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THE EFFECTS OF SPIN ORBIT COUPLING ON THE LANDAU LEVELS IN THE TILTED MAGNETIC FIELD

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The Effects of Spin Orbit Coupling on the Landau Levels in the Tilted Magnetic Field

The Hamiltonian in the presence of spin orbit coupling is

$$\hat{H} = \frac{\hbar^2 k^2}{2m} + \alpha(\vec{\sigma} \times \vec{k})\vec{z}$$

$\vec{\sigma}$ – Pauli matrices

α – the spin orbit coupling constant

$\vec{z}$ – a unit vector perpendicular to the surface

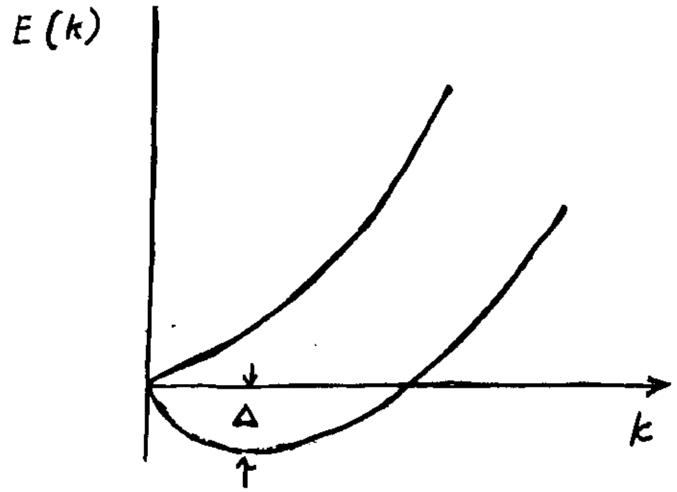
The electron energy dispersion related to the motion along the 2D plane is

$$E(k) = \frac{\hbar^2 k^2}{2m} \pm \alpha k$$

The essential energetical parameter is

$$\Delta = m\alpha^2 / 2\hbar^2$$

The schematic electron energy spectrum calculated from this model with an exaggerated spin orbital coupling constant α



The important characteristic of this dispersion relation is that the spins are degenerate at $k=0$ and the spin splitting linearly increases with wave vector k.

The zero-field spin splitting of electron energy levels in a quantu well can be caused by the lack of inversion symmetry either in the hos semiconductor bulk crystal or in the interface confinement potentia Both kinds of assymetry can exist in a system with a zinc-blende structure.

The spin splitting associated with the potential well assymetry has no explicit dependence on the well thickness. The constant α deduced from the k^3 contribution in conduction band Hamiltonian is proportional to

$$\left(\pi/d\right)^2 k$$

d - the layer width

J. Luo et al. observed the zero-field splitting caused by the spin orbital coupling associated with the well assymetry so that in the

InAs/GaSb system the contribution from the bulk assymetry to the constant α is effectively negligible.

In other 2D systems the contribution to α from the bulk assymetry can be comparable with that from the well assymetry.

The Landau spectrum in the magnetic field perpendicular to the 2D layer is given by

$$E_0 = \hbar\omega_c\delta, \quad E_n^{\pm}/\hbar\omega_c = n \pm \sqrt{\delta^2+\gamma^2 n} = \varepsilon_n^{\pm} \qquad n = 1, 2,$$

where the parameters are

$$\delta = 1/2 - \beta, \quad \beta = \frac{gm}{4m_0}$$

$$\gamma^2 = \frac{4\Delta}{\hbar\omega_c}$$

ω_c - cyclotron frequency

m_0 - free electron mass

g - g-factor

The characteristic parameters of different 2D systems are listed below

2D System	$\frac{m}{m_0}$	α, 10^{-10} eV.cm	Δ, meV	γ^2, $1/B_z[T]$
n- GaAs (D. Stein et al. ESR)	0.07	2.5	2.5×10^{-3}	6×10^{-4}
p- GaAs (H. Störmer et al., CR)	0.5	6	0.1	1.7
p - Si (S. Dorozbkin SdH)	0.35	1	2.5×10^{-3}	3×10^{-2}
n - InAs (J. Luo et al. SdH)	0.055	9	2.9×10^{-2}	5.5×10^{-2}
n- $In_xGa_{1-x}As$ (B. Das et al. SdH)	0.046			
$x = 0.65$		3.6	3.9×10^{-3}	6.2×10^{-3}
$= 0.60$		4.0	4.8×10^{-3}	7.7×10^{-3}
$= 0.53$		2.5	1.8×10^{-3}	2.9×10^{-3}

$$\alpha = (1 \div 9)\times10^{-10}\ \text{eV.cm}$$

$$\Delta \simeq (2\times10^{-3} \div 0.1)\ \text{meV}$$

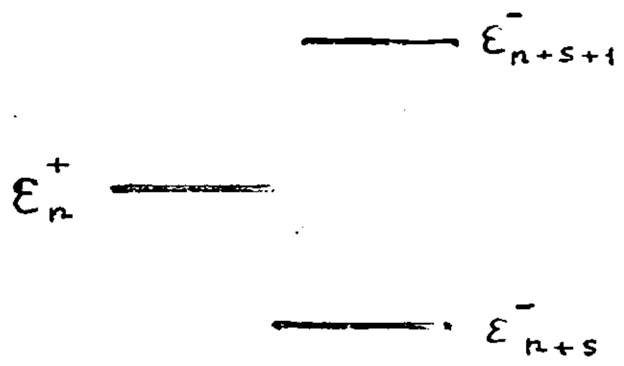
$$\gamma^2 = (6\times10^{-4} \div 1.7)\frac{1}{B_z[T]}$$

By changing the value of magnetic field B we can obtain two possibilities:

① It is possible that intersection of the levels belonging to the different Branches of the Landau Spectrum occurs

a) $$\varepsilon_n^+ = \varepsilon_{n+s}^- \ , \quad s = 1, 2, ..$$

② The energy level ε^+ is in the middle between two levels ε^- such as



b) $$2\varepsilon_n^+ = \varepsilon_{n+s+1}^- + \varepsilon_{n+s}^- \ , \quad s = 0, 1,$$

It seems that a very visible manifestation of the nonzero spin splitting will be two well-defined frequencies in the Shubnikov-de Haas (SdH) oscillations. This in turn will give rise to the series of beats in the amplitude of the SdH oscillations leading to a modulation of SdH amplitude given by

$$A \sim \cos \pi \frac{\Delta_{sp}(B)}{\hbar\omega_c}$$

Δ_{sp} is the energy separation between the spin-split Landau levels which is a function of the magnetic field B.

The most interesting case is at the condition that

$$\Delta_{sp} \neq 0 \quad \text{at} \quad B = 0$$

In our case

$$\Delta_{sp}(B=0) = 2\alpha k_F$$

k_F - the Fermi wave vector determined by the total number of electrons in the system

In the quasiclassical limit the condition b) is

$$\delta^2 + \gamma^2 n = \left(\frac{2s+1}{4}\right)^2$$

If we take

$$\delta^2 \ll 1 \,, \quad n \simeq \frac{E_F}{\hbar\omega_c} \gg 1$$

than the condition b) is equivalent to the relation

$$\frac{2\alpha k_F}{\hbar\omega_c} = s + 1/2 = \frac{\Delta_{sp}(B=0)}{\hbar\omega_c}$$

So the nodes in the beat pattern in the SdH oscillation will occur $(B \to 0)$ at half-integer values of $\frac{\Delta_{sp}(B=0)}{\hbar\omega_c}$

(B. Das et al.)

The basic assumption for applying the tilted magnetic field is that the electron spin splitting is determined by the total magnetic field but orbital motion is determined by the field component perpendicular to the 2D layer.

For a 2D system with spin orbit coupling we might expect the spin splitting to depend on the magnetic field (both magnitude and orientation) in a more complicated way

Landau spectrum in the tilted magnetic field

The basic relations

In the tilted magnetic field the wave function of the electron is of the form

$$\Psi(x,y) = e^{iky} \begin{pmatrix} \sum\limits_{n=0}^{\infty} a_n \varphi_n (x - k l_H^2) \\ \sum\limits_{m=0}^{\infty} b_m \varphi_m (x - k l_H^2) \end{pmatrix}$$

$\varphi_n(u)$ – the oscillatory wave function

The equations for coefficients a_n, b_m are given by:

$$(\varepsilon - n - 1/2 - \beta) a_n = \beta \, tg\theta \cdot a_n + \gamma \sqrt{n+1}\, b_{n+1}$$

$$(\varepsilon - n - 1/2 + \beta) b_n = \beta \, tg\theta \cdot b_n + \gamma \sqrt{n}\, a_{n-1}$$

$$\varepsilon = E/\hbar\omega_c$$

θ – the tilting angle

The equations can be transformed to the equations for coefficients a_n and b_n only

$$\alpha_n b_{n-1} - \beta_n b_n + \alpha_{n+1} b_{n+1} = 0$$

$$\alpha_n' a_{n-1} - \beta_n' a_n + \alpha_{n+1}' a_{n+1} = 0$$

And

$$\alpha_n(\varepsilon) = \frac{\beta\gamma\sqrt{n}\, tg\theta}{\varepsilon - n + 1/2 - \beta}$$

$$\beta_n(\varepsilon) = \varepsilon - n - 1/2 + \beta - \frac{\gamma^2 n}{\varepsilon - n + 1/2 - \beta} - \frac{\beta^2 tg^2\theta}{\varepsilon - n - 1/2 - \beta}$$

The expressions for the coefficients α_n', β_n' are similar.

The energy spectrum in the tilted magnetic field can be found from the condition that the correspondi[ng] determinant is equal to zero.

$$
t\begin{pmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdots\ \alpha_{n-1} & -\beta_{n-1} & \alpha_n & 0 & 0 & \cdots \\
\cdots\ 0 & \alpha_n & -\beta_n & \alpha_{n+1} & 0 & \cdots \\
\cdots\ 0 & 0 & \alpha_{n+1} & -\beta_{n+1} & \alpha_{n+2} & \cdots
\end{pmatrix} = 0
$$

In the case $\gamma \cdot tg\Theta = 0$:

1) $\gamma = 0$ – absence of spin orbital coupling

2) $\Theta = 0$ – the magnetic field is perpendicular to the 2D layer

the matrix is diagonal ($\alpha_n \equiv 0$) and the exact spectrum can be found from equations $\beta_n(\varepsilon) = 0$

In the general case $\gamma \cdot tg\theta \neq 0$ the spectrum can not be found exactly

The most interesting conclusions

A) In the case $\gamma \cdot tg\Theta \neq 0$ the intersection of the Landau levels is forbidden

The experimentally most interesting situation

$$\gamma^2 \ll 1 \quad \text{but} \quad \gamma^2 \cdot n \sim 1$$

B) It can be shown that in that case in ~~...~~ ~~...~~ energetical interval $\hbar\omega_c$ there are two Landau levels exactly

The results of the numerical calculations of the energy spectrum

The calculation were performed for two situations

I) The tilting angle Θ is fixed and the total magnetic field is changing

II) The total magnetic field is fixed while the tilting angle Θ is changing all calculations were performed with values of the parameters which are near those for the system

InAs/GaSb (J. Luo et al.)

$$\overline{B}_t = B/B_0 \qquad B = \frac{m}{m_0} \cdot \frac{\alpha k_F}{2\mu_B} \approx 1T$$

μ_B – the Bohr magneton

The energy is in the units of the cyclotron energy $\hbar\omega_c$

From the results of numerical calculations we can conclude that at the presence of spin orbit coupling the drastic changing of the spectrum in tilted magnetic field occurs at such total magnetic fields when the spin Seeman splitting is comparable with the cyclotron energy

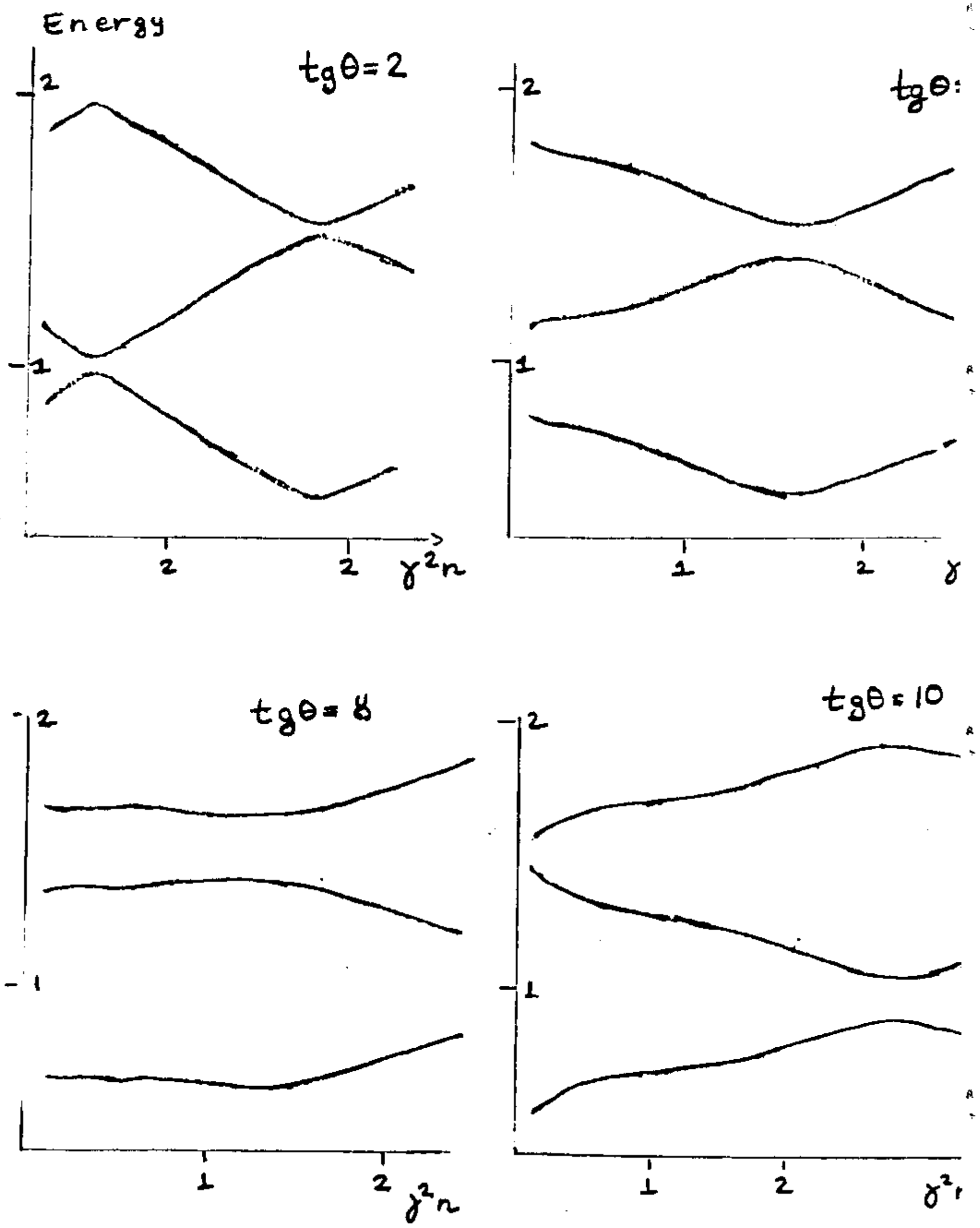
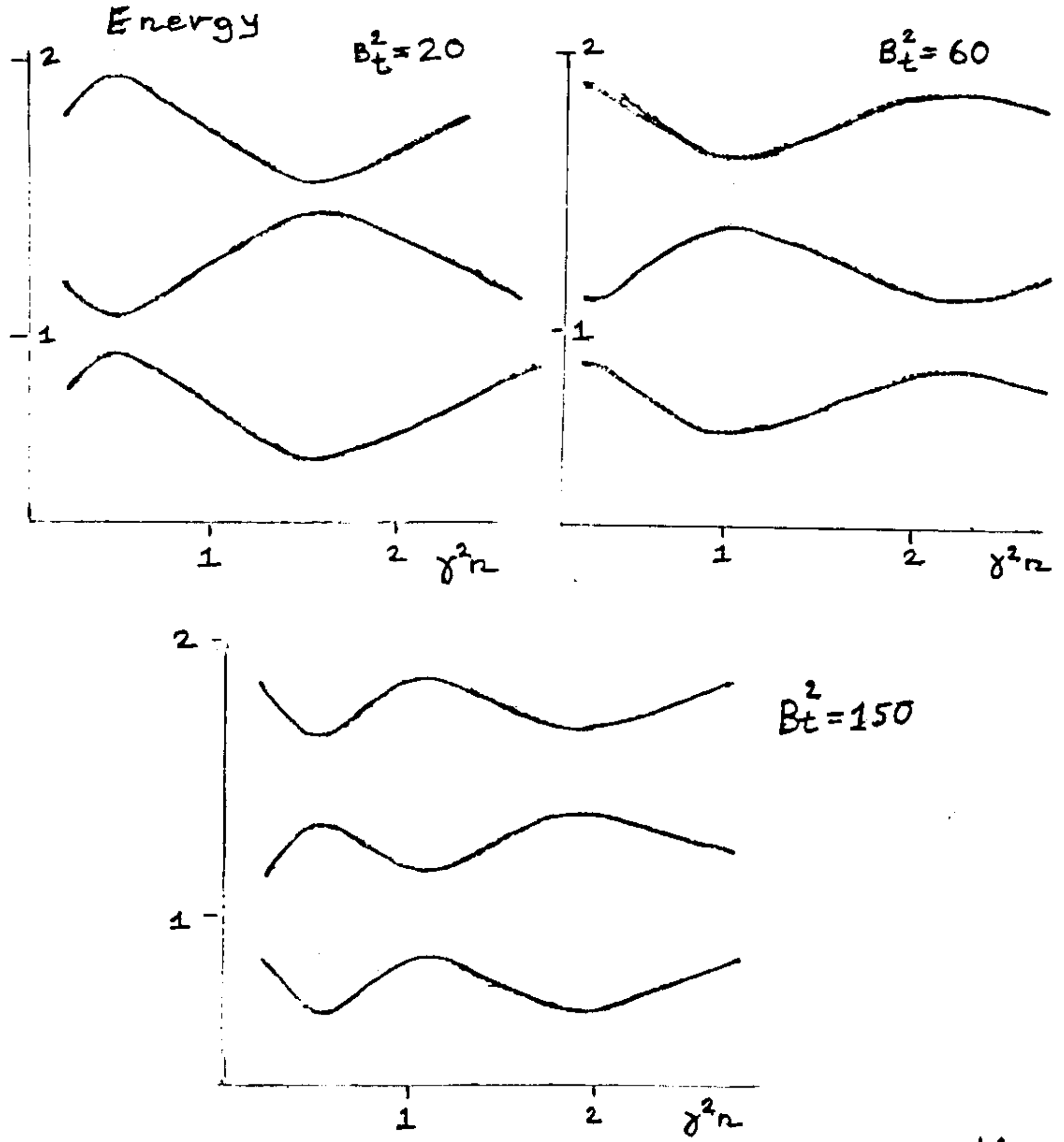


Fig. 1



$$tg\,\Theta = (B_t^2 \gamma^2 n - 1)^{1/2}$$

Fig. 2