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SPRING COLLEGE IN CONDENSED MATTER
ON
'PHYSICS OF LOW-DIMENSIONAL SEMICONDUCTOR STRUCTURES'
(23 April - 15 June 1990)

TRANSMISSION PROBABILITIES AND TRANSPORT

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Transmisson Probabilities and Transport

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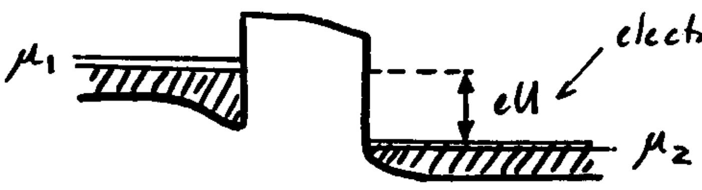
Introductory Remarks

Early connections between transmission and electric conduction; isolated scatterer:

Fowler and Nordheim (1928); Field-Emission
Frenkel (1930); Metal-Metal-Contacts / Grain Boundaries
Landauer (1957); Localized Defects / Grain Boundaries

Transmission through an entire sample
Landauer (1970); Disordered single channel conductor
Tsu and Esaki (1973); Supperlattice

Key issue; What counts? Electrostatic Potential Drop or Chemical Potential Drop?
Charge Neutrality versus zero Current Condition



μ_1, μ_2 chemical potential of current source and current drain

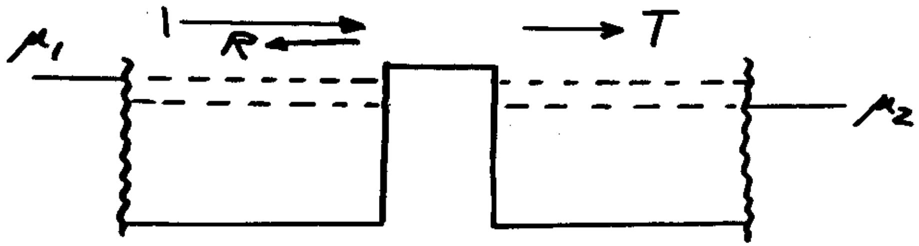
Below we emphasize chemical potentials measured at electron reservoirs

Voltage Drop Across Resistor?

Electrostatic Potential

Charge Neutrality Condition

The Landauer Formula

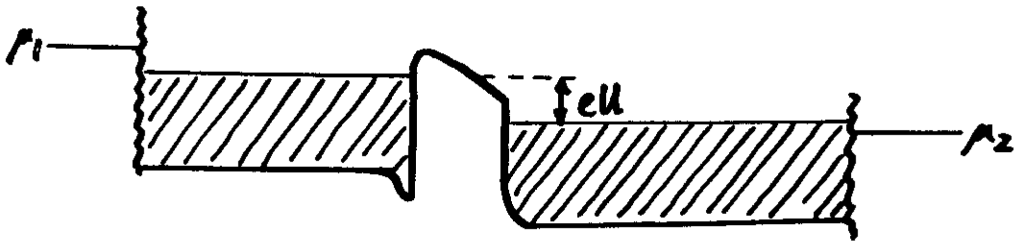


$$I_{injected} = e v_F \Delta n = e v_F (dn/dE)(\mu_1-\mu_2) = (e/h)(\mu_1-\mu_2)$$

$$dn/dE = (dn/dk)(dk/dE) = 1/hv_F \rightarrow$$

$$I = (e/h)\,T\,(\mu_1-\mu_2)$$

$$\Delta n = \frac{1}{hv_F}(1+R)\Delta\mu \qquad \Delta n = \frac{1}{hv_F}(1-R)\Delta\mu$$

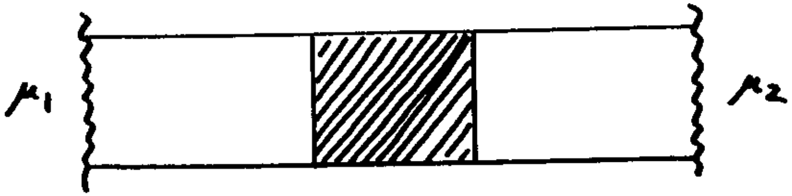


$$\Delta n_{tot} = \frac{1}{hv_F}[(1+R)-(1-R)]\Delta\mu = \frac{2R}{hv_F}\Delta\mu = \frac{2}{hv_F}eU \rightarrow$$

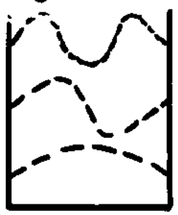
$$eU = R(\mu_1-\mu_2)$$

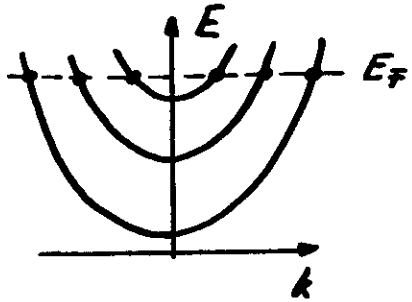
$$\boxed{\mathcal{R} = \frac{U}{I} = \frac{h}{e^2}\frac{R}{T}}$$

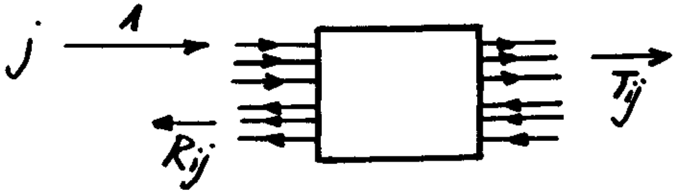
Many Channel Conductor



confining potential







$$I_{incident,j} = e v_j \Delta n_j = e v_j (dn_j/dE)(\mu_1-\mu_2) = (e/h)(\mu_1-\mu_2)$$

$$dn_j/dE = 1/h v_j$$

$$I_{transmitted,ij} = \frac{e}{h} T_{ij} (\mu_1-\mu_2)$$

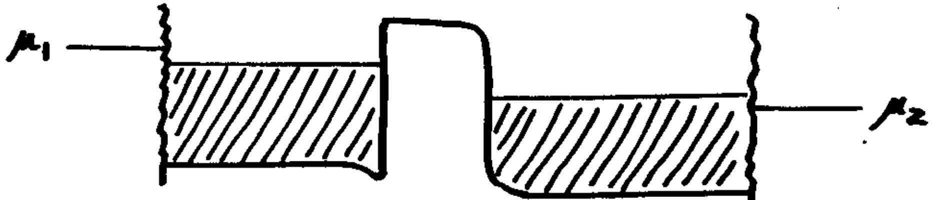
$$I_{tot} = \frac{e}{h} \sum_{ij} T_{ij} (\mu_1-\mu_2) = \frac{e}{h} T (\mu_1-\mu_2)$$

$$T \equiv \sum_{ij} T_{ij}$$

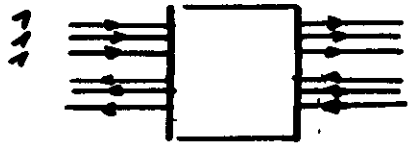
$$G = \frac{I}{(\mu_1-\mu_2)/e} = \frac{e^2}{h} T$$

Many-Channel Landauer Formula

Azbel (81); Büttiker, Imry, Landauer, Pinhas (85)



$$\Delta n = \frac{1}{h} \sum_i \frac{1}{v_i} (1+R_i) \Delta\mu \qquad \Delta n = \frac{1}{h} \sum_i \frac{1}{v_i} T_i \Delta\mu$$



$$T_i = \sum_j T_{ij}$$

$$R_i = \sum_j R_{ij}$$

$$\Delta n_{tot} = \frac{1}{h} \sum_i \frac{1}{v_i} (1+R_i-T_i) \Delta\mu = \frac{2}{h} \sum_i \frac{1}{v_i} eU$$

$$I = \frac{e}{h} T \Delta\mu$$

$$\boxed{\mathcal{R} = \frac{U}{I} = \frac{h}{e^2} \frac{1}{T} \frac{\sum_i \frac{1}{v_i}(1+R_i-T_i)}{2 \sum_i \frac{1}{v_i}}}$$

$$T_{ij}(B) = T_{ji}(-B) \quad \rightarrow \quad T_i(B) \neq T_i(-B)$$

$$R_{ij}(B) = R_{ji}(-B) \qquad R_i(B) \neq R_i(-B) \quad \rightarrow \mathcal{R}(B) \neq \mathcal{R}(-B)$$

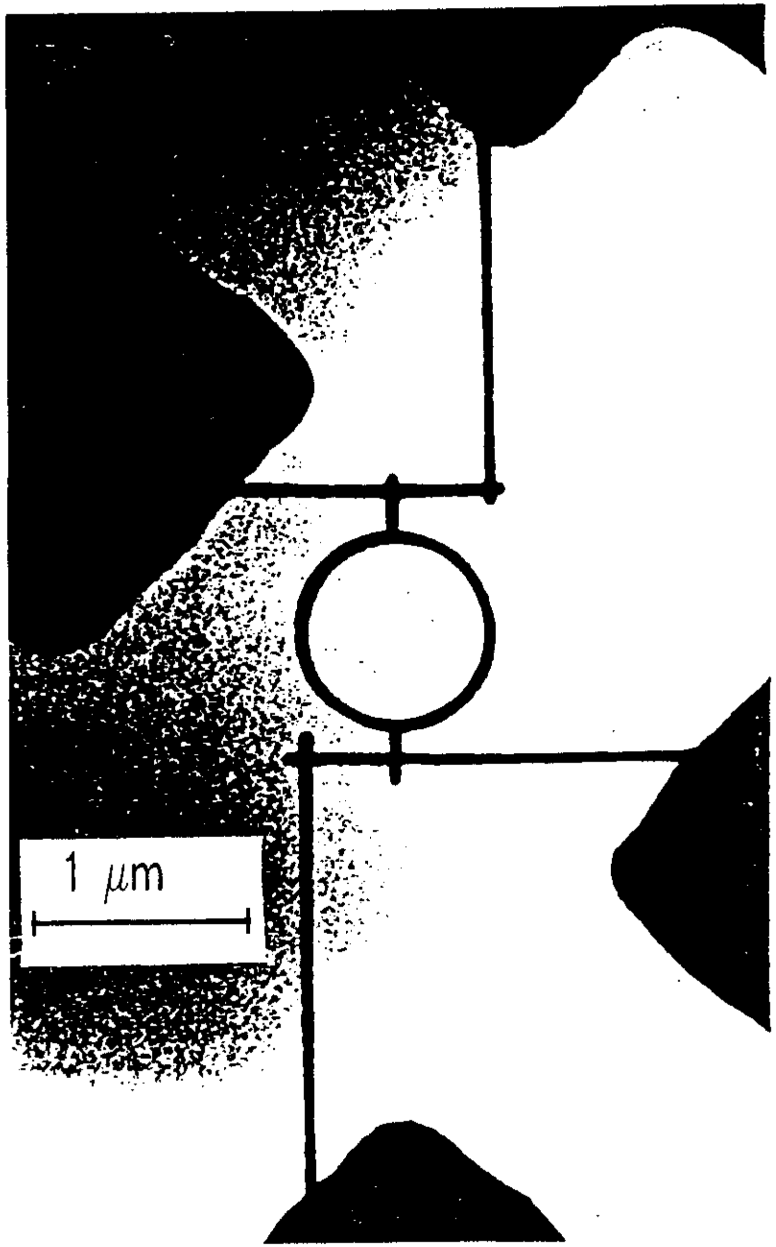
metallic conductor:

$$T_i \cong \ell/L \qquad R_i \cong 1 - \ell/L \quad \Rightarrow \quad \mathcal{R} \cong \frac{h}{e^2} \frac{1}{T}$$

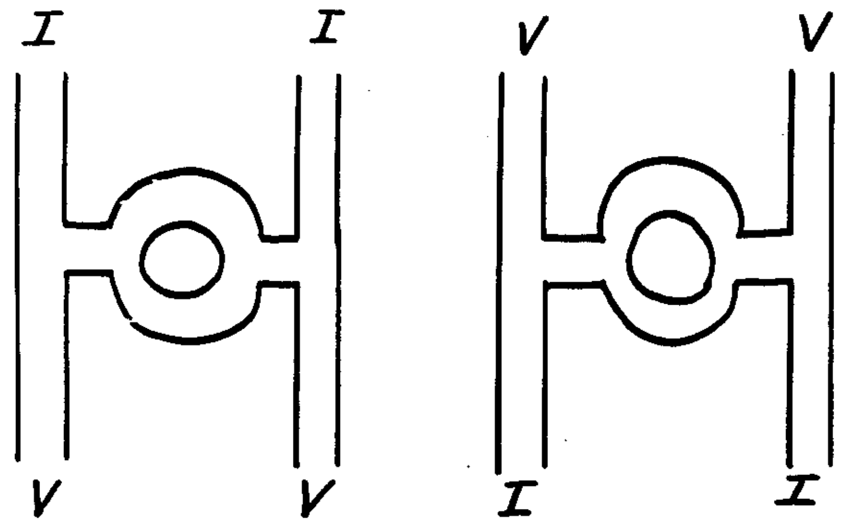
Voltage Drop Across Resistor ?

Difference of Electro-Chemical Potential

Zero Current Condition

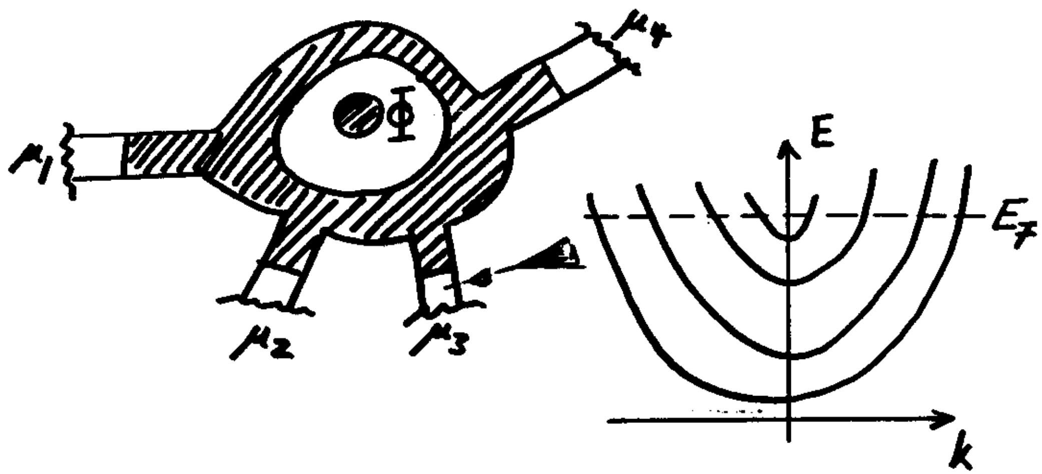


Four Probe Measurements
Benoit et al. (1986)



→ Must treat all probes (whether current or voltage probes) in an equivalent manner.

Four Probe Conductor
Büttiker (1986)



$T_{ij} = \Sigma_{mn} T_{ij,mn}$ $\quad R_{ii} = \Sigma_{mn} R_{ii,mn}$

$T_{ij,mn}(\Phi) = T_{ji,nm}(-\Phi)$; $\quad R_{ii,mn}(\Phi) = R_{ii,nm}(-\Phi)$

$$\boxed{T_{ij}(\Phi) = T_{ji}(-\Phi); \quad R_{ii}(\Phi) = R_{ii}(-\Phi)}$$

$$\boxed{I_i = \frac{e}{h}\left[(M_i - R_{ii})\mu_i - \Sigma'_j T_{ij}\mu_j\right]}$$

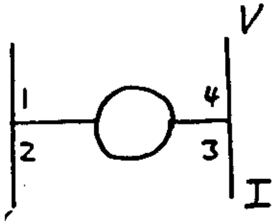
$I = I_m = -I_n, \quad I_k = I_\ell = 0 \rightarrow$

$$\boxed{R_{mn,k\ell} = \frac{\mu_k - \mu_\ell}{eI} = \frac{h}{e^2}\frac{(T_{km}T_{\ell n} - T_{kn}T_{\ell m})}{D}}$$

$$\boxed{R_{mn,k\ell}(\Phi) = R_{k\ell,mn}(-\Phi)}$$

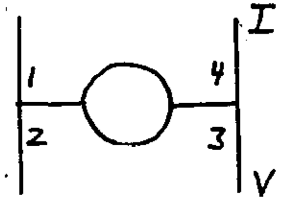
Benoit, Washburn, Umbach, Laibowitz and Webb (1986)

Reciprocity Theorem:
Consequence for Aharonov-Bohm Effect



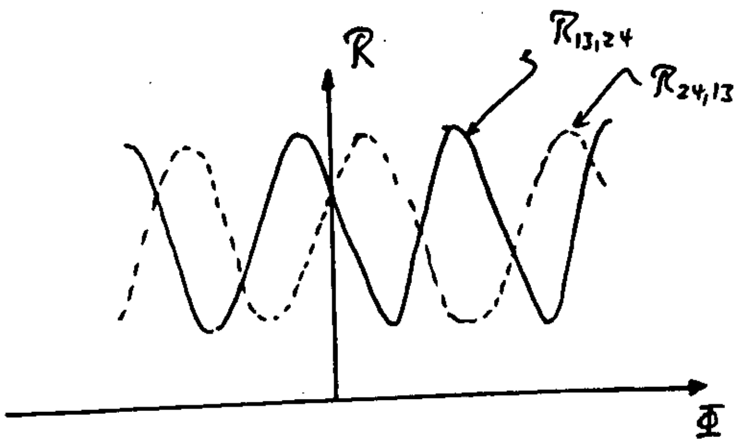
$$R_{13,24} = R_0 + \Delta R \cos\left(2\pi \frac{\Phi}{\Phi_0} - \varphi_\alpha\right) + \cdots$$

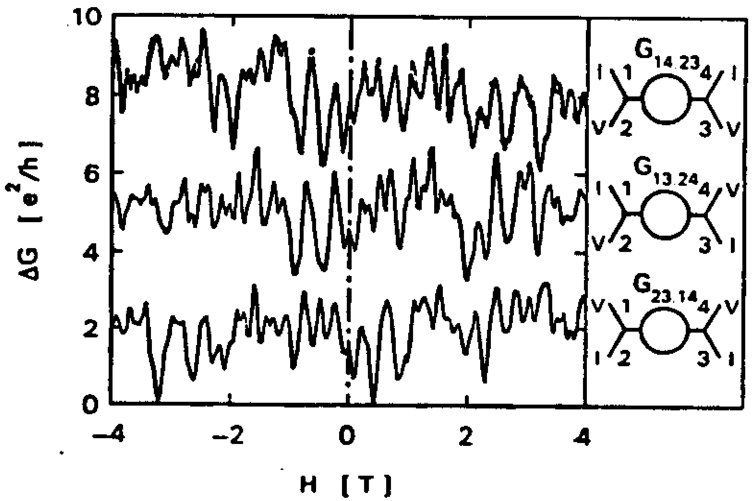
$$\Phi_0 = hc/e$$

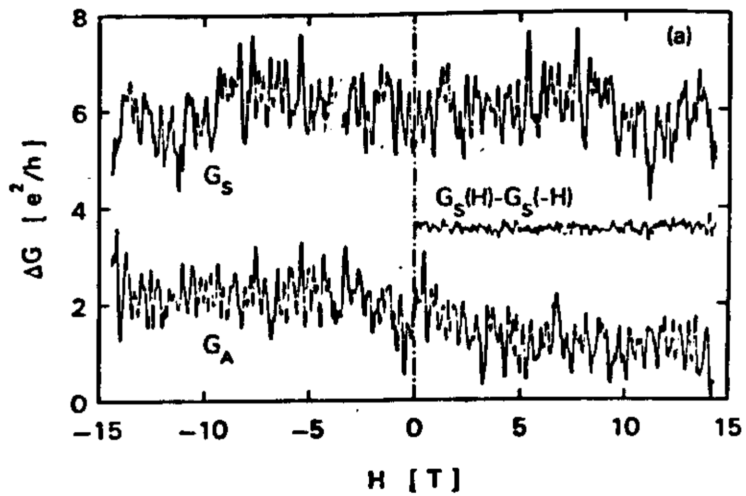


$$R_{24,13} = R_0 + \Delta R \cos\left(2\pi \frac{\Phi}{\Phi_0} + \varphi_\alpha\right)$$

since $R_{13,24} + R_{24,13}$ symmetric

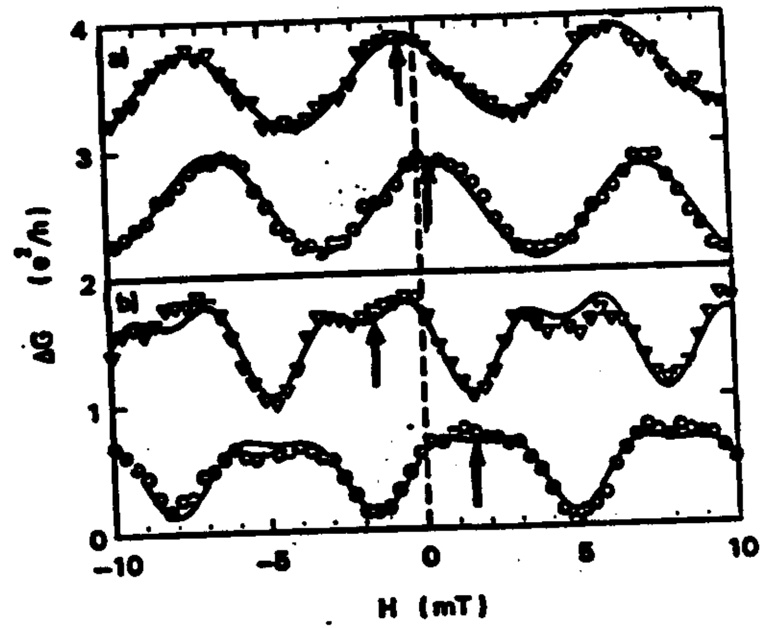






Reciprocity of Four-Terminal Aharonov-Bohm Oscillations

Benoit, Washburn, Umbach, Laibowitz, Webb (86)



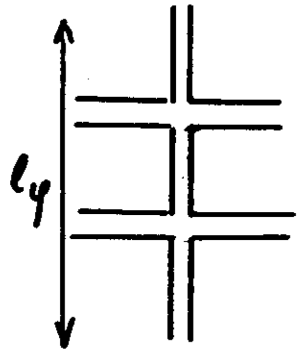
Resistance Fluctuations

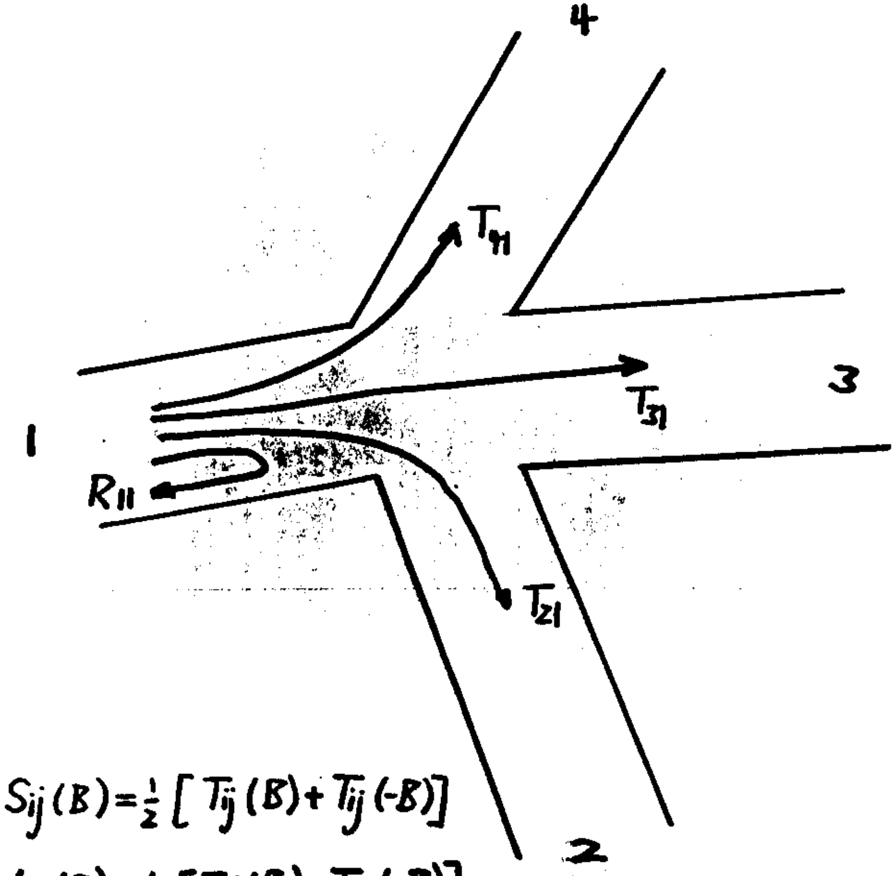
Benoit, Umbach, Laibowitz, Webb (86)

$$S = \frac{1}{2}\left[R_{ik,mn}(H) + R_{mn,ik}(H)\right]$$

$$A = \frac{1}{2}\left[R_{ik,mn}(H) - R_{mn,ik}(H)\right]$$

$$\boxed{\langle(\Delta S)^2\rangle = \langle(\Delta A)^2\rangle}$$





$$S_{ij}(B) = \frac{1}{2}\left[T_{ij}(B) + T_{ij}(-B)\right]$$

$$A_{ij}(B) = \frac{1}{2}\left[T_{ij}(B) - T_{ij}(-B)\right]$$

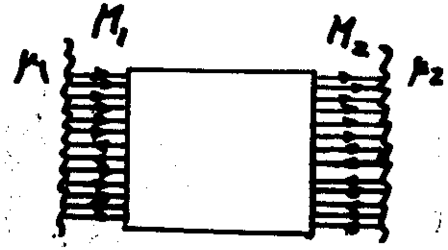
A four probe conductor is characterized by 6 symmetric and 3 antisymmetric transport coefficients

N - Probe Conductors

$$I_i = \frac{e}{h}\left[(M_i - R_{ii})\mu_i - \sum_{j\neq i} T_{ij}\mu_j\right]$$

$$T_{ij}(B) = T_{ji}(-B), \qquad R_{ii}(B) = R_{ii}(-B)$$

A. Two Probe Conductor



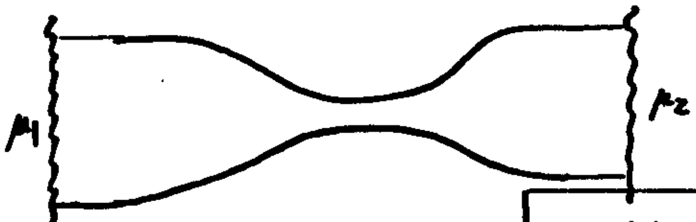
$$I_1 = \frac{e}{h}\left[(M_1 - R_{11})\mu_1 - T_{12}\mu_2\right]$$
$$I_2 = \frac{e}{h}\left[(M_2 - R_{22})\mu_2 - T_{21}\mu_1\right]$$

$$M_1 = R_{11}(B) + T_{12}(B) = R_{11}(-B) + T_{12}(-B) = R_{11}(B) + T_{12}(-B)$$
$\Longrightarrow$

$$\boxed{T \equiv T_{12}(B) = T_{12}(-B)}$$

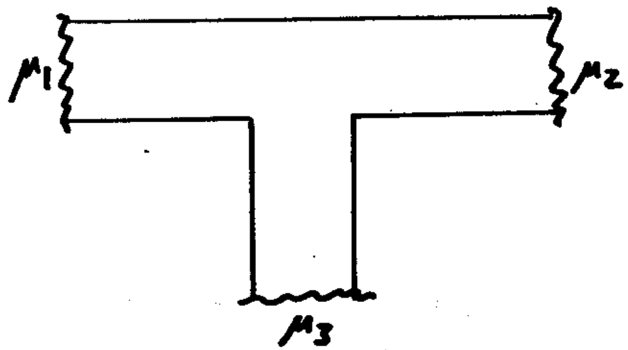
$$I_1 = \frac{e}{h}\,T(\mu_1 - \mu_2)$$
$$I_2 = \frac{e}{h}\,T(\mu_2 - \mu_1) \quad \Longrightarrow \quad \boxed{G = \frac{I}{(\mu_1 - \mu_2)/e} = \frac{e^2}{h}T}$$

Imry (1986)



$$\boxed{\min(M_1, M_2) \gg T}$$

B. Three Terminal Conductor



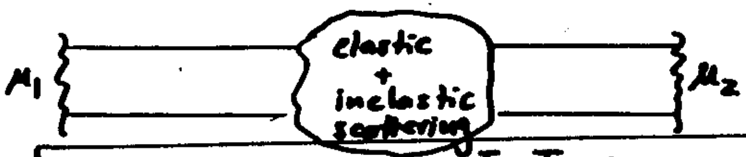
$$I_i = \frac{e}{h}\left[(M_i - R_{ii})\mu_i - \sum_{j(\neq i)} T_{ij}\mu_j\right], \quad i = 1,2,3$$

$$I_3 = 0; \quad \boxed{\mu_3 = \frac{T_{31}\mu_1 + T_{32}\mu_2}{T_{31} + T_{32}}} \qquad \boxed{W = \frac{1}{h}\frac{T_{31}T_{32}}{T_{31} + T_{32}}(\mu_1 - \mu_2)^2}$$

$$I_i = \frac{e}{h}\left[(M_i - \hat{R}_{ii})\mu_i - \sum_{j(\neq i)} \hat{T}_{ij}\mu_j\right], \quad i = 1,2$$

$$\hat{T}_{21} = T_{21} + \frac{T_{23}T_{31}}{T_{31} + T_{32}}, \quad \hat{R}_{11} = R_{11} + \frac{T_{13}T_{31}}{T_{31} + T_{32}} \qquad \hat{T} \equiv \hat{T}_{21} = \hat{T}_{12}$$

$$\hat{T}_{12} = T_{12} + \frac{T_{13}T_{32}}{T_{31} + T_{32}}, \quad \hat{R}_{22} = R_{22} + \frac{T_{23}T_{32}}{T_{31} + T_{32}}$$



$$\boxed{G = \frac{e^2}{h}\hat{T} = \frac{e^2}{h}\left(T_{21} + \frac{T_{23}T_{31}}{T_{31} + T_{32}}\right) = \frac{e^2}{h}(T_{el} + T_{in})}$$

Coherent and Incoherent Transmission

$$G = \frac{e^2}{h}\left(T_{21} + \frac{T_{23}\,T_{31}}{T_{31}+T_{32}} \right) = \frac{e^2}{h}\left(T_{coh} + T_{in} \right)$$

If $T_{23} = 0$ or $T_{31} = 0$

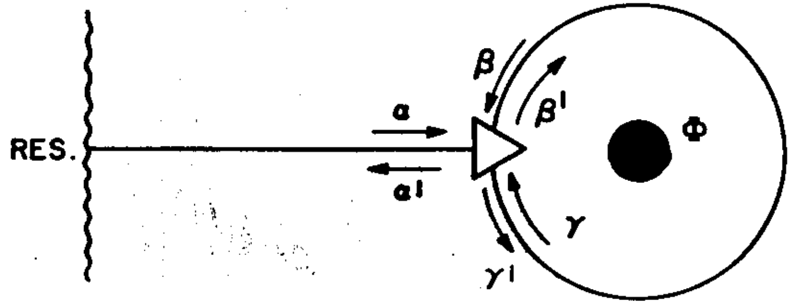
$$G = \frac{e^2}{h} T_{21} = \frac{e^2}{h} T_{coh}$$

If $T_{21} = 0$, $\rightarrow$

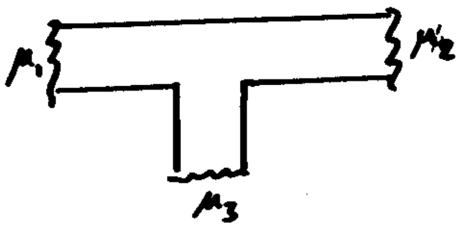
$$\frac{1}{G} = R = \frac{h}{e^2}\left(\frac{1}{T_{31}} + \frac{1}{T_{32}} \right) = R_1 + R_2$$

Normal Loop Coupled to an Electron Reservoir

M. Büttiker, Phys. Rev. B32, 1846 (1985)



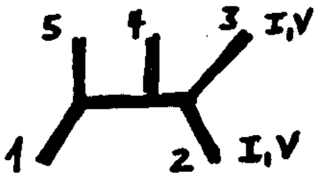
<u>Rigidity of Reziprocity Symmetry</u>



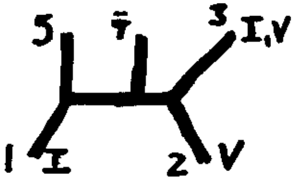
$\Longrightarrow$

T_{ij} , R_{ii}
symmetry & current conservation of N-probe conductor

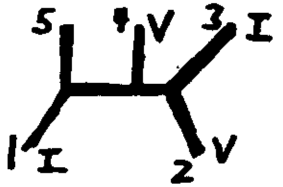
$\hat{T}_{ij}$, $\hat{R}_{ii}$
symmetry & current conservation of N-1-probe conductor



$R_{32,32}$

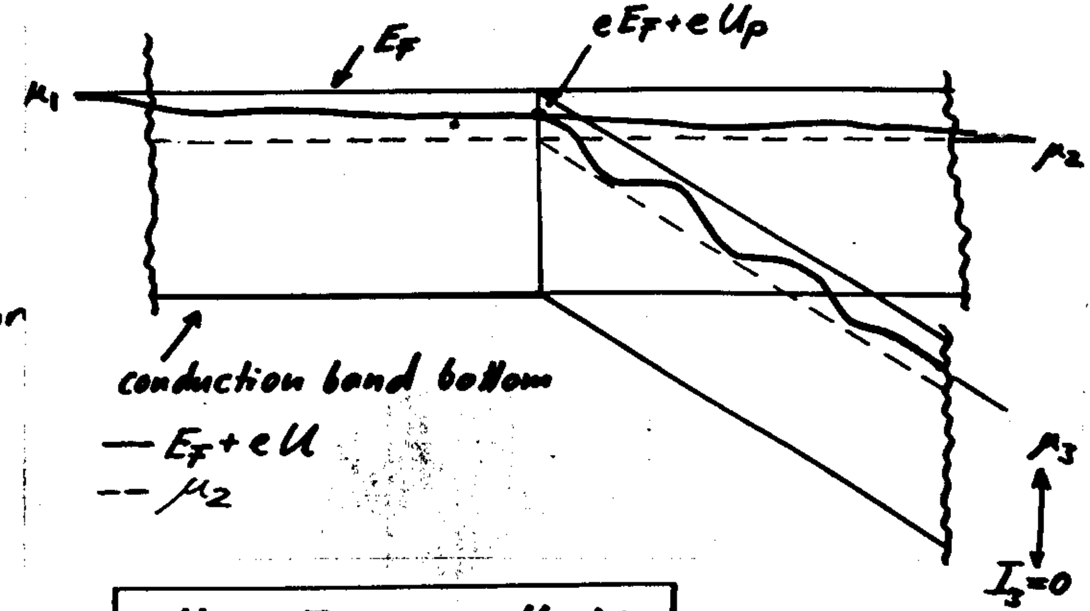


$R_{31,32}$



$R_{31,42}$

<u>Contact Potential of Voltage Probes</u>



$$\boxed{eU_p+eE_F-\mu_3=eU_c\neq 0}$$

Measurement of chemical potential yields <u>no</u> information about electro-static potential eU.

Literature

M. Büttiker, PRL 57, 1761 (1986)

M. Büttiker, IBM J. Res. Developm. 32, 317 (1988)

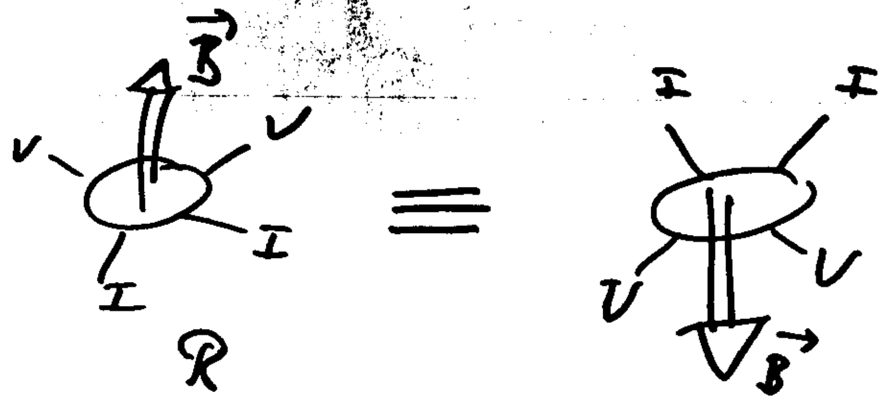
H. U. Baranger and A. D. Stone, Phys. Rev. B40, 8169(89)

I. B. Levinson, Sov. Phys. JETP 68, 58 (1989)

Summary

Voltage measurement:

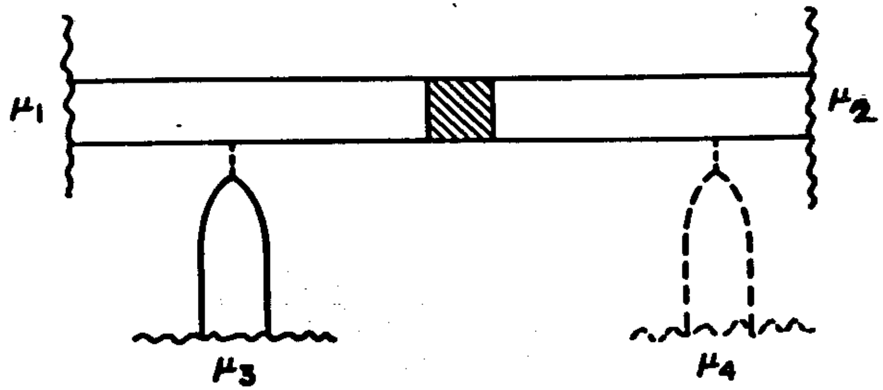
- At probes which permit carrier exchange with the conductor a chemical potential is measured (not an electrostatic voltage).
- Voltage measurement can be sensitive to the phase of electronic wave functions



Phase Sensitive Voltage Measurements

Conductor with (two) weakly coupled probes

(a)



$$R = \frac{eI}{\mu_3-\mu_4} = \frac{h}{e^2}\frac{1-T}{T};\qquad R_{12,34} = \frac{h}{e^2}\frac{1}{T}\frac{T_{31}T_{42}-T_{32}T_{41}}{(T_{31}+T_{32})(T_{41}+T_{42})}$$

(b)

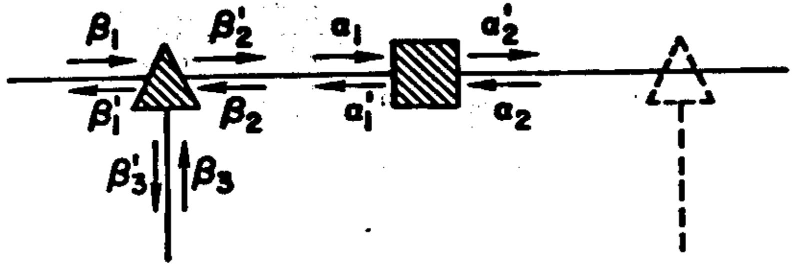
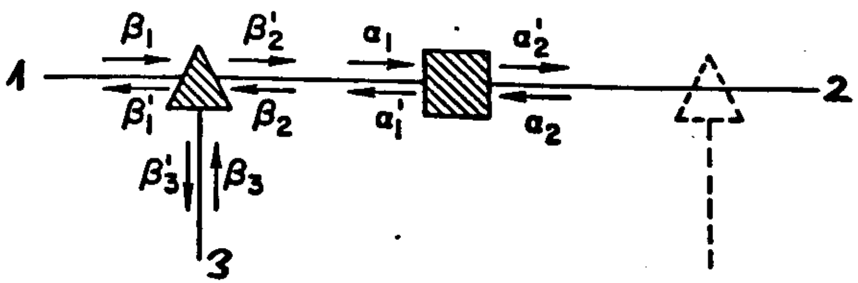


Fig. 1

Interference in a Measurement Probe



$$\begin{pmatrix}\beta_1'\\ \beta_2'\\ \beta_3'\end{pmatrix} = \underline{\underline{S}}\begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\end{pmatrix} \qquad \underline{\underline{S}} = \begin{pmatrix}-\varepsilon/2 & 1-\varepsilon/2 & \sqrt{\varepsilon}\\ 1-\varepsilon/2 & -\varepsilon/2 & \sqrt{\varepsilon}\\ \sqrt{\varepsilon} & \sqrt{\varepsilon} & -1+\varepsilon\end{pmatrix}$$

$$\varepsilon = 0 \Downarrow$$

$$\underline{\underline{S}} = \begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$$

$T_{31} = ?$

$\beta_1 = 1,\ \beta_3 = 0,\ \alpha_2 = 0 \rightarrow$

$\beta_2' = 1 \rightarrow \beta_2 = e^{2ik_F d} r \rightarrow$

$\beta_3' = \sqrt{\varepsilon}\,\beta_1 + \sqrt{\varepsilon}\,\beta_2 = \sqrt{\varepsilon}\,(1 + e^{2ik_F d} r)\ ; \quad r = R^{1/2} e^{i\Delta\phi}$

$\boxed{T_{31} = \varepsilon\,(1 + R + 2R^{1/2}\cos\chi_3);}\quad \chi_3 = 2k_F d + \Delta\phi + \Delta\varphi$

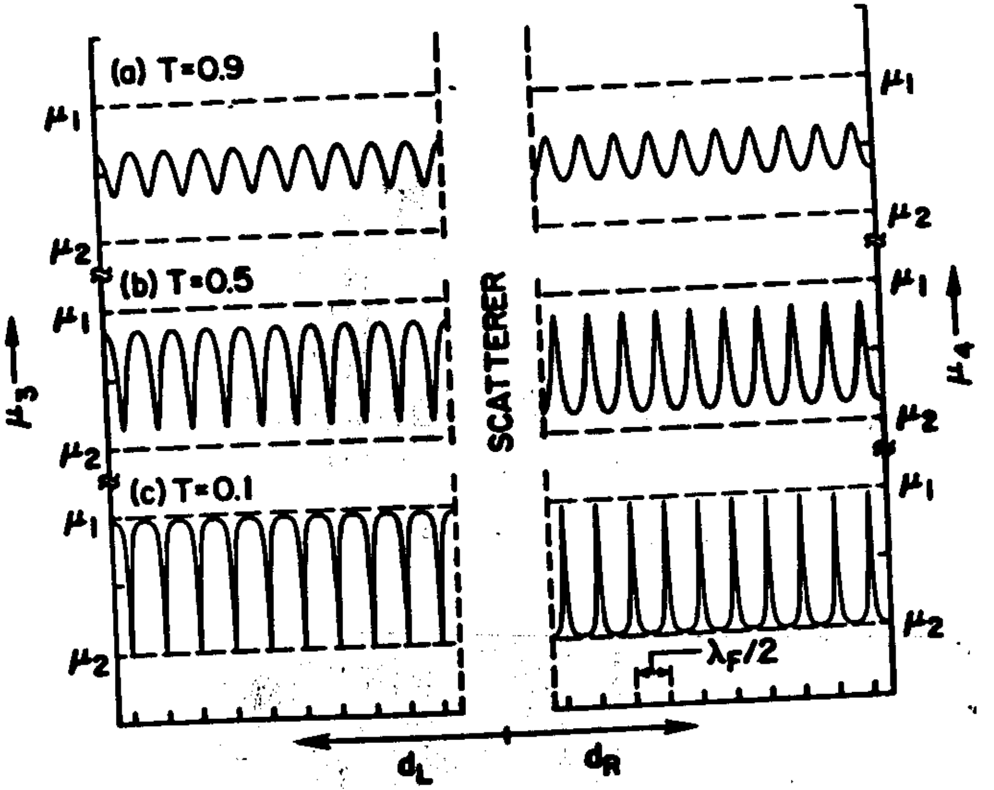
$\boxed{T_{32} = \varepsilon T}$

$$I_3 = 0 = \frac{e}{h}\left[-T_{31}\mu_1 - T_{32}\mu_2 + (1 - R_{33})\,\mu_3\right]$$

$$1 = R_{33} + T_{31} + T_{32} \rightarrow$$

$$\mu_3 = \frac{T_{31}\mu_1 + T_{32}\mu_2}{T_{31} + T_{32}} \rightarrow \boxed{\mu_3 = \frac{(1 + R + 2R^{1/2}\cos\chi_3)\mu_1 + T\mu_2}{2\,(1 + R^{1/2}\cos\chi_3)}}$$

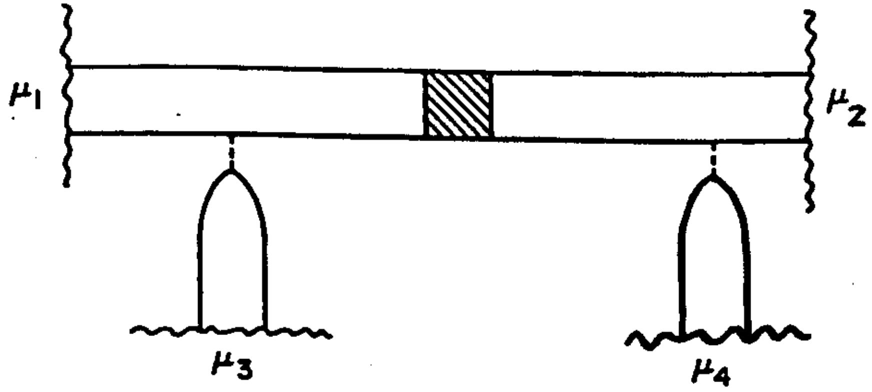
Chemical Potential Oscillations Near a Barrier



$$\boxed{\mu_3 = \frac{(1 + R + 2R^{1/2}\cos\chi_3)\mu_1 + T\mu_2}{2\,(1 + R^{1/2}\cos\chi_3)}} \qquad \chi_3 = 2k_F d_L + \cdots$$

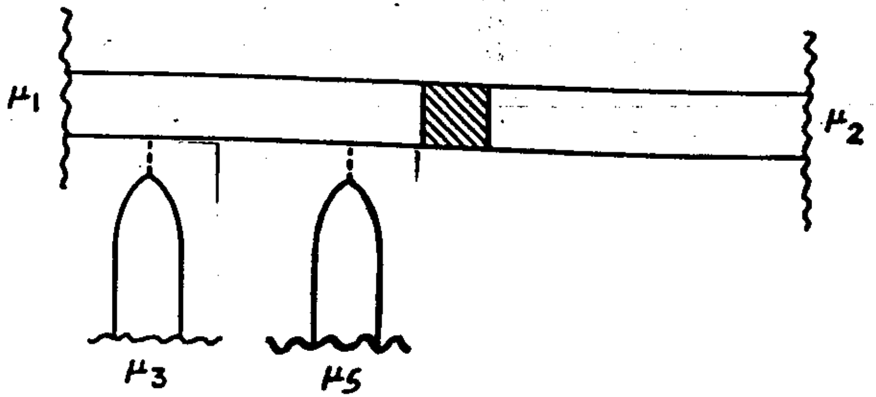
$$\boxed{\mu_4 = \frac{T\mu_1 + (1 + R + 2R^{1/2}\cos\chi_4)\mu_2}{2\,(1 + R^{1/2}\cos\chi_4)}} \qquad \chi_4 = 2k_F d_R + \cdots$$

Four Probe Resistances



$$I = \frac{e}{h} T (\mu_1 - \mu_2)$$

$$R_{12,34} = R_{34,12} = \frac{\mu_3 - \mu_4}{eI} \qquad -\frac{h}{e^2}\frac{R^{1/2}}{T} \le R_{12,34} \le \frac{h}{e^2}\frac{R^{1/2}}{T}$$



$$R_{12,35} = R_{35,12} = \frac{\mu_3 - \mu_5}{eI} \qquad -\frac{h}{e^2}\frac{R^{1/2}}{T} \le R_{12,35} \le \frac{h}{e^2}\frac{R^{1/2}}{T}$$

Resistance Oscillations due to Phase Sensitive Voltage Measurements

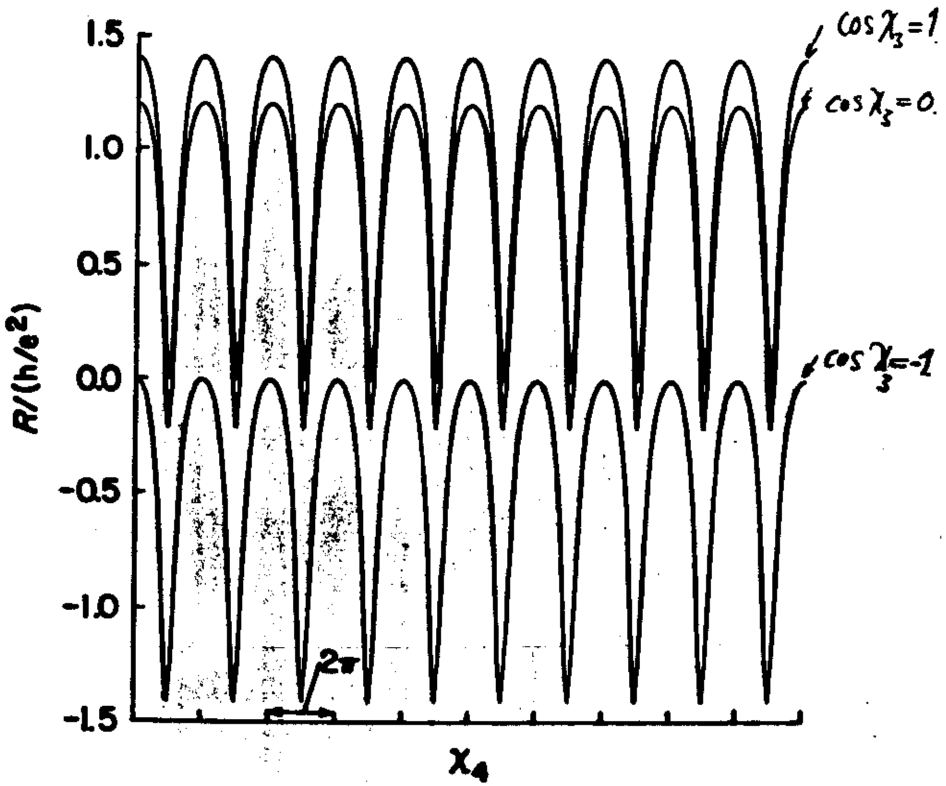
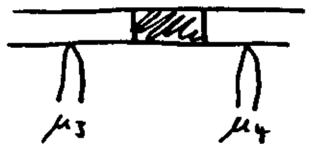


Fig. 3

$$-\frac{h}{e^2}\frac{R^{1/2}}{T} \le R \le \frac{h}{e^2}\frac{R^{1/2}}{T}$$

Voltage and Resistance Fluctuations



$$\langle \mu_i \rangle = \frac{1}{2\pi} \int d\chi_i \, \mu_i$$

$$eV \equiv \mu_3 - \mu_4$$

$$\boxed{\langle (eV - \langle eV \rangle)^2 \rangle = \frac{1}{2} \sqrt{T}\,(1-\sqrt{T})\,(\mu_1-\mu_2)^2}$$

$$\boxed{\langle (\mathcal{R} - \langle \mathcal{R} \rangle)^2 \rangle = \left(\frac{h}{e^2}\right)^2 \frac{1}{2} \; \frac{1-\sqrt{T}}{T^{3/2}}} \longrightarrow \infty$$

for $T \to 0$

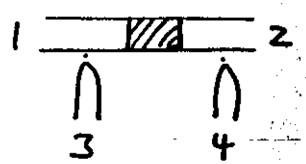
Phase - Averaged Chemical Potentials

$$\mu_3 = \frac{(1+R+2R^{1/2}\cos\chi_3)\,\mu_1 + T\mu_2}{2\,(1+R^{1/2}\cos\chi_3)}$$

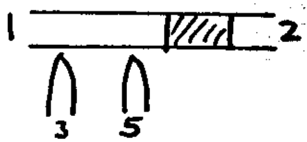
$$\mu_4 = \frac{T\mu_1 + (1+R+2R^{1/2}\cos\chi_4)\,\mu_2}{2\,(1+R^{1/2}\cos\chi_4)}$$

$$\langle \mu_i \rangle = \frac{1}{2\pi} \int d\chi_i \, \mu_i$$

$\langle \mu_{3,4} \rangle = \frac{\mu_1+\mu_2}{2}$ [illegible] $(\mu_1-\mu_2)$



$$\boxed{\langle \mathcal{R}_{12,34} \rangle = \frac{h}{e^2} \; \frac{1-\sqrt{T}}{\sqrt{T}}}$$



$$\langle \mathcal{R}_{12,35} \rangle = 0$$

Phase - Insensitive Chemical Potentials

$$\mu_3 = \frac{(1+R)\mu_1 + T\mu_2}{2} = \frac{\mu_1+\mu_2}{2} + \frac{1}{2}\,(1-T)\,(\mu_1-\mu_2)$$

$$\mu_4 = \frac{T\mu_1 + (1+R)\mu_2}{2} = \frac{\mu_1+\mu_2}{2} - \frac{1}{2}\,(1-T)\,(\mu_1-\mu_2)$$

$$\Longrightarrow \quad \boxed{\mathcal{R}_L = \frac{h}{e^2}\,\frac{1-T}{T}}$$

Landauer 1970

Comparison of Phase-Sensitive, Phase-Averaged and Phase Insensitive Voltage Measurements

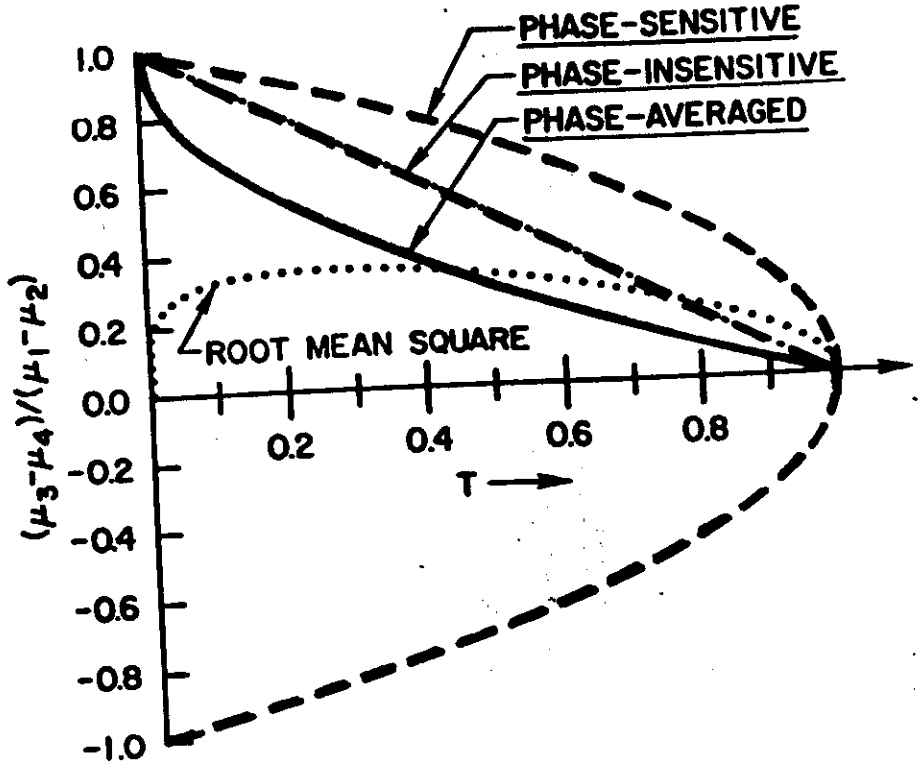


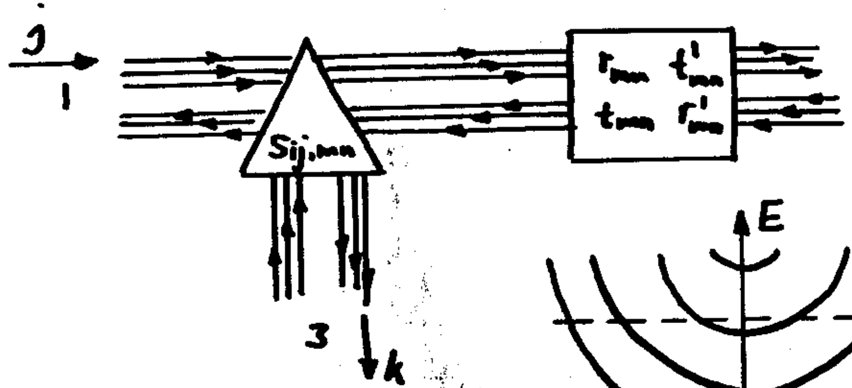
Fig 4

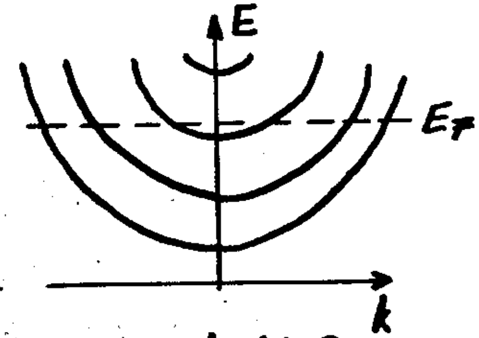
$$-\frac{h}{e^2}R^{1/2}/T \le \mathcal{R}_{12,34} \le \frac{h}{e^2}R^{1/2}/T$$

$$\langle \mathcal{R}_{12,34} \rangle = \frac{h}{e^2}\,\frac{1-\sqrt{T}}{T}$$

$$\mathcal{R}_L = \frac{h}{e^2}\,\frac{1-T}{T}$$

Many Channels: Multiple Periodic Oscillations





$$T_{31,kj} = \left| S_{31,kj} + \sum_{\ell} S_{32,k\ell}\, r_{\ell j}\, e^{i(\phi_\ell+\phi_j)} \right|^2$$

$$\phi_m = k_m d$$

- many periods (as function of d)
- extremly long if near threshold
- not in phase (self-averaging)
- wide variety depending on matrix elements $S_{ij,mn}$

M. Büttiker, Phys. Rev. B 40, 3409 (1989)
[I.B. Levinson, Sov. Phys. JETP 68, 68 (1989)]