



INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION



INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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**EXPERIMENTAL WORKSHOP ON HIGH TEMPERATURE
SUPERCONDUCTORS & RELATED MATERIALS
(BASIC ACTIVITIES)**

12 - 30 MARCH 1990

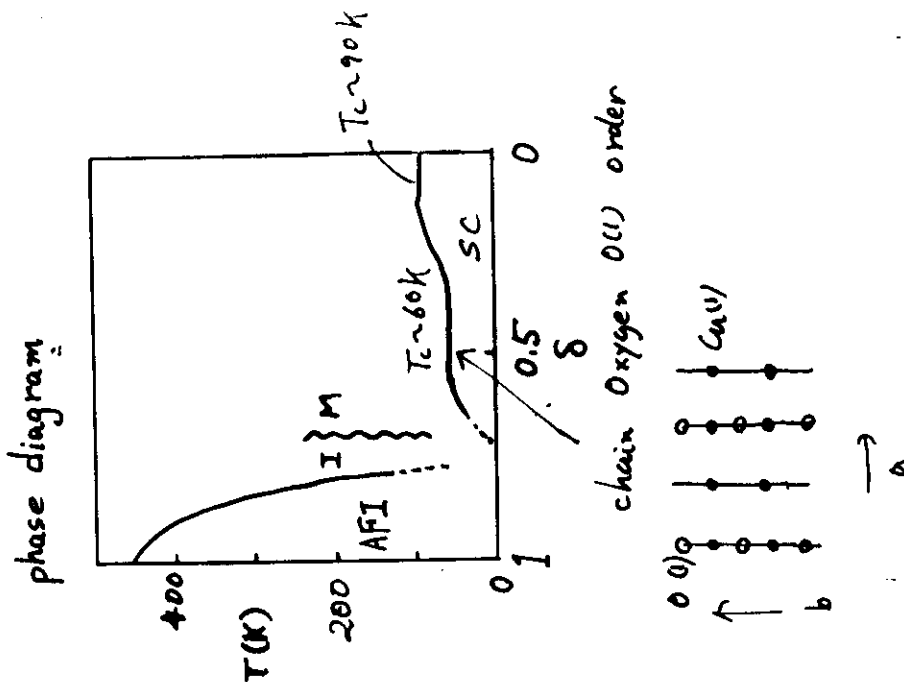
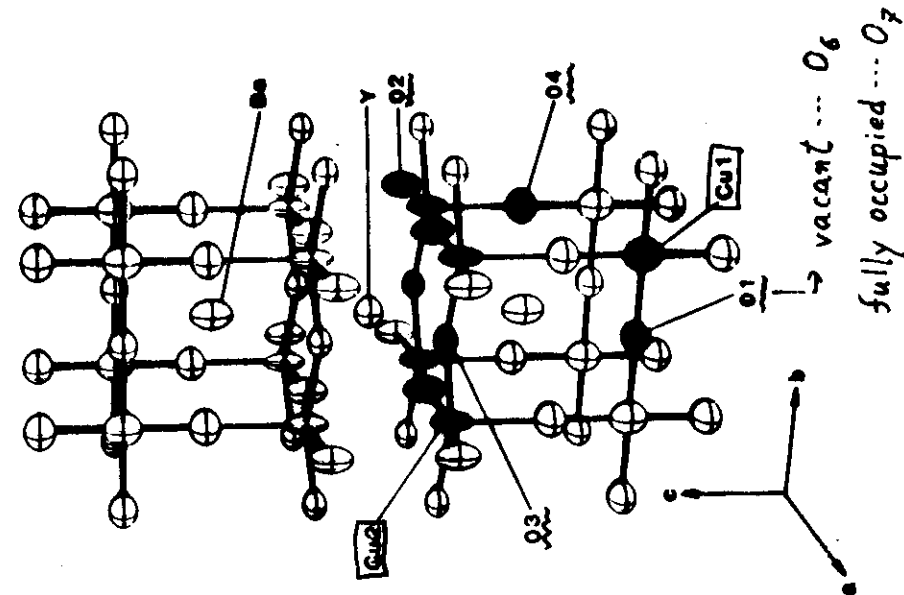
MAGNETIC NUCLEAR RESONANCE

Part II

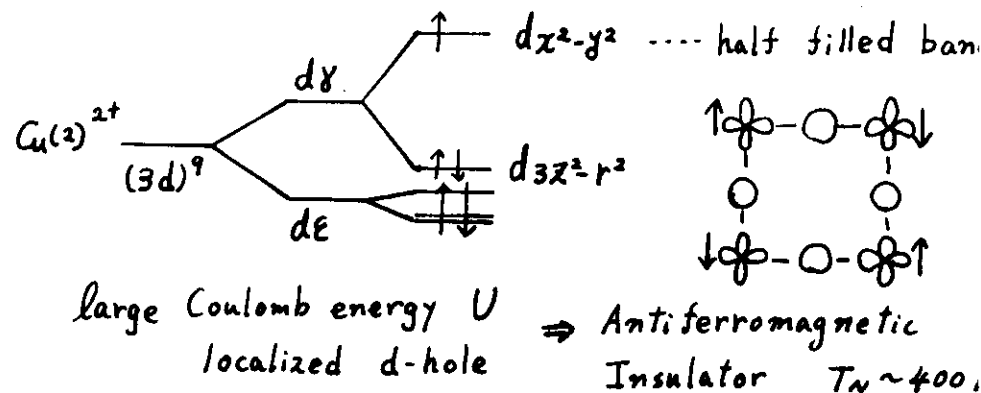
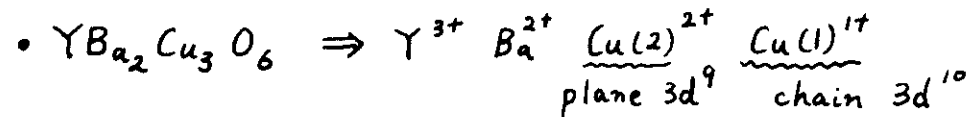
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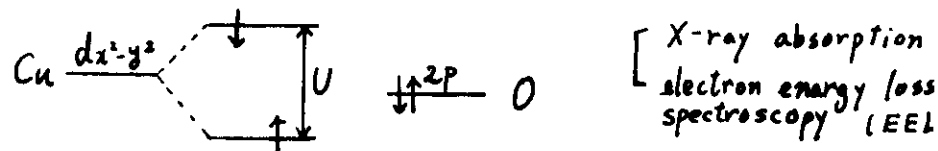
These are preliminary lecture notes, intended only for distribution to participants.



Electronic Structure



- upon doping $0 \xrightarrow{\delta} 1$ ($Cu(1) \rightarrow 2+$)
 - Holes are doped also in the plane. $Cu(2)O_2^{(2-)}$
 - destroy antiferro. order
 - metallic conduction $\Rightarrow$ superconductivity
 - Doped hole states are mainly of O-2p charac



[1] single or two component(s) spin in the CuO_2 planes:

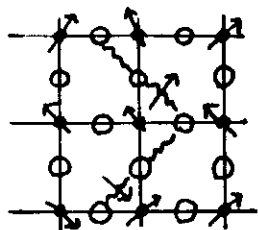
$$\chi_{\text{spin}} = \chi_{\text{Cu}} + \chi_{\text{hole}}$$

(Johnston: $\uparrow$ 2D localized $\uparrow$ Pauli-like)

$$L \quad \chi_{\text{spin}} = \chi_{\text{Cu-O}}$$

(ret 17)

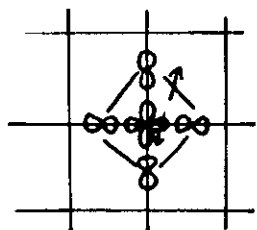
a) two band picture (Aharony et al, Emery et.al)



- 0 hole spins destroy Cu-Cu AF order. (frustration)
- $$-\uparrow-\phi-\uparrow- \quad J_{Cu-O} \geq 0 \quad \dots p_{\uparrow}$$
- $$(1) \quad \dots p_{\downarrow}$$
- Mobil holes (quasi particles)
carry spin, moving in the
 Cu d-spin background

- two distinct spin degrees of freedom

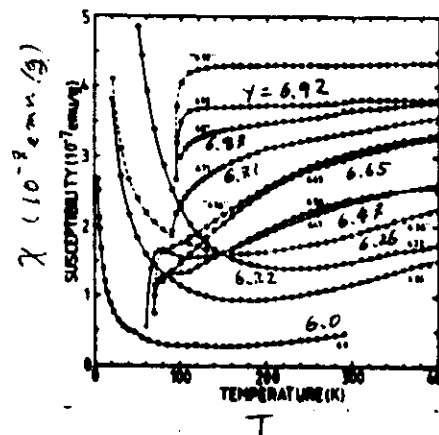
b) single band picture (Zhang and Rice, Sawatzky)
local singlet formation (ref. 18)



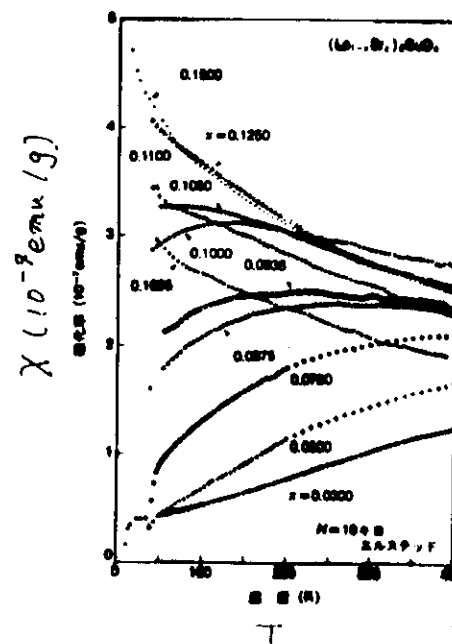
- oxygen holes make local singlet with Cu spin.
- Mobile holes do not carry spin
 $\Rightarrow$ equivalent to single band model.
 (t-J model)

$$YBa_2Cu_3O_y$$

(Nakazawa and
Ishikawa)
(ret. 19)


$$(La_{1-x}Sr_x)_2CoO_4$$

(Uchida et al.)



Ukagawa, Hasegawa, Hettner, Phys. Rev. B 39, 360 (1989) (ref. 23)

^{63}Cu NMR spectra in $\text{YBa}_2\text{Cu}_3\text{O}_{7-y}$ ($y=0.65$, $T_c=93\text{K}$)
85 MHz, 100 K

SPIN ECHO INTENSITY

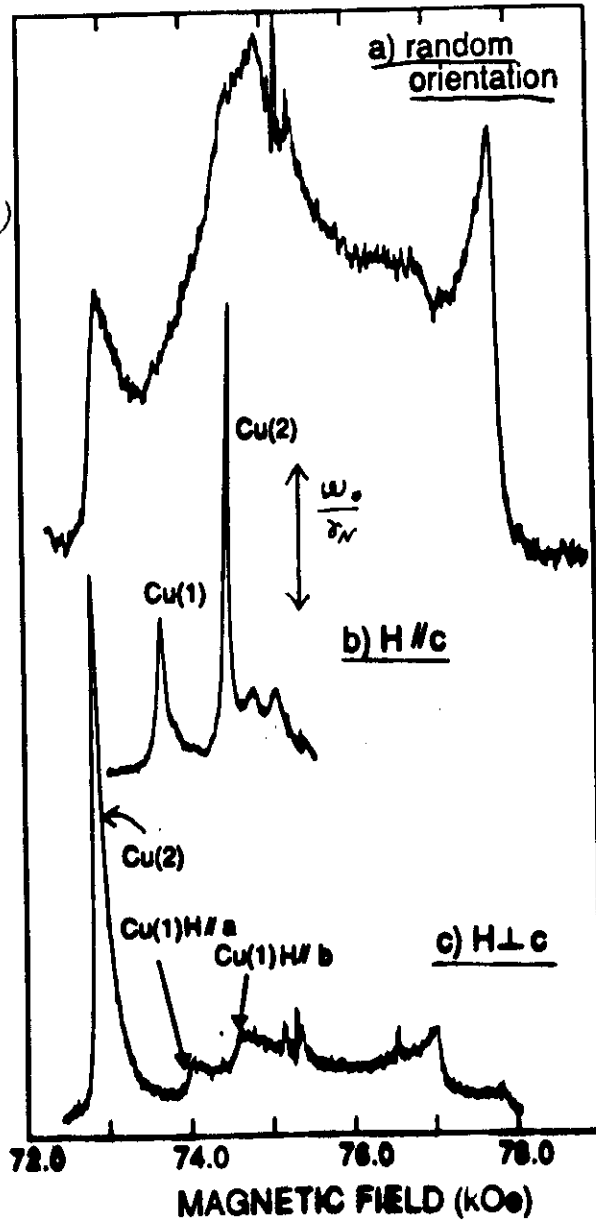
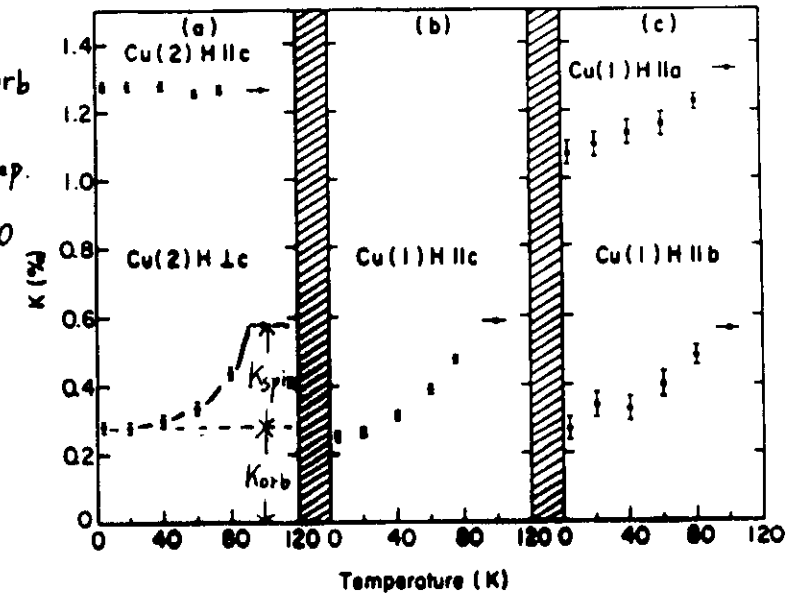


Fig 1

Cu Knight shift in YBCO_7 (Barnett et al, 90) (ref. 21)

$$K = K_{\text{spin}} + K_{\text{orb}}$$

$$\begin{cases} K_{\text{orb}}: T\text{-indep.} \\ K_{\text{spin}}(T \rightarrow 0) = 0 \end{cases}$$



	$K(T > T_c)$ (%)	$K(4.2\text{K})_{\text{orb}}$ (%)	K_{spin} (%)
Cu(1) c	0.59	0.25 ± 0.01	0.33 ± 0.01
Cu(1) a	1.32	1.08 ± 0.04	0.25 ± 0.04
Cu(1) b	0.56	0.27 ± 0.04	0.29 ± 0.04
Cu(2) c	1.27	1.28 ± 0.01	-0.01 ± 0.01
Cu(2) ab	0.58	0.28 ± 0.02	0.30 ± 0.02

Anisotropy of $1/T_1$

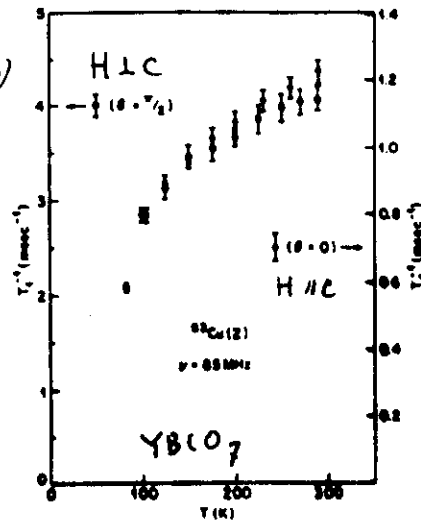
$$(1/T_1)_{H \parallel z} \propto (|A_x|^2 + |A_y|^2) \sum_f \frac{\text{Im} \chi(f, \omega)}{\omega_0}$$

simple expectation

$$\frac{(1/T_1)_{ab}}{(1/T_1)_c} = \frac{\{K_s(c)\}^2 + \{K_s(a,b)\}^2}{2\{K_s(a,b)\}^2} = \frac{1}{2}$$

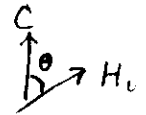
experiment: $\frac{(1/T_1)_{ab}}{(1/T_1)_c} = 3.6 \quad (\text{at } 100 \text{ K})$

(Walstedt et al.) (ref. 37)



Cu Knight shift

$$K_{\text{spin}}(\theta) = K_{\text{iso}} + K_{\text{ax}} (3 \cos^2 \theta - 1)$$



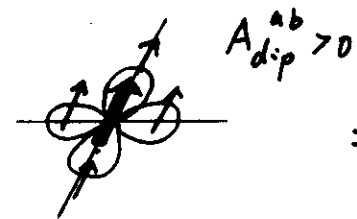
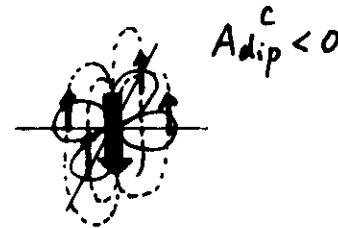
$$\begin{cases} K_{\text{iso}} = 0.23 \pm 0.02 \% \\ K_{\text{ax}} = -0.13 \pm 0.01 \% \end{cases}$$

- negative K_{ax} : consistent with the hyperfine field from a spin in $d_{x^2-y^2}$ orbital

$$\vec{A}_d = A_{\text{cp}} + \vec{A}_{\text{dip}}$$

($A_{\text{cp}} < 0$) $\Rightarrow$ modified by spin-orbit coupling

dipolar field



$$\Rightarrow K_{\text{ax}} < 0$$

- positive K_{iso} : incompatible with $\langle \vec{A}_d \rangle = A_{\text{cp}} < 0$
 $\Rightarrow$ transferred hyperfine field from neighboring Cu sites. (Mila, Rice)

$$K_{\text{ax}} = \frac{1}{3} (A_d^c - A_d^{ab}) \chi_d / N_A \mu_B$$

(ref. 2)

$$A_d^c - A_d^{ab} \approx 230 \text{ kOe}/\mu_B : (\pm 20\% \text{ characteristic of } \text{Cu}^{2+} \text{ d}_{x^2-y^2} \text{ state})$$

$$\chi_d = 9.2 \times 10^{-5} \text{ emu/mole.Cu}$$

spin susceptibility due to planar Cu d-holes.

Positive Kiso

- Mila and Rice (Physica 157, 561 (1991))

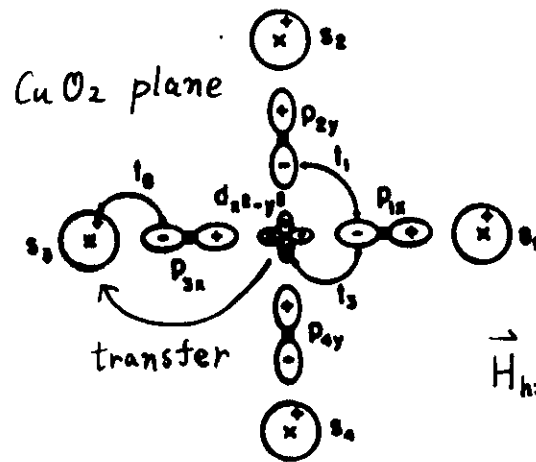


Fig. 1

$$|\phi\rangle = d_d |d_{x^2-y^2}\rangle + \alpha_p |p\rangle + \alpha_s |s\rangle$$

$$\vec{H}_{ht}(i) = \underbrace{\vec{A}_d \cdot \vec{S}_i}_{\text{on site}} + \underbrace{\sum_j \vec{B}_{ij} \cdot \vec{S}_j}_{\text{transfer}}$$

$$K_d = (A_d^d + 4B) \chi_d$$

it no pair spin correlations.

$$1/T_1)_d \propto \frac{1}{2} (A_d^{\rho 2} + A_d^{\sigma 2}) + 4B^2 \quad (\text{'add incoherently'})$$

hyperfine parameters $A_d^c = -170$, $A_d^{a,b} = 35$ $B = +2$ K/G

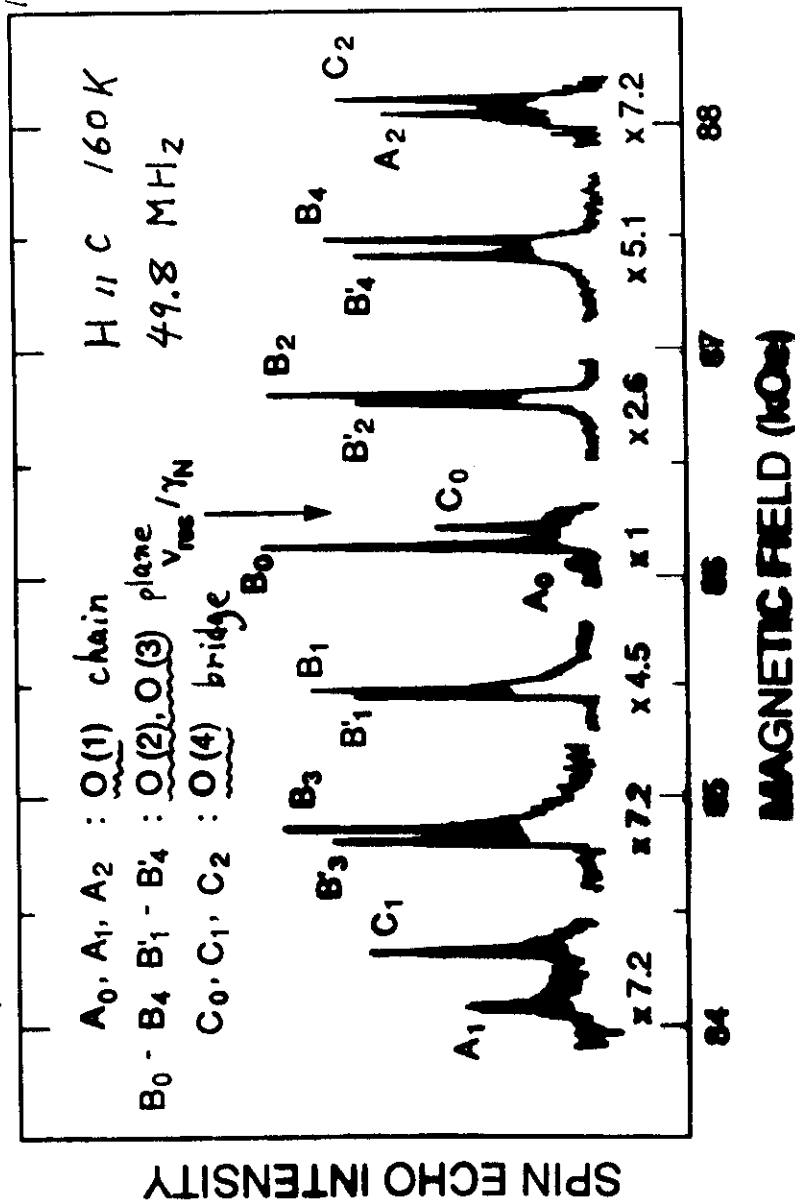
$$\Rightarrow K_c = 0$$

$$\frac{(1/T_1)_{ab}}{(1/T_1)_c} = 2.7$$

[better agreement by considering the AF correlations]

^{17}O NMR spectra in $YBa_2Cu_3O_{7-x}$ ($x=0.5$) 160K, 49MHz

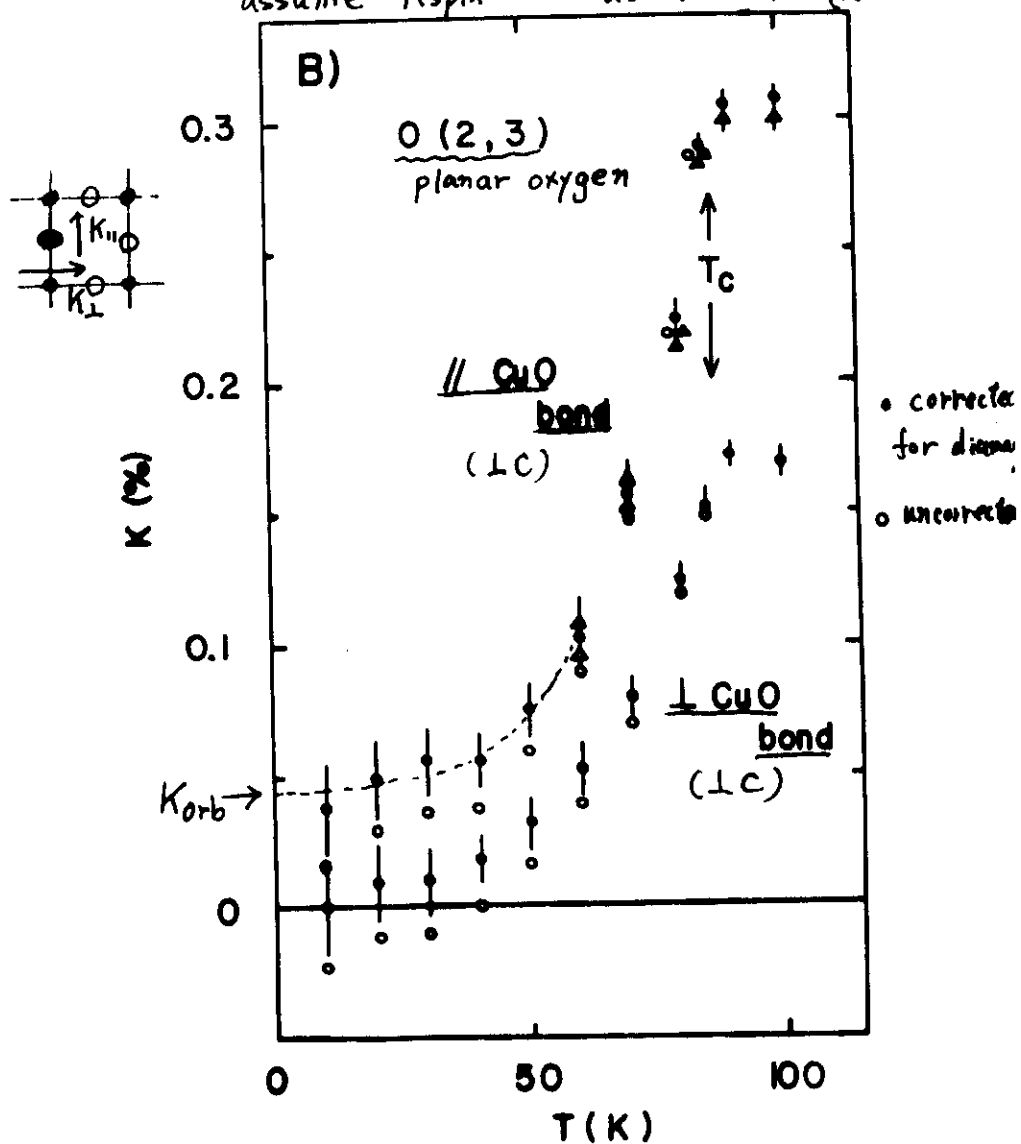
Takigawa, Hammel, Hettler, (ref. 15 above) 1989
Fisk, Thompson, et al. Phys. Rev. Lett. 61, 1365 (1988)



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$$K = K_{\text{spin}} + K_{\text{orb}} = \alpha \chi_{\text{spin}} + \beta \chi_{\text{orb}}$$

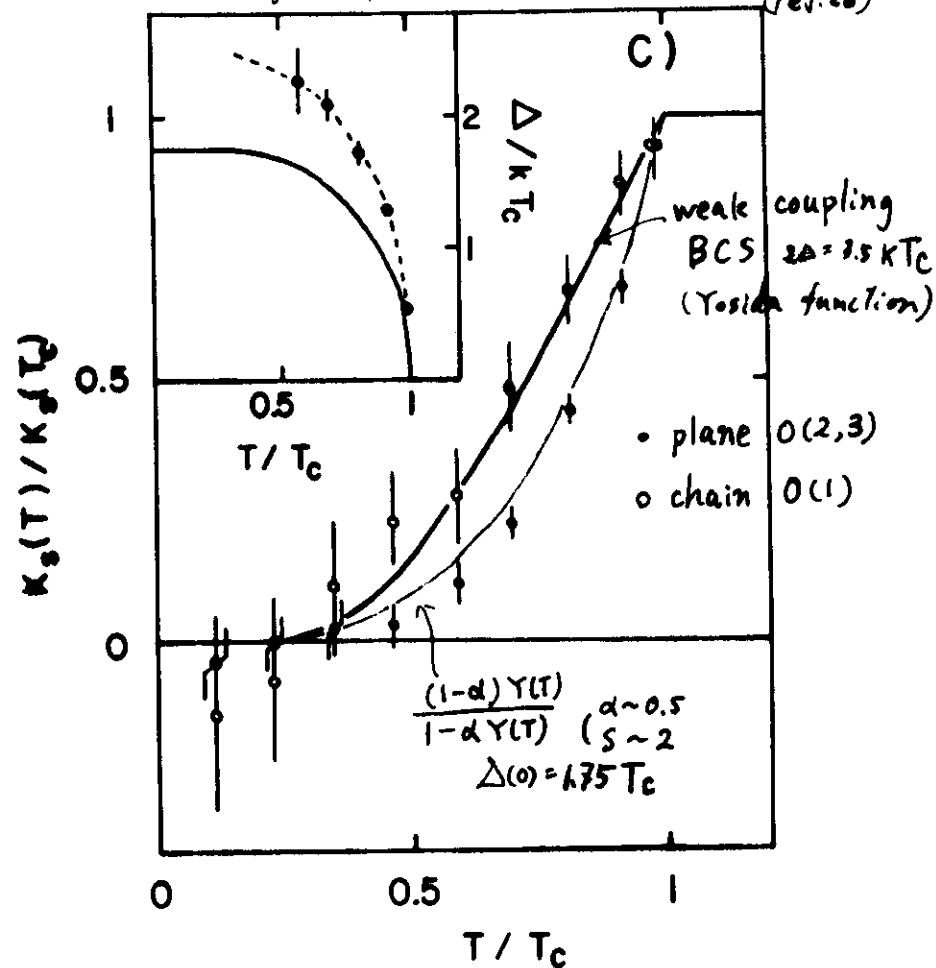
assume $K_{\text{spin}} \rightarrow 0$ as $T \rightarrow 0$. (ref. 26)



M. Takigawa
Fig. 2B)

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spin susceptibility in the superconducting state
more rapid decrease of planar χ_s than chain χ_s
S-wave pairing (ref. 26)

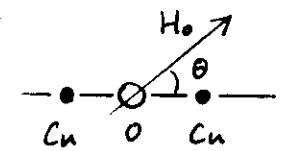


M. Takigawa
Fig. 2C)
reduction 45%

Oxygen Knight shift ($O(2,3)$ sites)

(red. 15)

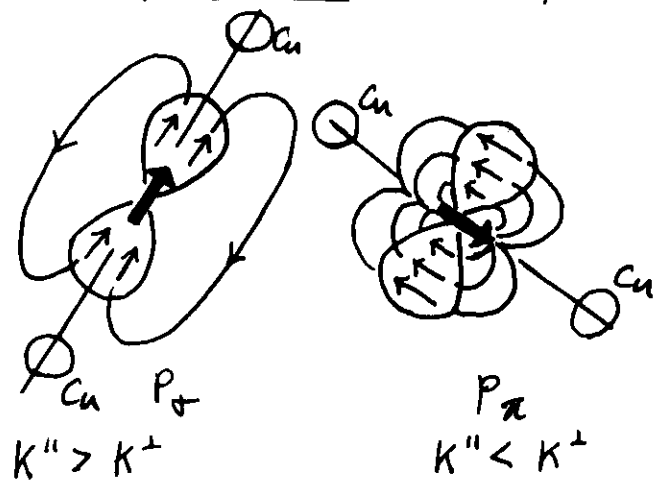
$$K_{spin}(\theta) = K_{iso} + K_{ax}(3\cos^2\theta - 1)$$



$$\begin{cases} K_{iso} = 0.19 \pm 0.02 \% \\ K_{ax} = 0.033 \pm 0.003 \% \end{cases}$$

K_{iso} : Fermi contact field from $2s$ state spin.

K_{ax} : dipolar field from a spin in $2p$ state.



Exp.
 $K'' > K^+$

spin density resides on $2p_{\sigma}$ orbitals.

spin susceptibility from Oxygen $2p$ -state.

$$\begin{cases} \chi_p = \frac{K_{ax}}{A_p} N_A \mu_B = 2.1 \times 10^{-5} \text{ emu/mole O} \\ (A_p = 90 \text{ KOe}/\mu_B) \\ \chi_d = 9.2 \times 10^{-5} \text{ emu/mole Cu} \end{cases}$$

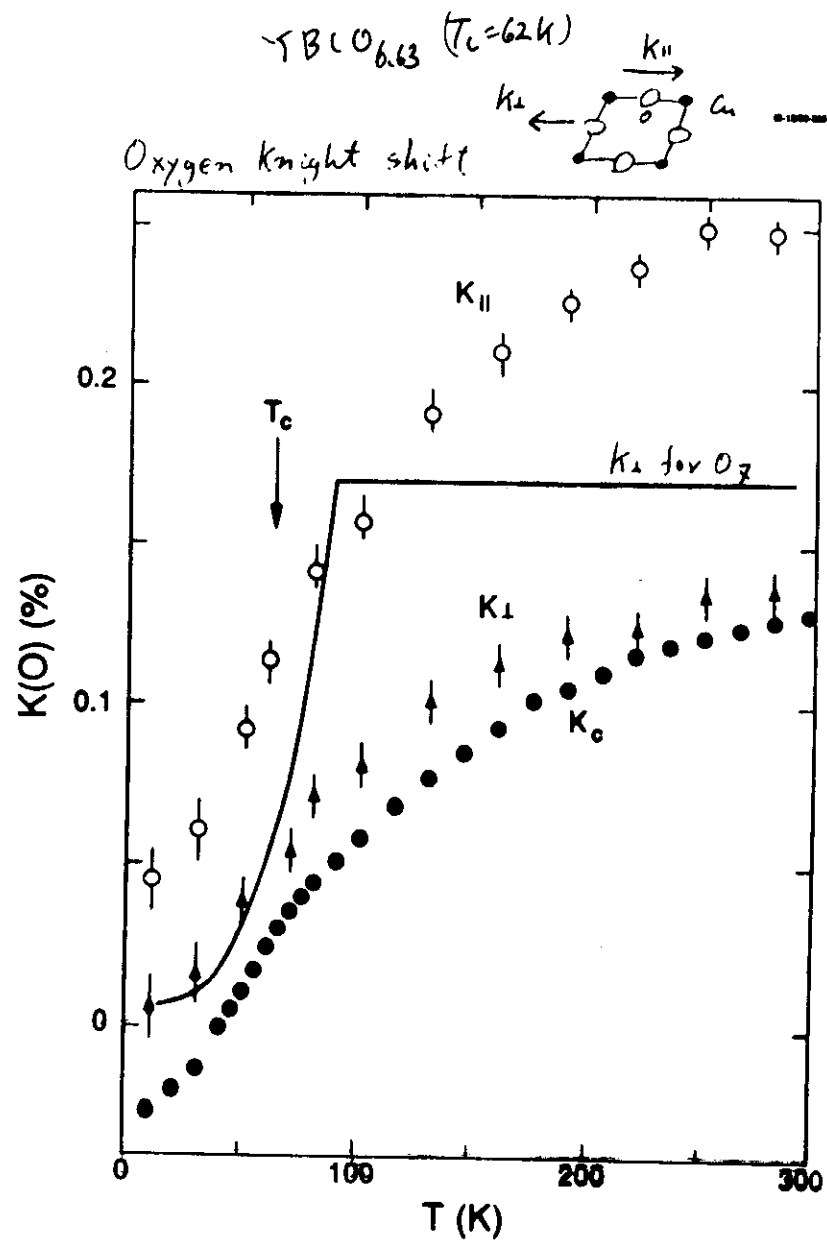
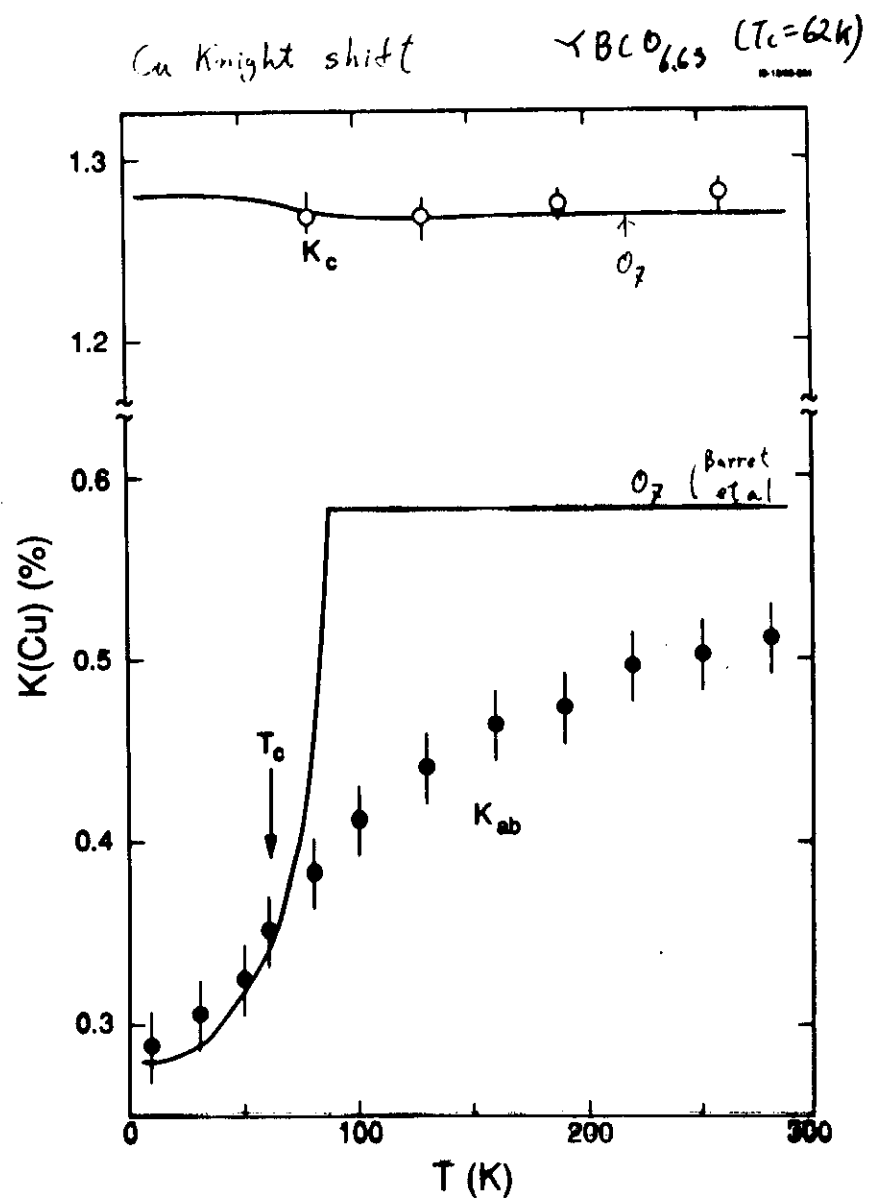
$$\chi_{Cu} : \chi_o = \chi_d : 2\chi_p \approx 7 : 3$$

• How much of χ_p is due to mobile doped holes?

$$\chi_p = \underbrace{\chi_{hole}}_{\text{mobile holes}} + f_{\sigma} \chi_{Cu}$$

covelency effect.
A part of Cu-d spin density is transferred to neighboring O $2p$ state.

$\chi_{hole} = 0$ in the single band picture, (local singlet)



Significant reduction of χ_{spin} with decreasing temperature.

speculations.

- T-dependent density of state $N(E_F)$
(pseudo-gap in $N(E)$ due to short range AF correlations, (Kampt, Schrieffer))
- preformed singlet pair (Warren et al.)
- spin-gap: a general property of a disordered quantum spin system.

$\chi_s \rightarrow 0$ as $T \rightarrow 0$

similar behavior

- $(LaSr)_2CuO_4$ (Kitaoka et al) (KCl)
- La_2CuO_{4+x} (Hammel et al) KCl
- $(TPh)Ba_2Cu_3O_7$ (Reyes et al) KCl

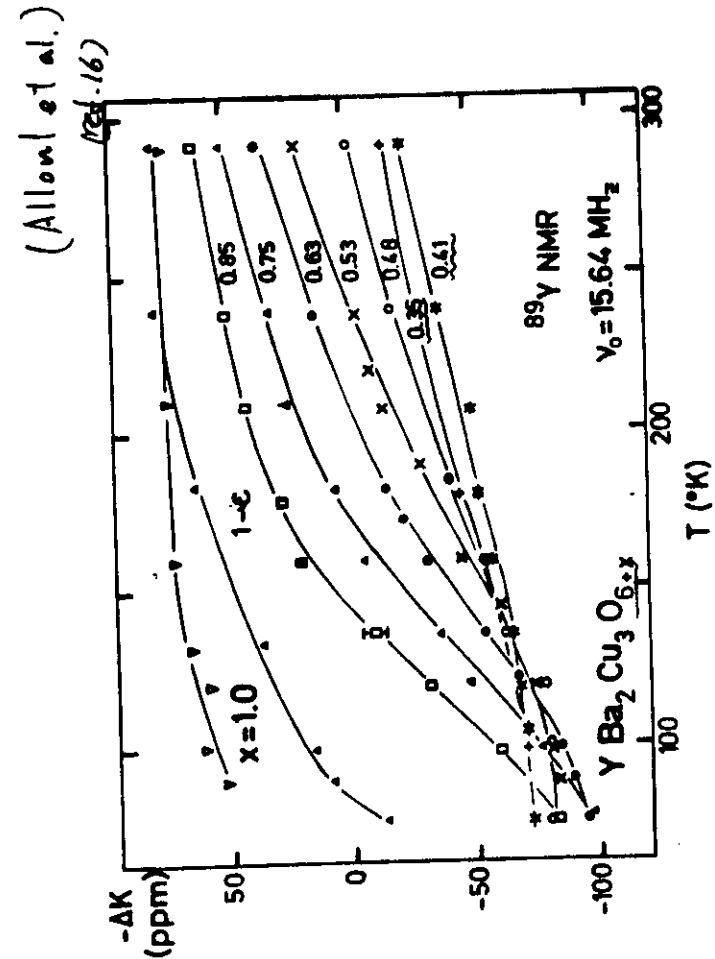
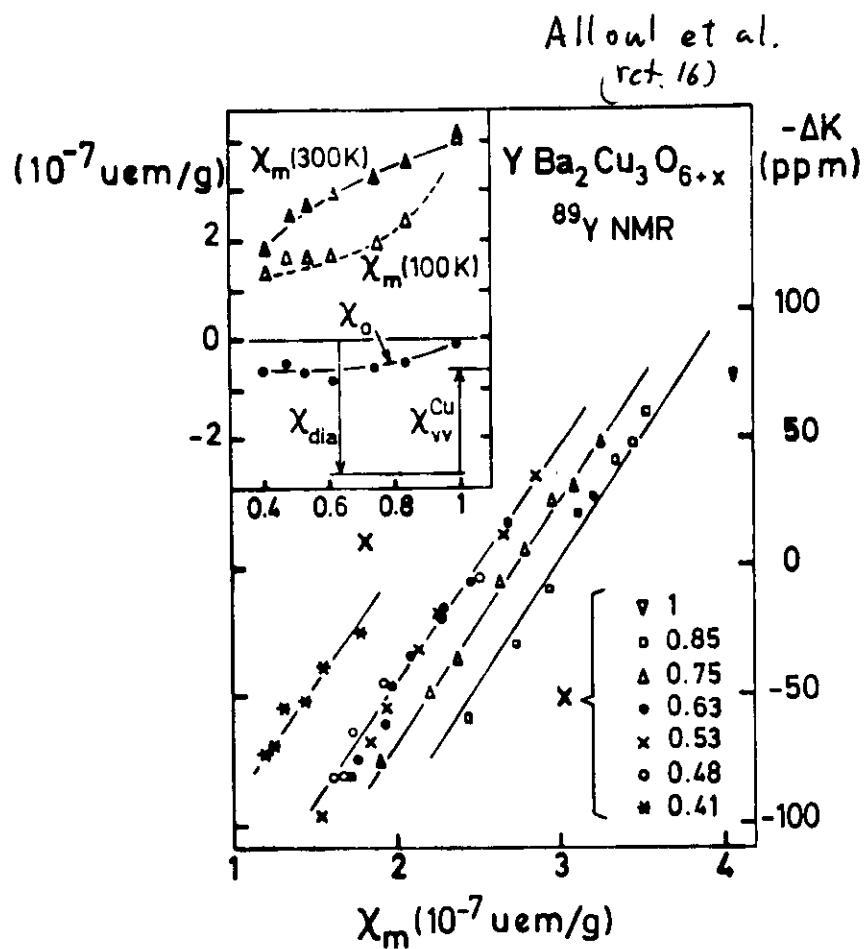


Fig. 1



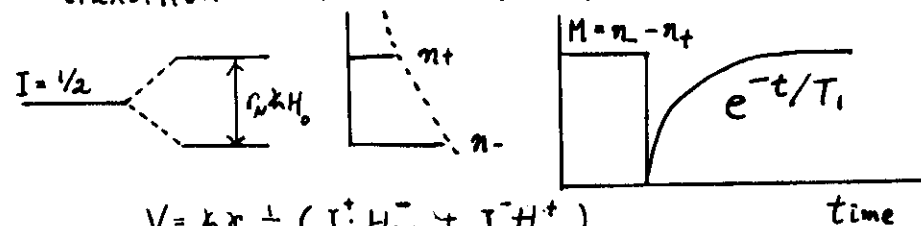
$\frac{\chi_p}{\chi_d}$: constant from 0.64 to 0.70

Fig. 1

[2] spin dynamics

nuclear spin relaxation : caused by fluctuations of $\vec{H}_h$

transition between Zeeman levels



$$V = h \gamma_N \frac{1}{2} (I^+ H_{h-} + I^- H_{h+})$$

$$1/T_1 \propto |H_{h+}(\omega_0)|^2 \quad (\vec{H}_h = A \vec{S}_i)$$

$$\propto A^2 \int_{-\infty}^{\infty} \langle S_i^+(t) S_i^-(0) \rangle e^{i\omega_0 t} dt$$

generally.

$$\frac{1}{T_1} = \frac{\gamma_N^2 k_B T}{2\mu_B^2} \sum_i |A_i|^2 \frac{\text{Im} \chi(i, \omega_0)}{\omega_0}$$

$$A_i = \sum_l A_{il} e^{i\vec{r}_l \cdot \vec{r}_i}$$

$\vec{r}$ -dependent hyperfine coupling (form factor)

$$\vec{H}_h = \sum_i A_i \vec{S}_i = \sum_i A_i \vec{S}_i$$

$$\langle H_{h+}^+(t) H_{h+}^-(0) \rangle = \sum_i |A_i|^2 \langle S_i^+(t) S_i^-(0) \rangle$$

- Lorentzian spin fluctuation spectrum

$$\frac{\text{Im} \chi(q, \omega)}{\omega} = \pi \frac{\chi_s \Gamma_s}{\omega^2 + \Gamma_s^2} \xrightarrow{\omega \rightarrow 0} \pi \frac{\chi_s}{\Gamma_s}$$

- Fermi liquid without strong magnetic correlation.

$$\chi_s / \mu_B^2 \sim 1/\hbar \rho_s \sim \rho(\epsilon_F)$$

$$\sum_q \text{Im} \chi(q, \omega_0) / \omega_0 \sim \pi \hbar \mu_B^2 \rho(\epsilon_F)^2 = \pi \hbar \chi_s^2 / \mu_B^2$$

↑
uniform (q=0) spin susceptibility

for non-interacting electron:

$$\sum_q \text{Im} \chi(q, \omega_0) / \omega_0 = \pi \hbar \chi_s^2 / (2 \mu_B^2) \quad (\text{exact})$$

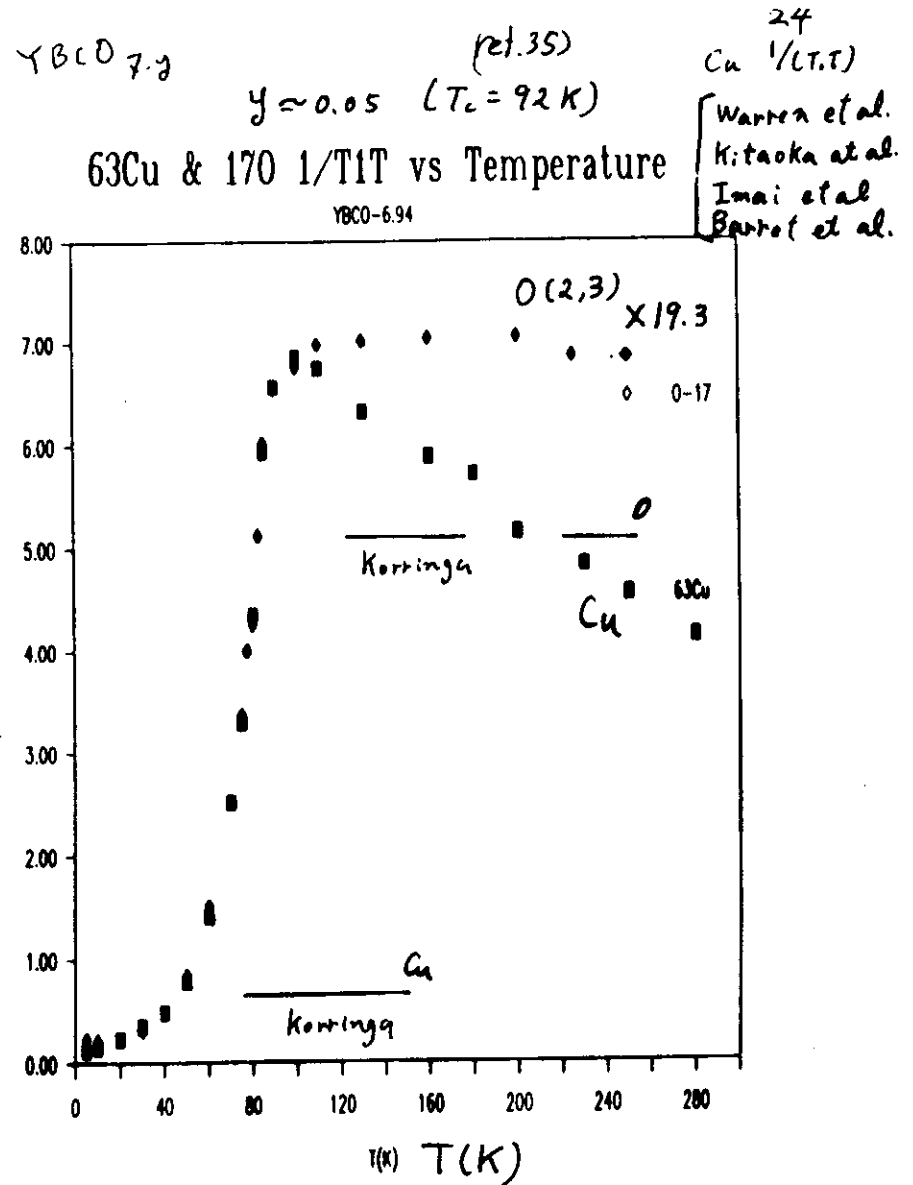
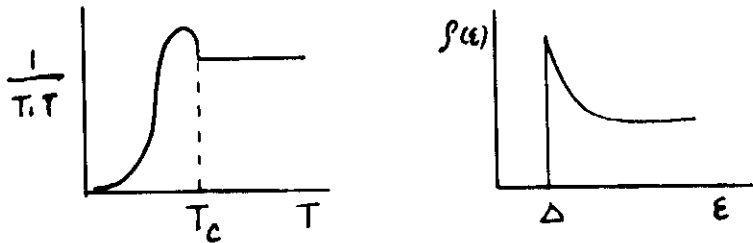
- If A_s is local (i.e. q-independent) and isotropic,

$$\frac{1}{T_1 \cdot T \cdot K^2} = \frac{\pi \hbar \gamma_N^2 k_B}{\mu_B^2} \equiv S \quad (\text{Korringa relation})$$

universal constant.

- Superconducting state

BCS (Hebel-Slichter) peak



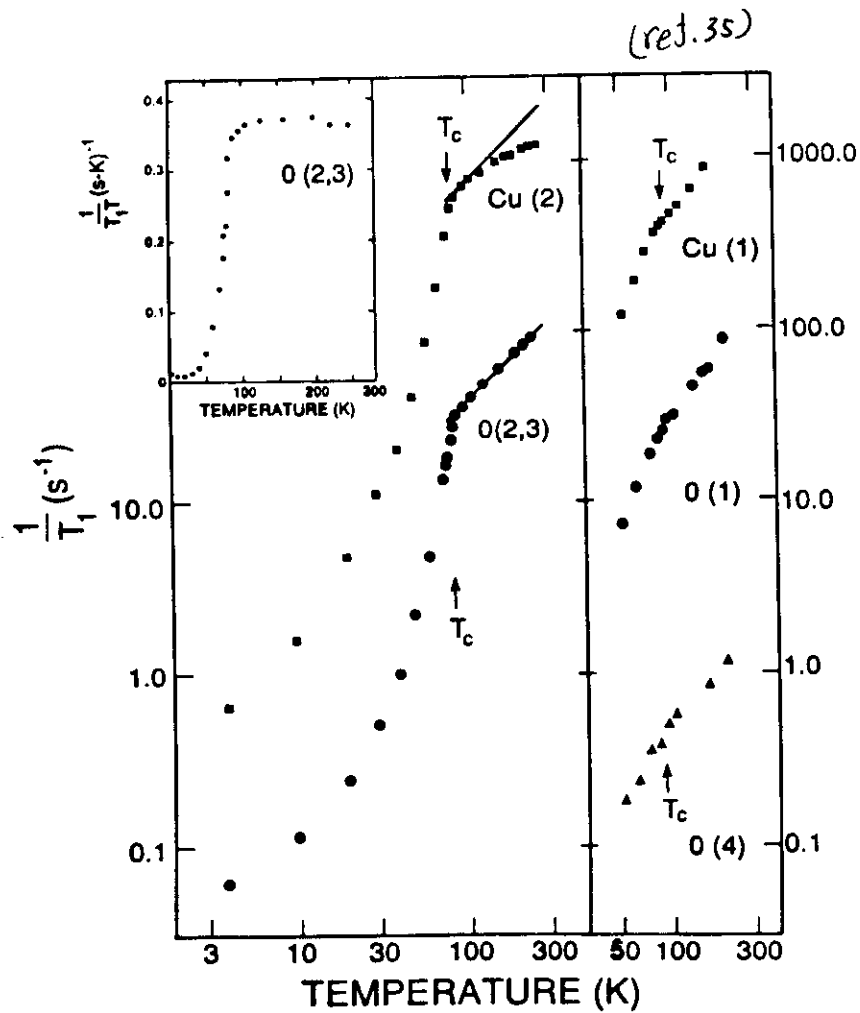


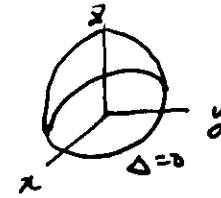
FIG. 2

Absence of Hebel-Slichter peak
(coherence)

possibilities

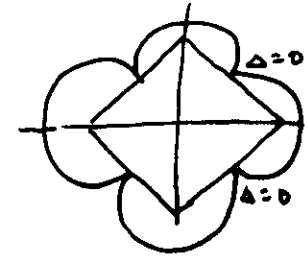
1) anisotropic superconducting gap

3-dim. $\Delta = \Delta(\vec{k})$
 $= \Delta_0 \cos \theta$



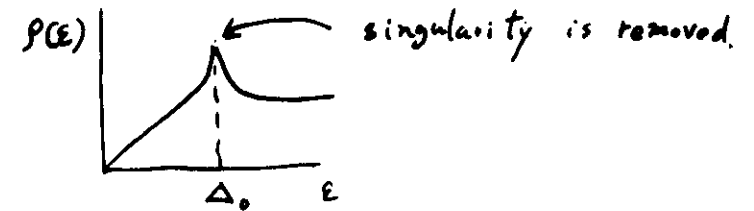
polar state.

2-dim $\Delta(\vec{k}) = \Delta_0 (\cos a k_x - \cos a k_y)$



d-wave pairing

quasi-particle density of state

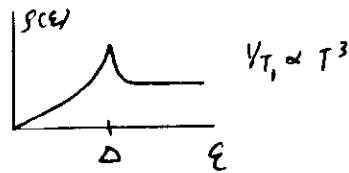


Absence of Hebel-Slichter peak

1) anisotropic (d-wave) pairing

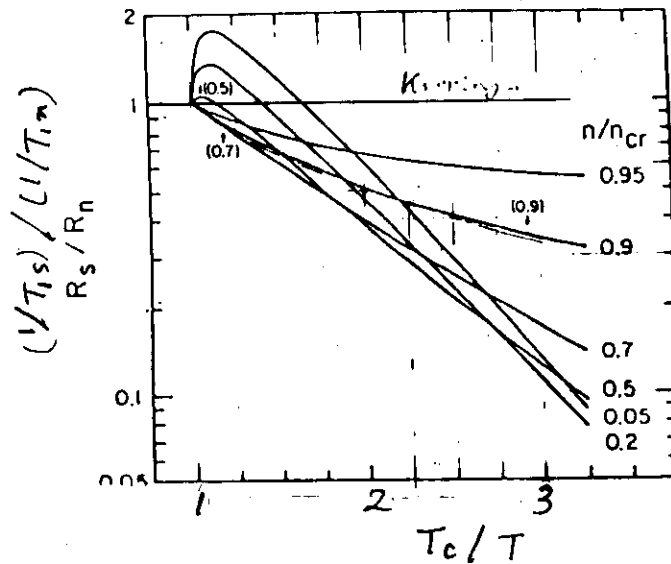
$\uparrow \downarrow \times$

(penetration depth, photoemission etc.)



2) pair breaking (magnetic impurity)

Griffin and Ambegaokar '64



$\Delta = \dots \int \vec{\sigma} \cdot \vec{s}$
impurity spin
break time revers.
symmetry

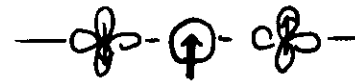
T-dependent pair breaking (by spin fluctuations)
(inelastic magnetic scattering, spin fluctuations)
significant pair breaking effect only near T_c .
(L. Coffey, Kuroda and Varma)

Enhancement of the Korringa product.

$$\frac{1}{T_c T K^2 S} \sim 11 \text{ Cu (15) } \left\{ \begin{array}{l} 1.4 \text{ O (2.5)} \\ \text{(Mills, Monien, Pines)} \end{array} \right.$$

• large enhancement at Cu $\Rightarrow \chi(q)$ has a peak at $q_0 \neq 0$

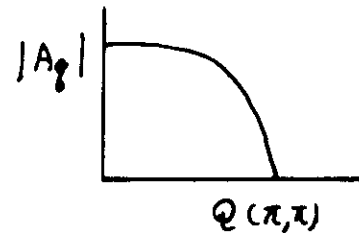
• oxygen relaxation



dominant source: O_{2s} - Cu_{3d} mixing

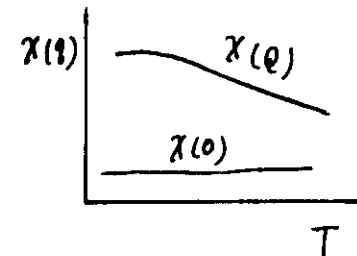
$$\vec{H}_{hf} = A (\vec{S}_i + \vec{S}_{i+1})$$

$$= \sum_{\vec{q}} A (1 + e^{i\vec{q} \cdot \vec{a}}) \cdot \vec{S}_{\vec{q}} \\ = A_{\vec{q}} \text{ (form factor)}$$



Small enhancement at 0

$\chi(q)$ must have a peak
at $Q = (\pi, \pi)$

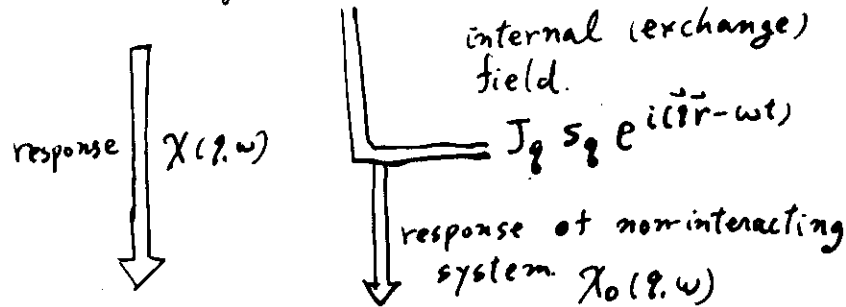


• Identical T-dependence of $\frac{1}{T_c T}$ at Cu and O below T_c .

$\downarrow$
[χ_q becomes T-independent below T_c
AF correlations persist below T_c]

Random Phase Approximation (RPA) (Dynamical Hartree-Fock method)

external field $H(\vec{r}, t) = H_0 e^{i(\vec{q} \cdot \vec{r} - \omega t)}$



magnetization $S(\vec{r}, t) = S_0 e^{i(\vec{q} \cdot \vec{r} - \omega t)}$

$$\begin{cases} S_0 = \chi_0(q, \omega) (H_0 + J_0 S_0) \\ \chi(q, \omega) = S_0 / H_0 \end{cases} \Rightarrow \frac{1}{\chi_0(q, \omega)} = \frac{1}{\chi(q, \omega)} + J_0$$

$$\chi(q, \omega) = \frac{\chi_0(q, \omega)}{1 - J_0 \chi_0(q, \omega)}$$

Hubbard model

$$H = \sum_k \epsilon_k a_{k\uparrow}^\dagger a_{k\uparrow} + \sum_i U n_{i\uparrow} n_{i\downarrow}$$

$$\frac{1}{2} N U - \frac{1}{2} \frac{U}{N} \sum_i (S_{i\uparrow}^\dagger S_{i\downarrow}^\dagger + S_{i\downarrow}^\dagger S_{i\uparrow}^\dagger)$$

$$S_{\vec{q}}^\dagger = \sum_k a_{k\uparrow}^\dagger a_{k+\vec{q}\downarrow}$$

Effect of magnetic correlations. (Ref. 14)
RPA (Moriya '63)

$$\chi(q, \omega) = \frac{\chi_0(q, \omega)}{1 - J \chi_0(q, \omega)}$$

$$\frac{1}{T, T} \propto \sum_q \frac{\text{Im} \chi(q, \omega_0)}{\omega_0} = \sum_q \frac{\text{Im} \chi(q, \omega_0) / \omega_0}{(1 - J \chi_0(q))^2} = \sum_q \left(\frac{\chi(q)}{\chi_0(q)} \right)^2 \frac{\text{Im} \chi_0(q)}{\omega_0}$$

$$K \propto \chi_s = \left(\frac{\chi_s}{\chi_0(0)} \right) \chi_0(0)$$

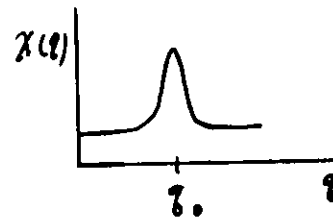
$$\Rightarrow \frac{1}{T, T K^2 S} = \frac{\langle (\chi(q) / \chi_0(q))^2 \rangle}{(\chi_s / \chi_0(0))^2}$$

nearly ferromagnetic



$$\frac{1}{T, T K^2 S} \ll 1$$

nearly antiferromagnetic



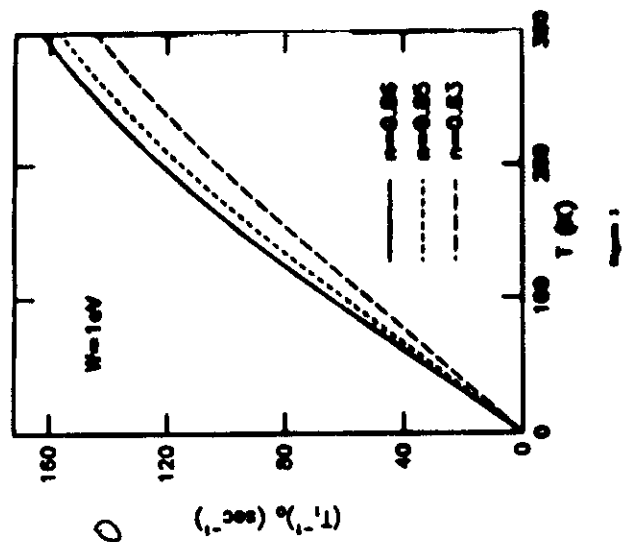
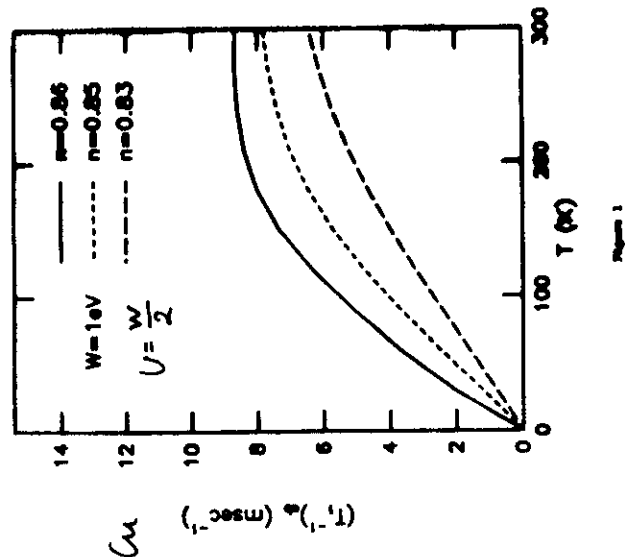
2-dim Hubbard model (square lat)

$$\frac{1}{T, T K^2 S} \gg 1$$

half filled $\chi(q)$

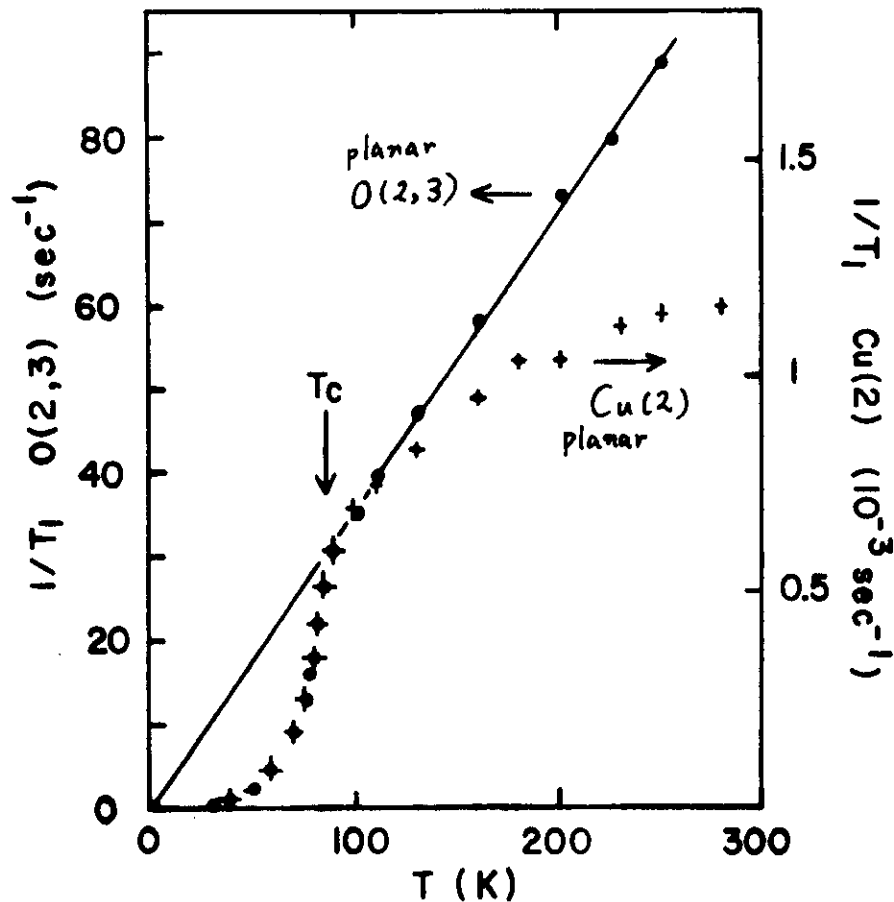
Van Hove singularity. q

Bulut, Hone, Scalapino, Bickers (ref. 25)
 2D single band Hubbard model/
 (tight binding)
 RPA



(ref. 27)
 Cu(2) data : same as NQR $1/T_1$ by { Warren et al
 Kitaoka et al
 Imai et al

H || c 7 tesla



Y: similar to O
 (Markot et al.)
 (Alloul et al.)
 M. Takigawa

Phenomenology (Millis, Monien and Pines) (ref. 24)

RPA expression $\chi(q, \omega) = \frac{\chi_0(q, \omega)}{1 - J\chi_0(q, \omega)}$

$$\chi''_{AF}(q, \omega) = \frac{\chi''(q, \omega)}{\{1 - J\chi'_0(q, \omega)\}^2 + \{J\chi''_0(q, \omega)\}^2} \quad \chi''_0(q, \omega) = \pi \frac{\omega}{P} \chi_0$$

$$J\chi'_0(q, \omega=0) = F_q$$

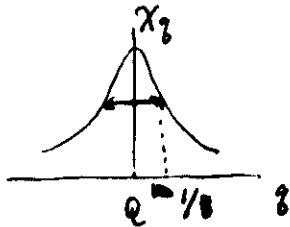
Strong AF correlation $\Rightarrow 1 - F_Q \ll 1$ $Q = (\pi/2, \pi/2)$
expansion around Q

$$F_Q = F_Q - g^2 \xi_0^2$$

Static susceptibility

$$\begin{aligned} \chi'(q, \omega=0) = \chi_q &= \frac{\chi_0(q)}{1 - F_q} \simeq \frac{\chi_0(Q)}{1 - F_Q} = \frac{\chi_0(Q)}{1 - F_Q} \frac{1 - F_Q}{1 - F_Q} \\ &= \chi_Q \frac{1 - F_Q}{1 - F_Q + g^2 \xi_0^2} = \chi_Q \frac{1}{1 + g^2 \xi_0^2 / (1 - F_Q)} \end{aligned}$$

definition of AF correlation length



$$\chi_q = \frac{\chi_q}{1 + g^2 \xi^2} \Rightarrow 1 - F_Q = (g/\xi)^2$$

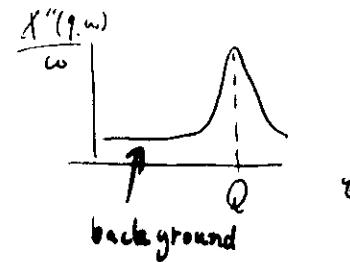
dynamical susceptibility.

$$\chi''_{AF}(q, \omega) = \frac{\pi \frac{\omega}{P} \chi_0}{(1 - F_q)^2 + \pi^2 \omega^2 J^2 \chi_0^2 / P} \quad (J\chi_0)^2 \sim 1$$

$$\begin{aligned} (1 - F_q)^2 &= 1 - F_Q + F_Q - F_q \\ &= (\xi_0/\xi)^2 + g^2 \xi_0^2 \\ &= (\xi_0/\xi)^2 (1 + g^2 \xi^2) \end{aligned}$$

$$\chi''_{AF}(q, \omega) = \frac{\pi \omega \chi_0}{P} \frac{(\xi/\xi_0)^4}{(1 + g^2 \xi^2)^2 + \frac{\pi^2 \omega^2}{P^2} (\frac{\xi_0}{\xi})^2}$$

$$\omega \rightarrow 0 \quad \frac{\chi''(q, \omega)}{\omega} = \frac{\pi \chi_0}{P} \frac{(\xi/\xi_0)^4}{(1 + g^2 \xi^2)^2}$$



$$\left(\frac{\chi''(q, \omega)}{\omega} \right)_{\omega \rightarrow 0} = \frac{\chi_0 Q}{P} \left(1 + \frac{(\xi/\xi_0)^4}{(1 + g^2 \xi^2)^2} \right)$$

difference for $\ln$ and 0 ---- hyperfine coupling

