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EXPERIMENTAL WORKSHOP ON HIGH TEMPERATURE SUPERCONDUCTORS & RELATED MATERIALS (BASIC ACTIVITIES)

12 - 30 MARCH 1990

MAGNETIC NUCLEAR RESONANCE

Part I

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Nuclear Magnetic Resonsnce in High-T_c Superconductors

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§ 1. INTRODUCTION TO THE NUCLEAR MAGNETIC RESONANCE PHENOMENA.

1-1) Motion of free nuclear spins in magnetic field.

1-2) Application of oscillatory magnetic field.

1-3) Effect of the surrounding "lattice".

§ 2. HYPERFINE INTERACTION IN SOLIDS

2-1) Derivation of the magnetic hyperfine interaction

2-2) Paramagnetic shift of resonance frequency.

2-3) Knight shift in metals and superconductors.

2-4) Nuclear spin-lattice relaxation (general formula).

2-5) Nuclear relaxation in metals and superconductors.

§ 3. NMR STUDIES OF $YBa_2Cu_3O_{7-\delta}$ --- I (STATIC PROPERTIES)

3-1) General properties --- crystal structure, phase diagram

3-2) Study of electronic structure and static magnetic properties

- Cu Knight shift
- Cu anisotropic nuclear relaxation
- study of hyperfine field
- Oxygen Knight shift
- Knight shift in the "T_c=60 K" phase

§ 4. NMR STUDIES OF $YBa_2Cu_3O_{7-\delta}$ --- II (DYNAMICAL PROPERTIES)

4-1) Cu, Oxygen and Yttrium nuclear spin relaxation

- absence of the Hebel-Slichter peak
- anomalous nomal state behavior

4-2) Theoretical analysis ----- effect of the antiferromagnetic spin fluctuations

In this short lecture, I will try to give a brief survey of the present status of the nuclear magnetic resonance (NMR) study on high-T_c superconductors. I will start with the introduction to the fundamental concepts of NMR (§1 and §2). First I shall explain the basic principles of the magnetic resonance phenomena (§1), which itself is a beautiful realization of the coherence of the quantum spin system. I hope this will help the audience to become familiar with this experimental technique.

The resonance phenamena, although being interesting by itself, would have been useless in condensed matter physics if the nuclear spins were isolated from surrounding electrons. It is the hyperfine interaction -- the magnetic interaction between nuclear spins and the electronic spin or orbital magnetic moments -- which makes NMR such a useful probe for the electronic properties of solids. In §2, I shall review the basic aspects of the hyperfine interaction and show how the various quantities which characterize the resonance, such as the shift of resonance frequency, line shape, nuclear spin lattice relaxation rate etc., reflect the magnetic properties of the electronic system. The thorugh knowlegde of the hyperfine interaction is crucial for the interpretation of any NMR data. I will not discuss the electric hyperfine interaction which is important to analize the quadrupole splitting of the NMR spectra and the NQR (nuclear quadrupole resonance) spectra. The typical behavior of the frequency shift (Kinght shift) and the nuclear relaxation rate in conventional metals and superconductors will be also discussed.

The rest of the lecture will be devoted to the recent experimental progress in the NMR study of high-T_c supeconductors. I shall focus on the results in $YBa_2Cu_3O_{7-\delta}$ since this system has been studied by far most extensively and has provided a wide variety of phenomena. There are two major issues to be disceeed. The first issue is concerned with the electronic structure and the

static magnetic properties of this system. The second issue is the dynamical behavior of the electron spins and the nature of magnetic excitations.

It is by now widely recognized that the two dimensional CuO_2 planes are the most fundamental consituent of the high-T_c materials and the spin and charge excitatious are associated with the Cu-3d and oxygen 2p states. However, the nature of these excitaions in the normal state is not well understood. An important problem is wherther the electron spins on the Cu-3d states and the oxygen 2p states have independent degrees of freedom or they are so strongly hybridized to form a single spin system. It is also important to know whether these electron spins have the character close to the localized moments such as the Cu^{2+} moments in the antiferromagnetic insulator $YBa_2Cu_3O_6$ or these are more like itinerant electron system which can be veiwd as a Fermi liquid.

The static NMR properties i.e. the Knight shift, which I discuss in §3, give useful information about the first problem. The Knight shift measurement in the superconducting state also gives some insight about the nature of the pairing states (s or d-wave, magnitude of the superconducting gap). The measurement of the nuclear relaxation relaxation rate which will be discussed in §4 is related to the second problem. There has been substantial theoretical progress in the understanding of the spin dynamics in the normal states. I would like to emphasize the importance of antiferromagnetic short range correlations among Cu spins which is the key concept to understand the nuclear relaxation behavior in this system. Another important feature of the relxation data is the absence of the so called Hebel-Slichter peak (coherence peak) in this system which is not fully understood yet.

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§1 Nuclear Magnetic Resonance Phenomena (ref. 1, 2)

1-1) motion of free nuclear spin in magnetic field

[nuclear spin $\hbar\vec{I}$
magnetic moment $\vec{\mu}$

$\vec{\mu} = \gamma_N \hbar \vec{I}$

γ_N : gyromagnetic ratio

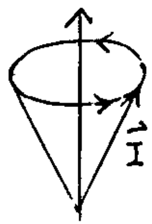
Energy (Hamiltonian) of nuclear spin in uniform magnetic field

$$\mathcal{H} = -\vec{H}\cdot\vec{\mu} = -\gamma_N \hbar\, \vec{H}\cdot\vec{I}$$

a) classical equation of motion

torque acting on a magnetic moment $\vec{\mu}\times\vec{H}$
= time derivative of angular momentum

$$\hbar\frac{d\vec{I}}{dt} = \vec{\mu}\times\vec{H} = \gamma_N\hbar\,(\vec{I}\times\vec{H})$$



$\vec{I}$ or $\vec{\mu}$

Larmor precession
frequency $\omega = \gamma_N H$

b) quantum theoretical treatment

Heisenberg equation $\dfrac{d\vec{I}}{dt} = \dfrac{i}{\hbar}[\mathcal{H}, \vec{I}]$ $\quad \mathcal{H} = -\gamma_N \hbar\, \vec{I}\cdot\vec{H}$

$= -\gamma_N \hbar H I_z$

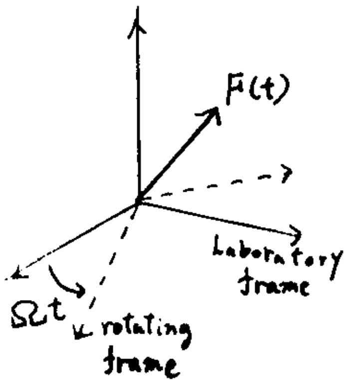
$$\begin{cases} \dfrac{dI_x}{dt} = -\gamma_N\, i\, H\, [I_z, I_x] = \gamma_N H I_y \\[2ex] \dfrac{dI_y}{dt} = -\gamma_N H I_x \\[2ex] \dfrac{dI_z}{dt} = 0 \end{cases} \Longrightarrow \frac{d\vec{I}}{dt} = \gamma_N (\vec{I}\times\vec{H})$$

taking expectation value $\dfrac{d\langle\vec{I}\rangle}{dt} = \gamma_N (\langle\vec{I}\rangle \times \vec{H})$

identical to the classical expression

1-2) Application of oscillatory magnetic field

a) concept of the rotating frame

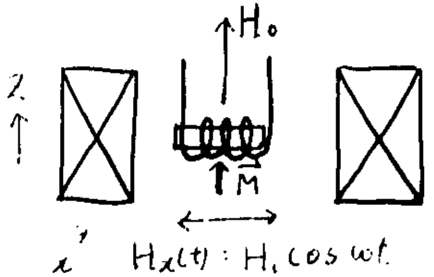


motion of $\vec{F}(t)$ with respect to the rotating frame

$$\frac{d\vec{F}}{dt} = \frac{\delta\vec{F}}{\delta t} + (\vec{\Omega}\times\vec{F})$$

↑ Lab. frame ↑ rotating frame

b) ensemble of free nuclear spins.

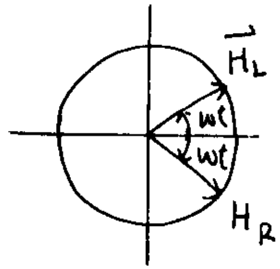


sample

$(\uparrow \nearrow \swarrow \uparrow \searrow)$

Boltzman distribution.

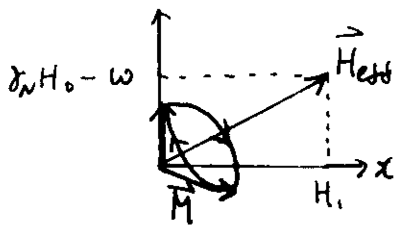
$\vec{M} = \langle\vec{\mu}\rangle \propto \dfrac{H_0}{k_B T}$



$\vec{H}_x(t) = \vec{H}_R + \vec{H}_L$ decomposition into two rotating field

In the rotating frame with $\Omega_z = -\omega$, H_R becomes a static field along $\vec{x}$

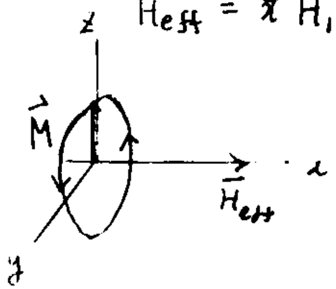
$$\frac{\delta\vec{M}}{\delta t} = \vec{M}\times\underbrace{[\vec{z}(\gamma_N H_0 - \omega) + \vec{x}\,\gamma_N H_1]}_{=\vec{H}_{eff}}$$



Larmor precession around $\vec{H}_{eff}$

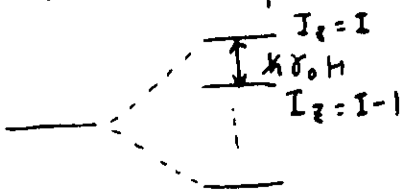
When $\omega = \gamma_N H_0$ (resonance)

$\vec{H}_{eff} = \hat{x} H_1$



$\vec{M}$ is rotating in the $\overline{yz}$ plane with frequency $\omega_1 = \gamma_N H_1$

- alternative picture of resonance



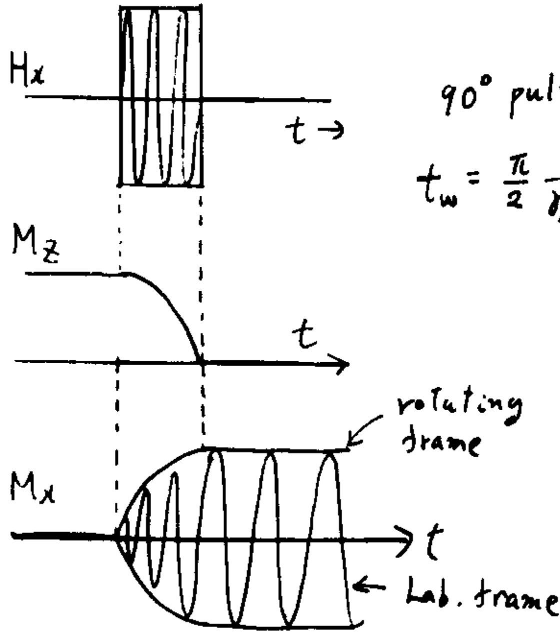
nuclear spin Zeeman levels

Transition between $|I_z = I\rangle$ and $|I_z = I-1\rangle$ is caused by transverse field $H_x = H_1 \cos\omega t$

$(\ V \propto -H_x I_x = -H_x (I_+ - I_-)/2\ )$

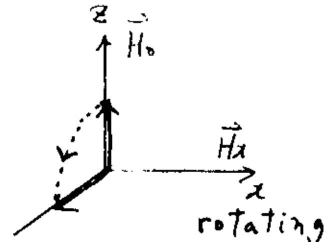
when $\hbar\omega = \hbar\gamma_0 H_0$

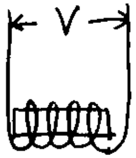
c) observation of resonance



90° pulse

$$t_w = \frac{\pi}{2} \frac{1}{\gamma_N H_1} \quad \left(t_w \omega_1 = \frac{\pi}{2} \right)$$

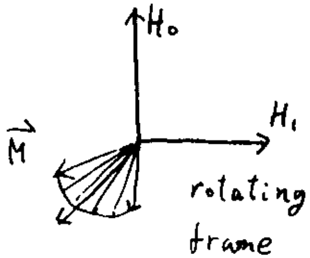




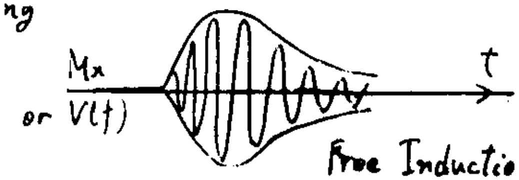
voltage induced accross the coil

$$V(t) \propto \frac{dM_x}{dt} = \omega M \propto \frac{H_0^2}{kT}$$

d) Inhomogeneous magnetic field.

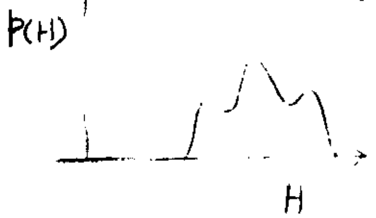


dephasing : Some spins precess fast, some spins precess slow.



Free Induction Decay

distribution of local magneti field
(inhomogeniety of magnet
anisotropic chemical shift
quadrupole interaction



P(H) : distribution function
local Larmor frequency
$\omega = \gamma_N H$

after $90°$ pulse
$$M_x(t) = \int P(H) \cos(\gamma_N H t)\, dH$$
$$M_y(t) = \int P(H) \sin(\gamma_N H t)\, dH$$

principles of FT-NMR

take data of $M_x(t)$, $M_y(t)$ $(t>0)$

$\Downarrow$

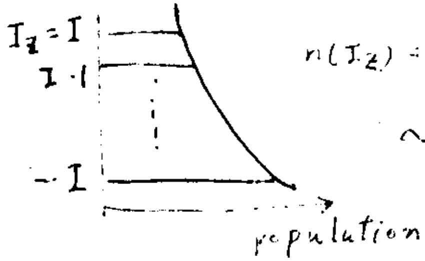
Flourier transform of $M_x(t) + i M_y(t)$
$$= \int P(H)\, e^{i\gamma_N H t}\, dH$$

$\Downarrow$

Obtain distribution function $P(H)$.

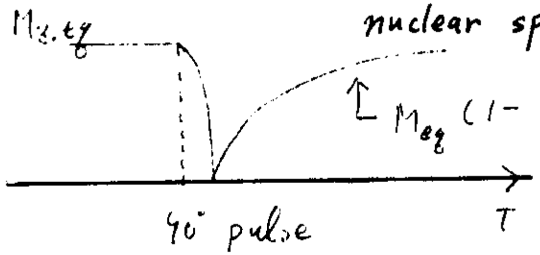
1-3) Effect of the interaction between nuclear spin system and external "lattice".
(electrons, phonons, etc)

$\Longrightarrow$ relaxation toward the Boltzman distribution.



$$n(I_z) = e^{-\frac{\gamma_N \hbar I_z H}{kT}}$$
$$\sim 1 - \frac{\gamma_N \hbar I_z H}{kT} \qquad M_{z,eq} \propto \frac{H}{kT}$$

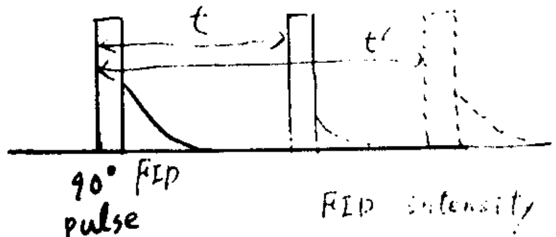
nuclear spin-lattice relaxation



$1/T_1$: relaxation rate

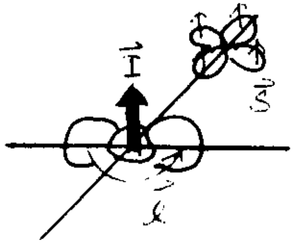
T_1 : relaxation time

measurement of T_1



FID intensity t after 90° pulse
$\propto (1 - e^{-t/T_1})$

§2 Hyperfine Interaction in Solids (ref. 1-6)
(Electron - Nucleus interaction)



Nuclear spins interact with surrounding electronic (spin or orbital) magnetic moment

⇓

magnetic hyperfine interaction

$\mathcal{H} = -\gamma_N\hbar\,\vec I\cdot\vec H_{hf}$ $\vec H_{hf}$: hyperfine field (electron operator)

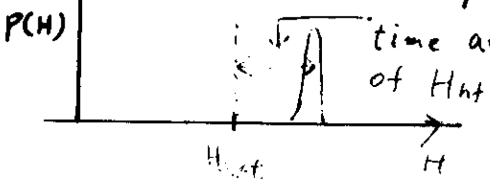
The effective field acting on the nuclear spin

$H_{eff} = H_{ext} + H_{hf}(\vec r, t)$

⇓ statistical average for electronic system

two major effects

1) shift of the resonance frequency



time average of $H_{hf}(\vec r, t)$

2) nuclear spin-lattice relaxation

2-1) Derivation of the Magnetic Hyperfine Interaction

- magnetic field produced by a nuclear moment

$$\vec H = \vec\nabla\times\vec A, \quad \vec A = \frac{\vec\mu_n\times\vec r}{r^3} = \vec\nabla\times\left(\frac{\vec\mu_n}{r}\right), \quad \vec\mu_n = \hbar\gamma_N\vec I$$

- Hamiltonian for electrons

$$\mathcal{H} = \frac{1}{2m}\left(\vec p + \frac{e}{c}\vec A\right)^2 + g\mu_B\,\vec S\cdot\vec\nabla\times\vec A$$

$\vec S$: electron spin

1st order in $\vec A$

$$\mathcal{H}_{el-n} = 2\mu_B\frac{\vec\mu_n\cdot\vec l}{r^3} + g\mu_B\,\vec S\cdot\vec\nabla\times\vec\nabla\times\left(\frac{\vec\mu_n}{r}\right)$$

$$\left[\begin{array}{l}\vec\nabla\times(\vec\nabla\times\vec B) = \vec\nabla(\vec\nabla\cdot\vec B) - \nabla^2\vec B \\ \nabla^2\left(\frac{1}{r}\right) = -4\pi\delta(\vec r)\end{array}\right.$$

$$\mathcal{H}_{el-n} = \frac{8\pi}{3}g\mu_B\gamma_n\hbar\,\delta(\vec r)\,\vec I\cdot\vec S$$ Fermi contact (s-states)

$$-\,g\mu_B\gamma_n\hbar\,\vec I\cdot\left[\frac{\vec S}{r^3} - 3\frac{\vec r(\vec S\cdot\vec r)}{r^5}\right]$$ Spin Dipolar (non-s state)

$$+\,2\mu_B\frac{\vec\mu_n\cdot\vec l}{r^3}$$ orbital (non-s state)

2-2) Shift of the resonance frequency

Effective field for nuclear spins

$$\langle \vec{H}_{hf} \rangle = \frac{8\pi}{3} 2\mu_B \langle \vec{s}\,\delta(\vec{r}) \rangle + 2\mu_B \left\langle \frac{3\vec{r}(\vec{s}\cdot\vec{r})}{r^5} - \frac{\vec{s}}{r^3} \right\rangle$$

$$+ 2\mu_B \left\langle \frac{\vec{l}}{r^3} \right\rangle$$

$\langle\ \rangle$: expectation value for particular states.

1st and 2nd term $\neq 0$ only for the unpaired electrons

last term $\neq 0$ only for the electron in the open shell.

Examples of finite $\langle H_{hf} \rangle_{av}$.

① ferromagnetic materials (Fe Co Ni ---)

$$M = g\mu_B \langle \vec{S} \rangle \neq 0$$

$$\langle \vec{H}_{hf} \rangle_{av} = \frac{8\pi}{3} g\mu_B |\psi(0)|^2 \langle \vec{S} \rangle = H_{int}$$

Resonance is observed at zero external field. at $\omega_N = \gamma_N H_{int}$

	ω_N (MHz)	H_{int} (kOe)
Co	230	227
Fe	46.5	337
Ni	28.5	75

② paramagnetic materials (linear response to external field)

$$M_{spin} = g\mu_B \langle \vec{s} \rangle = \chi_{spin} H_{ext}$$

$$M_{orb} = \mu_B \langle \vec{l} \rangle = \chi_{orb} H_{ext}$$

χ : magnetic susceptibility

$$\langle H_{hf} \rangle = \frac{8\pi}{3} \langle |\psi(0)|^2 \rangle \chi_{spin} H_{ext} + \cdots$$

$$+ \left\langle \frac{2}{r^3} \right\rangle \chi_{orb} H_{ext}$$

(Knight) shift

$$K = \frac{\langle H_{hf} \rangle}{H_{ext}} = \underbrace{\frac{8\pi}{3} \langle |\psi(0)|^2 \rangle}_{\text{hyperfine coupling constant}} \chi_{spin} + \cdots + \underbrace{\left\langle \frac{2}{r^3} \right\rangle}\chi_{orb}$$

in general,

$$K = A_s \chi_{s,spin} + \overleftrightarrow{A}_{p(d,f\cdots)} \chi_{p(d,f\cdots)\,spin}$$

$$+ B_{p(d,f\cdots)} \chi_{p(d,f\cdots)\,orb}$$

•

• core polarization effect.

Spin polarization of d, f (p) states produces an spin-dependent exchange potential for the inner core s-state $V_\uparrow(\vec{r}) \neq V_\downarrow(\vec{r})$

This produces spin polarization of I inner s-state with opposite direction.

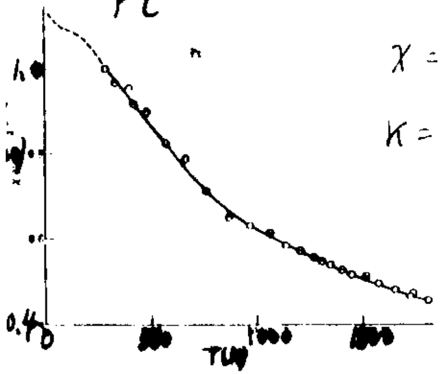
2-3) Knight shift in Metals and superconductors.

a) simple metals (T-independent Pauli para)

	Li	Na	Al	Cu	Sn	Pb
K(%)	0.026	0.114	0.164	0.24	0.78	1.54

b) transition metals (ref. 10)

Pt

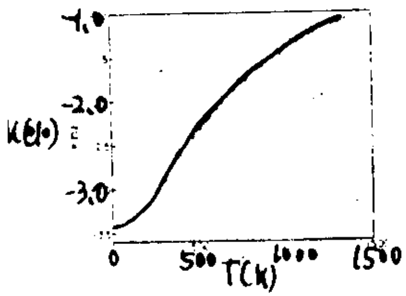


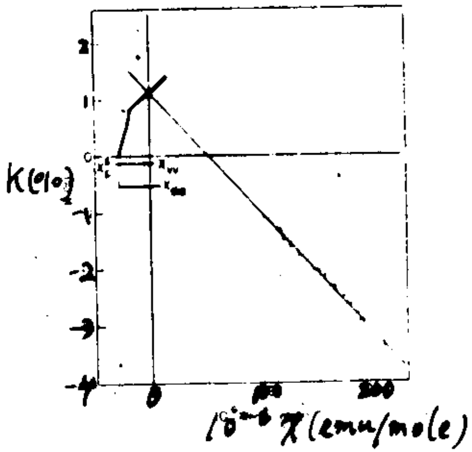
$\chi = \chi_{dia} + \chi_{s.p} + \chi_d(T) + \chi_{orb}$

$K = K_{s.p} + K_d(T) + K_{orb}$

$K_d(T) = A_d \chi_d(T)$

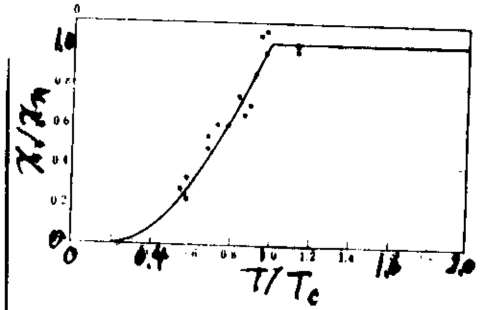
$A_d < 0$ core polarizatio





c) superconductors.

Al (ref. 11)



$\chi_{spin} = -2\mu_B^2 \sum_k \frac{\partial f_k}{\partial \epsilon_k}$

normal state

$\sum_k \frac{\partial f_k}{\partial \epsilon_k} = \sum_k \delta(\epsilon_F - \epsilon_k) = \rho(\epsilon_F)$

density of states

super.

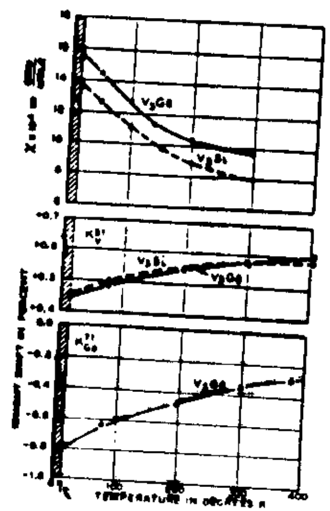
$\chi_{spin} = -2\mu_B \sum \frac{\partial f}{\partial E_k}$

$E_k = \sqrt{\epsilon_k^2 + \Delta^2}$: quasiparticle energy.

Δ : gap

$\chi_{spin} \propto e^{-\Delta/kT}$ $(T \to 0)$

A15 compounds (ref. 12)



T-dependent χ_s (narrow d-band)

$K_{spin} < 0$: core polarization

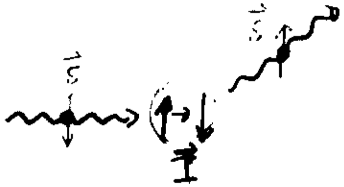
large $K(T=0)$: orbital shift.

$\propto \sum_{\substack{E_n > E_F \\ E_m < E_F}} \frac{|\langle m | l_z | n \rangle|^2}{E_n - E_m}$

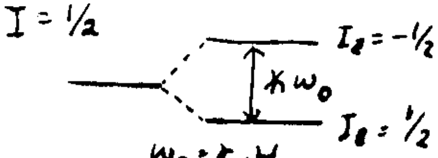
Van Vleck type

2-4) Nuclear spin-lattice Relaxation (genelar formula)

(Dynamical fluctuations of electronic magnetic moments)



typical process in metals



$\omega_0 = \gamma_N H_0$

$$\mathcal{H} = -\gamma_N \hbar \vec{I}\cdot\vec{H}_{hf}$$

$$= -\gamma_N \hbar \left[I_z H_{hf}^z + \frac{1}{2}\underbrace{\left(I_+ H_{hf}^- + I_- H_{hf}^+ \right)}_{} \right]$$

$$\left[\begin{array}{l} I_+ = I_x + i I_y \\ I_- = I_x - i I_y \end{array} \right.$$

perturbation causing the transition $I_z = -1/2 \longleftrightarrow 1/2$

Transition probability (Golden rule)

$$W_{1/2\leftrightarrow -1/2} = \frac{2\pi}{\hbar}\sum_{n,m} \left|\langle I_z = -1/2, m | -\frac{1}{2}\gamma_N\hbar I_- H_{hf}^+ | I_z = 1/2, n\rangle\right|^2 e^{-E_n/kT}\,\delta(E_n - E_m + \hbar\omega_0)$$

$\langle n|$, $\langle m|$: electronic states

$$= \frac{2\pi}{\hbar}\sum_{n,m} \left|\langle m | \frac{\gamma_N^2\hbar^2}{4} H_{hf}^+ | n\rangle\right|^2 e^{-E_n/kT}\,\delta(E_n - E_m + \hbar\omega_0)$$

$$\delta(E_n - E_m + \hbar\omega_0) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} dt\, e^{i\left(\frac{E_n - E_m}{\hbar} + \omega_0\right)t}$$

$$W_{1/2\to -1/2} = \frac{\gamma_N^2}{4}\int_{-\infty}^{\infty} dt \sum_{nm} |\langle m | H_{hf}^+ | n\rangle|^2 e^{+i\left(\frac{E_n - E_m}{\hbar} + \omega_0\right)t}\, e^{-E_n/kT}$$

$$= \frac{\gamma_N^2}{4}\int_{-\infty}^{\infty} dt \sum_{nm} \langle n | H_{hf}^- | m\rangle \langle m | H_{hf}^+(t) | n\rangle\, e^{i\omega_0 t}$$

$$\langle m | H_{hf}^+ | n\rangle^* = \langle n | H_{hf}^- | m\rangle$$

$$H_{hf}^+(t) = e^{\frac{i\mathcal{H}t}{\hbar}} H_{hf}^+ e^{-\frac{i\mathcal{H}t}{\hbar}}$$

(Heisenberg representation)

$$\langle m | H_{hf}^+ | n\rangle e^{i\left(\frac{E_n - E_m}{\hbar}\right)t} = \langle m | H_{hf}^+(t) | n\rangle$$

$$= \frac{\gamma_N^2}{4}\int_{-\infty}^{\infty} dt \langle H_{hf}^- H_{hf}^+(t)\rangle\, e^{i\omega_0 t}$$

$\langle\ \rangle$: statistical (thermal) average

$\langle H_{hf}^- H_{hf}^+(t)\rangle$: time correlation function of hyperfine field.

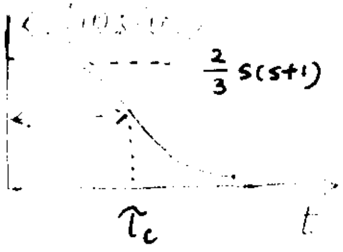
$$1/T_1 = 2 W_{1/2\to -1/2}$$

if $\vec{H}_{hf} = A\vec{S}_i$ (^{55}Mn in MnO etc)

$$1/T_1 = \frac{\gamma_N^2 A^2}{2}\int_{-\infty}^{\infty} \langle S_i^+(t) S_i^-(0)\rangle e^{i\omega_0 t} dt$$

relaxation rate is given by the auto spin correlation function.

- interacting localized moments.



τ_c : correlation time

$\hbar/\tau_c \sim J \gg \omega_0$

$$1/T_1 = \frac{\gamma_N^2 A^2}{2} \frac{2S(S+1)}{3} \tau_c$$

- relation to dynamical spin susceptibility

response to the space-time varing field

$$H(\vec{r},t) = H_q e^{i(\vec{q}\vec{r}-\omega t)}$$

$$\Downarrow$$

$$\boxed{\text{spin system}} \xrightarrow[\text{linear response}]{} s(\vec{r},t) = S_q e^{i(\vec{q}\vec{r}-\omega t)}$$

$\chi(q,\omega) = \frac{S_q}{H_q}$: dynamical susceptibility

$\mathrm{Im}\,\chi(q,\omega) \Rightarrow$ dissipation of the system

general correlation function (fluctuation)

$$g(q,\omega) = \int_0^\infty \langle S_q^-(0)\, S_q^+(t) \rangle e^{i\omega t} dt$$

$$\int \langle S^-(0) S^+(t) \rangle e^{i\omega t} dt = \sum_q g(q,\omega)$$

fluctuation – dissipation theorem.

$$g(q,\omega) = \frac{kT}{\omega} \mathrm{Im}\,\chi(q,\omega) \qquad (kT \gg \omega)$$

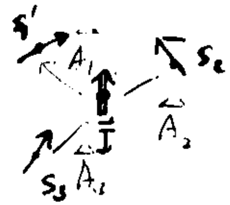
$$1/T_1 = \frac{\gamma_N^2 A^2}{2} k_B T \sum_q \frac{\mathrm{Im}\,\chi(q,\omega)}{\omega}$$

general case (Anisotropic A or χ, non local A)

$$1/T_1 = \frac{\gamma_N^2 k_B T}{2} \sum_q \left[|A_q^{xx}|^2 \frac{\mathrm{Im}\,\chi^{xx}(q,\omega)}{\omega_0} + |A_q^{yy}|^2 \frac{\mathrm{Im}\,\chi^{yy}(q,\omega)}{\omega_0} \right]$$

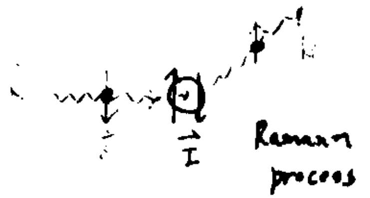
$H_0 \parallel z$

$$A_q = \sum_i A_i e^{i\vec{q}\vec{r}_i}$$



2-5) nuclear relaxation in metals and superconductors

a) Korringa relation (free electron)



$$\mathcal{H} = -\gamma_N \hbar A\, \vec{I}\cdot\vec{S}$$

$$1/T_1 = \frac{2\pi}{\hbar}\sum_{kk'} |\langle k'\uparrow| -\gamma_N\hbar A S^+/2 |k\downarrow\rangle|^2 f(\varepsilon_k)(1-f(\varepsilon_{k'}))\, \delta(\varepsilon_k - \varepsilon_{k'} + \hbar\omega_0)$$

$$= \pi\gamma_N^2\hbar A^2 \iint d\varepsilon d\varepsilon'\, \underbrace{f(\varepsilon)(1-f(\varepsilon'))}_{kT\,\rho(\varepsilon_F)}\, \underbrace{\delta(\varepsilon-\varepsilon')}_{\rho(\varepsilon_F)} \quad \omega_0 \to 0$$

$$= \pi\gamma_N^2\hbar A^2\, kT\, \{\rho(\varepsilon_F)\}^2$$

Knight shift $\quad K_s = A\chi_s \propto A\rho(\varepsilon_F)$

$$T_1 T K^2 = \left(\frac{\hbar}{4\pi k_B}\right)\frac{\gamma^2\mu_B^2}{\gamma_N^2} \qquad \text{Korringa relation.}$$

Intuitive picture

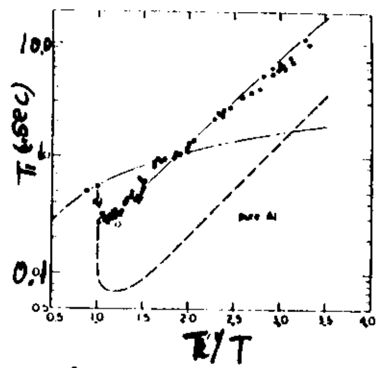
assume Lorentzian spectrum $\quad \frac{\mathrm{Im}\chi(q,\omega)}{\omega} = C\frac{\Gamma}{\omega^2+\Gamma^2}$

sum rule $\quad \int \frac{\mathrm{Im}\chi(q,\omega)}{\omega}d\omega = \pi\chi(q,\omega=0) = \pi\chi_q$

$C = \chi_q \qquad \left.\frac{\mathrm{Im}\chi(q,\omega)}{\omega}\right)_{\omega\to 0} = \chi_q/\Gamma \qquad \sum_q \frac{\mathrm{Im}\chi(q,\omega)}{\omega} \propto \{\rho(\varepsilon_F)\}^2$

$\chi_q \sim \rho(\varepsilon_F), \quad \Gamma \sim 1/\rho(\varepsilon_F)$

b) nuclear relaxation rate in superconducting state.



Al (ref. 13)

- coherence effect.

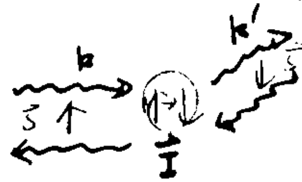
elementary excitation

$$\begin{cases} \gamma_{k\uparrow} = u_k c_{k\uparrow} - v_k c^+_{-k\downarrow} \\ \gamma^+_{-k\downarrow} = u_k c^+_{-k\downarrow} + v_k c_{k\uparrow} \end{cases}$$

scattering processes

$(k\uparrow \to k'\downarrow)$, $(-k'\uparrow \to k\downarrow)$

not independent



transition probability $\quad I_z = -1/2 \to 1/2$

$$|\langle n| a^+_{k'\uparrow}a_{k\downarrow} + a^+_{k\uparrow}a_{-k'\downarrow} |m\rangle|^2$$

$$\Longrightarrow \underline{(u_k u_{k'} + v_k v_{k'})}\,(\gamma^+_{k'\uparrow}\gamma_{k\downarrow} + \gamma^+_{-k\uparrow}\gamma_{-k'\downarrow})$$

$\frac{1}{2}\left(1+\frac{\varepsilon\varepsilon'}{EE'}+\frac{\Delta^2}{EE'}\right) \quad$ average on $k\,k'$

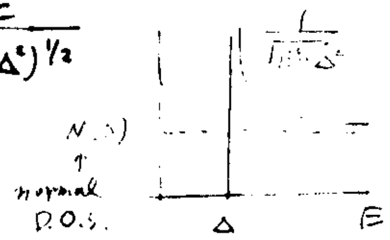
[see text book by J.R.Schrieffer] $\quad \Rightarrow \frac{1}{2}\left(1+\frac{\Delta^2}{EE'}\right)$: coherence factor

$$1/T_1 \propto \int_0^\infty N^s(E_i)\,N^s(E_f)\left(1+\frac{\Delta^2}{E_iE_f}\right) f(E_i)(1-f(E_f))\,\delta(E_i-E_f)\,dE_i$$

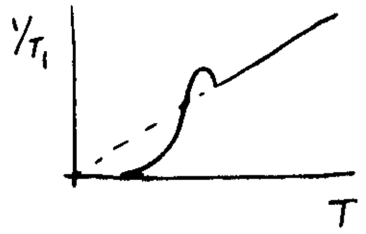
E: quasi particle energy $\quad E = \sqrt{\Delta^2+\varepsilon^2}$

- quasiparticle density of states.

$$N^s(E) = N(0)\frac{E}{(E^2-\Delta^2)^{1/2}}$$



- $1/T_1$ shows a peak below T_c ($\sim 0.9\,T_c$)



- at low T $\quad 1/T_1 \propto e^{-T/\Delta} \quad \Delta$: gap