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# QUANTUM WIRES AND QUANTUM DOTS

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# Quantum Wires and Quantum Dots

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Simple concepts and models for confined electronic states in semiconductors, and some representative experimental results, are presented. This is an outline of lectures to be presented 14-18 May in Trieste at the 1990 Spring College in Condensed Matter, on "Physics of low-dimensional semiconductor structures."

## 1. DIMENSIONALITY

Physicists and mathematicians have long thought about the way motions and relationships would change in systems with dimensionality different from that of the three-dimensional space we are accustomed to. Over the past decades it has become possible to fabricate structures that exhibit reduced dimensionality, and the subject of the 1990 Spring College deals largely with systems that already exist. Reduced dimensionality can arise in many different ways, depending on the physical processes being considered, and usually involves a sample or a physically defined region within a sample with one or more dimensions small compared to an appropriate physical scale length.

For transport processes, two important scale lengths are the mean free path, controlled by carrier scattering and the phase breaking length--which is often equal to the inelastic scattering length. At high temperatures, where phonon scattering dominates, both of these lengths are determined by the phonon scattering rate. At high carrier densities, carrier-carrier scattering may control the inelastic scattering or phase-breaking length, but does not--to lowest order--affect the transport mean free path because scattering among carriers in a simple band does not change the current.

At lower temperatures many transport processes depend on the inelastic scattering length, $L_{in} = (2dD\tau_{in})^{1/2}$, which is an average distance carriers diffuse between inelastic collisions in $d$ dimensions. The diffusion constant $D$ is usually controlled by elastic scattering processes like impurity scattering, and is approximately independent of temperature, while the inelastic scattering time is temperature dependent. At sufficiently low temperatures the inelastic scattering length can be of order 1 to10 $\mu$m, placing many samples in the low-dimensionality regime for transport. Corrections to transport properties at low temperatures occur for all dimensionalities, but are more easily observed in



lower dimensional systems. This subject, often called weak localization, had a rapid development beginning about ten years ago and has led to a large literature dealing with low-dimensional aspects. It is covered in other lectures at this Spring College.

The characteristic length we are concerned with here is the Fermi wavelength, $h/p_F$, where $h$ is Planck's constant and $p_F$ is the carrier momentum at the Fermi surface. A more useful characteristic length is $\hbar/p_F$, which is sometimes--but loosely--also called the Fermi wavelength. When a sample dimension becomes comparable to this distance, the motion in that direction becomes quantized, resulting in changes in the energy spectrum and in the dynamical properties of the system. When only one dimension of the sample is small compared to this distance we speak of a system that is (dynamically) two-dimensional. Many properties of such systems are described in the article by Ando et al.,[1] and are discussed in many of the lectures in this Spring College. The present discussion deals mainly with systems that are constrained in two or three dimensions, and are therefore dynamically one- or zero-dimensional, but we occasionally refer to results for the better-known two-dimensional systems for comparison. A dynamically one-dimensional system is often called a quantum wire, and a dynamically zero-dimensional system is often called a quantum dot or quantum box.

These lectures will cover some simple ideas, and examples of experimental results, for confined electron systems in semiconductors. The field, though still young, already has a large literature and many important results will be omitted. In some cases I cite one or two recent papers rather than the original work, because it is easier to follow a publication chain back in time than to start from the early papers and try to find later ones.

## 2. STRUCTURES AND FABRICATION

A brief summary of some of the structures that have been made or proposed for confining carriers to wire-like and dot-like configurations will be given here, and fabrication methods will be alluded to briefly. These subjects are covered in detail in other lectures. We can divide the confinement mechanisms into several categories, although actual structures usually involve more than one mechanism.

Compositional confinement arises from the barrier to carrier motion imposed by a change--either abrupt or graded--in chemical composition. One example is the barrier, about 3 eV, for electrons at the interface between Si and $SiO_2$. Another is the conduction band offset at the interface between GaAs and $Al_xGa_{1-x}As$. The conduction band offset is about 0.2 eV if the AlAs fraction $x$ is 0.3, which is in the direct-gap composition range for the alloy.

Electrostatic confinement was one of the first means used to create a true quantum-confined two-dimensional system, the inversion layer of a metal-oxide-silicon field-

effect transistor structure. The electric field attracts the carriers toward the gate, but the electrostatic potential together with the barrier at the interface creates a potential well whose effective width can be of order a few nm. Some of the extensive literature of this system is covered in Ref. 1. Several ingenious variations of this basic structure, involving narrow gates, split gates, grating gates, and other variants, have been used to shape the electrostatic confining potential which, when combined with one or more barriers due to compositional confinement, leads to an effectively one- or zero-dimensional system

Geometrical confinement is the simplest mechanism to consider, although not necessarily the easiest to realize. One or more sample dimensions are made suitably small either by an appropriate growth technique or, more commonly, by removing material from a larger sample, for example by etching. Posts of material with diameter of order 100 nm or less have been prepared from a GaAs-based heterostructure, and the combination of compositional and geometrical confinement has been used to fabricate structures for optical studies, including lasers, and for tunneling experiments.

Some physical or chemical processes, such as reactive ion etching or bombardment by particle beams, lead to changes in the surface or bulk properties of the material that have been used to confine carriers. While the effectiveness of this method has been demonstrated, there is little detailed understanding of the way the confinement induced by these processes should be modeled. This is an area that needs additional work before quantitative models can be formulated for such structures.

Selective growth on a terraced semiconductor surface, as obtained for example by using a surface slightly tilted from a high-symmetry direction like (001), can lead to a wire-like structure whose width can be controlled by the growth conditions and the terrace width and whose thickness can be as small as a monolayer. Selective growth can also be used on the etched or cleaved surface of a superlattice or other compositionally modulated structure to yield confinement to a wire, as suggested by Sakaki.[2]

An ingenious method of confinement using localized strains to confine excitons has been explored by Kash and coworkers for quantum dots and quantum wires.[3,4]

Many of the methods we have described can be used to fabricate wires or dots, or arrays of wires or dots that may or may not interact with each other. Alternatively they can be used to add spatial modulation, as for example to introduce linear or rectangular grating modulations onto a two-dimensional electron system. Such systems form a transition between one dimensionality and another. They have been widely studied experimentally, for example by groups in Hamburg[5] and at MIT,[6,7] and theoretically.[8–10]

## 3. ELECTRONIC STATES

The increasingly versatile fabrication methods being devised make it possible to create structures with a wide range of confining mechanisms and geometries. In this section I shall describe some geometries that vary from textbook simplicity to more realistic cases, and discuss the electronic states in those structures.

The simplest geometry for which explicit results can be obtained easily is a rectangular geometry with a flat potential inside (taken to be zero) and infinitely high, mathematically sharp walls. Here, and throughout most of this chapter, we use the effective mass approximation and assume that the respective effective masses for motion in the $k_x$, $k_y$, and $k_z$ directions are $m_x$, $m_y$, and $m_z$. The wave functions are approximated as products of Bloch functions, taken near the relevant band edge, and envelope functions which are presumed to be slowly varying on an atomic scale although that approximation is severely tested at times. For most purposes the underlying Bloch function is omitted but it must be included for some purposes, such as the calculation of optical absorption. In this effective mass approximation the envelope wave function of a state with quantum numbers $i$ and $j$ and wave vector $k_x$ in a rectangular wire of dimensions $a$ and $b$ is taken to be

$$\zeta_{i,j,k_x} = (4/ab)^{1/2} \sin(\pi i y/a) \sin(\pi j z/b)\, e^{ik_x x}, \qquad i,j = 1, 2, 3, \ldots. \tag{1}$$

Motion along the wire is taken to be free-electron-like, leading to subbands of states with energies

$$E_{i,j}(k_x) = \frac{i^2\pi^2\hbar^2}{2m_y a^2} + \frac{j^2\pi^2\hbar^2}{2m_z b^2} + \frac{\hbar^2 k_x^2}{2m_x}. \tag{2}$$

The subband energies for $k_x = 0$ are nondegenerate (not including spin) in general, but for a square wire with $a = b$ the states $i,j$ and $j,i$ will be degenerate when $i \neq j$. The density of states per unit energy for the one-dimensional subband with quantum numbers $i$ and $j$ in a wire of length $L$ is

$$\rho(E) = 2{\cdot}2\,(L/2\pi)dk_x/dE = (L/\pi)(2m/\hbar^2)^{1/2}(E - E_{i,j})^{-1/2}, \tag{3}$$

where the first factor 2 arises because there are states with both positive and negative values of $k_x$ and the second factor 2 is the spin degeneracy. The inverse square root behavior is to be contrasted with the two-dimensional result that the density of states is a constant above the band edge and the three-dimensional result that the density of states varies as the square root of the energy measured from the band edge.

Somewhat surprisingly, a closed-form solution for the wave functions and energy levels of a rectangular wire with a finite barrier is not available. We shall consider a treatment for such a case below.

If the simple rectangular wire is truncated to become a rectangular parallelepiped, then the envelope wave function will have sinusoidal factors in all three space dimensions, by obvious extension of Eq. (1), and the energy levels will be discrete.

Another simple case is a wire with a circular cross section. The Schrödinger equation in polar coordinates takes the form

$$-\frac{\hbar^2}{2}\left[\frac{1}{R}\frac{\partial}{\partial R}\frac{R}{m}\frac{\partial}{\partial R}+\frac{1}{R^2}\frac{\partial^2}{\partial\phi^2}\right]\zeta(R,\phi)=[E-V(R,\phi)]\zeta(R,\phi), \qquad (4)$$

where we have assumed free-carrier motion in the $x$ direction, taken the mass in the $y$-$z$ plane to be isotropic, and used $R=(y^2+z^2)^{1/2}$. If the potential inside the wire is zero and the barrier at radial distance $R_0$ from the center is infinite, then the envelope functions are

$$\zeta_{l,n,k_x}=(\pi L)^{-1/2}R_0^{-1}[J_l(j_{l,n}R/R_0)/J_{l+1}(j_{l,n})]\,e^{il\phi}\,e^{ik_x x}, \qquad l=0,1,2,..., \qquad (5)$$

where $J_l$ is the Bessel function of order $l$ and $j_{l,n}$ is its $n$th zero. The energy levels are

$$E_{l,n}(k_x)=\frac{\hbar^2 j_{l,n}^2}{2mR_0^2}+\frac{\hbar^2 k_x^2}{2m}. \qquad (6)$$

The subbands with angular momentum $l$ are nondegenerate if $l=0$ and are doubly degenerate for nonzero values of $l$.

It is instructive to compare the integrated density of states for a square wire and a circular wire of the same cross-sectional area $a^2=\pi R_0^2$. The results are shown in Fig. 1, and demonstrate, as expected, that the asymptotic density of states at large quantum numbers depends mainly on the area and only weakly on the geometry. Perhaps less well known is the effect of the boundary conditions, which should not affect the asymptotic density of states for a sufficiently large system. To illustrate that effect, Fig. 1 also shows the cumulative number of subbands and the integrated density of states for the same square and circular wires if the boundary condition on the envelope wave function were the vanishing of its normal derivative, rather than of its value. The envelope wave functions are then simple variants of those given in Eqs. (1) and (5). Note that the lowest envelope wave function for both geometries is then simply a constant, and the associated subband starts at zero energy. The results for these two extreme limits of physically realistic boundary conditions are instructive. For large quantum numbers the ratios of the results to the corresponding asymptotic values, which are shown as dashed lines in Fig. 1, approach unity, but the magnitude of the differences continues to increase.

Level broadening and thermal smearing must be small enough compared to the energy differences between subbands to make it possible to observe the energy level structure associated with the confinement, so small dimensions, which maximize the energy scale, and low temperatures, which minimize both thermal smearing and level broadening, are normally required.

In most realizations of wire and dot structures the barrier at the boundaries should be treated as finite, and the approximation that the envelope wave function vanishes at the boundary is likely to fail. That is the case, for example, when the confining potential is provided by a band offset as in a GaAs-(Al,Ga)As heterostructure. In such cases a more general Schrödinger equation must be solved. The general question of boundary conditions at a heterointerface is described by other lecturers, and the discussion here deals only with the simplest case, as in heterojunctions with $x \le 0.4$, for which both materials have their conduction band minimum at the center of the Brillouin zone. In that case the boundary condition to be used, chosen to preserve current as calculated from the envelope function, is[11]

$$m_1^{-1}\zeta'_1=m_2^{-1}\zeta'_2, \qquad (7)$$

where $m_1$ and $m_2$ are the respective effective masses and the prime denotes the derivative in the direction normal to the interface. The envelope function and its derivatives in the interface plane are continuous.

For the circular wire, explicit solutions involving Bessel and Neumann functions can be written down for the finite barrier case. The eigenvalues are found from the solutions of a transcendental equation for the matching at the boundary, using Eq. (7). In addition to the discrete solutions, which vanish at infinity, there is also a continuum of solutions with energies greater than the barrier height. The density of states in the continuum has broad maxima corresponding to resonances or virtual states.

In addition to the simple potentials already described, there has been considerable interest in parabolic potentials, which arise naturally in electrostatic confinement and can be fabricated via compositional variations in quantum wells. The parabolic potential has a number of special properties, such as an energy spectrum with equally spaced levels. Also, a linear perturbation leaves the potential parabolic, with only a shift in the position of the minimum. Some other properties of parabolic quantum wells will be discussed later, including the energy spectrum in a magnetic field.

Once we leave simple models like those just described, calculation of the energy level structure becomes more complicated. The problem can be divided into two parts: a description of the potential, and the calculation of the energy levels and other proper-

ties of the electronic system. If the system has carriers present, these two problems are coupled and must be solved self-consistently.

In some cases the potential is known, as for example in a structure that uses geometrical or compositional confinement. In other cases, particularly those involving electrostatic confinement, determination of the potential may require analytical or numerical methods. An example of a numerical calculation is given below. Analytical methods for describing a wire constrained by lateral diffusions were presented by Shik,[12] a detailed analysis of wires near threshold has been given by Davies,[13] and an approximate treatment above threshold has been given by Shikin.[14]

The Schrödinger equation for carriers in a wire must be solved in two space dimensions, with free-carrier motion assumed in the third direction. Exact solutions can be obtained for only a few cases, some of which have already been mentioned. To make further progress, it is often useful to formulate the problem as a set of coupled one-dimensional equations, as done for example by Brum and Bastard[15,16] in treating a rectangular GaAs wire buried in $Al_xGa_{1-x}As$. If the normalized z-dependent solutions $\chi_n(z)$ for an infinitely wide strip and the corresponding eigenvalues $E_n$ are known, then the wave functions in the wire can be expanded in terms of a complete set of solutions:

$$\zeta^{(p)} = L^{-1/2}\, e^{ik_x x} \sum_n \alpha_n^{(p)}(y)\, \chi_n(z). \tag{8}$$

After the free-electron behavior in the $x$ direction is separated out, the Schrödinger equation now becomes a set of coupled equations over index $m$ (which could, in principle, include a continuum)

$$-\frac{\hbar^2}{2m}\,\frac{\partial^2 \alpha_m^{(p)}(y)}{\partial y^2} + \sum_n \Delta V_{nm}(y)\, \alpha_m^{(p)}(y) = [E^{(p)} - E_m]\, \alpha_m^{(p)}(y), \tag{9}$$

where the spatial variation of the effective mass has been dropped for simplicity. The matrix element which enters in Eq. (9) is

$$\Delta V_{nm}(y) = \int \chi_m(z) \Delta V(y,z) \chi_n(z) dz, \tag{10}$$

where $\Delta V(y,z)$ is the perturbing potential. If the lateral dimension of the wire is larger than the thickness of the layer whose eigenstates are used for the expansion above, only a few coupled one-dimensional equations are needed to get accurate solutions. The case considered by Brum and Bastard,[15] in which carriers were supplied from a plane of donors outside the wire, also required a self-consistent treatment to determine the charge in the wire and the potential.

Numerical self-consistent calculations for electronic states in wires have been carried out by Laux and Stern[17] for electrons in silicon and by Laux et al.[18] and Kojima et al.[19] for electrons in GaAs. Numerical aspects of such calculations are discussed, for example, by Laux[20] and by Kerkhoven et al.[21] Numerical simulations for wires in GaAs show structure in good qualitative agreement with capacitance studies.[22]

Energy levels in a wire formed in a double-gate metal-oxide-silicon structure, taken form the work of Laux and Stern,[17] are shown in Fig. 2. The potential giving rise to these levels, shown in Fig. 3, is approximately parabolic for gate voltages below threshold, and then attains a flat bottom as electrons fill the well. As more and more electrons are added, the flat region widens. The flattening of the potential as carriers are added also occurs in parabolic quantum wells, a subject that has seen a spurt of activity.[23 – 25] In that case the underlying parabolic potential is usually realized by suitably grading the composition in the well.

Most calculations for electron states in wires have used the effective mass approximation and the Hartree approximation, in which each electron moves in the average potential of all electrons. That approximation ignores exchange and correlation effects, which lead to substantial changes in the energy levels in two-dimensional electron systems.[1] Corresponding, and perhaps even larger, changes are expected in wires and dots, although Laux et al.[15] found that a local density treatment of exchange and correlation effects led to relatively small changes in the energy spacings connected with lateral quantization. The effects of electron-electron interaction in a dot with two electrons have been investigated by Bryant,[26] and can be substantial.

There have been a number of calculations for electronic states in quantum dots, generally using model potentials, some of which are cited elsewhere in this chapter. Here we want to mention the self-consistent Hartree calculations by Kumar et al.,[27] which were motivated by the capacitance experiments of Smith et al.[28] and Hansen et al.[29] They used GaAs-(Al,Ga)As heterostructure samples with a GaAs cap layer patterned with electron beam lithography to form an array of mesas on a square lattice, with a metallic gate deposited over the entire array. A negative voltage on the gate depletes the charges from donors in the $Al_xGa_{1-x}As$ except under the middle of the mesas, where there are isolated groups of electrons which constitute the quantum dots. The capacitance and more particularly its derivative with respect to gate voltage show oscillatory structure associated with quantization in the dots. The period of the structure increases as the dot size decreases. Kumar et al.[27] have carried out numerical self-consistent solutions of the Poisson and Schrödinger equations (in the Hartree approximation) for this structure both with and without a magnetic field. They find that the effective potential acting on electrons under a 300 nm square mesa has a nearly circular shape and a diameter of about 100 nm. The lateral potential variation is approximately parabolic when there are no electrons in the dot and flattens out as electrons are introduced, but not as much as for the wires mentioned above.

Most of the discussion in the present chapter deals with conduction band states. For states in the valence band a more elaborate treatment is usually required, because of the degeneracy at the top of the valence band in most of the semiconductors of interest, with its accompanying nonparabolicity and anisotropy. See, for example, the papers by Citrin and Chang[30] and by Bastard et al.[16]

## 4. PHONONS

Where the confinement of electrons to a wire or dot is of electrostatic origin, there is no significant change in the lattice forces at the boundaries and therefore no significant change in the phonon spectrum. If, however, the confinement arises through a heterostructure, then one may need to consider the differences in lattice forces on crossing the boundaries of the wire or dot. That subject has been studied extensively in the case of quantum wells. One can use a continuum model, as discussed for example by Babiker,[31] or a microscopic model as discussed, for example, by Akera and Ando.[32] Significant changes in the phonon spectrum can arise when there is a large acoustic mismatch at the interface, and this leads to changes in the detailed description of phenomena such as particle capture into a quantum well via electron-phonon interaction.[33] This subject has not yet been widely pursued for quantum wires or dots.

## 5. CHARGES IN QUANTUM WIRES AND QUANTUM DOTS

Some experiments with quantum wires and quantum dots use samples as pure as possible, but others require the presence of carriers. In addition to doping with donor or acceptor impurities within the region of interest, one can introduce carriers in several other ways. Remote impurities, located for example within the barrier region of a heterostructure, can release carriers which then appear in regions of lower potential energy. Voltages applied at external electrodes (or gates) can modify the charge in the region of interest. Charge can also be induced dynamically, either by thermionic injection over a barrier or by tunneling injection through a sufficiently thin barrier.

Detailed discussion of charging effects depends on the specific geometry, and would require more space than is available here. Several of the calculations noted above include charging as an essential element. We want to mention here only two aspects of this problem. The first concerns the nature of the deep donor in $Al_xGa_{1-x}As$, also called the DX center.[34] This center is a deep donor for $x > 0.2$ and gives a resonant state in the conduction band for smaller AlAs fractions, in addition to the usual shallow hydrogenic state. When deep it has the unusual property that electrons excited out of the deep level, optically or otherwise, are prevented from returning to the lowest electronic state by a barrier that is attributed to lattice relaxation. Temperatures of order 150°K or higher, or electrons which attain sufficient energy to surmount the barrier, can restore the system to equilibrium, but the metastable state can be maintained indefinitely at low temperatures even after the exciting light is turned off. This effect generally leads to an increase in the carrier density and conductivity of the channel--be it quantum well, heterojunction, or wire--and is called persistent photoconductivity. Many experiments, especially on samples with very high mobility, are carried out with some infrared illumination to induce this effect, and it is not always clear whether reported results are obtained in the dark or involve such illumination. In modeling heterostructures with deep donors, I have found it useful to treat them as conventional deep donors in dealing with experiments carried out in the dark. For a system that has been saturated with light to ionize all the donors, the persistent photoconducting state is most easily simulated by replacing the donor degeneracy factor, usually 2, by 0. If not covered in earlier lectures, simple concepts of charge transfer in heterojunctions in the dark[35] and in the persistent photoconducting state[36] will be reviewed.

One other aspect of charging in small systems should be mentioned, namely the consequences of the electrostatic energy of small structures for the charging process in such devices. The energy of a capacitor is $Q^2/2C$, where $C$ is the capacitance and $Q$ is the charge, and the energy change on adding one electron can be significant. The capacitance of an isolated conducting sphere of radius R in a medium of permittivity $\varepsilon$ is $C = 4\pi\varepsilon R$ or about $1.4 \times 10^{-16}$ F for a sphere of radius 0.1$\mu$m in GaAs. The electrostatic energy to place the first electron charge on such a sphere is about 0.6 meV, and increases as the charge on the sphere increases. Similar considerations apply to small planar capacitor structures. These energies lead to deviations from simple ohmic behavior: the charging takes place in discrete steps, with no current flow until the voltage is large enough to overcome the charging energy. This phenomenon, known as the Coulomb blockade, has been seen both in conventional junctions and in Josephson junctions. It has been known for many years and has a growing literature, of which only one very recent example is cited.[37]

For quantum dots in semiconductors, the charging energy and the internal electronic energy presumably both enter in the experiments of Ilansen et al.,[29] who found structure in the capacitance when an array of dots is charged from a nearby electrode, as illustrated in Fig. 4. The relation of their results to the Coulomb blockade was noted by Silsbee and Ashoori.[38]

## 6. DIELECTRIC RESPONSE, SCREENING, AND PLASMONS

The linear response of a system to external electric fields of wave vector $q$ and radian frequency $\omega$ is given by the dielectric constant $\kappa(q, \omega) = \varepsilon(q, \omega)/\varepsilon_0$, where $\varepsilon_0$ is the permittivity of free space. The dielectric constant is complex in general, and its imaginary part corresponds to energy loss processes. When written in terms of a single wave vector, the dielectric response applies to a homogeneous medium, and it must therefore be used cautiously in lower-dimensional systems. A more generally applicable response function is the polarizability, which gives the polarization $P$, generally a function of position, induced in the system of interest by an electric field. There have been many studies of the dielectric response of quasi-one-dimensional systems, for example the work of Friesen and Bergersen.[39]

The real part of the polarizability of a wire with only the lowest subband occupied is[40]

$$\chi(q, \omega) = \frac{m}{\pi\hbar^2 q} \ln \left| \frac{(k_F - \frac{q}{2})^2 - (\frac{m\omega}{\hbar q})^2}{(k_F + \frac{q}{2})^2 - (\frac{m\omega}{\hbar q})^2} \right| \tag{11}$$

where $q$ is the wave vector along the wire and $k_F$ is the Fermi wave vector. This leads to an effective permittivity in the wire of

$$\varepsilon(q, \omega) = \varepsilon_L[1 + V_{\text{eff}}(q)\chi(q, \omega)], \tag{12}$$

where $\varepsilon_L$ is the permittivity of the medium in which the electrons move and $V_{\text{eff}}$ is an effective Coulomb interaction, which depends on the shape of the charge density in the wire. For a wire of radius $R_0$ with only one subband occupied, Gold and Ghazali[41] give at long wavelengths

$$V^{\text{eff}}(q) \sim -(e^2/2\pi\varepsilon_L)\ln(qr_0/2). \tag{13}$$

The dispersion relation for plasmons, the collective longitudinal excitations that correspond to vanishing of the dielectric constant, is approximately given, for a single wire with only the lowest subband occupied, by[42,42]

$$\omega_p(q) \sim (ne^2/2\pi m\varepsilon_L)^{1/2} q [\ln(qR_0/2)]^{1/2}. \tag{14}$$

For arrays of wires it is necessary to consider the mutual interaction of the wires with each other. The resonant frequency has been shown to be equal to the harmonic oscillator frequency if the bare potential in which the electrons move is parabolic.[24,43] A similar result obtains for dots[44] if the bare potential is parabolic, as is approximately the case for electrostatic confinement for both wire and dots.



The boundaries of a wire or disk lead to a new set of modes which have been studied in the presence of a magnetic field and are called called edge magnetoplasmons. They have been observed in GaAs[45,46] and for electrons on liquid helium.[47,48] Some of the related theoretical work is cited in Ref. 46.

Many bulk materials have highly anisotropic conductivity and can be treated for some purposes as three-dimensional arrays of wires. Such systems, often called one-dimensional systems, have some overlap in underlying physics with the systems being considered here, but have historically been considered by a different group of workers. There is remarkably little overlap in the literature of these two fields. I will not try to bridge that gap here.

## 7. TRANSPORT PROPERTIES

Since many of the aspects of transport involving low-dimensional structures are covered in other lectures at the Spring College, this section is limited to a few specific aspects.

Quantum corrections to transport, such as those that lead to logarithmic corrections in the weak localization regime, become increasingly important for one-dimensional transport. It is now well known that at absolute zero all states in a twodimensional system are (weakly) localized even for arbitrarily weak disorder, and that in one dimension all states are strongly localized, i.e. the states decay exponentially. Thus the conduction can have a weak-localization regime, an activated or variable-range hopping regime, and in some cases a direct tunneling regime. There are many interesting experimental and theoretical results related to localization in low-dimensional systems, some of which may be discussed in other lectures. A particularly interesting question is the transition from weak to strong localization in two-dimensional electron systems. That question has been extensively studied in silicon inversion layers,[1] and there have been some recent experimental[49] and theoretical[50,51] papers dealing with the onset of localization in GaAs heterojunctions.

At sufficiently high temperatures the quantum corrections to transport become smaller, and a Boltzmann-like treatment may be applied. In this regime the main scattering mechanisms that limit transport in wires are basically the same as for two-dimensional systems, and include Coulomb scattering, interface roughness scattering, and phonon scattering. Some simple concepts bearing on low-temperature transport in two-dimensional electron systems will be reviewed in the present lectures, including Coulomb scattering[1,52] and the temperature dependence of mobility induced by the temperature dependence of screening[53,54] and by thermal averaging over the Fermi distribution.[55]

One kinematical difference between one-dimensional transport and transport in higher dimensions is that the scattering within a single one-dimensional subband can be only forward or backward. For elastic scattering, only the backward mode changes the current. For a given value of the wave vector $k_F$ at the Fermi surface, this can mean a significant reduction in the cross section, because the only relevant wave vector transfer is $2k_F$, rather than a continuous range of possible wave vector transfers in two or three dimensions. Since the scattering cross section for Coulomb scattering decreases rapidly with increasing momentum transfer, this might be expected to lead to a significant reduction in scattering rate--and a concomitant increase in mobility--in one-dimensional systems, as pointed out by Sakaki.[2] Experimental support for this conclusion is not at hand, but quantitative tests can be expected as better structures are made and studied.

In quantum wires whose sides are determined by electrostatic confinement, there is reason to believe that edge roughness will not be as severe as might be expected from the somewhat ragged edges often found in the gates that provide the electrostatic confinement. Kumar et al.[56] showed that in typical heterostructures the higher Fourier components of the roughness of a gate edge are significantly attenuated because of the spatial separation between the surface where the gate is located and the buried interface where the electrons of interest are found. Evidence that boundary scattering in narrow channels is largely specular had already been presented by van Houten et al.[57]

Many authors have given Boltzmann-like treatments of transport in semiconductor wires, including for example Lee and Spector,[58] Riddoch and Ridley,[59] Fishman,[60,61] Weng and Leburton,[62] Das Sarma and Xie,[63] and Gold and Ghazali.[41] Detailed comparisons of theory and experiment are not yet at hand.

In most realistic quasi-one-dimensional situations, more than one subband will have carriers in it and scattering between subbands must be included. Intersubband scattering processes must be treated in a consistent way, which leads to some nontrivial couplings that are sometimes ignored. The formal treatment for such processes has been given in a two-dimensional context by Siggia and Kwok.[64]

An isolated quantum dot cannot show conventional transport, because it has no contacts. There are nevertheless very interesting effects involving dot-like structures coupled to each other or to quantum wires. Some of these effects may be covered in other lectures at the Spring College. Reed et al.[65] have reported structure in the current through semiconductor pillars with tunneling through a quantum dot, and have attributed the structure to resonances with energy levels in the dot. The interaction in arrays of dots weakly coupled to each other has also been studied.[66,67]



## 8. BOUND STATES

As the dimensionality of a system decreases, the binding energy of electrons to impurities and the analogous exciton binding energy increase. For short-range potentials, it is known that in three dimensions there is a minimum strength required before an attractive potential has a bound state. In two dimensions an arbitrarily weak potential has a bound state, although the binding energy is extremely small for a weak potential. Bound states in wires and dots have been examined by a number of authors,[41,68,69] who show that increasing confinement increases the binding.

One of the best-known results is that the binding energy of an electron of effective mass $m$ to a Coulomb center of charge $e$ in two dimensions is four times as large as the corresponding three-dimensional result, given by the effective Rydberg, $me^4/32\pi^2\epsilon^2\hbar^2$. Here $m$ is the effective mass, $\epsilon$ is the permittivity of the medium in which the electron plane is embedded, and we assume that the attractive center is also located in the plane. For electrons confined to a wire or dot of small radius, the binding energy diverges logarithmically or as the inverse of the radius, respectively.

The same considerations that enhance binding to a fixed attractive center also enhance the attraction between an electron and a hole, as studied for example by Keldysh[70] in a quasi-two-dimensional context. Excitonic effects are central to the understanding of optical properties of wires and dots.

## 9. OPTICAL PROPERTIES

We have already seen that localization effects and energies of binding to attractive potentials become stronger as the dimensionality decreases. Similarly, the excitonic effects lead to a strong enhancement of the oscillator strength near the absorption edge as the dimensionality decreases. This enhancement of optical effects has a number of important consequences, some of which will be briefly noted here.

The properties of semiconductor microcrystals in glassy matrices have been actively studied both theoretically[71] and experimentally.[72] Excitonic effects are very important but can be quenched as the dot size becomes comparable to the exciton radius, when confinement effects inhibit the ability of the electrons and hole to take full advantage of the Coulomb attraction. An interesting regime is one in which confinement effects are strong for the light-mass electron but not yet substantial for the hole,[73] leading to a situation reminiscent of the Born-Oppenheimer regime for molecules. Excitons in disk-shaped zero-dimensional structures have been studied theoretically by Bryant.[74]

There has been recent interest in biexcitons, a stable configuration of two electrons and two holes, in low-dimensional systems. We cite only one recent paper reporting an experimental observation and its relation to theory.[75]

The enhancement of oscillator strength and of excitonic effects in low-dimensional systems makes excitonic effects easier to observe at room temperature even in materials like GaAs, where they can only be seen well at low temperatures in the bulk. This enhancement also strengthens lasing action, and has led to a significant effort to make and to study quasi-one-dimensional lasers, as noted briefly below.

The changes in the energy spectrum as carriers in a quantum well are further confined to quantum wires and quantum dots lead to observable changes in optical spectra, and these have been studied for some time. For recent accounts, see for example Refs. 3 and 76-78.

## 10. MAGNETIC FIELD EFFECTS

A magnetic field in a bulk semiconductor leads to quantization of the motion perpendicular to the field into Landau levels, whose energy spacing is the cyclotron energy

$$E_c = \hbar\omega_c = e\hbar B/m, \tag{15}$$

where $B$ is the magnetic induction (loosely called the magnetic field hereafter) and $m$ is the effective mass. There is a free-electron continuum for motion along the field, so each Landau level is the bottom of a one-dimensional band with the inverse-square-root singularity in density of states noted earlier. In two-dimensional systems this degree of freedom is also quantized, and the density of states becomes a series of peaks. Some of the very rich physics of such systems has been reviewed,[1] and other aspects will be discussed in lectures at this Spring College. If the dimensionality is reduced further, the situation becomes more complicated, because the cyclotron motion is basically two-dimensional. Nevertheless there are substantial changes in the electronic structure when a magnetic field is applied, depending on the particular structure.

There is one case for which explicit solutions are available. It is the case of a circular disk with a parabolic potential, for which the potential energy in Eq. (4) is $V(R) = (k/2)R^2$. The solutions were obtained in 1930 by Darwin.[79] We quote only the eigenvalues,

$$E_{l,n} = (2n + |l| + 1)\hbar(\omega_0^2 + \omega_L^2)^{1/2} + l\,\hbar\omega_L, \tag{16}$$

where $\omega_0 = (k/m)^{1/2}$ is the harmonic oscillator radian frequency and $\omega_L = \omega_c/2 = eB/2m$ is the Larmor radian frequency.

Optical properties of quantum dots in a magnetic field have been studied by several groups.[76 - 78] The results are qualitatively consistent with the behavior to be expected from the energy level scheme found by Darwin,[79] as might be expected since the samples involved in those experiments used electrostatic confinement, which gives rise to a nearly parabolic lateral potential. Recent theories show that the optical response in such cases is at the oscillator frequency of the bare potential, and is unaffected by electron-electron interaction.[24,43,44]

If the carrier density in a wire or dot is fixed, then the change in energy level structure in a magnetic field leads to magnetic depopulation of levels, as studied extensively by Berggren and coworkers,[80] and by others.[81]

Magnetotransport effects in two-dimensional structures include weak localization effects, with their rich literature. For wires, there is now a remarkable set of experimental results that show that the actual contact configuration is important, rather than simply the dimensionality. These aspects are discussed in other lectures of this Spring College.

Omitted from this discussion is the subject of magnetization, which was the focus of the original Landau paper[82] and which has been treated by many authors, including Darwin[79] and Sivan and Imry.[83] We shall not pursue that topic here.

## 11. PROSPECTS FOR DEVICE APPLICATIONS

The trend of semiconductor device technology has been to work for ever smaller devices, in the interest of increased speed and lower overall cost. Commercial devices are still large on the distance scale envisioned here, and are not likely to reach the scale for which lateral quantum confinement effects become important for some time. There are several physical limitations that may slow the trend toward miniaturization before the quantum range is reached. One is the presence of the potential fluctuations[84] and the statistical variations[85] in characteristics from device to device that are expected to be increasingly significant as devices get smaller. Another is the reduced current carrying capacity of small devices. On the other hand, certain novel properties, such as the ability to influence the properties of a pair of contacts from a remote location, as has been demonstrated in the ballistic range of dimensions, may lead to new classes of devices and applications.

Perhaps the applications most likely to be realized quickly are those involving optical properties, and this direction has been actively pursued. Optical nonlinearities, important for many applications, have been analyzed for example by Hanamura,[86] and some of the experimental and theoretical work has been summarized in the book edited by Haug and Bányai.[87] Semiconductor lasers lend themselves well to miniaturization. Quantum wire lasers have been reported, for example, by Kapon et al.[88] The integration of such devices with electrical devices is another area being actively pursued.

## ACKNOWLEDGMENTS

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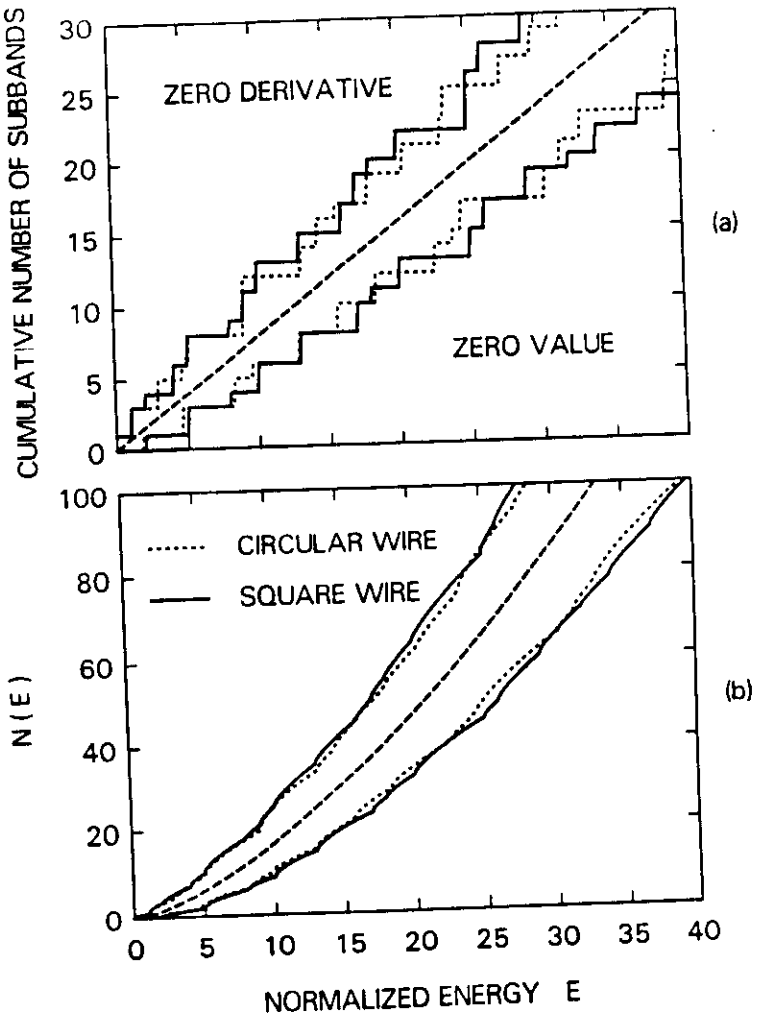
## FIGURE CAPTIONS

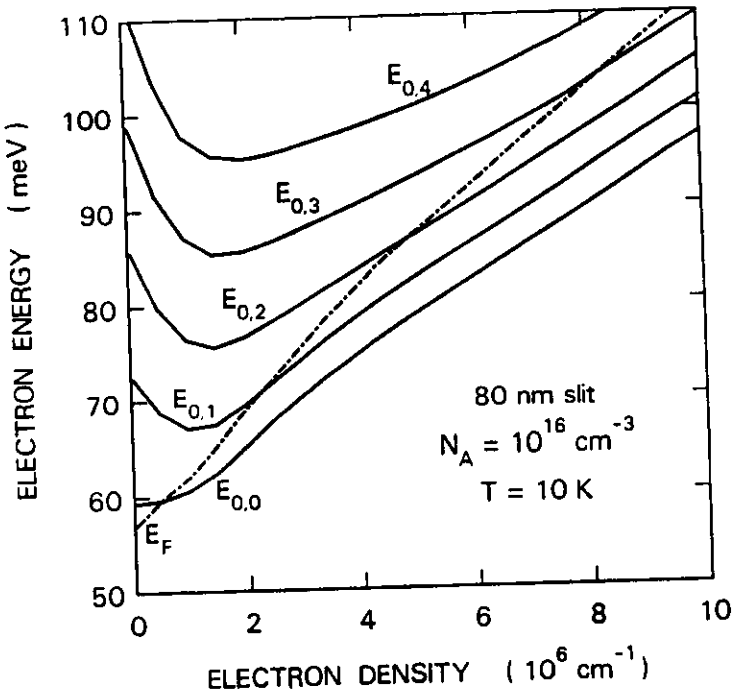
Fig. 1. (a) Cumulative number of subbands with normalized energy less than $E$ in a square wire of area $a^2$ (full curve) and in a circular wire with the same area (dotted curve). The energy unit is $\pi^2\hbar^2/2ma^2$, which equals 0.54 meV when $a = 100$ nm and $m = 0.07\, m_0$. The lower pair of curves corresponds to vanishing of the envelope wave function at the boundary, while the upper pair corresponds to vanishing of its derivative. The asymptotic result for a very large system, equal to $\pi E/4$ in these units, is given by the dashed line. (b) Integrated number of states per unit length (arbitrary units) with normalized energy less than E for the square and circular wires. The curves have the same significance as in (a), and the asymptotic value, shown dashed, is $\pi E^{3/2}/6$. Spin degeneracy is not included in this figure.

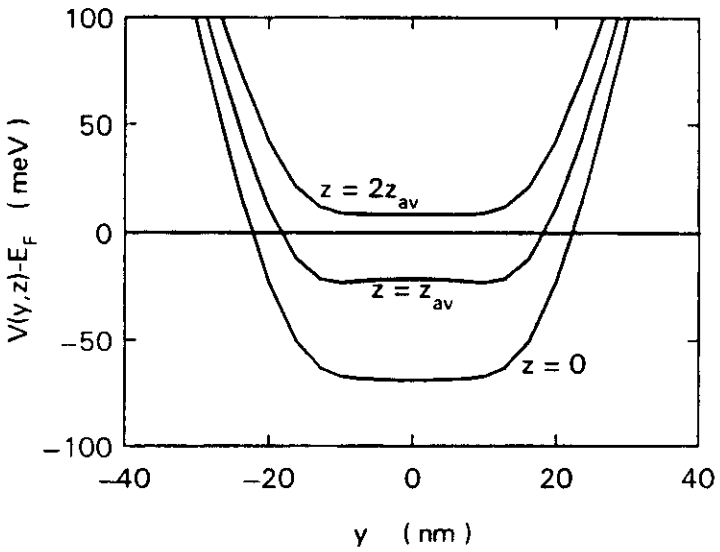
Fig. 2. Energy levels for the first five lateral modes versus density of channel electrons for electrons in a two-gate metal-oxide-silicon structure with an 80 nm slit in the gate closest to the silicon. The states are labeled by integers giving the number of nodes in the directions perpendicular and parallel to the interface, respectively. The dot-dash curve gives the Fermi level position. All energies are measured from the conduction band edge at the Si-$SiO_2$ interface under the center of the slit. (After Laux and Stern, Ref. 17)

Fig. 3. Lateral potential profiles at three distances from the Si-$SiO_2$ interface (at $z = 0$) for the structure whose energy levels are shown in Fig. 2. The curves correspond to a channel density of $2 \times 10^6$ $cm^{-1}$, close to the point where the second subband is beginning to be occupied. $z_{av}$ is the average distance of the channel electrons from the Si-$SiO_2$ interface. Note the flattening of the bottom of the potential, which is approximately parabolic in the absence of charge in the channel. (After Laux and Stern, Ref. 17)

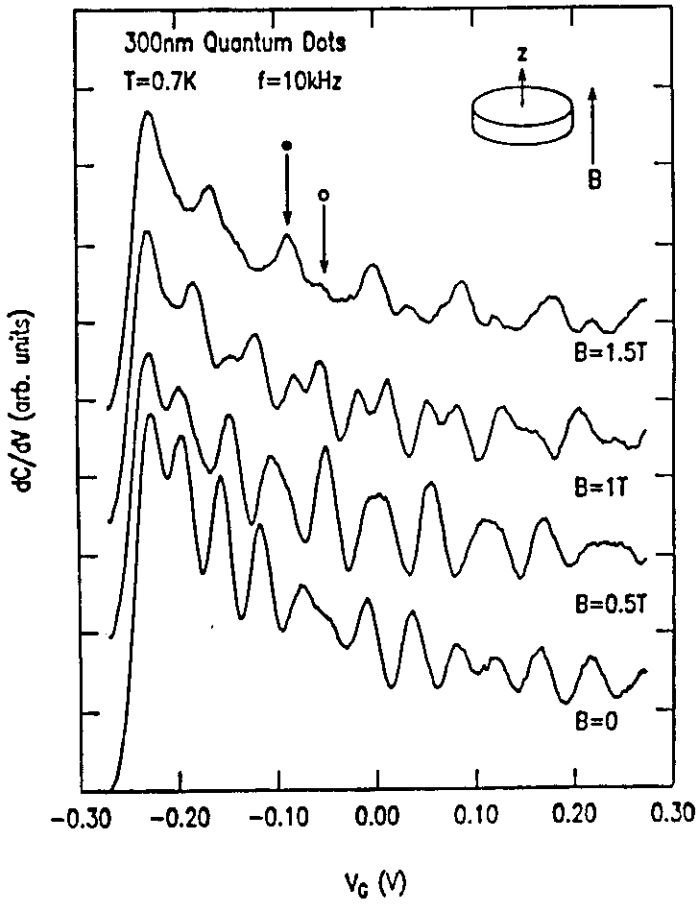
Fig. 4. Derivative of capacitance with respect to gate voltage for an array of quantum dots in GaAs. The dots are defined by 300 nm square mesas in the GaAs cap of a GaAs heterostructure. (After Hansen et al., Ref. 29)


CUMULATIVE NUMBER OF SUBBANDS
ZERO DERIVATIVE
ZERO VALUE
(a)
CIRCULAR WIRE
SQUARE WIRE
N(E)
(b)
NORMALIZED ENERGY E


ELECTRON ENERGY (meV)
$E_{0,4}$
$E_{0,3}$
$E_{0,2}$
$E_{0,1}$
$E_{0,0}$
$E_F$
80 nm slit
$N_A = 10^{16}$ cm$^{-3}$
T = 10 K
ELECTRON DENSITY ($10^6$ cm$^{-1}$)


100
50
0
−50
−100
−40
−20
0
20
40
V(y,z)-E_F ( meV )
y ( nm )
z = 2z_av
z = z_av
z = 0




300nm Quantum Dots
T=0.7K
f=10kHz
z
B
B=1.5T
B=1T
B=0.5T
B=0
dC/dV (arb. units)
−0.30
−0.20
−0.10
0.00
0.10
0.20
0.30
V_G (V)