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SPRING COLLEGE IN CONDENSED MATTER
ON
'PHYSICS OF LOW-DIMENSIONAL SEMICONDUCTOR STRUCTURES'
(23 April - 15 June 1990)

TRANSMISSION PROBABILITIES AND THE QUANTUM HALL EFFECT

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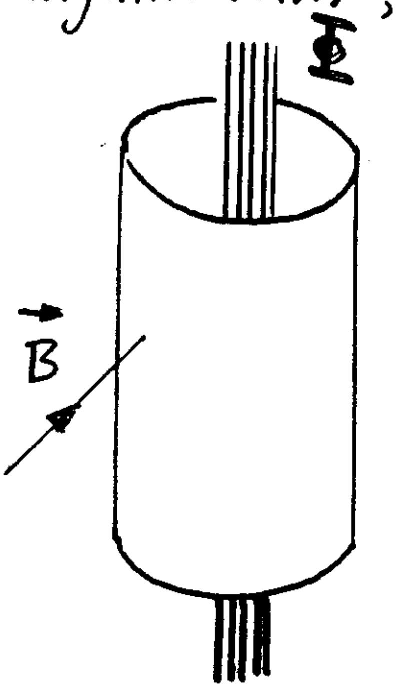
Transmission Probabilities and the Quantum Hall Effect

M. Büttiker

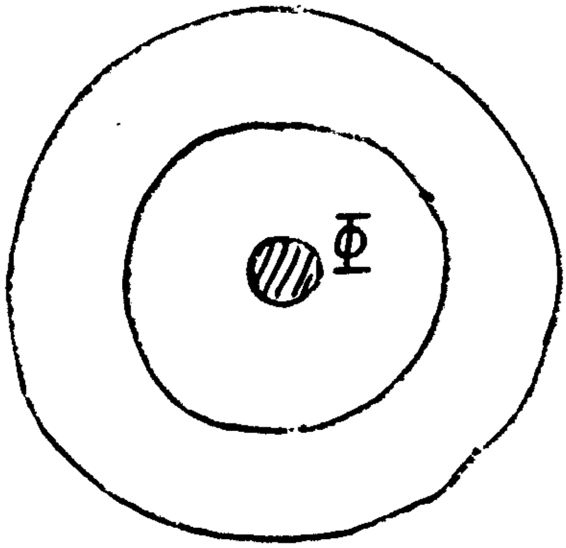
IBM

Closed Conductors

Laughlin (1981);

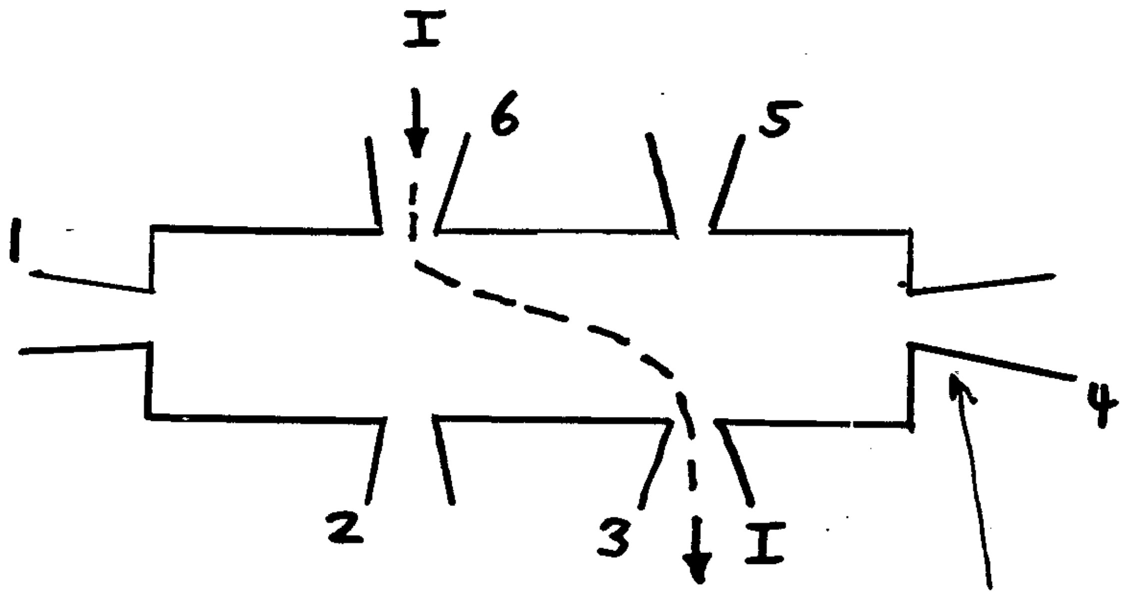


Halperin (1982);



$l_\varphi \leq 2\pi R$

Open Conductors



phase breaking (dissipative)

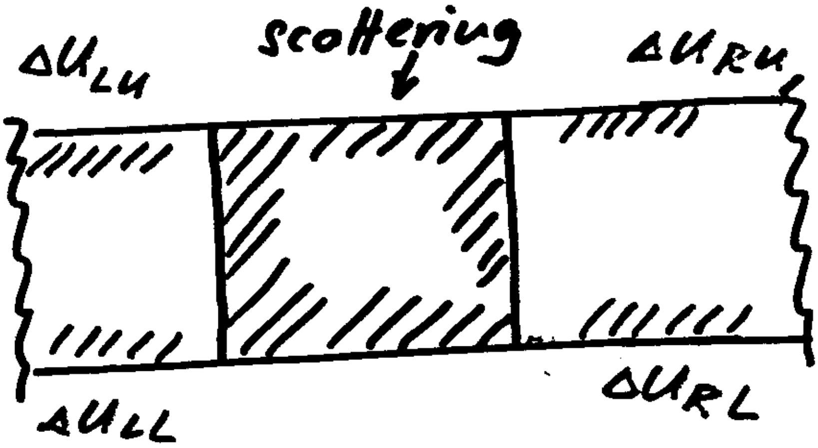
$$R_{mn,kl} = \begin{cases} 0 \\ \pm \frac{h}{e^2}\frac{1}{N} \end{cases}$$

$l_\varphi \simeq 1\mu m$

$l_{in} \gg 50\mu m$

Motivation

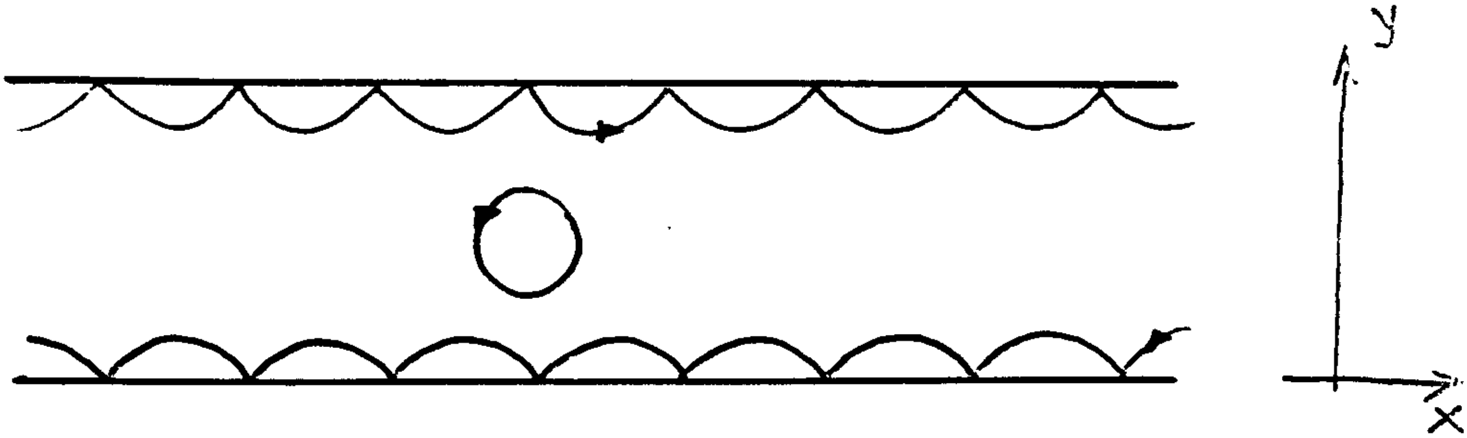
Streda, Kucera, MacDonald (1987)



calculate potentials from piled up charges.

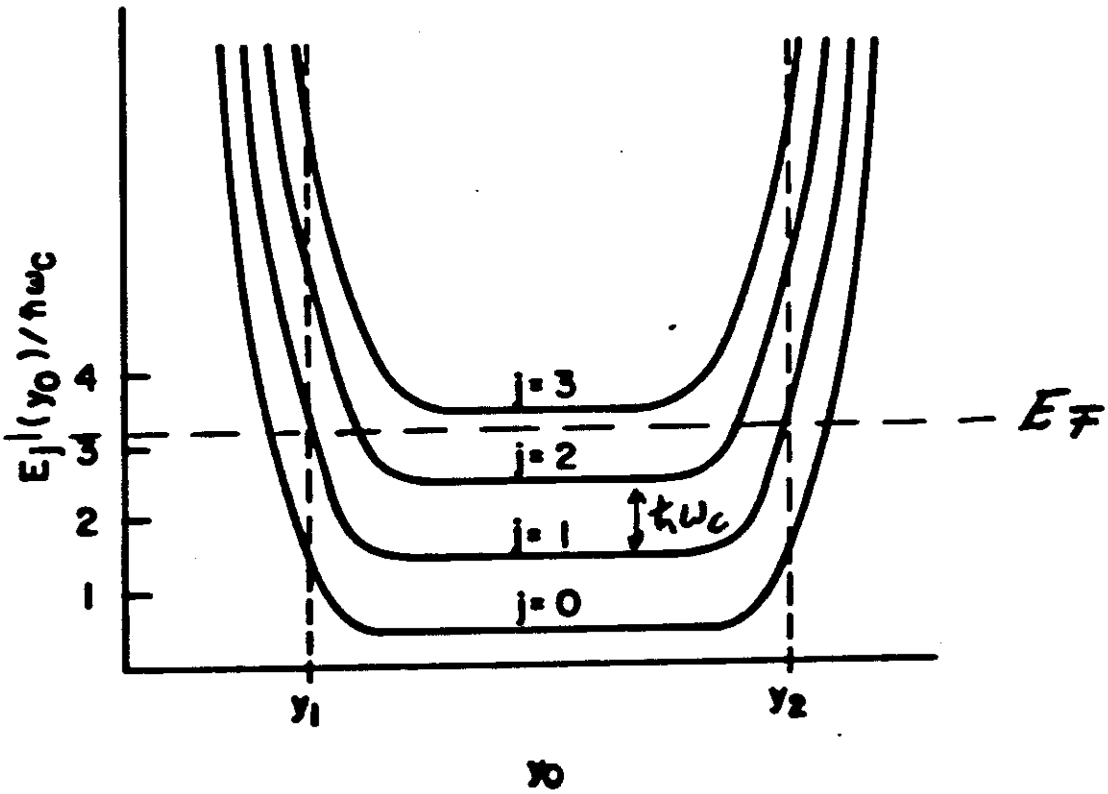
[Jain, Kivelson, 1988]

Transport in High Magnetic Fields



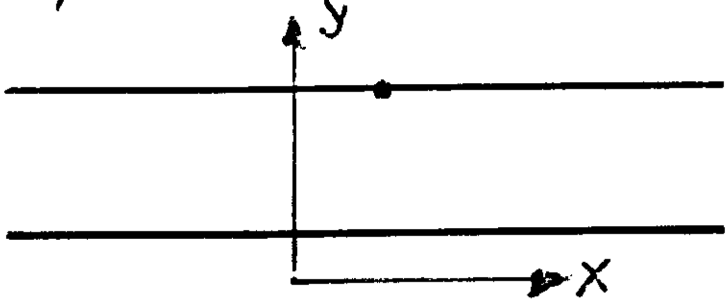
$r_c = v_F/\omega_c \qquad \omega_c = eB/mc$

Halperin (1981)



Motion in High Field Perfect Conductors

Haiperin (82)



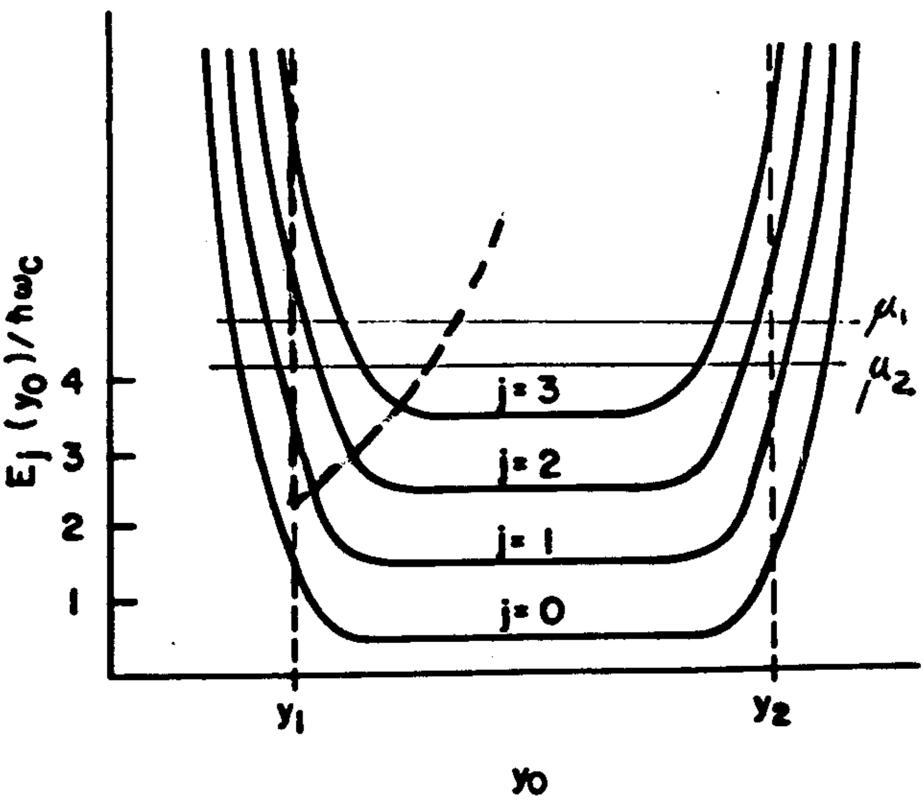
$\vec{A} = (By, 0, 0)$

$H = \frac{1}{2m}\left(\left[P_x + \frac{e}{c}By\right]^2 + P_y^2\right) + V(y)$

$\psi_k = e^{ikx} f_k(y)$

$$\boxed{E f_k = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2} + \frac{m}{2}\omega_c^2 (y_0 - y)^2 + V(y)\right) f_k}$$

$\omega_c = |eB|/mc, \quad y_0 = \frac{\hbar k}{m}\frac{1}{\omega_c} = k\ell_B^2; \quad \ell_B = (\hbar c/|eB|)^{1/2}$

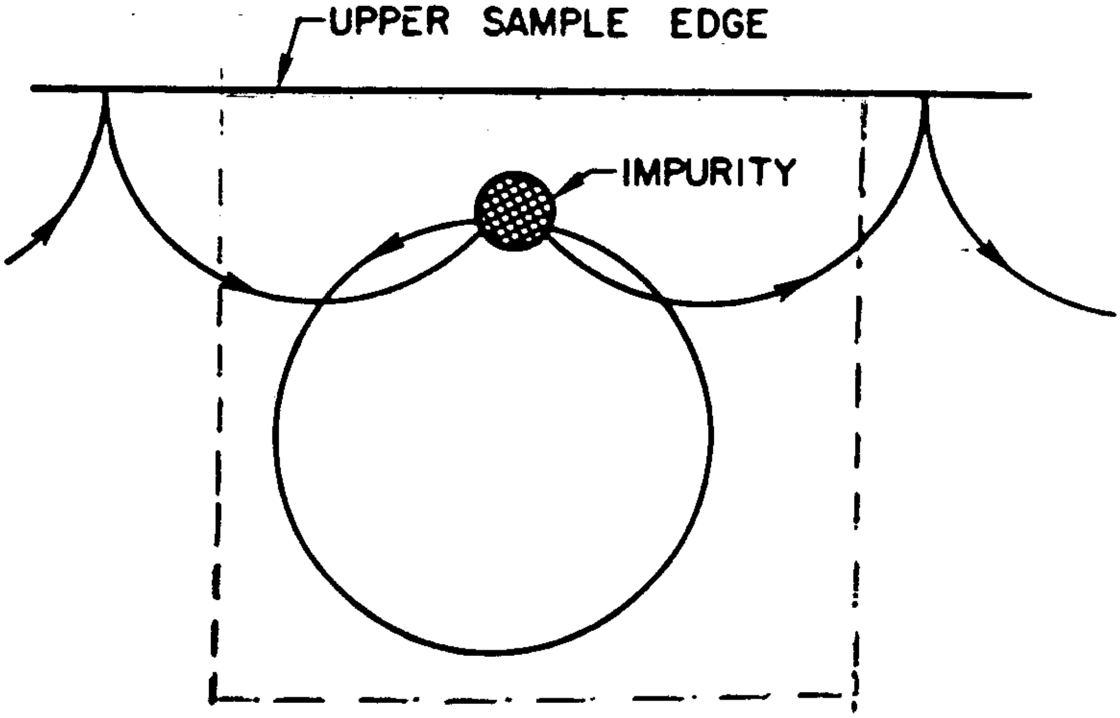


$V_j = \frac{1}{\hbar}\frac{dE}{dk} = \frac{1}{\hbar}\frac{dE}{dy_0}\frac{dy_0}{dk}$

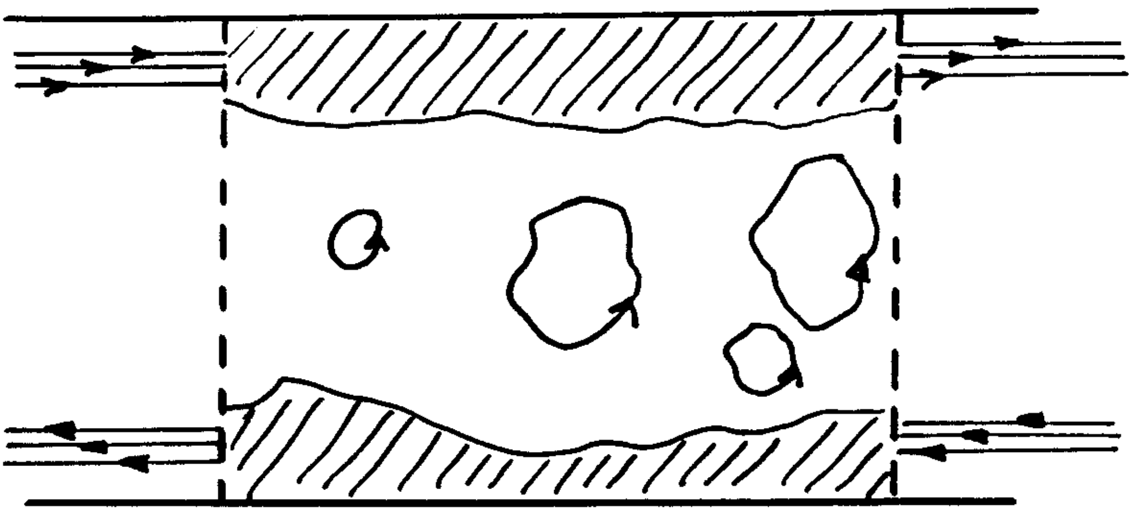
$\frac{dn}{dE} = \frac{1}{2\pi\hbar V_j}$

$$\boxed{\begin{aligned} I_j &= e\, v_j \left(\frac{dn}{dE}\right)_j (\mu_1 - \mu_2) \\ &= \frac{e}{h}(\mu_1 - \mu_2) \end{aligned}}$$

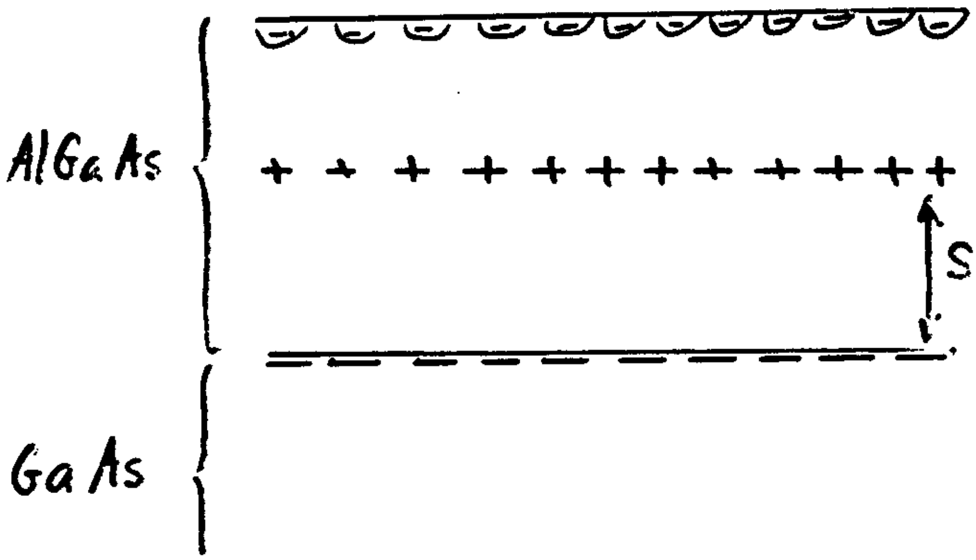
Suppression of Backscattering in High Magnetic Fields



$l_e > r_c$



GaAs – $Al_x Ga_{1-x} As$ – Heterostructure



$s \simeq 80\,Å - 1300\,Å$

$$\langle V(r)\, V(r+d) \rangle = V_0^2\, e^{-|d|^2/\zeta^2} \quad ; \quad \zeta \cong s$$

$$\ell_B = \left(\frac{\hbar c}{eB}\right)^{1/2} \qquad \left[\, 2\pi B \ell_B^2 = \Phi_0 = \frac{hc}{e} \,\right]$$

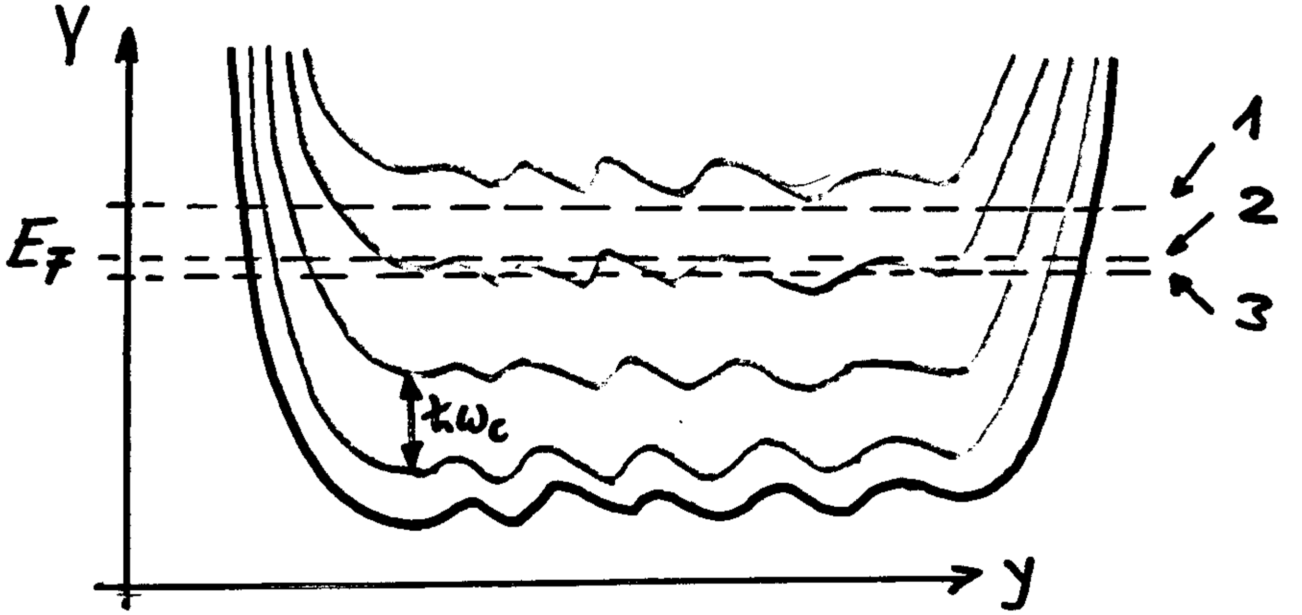
$\ell_B \cong 125\,Å$ at 4 Tesla

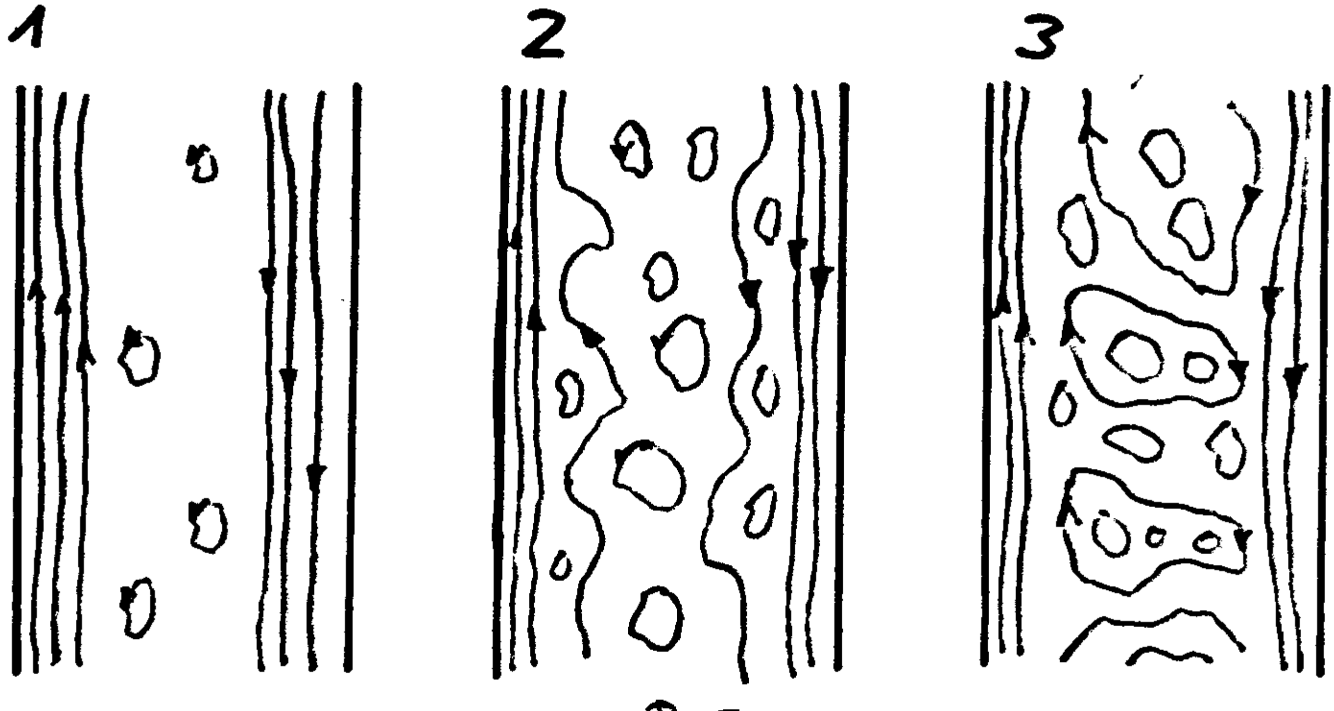
High Field Transport in Weakly Fluctuating Potential

Potential $V(x,y)$ $\qquad |\nabla V(x,y)|\, \ell_B \ll \hbar\omega_c$

Landau levels $\hbar\omega_c(n+1/2)$ $\qquad \omega_c = \frac{eB}{mc}$, $\ell_B = (\hbar c/eB)^{1/2}$

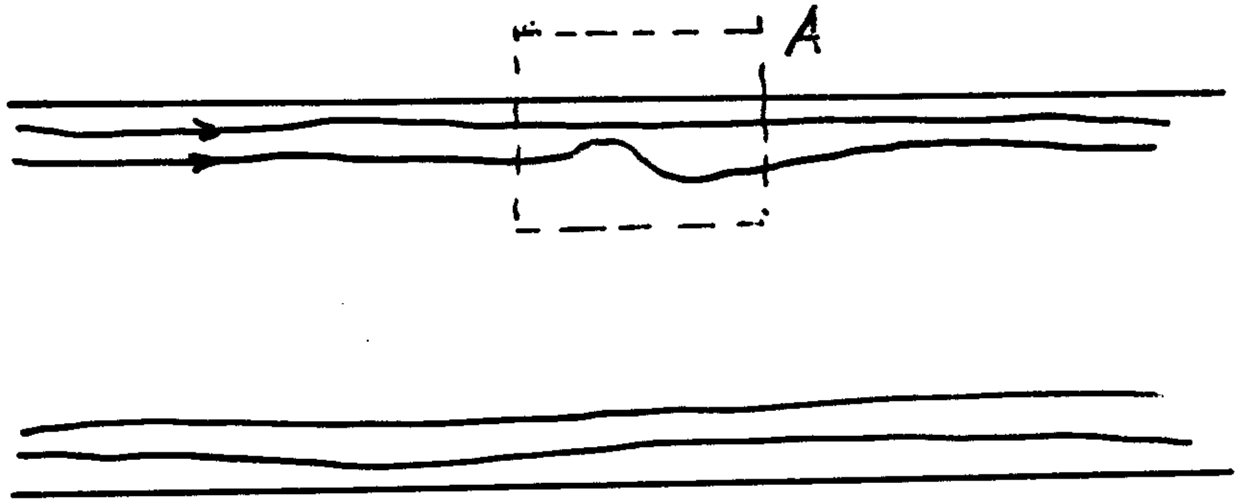
$E_F = \hbar\omega_c(n+1/2) + V(x,y)$ $\qquad v_d = c\,\frac{E}{B}$, $E = -\nabla V$

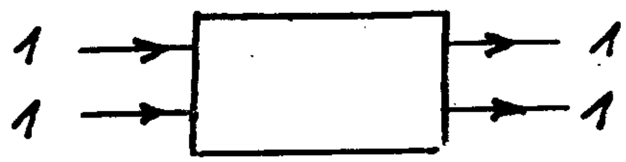




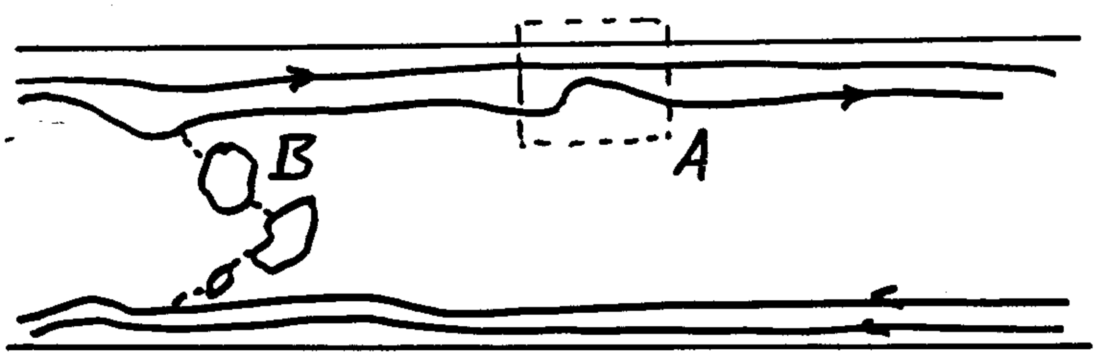
Immunity of Edge States to Disorder

Halperin (82), Büttiker (1988)



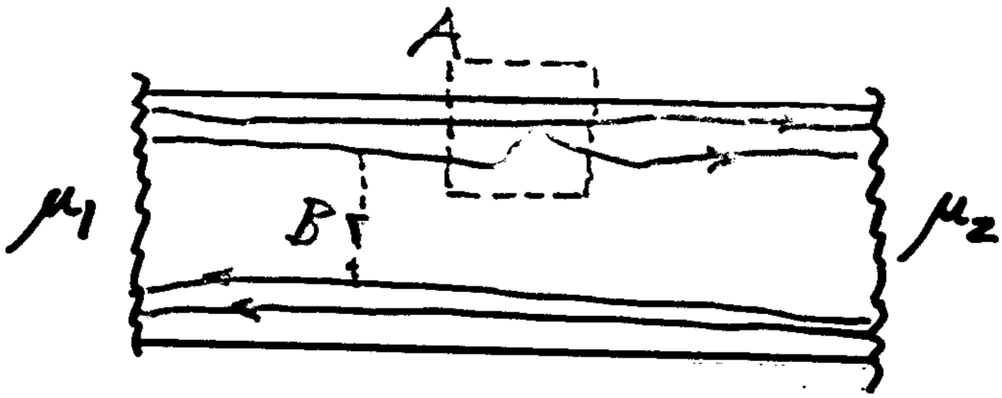


$$T = T_{11} + T_{12} + T_{21} + T_{22} = 2$$

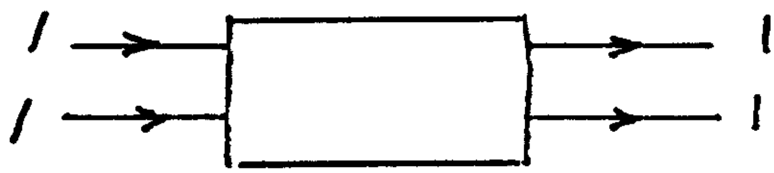


- if edge states are populated equally and in the absence of backscattering the transmission probabilities are <u>integers.</u>

Two Terminal Conductance



(1) equilibrium filling
(2) absence of backscattering



$$T = T_{11} + T_{21} + T_{12} + T_{22} =$$

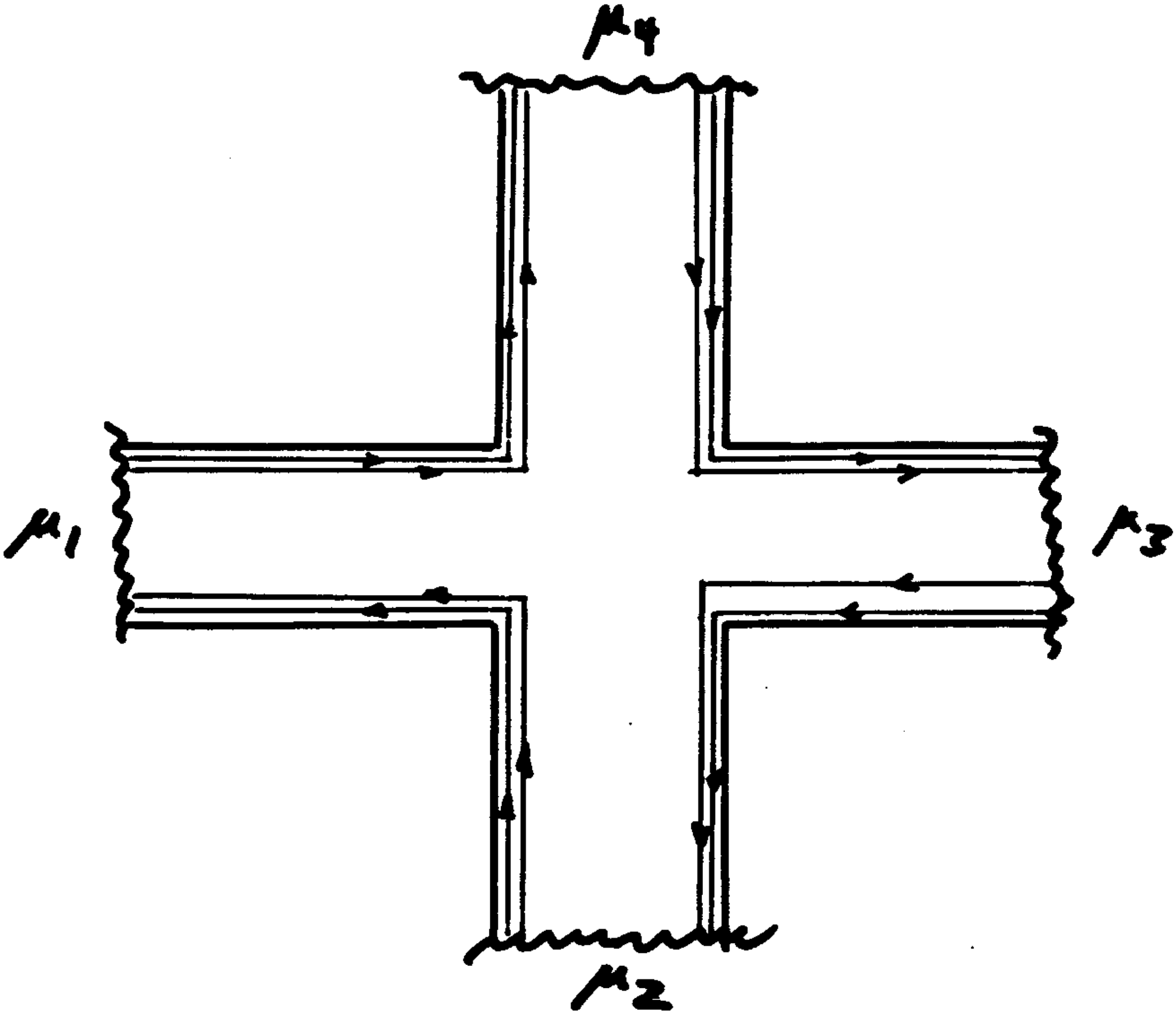
$\Rightarrow$ perfect transmission along edge channels (independent of residual impurities, surface roughness, ...)

$\Rightarrow$

$$\boxed{G = \frac{e^2}{h} T = \frac{e^2}{h} N}$$

Quantum Hall Effect in Open Conductors

Büttiker, 1988



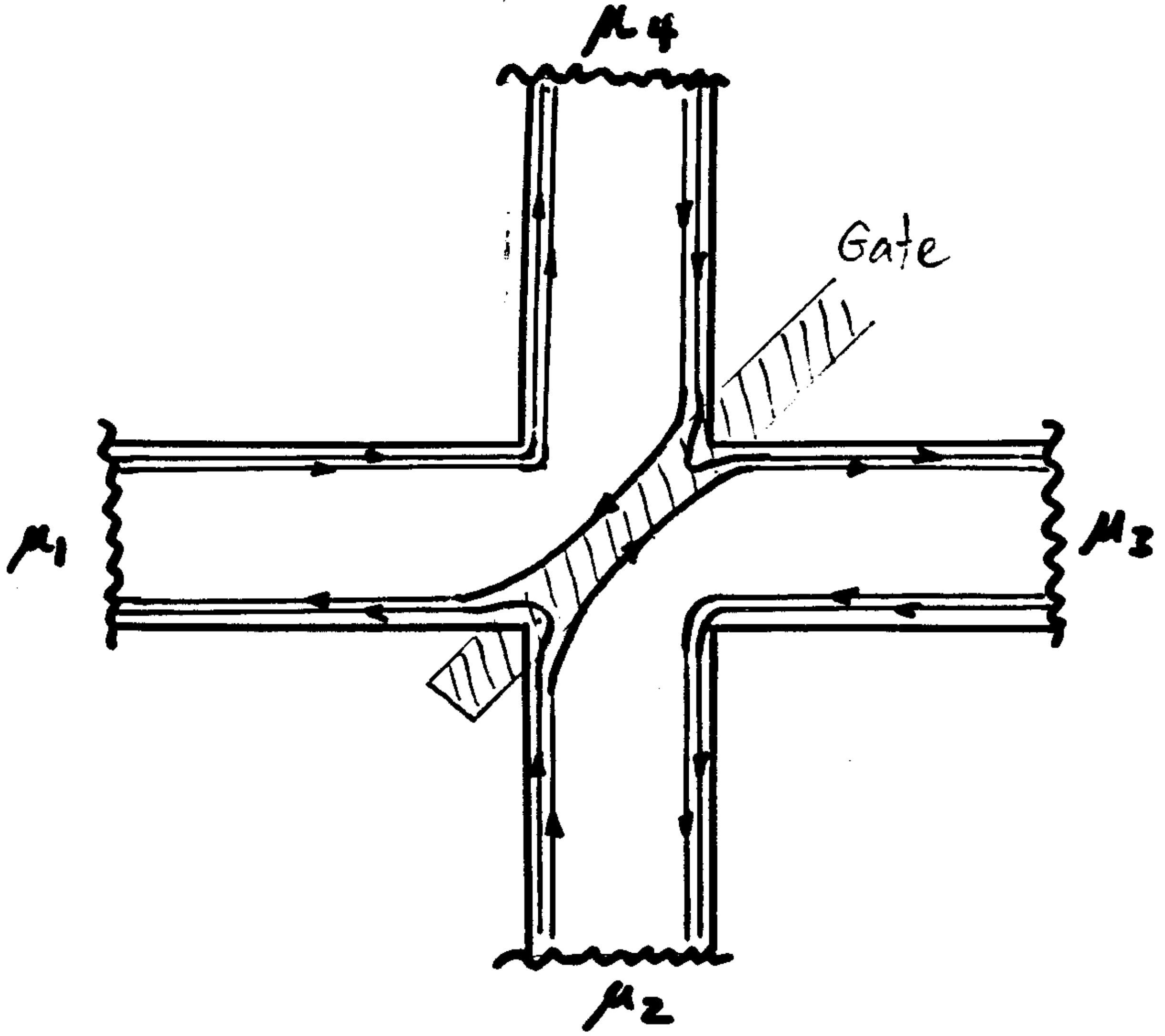
$$T_{41}=N, \quad T_{34}=N, \quad T_{23}=N, \quad T_{12}=N,$$

$$R_{13,42}=\frac{h}{e^2}\,\frac{T_{41}T_{23}-T_{43}T_{21}}{D}=\frac{h}{e^2}\,\frac{NN-0\cdot 0}{N^3}=\frac{h}{e^2}\,\frac{1}{N}$$

$$R_{42,13}\,(B)=R_{13,42}\,(-B)=-\frac{h}{e^2}\,\frac{1}{N}$$

$$R_{14,23}=\frac{h}{e^2}\,\frac{T_{31}T_{34}-T_{24}T_{21}}{D}=\frac{h}{e^2}\,\frac{0\cdot N-0\cdot 0}{N^3}=0$$

Scattering along a Diagonal Path



- Hall resistances and

$$R_{13,42}(B) = \frac{h}{e^2}\frac{1}{N-K} \qquad R_{13,42}(-B) = R_{42,13}(B) = -\frac{h}{e^2}\frac{N-2K}{N(N-K)}$$

Longitudinal resistances are simultaneously quantized

$$R_{12,43}(B) = R_{12,43}(-B) = \frac{h}{e^2}\frac{K}{N(N-K)} = \frac{h}{e^2}\left(\frac{1}{N-K} - \frac{1}{N}\right)$$

$$R_{13,42} - R_{12,43} = \frac{h}{e^2}\frac{1}{N}$$

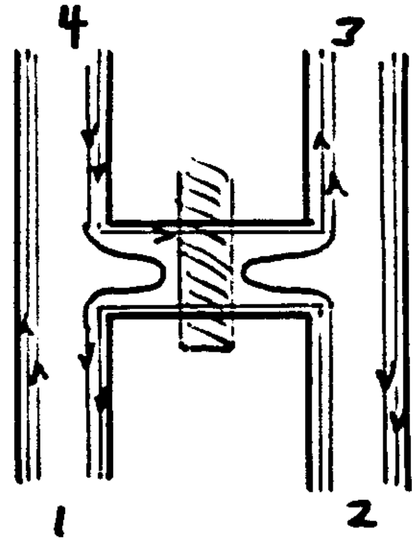
Washburn, Fowler, Schmid, Kern (1988)
Haug, MacDonald, Streda, von Klitzing, (1988)

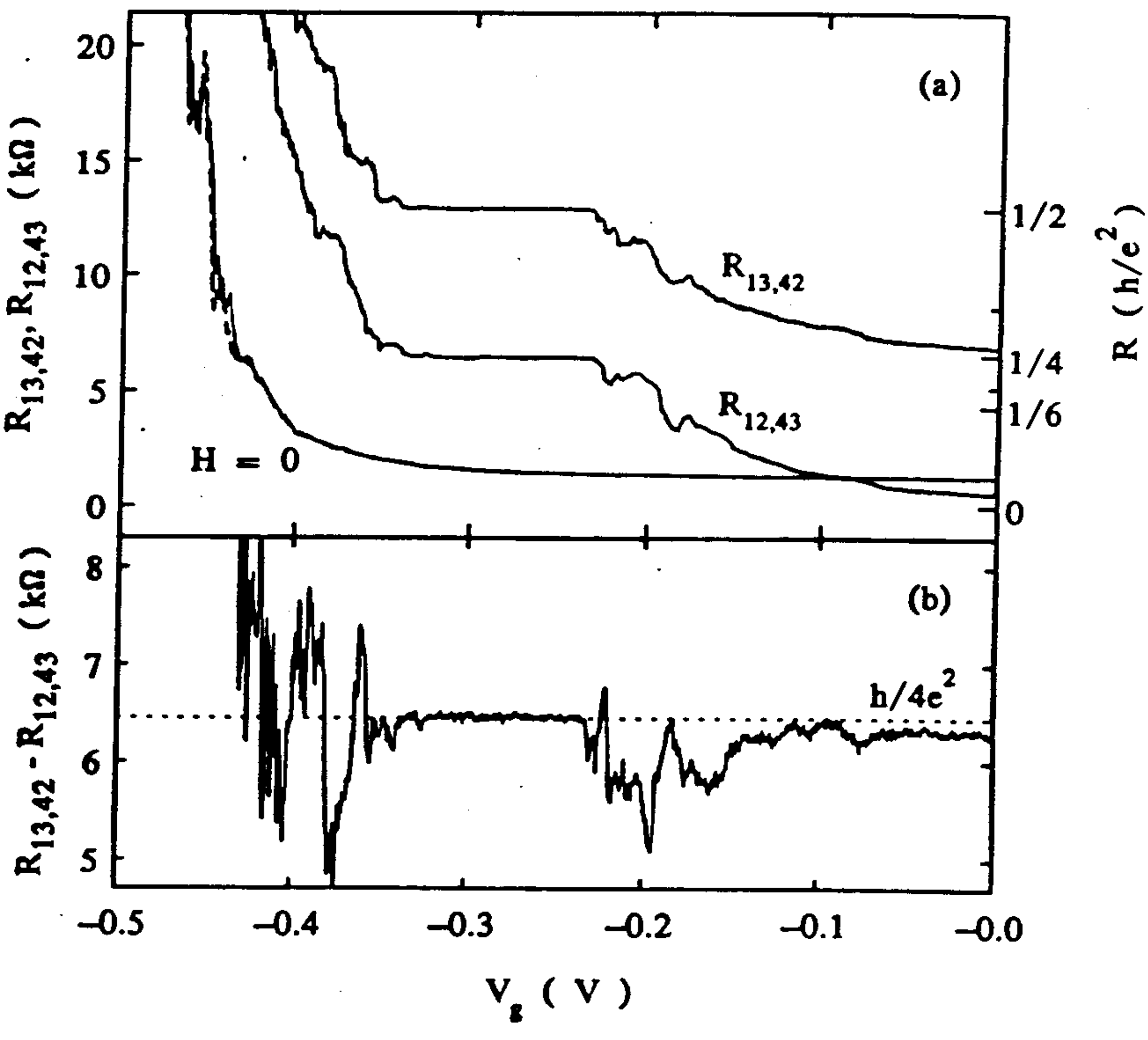
$N=4, \quad K=0, \quad K=2$

$$\mathcal{R}_{13,42} = \frac{h}{e^2}\frac{1}{N-K}$$

$$\mathcal{R}_{12,43} = \frac{h}{e^2}\frac{K}{N(N-K)} \qquad \frac{2}{4\cdot 2} = \frac{1}{4}$$

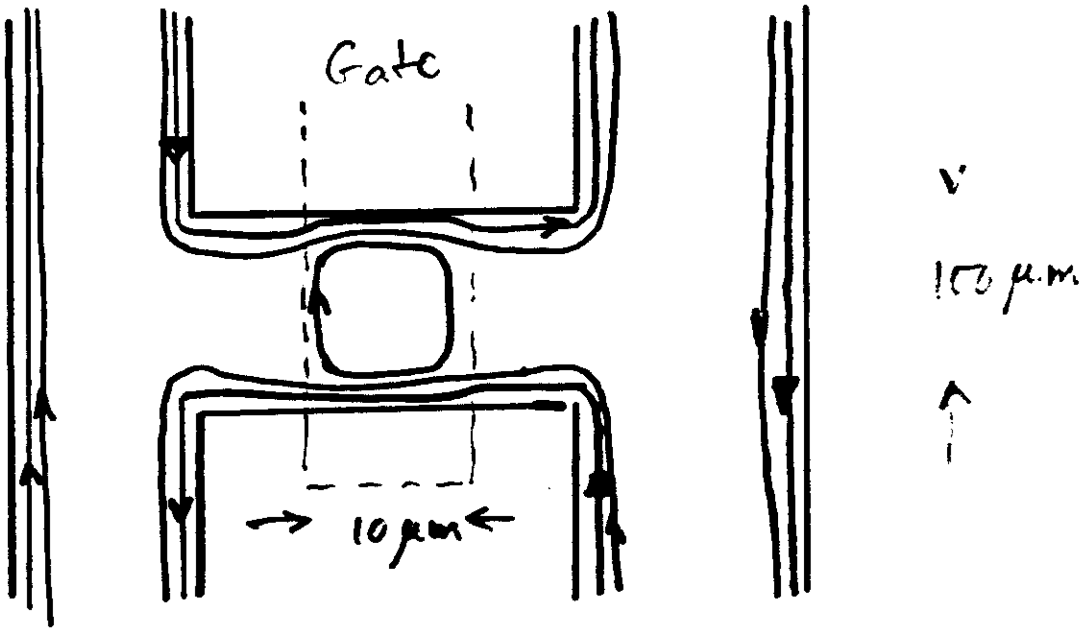
$$\mathcal{R}_{13,42} - \mathcal{R}_{12\,43} = \frac{h}{e^2}\frac{1}{N}$$





Reverse Biased Gate

Haug et al. (Surface Science)



$\underline{\nu > \nu_{g-}}$

$$R_L = \frac{h}{e^2}\,\frac{\nu - \nu_{g-}}{\nu\,\nu_g}$$

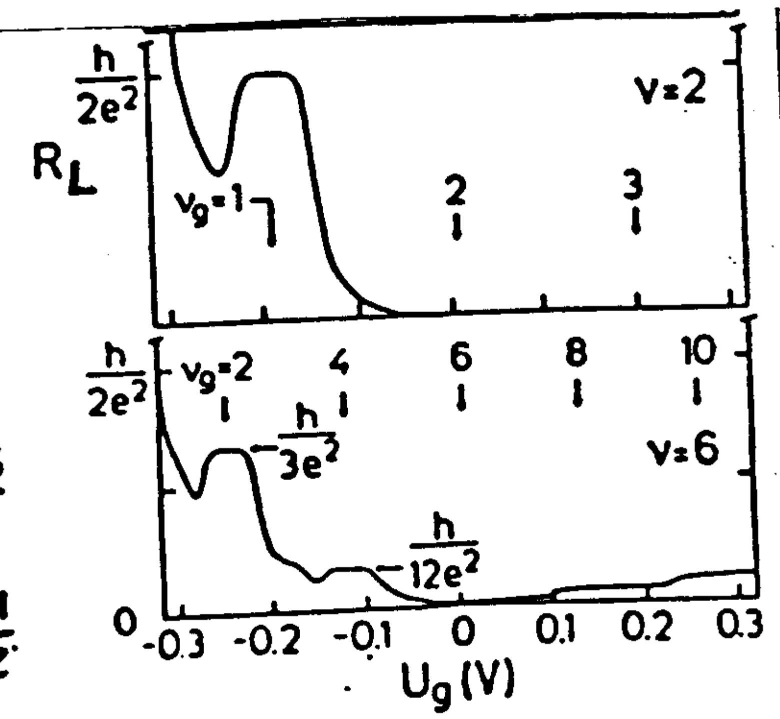
$\underline{\nu < \nu_{g-}}$

if coupled

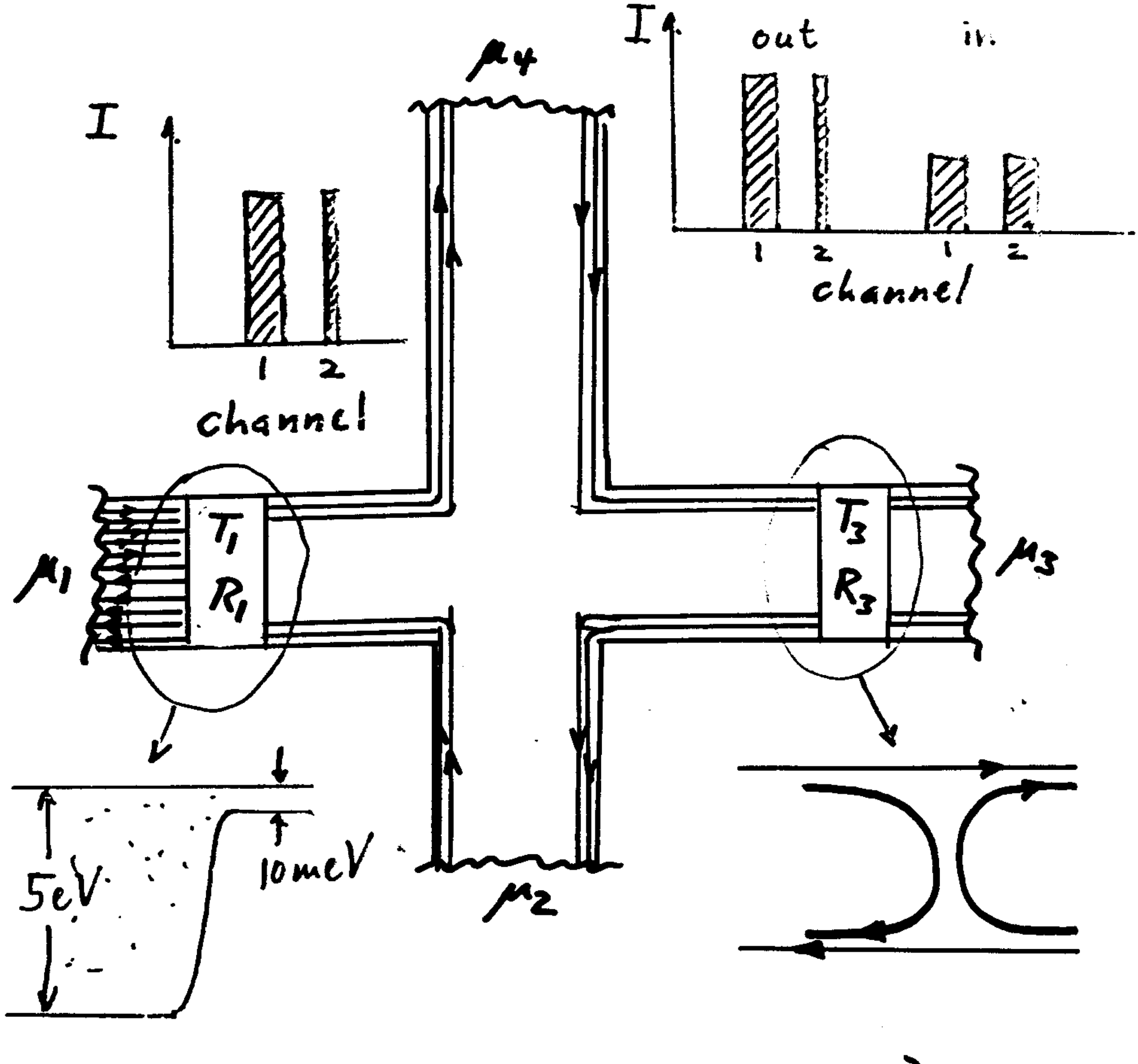
$$R_L = \frac{h}{e^2}\,\frac{1}{\nu(\nu-1)} \overset{\nu=2}{=} \frac{h}{e^2}\,\frac{1}{2}$$

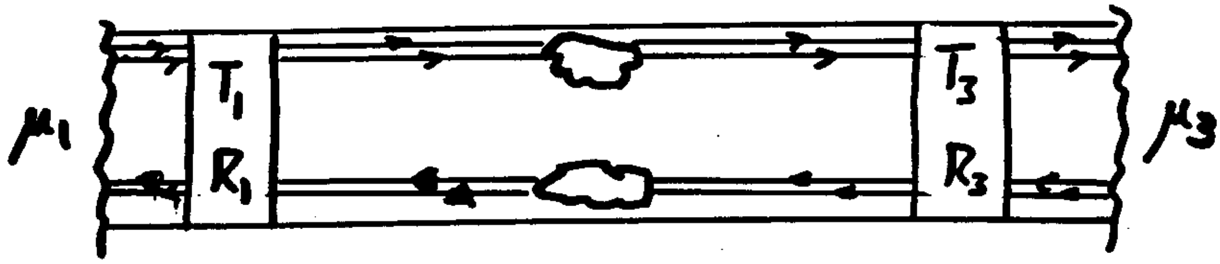
or

$$R_L = \frac{h}{e^2}\,\frac{2}{\nu(\nu-2)} \overset{\nu=6}{=} \frac{h}{e^2}\,\frac{1}{12}$$



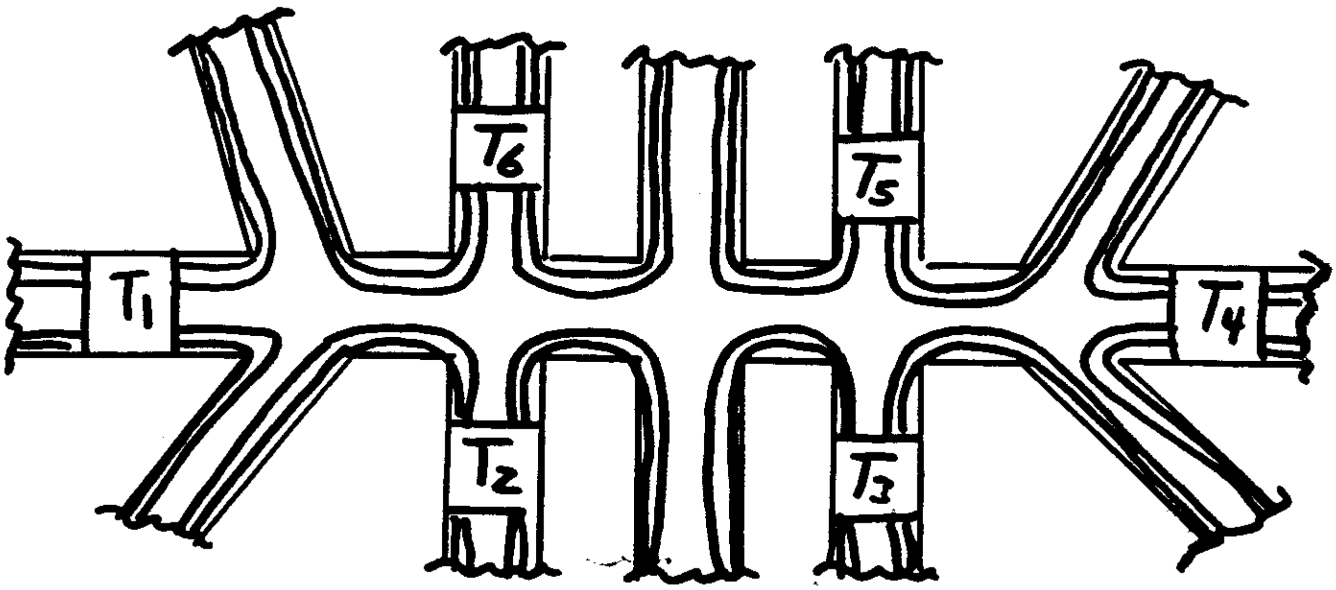
Contacts with Internal Reflection

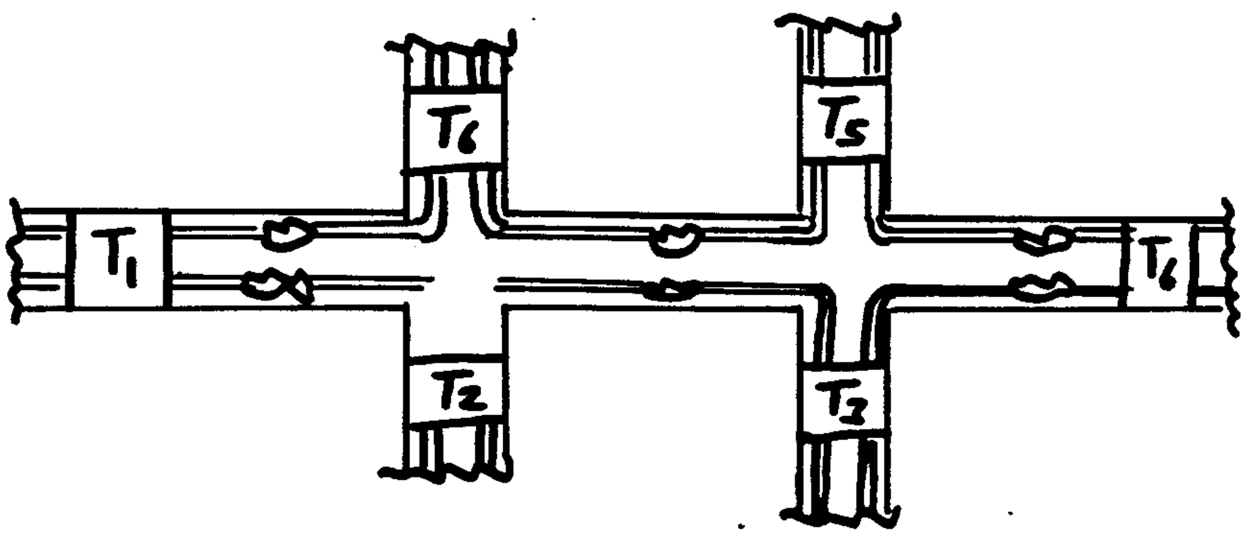




$$R_{13,13} = \frac{h}{e^2}\left(\frac{N-T_1}{NT_1} + \frac{N-T_2}{NT_2} + \frac{1}{N}\right)$$

Contacts with Internal Reflection Alternating with Ideal Contacts



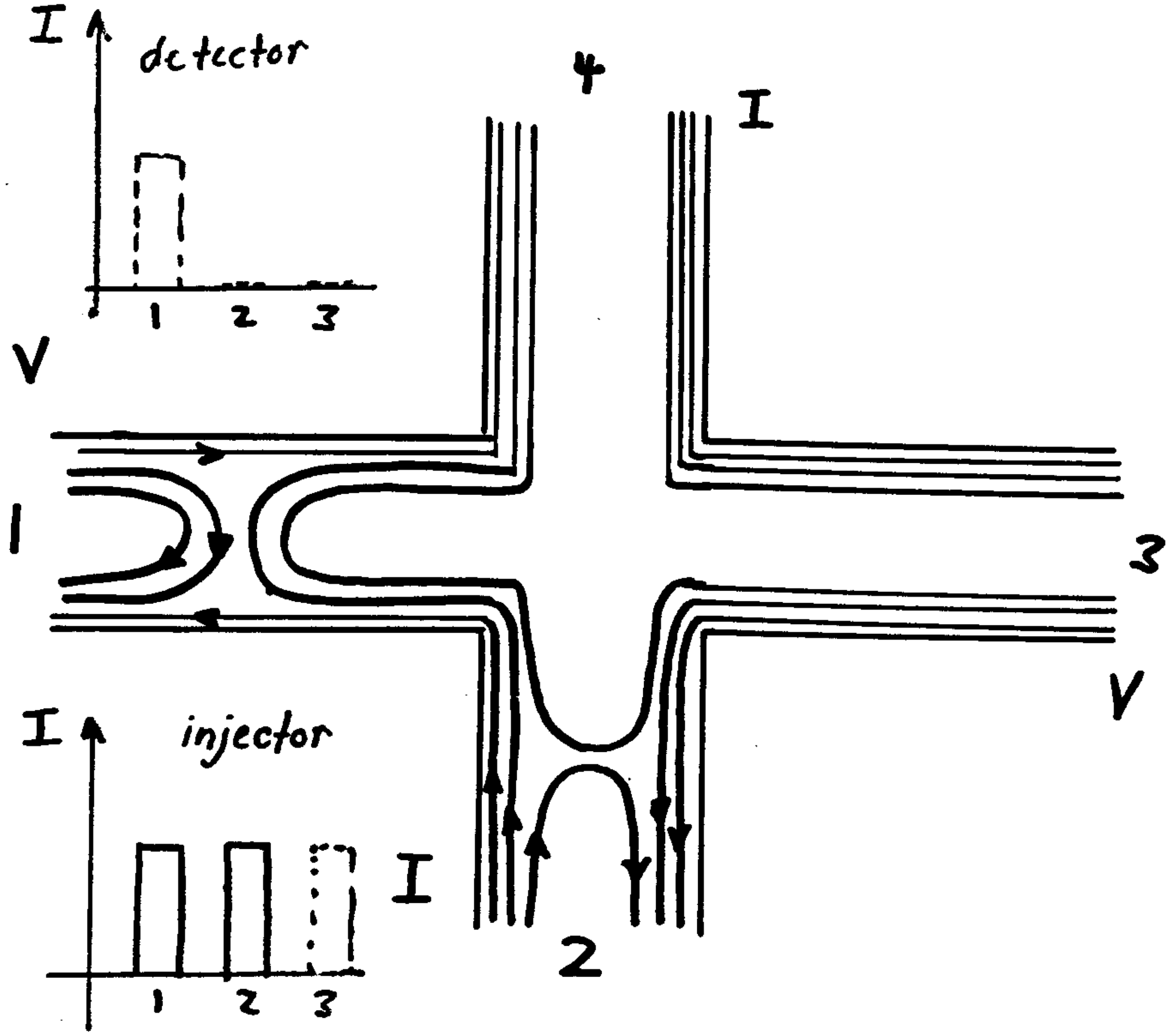


⟹

We have an ordinary QHE whenever edge states are equilibrated

Adjoning Contacts with Internal Reflection

van Wees et al. (1989)



$$\mathcal{R}_{24,13} = \frac{h}{e^2} \frac{T_{12}}{T_1 T_2}$$

if T_1, T_2, quantized, i.e. $T_1 = K$, $T_2 = L$, $\Rightarrow T_{12} = \min(T_1, T_2)$

$\Longrightarrow$

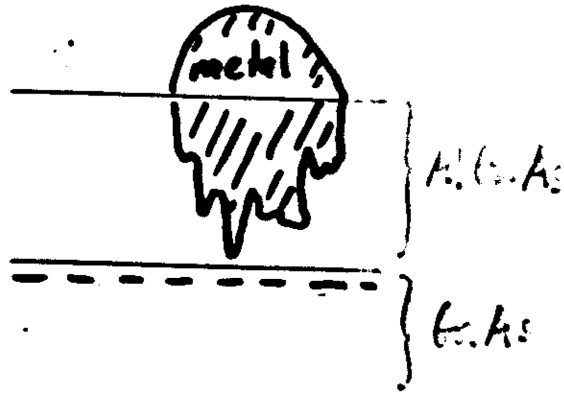
$$\boxed{\mathcal{R}_{24,13} = \frac{h}{e^2} \frac{1}{\max(T_1, T_2)}}$$

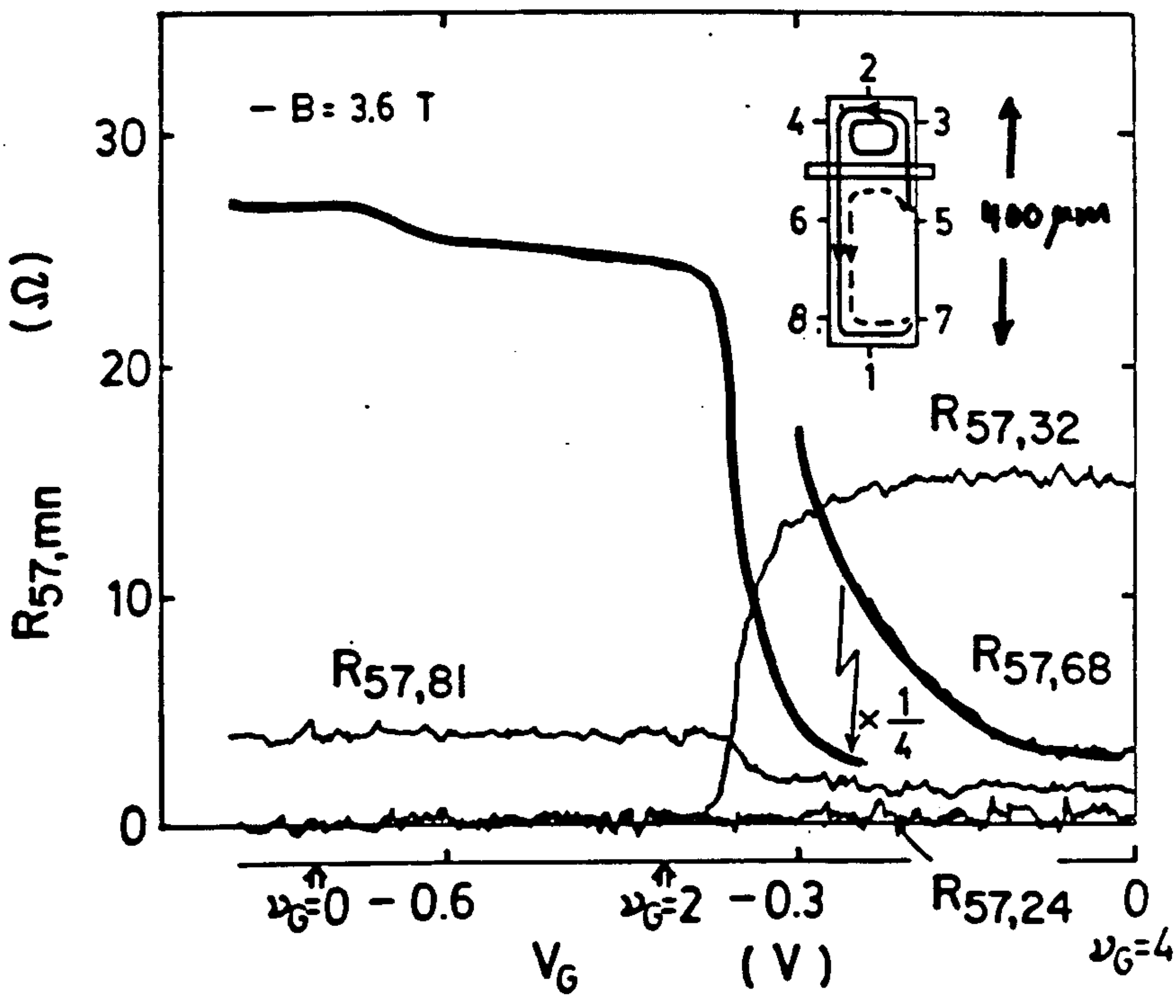
$d = 1.5\,\mu m$

Mesoscopic Effects on Macroscopic Length Scales

Komiyama et al., Solid State Commun., 73, 91 (1990)

$R_{contact} \simeq 0.5 \sim 8.7\ k\Omega$

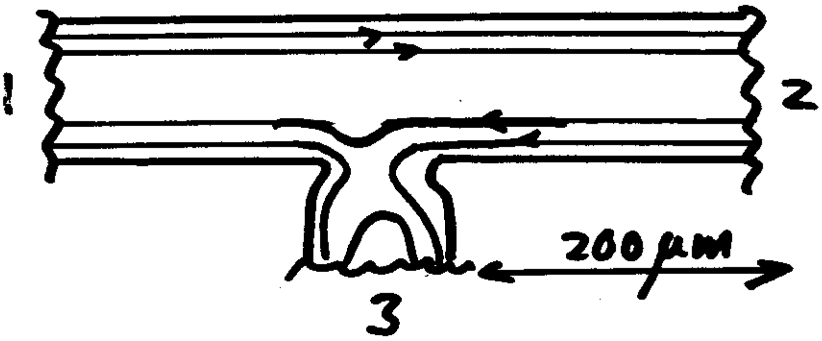




$\mu = 1.3 \cdot 10^6\ cm^2/Vs$, $n \cong 3.5 \cdot 10^{11}/cm^2$

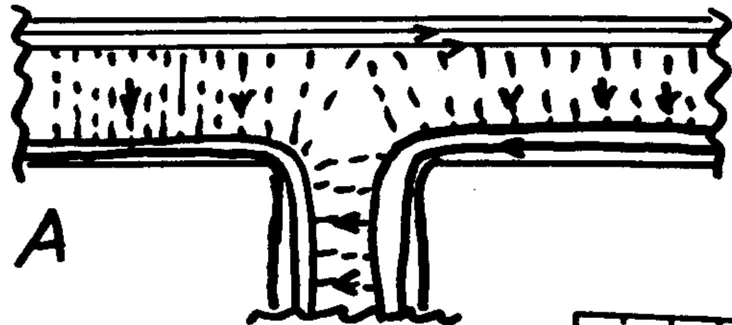
Suppression of Shubnikov-de Haas Resistance Oscillation

van Wees et al. (1989)

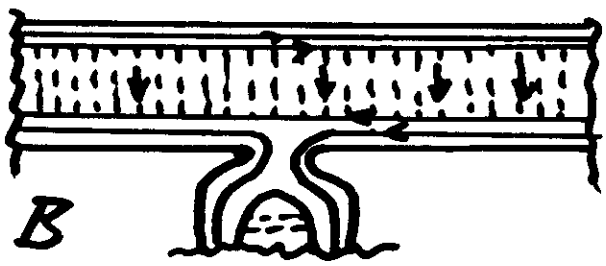


$$R_{12,13} = \frac{h}{e^2}\frac{1}{N}$$

$$R_{12,32} = 0$$



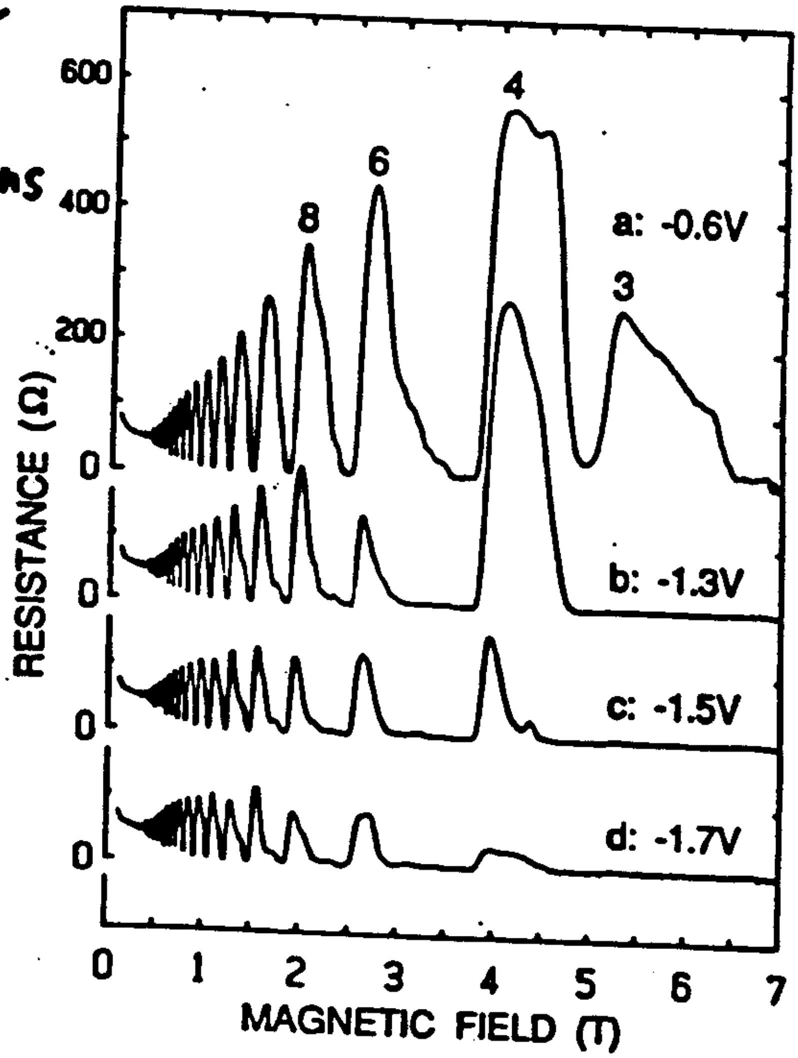
A



B

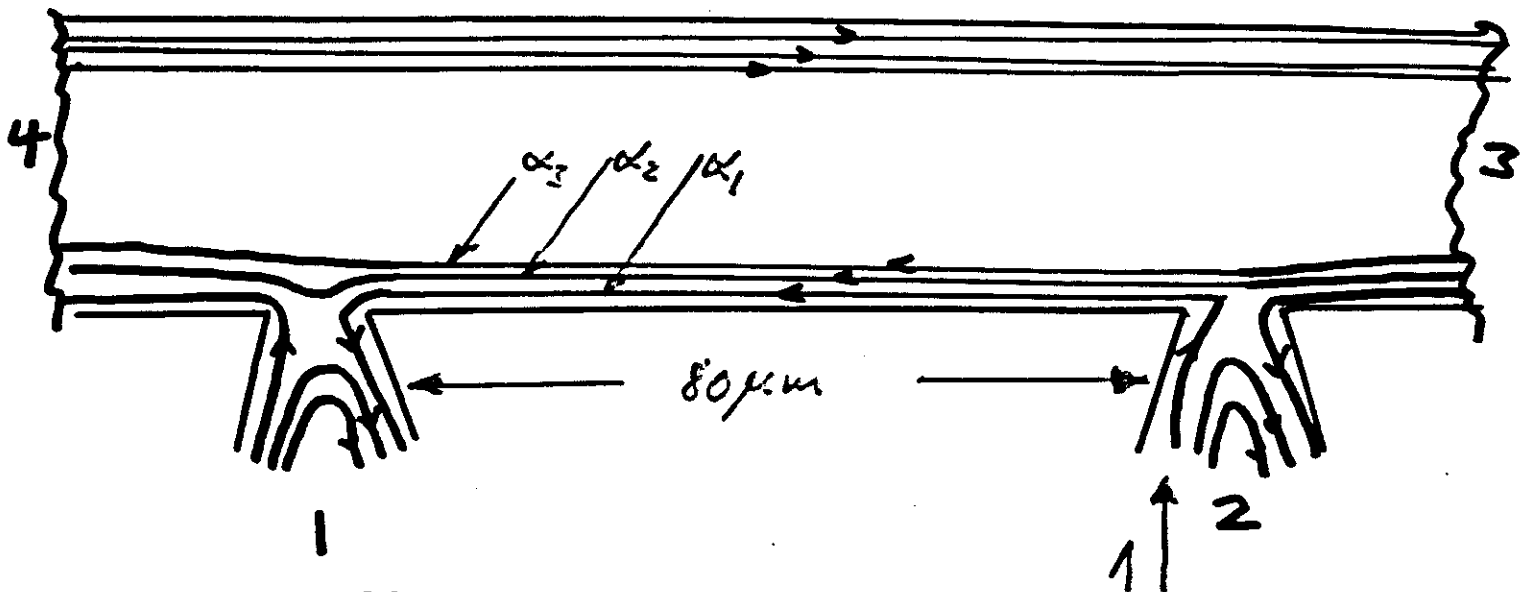
A → SdH oscillations

B → no SdH oscill.



Selective Equilibration of Edge States

Alphenaar et al (90)



$$R_{24,13} = \frac{h}{e^2} \frac{T_{12}}{T_1 T_2} \quad ;$$

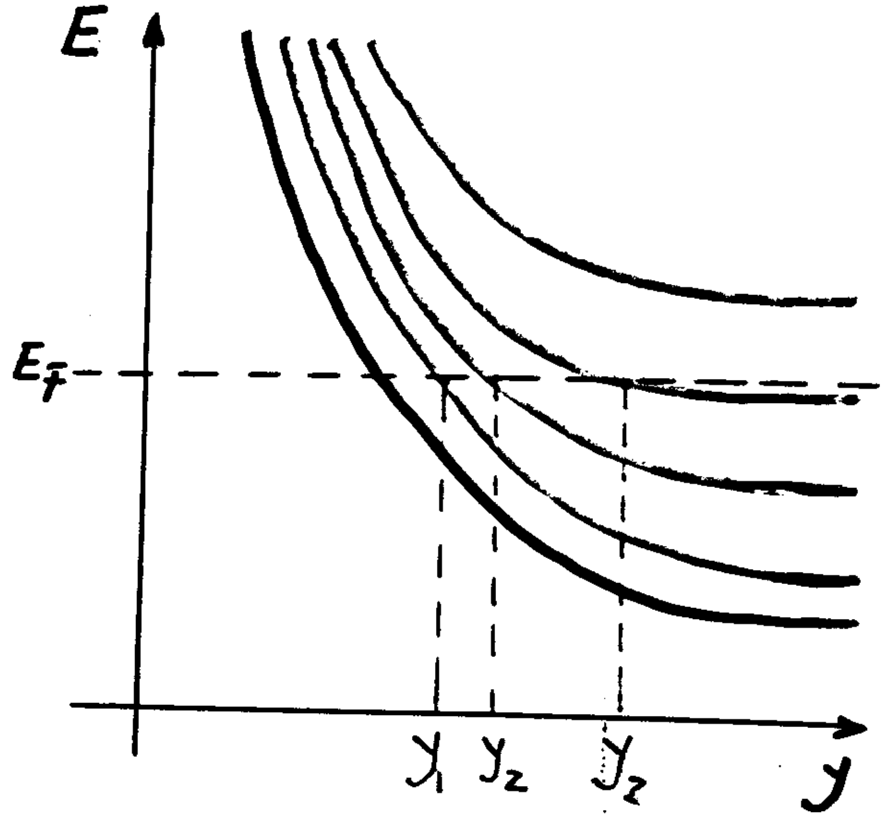
$$T_1 = N_1 \, , \quad T_2 = 1 \, , \quad T_{12} = \sum_{k=1}^{N_1} \alpha_k \, ;$$

$$\boxed{R_{24,13} = \frac{h}{e^2} \frac{1}{N_1} \sum_{k=1}^{N_1} \alpha_k}$$

$d = 80\,\mu m$, $T = 0.45\,K$

$\alpha_1 = 0.48$, $\alpha_2 = 0.44$, $\alpha_3 = 0.08$

Suppression of Elastic Inter-Edge Scattering



$$\delta V(x,y) = \frac{V_0}{\sqrt{\pi}}\frac{1}{S}\; e^{-\frac{1}{2S^2}(x-x_0)^2}\; e^{-\frac{1}{2S^2}(y-y_0)^2}$$

$$\boxed{\frac{1}{\tau_{n,n+1}} = \frac{1}{\tau_0}\, e^{-\frac{1}{2\ell_B^2}(y_n-y_{n+1})^2}\, e^{-\frac{S^2}{2\ell_B^4}(y_n-y_{n+1})^2}}$$

Martin & Feng
Ohtsuki and Ono (89); Glazman and Jonson (89)

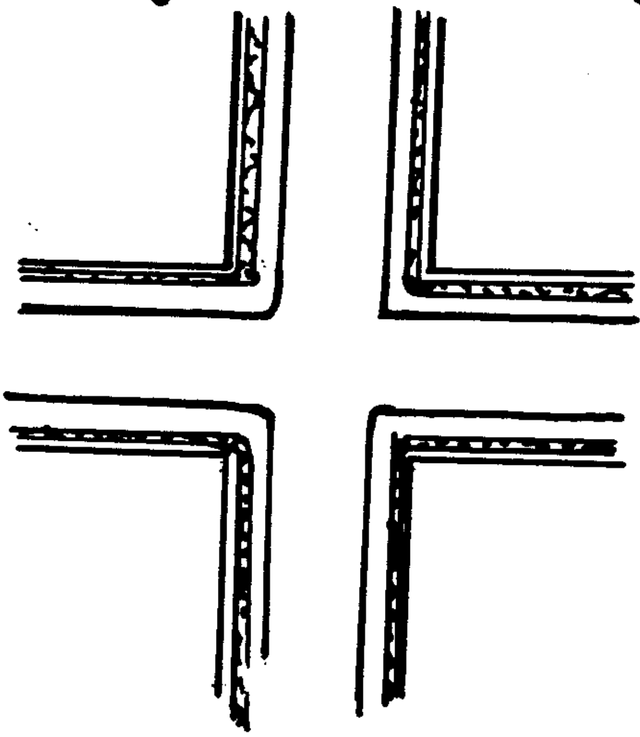
if $\sqrt{S(y_n-y_{n+1})} > 1.7\,\ell_B \Rightarrow \tau_{n,n+1} \sim 10^2\,\tau_0$ or

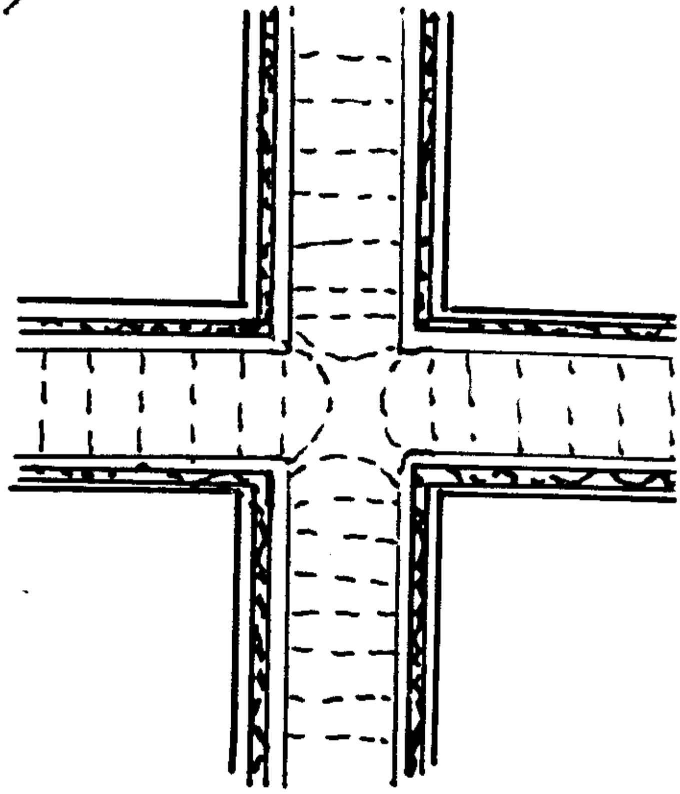
$\ell_{n,n+1} \sim 10^2\,\ell_0$, $\quad \ell_0 \cong 10\,\mu m \Rightarrow \ell_{n,n+1} \sim 0.1\ mm$

Consequences

$R_{xx} \neq \rho_{xx} L/W$

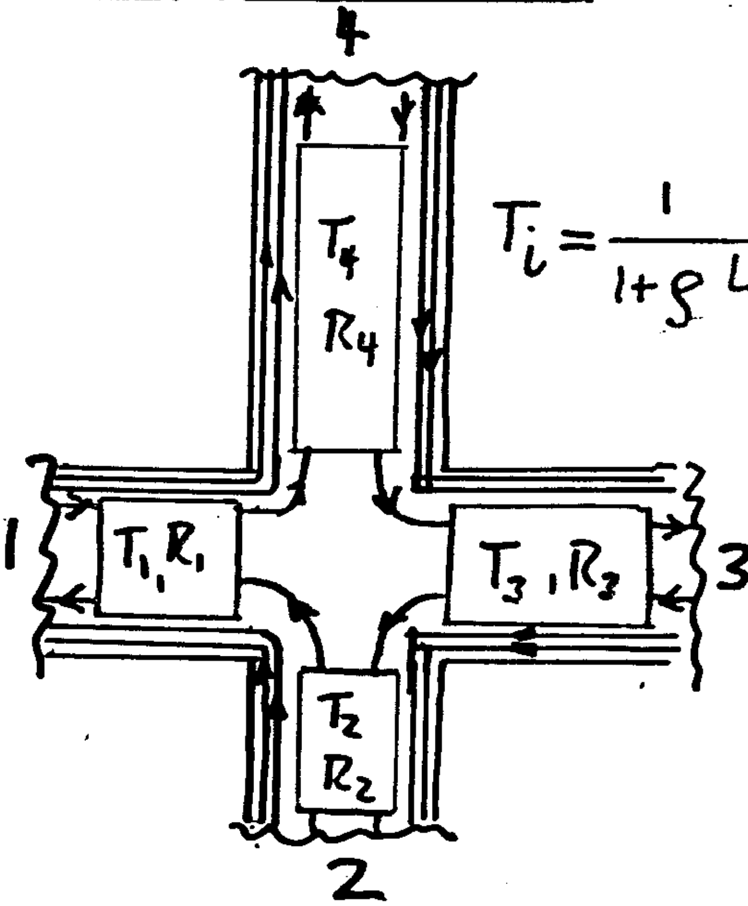
- van Houten et al,
- Haug and von Klitzing, ('89)



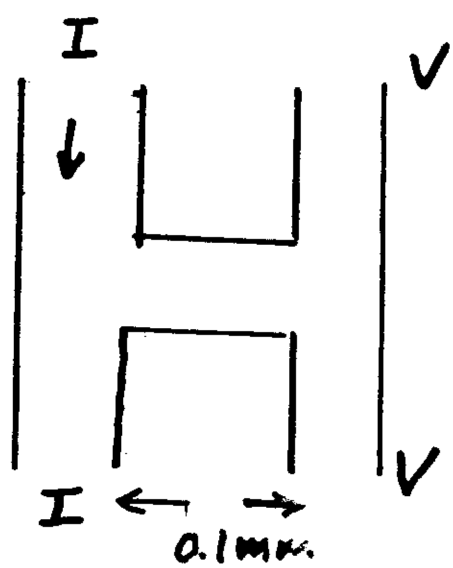


Mc Euen et al.

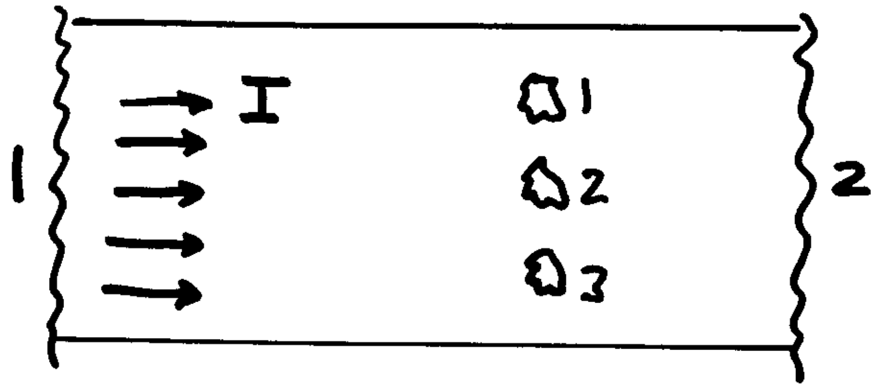
Model of Szafer et al.



$$T_i = \frac{1}{1 + \rho\, L_i/W_i}$$

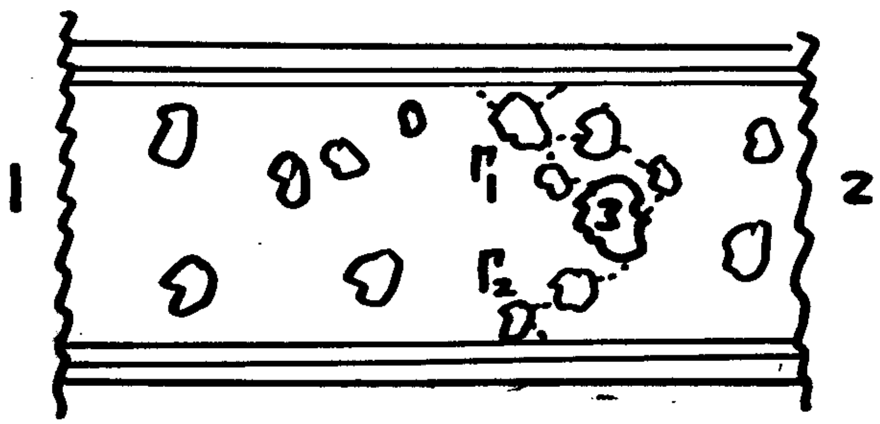


Interior Contacts



$$\Delta I = \sigma_{xy}\,\Delta V \qquad \sigma_{xy} = \frac{e^2}{h} N$$

Why wrong?



$$\mu_3 = \frac{\Gamma_1 \mu_1 + \Gamma_2 \mu_2}{\Gamma_1 + \Gamma_2}$$

$\Gamma_1 = 10^{-7}$, $\Gamma_2 = 10^{-8} \Rightarrow \mu_3 \cong \mu_1$
$\Gamma_2 = 10^{-8}$, $\Gamma_1 = 10^{-7} \Rightarrow \mu_3 \cong \mu_2$

can change potential of an interior contact with changing current pattern appreciably