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SPRING COLLEGE IN CONDENSED MATTER
ON
'PHYSICS OF LOW-DIMENSIONAL SEMICONDUCTOR STRUCTURES'
(23 April - 15 June 1990)

THE LINEAR MAGNETORESISTANCE OF VERY PURE SIMPLE METALS

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The Linear Magnetoresistance of Very Pure Simple Metals

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exponentially rapidly, this being the emission threshold. Even here, however, the real part is relatively smooth. For weak coupling, the [illegible] term in $\mathrm{Re}Z_\nu$ dominates; for stronger coupling, $\mathrm{Re}Z_\nu$ $\mathrm{Im}Z_\nu$ [illegible]. Thus $|Z_\nu|^2$ can be taken to be relatively smooth compared to $N(\epsilon, n, \rho_1^1, \mathcal{T})$.

[1] $Z_\nu(\epsilon)$ here is to be distinguished from $\overline{Z}_\nu(\epsilon)$ in Appendix B. No confusion should arise from this.

[1] J. W. Hodby, J. A. Borders, and F. C. Brown, J. Phys. C 3, 335 (1970).

PHYSICAL REVIEW B VOLUME 3, NUMBER 6 15 MARCH 1971

Magnetoresistance of Very Pure Polycrystalline Aluminum†

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The behavior of the resistance of polycrystalline aluminum wires as a function of magnetic field and purity at temperatures of 4, 15, and 19.6 K is reported. Both longitudinal and transverse configurations were measured. The residual resistance ratios of the specimens varied from 1600 to 31 000. The measured magnetoresistance ($\Delta R/R_0$) is separated into a saturating and a linear part. The value of the saturating component is high at 19.6 K but is shown to be less than 6, even in the limit of infinite specimen purity. The linear component varies with both temperature and purity. Possible sources for the large saturating magnetoresistance values and for the variations observed in the linear portion are discussed. An analysis scheme is presented which allows prediction of the saturating component from zero-field resistance values. A deviation from Matthiessen's rule observed here, and by several other experimenters, is presented and discussed.

I. INTRODUCTION

The magnetoresistance of aluminum has been studied extensively.[1-5] Both single- and polycrystalline specimens have been measured. Most of the measurements were made only at 4 K on specimens of relatively low purity. Frequently, the specimens used were very small in at least one dimension, leading to the possibility of size effects. Several experiments, however, have been performed on large high-purity specimens and at temperatures up to 20 K.[6,7] These measurements indicate that the magnetoresistance ($\Delta R/R_0$) rises dramatically with temperature, reaching as much as four times the value measured at 4 K.

The experiment reported here was designed to cover a range both of temperature (4–20 K), and of specimen purity [residual resistance ratio (RRR) = 1000 – 30 000]. Magnetoresistance measurements were made both in the transverse and longitudinal configurations. We hoped, by this technique, to arrive at a phenomenology which would characterize the magnetoresistance of aluminum, at least in the form of polycrystalline wires, over this range.

It has become almost axiomatic that the more simple metals, in the free-electron sense, exhibit magnetoresistance effects which are at odds with theory.[8] Aluminum,[1-7] indium,[9] potassium,[10] and sodium[10] all show a linear magnetoresistance at high fields. A typical curve for aluminum is shown in Fig. 1. Furthermore, no simple metal which has been investigated over a wide range of purity and temperature has been observed to obey Kohler's rule. This indicates that the relative effects of different scattering mechanisms are more complex than the rule anticipates. More recent theoretical treatments such as those by Young,[11,12] and Pippard,[13] although promising some success in particular cases, have not yet shown wide applicability.

The Fermi surface of aluminum is well known and theoretical calculations of the major features have been adequately confirmed by de Haas-van Alphen and other experiments.[14] In one instance, transverse-magnetoresistance rotation diagrams for several crystal orientations were calculated, based on early models of the surface; however, agreement with available experimental data was not good.[15]

Recently, a good deal of discussion has taken place as to the presence or absence of magnetic breakdown effects which could lead to extended orbits on the Fermi surface.[7,13,14,16] The situation is still not totally clear, but it seems that magnetic breakdown may well occur in aluminum with the field along the ⟨100⟩ direction.

An extended orbit configuration, whatever its cause, would be expected to lead to a significant anisotropy of single-crystal transverse-magnetoresistance rotation diagrams. Early experiments showed no such large anisotropy,[2] whereas more recent work on higher-purity aluminum[7] does show a considerable effect.

Finally, the creation of a significant linear mag-

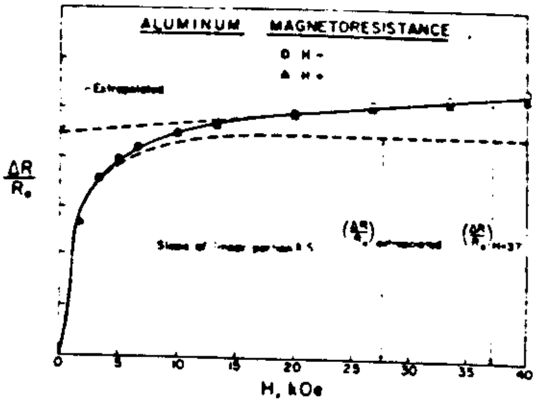

FIG. 1. General behavior of the magnetoresistance and definition of terms.

netoresistance in a polycrystalline specimen by the proper combination of crystallites with quadratic and saturating magnetoresistance does not appear likely, even in metals known to have open orbits.[17]

The remainder of the paper is constructed as follows. Section II describes the specimen preparation and the electrical measurements. A discussion is included on size-effect corrections. The experimental results are presented in Sec. III. Section IV gives a description of the analysis and a discussion of the results. Finally, Sec. V presents our conclusions and some suggestions for further work.

II. TECHNIQUES

A. Magnet, Dewar, and Voltage Detection Systems

The magnet used for these measurements is a conventional superconducting solenoid with a 1-in. bore. The maximum field used was 40 kOe. The field homogeneity is 0.5% over the specimen region.

Voltages are detected with a dc system described in an earlier publication.[18] The system detects signals at the nanovolt level with an imprecision of 50 picovolts. It can operate at levels as high as 500 nV and in the presence of microvolt thermals. Specimen current was chosen [illegible] nal level [illegible] 50 nV, an optimum value for the detection system, in all cases where this was possible. The current ranged from 10 to 500 mA, depending on the specimen measured and the temperature. In all cases measurements of current dependence of the resistance with the magnet off confirmed that magnetoresistive effects of the specimen current were negligible.

An isolation Dewar within the magnet bore contains the cryogenic fluid, hydrogen or helium. This Dewar can be pumped for temperature control. The specimen, enclosed in a vacuum can with a low pressure of helium gas for heat transfer, is immersed in the cryogen and centered in the magnet bore. A system of gear-driven push rods is used to align the specimen with respect to the magnetic field.

B. Specimens

Table I gives a summary of several properties of the specimens. Figure 2 shows the mounting configuration. The tight winding of the transverse specimen on the aluminum form is made necessary by the small magnet bore. The longitudinal specimen is attached with varnish to an aluminum pallet for the measurements.

The slugs for the wires are prepared from the bulk material by compressing to ~0.1-in.-thick plate, sectioning into strips, chemically cleaning, annealing, and finally swaging to the desired diameter. The wires are chemically cleaned and annealed after passing through every other die. The cleaning solution used (70% H_3PO_4, 25% H_2SO_4, 5% HNO_3) will give either an etch (2 min at 60 °C) or a polish (seconds at 100 °C). The final treatment of the specimens varies; some low-purity specimens are etched, some polished. All high-purity specimens are etched. The anneal procedure is the same in all cases, 1 h at 300 °C and a furnace cool. The transverse specimens are given a final anneal after winding.

The longitudinal and transverse specimens of a given purity are not necessarily from the same wire, as is indicated by their designation. In the instances where the two specimens are from the same wire and of high purity (4.0-T, L of the table and several not reported here), the winding of the transverse specimen caused a slight (< 10%) increase in its residual resistance. The cause of this increase appears to be a slight offsetting of the rather large grains during winding. Grain diameter in wires Nos. 3 and 4 tends to be 1 – 2 mm; the less pure wires have significantly smaller grains.

The wire processing described, exclusive of the winding, maintains the purity of the bulk material for all purities except the highest. The purity of the bulk aluminum is measured by an eddy current decay method before processing is begun. The highest-purity aluminum has an eddy current ratio near 35 000. Our dc measurement gives 31 000 in the wire. Errors in the two different size-effect corrections involved are such that the difference may not be meaningful.

Copper current caps are lightly crimped to the wires and filled with a low-melting-point solder. The current leads are then soldered into the caps. Potential leads of No. 36 copper wire are looped around the specimen, twisted and the joint is covered with conducting epoxy. This type of contact has proven to be reliable on repeated cycling from room temperature to 4 K and results in minimum

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Linear magnetoresistance due to sample thickness variations: Applications to aluminum

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(Received 23 October 1984)

A model of transverse magnetoresistance (MR) in metals due to sample-thickness variations is presented. It predicts larger magnetoresistance then do previous models. The model is applied to magnetoresistance data on well annealed, polycrystalline aluminum plates which are wedge shaped, or which contain surface defects such as steps, grooves, or projections. For wedge-shaped samples or samples containing a surface step, the model predicts magnetically induced voltages which differ on opposite sides of the sample, and which are not strictly linear in magnetic field strength B. Both phenomena occur with the predicted magnitudes. For grooves or projections which extend completely across the width of the sample, the model predicts a MR which is linear in B (LMR) and directly proportional to both the groove (projection) depth (height) and the sample width. The data are found to be in quantitative agreement with prediction. The prediction and observation of a very large LMR for large surface defects provides at least a partial resolution of a disagreement in the literature concerning the magnitude of LMR in single-crystal Al samples when $\mathbf{B}$ is directed along the [110] crystallographic axis. Thermal magnetoresistance measurements are shown to be consistent with the electrical measurements. Measurements are also reported on (i) the angular variation of MR when $\mathbf{B}$ is rotated away from the perpendicular to the sample surface; (ii) MR for surface defects which extend only part way across the sample; and (iii) MR for surface defects in unannealed Al plates.

I. INTRODUCTION

Linear magnetoresistance (LMR) in simple metals such as K, Al, and In is a phenomenon which has puzzled physicists for decades. On very general grounds,[1] the magnetoresistance of a homogeneous uncompensated metal with no open orbits in k space should saturate with increasing magnetic field to a constant value ρ_s. There is continuing disagreement over whether the observed LMR is intrinsic or extrinsic. The most ably defended potential intrinsic source is Overhauser's charge-density-wave model (Ref. 2). Proposed extrinsic sources include magnetic-field nonuniformity along the sample length,[3] dislocations in the sample,[4] and the presence of volume defects (voids) in the sample. Macroscopic voids have been shown to produce an LMR proportional to their volume fraction in the sample.[8,9] In carefully prepared samples, however, this volume fraction is too small to produce the observed LMR. Although surface defects were also previously suggested as a source of LMR,[6] extension of the volume-void theory of LMR to surface defects yields an LMR too small to explain published data. Moreover, the volume-void theory of Refs. 5–7 is only applicable to defects which are small with respect to sample dimensions, because the change in Hall field can be neglected in this case. For sample-thickness variations stretching across the whole width of a sample this approximation is no longer valid.

In a previous letter,[10] we showed that surface imperfections of this latter kind produce a transverse LMR which is larger than expected from the void theories. We showed also how these LMR can be understood quantitatively. In this paper, we provide further details of both our data and our model for surface-defect-induced LMR. The paper is organized as follows. In Sec. II, we first outline the volume-defect theory of LMR as background. We apply it to surface defects, and demonstrate its inadequacy. We then describe our new model and derive the equations needed to analyze our data. In Sec. III, we describe our experimental technique and procedures. In Sec. IV, we present and analyze our data. Section V contains a summary of the most important results, and our conclusions.

II. THEORY

We are interested in the transverse magnetoresistance ρ_{xx} of a sample whose thickness varies along the direction of an applied magnetic field $\mathbf{B}=B_z\hat{\mathbf{z}}$. We limit ourselves to consideration of a homogeneous, uncompensated metal with no open orbits. In the high-field limit, $\beta=\omega_c\tau\gg 1$, the transport equations for such a metal relating the electric field $\mathbf{E}$ to the current density $\mathbf{J}$ are

$$\begin{aligned} E_x &= \rho_s(J_x+\beta J_y)\,, \\ E_y &= \rho_s(-\beta J_x+J_y)\,, \\ E_z &= \rho_s J_z\,. \end{aligned} \tag{1}$$

Here $\omega_c=eB/\hbar c$ is the cyclotron frequency and ρ_s is the saturation magnetoresistance of the homogeneous metal. In Eqs. (1), E_i and J_i ($i=x,y$) are functions of x, y, and z. As our measure of the LMR we use the dimensionless Kohler slope S:

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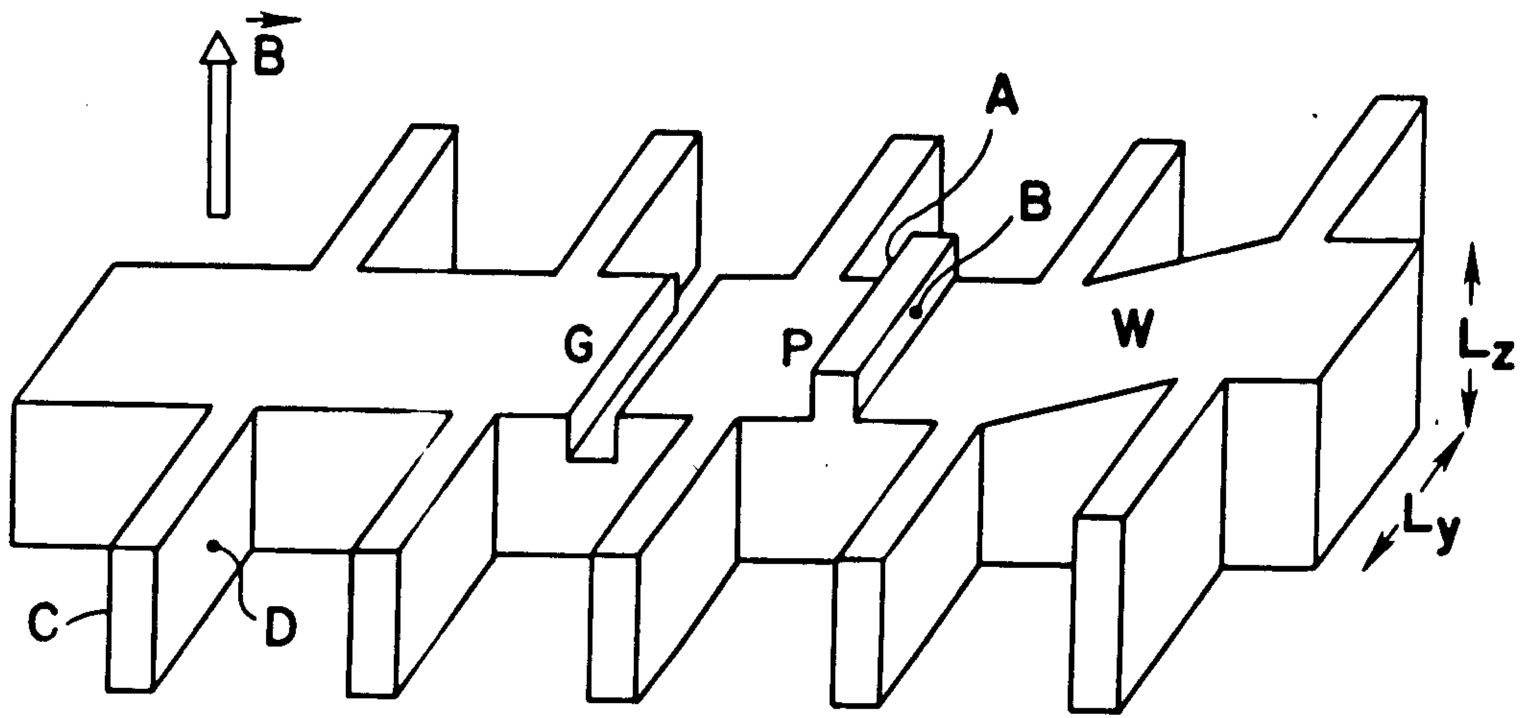
G. J. C. L. Bruls, J. Bass, A. P. van Gelder, H. van Kempen, and P. Wyder, Phys. Rev. $\underline{B32}$, 1927 (1985)



$L = 5\,cm$, $l_c = 0.3\,mm$, $l_{in} \simeq 0.03\,mm$ at $T = 4K$

Fig. 1.

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LETTER TO THE EDITOR

PRB38, 8342 (88)

Current–voltage reciprocity in the magnetoresistance of simple metals

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Abstract. High-field magnetoresistance measurements have been made on unoriented single crystals of potassium and polycrystalline samples of indium and aluminium in fields up to 7.5 T. The measurements show that a reversal of the applied magnetic field may be replaced by an interchange of current and potential leads on the sample, no matter what the position of the contacts and the shape of the sample. Onsager's theory of reciprocal relations in irreversible processes provides a proof of this phenomenon.

1. Introduction

Usually the high-field magnetoresistance is measured using the four-terminal method. The sample under investigation needs four contacts, two as source and sink for the current and two for measuring the voltage difference due to that current. To obtain the real magnetoresistance two reversals are necessary: a reversal of current to eliminate the effects of thermal EMFs and a reversal of magnetic field to eliminate the contribution of Hall fields. Our experiments show that the effect of this last reversal can also be obtained by interchanging the current and potential leads. A preliminary report of our experimental findings was given by Schroeder *et al* (1986). A theoretical proof was first given by Casimir (1945), using Onsager's (1931) theory of irreversible processes. Recently, Büttiker (1986) gave a derivation of the basic Onsager relations, starting from current conservation and time-reversal invariance. The theory indicates that $R_{+H}(1,2/3,4)$ should be equal to $R_{-H}(3,4/1,2)$, if $R_{+H}(1,2/3,4)$ is defined as the resistance ($\propto$ voltage) between contacts 1 and 2 for a [illegible] current flowing from contact 3 to contact 4, the magnetic field being in an arbitrary positive direction.

2. Experiment

We have verified the expected relations by measurements on samples of different shape, sometimes with a very low symmetry. The samples consisted of potassium, indium and

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Magnetoserpentine Effect in Single-Crystal Potassium

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Analysis of four-terminal magnetoresistance data (published recently) from single-crystal cylinders of potassium indicates that open orbits, created by a charge-density-wave structure, are present. Apparent *negative* resistance for some orientations of the magnetic field shows that large charge-density-wave Q domains can cause the current to follow a serpentine path having a reversed direction between the voltage contacts.

PACS numbers: 72.15.Gd, 61.55.Fe

Twenty years ago Garland and Bowers[1] studied the transverse magnetoresistance of single-crystal cylinders of Na and K versus magnet angle. Their work was never published because the results were (then) incomprehensible. Recently Soethout *et al.*[2] have reported high-field data on single crystals of K which deserve attention and explanation. Their research was focused on verifying that equal resistance values are obtained if current and voltage leads are interchanged and **H** is reversed.[3] Confirmation was achieved by measurements on K, In, and Al. The extraordinary data found for K, which I describe and explain below, appeared to be a serendipitous discovery.

The K crystals were 3 cm long and 2 mm in diameter,

i.e., the resistivities at 4.2 K were $\simeq 2\times 10^{-9}\,\Omega$ cm. A magnetic field ($H=7.5$ T) was oriented perpendicular to the cylinder axis, and the angular dependence of the resistance was found for all directions of **H** in the perpendicular plane. Thermal emf's were eliminated by current reversals, and Hall voltages were eliminated by **H** reversals.

The 7.5-T resistances (versus magnet angle) of the two crystals are shown in Figs. 1 and 2. The horizontal dashed lines (for $H=0$) are the anticipated high-field values for a simple metal having a (simply connected) spherical Fermi surface. Such simplicity is clearly not attributable to K. The large (30–40-fold) resistance enhancement and the peaked structure can only arise

gle. I have calculated the voltage across the contacts B, C as a function of R/r. The resulting behavior of the apparent resistance is shown in Fig. 4. The voltage changes sign near $R/r \simeq 50$. For larger values the current follows a serpentine path from A to C, back to B, and then to D.

The foregoing calculation demonstrates that the "resistance" behavior shown in Fig. 1 can result from a propitious (but possibly rare) Q-domain structure of a single crystal. Open-orbit resistivity peaks have maxima proportional to $(\omega_c\tau)^2$, where ω_c is the cyclotron frequency and τ the electron scattering time. Since $\omega_c\tau \simeq 200$ at $H=7.5$ T in pure K, the resistivity ratio needed for the serpentine effect is easily achieved.

Juxtaposition of the data of Figs. 1 and 2 proves that two identical crystals of K can exhibit vastly disparate properties. For metals having a CDW broken symmetry,

KOH crust. Differential thermal stress can then cause nucleation and growth of a new Q-domain pattern for each thermal cycle. Consequently, inadvertent preferred orientations can occur.[15] This is particularly relevant in

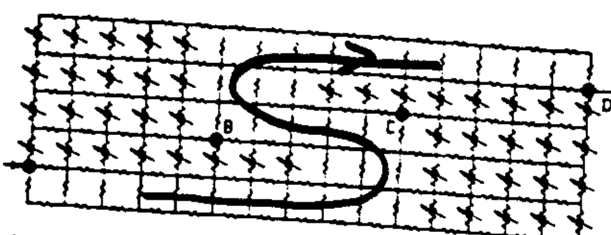

FIG. 3. Resistor network modeling a K single crystal having ree large Q domains. The two domains having the variable sistors (ganged together) allow imitation of an open-orbit sistivity peak. The other resistors have fixed resistance r.

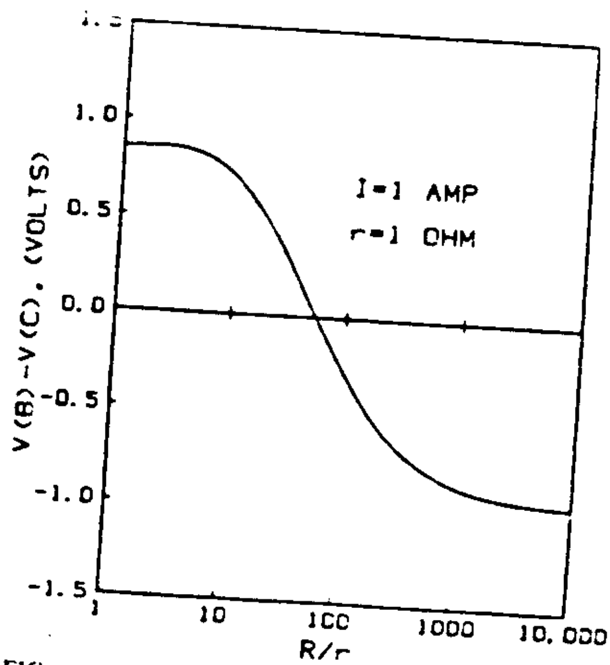

FIG. 4. Voltage across the contacts B, C of Fig. 3 vs R/r, the ratio of resistance values used in the network of Fig. 3. The curve shown results from an exact calculation.

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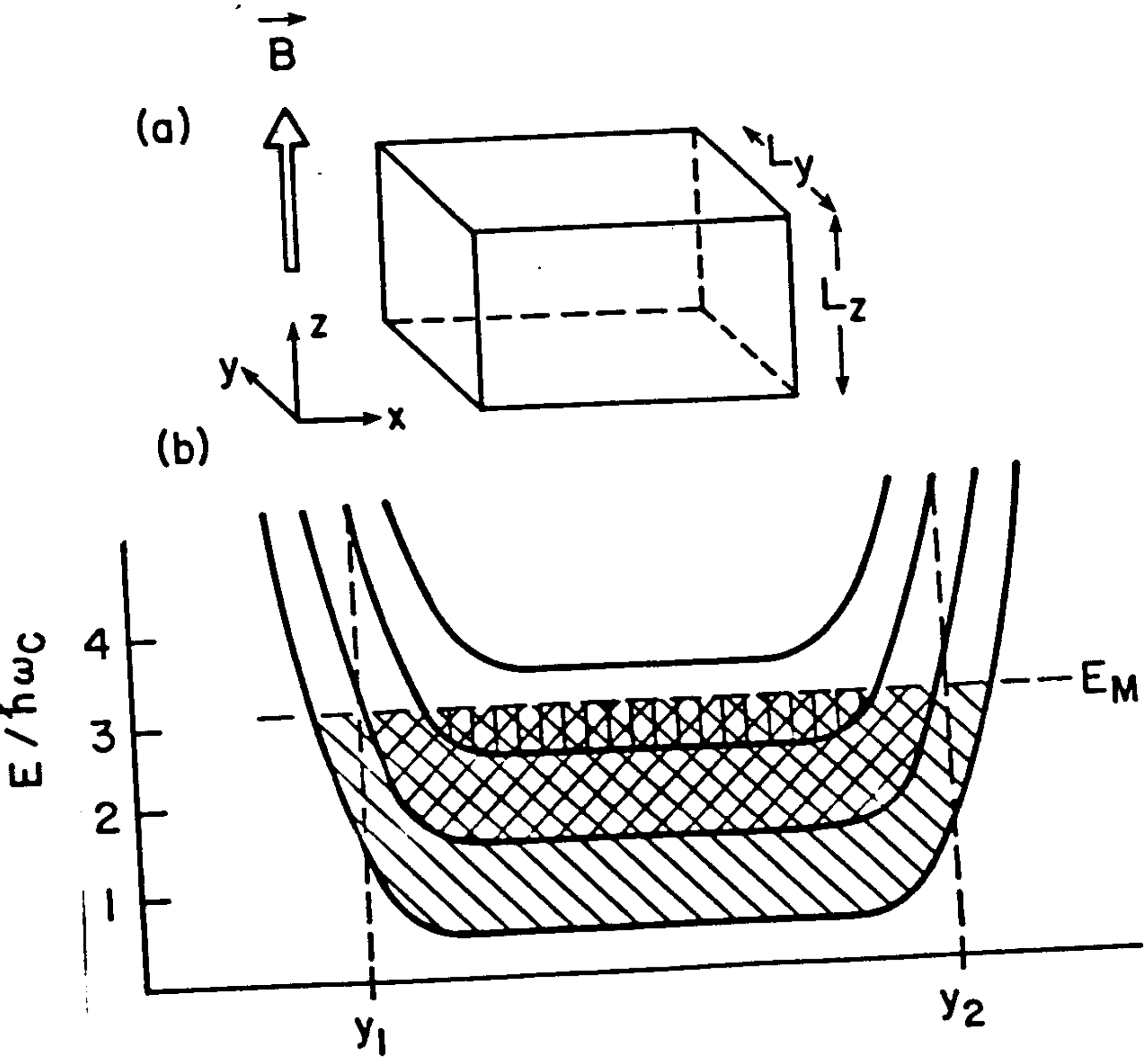
Spectrum of Perfect Conductor

A. Monolayer of thickness λ_F

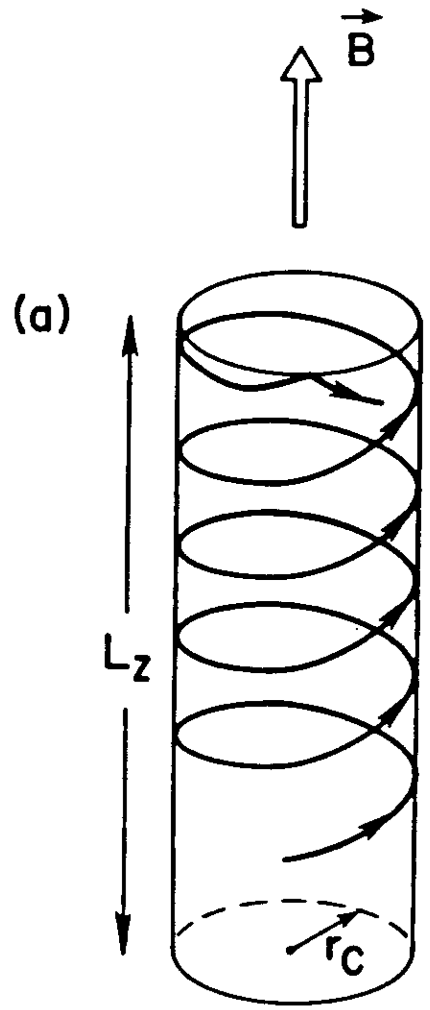
$E_\ell(y) \leftrightarrow$ 2DEG

B. Metal of thickness L_z

$$E_{\ell n}(y) = E_\ell(y) + \hbar^2\pi^2 n^2/2m L_z^2$$



Helical Orbits



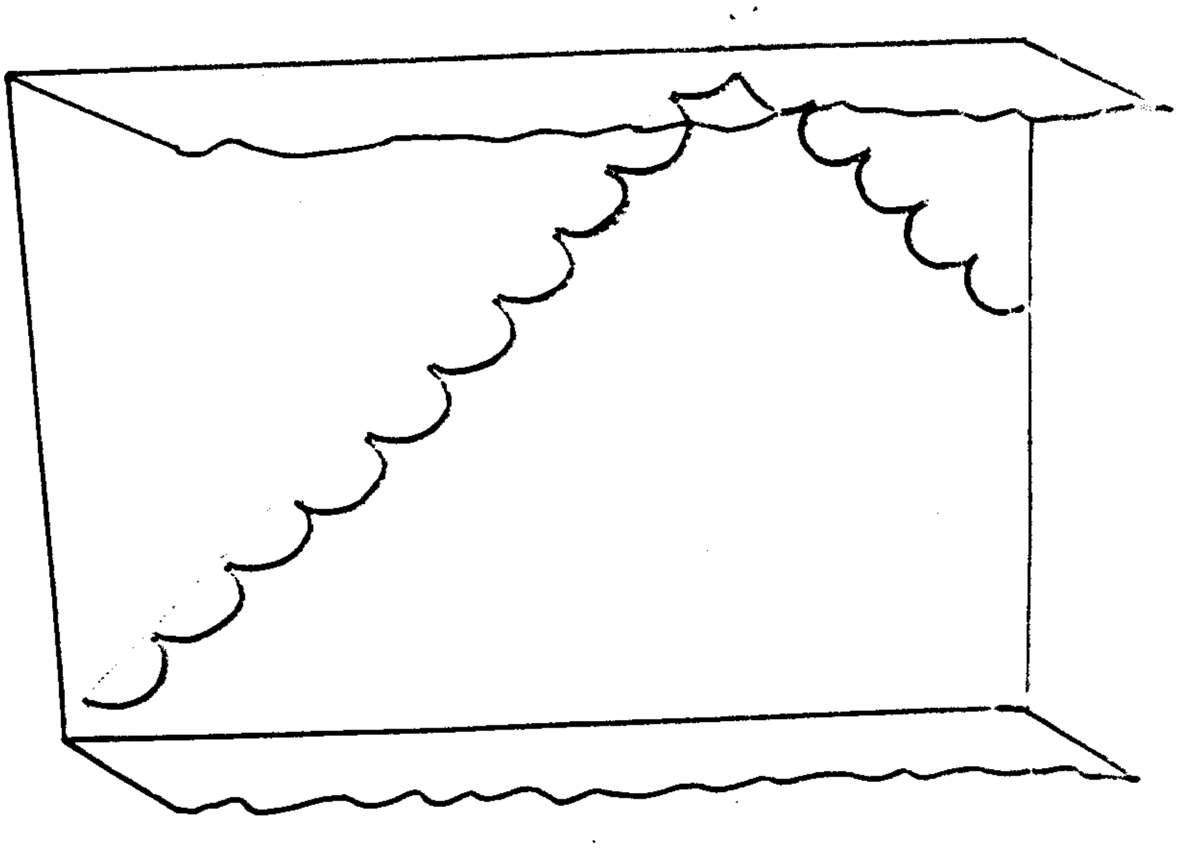
$r_C = \hbar k_\ell / m\omega_C$

$V_{z,n} = \hbar n \pi / m L_z$

$$V_H = 2\pi \ell_B^2 \,(\ell' + 1/2)\, L_z \approx 2\pi \ell_B^2 E_F / \hbar\omega_c$$

$$n_i V_H \approx 1 \implies B_c = 860 \text{ Gauss} \qquad (E_F = 5.6\,\text{eV},\ \ell_c \approx 0.3\,\text{mm},\ L_z = 1\,\text{mm})$$

Skipping Orbits



Current of Perfect Piece

$I = (e/h)\, N\, (\mu_1 - \mu_2)$; N levels with $E_{\ell n} < E_F$.

$E_{\ell n}(y) = E_\ell(y) + E_z$; $E_z = \hbar^2 \pi^2 n^2 / 2 m L_z^2$ $\rightarrow$

$$\frac{dn}{dE_z} = (m L_z^2 / 2\pi^2 \hbar^2)^{1/2} E_z^{-1/2} \rightarrow$$

$$\boxed{\begin{aligned} N_\ell &= (2 m L_z^2 / \hbar^2)^{1/2} \left(E_F - \hbar\omega_c(\ell + 1/2)\right)^{1/2} \\ N &= \sum_{\ell=0}^{\ell=\ell'} N_\ell \end{aligned}}$$

$$\frac{2}{3}\left(E_F / \hbar\omega_c\right)^{3/2} = \left(\frac{1}{\hbar\omega_c}\right)^{1/2} \sum_\ell \left(E_F - \hbar\omega_c(\ell + 1/2)\right)^{1/2}$$

$\Rightarrow$

$$N = \frac{2}{3} (2 m L_z^2 / \hbar^2)^{1/2} E_F^{1/2} (E_F / \hbar\omega_c)$$

$$n = (1/6\pi^2)(2 m E_F / \hbar^2)^{3/2} ; \quad \ell_B = (\hbar c / eB)^{1/2}$$

$\Rightarrow$

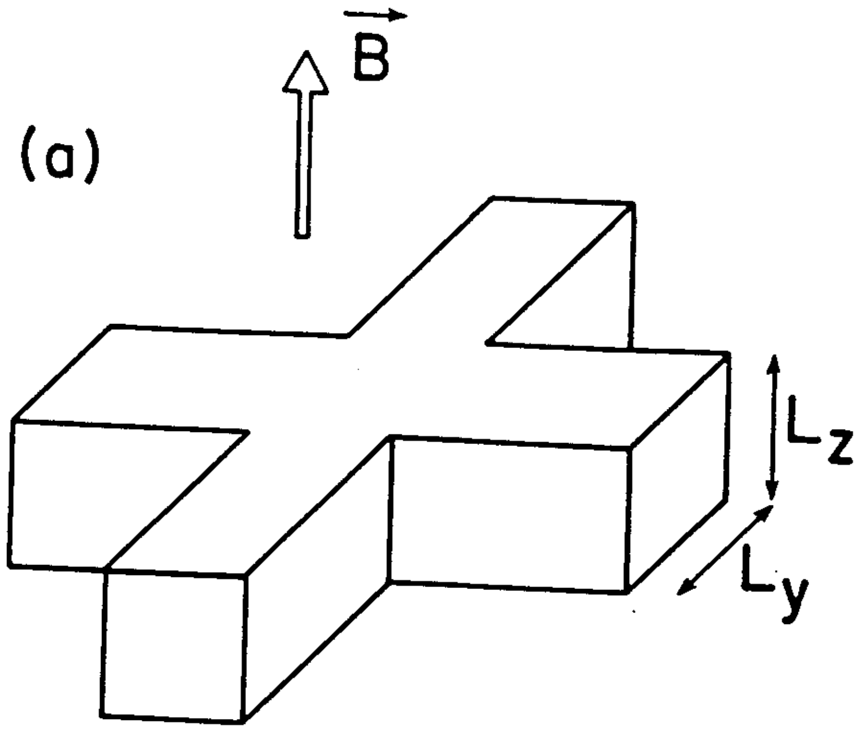
$$\boxed{N = 2\pi \ell_B^2 L_z n}$$

$L_z n$ two dimensional sheet density

$\frac{1}{2\pi \ell_B^2}$ density which can be accomodated in one Landau level

$B = 1$ Tesla , $n = 6 \cdot 10^{22}\,cm^{-3}$, $L_z = 1mm$, $\Rightarrow N = 4.1 \times 10^{8}$

Hall Resistance



$$I_i = (e/h)\,[(N_i - R_{ii})\mu_i - \sum T_{ij}\mu_j]$$

$$R_H = \frac{h}{e^2}\frac{1}{N} \qquad N = 2\pi l_B^2 L_z n \propto \frac{1}{B}$$

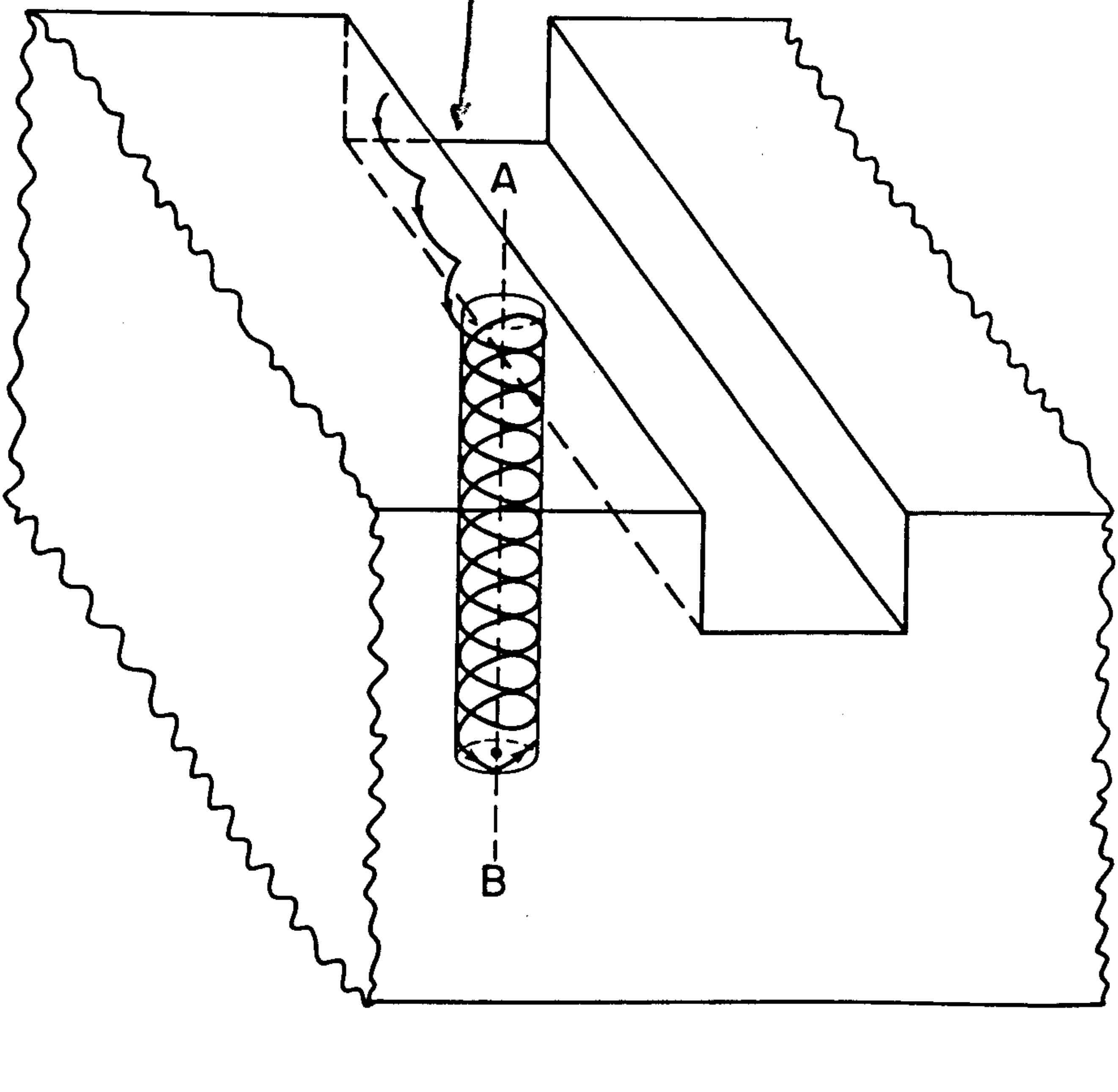
Scattering at a Groove

$T = N_2$

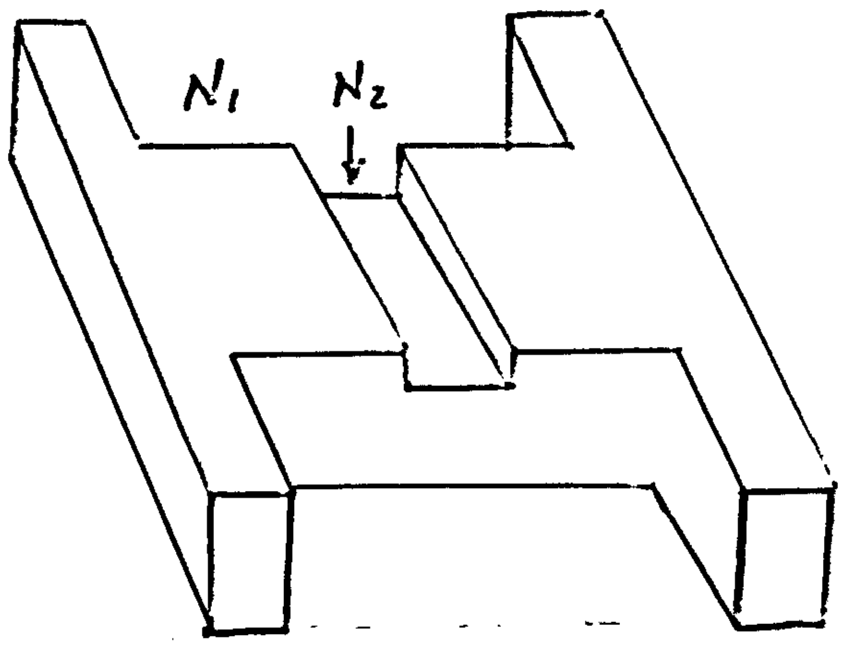
$R = N_1 - N_2$

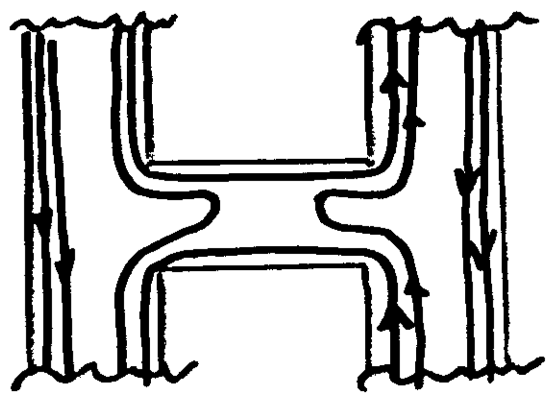
$N_1 = N(L_1)$ $\quad$ $N_2 = N(L_2)$

(b)



Linear Magnetoresistance due to Groove



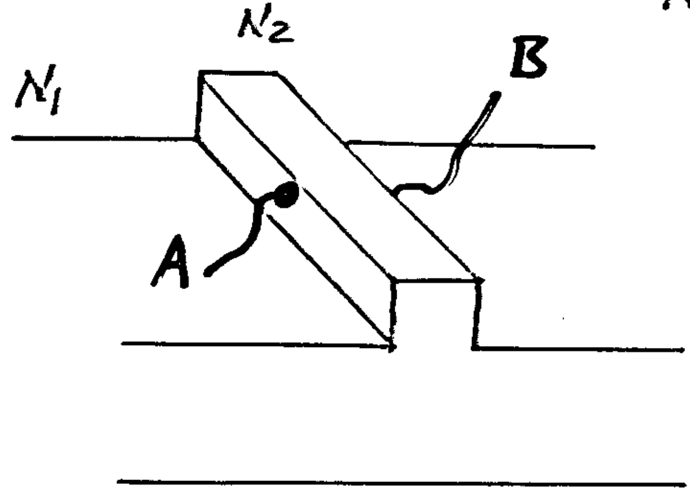


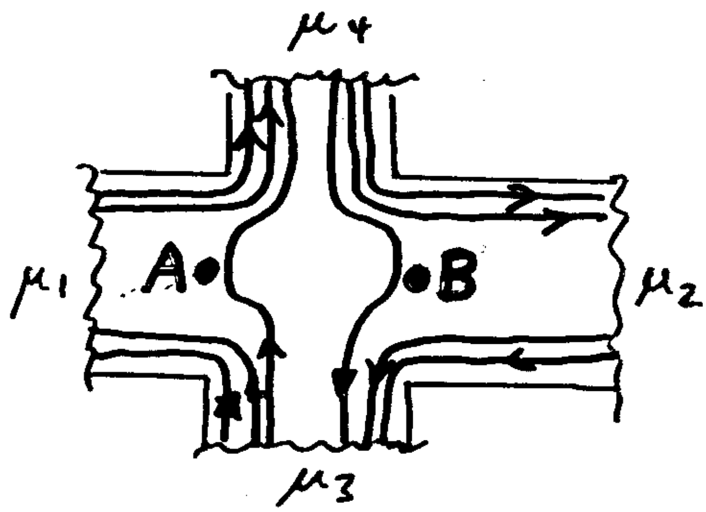
$$R_L = \frac{h}{e^2}\left(\frac{1}{N_2} - \frac{1}{N_1}\right) = \frac{h}{e^2}\,\frac{N_1 - N_2}{N_1 N_2} = \frac{h}{e^2}\,\frac{L_1 - L_2}{N_1 L_2}$$

$$\boxed{\frac{R_L}{R_H} = \frac{L_1 - L_2}{L_2}} \Longrightarrow \boxed{R_L \propto R_H \propto B}$$

Scattering at Protrusion

$N_2(L_2) > N_1(L_1)$





$T = N_1 N_2 / (2N_2 - N_1)$; $\quad R = N_1 (N_2 - N_1)/(2N_2 - N_1)$

$\Longrightarrow$

$$\boxed{\mathcal{R}_L / \mathcal{R}_H = (L_2 - L_1)/L_2}$$

$$\boxed{\mu_A - \mu_B = \mu_4 - \mu_3 = -\frac{L_1}{2L_2 - L_1}(\mu_1 - \mu_2) < 0}$$

The easiest way to simulate a large Δ is merely to rotate a sample by 90° so that the magnetic field points along the potential arms rather than perpendicular to the plane of the arms. This is not quite the same as having long projections or deep grooves in the proper geometry, since now the voltage is being measured on arms which point along the field direction. However, as we will now see, the results are similar to what is expected for the proper geometry.

Figure 11 shows the magnetoresistivity for a sample containing no deliberately introduced surface defects, with the field oriented both in and perpendicular to the plane of the arms. With the field perpendicular to the plane of the arms we see only the small LMR appropriate to a flat sample. With the field along the arms, however, we see a much larger LMR, with a value of S about half of L_y/L, the upper bound indicated above. The magnitude of this LMR is about the same on the two sides of the sample and also approximately symmetric under field and current reversal.

To see whether or not the behavior shown in Fig. 11 was associated solely with electrical magnetoresistivity, the thermal magnetoresistivity $\rho_{th}(B)$ was measured on a sample of the same shape,[19] prepared from the same Al plate as the one used for Fig. 11. Data for parallel and

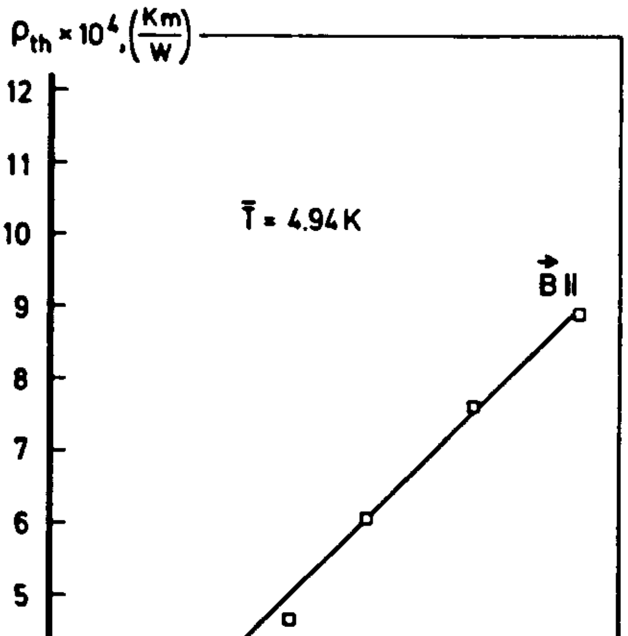


FIG. 12. Thermal resistivity ρ_{th} under conditions similar to those in Fig. 11.

perpendicular fields are shown in Fig. 12. For parallel fields, where the effect is large, it is possible to calculate the Wiedemann-Franz ratio, $(1/T)[\rho_{el}(B)/\rho_{th}(B)]$, using the two linear slopes. Its value of $(2.2\pm0.2)\times10^{-8}$ $(V/K)^2$ is in good agreement with the Lorenz number 2.44×10^{-8} $(V/K)^2$.

We take this consistency check, plus the fact that S for the electrical LMR is comparable to the upper bound of Eq. (25), as additional evidence for the basic validity of our model. We can also use this geometry, and these results, to study further the fundamental physics underlying surface LMR.

It is easily demonstrated that the average electric field throughout the entire sample should be independent of the magnitude of the magnetic field B, provided that ρ_s is itself independent of B. We begin with the first of Eqs. (1),

$$E_x=\rho_s(J_x+\beta J_y)\ . \tag{1}$$

If we average this equation over the entire sample volume, we find

$$\langle E_x\rangle=\rho_s\langle J_x\rangle\ , \tag{26}$$

since $\langle J_y\rangle=0$. Because we maintain $\langle J_x\rangle$ constant as we increase the magnetic field, Eq. (26) predicts that $\langle E_x\rangle$ will be independent of magnetic-field strength B. This result is not only valid for $\mathbf{B}$ perpendicular to the plane of the sample arms (where indeed we observe a nearly constant ρ_s with $S\simeq10^{-3}$ in well-annealed samples with no deliberately introduced surface defects), but also for $\mathbf{B}$ parallel to the arms (where we observe a huge LMR with $S>10^{-1}$). From Eq. (26) we must infer for $\mathbf{B}$ parallel to the arms that the large electric fields giving rise to the huge LMR observed in the body of the sample must be balanced off by large oppositely directed fields in the arms.

To search for such a field, we attached potential leads across one arm of the sample as indicated in the inset of Fig. 13, and used these leads to measure the potential differences across this arm as a function of the magnitude of B. For B directed perpendicular to the plane of the

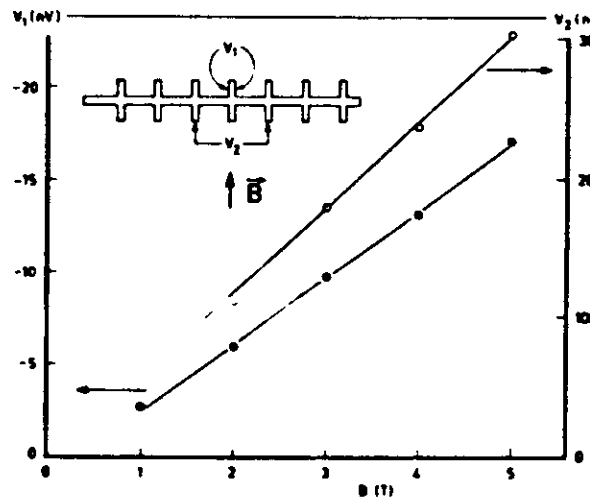


FIG. 13. Voltage V_1 (across a sample arm) and V_2 [between two different sample arms (see inset)] as functions of magnetic field strength B. Note that the data for V_1 and V_2 have opposite signs.

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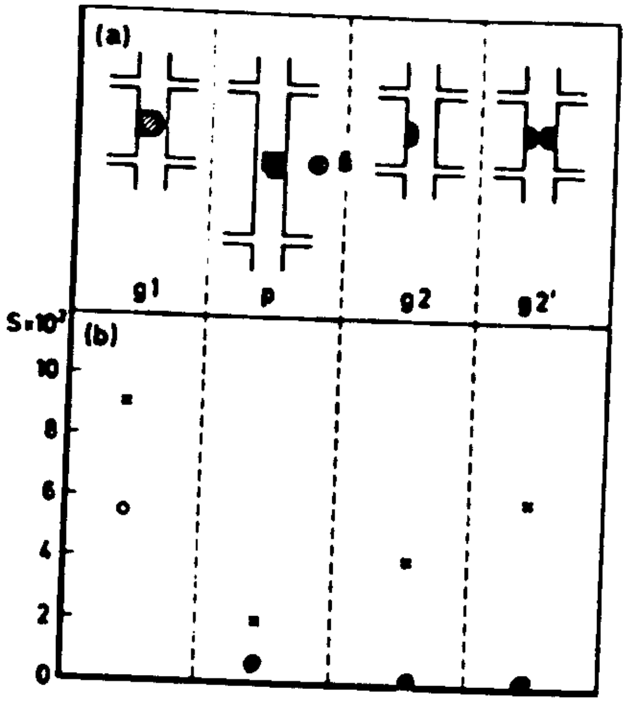

Fig. 16. (a) Shapes and relative sizes of partial grooves (g) and projections (p).

(b) Kohler slopes S for the partial defects (open circles) compared with expectation for full defects (crosses) having the same Δ with magnetic field $\vec{B}$ perpendicular to the plane of the arms.

upon annealing, but sometimes decreased. Such divergent behaviour is not what would be expected for simple additivity. When "net" LMRs were calculated by subtracting out the LMRs for flat pieces on the same annealed or unannealed sample, then these "net" LMRs almost always increased upon annealing. The various behaviours just described are illustrated in Fig. 17. We currently have no explanation for the increase in "net" LMR generally observed upon sample annealing.