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SPRING COLLEGE IN CONDENSED MATTER
ON
'PHYSICS OF LOW-DIMENSIONAL SEMICONDUCTOR STRUCTURES'
(23 April - 15 June 1990)

MAGNETIC AND TRANSPORT PROPERTIES OF
SEMICONDUCTING La_2CuO_4

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These are preliminary lecture notes, intended only for distribution to participants.

Hopping magnetoresistance of quasi-2D semiconducting $\text{La}_2\text{CuO}_{4+\delta}$.

1. $\text{La}_2\text{CuO}_{4+\delta}$ - general experiments - magnetism and transport.
2. Magnetoresistance - experiment
3. Magnetoresistance - simplified model - theoretical illustration
4. Extended percolation problem
5. Magnetoresistance - realistic model for La_2CuO_4

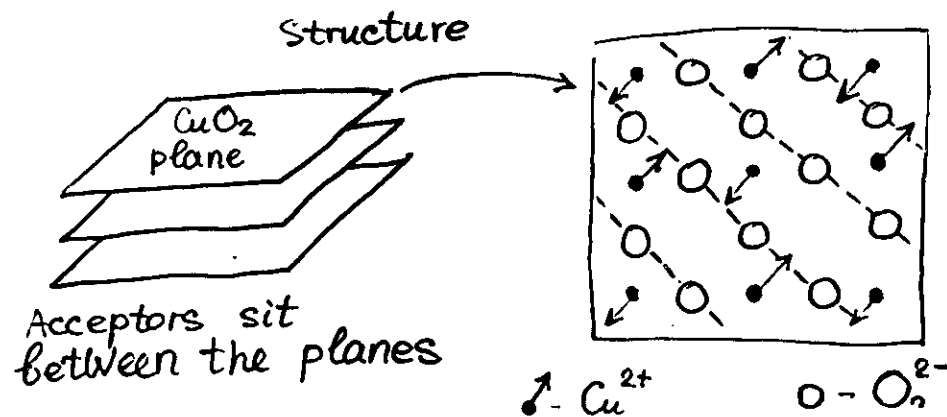
6. Conclusions

Experiment	Theory	
MIT Group	LANL Group	Alexandre O. Gogolin
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et al		

$\text{La}_2\text{CuO}_{4+\delta}$ - single crystals

①

1. For $\delta \leq 0.015$ $\text{La}_2\text{CuO}_{4+\delta}$ homogeneously doped p-type semiconductor ($p \approx \delta/2$, acceptors O_2^{2-} ?)
2. Quasi-2D antiferromagnet, $T_N(\delta=0) \approx 300$, T_N rapidly decreases with δ .



3. Anisotropic transport (Quasi-2D)
At low T - VR Hopping (3D-cluster)

Mott law

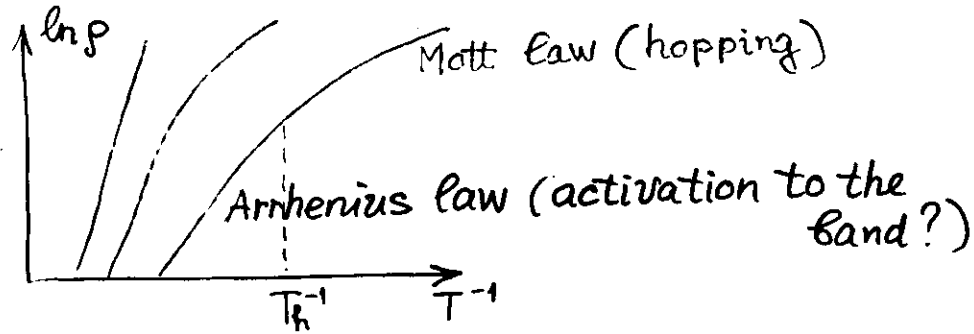
$$\rho \approx \rho_0 (T/T_0)^{1/2} \exp \{ (T_0/T)^{1/4} \}$$

$$T_0 \sim 10^6 \text{ K} \quad T_0 \approx 20/g(\epsilon_F) a_{||}^2 a_{\perp}$$

$$g \sim 10^{20} \text{ cm}^{-3} \text{ eV}^{-1}, \quad a \sim 10 \text{ \AA}$$

$$\rho_{10}/\rho_{110} \sim 10 \text{ (moderate anisotropy)}$$

For good crystals:



$$\rho \propto \exp(T_0/T)^{1/4} \quad T > T_h$$

$$\rho \propto \exp(E/T) \quad T > T_h$$

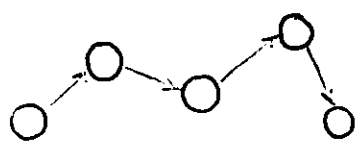
For Sr or Li doped samples $T_h > 300 \text{ K}$
 For the best crystal ($\delta \approx 0$) $T_h < 4 \text{ K}$, $E \sim 70 \text{ meV}$
 For typical $\delta \sim 0.01$ $T_h \sim 50 \text{ K}$, $E \sim 40 \text{ meV}$

Anisotropy of E : $E_{||} > E_{\perp}$

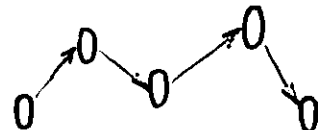
Strong anisotropy of ρ ($\rho_{\perp}/\rho_{||} \sim 500$)

Magnetoresistance in the hopping regime

Usual picture



$H=0$



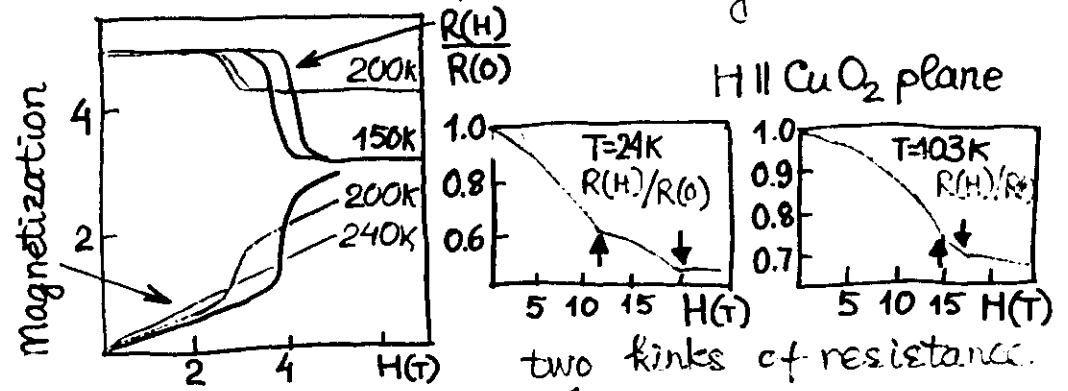
$H \neq 0$

wave functions shrink

Giant positive magnetoresistance $\rho \uparrow$

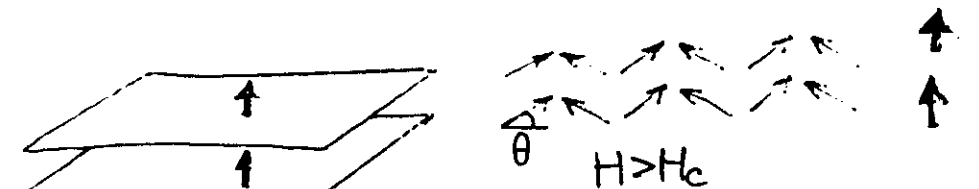
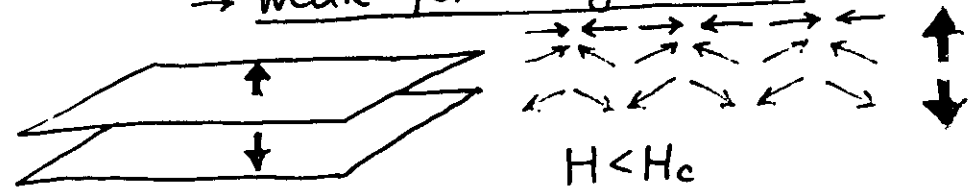
$$\rho \propto [1/(H/H_0)^2]$$

In $\text{La}_2\text{CuO}_{4+\delta}$: Negative magnetoresistance



Are associated with magnetic phase transitions

Small orthorhombic distortions ($\phi \sim 10^{-2}$) $\rightarrow$
 $\rightarrow$ Dzyaloshinsky-Moriya interaction $\rightarrow$
 $\rightarrow$ small canting of spins ($\theta \approx 3 \cdot 10^{-3}$) $\rightarrow$
 $\rightarrow$ weak ferromagnetism



The main Idea

It is not the external field $\vec{H}$ itself that affects R , but the local molecular fields $\vec{B}_i$.

The role of $\vec{H}$ is only the reorientation of $\vec{B}_i$.

Illustrative model

1. Ising antiferromagnet ($\sigma_i = \pm 1$)
2. Some fraction of sites are donors with energy levels

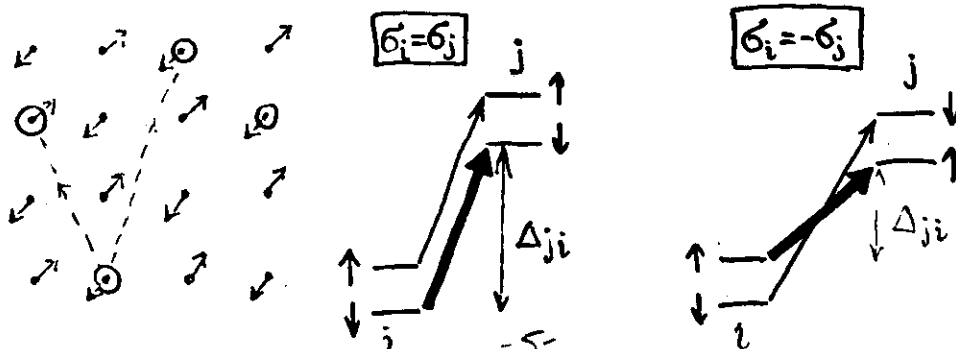
$$E_i = \epsilon_i + B_i \sigma_i$$

$\sigma = \pm 1$ - spin of electron
 $B_i = B \sigma_i$ - local molecular field

ϵ_i - is random

3. σ fluctuates faster than electron hops to another site

4. Hops occur without spin reversals.



(4)

Average hopping rate

(5)

$$\Gamma_{ij} = \exp(-2r_{ij}/a) f_F(\epsilon_i) [1 - f_F(\epsilon_j)] \cdot \sum_{\sigma=\pm 1} F(\sigma) w(\Delta_{ij}) \begin{cases} N_{ph}(\Delta_{ij}) & (\Delta_{ij} > 0) \\ 1 + N_{ph}(\Delta_{ij}) & (\Delta_{ij} < 0) \end{cases}$$

$$F(\sigma) = \exp(-\sigma \delta B/T) / 2 \cosh(B/T)$$

$$\Delta_{ij} = (\epsilon_j - \epsilon_i) + B \sigma (\sigma_j - \sigma_i)$$

$$w(\Delta) = |\text{matrix element}|^2$$

since $\Delta_{ij}^{\uparrow\downarrow} < \Delta_{ij}^{\uparrow\uparrow} \Rightarrow \Gamma_{ij}^{\uparrow\downarrow} < \Gamma_{ij}^{\uparrow\uparrow}$

Intermediate conclusions

1. Hopping rate depends on whether B_i and B_j (Molecular fields on the initial and final state) are parallel or antiparallel
2. In AF state $B_i = B_j$ for one half of the hops and $B_i = -B_j$ for another half
3. In external field $H > H_{sf}$ spin flip transition occurs and $B_i = B_j$ for all hops.
4. So resistivity ρ drops abruptly at $H = H_{sf}$

How to calculate resistivity in practice?

$$\Gamma_{ij}^{\uparrow\uparrow} = \Gamma_{ij}^{\downarrow\downarrow} \propto w(|\epsilon_j - \epsilon_i|)$$

$$\Gamma_{ij}^{\uparrow\downarrow} = \Gamma_{ij}^{\downarrow\uparrow} \propto w(|\epsilon_j - \epsilon_i| - 2B)$$

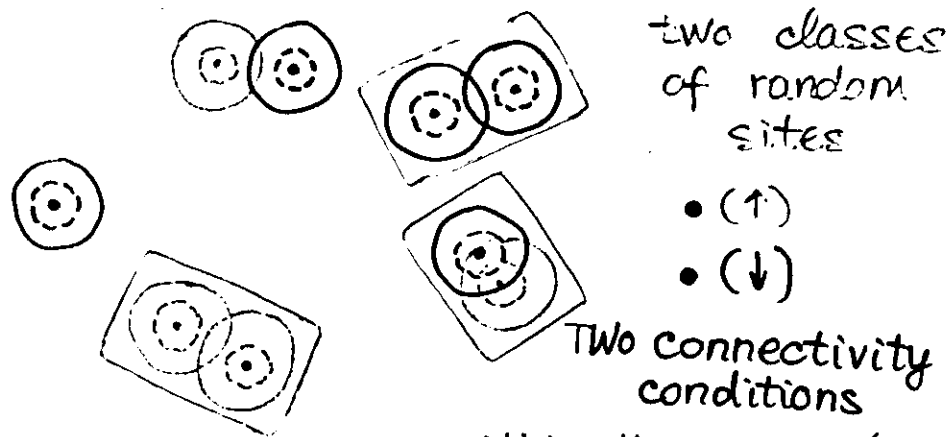
Suppose $w(\Delta) \approx \exp(-\alpha|\Delta|)$ $\alpha > 0$
(For polaronic hops $\alpha = 1/2T$)

Then

$$\Gamma_{ij}^{\uparrow\downarrow} = \Gamma_{ij}^{\uparrow\uparrow} \exp(-\Delta\xi) ; \quad \Delta\xi = 2B\alpha$$

$$\Gamma_{ij}^{\uparrow\uparrow} \approx \exp(-\xi_{ij}(r_{ij}, \epsilon_i, \epsilon_j))$$

Extended percolation problem



within the same class:

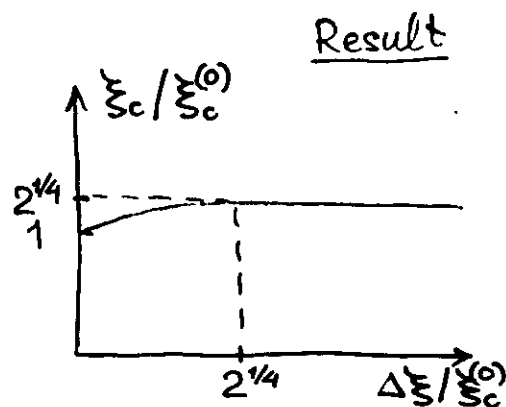
$$\xi_{ij} > \xi_{ij} < \xi_c$$

interclass connectivity

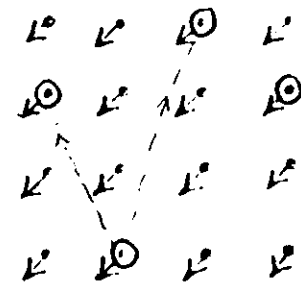
$$\xi_{ij}, \xi_{ij} < \xi_c + \Delta\xi$$

ξ_c corresponds to the appearance of infinite cluster of connected sites

⑥



$\xi_c^{(0)}$ - in ferromagnetic case
(conventional percolation problem with single class)



$$1. \text{ For } \Delta\xi > 2^{1/4} \xi_c^{(0)}: \quad \xi_c = 2^{1/4} \xi_c^{(0)}$$

(Independent percolation via each class. Interclass hops are suppressed.)

$$2. \text{ For } \Delta\xi \ll \xi_c^{(0)}: \quad \xi_c \approx \xi_c^{(0)} + \Delta\xi/2$$

$$P(H < H_{sf}) / P(H > H_{sf}) = e^{\Delta\xi/2} > 1$$

Realistic model for La_2CuO_4

1. $\vec{B}_i$ must feel WF transition

We suggest $B_i \propto [\vec{n}_i \times \vec{\omega}_i]$
vector $\vec{n}$ - antiferromagnetic order parameter

Dzyaloshinsky vector $\vec{\omega}_i \propto$ vector of orthorhombic rotation

In AF phase Then

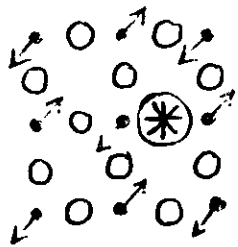
in WF phase

$$\begin{array}{l|l} B_i = B_j \text{ for } ij \text{ in the same plane} & B_i = B_j \text{ for all } ij \\ B_i = -B_j \text{ for } ij \text{ in the adjacent} & \end{array}$$

What is needed for $\vec{B}_i = [\vec{n}_i \times \vec{\omega}_i]$?

The symmetry of acceptor, that forbids $\vec{B}_i \sim \vec{n}_i$

The simplest example: symmetry of Oxygen site:



Qualitatively: everything is Ok

Quantitatively: a serious problem...

In the bulk $|\omega| \approx 8K$ ($\omega \approx \theta \approx 3 \cdot 10^{-3}$)

If $\omega_i \sim \omega_0$ then for $T \sim 100K$ $\Delta P/P \sim 10^{-3}$!

Moreover:

1. Whatever the mechanism of MR be,
 $\Delta P/P \sim \theta^2$ at $\theta \ll 1$

2. Experimentally $\Delta P/P \sim 1$

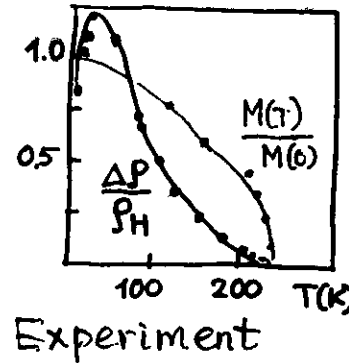
This contradiction can be resolved if
 $\omega_i \gg \omega_0$ due to polaronic effect

For $\phi_{loc} \sim 1 \rightarrow \omega_{loc} \sim 100-200K$
providing $\Delta P/P \sim 1$!

Large local distortions appear spontaneously
The small uniform $\vec{\omega}_0$ controls
only their signs.

⑧

Temperature dependence of $\Delta P/P$



Experiment

The simplest theory-
Mean Field

$$\vec{n} \rightarrow M(T) \equiv \langle \vec{n} \rangle_T$$

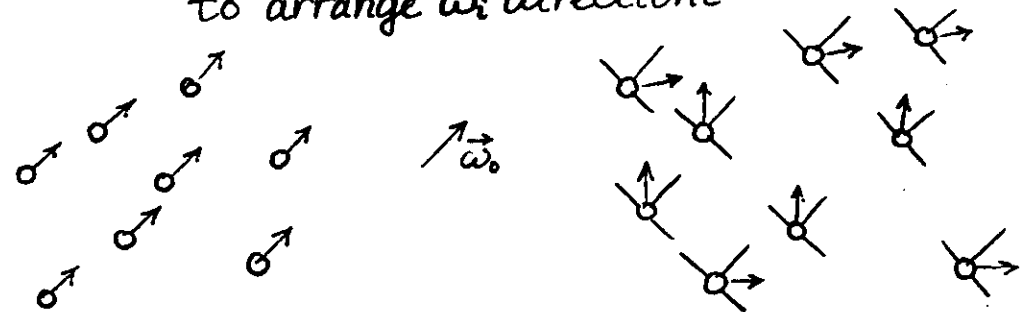
$$\frac{\Delta P}{P} \approx \sqrt{ch \frac{M(T)\omega}{2T}} - 1$$

For small M (near Neel point T_N)

$$\frac{\Delta P}{P} \approx \frac{M^2(T)\omega^2}{16T^2}$$

The drop of $\Delta P/P$ at low T may be due to
reentrant behaviour of $M(T)$.

Several theoretical possibilities
to arrange $\vec{\omega}_i$ directions



The same results for $P(H,T)$ at $H \perp CuO_2$
Different results at $H \parallel CuO_2$

Crucial experimental facts for $H \parallel \text{CuO}_2$

1. ρ decreases with H monotonously
2. There are two kinks in $\rho(H)$ at $H_1(T)$, $H_2(T)$ but no jumps
3. Overall drop of ρ : $(\rho(H=0) - \rho(H > H_2))/\rho(H=0)$ coincides with that at $H \perp \text{CuO}_2$

Only one type of $\vec{a}_i$ -arrangement is compatible with all three facts

Conclusions

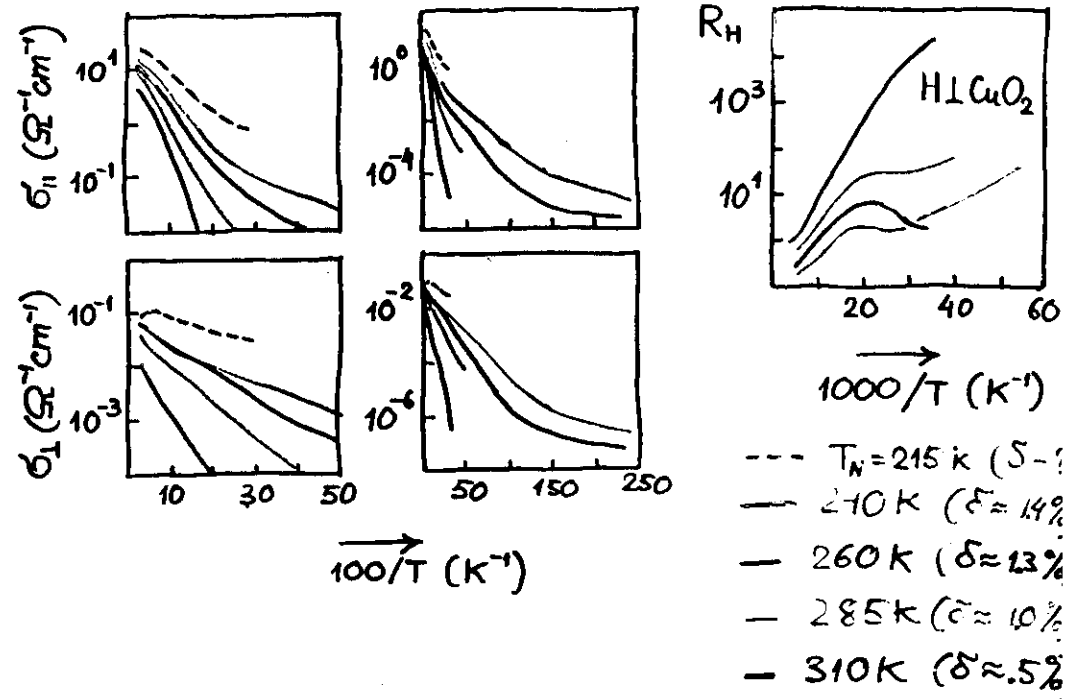
1. VRH, highly unusual MR.
2. Magnetic phase transitions $\rightarrow$ features in MR
3. Probability of hop $i \rightarrow j$ depends on relative orientation of molecular fields $\vec{B}_i$ and $\vec{B}_j$
4. Small external $\vec{H}$ governs the orientations of $\vec{B}_i, \vec{B}_j$
5. Extended percolation problem.

6. Quantitative explanation of MR in $\text{La}_2\text{CuO}_{4+x}$ needs strong local distortions (bound polarons)

7. Exptl MR $\rightarrow$ symmetry of acceptor state and local distortions.

Polaronic nature of transport in La_2CuO_4

T-dependence of conductivity:



Low temperature ($T < T_h$): hopping:

$$\sigma_{\parallel, \perp}^{(\text{Hop})} \approx B_{\parallel, \perp}^{(\text{Hop})} \exp \left\{ - (T_0/T)^\alpha \right\}$$

(Mott law? $\alpha \approx 1/4$)

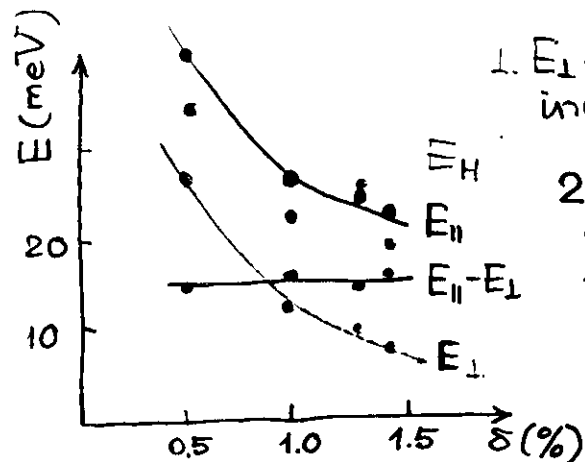
"High" temperature ($T > T_h$) activation law:

$$\sigma_{\parallel, \perp}^{(\text{act})} \approx B_{\parallel, \perp}^{(\text{act})} \exp \left\{ - E_{\parallel, \perp} / T \right\}$$

Problem: $\sigma_{\parallel} \gg \sigma_{\perp}$ but $E_{\parallel} > E_{\perp}$!!?

WHY $E_{||} > E_{\perp}$??

Any additional interlayer disorder would lead to $E_{\perp} > E_{||}$

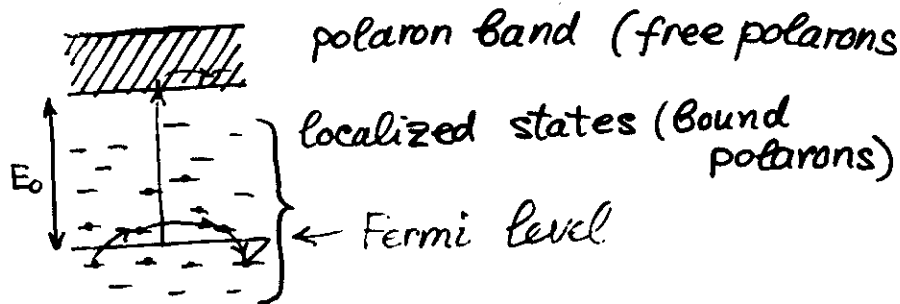


1. $E_{\perp}$ decreases with increasing δ

2. $E_{||} \approx E_{\perp} + W_H$,
 W_H is δ -independent

3. $E_H \approx E_{||}$ (?)
(at dispersion is large)

Physical pattern



At low T: hopping via bound polaronic states in the vicinity of Fermi level

At higher T: activation to the band; concentration of polarons in the band $n \sim e^{-E_0/T}$
 E_0 decreases with increasing δ (as the system approaches IM transition)

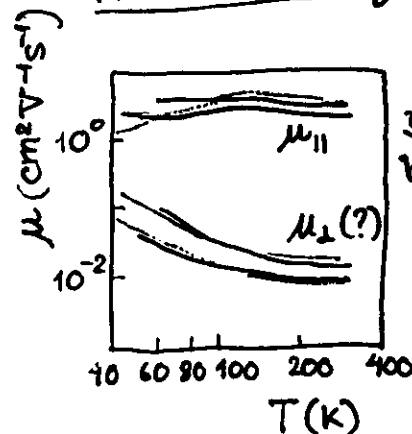
(3)

Hall coefficient R_H :

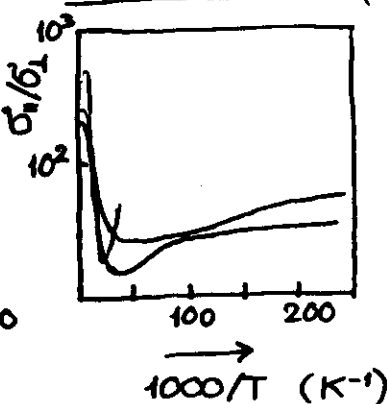
at $T > T_H$

$$R_H \propto \exp(E_H/T) \quad E_H \approx E_{||}$$

Hall mobility



Anisotropy of conductivity



— $\delta = 1.4\%$
— $\delta = 5.5\%$
— $\delta = 13\%$
— $\delta = 18\%$

anisotropy $\begin{cases} \sim 10 & \text{in hopping regime} \\ \sim 500 & \text{in "band" regime} \end{cases}$

A maximum of R_H at crossover T

In the "band" regime $\begin{cases} \mu_{||} \sim \sigma_{||} R_H \\ \mu_{\perp} \sim \sigma_{\perp} R_H \end{cases}$

Hall mobility is almost T and δ independent

μ is very small ($\mu_{||} \sim 3 \text{ cm}^2/\text{V}\cdot\text{s}$)

WHAT DOES IT MEAN?

MIT

$$m^* \sim 2-3 m_e$$

$$\tau \sim 10^{-14} \text{ s} \sim \hbar/J$$

Magnetic scattering (??)

$$\mu = e^2 \tau / m^*$$

LANL

$$\tau \sim 10^{-12} \text{ s (as usual)}$$

$$m^* \sim 10^2 m_e$$

Polaronic effect

THE MAIN IDEA

How does the hopping in the band proceed?

- (?) {
1. Along the CuO_2 -plane: thermally activated hops
 2. Interplane hops: coherent quantum tunneling

Hopping probabilities:

$$\begin{cases} w_{||} \sim \exp(-W_{||}/T) \\ w_{\perp} \sim \exp(-W_{\perp}/\omega_{\perp}) - T\text{-independent} \end{cases}$$

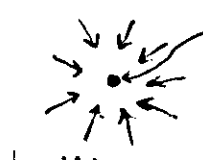
$$W_{\perp} \ll W_{||}$$

$$\begin{cases} \sigma_{||} \sim n w_{||} \sim \exp\{-(E_0 + W_{||})/T\} \\ \sigma_{\perp} \sim n w_{\perp} \sim \exp\{-W_{\perp}/\omega_{\perp}\} \exp\{-E_0/T\} \end{cases}$$

Identification: $\begin{cases} E_0 + W_{||} \equiv E_{||} \\ E_0 \equiv E_{\perp} \end{cases}$ enables one to explain experimental data

How can it be?

Brief summary of polaronic hops theory:

Polaron  Electron

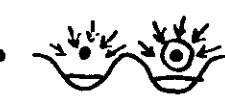
1. Energy gain
 W_p (polaronic shift) $\gg$
 ω_D - Debye frequency

(4) Small polarons (radius $r_p \ll a_0$)

non adiabatic hops

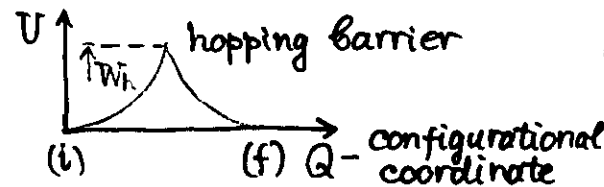
•  • initial state (i)

•  fluctuation (thermal or quantum)

•  hop: electron's wave-function changes abruptly

• •  final state (f)

Energy diagram



$$W_h \sim W_p : \omega_{tr} \sim \omega_D$$

tunneling probability:

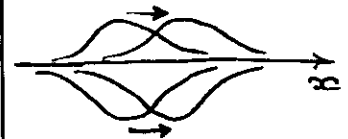
$$w_{tun} \sim \exp(-4W_h/\omega_{tr})$$

$$\text{polaron mass } M_s^* \sim w_{tun}^{-1}$$

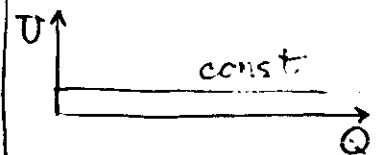
activation probability:

(5) Large polarons ($r_p \gg a_0$)

adiabatic motion.



Energy is invariant with respect to continual translations



$W_h = 0, \omega_{tr} = 0$
translational zero-mode

$M_e^* (W_p/\omega_D)^d$ is not exponentially large

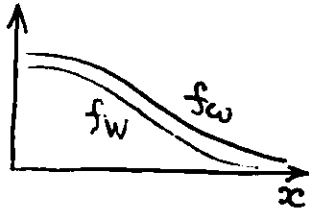
In general case:

$$w_{\text{hop}} \propto \begin{cases} \exp(-AW_h/\omega_{\text{tr}}) & (T < T_c \sim \omega_{\text{tr}}) \\ \exp(-W_h/T) & (T > T_c) \end{cases}$$

where $A = \text{const} \sim 4$

$$W_h = W_p f_w(r_p/a_0)$$

$$\omega_{\text{tr}} = \omega_D f_\omega(r_p/a_0)$$



$$f_w(0), f_\omega(0) \sim 1$$

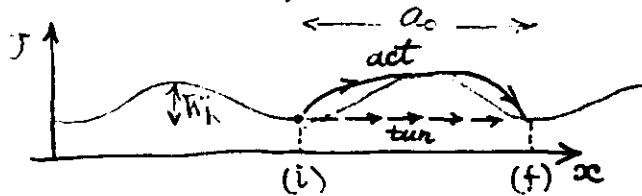
$$f_w(\infty), f_\omega(\infty) = 0$$

"Nearly large polaron" (NLP)

$r_p \gg a_0$, though finite

Physical pattern:

NLP = Large polaron (a particle with mass M_e^*) in a small periodic potential:



$$U(x) = \text{const} + W_h \sin^2(\pi x/a_0)$$

$U(x)$ is a reminiscence of the lattice

$$W_h \sim \int dx e^{2\pi i x/a_0} F(x/r_p) \sim W_p \exp(-\pi^2 r_p/a_0)$$

Since F is smooth.

Barrier W_h can be penetrated either by tunneling ($\rightarrow\rightarrow\rightarrow$) or by activation ($\longrightarrow$)

(6) Shrödinger equation for large polaron:

$$\left\{ -\frac{1}{2M_e^*} \frac{d^2}{dx^2} + U(x) \right\} \psi(x) = E \psi(x)$$

Probability of tunneling:

$$w_{\text{tun}} \propto \exp\{-S\}; S - \text{under-the-barrier action}$$

$$S = 2 \int_0^{\sqrt{2M_e^* U(x)}} dx = \frac{4\sqrt{2}}{\pi} a_0 \sqrt{W_h M_e^*}$$

$$\text{since } \omega_{\text{tr}} = \frac{\pi}{a_0} \sqrt{\frac{2W_h}{M_e^*}}; \text{ we have } S = \frac{8W_h}{\omega_{\text{tr}}} (A=8)$$

$$f_w \propto \exp\{-\pi^2(r_p/a_0)\}; f_\omega \propto \exp\{-\frac{\pi^2}{2}(r_p/a_0)\}.$$

$$M_{NL}^* \sim M_e w_{\text{tun}}^{-1}$$

Intermediate conclusion

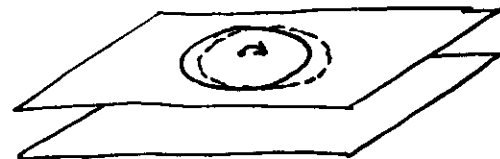
For Nearly Large Polaron.

$$1. W_h \ll W_p$$

$$2. T_c (\text{temperature of crossover from tunneling to activation}) \ll \omega_D$$

Having in mind La_2CuO_4

Consider quasi-2D polaron:



$$\begin{cases} r_{\parallel} \gg a_{\parallel} \\ r_{\perp} < a_{\perp} \end{cases}$$

A disk: $\begin{cases} \text{NLP in the plane} \\ \text{Small polaron in } \perp \text{ direction} \end{cases}$

so
hops in the plane are adiabatic and

$$T_{c||} \sim \omega_{tr||} \ll \omega_D$$

$$W_{||} \ll W_P$$

interplane hops are nonadiabatic and

$$T_{c\perp} \sim \omega_{tr\perp} \sim \omega_D$$

$$W_{\perp} \sim W_P$$

In the temperature interval $T_{c||} < T < T_{c\perp}$
we have what we wanted!

$$\left\{ \begin{aligned} W_{||} &\propto \exp \{ -W_{||}/T \} \\ W_{\perp} &\propto \exp \{ -AW_{\perp}/\omega_{tr\perp} \} \end{aligned} \right.$$

WHY DO WE THINK IT IS SO FOR La_2CuO_4 ?

$$1. E_{||} - E_{\perp} = W_{||} > 0$$

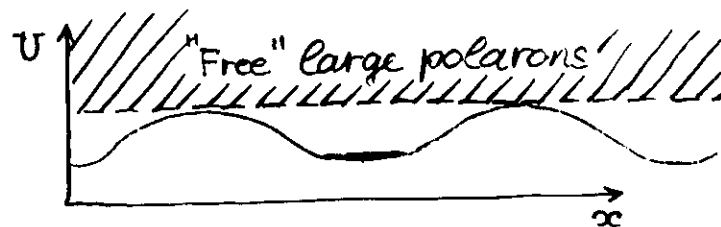
$$2. W_{||} \text{ is } \delta\text{-independent}$$

$$3. W_{||} \sim 15 \text{ meV is small (although } W_P \text{ is unknown, one can expect } W_{||} < W_P)$$

$$4. \text{ Hall effect}$$

a) Note, that small polaron theory predicts
 $E_H > E_{||}$

(8) Hall effect (in-plane) in terms of
large polarons:



Above-the-barrier polarons, those contribute to conductivity at $T > T_{c||}$

contribute also to Hall effect

So one can expect $E_H = E_0 + W_{||} \equiv E_{||}$

? It is unclear yet, why experimental E_H is a few meV less than $E_{||}$

CONCLUSION

1. Low mobility can be explained in terms of polaronic effect.
2. The polaron must be anisotropic: moderately large in the plane and small in $\perp$ direction.
3. $E_{||} > E_{\perp}$ because in-plane hops are thermoactivated though interplane hops are quantum.
4. Model predicts $E_H \approx E_{||}$
5. Magnetoresistance data favours polaronic effects.

Questions

What is microscopic nature of polaron?
How to obtain $T_{c||}$ from experimental data?

①
Magnetic order of the Quasi-2D-Antiferromagnet
with frustrating impurities ($\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$)

Coauthors

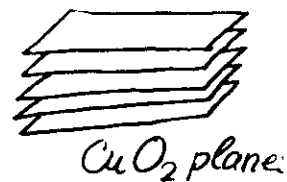
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3. Magnetic structure of Sr acceptors
4. Theory of interacting magnetic impurities
(neutral acceptors)
5. Reentrant transition

②
 La_2CuO_4 is a Quasi-2D Heisenberg AF; $S=1/2$

1. Cu^{2+} : localized spins; $S=1/2$
2. In-plane exchange $J \sim 1200\text{K}$
3. Net interplane exchange $J' \sim .05\text{K}$
4. Magnetic anisotropy is small
(magnon gap $\sim 10 \div 20\text{K}$)



CuO_2 plane

Spin Hamiltonian

2D $\vec{n}$ -field

$$\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \xrightarrow{\text{at } T \ll J} \rho/2 \int d^2r (\nabla \vec{n})^2$$

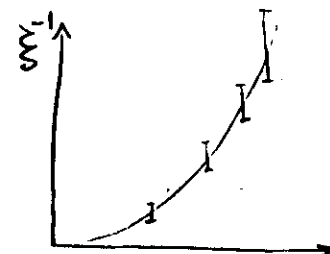
(Chakravarty, Halperin, Nelson, 1988)

ρ is a spin stiffness (renormalized, $\rho < \rho^0$)
 $\vec{n}$ is an AF order parameter ($|\vec{n}| \equiv 1$)

2D-correlation radius

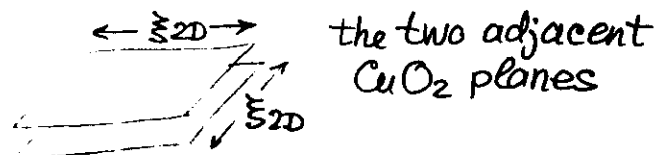
$$\xi_{2D}(T) \sim \exp(\pi \rho / T)$$

(Polyakov, 1977)



(Ukeshima et al 1988)

③ Neel Temperature

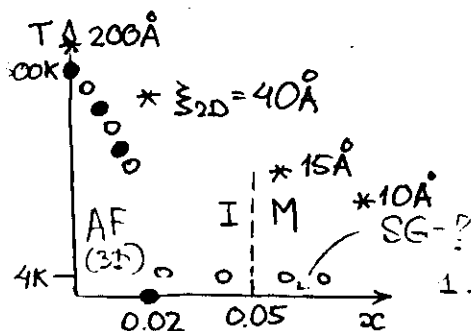


$$J' \xi_{2D}^2(T_N) \sim T_N \xrightarrow{J' \ll J} \xi(T_N) = \xi_c \equiv \sqrt{2\pi\rho/J'}$$

$$T_N \approx 2\pi\rho/\ln \xi_c$$

actually $\ln \xi_c \approx 5$

The phase-diagram of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$



- - neutron scattering, magnetic sublattice peak
- - μSR (quenched moments?)
- * - $\xi_{2D}(x, T)$; inelastic neutron scattering

1. Nontrivial magnetic behaviour occurs in the insulating state

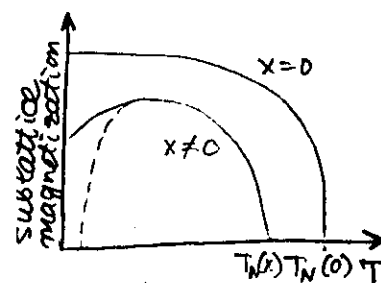
2. T_N drops rapidly with x :
 $T_N \equiv 0$ for $x > x_c$, $x_c \approx 0.02$

3. Why x_c is so small

4. T_N vs x -dependence of ξ_{2D}

5. Spin Glass?

③



Quasi-reentrant behaviour? ④

Endoh et al. 1988

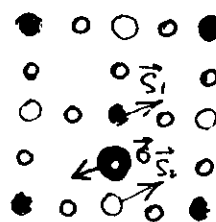
Theory of lightly doped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

1. Sr^{2+} ions are acceptors (substitute La^{3+} and produce holes)
2. At low T and $x < 0.05$ all the holes are localized (bound to acceptors?)

note: law for conductivity, [Kas'ner et al.]

What is magnetic structure of a neutral Sr acceptor?

1. Simple model [Aharony et al. 1988]



○, • - Cu^{2+} ions in two magnetic sublattices

○ - O^{2-} ions

The hole is localized in the single Oxygen site.

AF bond ($\vec{S}_1 \vec{S}_2$) $\rightarrow$ Ferro bond ($\vec{S}_1 \vec{S}_2$) ⑤

$\uparrow$ due to

$$J_{\text{Cu}^{2+}\text{-hole}} = K(\vec{S}_1 \vec{\sigma} + \vec{S}_2 \vec{\sigma}); K \sim t \gg J$$

Mechanism of Aharony et al:

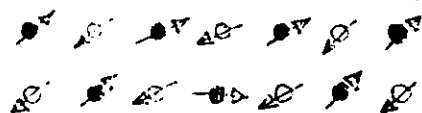
Destruction of AF order $\Leftrightarrow$ percolation via
Ferro bonds

1. Does not explain such a small x_c
2. Predicts Ferro order at $x > x_c$

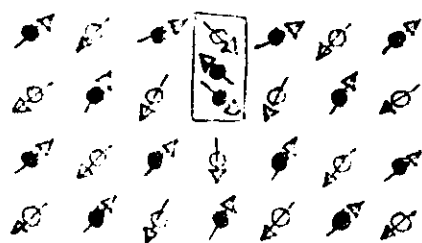
Alternative mechanism:

Destruction of AF order $\Leftarrow$ acceptor induced
long range perturbations of AF order of matrix.

Analogy: "Transverse Spin Glass" Villain 1979



Frustration induced by the
localized hole



$\downarrow$ $\vec{S}$ - the total Spin of
acceptor

$\uparrow$ $\vec{\mu}$ - the frustrating
moment

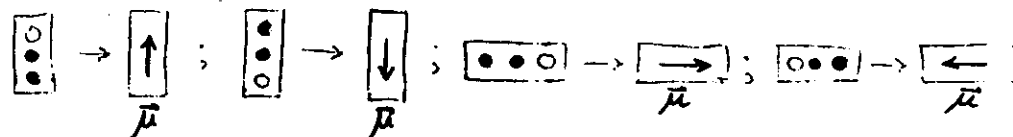
At large distances

⑥

$$S\vec{n} \approx (1/2\pi) \vec{S} (\vec{\mu} \vec{r}) / r^2$$

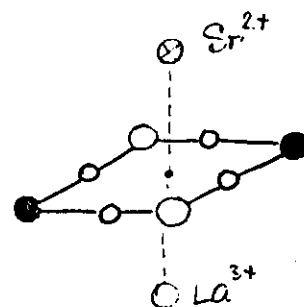
2D-dipolar component of frustration

Four possible directions of $\vec{\mu}$:



More realistic model of Sr acceptor

Gogolin, Ioselovich, 1989



1. Four O^{2-} sites are available
on equal ground
2. Four Cu^{2+} spins are involved

The Ground State

If Emery model is accepted Emery 1985
to O-Cu-O hopping, then

1. The total spin of Gr. St. is minimal ($S=1/2$)
(c.f. magn. polaron!)

2. The symmetry group representation is E : ⑦
(orbitally degenerate p-type state)

Symmetry consideration of $\vec{\mu}$

1. $\mu=0$ for the unbroken Tetragonal symmetry

2. $\mu \sim (V/\Delta)\phi$

ϕ - is the orthorhombic rotation due to:

{ Global Orthorhombic phase of matrix
or/and
Local (pseudo) Jahn-Teller effect

V is vibronic constant

$\Delta \sim t$ is splitting

For the realistic V and Δ μ is large!

Phenomenology

Classical description [Glazman, Iosevich, 1989]

For large r : $\Delta \delta \vec{n} = 0$

Multipole expansion:

$$\vec{n} = \vec{n}_0 + \frac{1}{2\pi} \vec{S} \left\{ \vec{\mu} \vec{r} / r^2 + A_{2\beta} Q_{2\beta} / r^4 + \dots \right\}$$

If $\vec{\mu}$ is not symmetry-forbidden:

⑧

2D-Hamiltonian

$$\mathcal{H} = p/2 \int d^2r (\nabla \vec{n})^2 + p \sum_i \vec{S}_i \cdot \vec{\mu}_i \nabla_{\vec{r}} \vec{n}_i(\vec{r}) |_{\vec{r}=\vec{r}_i}$$

2D- $\vec{n}$ -field with dipolar magnetic impurities

$\vec{n}(\vec{r})$, $\vec{S}_i$ - dynamical variables

$\vec{\mu}_i$, $\vec{r}_i$ - quenched disorder

Let's suppose $\xi_{2D} \gg r_0 \sim x^{-1/2}$ - the mean distance between acceptors

Then $\xi_{2D}(T, x)$ makes sense

Our plan is

1. To find $\xi_{2D}(T, x)$ from 2D-Hamiltonian
2. To find $T_N(x)$ from the Eq. $\xi_{2D}(T_N, x) = \xi_c$

Perturbation theory (small concentration x) ⑨

Fluctuations of order parameter $\vec{n}(\vec{r})$

$$\langle (\delta \vec{n})^2 \rangle \approx \langle (\delta \vec{n})^2 \rangle_{\text{thermal}} + \langle (\delta \vec{n})^2 \rangle_{\text{acceptor induced}}$$

$$\langle (\delta \vec{n})^2 \rangle_{\text{thermal}} \approx (T/\pi p) \ln L$$

$$\langle (\delta \vec{n})^2 \rangle_{\text{acceptor induced}} \approx \left\langle \left\{ \frac{1}{2\pi} \sum_i (\vec{\mu}_i \vec{n}_i) r_i^{-2} \vec{S}_i \right\}^2 \right\rangle_{\vec{\mu}_i, \vec{S}_i} \approx (x \mu^2 s^2 / 4\pi) \ln L$$

Both contributions diverge logarithmically at large spatial scales L :

$$\begin{aligned} \langle (\delta \vec{n})^2 \rangle &\approx (T/\pi p_{\text{eff}}) \ln L \\ p_{\text{eff}} &= p(1 - x \mu^2 s^2 p / 4T) \end{aligned}$$

We claim:

1. The perturbation theory small parameter is $z \equiv x \mu^2 s^2 p / 4T$

2. For arbitrary z :

$$\begin{aligned} \xi_{2D} &\propto \exp \{ 2\pi p_{\text{eff}} / T \} \\ p_{\text{eff}}(x, T) &= p_e \phi(z) \quad (\phi(0) = 1) \end{aligned}$$

The Proof

⑩

1. Introduce the intermediate scale L_0

$$r_0 \sim x^{1/2} \ll L_0 \ll \xi_{2D}$$

2. Integrate out the components of $\vec{n}(\vec{r})$ with the scales $< L_0$

Then

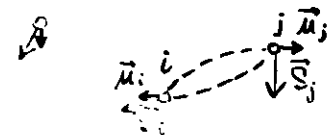
$$\mathcal{H} \rightarrow \mathcal{H}_1 = p/2 \int d^2 \vec{r} (\nabla \vec{n})^2 + p \sum_i \vec{S}_i^a \mu_i^a \nabla_a \vec{n}_a(\vec{r})|_{\vec{r}=\vec{r}_i} + \dots + \mathcal{H}_{\text{acc}} \{ \vec{S}_i \}$$

Now $\vec{n}$ - with the scales $> L_0$

$$\mathcal{H}_{\text{int}} \{ \vec{S}_i \} = p/2 \sum_{i \neq j} (\vec{S}_i \vec{S}_j) r_{ij}^{-2} \{ (\vec{\mu}_i \vec{\mu}_j) - 2(\vec{r}_{ij} \vec{\mu}_i)(\vec{r}_{ij} \vec{\mu}_j) r_{ij}^{-2} \}$$

The dipole-dipole interaction of acceptors

via short-range part of $\vec{n}$ -field



2D random magnet

$\vec{S}_i$ - magnetic moments (they can rotate)

μ_i are randomly quenched

Characteristic energy $U \sim x \mu^2 s^2 p / 4$

3. Integrate out $\vec{S}_i$ (11)

$\vec{n}$ is a small quasiuniform external field

Linear response formalism:

$$\mathcal{H} \rightarrow \frac{f}{2} \int d^2r (\nabla \vec{n})^2 + F_{\text{imp}}^{(0)}(x, T) - \frac{\tilde{\chi}(x, T)}{2} \int d^2r (\nabla \vec{n})^2$$

Generalized susceptibility:

$$\tilde{\chi}(x, T) = (P/V)(P/4T) \sum_{ij} \langle (\vec{\mu}_i \cdot \vec{\mu}_j) \langle \vec{S}_i \vec{S}_j \rangle_T \rangle_{\text{config.}}$$

$$\tilde{\chi}(x, T) = P f(U/T)$$

For $z \gg 1$ $f(z) \approx 1/z$ (Curie-Weiss law)

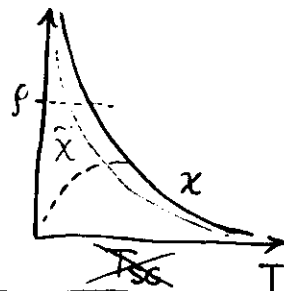
For $z \sim 1$ $f(z)$ is some sophisticated function

So

$$\mathcal{H} \rightarrow F_{\text{imp}}^{(0)}(x, T) + P_{\text{eff}}/2 \int (\nabla \vec{n})^2 d^2r$$

$$P_{\text{eff}} = P - \tilde{\chi}(x, T) = P \phi(U/T)$$

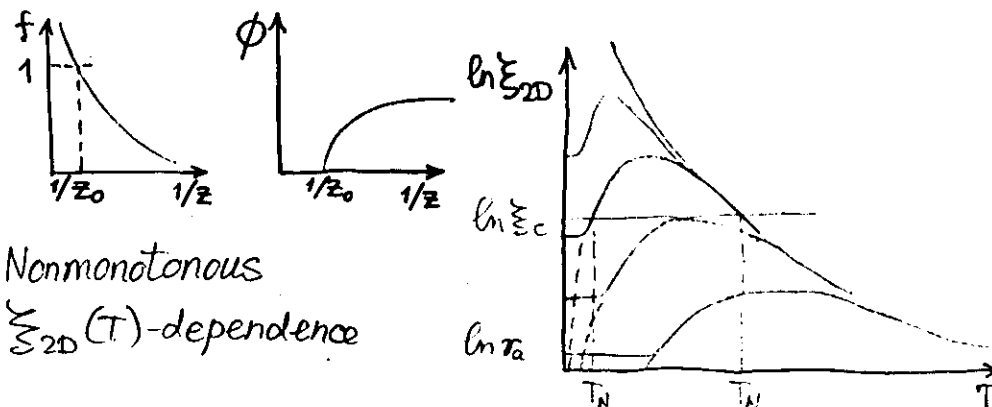
$$\phi(z) = 1 - f(z)$$



There is no SG transition!!
(Binder, Yang 1986)

Correlation radius and Neel temperature (12)

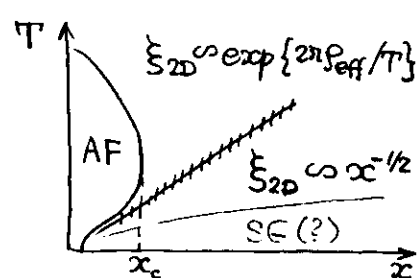
$$\ln \xi_{2D}(x, T) = 2\pi P_{\text{eff}}(x, T)/T$$



Two solutions - reentrant transition

Phase diagram

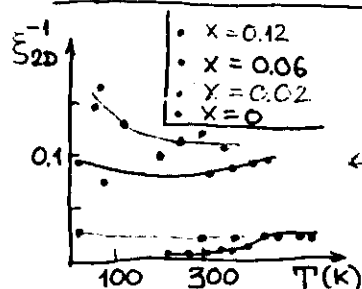
(13) 9



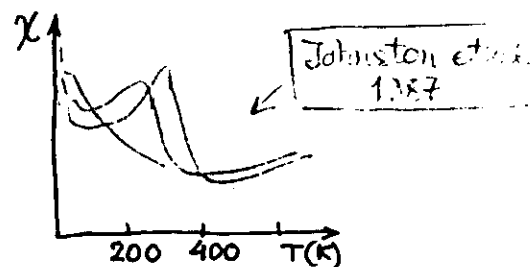
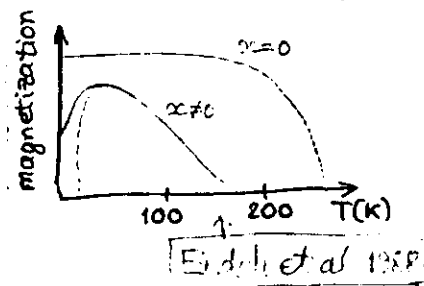
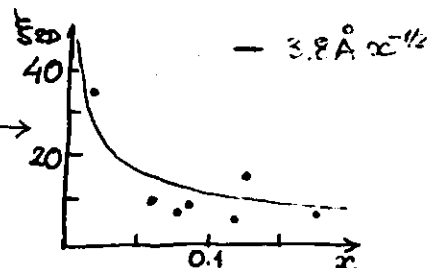
1. Reentrant transition
2. $x_c \sim 1/\mu^2 \ln \xi_c$ is below the percolation threshold
3. Below the ~~xxxx~~ line - the instability of AF state:

1. Spin-glass transition may arise due to magnetic anisotropy. $\xi_c \sim x^{1/2}$

Experimental evidence (?) of reentrant behaviour



Birgenau et al 1980



Abstract

The effect of impurity-induced states on the long range order in a lamellar antiferromagnet (AF) is studied and the magnetic phase-diagram of a lightly doped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ is proposed. It is shown that long range magnetic perturbations and the layered structure cause the shrinkage of AF domain on the phase diagram and lead to the reentrant AF transition. A nonmonotonous dependence of the correlation length ξ_{2D} on temperature T is obtained; the dependence $\xi_{2D}(x)$ is exponential for high T and $\xi_{2D} \propto x^{1/2}$ for low T .

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Introduction

The antiferromagnetism of high- T_c copper oxides is caused by the superexchange between localized copper spins. The high anisotropy of these compounds provides the small value of an interplane exchange integral I' as compared with the in-plane one I [1]. Doping creates holes in the p-shells of the Oxygen ions of CuO_2 -planes [2]. At low temperatures and low doping level x these holes are bound to acceptors. This is clear from the Mott type of conductivity [3].

The localized holes produce the magnetic defects, which frustrate the order in AF matrix [4]. In fact, the AF order in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ is destroyed by a surprisingly small dopings: $x_2=0.02$ [1]. We show in this paper that the strong doping-sensitivity of magnetic order may be due to the long range of magnetic perturbations combined with the strong anisotropy. The doping-dependence of AF order in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ is considerably weaker ($x_2=0.4$). This can originate from the higher symmetry of defects in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, which may forbid the long range interactions.

A localized hole induces a defect domain (of size D) with a novel magnetic order [4,5]. The interaction of domain boundary with the outer spins produces a perturbation in AF matrix which drops off as r^{-1} . This kind of perturbation was considered by Villain [6] in connection with a spin-glass problem and later was applied to LaSrCuO by Aharony et al [4]. It is the long range interaction and the small parameter I'/I that makes possible for

us to obtain the boundary of the 3D-AF-region on T - x diagram (Fig.1), i.e. to determine the x -dependence of Neel temperature $T_N(x)$. The long range order is greatly distorted by acceptors only if $x > x_1 - I'/I$. For $x > x_2 - (D^2 \ln(I/I'))^{-1}$ the long range order is destroyed at all temperatures. Since the domain size $D \geq 1$ and $I/I' \gg 1$, the value $x_2 \gg x_1$. In a wide interval $x_1 < x < x_2$ a reentrant transition arises, its origin is the magnetic defect-defect correlation enhanced at low T .

The impurity-induced magnetic phase transition is not of the percolational type: $x_2 \ll x_c$ (where $x_c D^{-2}$ is a threshold for the percolation through the defect domains). At this point our concept of the transition differs from that of Aharony et al [4].

The x -dependence of the in-plane correlation length ξ_{2D} , as well as T_N , is of high interest for the interpretation of experimental data. The defect-induced disorder modifies the dependence $\xi_{2D}(T)$. Under the finite x this dependence becomes nonmonotonous. Such a kind of behaviour is of particular importance, because it forms the source for the reentrant transition. At low temperatures one has $\xi_{2D} \sim x^{-1/2}$, which agrees with the experiments [1]. At relatively high temperatures (above AF transition) the function $\xi_{2D}(x)$ is an exponential one.

The effective Hamiltonian of magnetic system

To determine the in-plane correlation length ξ_{2D} we have to derive a Hamiltonian of the long-wave-length 2D-fluctuations of $\mathbf{n}(\mathbf{r})$ ($\mathbf{n}$ is the unit vector of antiferromagnetism). For a pure AF this Hamiltonian has a well-known form:

$$H_0 = f d^2 r (\nabla n)^2 \quad (1)$$

where $p-I$ is a spin stiffness.

The defect domain carries a finite spin M . The value of M (as well as D) can be obtained in microscopic model calculation [5]. The interaction energy of a defect and AF matrix is minimal when $\text{Min}_0 (n_0 - n(r \rightarrow \infty))$ [6]. The perturbation of vector n at large r is:

$$\delta n = n(r) - n_0 = (\mu/2\pi) m(e r) / r^2 \quad (2)$$

which corresponds to the field of a dipole in two dimensions. Here $m = M/M$, e is the unit vector, parallel to one of the tetragonal axes (a or b). Four possible directions of e correspond to the four nonequivalent positions of a defect in the magnetic lattice. Asymptotics (2) and the randomness of e are universal and do not depend on the specific form of a microscopic Hamiltonian. The frustrating dipolar momentum modulus μ is the only model-dependent parameter, it is determined by a defect structure. The 2D-law (2) is valid on the scale $D \ll r \ll r_{3D}$; for $r \approx r_{3D} = (I/I')^{1/2}$ the interplanar exchange transfers the perturbation onto the adjacent planes and leads to a 3D-law: $\delta n \sim r^{-2}$.

Now we shall construct a long-wave-length Hamiltonian of n - m interaction, which produces the same perturbation δn , as (2):

$$H = H_0 + H_{\text{int}}; H_{\text{int}} = \rho \mu \sum_i (e_i \nabla) (n(r_i) m_i) \quad (3)$$

where r_i are the positions of the acceptors. It can be easily shown that (2) minimizes the energy (3) in the case of single acceptor. Eq(3) holds for the case of a finite acceptor

concentration x if the average distance $r_a x^{-1/2}$ between acceptors satisfies the condition: $D \ll r_a \ll r_{3D}$.

Thus the fluctuations of unit vectors $n(r)$ and m are governed by the Hamiltonian (3) with randomly distributed r_i and e_i . It is well-known that in 2D-theory with a Hamiltonian (1) the fluctuations of n diverge logarithmically in the thermodynamic limit: $\langle \delta n^2 \rangle \sim \ln L$, where L is the size of the system. We shall demonstrate that the defect-induced fluctuations are also logarithmically divergent². The calculation up to the second order in H_{int} yields:

$$\begin{aligned} \langle \delta n^2 \rangle &= \langle \delta n^2 \rangle_0 + \langle \delta n^2 \rangle_2; \quad \langle \delta n^2 \rangle_0 = (T/\pi \rho) \ln L, \\ \langle \delta n^2 \rangle_2 &= (\mu/2\pi)^2 \langle (\sum_i (e_i r_i) r_i^{-2} m_i)^2 \rangle \approx x \mu^2 / 4\pi \ln(L/r_a). \end{aligned} \quad (4)$$

The distances are measured in lattice-spacing units. Indices 0 and 2 denote the orders of the perturbation theory; the first-order term is obviously zero. The result for the second order is valid for $r_a \ll L$, when the sum may be substituted by the integral. The crossing terms in $(\sum \dots)^2$ are neglected because they are of x^2 order. The result can be rewritten in the form:

$$\langle \delta n^2 \rangle \approx (T/\pi \rho_{\text{eff}}) \ln L; \quad \rho_{\text{eff}} = \rho(1 - x \mu^2 / 4T) \quad (5)$$

So, at least for the small values of x , the influence of magnetic defects on $\langle \delta n^2 \rangle$ can be taken into account by means of small renormalization of a spin-stiffness ρ .

² Perturbation theory results are valid obviously only if $\langle \delta n^2 \rangle$ obtained is small as compared with unity.

Our idea is that an appropriate renormalization of ρ in the simplest Hamiltonian (1) is sufficient for taking into account the impurity effects on n at arbitrary x . We show in the following, that Eq(5) for ρ_{eff} presents the two first terms of expansion for the effective stiffness (see (11)).

According to this idea we shall integrate out the fluctuations of n with the spatial scales $r < L_0$, where $r_a \ll L_0 \ll r_{3D}, \xi_{2D}$. Applying the standard procedure to the Hamiltonian (3) we obtain after the Orenormalization:

$$H_1 = H_0 + H_{\text{int}} + H(m); \quad H(m) = (\rho\mu^2/2) \sum_{i,j} (I_{ij} m_i m_j);$$

$$I_{ij} = r_{ij}^{-2} ((e_i e_j) - 2(r_{ij} e_i)(r_{ij} e_j) r_{ij}^{-2}) \quad (6)$$

The term $H(m)$, which is generated by the short-wave-length part of H_{int} in (3), describes the m - m interaction via n -field. This term is L_0 -independent due to the condition $r_a \ll L_0$. The characteristic value of random m - m exchange in $H(m)$ is $U = \rho\mu^2 x/4$. The renormalizations of ρ and μ in H_0 and H_{int} are negligible since $\ln L \ll \ln \xi_{2D}$.

After the above procedure Vn becomes a small and slowly varying function in r -space. Since that one can easily perform the integration over m_i . In zeroth order in Vn this integration gives the free energy $F_0(T)$ of a random magnet with a Hamiltonian $H(m)$. The smallness of Vn enables us to consider H_{int} in the framework of the linear response formalism and to obtain a quadratic in Vn contribution to the free energy of the random magnet mentioned above:

$$F = F_0 - (\tilde{\chi}/2) \int d^2 r (Vn)^2, \quad (7)$$

where Vn appears as a generalized external field and $\tilde{\chi}$ is a corresponding susceptibility:

$$\tilde{\chi} = (\rho/V) (\rho\mu^2/4T) \sum_{i,j} \langle (e_i e_j) \langle (m_i m_j) \rangle_T \rangle_c. \quad (8)$$

Here V is the volume, $\langle \dots \rangle_T$ and $\langle \dots \rangle_c$ denote the thermodynamic (with the Hamiltonian $H(m)$) and the configurational averaging respectively. Eq(7) is just a formal expansion of a free energy F in small "external field" Vn . It does not require any restrictions on the Hamiltonian $H(m)$. In particular, the assumption of the Gaussian type of fluctuations is not needed. All the problems, connected with the non-Gaussian field m are reduced to the two unknown functions $F_0(T)$ and $\tilde{\chi}(T)$. At high T the free energy F can be obtained by a straight-forward calculation resulting in (7) with $\tilde{\chi}$ of Curie-Weiss type.

The spatial dispersion of $\tilde{\chi}$ occurs on the scale r_a and can be neglected since $L_0 \gg r_a$. Random m - m interactions in $H(m)$ do not break the isotropy neither in spin, nor in real space; hence $\tilde{\chi}$ is a scalar function. As it follows from (6) and (8), $\tilde{\chi}$ depends on x and T only via ratio $U/T = \rho\mu^2 x/4T$; so one has $\tilde{\chi} = \rho(1 - \phi(\rho\mu^2 x/4T))$, where $\phi(z)$ is a universal dimensionless function.

After the integration over m_i , the free energy (7) replaces the last two terms in H_1 (Eq(6)). Thus, as we have just expected, the effective Hamiltonian of long-wave-length fluctuations of n coincides with H_0 , except for the renormalization of the spin-stiffness:

$$H_{\text{eff}} = (\rho_{\text{eff}}/2) \int d^2r (v_n)^2; \quad \rho_{\text{eff}} = \rho \phi(\rho \mu^2 x/4T) \quad (9)$$

The lowest order perturbation theory results (4) and (5) coincide with the first two terms of expansion for $\phi(z)$ in (9).

The 2D-correlation length

The correlation length for the 2D-n-field with the Hamiltonian (1) or (9) is well-known [7]:

$$\xi_{2D} \sim \exp(2\pi\rho_{\text{eff}}/T) \quad (10)$$

Thus, a problem of x- and T-dependences of ξ_{2D} is reduced to that of ϕ shape.

At large z a function $\phi(z)$ follows the Curie-Weiss law:

$$\phi(z) = 1 - z \quad (11)$$

This limit corresponds to relatively high temperatures case (T exceeds the m-m interaction U). The correction to (11) can be obtained from the cluster expansion of $\tilde{\chi}$. The first correction (of order of z^2) could arise from the two-acceptor clusters:

$$\psi^{(1)}(z) = -z^2 \int d\mathbf{e}_1 d\mathbf{e}_2 (e_1 e_2) \int d^2r_{12} \frac{\int d\mathbf{m}_1 d\mathbf{m}_2 (m_1 m_2) \exp(-I_{12}(m_1, m_2))}{\int d\mathbf{m}_1 d\mathbf{m}_2 \exp(-I_{12}(m_1, m_2))} \quad (12)$$

where r_{12} is a separation of acceptors in the cluster, normalized by a factor $(\rho \mu^2/2T)^{1/2}$. It can be easily shown, that the integrand in (12) changes its sign under the rotation of r_{12} by the

angle $\pi/2$. Hence $\phi^{(1)} = 0$ and the first nonzero correction to (12) is of order of z^3 .

The shape of $\phi(z)$ at $z \sim 1$ is determined by the qualitative behaviour of the impurity spin system at intermediate temperatures T-U. The properties of 2D-planar magnet with random dipolar interactions are not known exactly so far. However, there are strong arguments (see [8]) in favour of zero spin-glass-transition temperature for such a system. Hence one can expect the magnetic susceptibility χ to increase monotonously without a saturation under lowering T. The generalized susceptibility $\tilde{\chi}$ differs from χ by a factor $(e_i e_j)$ in (8). However, there is no reason for a qualitative difference between shapes of χ and $\tilde{\chi}$ in the paramagnetic phase. So we assume the monotonous increase of $\tilde{\chi}(T)$ or, in other words, the monotonous decrease of $\phi(z)$.

At large T the renormalization of ρ_{eff} can be neglected and $\xi_{2D}(T)$ increases with lowering T. On the other hand ρ_{eff} goes to zero and $\xi_{2D}(T)$ decreases when T tends to U/z_0 (where z_0 satisfies an equation: $\phi(z_0) = 0$). So $\xi_{2D}(T)$ is a nonmonotonous function. Asymptotics (11) enables one to write down an analytical expression:

$$\xi_{2D}(T, x) \sim \exp((2\pi\rho/T) - (\pi\rho^2\mu^2 x/2T^2)) \quad (13)$$

which reveals the exponential decrease of ξ_{2D} with doping. Eq(13) is exact at $z \ll 1$ (when the second term in (13) is much less, than the first one). For $z \sim 1$ Eq(13) provides a qualitative illustration of nonmonotonous temperature dependence of ξ_{2D} .

There is an instability ($\rho_{\text{eff}} < 0$) in the Hamiltonian (9) for

$z \gg z_0$. Formally, it means the divergence of fluctuations, and the shorter is the wave-length, the stronger is the divergence. Physically, the divergence should be cut at the shortest relevant scale, i.e. at the mean interdefect spacing r_a . Hence, the correlation length $\xi_{2D} \sim r_a x^{-1/2}$ at $T < x \rho \mu^2 / 4z_0$ (i.e. below the broken line in Fig.1). Square root x -dependence of ξ_{2D} and it's weak temperature variations at low T and $x > 0.02$ were found in the experiments [1,9]. In our opinion, the explanation of these results within the percolation theory is unlikely.

The above study of ξ is related to a strictly 2D AF. In a real case of 3D AF with a strong anisotropy the system undergoes a phase transition. The results of this section are relevant to the experiment only outside the region of AF-phase. However, the results for ξ_{2D} and the condition $I/I' \ll 1$ enable us to determine the phase boundary.

The 3D-Neel temperature

The Neel temperature T_N of an undoped quasi-2D-antiferromagnet is defined approximately by a relation: $T_N \sim I' \xi_{2D}^2(T_N)$. In a strictly-2D-AF an infinitesimal concentration of quenched dipolar defects destroys the long-range order [6] even at $T=0$ (i.e. $\xi_{2D}(T=0)$ is finite). So, in a doped quasi-2D-AF one can expect the suppressing of long-range order for rather small x . The above relation for T_N can be easily generalized for this case: $T_N \sim I' \xi_{2D}^2(T_N, x)$, or in the leading logarithmic approximation, we may obtain $T_N(x)$ from the equation:

$$\xi_{2D}(T_N, x) = r_{3D} (I/I')^{1/2} \quad (14)$$

Using (9) and (10), one can rewrite Eq(14) in the form:

$$\phi(y/\tau) = \tau \quad (15)$$

where $y = x/x_0$, $\tau = T_N/T_N(0)$ are normalized concentration and transition temperature; $T_N(0) = 2\pi\rho/\ln r_{3D}$, $x_0 = 8\pi/\mu^2 \ln r_{3D}$. For low concentrations $y \ll 1$ Eq(11) is valid and (15) can be solved:

$$\tau = 1 - y - y^2 \quad (16)$$

The last term in (16) is within the calculation accuracy because of absence of z^2 terms in the $\phi(z)$ expansion.

The presence of the reentrant transition is a direct consequence of nonmonotonous temperature dependence of ξ_{2D} . Eq(14) in this case may have two solutions for T_N , the low-temperature solution describes a reentrant transition (Fig.3).

We have no accurate analytical expression for $\phi(z)$ at $z=1$, but the shape of $\tau(y)$ -function can be qualitatively established with the help of graphic analysis of Eq(15) (see Fig.2). Above the critical concentration y^* Eq(15) has no solutions (the AF ordering is impossible for all T 's); below y^* there are two solutions that correspond to the reentrant behaviour of phase transition. In the dimensional variables a critical concentration is $x_2 = 8\pi y^*/\mu^2 \ln r_{3D}$.

The physical origin of the discussed phenomena lies in the π -m correlations enhancement at low temperatures. However, there is a

limitation on the concentration x , under which these correlations occur: $x > x_1$, i.e. $x > x_1$. At $x < x_1$ it is only 3D-tails of perturbations that overlap. Hence there is no divergence in the impurity-induced fluctuations, no ρ -renormalization and no reentrant transition for $x < x_1$.

Estimate a value of x_2 , extrapolating the Curie-Weiss law (11) onto the $z=1$ region. The solution of (15) with $\phi(z)=1-z$ gives:

$$\epsilon(y) = (1 \pm \sqrt{1-4y})/2, \quad y^* = 1/4 \quad (17)$$

Using the values $T_H(0)=300$ K, $2\pi\rho=1200$ K [10] one can see, that a value $x_2=0.02$ corresponds to $\mu=10$. The estimate $\mu=6D$ [5] shows, that the experimentally observed x_2 can be interpreted in terms of defects with realistic size D of 1-2 lattice spacings.

There is no direct experimental evidence for a reentrant transition in LaSrCuO so far. But several neutron-scattering experiments (especially the suppression of the sublattice magnetization at low T) [1], as well as resistivity measurements [10] indicate the presence of a reentrant behaviour.

The paramagnetic state of the defect spins is preserved even on the AF background. Hence the impurity contribution χ to the magnetic susceptibility may be comparable to that of AF-matrix despite the low doping.

Discussion

The true long-range order is destroyed below the line of the reentrant transition. Though at $T < U$ the length ξ of spin-glass

correlations in the impurity-spin system increases. So, the lowering of T can lead to the strong frequency dispersion of the susceptibility. 3D-interactions and the magnetic anisotropy, which are inevitably present in the crystal, can result in 3D-Ising-like behaviour at the lowest T 's. Consequently, a 3D-spin-glass transition arises.

In conclusion we have studied theoretically the magnetic properties of the doped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$. The quasi-2D nature of the structure enabled us to determine temperature and concentration dependences of the magnetic correlation length ξ . At low dopings ξ depends on x exponentially, at relatively high x the length $\xi \sim x^{-1/2}$. The T -dependence of ξ is nonmonotonous. Consequently, a reentrant AF transition in the doped LaSrCuO should occur. The theory developed here can be applied to any layered antiferromagnet with dipolar defects.

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Figure captions

Fig.1 The magnetic phase-diagram of a lightly doped lamellar antiferromagnet. AF- antiferromagnetically long-range-ordered phase of a matrix; PM- paramagnetic phase, x_c is a percolation threshold. The region of strong fluctuations ($\xi_{2D}^{-1/2}$) lies below the broken line; the suppositive region of impurity spin-glass (SG) is shadowed.

Fig.2 A graphical solution of Eq(15). The three stright lines shown are defined by the equation $\phi=y/z$ with: 1)- $y>y^*$ (no solutions), 2)- $y=y^*$ (one solution), 3)- $y<y^*$ (two solutions). It is clear, that the proposed picture holds for any shape of $\phi(z)$ as long as a finite value z_0 exists ($\phi(z_0)=0$).

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Fig 1

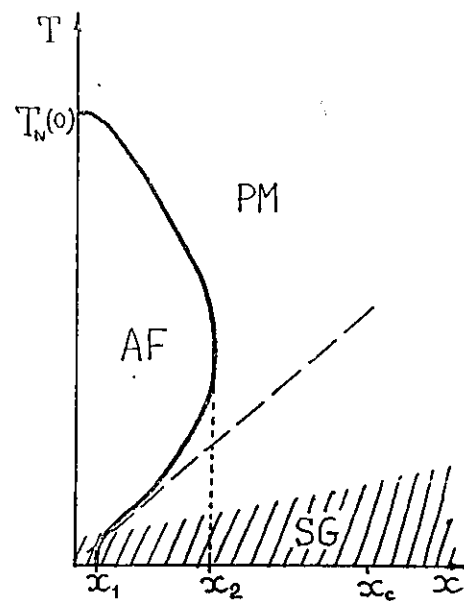


Fig 2

