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SMR/455 - 11

EXPERIMENTAL WORKSHOP ON HIGH TEMPERATURE SUPERCONDUCTORS & RELATED MATERIALS (BASIC ACTIVITIES)

12 - 30 MARCH 1990

INTRODUCTION TO SUPERCONDUCTIVITY

PART II

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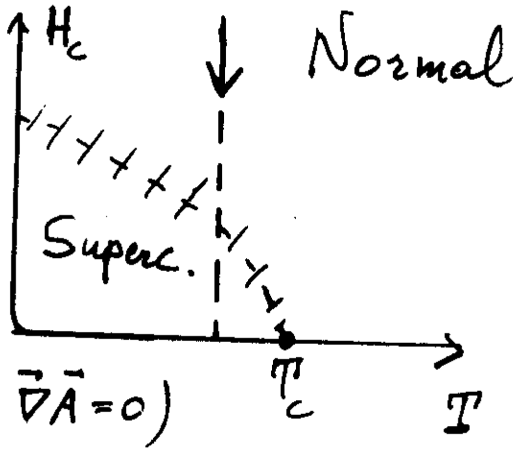
These are preliminary lecture notes, intended only for distribution to participants.

Nucleation in bulk samples :



H along z

$\vec{A} \equiv (0, Hx, 0)$ (gauge chosen $\vec{\nabla}\vec{A} = 0$)

ψ is very small if nucleating a superconducting domain : linearize the G.L. eqn. :

$$\xi^2 \left(\vec{\nabla} + \frac{i}{\Phi_0} \vec{A} \right)^2 \psi = \psi$$

assume : $\psi = e^{i k_y y} e^{i k_z z} f(x)$

$$- f'' + \left(\frac{H}{\Phi_0} \right)^2 (x - x_0)^2 f = \left(\frac{1}{\xi^2} - k_z^2 \right) f$$

$$x_0 = \frac{k_y \Phi_0}{H}$$

similar to the Schrödinger eqn for a particle in a constant magnetic field :

$$\omega_c = \frac{2eH}{m^* c} \quad : \quad \left(n + \frac{1}{2} \right) \hbar \omega_c = \frac{\hbar^2}{2m^*} \left(\frac{1}{\xi^2} - k_z^2 \right)$$

highest H is for $n = 0$, $k_z = 0$

$$\frac{1}{2}\omega_c = \frac{\hbar}{2m^*\xi^2} \quad \Longrightarrow \quad \bar{H} = \frac{\Phi_0}{\xi^2}$$

because $\quad \varkappa = \dfrac{\Phi_0}{\sqrt{2}\, H_c \xi^2}$

$$\bar{H} = \frac{\Phi_0}{\xi^2} = \sqrt{2}\,\varkappa\, H_c$$

$\varkappa^2 < \frac{1}{2} \qquad H_c > \bar{H}$: H_c comes first $\Rightarrow$ type I

$\varkappa^2 > \frac{1}{2} \qquad H_c < \bar{H} \equiv H_{c2} = \dfrac{\Phi_0}{\xi^2} \Longrightarrow$ type II

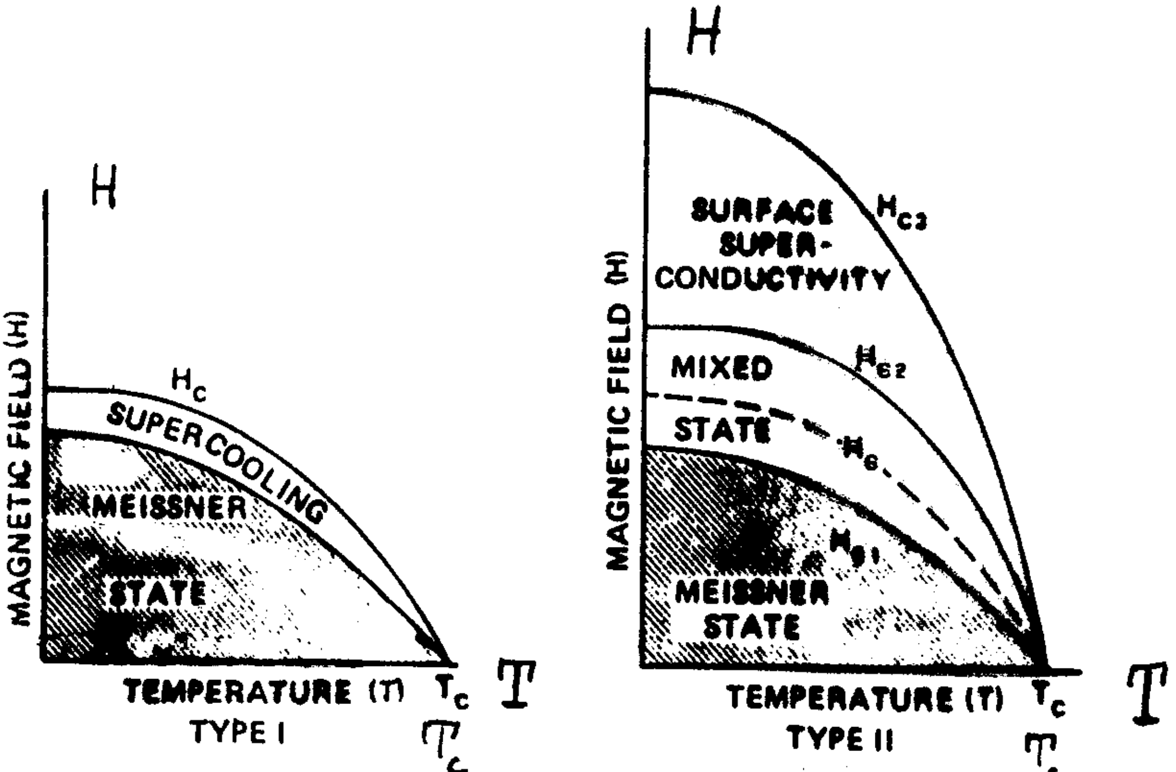


FIG. V.5. Phase diagram for type I and type II superconductors.

c19

... in an external magnetic field : Intermediate state

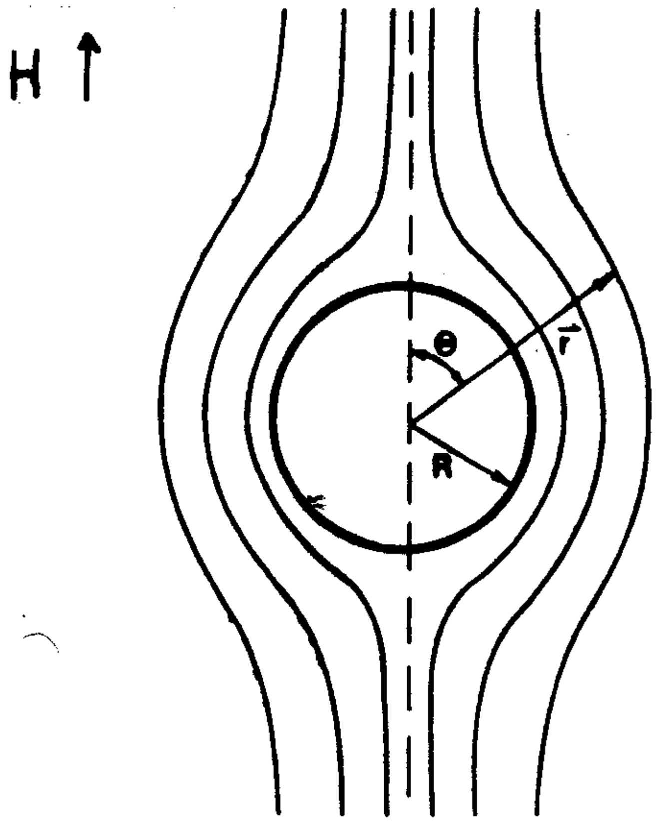
Outside a sphere: $\vec{B}\cdot\vec{n} = 0$ at $r = R$

$\vec{B} \to \vec{H}$ as $r \to \infty$

$$\vec{\nabla}\cdot\vec{B} = \vec{\nabla}\times\vec{B} = 0 \implies \vec{B} = \vec{H} + \frac{HR^3}{2}\vec{\nabla}\left(\frac{\cos\theta}{r^2}\right)$$

$$B_\theta(R) = \frac{3}{2} H \sin\theta$$

$\Rightarrow$ for $\frac{2}{3} H_c < H < H_c$ some portions of the sphere superconducting, some normal



Configuration

(b)

depends on the wall energy

(a)

FIG. VIII.15. Intermediate state patterns constructed from field mappings made with a thin bismuth wire in a 0.2 mm gap between two tin hemispheres of diameter 4 cm.

Fraction of normal material :

$$\frac{\sigma_{norm}}{\sigma_{norm} + \sigma_{superc.}} = \frac{B}{H_c}$$

B $\longrightarrow \infty$ $\vec{B} = \vec{H}$

$H \uparrow$

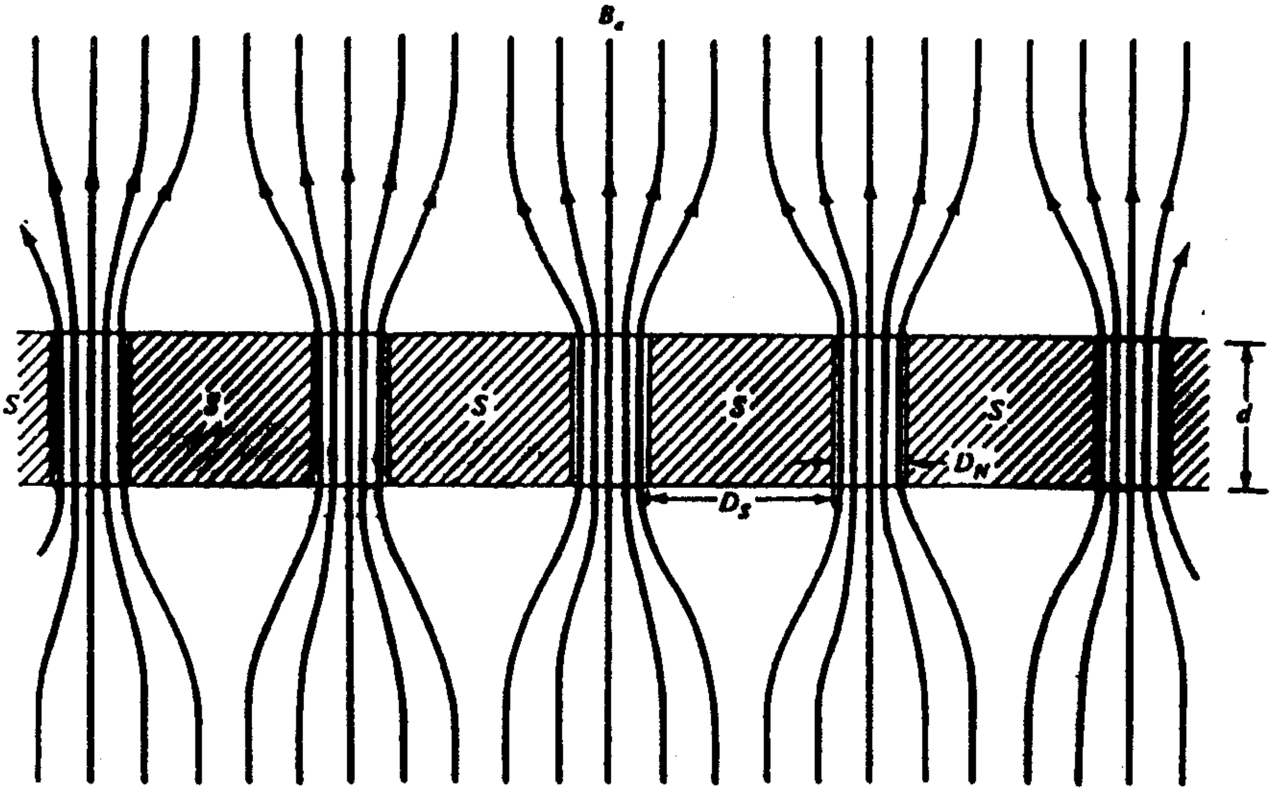


FIGURE 3-6
Schematic diagram showing magnetic flux channeling through the normal laminae in the intermediate state. Flux density is B_a at large distances, and zero or h_n ($\approx H_c$) in the cross section of the slab.

Vortex solution of the G.L. eqns :

3 along the z axis : cylindrical coordinates (r, ϑ, z)

Form of the solution is :

$$\begin{cases} \psi(r) = n_s^{1/2} f(r)\, e^{i m \vartheta} \\ \vec{A} \equiv (0, A_\vartheta(r), 0) \end{cases}$$

single valuedness of $\psi \Rightarrow m = \text{integer}$

Far from the vortex core : $|\psi|^2 = n_s$

$$J_\vartheta = \frac{\hbar e^*}{m^*}\left(\frac{m}{r} - \frac{A_\vartheta}{\Phi_0}\right)|\psi|^2 \longrightarrow 0$$

or, integrating along a line :

$$\Phi(B) = \int_\Gamma \vec{A}\cdot d\vec{\ell} = m\,\Phi_0 \underbrace{\int r\,d\vartheta\,\frac{1}{r}}_{= 2\pi}$$

The vortex carries an integer number of flux lines $\phi_0 = \dfrac{hc}{2e}$

d1

In the G.L. free energy: $(r > r_{core})$

$$\frac{H_c^2 \xi^2}{4\pi n_s} \left| \left(\vec{\nabla} + \frac{i}{\Phi_0} \vec{A} \right) \psi \right|^2 = 2\pi \lambda^2 c^2 J_\vartheta^2 =$$

$$= \frac{1}{8\pi} \lambda^2 (\mathrm{curl}\, B)^2$$

$$\mathcal{F}_{r > r_c} = \frac{1}{8\pi} \left\{ \lambda^2 (\mathrm{curl}\, B)^2 + B^2 - H_c^2 \right\}$$

Minimization with respect to B yelds:

London eqn : $\lambda^2 \,\mathrm{curl}\,\mathrm{curl}\, B + B = 0$

except at the origin:

boundary condition $\Phi(B) = m \phi_0$

becomes:

$$\frac{\partial^2 B}{\partial r^2} - \frac{1}{r} \frac{\partial B}{\partial r} - \frac{1}{\lambda^2} B = 0 \qquad + \text{b.c.}$$

solution: $\vec{B}(r) = \hat{z} \dfrac{\phi_0 m}{\lambda^2} K_0\left(\dfrac{r}{\lambda}\right) \qquad \left(\dfrac{r}{\xi} > 1\right)$

Bessel function $\sim e^{-\frac{r}{\lambda}}$ $r \to \infty$

$\sim \ln \frac{r}{\lambda}$ $r \to 0$

d 2

Asymptotic behavior :

$$f(r) \to 1 + \Theta\left(e^{-\frac{r}{\xi}}\right)$$

$$A_\vartheta(r) \to \frac{m}{r}\left(1 + \Theta\left(e^{-\frac{r}{\lambda}}\right)\right)$$

Only parameter is

$$\varkappa = \frac{\lambda}{\xi}$$

Multi-vortex solution :

$\varkappa < \frac{1}{\sqrt{2}}$ vortices are unstable ; they collapse one over another

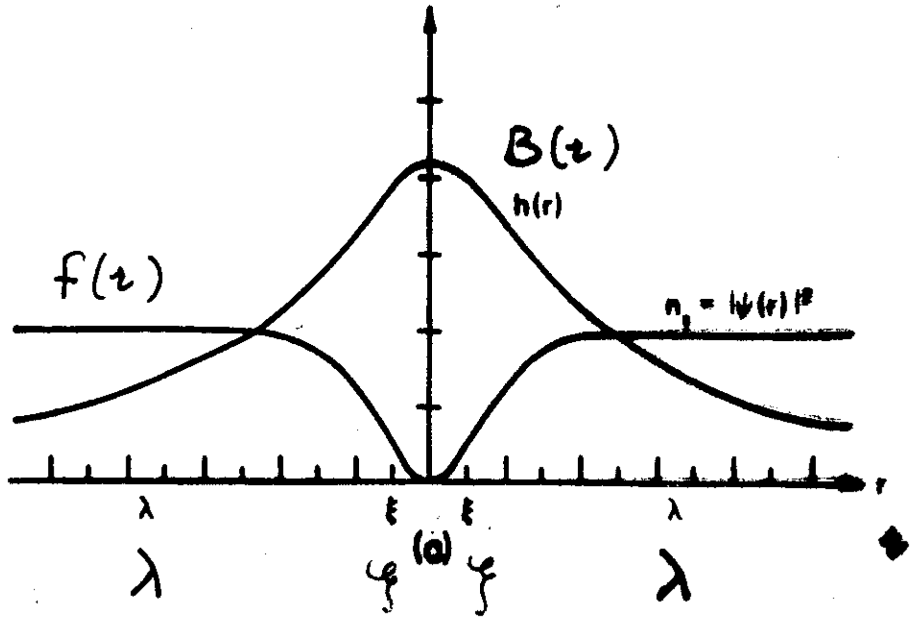
$\varkappa > \frac{1}{\sqrt{2}}$ vortices are stable ; they repel each other

$\varkappa = \frac{1}{\sqrt{2}}$ they do not interact. (Jacobs & Rebbi '79)

d.3

Isolated vortex in a type II superconductor

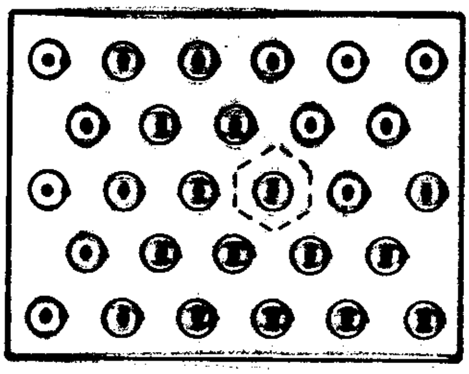
$\lambda >> \xi$



H

Vortex lattice

type II supercond.



(b)

FIG. VIII.17. (a) Structure of isolated vortex. The normal core and the decay of the magnetic field away from center are clearly evident. Curves shown for $\kappa = 10$. (b) Quantized vortex lattice of a type II superconductor. Figure shows normal cores surrounded by circulating currents. Dashed line indicates unit cells of lattice. The area of this unit cell contains one quantum Φ_0 of magnetic flux.

One flux quantum Φ_0

in each unit cell

d4

type II superc. : $\lambda \gg \xi$

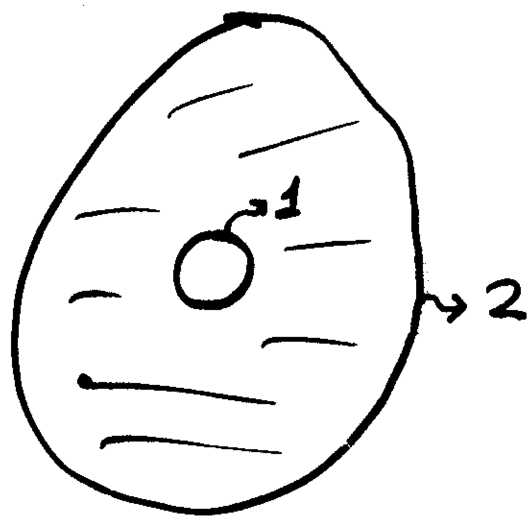
Energy of an isolated vortex line (per unit length)

The normal core is excluded :

$$\epsilon = \frac{1}{8\pi} \int_{\substack{\text{outside} \\ \text{the core}}} \left(B^2 + \lambda^2 |\operatorname{curl} B|^2 \right) dS$$

$$= \cancel{\frac{1}{8\pi} \int_{\substack{\text{core} \\ \text{excluded}}} \left(\vec{B} + \lambda^2 \operatorname{curl} \operatorname{curl} \vec{B} \right) \cdot \vec{B}\, dS} \; + \qquad \text{vanishes due to London eqn}$$

$$+ \frac{\lambda^2}{8\pi} \oint_{\substack{\text{boundary} \\ 1+2}} \left(\vec{B} \times \operatorname{curl} \vec{B} \right) \cdot d\vec{\ell}$$



$$\epsilon = \frac{\lambda^2}{8\pi} \; B \frac{dB}{dr} \cdot 2\pi r \Big|_{r=\xi}$$

$$= \left(\frac{m \Phi_0}{2\lambda} \right)^2 \ln \frac{\lambda}{\xi}$$

Energy spent by the sources to create a vortex with $\Phi = m \phi_0$

d5

two remarks :

+) because $m\,\phi_0$ appears squared, vortices including just one flux line are favoured

++) because $\phi_0 = \sqrt{2}\, H_c \lambda \xi$

$$\epsilon = \frac{H_c^2}{8\pi} \cdot \pi \xi^2 \cdot 4 \ln \kappa$$

= to the condensation energy lost in the vortex core $\times\, 4 \ln \kappa$

First vortex appears in the sample

when $$G_s\Big|_{\substack{\text{no}\\ \text{flux}}} = G_s\Big|_{\substack{\text{+ first}\\ \text{vortex}}}$$ for $H = H_{c1}$

$$F_s = F_s + \epsilon L - \frac{\vec{H}_{c1}}{4\pi} \cdot \int \vec{B}\, dv$$

or when :

$$\epsilon = \frac{H_{c1}}{4\pi}\, \phi_0$$

definition of H_{c1}

Interaction energy between two vortex lines: 1, 2

(per unit length)

|| arises from a term in the free energy:

$$E^{int}_{1,2} = \frac{\Phi_0}{4\pi} \underbrace{B_1(r_2)}$$

field due to line 1 evaluated at point 2

gives place to a repulsive force:

$$f_{2,x} = -\frac{\partial E^{int}_{1,2}}{\partial x_2} = -\frac{\Phi_0}{4\pi} \frac{\partial B_1(r_2)}{\partial x_2}$$

force on vortex 2 along x direction

$$= \frac{\Phi_0}{c} J_{1,y}(r_2)$$

y-component of current density due to vortex 1 evaluated at r_2

d 7

Growing number of vortices arranges in a

Triangular lattice

nearest neighbour distance is $\quad a_{\Delta} = 1.075 \left(\dfrac{\phi_0}{B} \right)^{1/2}$

This value makes the Gibbs function (per unit volume) reach a minimum :

$$G - G_s \Big|_{\substack{no \\ flux}} = \frac{B}{\phi_0} \epsilon + \sum_{i>j} E^{int}_{i,j} \cdot \frac{1}{\underset{\text{surface}}{S}} - \frac{BH}{4\pi}$$

For fixed H, the value of $B(H)$ which correspond to the equilibrium configuration is given by :

$$\frac{\partial G}{\partial B} = 0$$

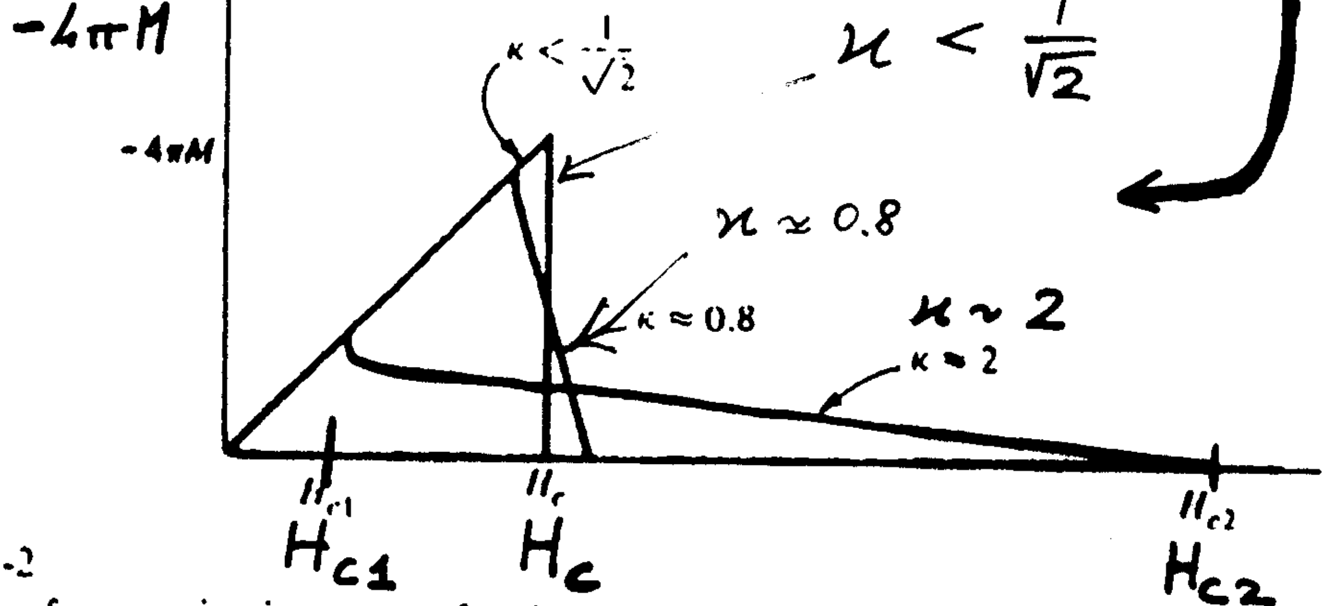
d8

$$2\pi H_{c2} \xi^2 = \phi_0$$ definition of H_{c2}

$\sim$ closest packing of vortices

the area under the curve is the condensation energy

$$-\int \vec{M} \cdot d\vec{H} = H_c^2 / 8\pi$$



5-2

…on of magnetization curves for three superconductors with the same value …odynamic critical field H_c, but different values of κ. For $\kappa < 1/\sqrt{2}$, the …ductor is of type I and exhibits a first-order transition at H_c. For $\kappa > 1/\sqrt{2}$, …nductor is type II and shows second-order transitions at H_{c1} and H_{c2} (for …ked only for the highest κ case). In all cases, the area under the curve is …nsation energy $H_c^2/8\pi$.

Superconducting magnets : flux flow

In a "dirty" superconductor : $\xi \sim \xi_0 \ell$ mean free path

$$H_{c2} = \frac{\Phi_0}{\xi^2} \simeq \frac{3c}{e} \frac{k_B T_c}{v_F \ell} \simeq 10 \text{ Tesla}$$

A current gives rise to a force on the vortices :

$$\vec{f} = \vec{J} \times \frac{\phi_0}{c} \hat{z}$$ z direction

flux lines move transverse to the current unless pinning occurs

Flux trapped inside the cylinder is reduced at the rate :

$$\phi_0 \frac{dm}{dt}$$

An e.m.f. is induced against the flux reduction :

$$\mathcal{E} = -\frac{1}{c} \frac{d\Phi}{dt} = \frac{1}{c} B \, 2\pi R v$$

outward velocity of the flux density

d10

Trapped flux : $\Phi = m\,\phi_0$

flux lines escape at the rate :

$$\phi_0 \, \frac{dm}{dt}$$

$\vec{J}_s$:

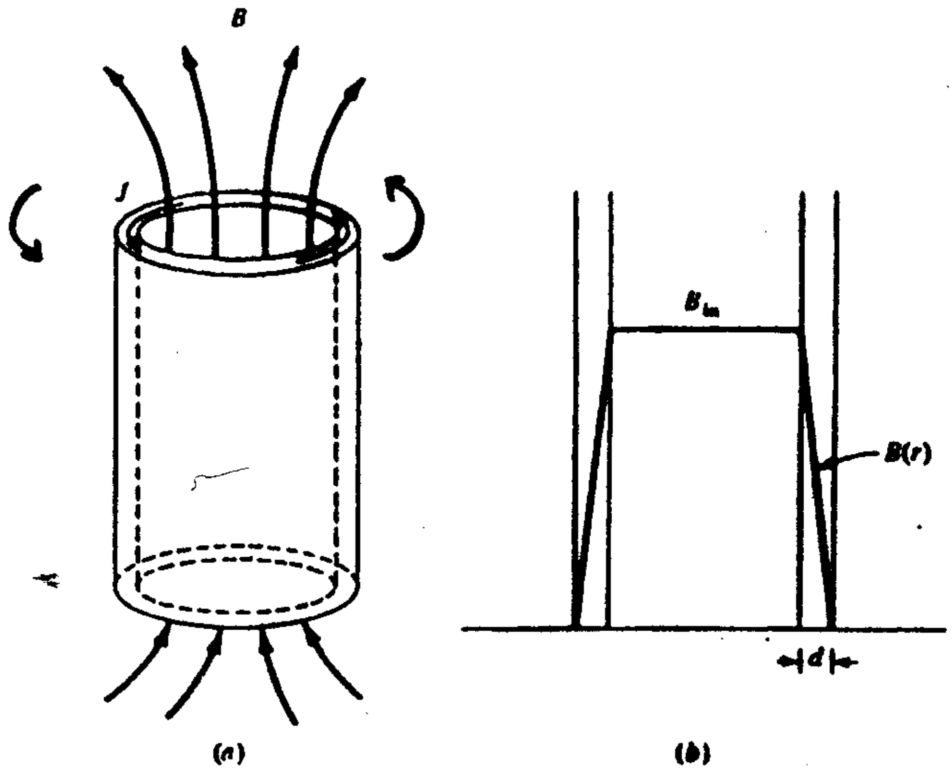


FIGURE 5-3
Flux trapped in hollow cylinder of type II superconductor. (*a*) Sketch of overall geometry. (*b*) Local flux-density profile.

The velocity of flux lines v is constant

viscous drag : electronic excitations in the normal core of the vortices :

$$\vec{f} = \frac{J}{c}\phi_0 \,\hat{x} = \eta \vec{v}$$

Energy dissipation at the rate $\vec{E} \cdot \vec{J}$

$\Longrightarrow$ resistivity : $\rho_f = \frac{E}{J} = \frac{B\,\phi_0}{\eta\, c^2}$

η : e.g. Bardeen - Stephen model:

fully normal vortex core of radius ξ

$$\eta \approx \frac{\phi_0 H_{c2}}{\rho_n c^2} \Longrightarrow \frac{\rho_f}{\rho_n} \approx \frac{B}{H_{c2}}$$

ρ_n: normal resistivity

ρ_f joins smoothly onto ρ_n at H_{c2}

$\Rightarrow$ second order phase transition

Inhomogeneities in the sample

extended more than ξ can cause

pinning of the flux lines

$\Longrightarrow$ no energy losses

except for

thermally activated hopping from one pinning site to another

called flux creep

d 12

"Dirty" type II superconducting films:

thickness $d < \xi, \lambda \Rightarrow$ all quantities only depend on r, ϑ (not on z)

Dirty limit $\equiv$ London limit
take $|\psi| = \text{const}$, only phase varies

Interaction energy between two vortex lines:

$$E^{int}_{1,2} = \frac{\phi_0}{4\pi} B_1(r_2)$$

$$\longrightarrow K_0\left(\frac{r_{12}}{\lambda_p}\right) \sim \ln \frac{r_{12}}{\lambda_p} \qquad r_{12} < \lambda_p$$

$r_{12} \equiv |\vec{r}_1 - \vec{r}_2|$

$$\lambda_p = \frac{2\lambda^2}{d} \qquad \sim e^{-r_{12}/\lambda_p} \qquad r_{12} > \lambda_p$$

penetration length in 2D for perpendicular fields.

$$\lambda_p = \frac{R_\square}{1k\Omega} \frac{1°K}{T_{c0}} \sim 0.1 \text{ cm} \qquad R_\square = \rho/d$$

sheet resistance

Mean field transition temperature

d 14

Take a disk of radius $R < \lambda_p$

Analogy with : interacting point-like charges

Poisson equation

$$\nabla^2 V(r) = 4\pi\,\delta(r - r_2)$$

electrostatic potential due to charge at r_2

In 2D solution is $V(r) = \ln|r - r_2|$

The problem maps onto that of a 2D coulomb gas.

H = 0 fluctuations consist in : creation of vortex – antivortex pairs

→ no effect on conductivity

For temperatures $T > T_{KT}$

pairs unbind

d15

... temperature an isolated vortex can exist ?

$$\Delta F_{1\,\text{vortex}} = U_{1\,\text{vortex}} - TS$$

$$= \left(\frac{\Phi_0}{2\lambda}\right)^2 d \ln \frac{R}{\xi} - T k_B \ln N_1$$

N_1 : # of ways the vortex can be put into the sample $\approx R^2/\xi^2$

$$\Delta F_{1V} \leq 0 \implies \left(\frac{\phi_0}{2\lambda}\right)^2 d \leq 2 k_B T$$

defines a temperature
$$T_{KT} = \frac{1}{2k_B}\left(\frac{\Phi_0}{2\lambda}\right)^2 d$$

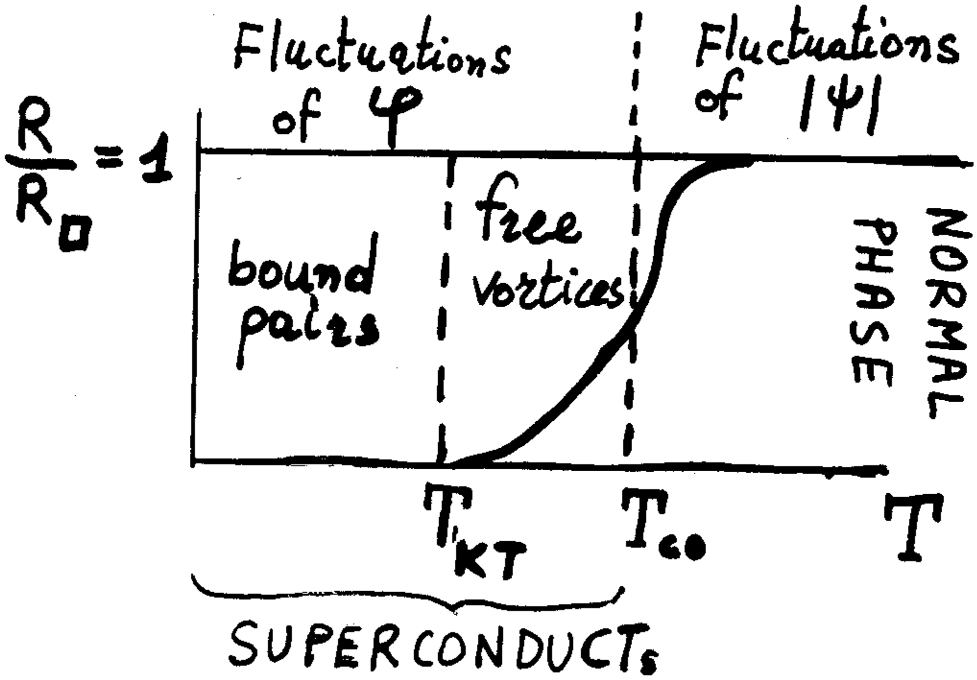
$$T_{KT}/T_{co} = 2.18\,\frac{\hbar/e^2}{R_\square}$$

For $T > T_{KT}$ vortices and anti-vortices are free

dissipation due to free vortices

$$\frac{R}{R_\square} = 2\pi\xi^2 N_{FV}$$

number of free vortices



d16

One dimensional wire : <u>critical current</u>

Cross section of the wire : $\sigma \ll \xi^2, \lambda^2$

its length $L \to \infty$

A current carrying equilibrium state :

$$\frac{\delta F}{\delta \psi^*} = 0 \quad \Rightarrow \quad \xi^2 \left| \left(\frac{d}{dx} + \frac{i}{\Phi_0} A_x \right) \psi \right|^2 + \left(1 - \frac{1}{n_s} |\psi|^2 \right) \psi = 0$$

$$\psi(x) = n_s^{1/2} f(x') \, e^{i\eta(x')} \qquad x' = \frac{x}{\xi}$$

$$\frac{\delta F}{\delta \vec{A}} = 0 \quad \Rightarrow \quad J_x = \frac{c}{\Phi_0} H_c^2 \xi^2 f^2(x') \left(\frac{d}{dx'} + i \frac{1}{\Phi_0} A_x \right) \eta$$

Neglect the magnetic field generated by the supercurrent $\quad A_x = 0$

Constant current (dimensionless) $\quad j = f^2(x') \dfrac{d}{dx'} \eta(x')$

$$\frac{d^2 f}{dx^2} + f - f^3 - \frac{1}{f^3} j^2 = 0$$

with given boundary conditions at the extrema :

$$\Delta \eta = \kappa L / \xi$$

definition of κ

d17

Solution : $\psi(x') = f_\varkappa \, e^{i(\eta_0 + \varkappa x)}$

$$f_\varkappa = (1-\varkappa^2)^{1/2}$$

$$j = \varkappa(1-\varkappa^2)$$

Maximum value of J is for $\varkappa^2 = 1/3$

The free energy for this solution :

$$\frac{F}{\sigma L} = \frac{H_c^2}{8\pi}\left\{(1-f_\varkappa)^2 + 2\frac{j^2}{f_\varkappa^2} - 1\right\} =$$

$$= \frac{H_c^2}{8\pi}\left\{(\varkappa^2-1)f_\varkappa^2 + \frac{1}{2}f_\varkappa^4\right\}$$

Its minimum disappear for $\varkappa^2 > 1/3$

Critical current is for $j_c = \dfrac{2}{3\sqrt{3}}$

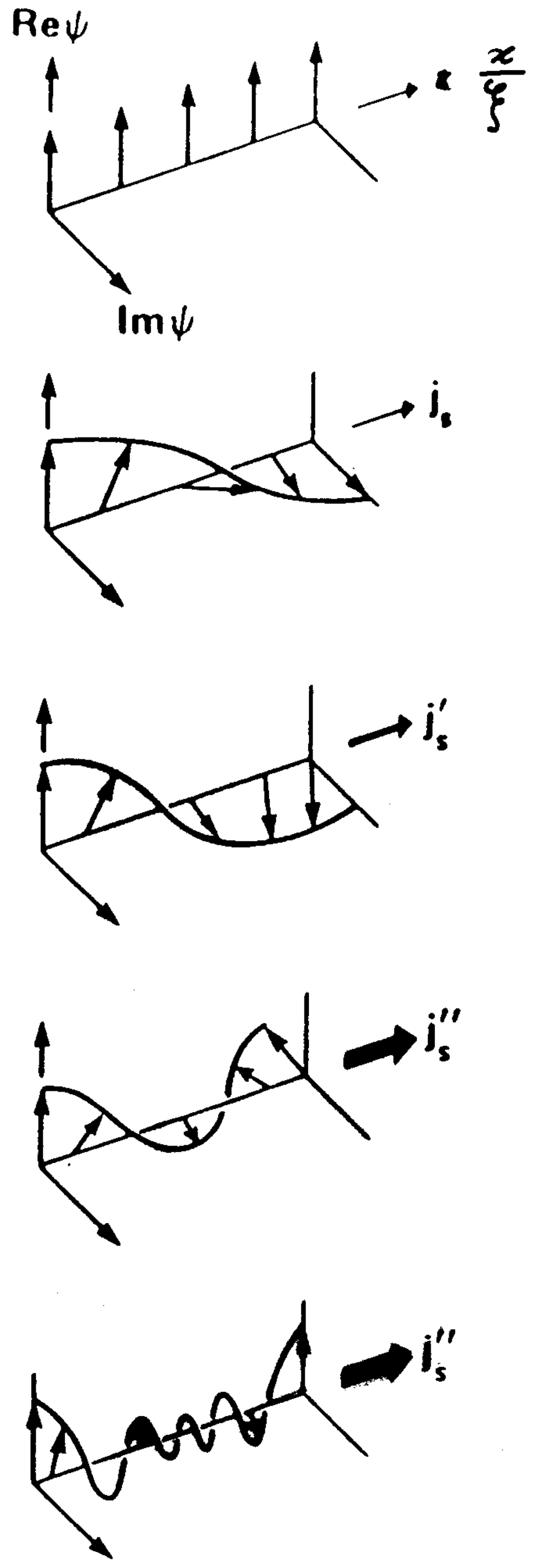
d18

Plot of the solution : $\psi \propto (1-\kappa^2)^{1/2} e^{i\kappa \frac{x}{\xi}}$

$j = 0$

increasing current:

$j \neq 0$



$j = f^2(x') \frac{d}{dx'} \eta(x') =$
$= \text{constant}$

Close to T_c : Resistive current carrying state

phase slips

boundary conditions can be satisfied also for $\Delta\eta \longrightarrow \Delta\eta + 2\pi m$

m → integer

The free energy has various minima corresponding to different m

Energy difference :

$$\delta F = \frac{dF}{d\varkappa}\,\delta\varkappa \qquad \delta\varkappa = \frac{2\pi}{L}\xi$$

$$\frac{d}{d\varkappa}F = \cancel{\frac{\delta F}{\delta f_\varkappa}} + \frac{\partial F}{\partial \varkappa} = \sigma L \frac{H_c^2}{4\pi}\, 2\varkappa f_\varkappa^2$$

F is stationary

$$\delta F = \frac{\hbar}{2e} I \cdot 2\pi$$

Working at constant current bias implies that :

$$F(\Delta\eta) \longrightarrow G(I) = F - \frac{\hbar}{2e} I \eta$$

d 20

Note that, because $j = f(x') \dfrac{d}{dx'} \eta(x')$

current can be kept constant <u>increasing</u> the windings of η and <u>decreasing</u> the magnitude of f

At a given time, <u>modulus</u> of ψ can vanish and <u>phase</u> jump by 2π

Voltage arises :

$$V = \frac{\hbar}{2e} \frac{d}{dt} \Delta\eta = \frac{\hbar}{2e}\, 2\pi \left(\Gamma^{+} - \Gamma^{-} \right)$$

$\Gamma^{\pm}$: transition rates due to thermal fluctuations towards neighboring minima

$$\varkappa \underset{-}{\overset{+}{\rightleftarrows}} \varkappa - \frac{2\pi\xi}{L}$$

Energy cost of a phase slip : $\sqrt{2}\, H_c^2(T)\, \sigma\, \xi(T) / 3\pi$

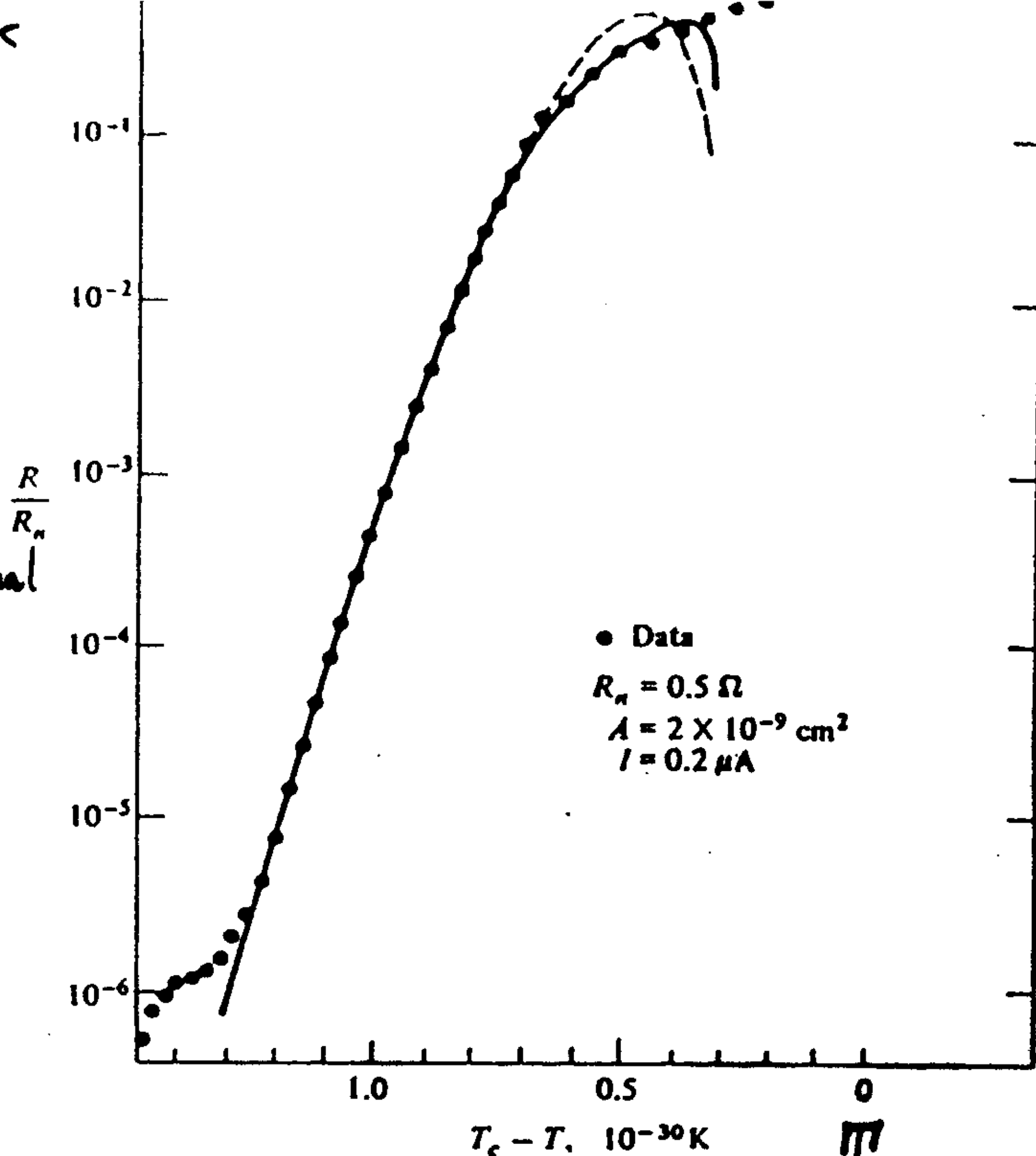
$\sim$ condensation energy within a length $\sim \xi$

d21

$T_c - T \sim 1\ \text{mK}$

100 phase slips/sec.

$$\frac{R}{R_{Normal}}$$



T_c T

FIGURE 7-2
Decrease of resistance below T_c in a superconducting tin whisker, as measured by Newbower et al. Solid curve is LAMH theory, with only T_c as adjustable parameter. Dashed curve is LAMH theory if parallel normal conduction channel is omitted. "Foot" at $R/R_n \approx 10^{-6}$ is believed to be caused by contact effects.

Resistance : V/I

$$R \sim \frac{\pi\hbar}{2e^2}\,\frac{2e}{I}\left(\Gamma^+ - \Gamma^-\right)$$

quantum resistance $\sim 6.5\ k\Omega$

Magnetic impurities

Time reversal is not a good symmetry any longer

Scattering has pair breaking effect

of strength $2\alpha = \dfrac{\hbar}{\tau_K}$

τ_K : time required for the relative phase of the two electrons to be randomized by the perturbation

T_c is depressed :

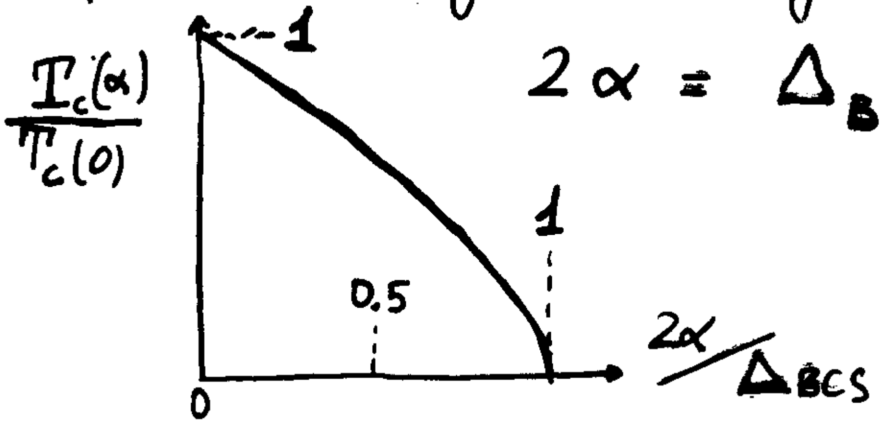
$$\ln \frac{T_c(\alpha)}{T_c(0)} = \psi(1/2) - \psi\left(1/2 + \frac{\alpha}{2\pi k_B T_c(0)}\right)$$

digamma functions

(Abrikosov, Gorkov theory)

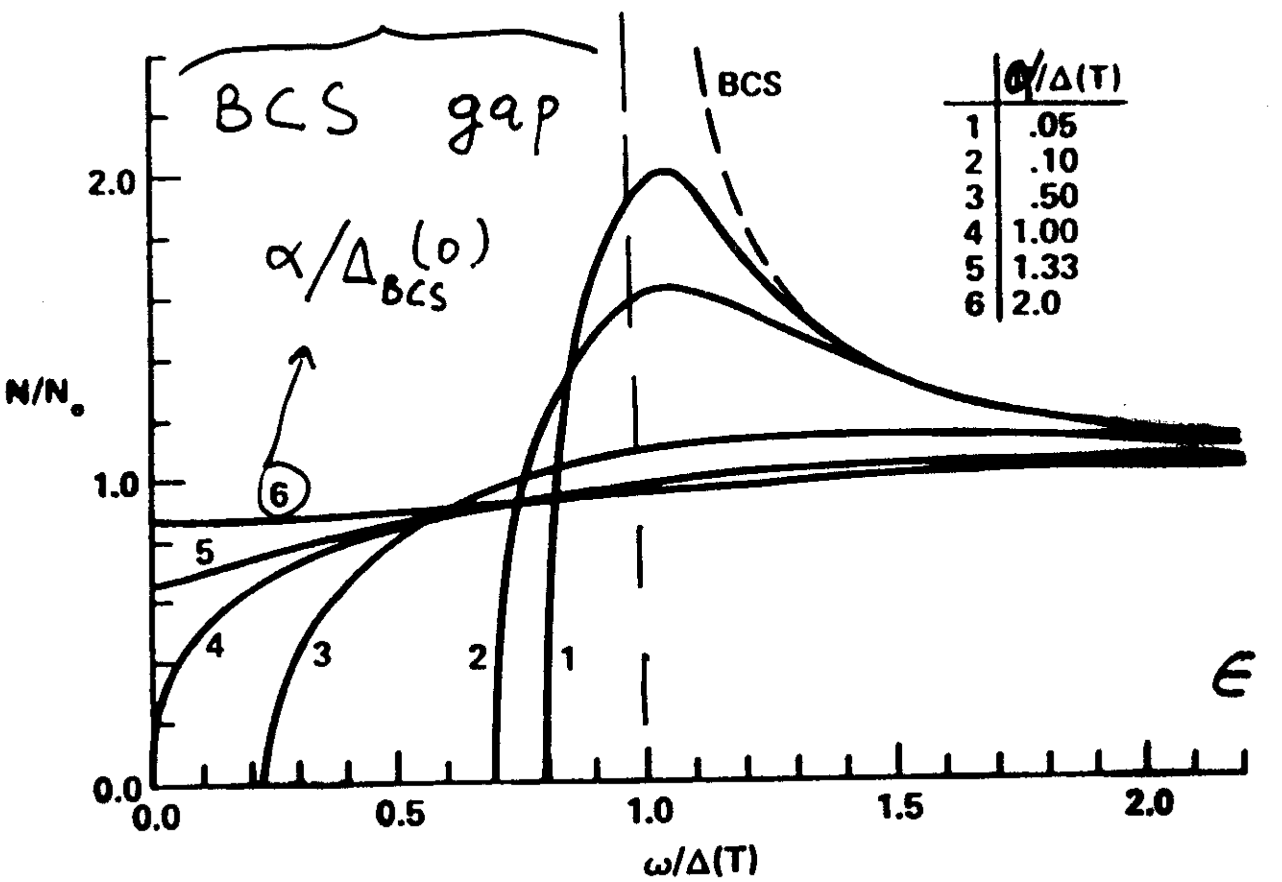
Superconductivity is destroyed if

$$2\alpha = \Delta_{BCS}(0)$$



d 23

Density of states $N_s(\epsilon)/N_n(0)$



/II.25. Density of states in energy as a function of energy for various values e parameter Γ. After Skalski *et al.*[22]

The gap in the excitation spectrum tends to disappear

<u>Gapless</u> superconductivity

Note: $\underline{\underline{\psi}}$ ($\equiv$ ordering) is not the same as $\underline{\underline{\Delta}}$

Errata corrige in Part I

page 31 :

$$1 = V N(0) \int_0^{\hbar\omega_D} d\epsilon \, \frac{\tanh \beta E/2}{E}$$

$$k_B T_c = 1.13 \, \hbar\omega_D \, e^{-1/N(0)V}$$

page 38 : read $2\Delta(0) / k_B T_c$ everywhere

pages 45 and 46 are exchanged

page 58 : N is the number of pairs

References:

M. Tinkham "Introduction to Superconductivity" Mc Graw-Hill book company (1975)

P.G. De Gennes : "Superconductivity in Metals & Alloys", Benjamin (1966)

G. Grimvall : "The electron phonon interaction in metals" North Holland (1981)