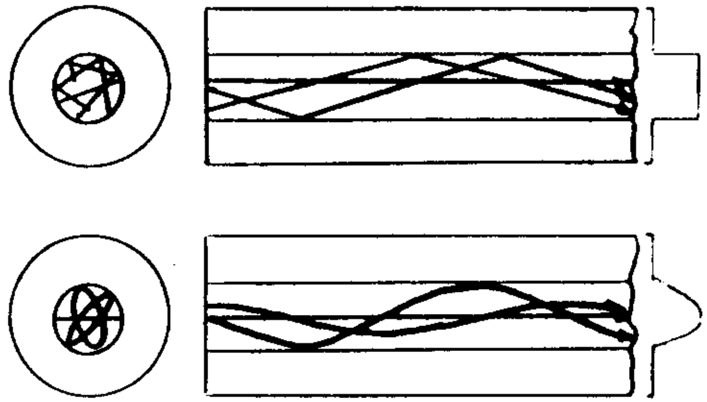
$$2a \cdot \sin\vartheta = m\lambda \qquad (m = 0, 1, 2, 3, \ldots)$$

$\Longrightarrow$ ONLY DISCRETE MODES PROPAGATE

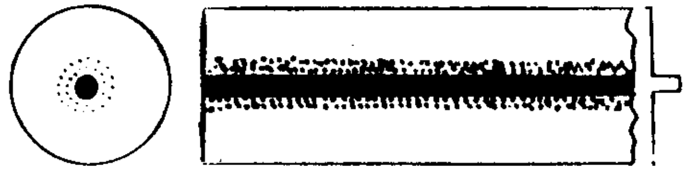
WITH PROPAGATION CONSTANT DEPENDING ON $\vartheta = \vartheta_m$

FIBRE TYPES

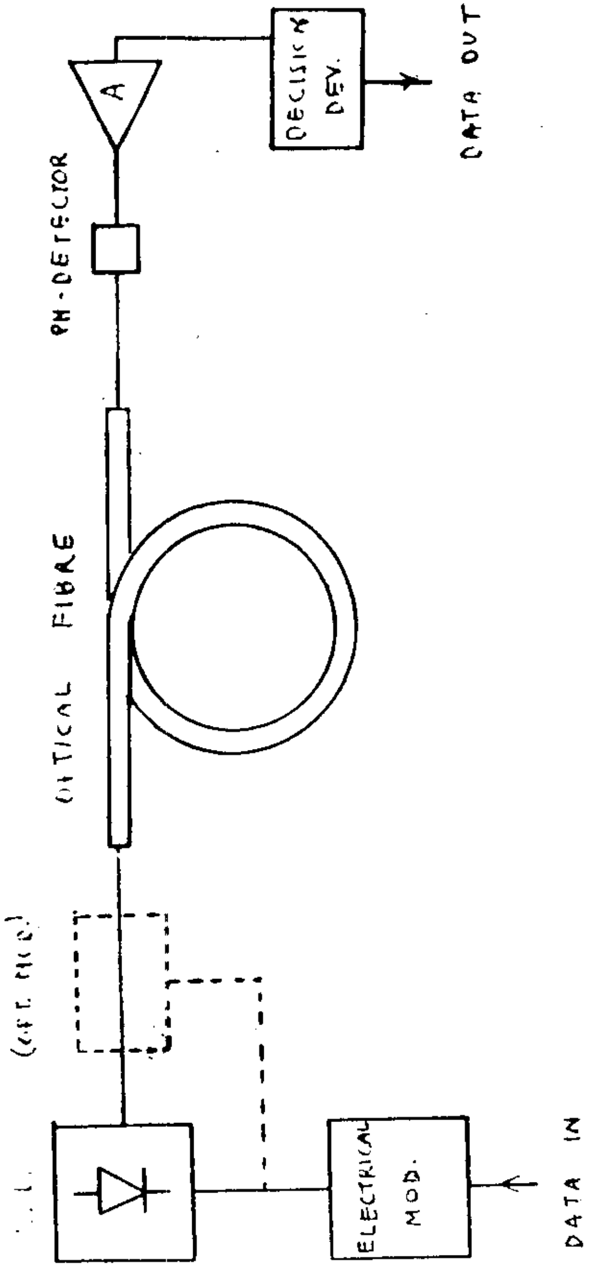
MULTIMODE FIBRE



SINGLE-MODE FIBRE



BLOCK STRUCTURE OF A DIGITAL OPTICAL COMMUNICATION SYSTEM



ATTENUATION CAUSES

INTRINSIC

- UV ABSORPTION
- IR ABSORPTION
- RAYLEIGH DIFFUSION

DEPENDENT ON THE GLASS CHEMICAL COMPOSITION AND STRUCTURE.
DEFINE THE OVERALL SPECTRAL TRANSMISSION WINDOW.
UNAVOIDABLE

IMPURITIES

- METALLIC IONS
- OH GROUPS

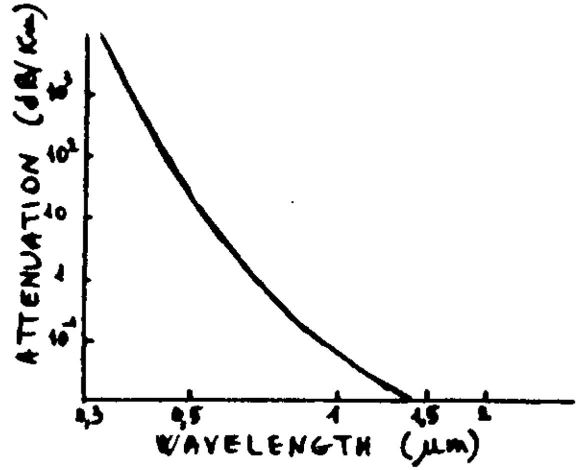
⇒ ABSORPTION BANDS AT CERTAIN WAVELENGTHS

[illegible]

ABSORPTION : INTRINSIC

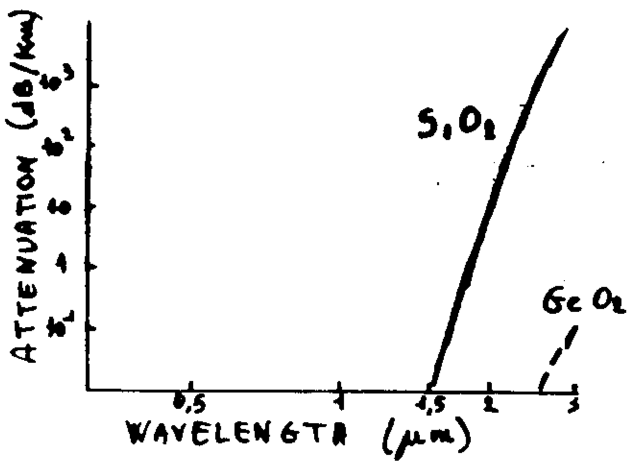
A) ULTRAVIOLET

DUE TO ELECTRONIC TRANSITIONS



B) INFRARED

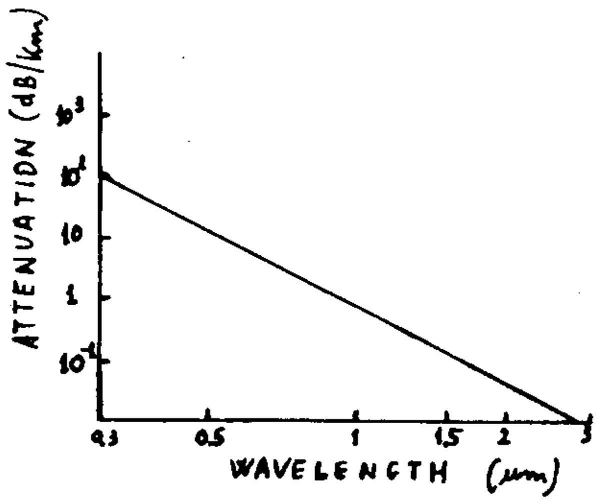
DUE TO MOLECULAR VIBRATIONS



INTRINSIC SCATTERING

- RAYLEIGH

DUE TO INHOMOGENEITIES OF SMALL DIMENSIONS WITH RESPECT TO WAVELENGTH [illegible] AT THE MOLECULAR [illegible]



ATTENUATION PROPORTIONAL TO λ^{-4}

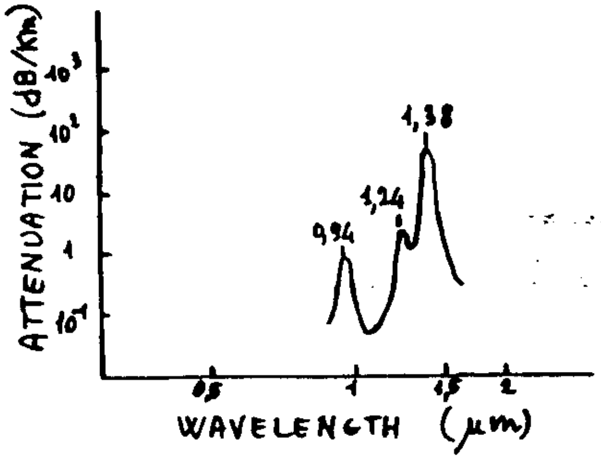
ABSORPTION : IMPURITIES

A) OH^- GROUPS

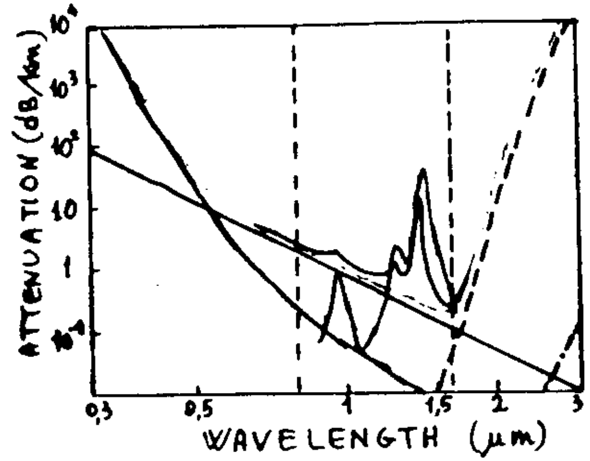
B) TRANSITION METALS IONS :

(Fe^{++}, Ni^{+++}, Cu^{++}, Cr^{+++}, Mn^{+++}, V^{+++})

ATTENUATION DUE TO 1 ppm OF OH^-



TOTAL ATTENUATION



UV ABSORPTION ————

IR ABSORPTION – – – – – · – ·

RAYLEIGH SCATTERING ————

OH ABSORPTION ————

TOTAL INTRINSIC ATTENUATION - -

MEASURED ATTENUATION

OF A CSELT FIBRE ————

INFORMATION TRANSMISSION CAPACITY

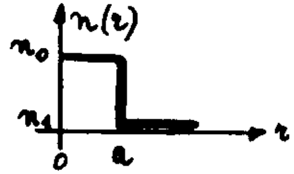
PULSE DISPERSION

* Modal Dispersion
  - Mode Coupling
  - Mode Filtering
  - Optimized Profile

* Chromatic Dispersion
  - Waveguide Dispersion
  - Material Dispersion
  - Zero-Dispersion

STEP INDEX FIBRES

- $k^2(r) = \left(\frac{2\pi}{\lambda}\right)^2 n^2(r) \rightarrow n^2(r) = \begin{cases} n_0^2, & r<a \\ n_1^2, & r>a \end{cases}$



- $LP_{\nu\delta}$:

$$\Xi_t \propto \begin{cases} \phi_0(r) = J_\nu(\kappa r), \; r<a & - \; \left[\kappa = \sqrt{\left(\frac{2\pi}{\lambda}n_0\right)^2-\beta^2}\right] \\ \phi_1(r) = K_\nu(\gamma r), \; r>a & - \; \left[\gamma = \sqrt{\beta^2-\left(\frac{2\pi}{\lambda}n_1\right)^2}\right] \end{cases}$$

- $a^2(\kappa^2+\gamma^2) = V^2$

$$V = \frac{2\pi}{\lambda} a \sqrt{n_0^2-n_1^2} = \frac{2\pi}{\lambda} a A$$

NORMALIZED FREQUENCY

CHARACTERISTIC EQUATION

$$\frac{\phi_0'(a)}{\phi_0(a)} = \frac{\phi_1'(a)}{\phi_1(a)} \Rightarrow \beta(V)$$

for each $LP_{\nu\delta}$ mode

- $\nu = 0$ $\begin{cases} EH_{0\delta} \equiv \underline{TE_{0\delta}} : E_z = E_r = H_\phi = 0 \\ HE_{0\delta} \equiv \underline{TM_{0\delta}} : H_z = H_r = E_\phi = 0 \end{cases}$

- $\nu \neq 0$ $\begin{cases} EH_{\nu\delta} : H_z = -i\sqrt{\frac{\varepsilon}{\mu}}\, E_z ; E_r = -iE_\phi ; H_r = -iH_\phi \\ HE_{\nu\delta} : H_z = i\sqrt{\frac{\varepsilon}{\mu}}\, E_z ; E_r = iE_\phi ; H_r = iH_\phi \end{cases}$

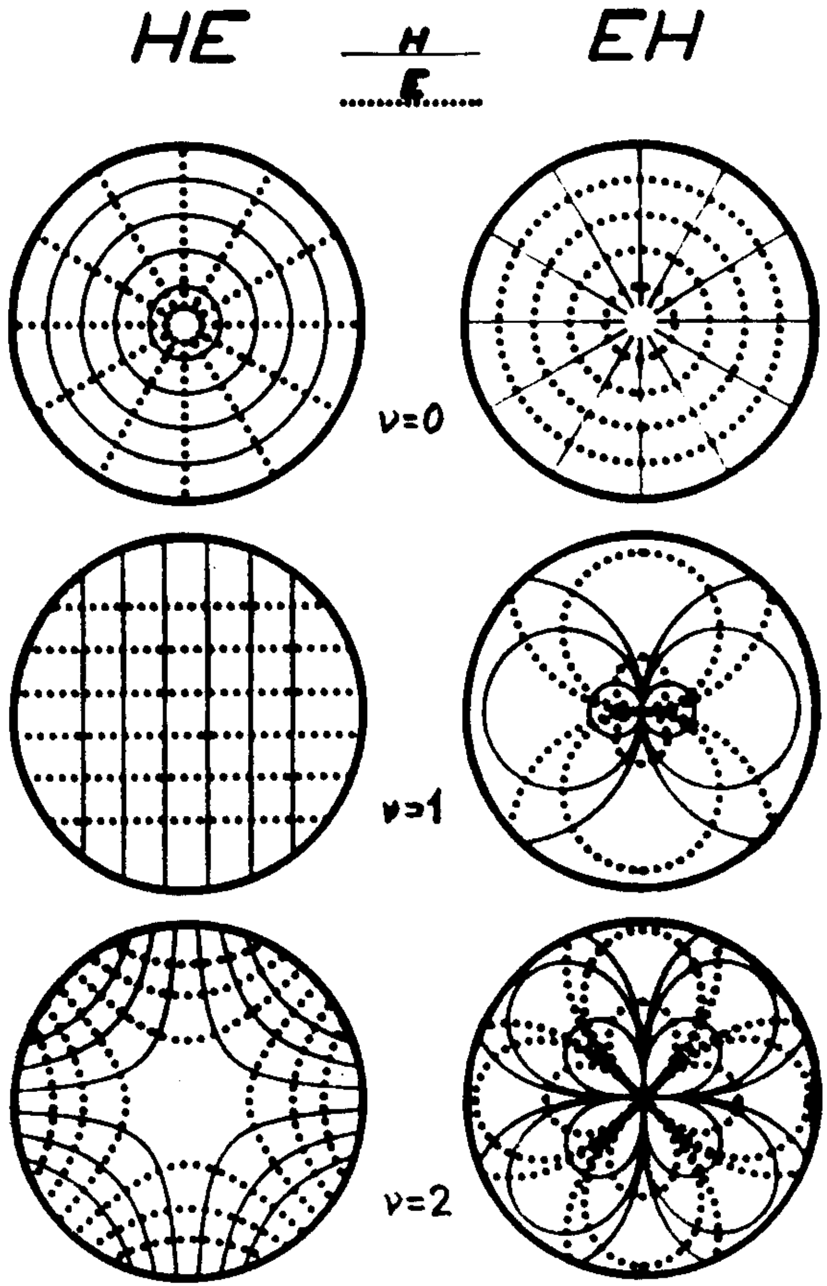
$$H_r = -\sqrt{\frac{\varepsilon}{\mu}}\, E_\phi \quad ; \quad H_\phi = \sqrt{\frac{\varepsilon}{\mu}}\, E_r$$

$$|H_z| \ll |H_r|, |H_\phi| \quad ; \quad |E_z| \ll |E_r|, |E_\phi|$$

$$\vec{H} \simeq \sqrt{\frac{\varepsilon}{\mu}}\, \hat{z} \times \vec{E}$$

LINES OF FORCE OF $\vec{E}$: $r(\phi)$

(Real Parts) $$\frac{E_r}{E_\phi} = \frac{r'}{r} = \begin{cases} -\tan \nu\phi \rightarrow EH_{\nu\delta} \\ +\tan \nu\phi \rightarrow HE_{\nu\delta} \end{cases}$$



MODE DEGENERACY : LP MODES

- $HE_{\nu+1\,\delta}$ and $EH_{\nu-1\,\delta}$ FOLLOW THE SAME EQUATION

→ THEY HAVE THE SAME β and PROPERTIES

▼

COMPOSED MODE :

LINEARLY POLARIZED

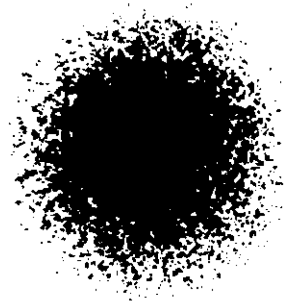
- $\nu = 0$ ONLY ONE MODE $LP_{0\delta} \equiv HE_{1\delta}$

- $\nu = 1$ THREE MODES COMBINED :

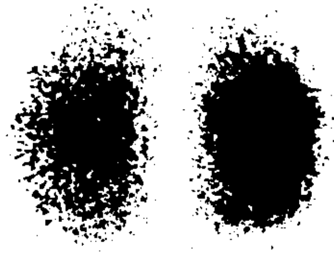
$$LP_{1\delta} \equiv \begin{cases} HE_{2\delta} + TE_{0\delta} \\ HE_{2\delta} + TM_{0\delta} \end{cases}$$

PHYSICAL MEANING OF ν and δ

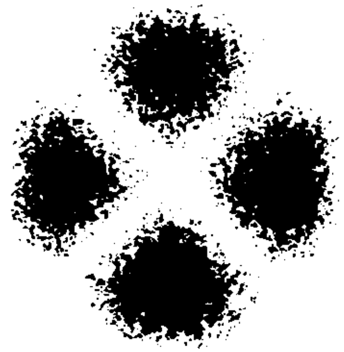
- **2ν** NUMBER OF MAXIMA OF THE FIELD INTENSITY IN A TURN AROUND THE FIBRE AXIS
- **δ** NUMBER OF MAXIMA OF THE FIELD INTENSITY IN RADIAL DIRECTION



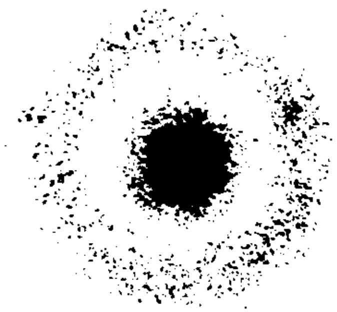
LP_{01}
(HE_{11})



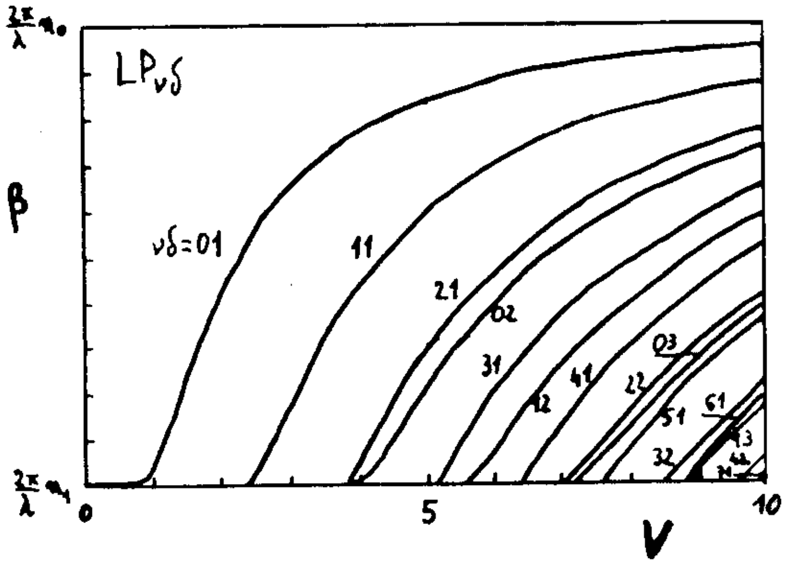
LP_{11}
$(TE_{01} + TM_{01} + HE_{2}$



LP_{21}
$(EH_{11} + HE_{31})$



LP_{02}
(HE_{12})



CUT-OFF

$LP_{\nu\delta}$ MODE IS GUIDED ONLY IF:

$$V > V^{co}_{\nu\delta} \quad \text{or} \quad \lambda < \lambda^{co}_{\nu\delta} = \frac{2\pi a A}{V^{T}_{\nu\delta}}$$

$\lambda^{co}_{\nu\delta}$: CUT-OFF WAVELENGTH OF $LP_{\nu\delta}$ MODE

IF $V < V^{co}_{11}$ ($\lambda > \lambda^{co}_{11}$) FIBRE GUIDES ONLY LP_{01} (HE_{11}) MODE : FUNDAMENTAL MODE

⇒ FIBRE IS **SINGLE-MODE**
(OTHERWISE IT IS **MULTIMODE**)

NORMALIZED FREQUENCY:

$$V = \frac{\pi}{\lambda} \cdot (\text{CORE DIAMETER}) \cdot (\text{NUMERICAL APERTURE})$$

⟹ NUMBER OF GUIDED MODES $\propto V^2$

- EACH LP_{nl}-MODE IS GUIDED ONLY IF:

$$V > V_{cutoff}(LP_{nl})$$

- ONLY THE FUNDAMENTAL LP_{01}-MODE MAY HAVE NO CUT-OFF

▶ MULTIMODE FIBRE

(PROPAGATES MANY MODES: ~200 TYP.)

▶ SINGLE-MODE FIBRE

(PROPAGATES ONE MODE ONLY):

CONDITION: $V < V_{cutoff,min}$ ⟹

$$\lambda > \lambda_{cutoff,min}$$

SINGLE-MODE FIBRES

CUT-OFF WAVELENGTH

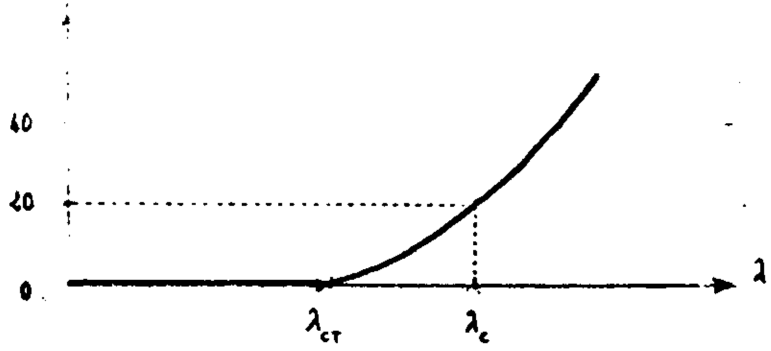
SINGLE-MODE FIBRE if $V < V_{11}^{T}$

$V = \frac{2\pi a}{\lambda} A$ (A - numerical apeture $\sqrt{n_0^2 - n_1^2} \simeq 2n_1\Delta n$)

$V_{11}^{T} = \frac{2\pi a}{\lambda_{cT}} A \quad \Longrightarrow \quad \lambda > \lambda_{cT}$

● - higher-order mode <u>theoretical</u> cut-off wavelength

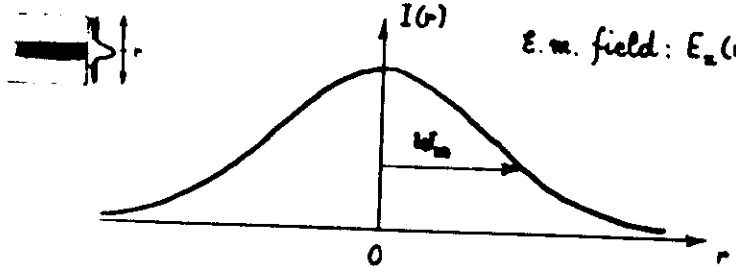
Higher-order mode radiation loss (dB/m)



MULTIMODE | ? |

● - higher-order mode <u>effective</u> cut-off wavelength

NEAR-FIELD INTENSITY: $I(r) \propto E_x^2(r), H_y^2(r)$

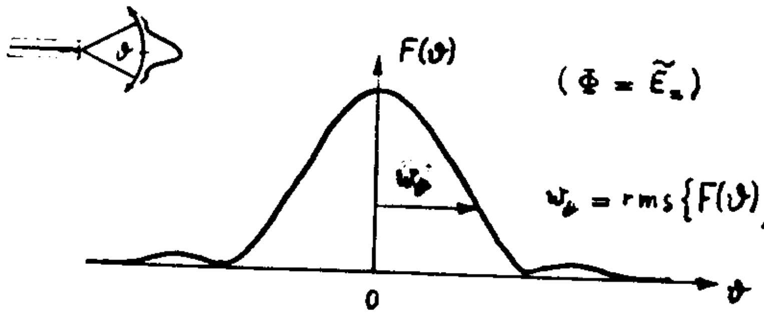


E.m. field: $E_x(r), H_y(r)$

* <u>Mode (field) radius (spot-size) 1</u>

$$\boxed{w_m = \sqrt{2} \cdot rms\{I(r)\}}$$

FAR-FIELD INTENSITY: $F(\vartheta) \propto |\Phi(\vartheta)|^2$



$(\Phi = \tilde{E}_x)$

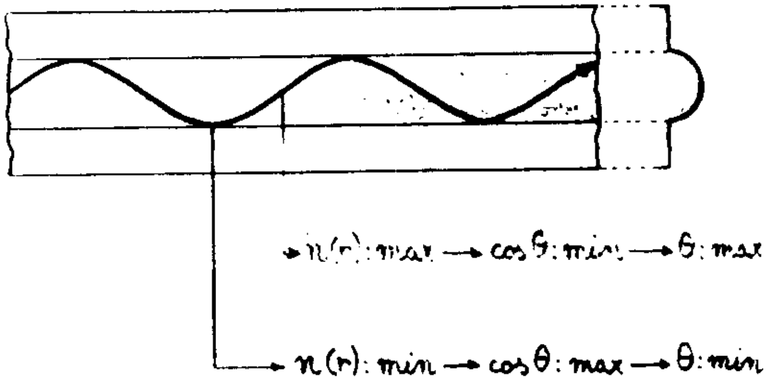
$w_\vartheta = rms\{F(\vartheta)\}$

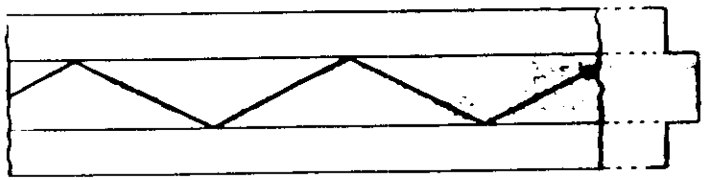
* <u>Mode radius (spot-size) 2</u>

$$\boxed{w_j = \sqrt{2}\,\frac{\lambda}{2\pi w_\vartheta}}$$

MULTIMODE FIBRES

$$\beta = \frac{2\pi}{\lambda}\, n(r) \cos\theta = \text{constant}$$





$n(r) = \text{const.} \to \theta = \text{const.}$

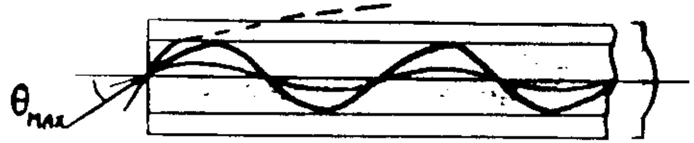
FIBRE NUMERICAL APERTURE

$$A = n \cdot \sin\theta_{MAX}$$

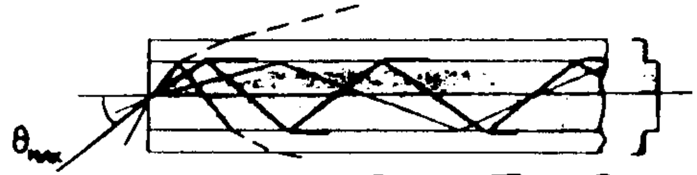
$$\left(\text{Cutoff Cond.: } \beta = \frac{2\pi}{\lambda} n(r)\cos\theta_{MAX} = \frac{2\pi}{\lambda} n_{MIN}\right)$$

$$\triangleright \quad \boxed{A(r) = \sqrt{n^2(r) - n^2_{MIN}}}$$

$$\left(A_M = \sqrt{n^2_{MAX} - n^2_{MIN}} = \begin{cases} \sim 0.1 & \text{Single-Mode} \\ \sim 0.2 & \text{Multimode} \end{cases}\right)$$



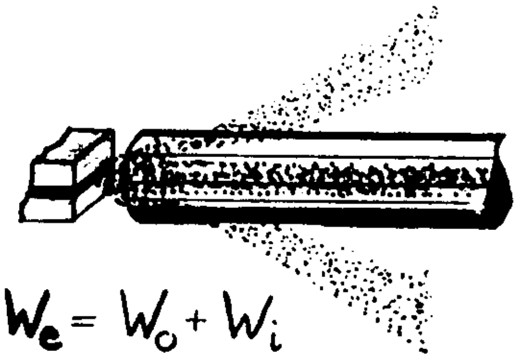
Graded-Index Prof. Fibres: Contin. Intern. Refraction



Step-Index Prof. Fibres: Total Intern. Reflection

$$\triangleright \quad V = \frac{2\pi}{\lambda}\, a\, A_M$$

LAUNCHING AND COUPLING PROBLEMS



$$W_e = W_o + W_i$$

EMITTED POWER ⟹ **GUIDED POWER** + **RADIATED (LOST) POWER**

- LAUNCHING EFFICIENCY: $\Lambda = W_o / W_e$ (% typ.)
- COUPLING LOSS: $L = W_i / W_e$ (dB typ.)

SINGLE-MODE FIBRES:

$I_e(r)$ – Emitted power radial distribution

$\boxed{\Lambda \text{ DEPENDS ON THE “LIKENESS” OF } I_e(r) \text{ AND } I(r)}$

$\Lambda = [\text{Overlap integral between } I_e \text{ and } I]^2$

* TYPICAL Λ-VAL[illegible] FOR IN[illegible] L[illegible]-FI[illegible] C[illegible]:

~20% f[illegible] [illegible]

~80% [illegible]

SOURCE-FIBRE COUPLING

- **LAUNCHING IN MULTIMODE FIBRES**

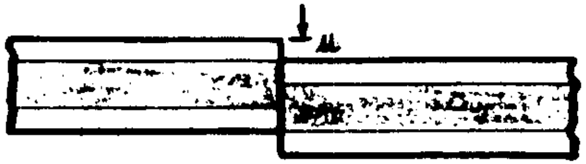
$$\Lambda \propto \frac{A_N^2}{A_{sorg}^2} \cdot R_e \cdot f(\text{PROF.}) \cdot a^2$$

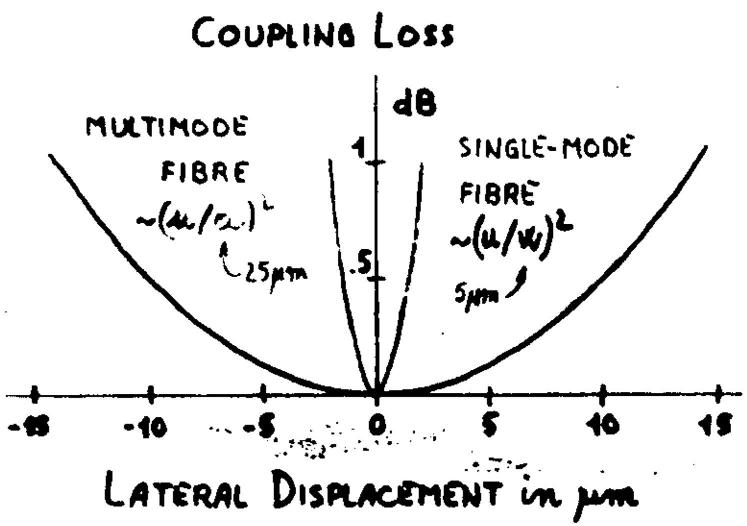
$$f(\text{PROF.}) = \begin{cases} 1/2 & \text{PARABOLIC PROFILE} \\ 1 & \text{STEP PROFILE} \end{cases}$$

- **TYPICAL VALUES:**

$$\Lambda = \begin{cases} \sim 2\% - \text{LED SOURCES} \\ \sim 50\% - \text{SEMICONDUCTOR LASER SOURCES} \end{cases}$$

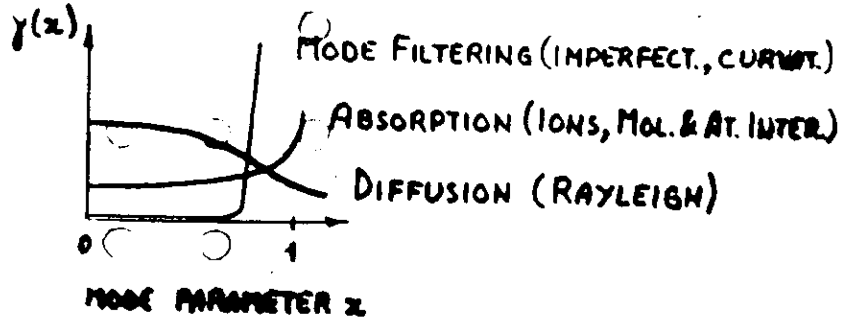
COUPLING PROBLEMS





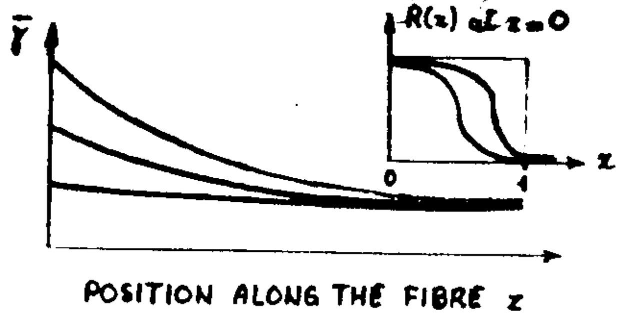
POWER ATTENUATION

* Each Mode Is Attenuated In A Different Way: $\gamma(x)$ – x-Mode Attenuation Constant



* Average Attenuation: $\bar{\gamma} = \langle \gamma(x) \rangle$ In $R(x)$
  * $\bar{\gamma}$ Depends On Launching Conditions: $R(x)$ At $z=0$
  * $\bar{\gamma}$ Depends On z-Coordinate Too

for large z: $\bar{\gamma} \to \gamma_{min}(x)$



BS

MODE CONVERSION

- CAUSED BY MICROBENDING,
  SMALL IMPERFECTIONS,
  - - - - - -
- PRODUCES: OPTICAL LOSSES,
  (CONVERS. TO RADIAT. MODES)
  MODIFICATION ON DISTRIB.
  (CONVERS. VS. GUIDED MODES)

- SINGLE-MODE FIBRES

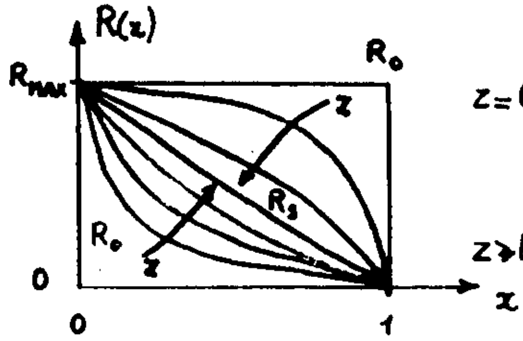
– MODE CONVERSION PRODUCES LOSSES ONLY

– LOSS COEFFICIENT $\propto w^p$ $(p \sim 2-4)$

(INCREASING FUNCTION OF MODE RADIUS)

MULTIMODE FIBRES

- for every launched $R(x)$, a steady state is reached after a length L_s: $R_s(x)$, where each mode has the same attenuation: γ_s

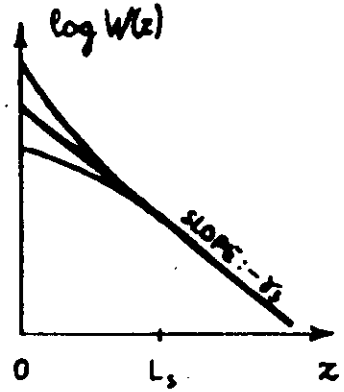


$z = 0 : R_0(x)$

⇓

$z \gg L_s : R_s(x)$

- $R_s(x)$: steady state mode distribution
- L_s : steady state length
- γ_s : steady state attenuation coefficient



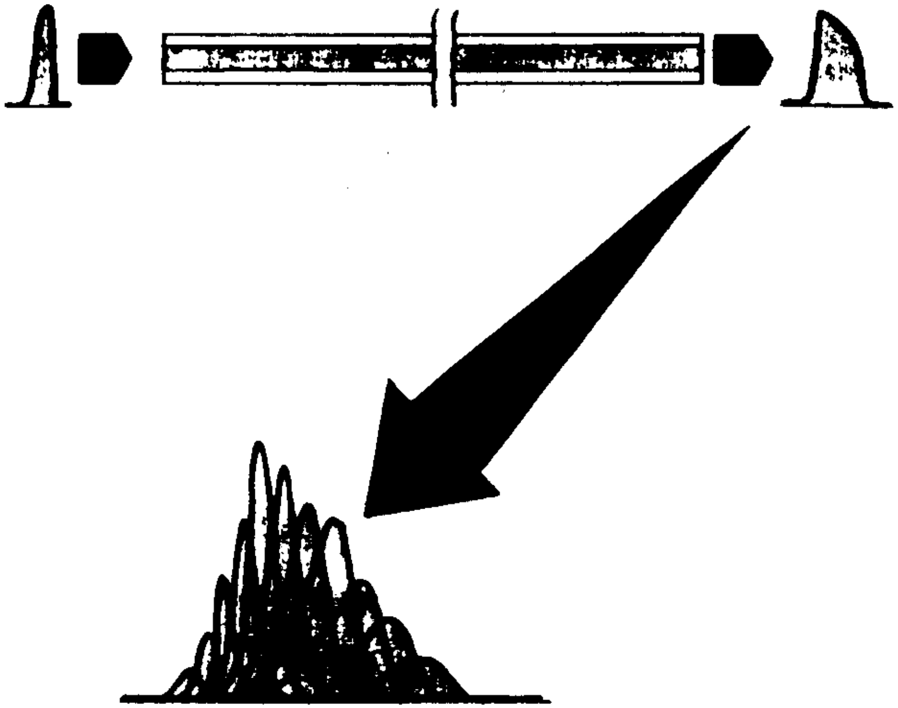
for $z \gg L_s$:

$R(x) \sim R_s(x) \cdot e^{-\gamma_s z}$

⇓

$W(z) \sim e^{-\gamma_s z}$

PULSE DISTORTION

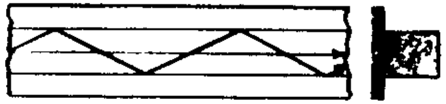


* travelling time depends on modes:
  ▶ INTERMODAL DISPERSION
* travelling time depends on λ:
  ▶ INTRAMODAL DISPERSION

INTERMODAL DISPERSION

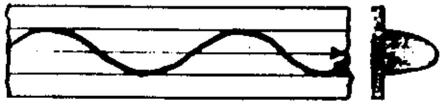
(TRAVELLING TIME DEPENDS ON MODES)

* STEP PROFILE



$\Delta t \propto (\text{Num. Apert.})^2$
~ 40 ns/km
BW ~ 5 MHz·km

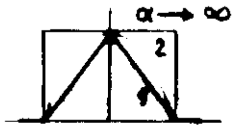
* PARABOLIC PROFILE

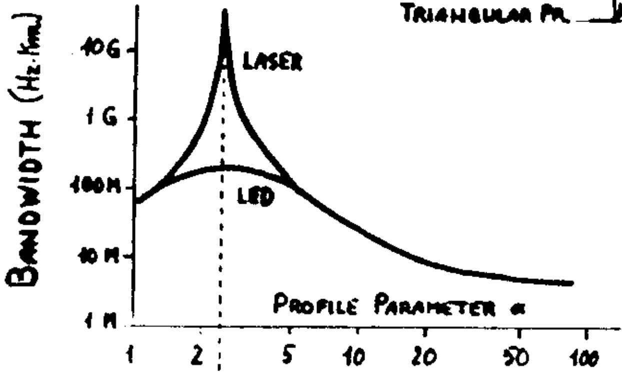


$\Delta t \propto (\text{Num. Apert.})^4$
~ 0.1 ns/km
BW ~ 500 MHz·km

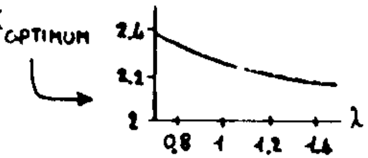
* "α" - PROFILES

STEP PROFILE
PARABOLIC PROF.
TRIANGULAR PR.

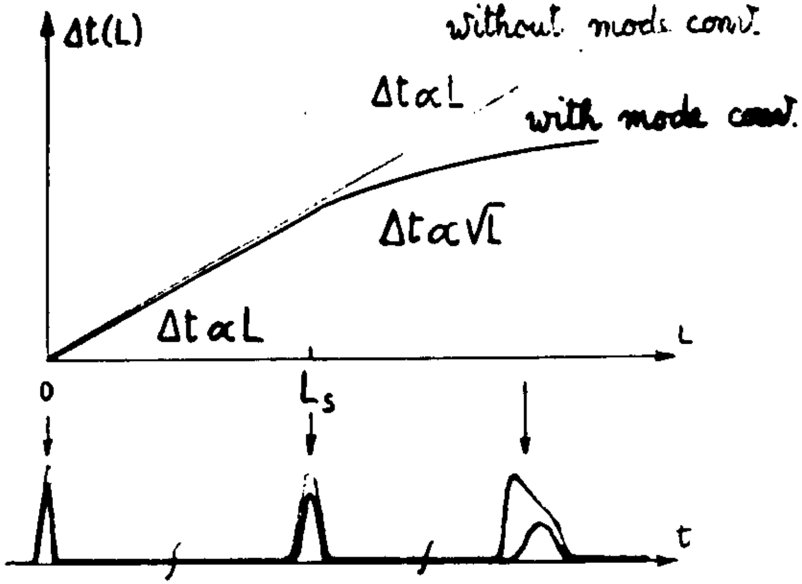


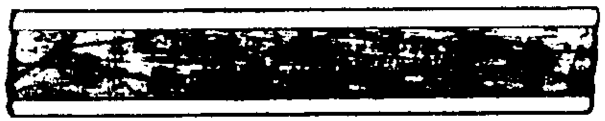


$\alpha_{OPTIMUM}$



MODE CONVERSION EFFECTS ON PULSE DISTORTION





* Statistical equalization of optical path:

$$\Delta t \propto L^{\gamma} \; : \; \gamma = 0.5 - 1 \quad ?$$

CHROMATIC DISPERSION

MODE TRAVELLING TIME DEPENDS on λ

τ : TRAVELLING TIME IN 1 km OF FIBRE

$D = d\tau / d\lambda$: CHROMATIC DISPERSION

PULSE BROADENING:

$$\Delta t = L \cdot \Delta\lambda \cdot D$$

L : FIBRE LENGTH
$\Delta\lambda$: SPECTRA WIDTH OF THE OPT. SOURCE

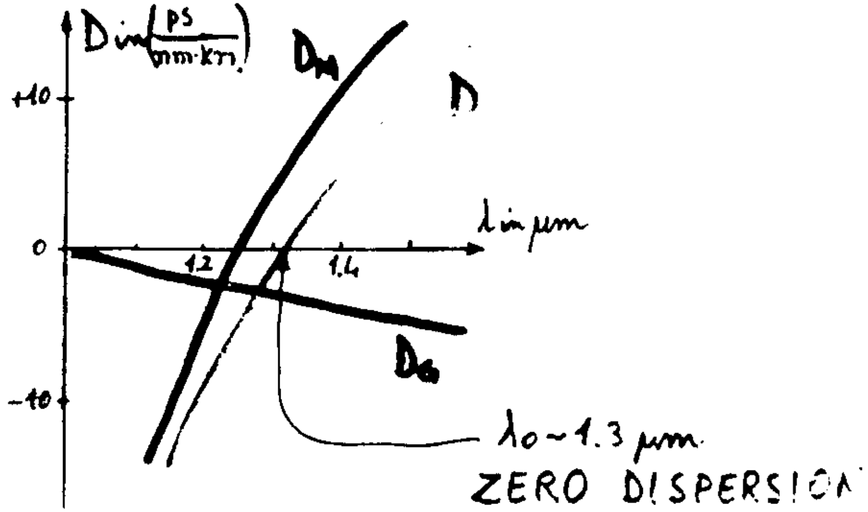
BANDWIDTH OF SINGLE-MODE FIBRES

	λ in μm	D in $\frac{ps}{nm \cdot km}$	BW in GHz·km
MULTIMODE LASER: $\Delta\lambda \sim 2$ nm	1.3	~ 1	~ 100
	1.5	~ 10	~ 10
SINGLE-MODE LASER: $\Delta\lambda \sim 0.1$ nm	1.3	~ 1	~ 2000
	1.5	~ 10	~ 200

- τ DEPENDS on λ:
  1. THROUGH THE <u>GUIDE</u> EFFECT
  2. BECAUSE THE <u>MATERIAL</u> IS DISPERSIVE $(n = n(\lambda))$

➡ D HAS TWO CONTRIBUTIONS:

1. GUIDE DISPERSION : D_G
2. MATERIAL DISPERSION: D_M



DISPERSION OPTIMIZATION