INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4.SMR/453-9

TRAINING COLLEGE ON PHYSICS AND CHARACTERIZATION OF LASERS AND OPTICAL FIBRES

(5 February - 2 March 1990)

BASIC LASER PHYSICS

TRANSITION CROSS SECTION AND SPONTANEOUS DECAY

O. Svelto

Centro di Elettronica Quantistica
e Strumentazione Elettronica
Istituto di Fisica del Politecnico
Milano, Italy

MAIN BUILDING STRADA COSTIERA, 11 TEL. 22401 TELEFAX 224163 TELEX 460392 ADRIATICO GUEST HOUSE VIA GR[illegible] [illegible]
MICROPROCESSOR LAB. VIA BEIR[illegible] TEL. 224471 TELEFAX 224163 TELEX 460392 GALILEO GUEST HOUSE VIA BE[illegible] [illegible]

Basic Laser Physics

O. Svelto

Second Lecture: Transition Cross Section and Spontaneous Decay

1. THE TRANSITION CROSS-SECTION

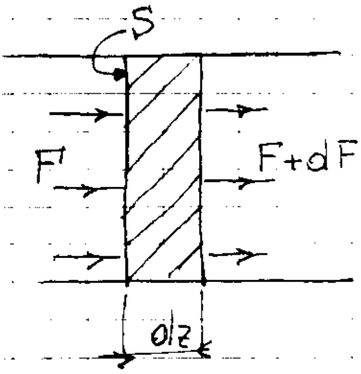
1.1 Definition

$$W = \sigma F$$

More generally ϕ = photon density = F/c

$$W = (c\phi)\sigma$$

1.2 Main property



$$dF \times S = W N_2 (S \times dz) - W N_1 (S \times dz)$$

$$\boxed{dF = W(N_2 - N_1)\,dz = \sigma F (N_2 - N_1)\,dz}$$

1.3 Thermal Equilibrium

$$\frac{N_2^e}{N_1^e} = \exp - \frac{E_2 - E_1}{kT}$$

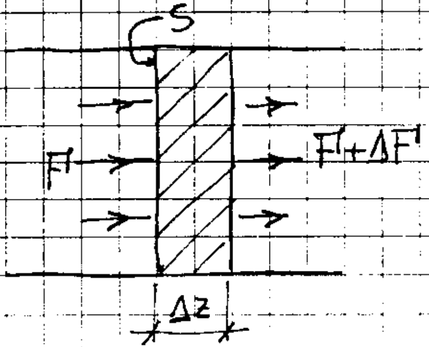
$kT \simeq (1/40)\,\mathrm{eV}$ $\quad E_2 - E_1 \simeq 1\,\mathrm{eV}$ (in the visible)

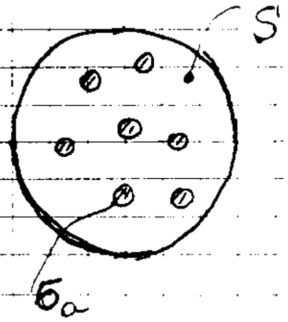
$$(N_2^e / N_1^e) \simeq \exp(-40) \ll 1$$

$$dF = -\sigma N_1 F dz = -\sigma N_t F dz$$

1.4 Intepretation of σ [1]

$$\Delta F = -\sigma N_t F \Delta z$$





$$\frac{\sigma_a (N_t S \times \Delta z)}{S} = -\frac{\Delta F}{F} \Longrightarrow \quad \sigma_a = \sigma$$

1.5 Measurement of σ [2]

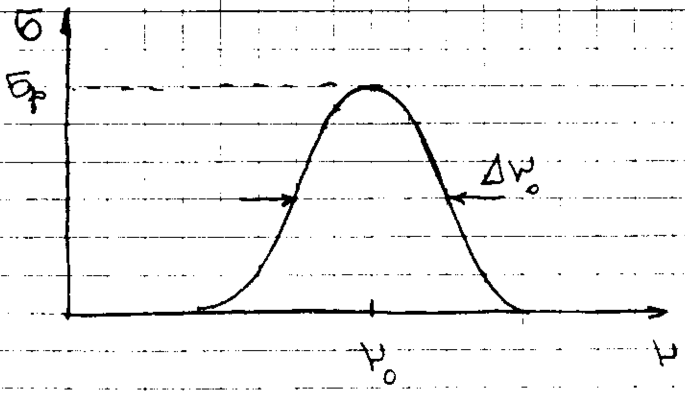
$$F(\ell) = F(0)\exp[-\sigma N_t \ell] = F(0)\exp[-\alpha \ell]$$

$$\alpha = \sigma N_t$$

$$\alpha = -\frac{1}{\ell}\ln\frac{F(\ell)}{F(0)} = -\frac{1}{\ell}\ln\frac{I(\ell)}{I(0)}$$

Instument : absorption spectrophotometer

1.6 Frequency Dependence



Peak value σ_p

For the strongest transitions (electric-dipole allowed transitions) $\sigma_p \simeq$ atom's area $\simeq 10^{-16}\ \mathrm{cm}^2$

For weaker transitions (forbidden by electric dipole) $\sigma_p \simeq 10^{-20}\ \mathrm{cm}^2$ or even lower

Ranges of frequency broadening [3]

Type	Gas	Liquid	Solid
Natural	1 kHz ÷ 10 MHz	Negligible	Negligible
Collision	5 ÷ 10 MHz/Torr	~ 300 cm^{-1}	–
Phonons	–	–	~ 10 cm^{-1} [300 °K]
Doppler	50 MHz - 1 GHz	Negligible	–
Local field	–	~ 500 cm^{-1}	1-500 cm^{-1}

2 SPONTANEOUS DECAY

2.1 Radiative (Spontaneous Emission)

$$\left(\frac{dN_2}{dt}\right)_r = -\frac{N_2}{\tau_{sp}}$$

2.2 Non-radiative [4]

$$\left(\frac{dN_2}{dt}\right)_{nr} = -\frac{N_2}{\tau_{nr}}$$

2.3 Combined Effect

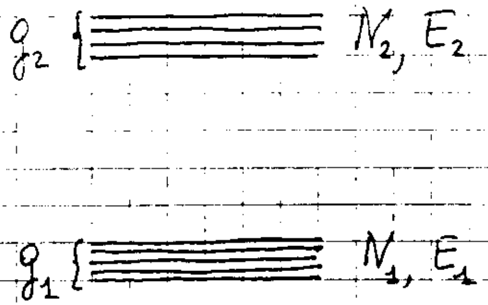
$$\frac{dN_2}{dt} = \left(\frac{dN_2}{dt}\right)_r + \left(\frac{dN_2}{dt}\right)_{nr} = -N_2\left[\frac{1}{\tau_{sp}} + \frac{1}{\tau_{nr}}\right] = -\frac{N_2}{\tau}$$

$$\frac{1}{\tau} = \frac{1}{\tau_{sp}} + \frac{1}{\tau_{nr}}$$

2.4 Overall Time Change of Level 2

$$\frac{dN_2}{dt} = -W(N_2 - N_1) - \frac{N_2}{\tau}$$

3. DEGENERATE LEVELS [4]



$$N_{2j} = \frac{N_2}{g_2}$$

$$N_{1i} = \frac{N_1}{g_1}$$

3.1 Thermal Equilibrium

$$\frac{N_{2J}}{N_{1i}} = \exp - \frac{E_2 - E_1}{kT}$$

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp - \frac{E_2 - E_1}{kT}$$

3.2 Cross-sections and Lifetime

$$\frac{dN_2}{dt} = \sum_{1\,i}^{g_1} \sum_{1\,J}^{g_2} \left[- W_{Ji} N_{2J} + W_{iJ} N_{1i} - \frac{N_{2J}}{\tau_{Ji}} \right]$$

$$\frac{dN_2}{dt} = - W \left(\frac{N_2}{g_2} - \frac{N_1}{g_1} \right) - \frac{N_2}{\tau}$$

$$W = \sum_i \sum_J W_{Ji} = \sum_i \sum_J W_{iJ}$$

$$\frac{1}{\tau} = \frac{\sum_i \sum_J (1/\tau_{Ji})}{g_2}$$

$$\boxed{\sigma_{21} = W / g_2 F \qquad \sigma_{12} = W / g_1 F}$$

$$\frac{dN_2}{dt} = - \left(\sigma_{21} N_2 - \sigma_{12} N_1 \right) - \frac{N_2}{\tau}$$

$$g_2 \sigma_{21} = g_1 \sigma_{12}$$

3.3 Variation of Photon Flux

$$dF = (\sigma_{21} N_2 - \sigma_{12} N_1) F \, dz$$

Thermal equilibrium

$$dF = -\sigma_{12} N_t F \, dz$$

REFERENCES

[1] O. Svelto : "Principles of Lasers", third Edition, Plenum Press N.Y. 1989, pp. 42-43

[2] Ref. [1] pp. 43-44

[3] Ref. [1] pp. 40-42

[4] Ref. [1] pp. 70-73

of the random distribution of atoms surrounding a given atom. The width of the line shape will now depend on the mean square fluctuation of the local electric field.

The Doppler linewidth $\Delta\nu_0^* = \Delta\omega_0^*/2\pi$ at $T = 300$ K for the Ne line of wavelength $\lambda = 0.6328$ μm (one of the lines that exhibit laser action in Ne; see Chapter 6) can be calculated from (2.78) to be

$$\Delta\nu_0^* = 1.7\,\text{GHz} \tag{2.81}$$

A comparison of this value with those evaluated for collision broadening [see (2.66)] and natural broadening shows that, in the example considered, Doppler broadening is much greater than natural broadening and this, in turn, is much greater than collision broadening. This is not always the case, however, since collision broadening dominates Doppler broadening at sufficiently high gas pressures (e.g., CO_2 laser at atmospheric pressure; see Chapter 6).

2.3.3.3. *Concluding Remarks and Examples*

In the previous two sections we have introduced a few notable examples of both homogeneous and inhomogeneous line-broadening mechanisms. We have seen that, at least for the cases considered, the shape of homogeneous line is always Lorentzian while that of inhomogeneous line is always Gaussian. For the example considered, of the Ne atom, the inhomogeneous broadening dominated the homogeneous broadening. We now consider a further example, in this case of predominantly homogeneous broadening. Figure 2.9 shows the dependence of laser linewidth on temperature for a ruby and a Nd^{3+}:YAG crystal. Ruby is a crystal of Al_2O_3 doped with Cr^{3+} ions, which substitute for

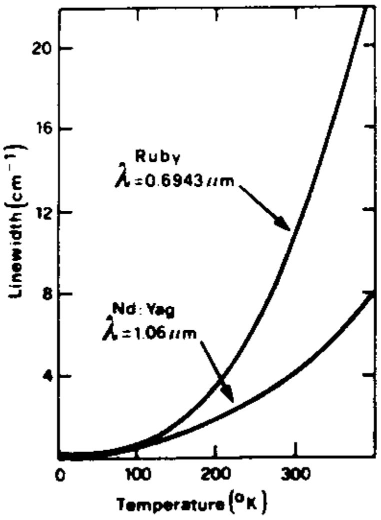


FIG. 2.9. Laser linewidth versus temperature for ruby (after Schawlow[15]) and Nd:YAG (after Siegman[16]).

some of the Al^{3+} ions in the lattice (fraction of Al^{3+} replaced by Cr^{3+} ions is typically ~0.05%). The Nd^{3+}:YAG crystal consists of YAG (an acronym for yttrium aluminum garnet, $Y_3Al_5O_{12}$) doped with Nd^{3+} ions, which substitute for some of the Y^{3+} ions in the lattice (fraction of Nd^{3+} ions is ~1%). The laser transition corresponds to one of the Cr^{3+} transitions in ruby (λ = 694.3 nm) and one of the Nd^{3+} transitions in Nd^{3+}:YAG (λ = 1.06 μm). In each case the laser linewidth is predominantly due to collisions of the ions with the lattice phonons. This explains the rapid increase of linewidth with temperature. It should be noted that the shape of the line again turns out to be approximated well by a Lorentzian curve. The residual linewidth observed as $T \rightarrow 0$ (which can be barely seen in Fig. 2.9) arises from inhomogeneous broadening due to inhomogeneities of the crystal field at the site of the Cr^{3+} or Nd^{3+} ions.

When two or more broadening mechanisms contribute a comparable broadening effect, the overall line shape is given by a convolution between the processes of the type indicated in (2.69). It can be shown that the convolution of a Lorentzian line of width $\Delta\nu_1$ with another Lorentzian line of width $\Delta\nu_2$ gives again a Lorentzian line whose width is $\Delta\nu = \Delta\nu_1 + \Delta\nu_2$. The convolution of a Gaussian line of width $\Delta\nu_1$ with another Gaussian line of width $\Delta\nu_2$ is again a Gaussian line of width $\Delta\nu = (\Delta\nu_1^2 + \Delta\nu_2^2)^{1/2}$. It is, therefore, always possible to reduce the problem to the convolution of a single Lorentzian line with a single Gaussian line, and this integral (which is known as the Voigt integral[14]) is tabulated. Sometimes, however (e.g., as in the previously discussed cases for Ne), one mechanism is predominant: in this case, it is then possible to talk about either a Lorentzian or a Gaussian line.

As a conclusion to the entire Section 2.2.3 we show in Table 2.1 the actual range of linewidths for the various line-broadening mechanisms considered. Note that, since, as we shall see in Section 2.3, $\tau_{sp} \propto \nu_0^3$, $\Delta\nu_{nat}$ decreases with decreasing frequency, becoming negligible, at the frequency of, for example, the CO_2 laser transition (λ = 10.6 μm). Note also that, according to (2.78), $\Delta\nu_0^*$ for Doppler broadening decreases with decreasing ν_0 but less quickly

TABLE 2.1. Range of Frequency Broadening for the Various Line-Broadening Mechanisms[a]

	Type	Gas	Liquid	Solid
Homogeneous	Natural	1 kHz-10 MHz	Negligible	Negligible
	Collisions	5-10 MHz/Torr	~300 cm^{-1}	—
	Phonons	—	—	~10 cm^{-1} (300 K)
Inhomogeneous	Doppler	50 MHz-1 GHz	Negligible	—
	Local field	—	~500 cm^{-1}	1-500 cm^{-1}

[a] Note that, in some cases, the frequency is expressed in cm^{-1}. To calculate the actual frequency these numbers must be multiplied by the light velocity in vacuum (~3×10^{10} cm/s).

($\Delta\nu_0^* \propto \nu_0$). It should also be noted finally that, in liquids, inhomogeneous broadening arising from local field inhomogeneities is probably the predominant broadening mechanism. Because of the very high collision rate in a liquid, however, collision broadening is also quite significant, and it is sometimes difficult to distinguish between homogeneous and inhomogeneous broadening in this case.

2.3.4. Transition Cross Section, Absorption, and Gain Coefficient

Having calculated the transition rate W, we can now go on to define and calculate other parameters that are often used to describe the given transition.

The first of these parameters is the transition cross section σ, which has already been briefly discussed in Chapter 1 [see (1.4) and (1.6)]. We have seen that, for a uniform plane wave, the transition rate is proportional to the intensity, and we can therefore define the cross section σ as

$$\sigma = W/F \tag{2.82}$$

where $F = I/h\nu$ is the photon flux of the incident e.m. wave. From (2.40) we then obtain the expression for σ as

$$\sigma = \frac{2\pi^2}{3n\varepsilon_0 c_0 h}|\mu|^2 \nu g_t(\Delta\nu) \tag{2.83}$$

We see from (2.83) that σ depends only on the material parameters ($|\mu|^2$ and g_t) and on the frequency ν of the incident wave. A knowledge of σ as a function of ν is therefore all that is needed to describe the interaction process. The transition cross section σ is therefore a very important and widely used parameter for the transition. A physical interpretation of its significance can be obtained from equation (1.7). For simplicity, we assume all the atoms to be in the ground level so that $N_2 = 0$ and $N_1 = N_t$, where the symbol N_t denotes the total population of the system. From (1.7) we then get

$$dF = -\sigma N_t F\, dz \tag{2.84}$$

Let us now suppose that we can associate with each atom an effective absorption cross section σ_a, in the sense that, if a photon enters this cross section, it will be absorbed by the atom (Fig. 2.10). If we call S the cross-sectional area of the e.m. wave in the material, the number of atoms in the element dz of the material irradiated by the wave (see also Fig. 1.2) is $N_t S\, dz$, thus giving a total absorption cross section of $\sigma_a N_t S\, dz$. The fractional change (dF/F) of photon flux in the element dz of the material is therefore

$$\frac{dF}{F} = -\frac{\sigma_a N_t S\, dz}{S} \tag{2.85}$$

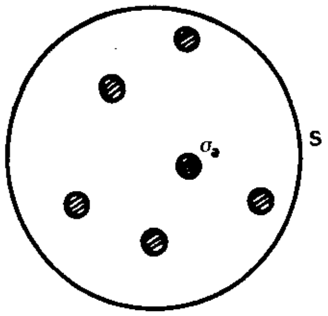


FIG. 2.10. Effective absorption cross section σ_a of atoms in a beam of cross section S.

A comparison of (2.85) with (2.84) shows that $\sigma = \sigma_a$, so that the meaning we can attribute to σ is that of an effective absorption cross section as defined above.

Another way of describing the interaction of radiation with matter is obtained by defining a coefficient α as

$$\alpha = \sigma(N_1 - N_2) \tag{2.86}$$

If $N_1 > N_2$, the quantity α is referred to as the absorption coefficient of the material. Using (2.83) the following expression for α is obtained:

$$\alpha = \frac{2\pi^2}{3n\varepsilon_0 c_0 h}(N_1 - N_2)|\mu|^2 \nu g_t(\Delta\nu) \tag{2.87}$$

Since α depends on the population of the two levels, it is not the most suitable parameter to describe the interaction in a situation where the level populations are changing, such as, for example, in a laser. Its usefulness, however, lies in the fact that the absorption coefficient α can often be directly measured. From (1.7) and (2.86) we in fact get $dF = -\alpha F\,dz$. The ratio between the photon flux after traversing a length l of the material and the incident flux is therefore $F(l)/F(0) = \exp(-\alpha l)$. By experimentally measuring this ratio with a sufficiently monochromatic radiation, we can obtain the value of α for that particular incident wavelength. The corresponding transition cross section is then obtained from (2.86) once N_1 and N_2 are known. If the medium is in thermodynamic equilibrium, N_1 and N_2 can be obtained (given a knowledge of the total population $N_t = N_1 + N_2$) with the help of (1.8). The instrument used to measure the absorption coefficient α is known as an absorption spectrophotometer. We note, however, that an absorption measurement obviously cannot be performed for a transition in which level 1 is empty. This, for instance, occurs when level 1 is not the ground level and its energy above the ground level energy is much larger than kT. As a final observation we note that if $N_2 > N_1$, the absorption coefficient α, defined by (2.86), becomes negative and, of course, the wave gets amplified rather than absorbed in the material. In this case it is customary to define the new quantity

$$\alpha_g = -\alpha = \sigma(N_2 - N_1) \tag{2.88}$$

which is positive and is called the gain coefficient. We will also define

$$g = \alpha_g l \tag{2.88a}$$

where l is the length of the active material. The quantity g is called the (logarithmic) gain of the material.

We now summarize the discussion of this section. Three transition coefficients, W, σ, and α have been introduced. They represent three different ways of describing the absorption and stimulated emission phenomena. The relative merits of the three parameters can be summarized as follows: (1) The transition rate W has a simple physical meaning [see equations (1.3) and (1.5)] and it is also directly obtained from the quantum mechanical calculation. (2) The transition cross section σ depends only on the characteristics of the given material. (3) The absorption coefficient α is a parameter that can often be experimentally measured in a straightforward way.

2.4. SPONTANEOUS EMISSION

The purpose of this section is to calculate the probability A of spontaneous emission, A being defined as in equation (1.2). Unfortunately, as discussed later in this section, the semiclassical treatment of the interaction between radiation and matter does not permit a correct prediction and understanding of the phenomenon of spontaneous emission. It is, nevertheless, instructive to first develop a treatment of this phenomenon based on semiclassical considerations. The results obtained will then be compared with the exact treatment, based on quantum electrodynamics, in which both atoms and radiation are quantized.

2.4.1. Semiclassical Approach

Let us first consider, from a purely classical viewpoint, an electric dipole oscillating at frequency ω_0. If the positive charge is assumed to be fixed, the instantaneous position $\mathbf{r}$ of the negative charge, with reference to a coordinate system whose origin is centered on the positive charge, can be written as

$$\mathbf{r} = \mathbf{r}_0 \cos(\omega_0 t + \phi) = \mathrm{Re}[\mathbf{r}'_0 \exp(i\omega_0 t)] \tag{2.89}$$

From the above discussion of ASE it is apparent that, strictly speaking, a threshold for ASE does not exist. Since, however, the power P emitted by ASE increases rapidly with inversion {approximately as $[\exp(\sigma_0 N_2 l)]/(\sigma_0 N_2 l)$; see (2.150)}, when the "threshold" conditions given by (2.153) or (2.153a) are exceeded, the ASE phenomenon rapidly becomes the predominant decay mechanism for the active medium. The absence of a true threshold is therefore a feature that distinguishes ASE from superfluorescence. A second distinguishing feature is that, for superfluorescence, the length of the active material must be smaller than a critical cooperative length l_c, while no such limitation exists for ASE. A third distinctive feature is that the emission solid angle for ASE is established by geometrical considerations and it is usually much greater than for superfluorescence, which is determined by diffraction. Note finally that the phenomenon of ASE may be used to advantage to obtain a reasonably directional emission in some high-gain laser oscillators (e.g., nitrogen or excimer lasers), while it can be a nuisance in high-gain laser amplifiers (e.g., excimer, dye, or neodimium) since it depopulates the available inversion.

2.8. *DEGENERATE LEVELS*

So far we have considered only the simplest case, in which both levels 1 and 2 are nondegenerate. Let us briefly see what happens when the levels are degenerate, a situation that very often occurs in practice. This is depicted in Fig. 2.22, where level 1 is assumed to be g_1-fold degenerate and level 2, g_2-fold degenerate. Let us write N_1 for the total population of the manifold of degenerate lower levels and N_2 for the higher levels. We will use N_{2j} and N_{1i} to indicate the population of a particular level of the upper or lower manifolds, respectively.

The simplest case we might consider is that of levels in thermal equilibrium. In this case, the population of each sublevel of both upper and lower state must obey the usual Boltzmann equation, thus

$$N^e_{2j} = N^e_{1i} \exp[-(E_2 - E_1)/kT] \tag{2.154}$$

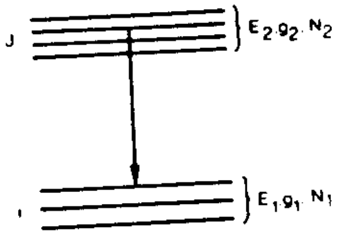


FIG. 2.22. Two-level system with degeneracy g_1 and g_2 corresponding to each level.

Since, however, the sublevels of, e.g., level 1 are also in thermal equilibrium, their populations must all be equal, thus

$$N_{1i}^{e} = N_{1}^{e}/g_{1} \tag{2.155a}$$

Similarly we have

$$N_{2j}^{e} = N_{2}^{e}/g_{2} \tag{2.155b}$$

From (2.154) and (2.155) we then get

$$N_{2}^{e} = N_{1}^{e}\left(\frac{g_{2}}{g_{1}}\right)\exp[-(E_{2}-E_{1})/kT] \tag{2.156}$$

A comparison of (2.156) with (1.8) then shows that the latter equation holds not only for nondegenerate levels but also for degenerate levels with the same degeneracy (i.e., $g_1 = g_2$).

Let us now see how the expressions for transition cross section, gain, and absorption coefficient need to be modified in the case of degenerate levels. For this purpose we consider an e.m. wave passing through a material with given populations in the two levels, and we address the question of how (2.131a) and (2.131b) are to be modified. Obviously, (2.131a) is still valid. The rate of change of the total population N_2 of the upper level must now include all possible transitions between levels i and j. Therefore,

$$\left(\frac{dN_{2}}{dt}\right) = -\sum_{i=1}^{g_{1}}\sum_{j=1}^{g_{2}}\left(W_{ji}N_{2j} - W_{ij}N_{1i} + \frac{N_{2j}}{\tau_{ji}}\right) \tag{2.157}$$

On the other hand, if there is rapid relaxation between the sublevels, all sublevels of the upper state will again be equally populated, and the same will apply to the sublevels of the lower state. Therefore

$$N_{2j} = N_{2}/g_{2} \tag{2.158a}$$

$$N_{1i} = N_{1}/g_{1} \tag{2.158b}$$

Upon substitution of (2.158) into (2.157), we get

$$\dot{N}_{2} = -W\left(\frac{N_{2}}{g_{2}} - \frac{N_{1}}{g_{1}}\right) - \frac{N_{2}}{\tau} \tag{2.159}$$

where we have defined

$$W = \sum_{i=1}^{g_{1}}\sum_{j=1}^{g_{2}} W_{ij} = \sum_{i=1}^{g_{1}}\sum_{j=1}^{g_{2}} W_{ji} \tag{2.160}$$

and

$$\frac{1}{\tau} = \frac{\sum_{i=1}^{g_{1}}\sum_{j=1}^{g_{2}}(1/\tau_{ji})}{g_{2}} \tag{2.161}$$

The change in photon flux dF when the beam travels a distance dz in the material (see Fig. 1.2) can now be written, from (2.159), as

$$dF = W\left(\frac{N_2}{g_2} - \frac{N_1}{g_1}\right) dz \tag{2.162}$$

We can then define a stimulated emission cross section, σ_{21}, and absorption cross section, σ_{12}, as [compare with (2.82)]

$$\sigma_{21} = W/(g_2 F) \tag{2.163a}$$

$$\sigma_{12} = W/(g_1 F) \tag{2.163b}$$

from which we obviously have

$$g_2\sigma_{21} = g_1\sigma_{12} \tag{2.164}$$

When $(N_1/g_1) > (N_2/g_2)$ equation (2.162) with the help of (2.163b) can be put in the familiar form $dF = -\alpha F\,dz$ provided we define the absorption coefficient α as

$$\alpha = \sigma_{12}\left(N_1 - N_2\frac{g_1}{g_2}\right) \tag{2.165}$$

Similarly, when $(N_2/g_2) > (N_1/g_1)$, equation (2.162) with the help of (2.163a) can be put in the familiar form $dF = gF\,dz$ provided we define the gain coefficient g as

$$g = \sigma_{21}\left(N_2 - N_1\frac{g_2}{g_1}\right) \tag{2.166}$$

The reason for defining σ_{21} and σ_{12}, respectively, as in (2.163a) and (2.163b) is now apparent. When in fact $N_1 \gg N_2$ (as usually applies to absorption measurements involving optical transitions) (2.165) simply reduces to $\alpha = \sigma_{12}N_1$. Conversely, when $N_2 \gg N_1$ (as applies in a four-level laser), (2.166) simply reduces to $g = \sigma_{21}N_2$.

Another case of interest is where the upper (2) or lower (1) levels actually consist of sublevels (themselves degenerate) differing in energy, but with a relaxation between sublevels that can again be considered instantaneous. In this case, thermalization between each of the sublevels of levels 1 and 2 will occur and, instead of (2.158), we write

$$N_{2j} = z_{2j}N_2 \tag{2.167a}$$

$$N_{1i} = z_{1i}N_1 \tag{2.167b}$$

where $z_{2j}(z_{1i})$ is the fraction of total population of level 2 (level 1) that, according to Boltzmann's statistics, is found in sublevel $j(i)$ (*the level partition function*). With the further assumption that the stimulated transition is from a given sublevel (say l) of level 2 to a given sublevel (say m) of level 1, equation (2.157), with the help of (2.167a) and (2.167b), can be written as

$$\frac{dN_2}{dt} = -W_{21}N_2 + W_{12}N_1 - \frac{N_2}{\tau} \tag{2.168}$$

where we have defined the effective rates of stimulated emission, W_{21}, stimulated absorption, W_{12}, and spontaneous decay $(1/\tau)$, respectively, as

$$W_{21} = z_{2l}W_{lm} \tag{2.169a}$$

$$W_{12} = z_{1m}W_{ml} \tag{2.169b}$$

$$\frac{1}{\tau} = \sum_{ji} \frac{z_{2j}}{\tau_{ji}} \tag{2.169c}$$

Note that, according to (2.169a) and (2.169b), we can define an effective stimulated emission cross section, σ_{21}, and an effective absorption cross section, σ_{12}, as

$$\sigma_{21} = z_{2l}\sigma \tag{2.170a}$$

$$\sigma_{12} = z_{1m}\sigma \tag{2.170b}$$

where $\sigma = \sigma_{lm} = W_{lm}/F$ is the actual cross section of the given transition.

2.9. MOLECULAR SYSTEMS

The purpose of this section is to specialize some of the results and considerations of previous sections to the particular case of molecular media, since these play a very important role in the laser field. Here again we will limit our description to the coarse features of the complex phenomena that occur. Our discussion does, however, provide the basis for a deeper understanding of the laser physics of systems such as molecular gas or dye lasers. For a more extensive treatment of this subject the reader is referred to specialized texts.(17)

2.9.1. *Energy Levels of a Molecule*

The total energy of a molecule consists generally of a sum of four contributions: (1) electronic energy E_e, due to the motion of electrons about