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SMR.459 28

SPRING COLLEGE IN CONDENSED MATTER ON:
"PHYSICS OF LOW-DIMENSIONAL STRUCTURES"

(23 APRIL - 15 JUNE 1990)

QUANTUM ADIABATIC ELECTRON TRANSPORT IN BALLISTIC CONDUCTORS

(PART I)

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QUANTUM ADIABATIC TRANSPORT IN BALLISTIC CONDUCTORS

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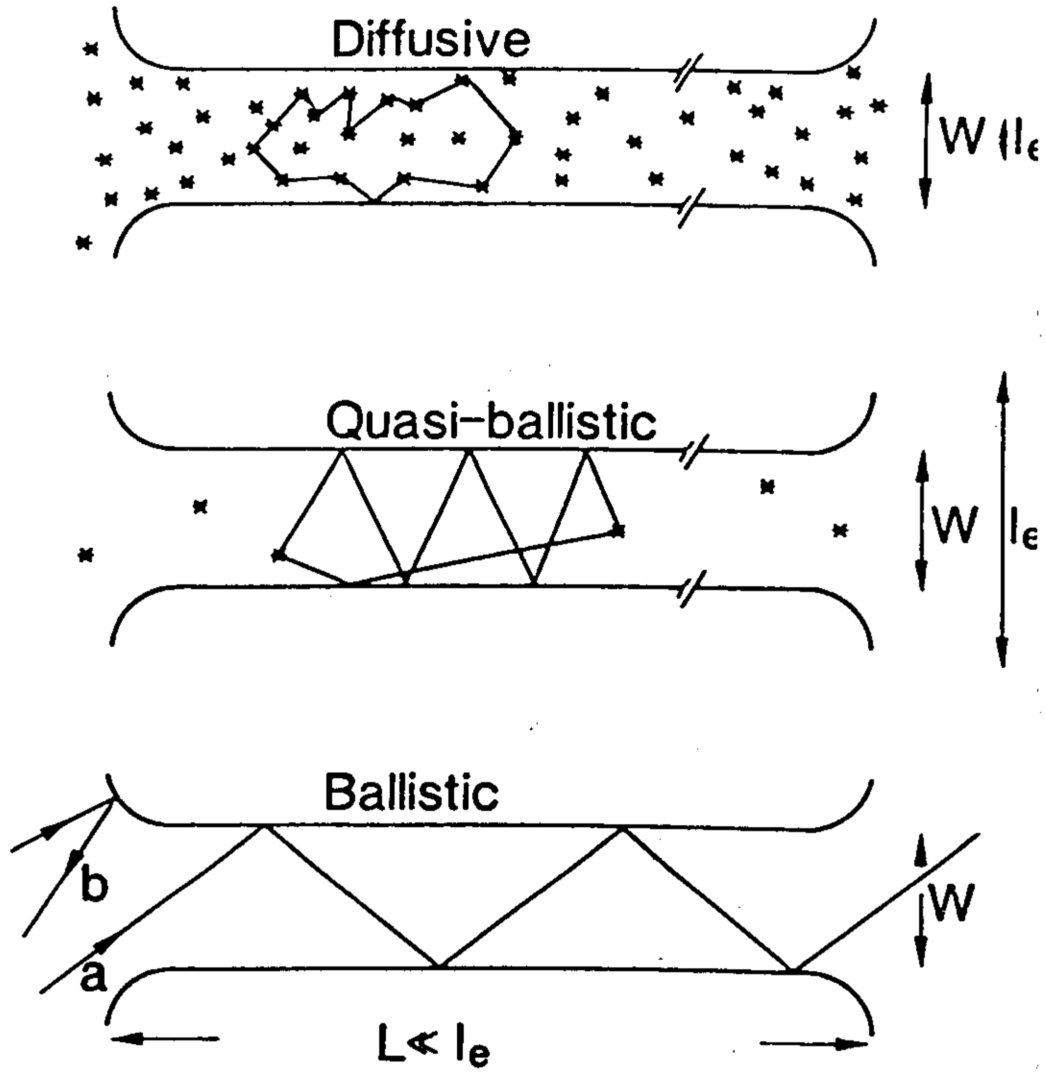
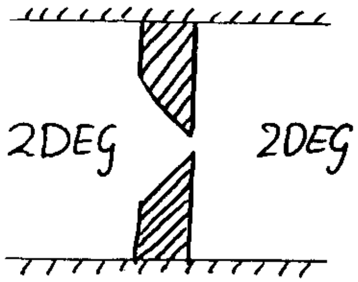
TRANSPORT REGIMES



Quantum Ballistic if $\lambda_F \sim W$

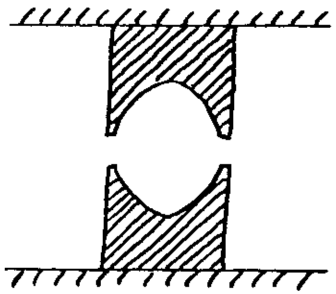
Outline

quantum point contact (QPC)



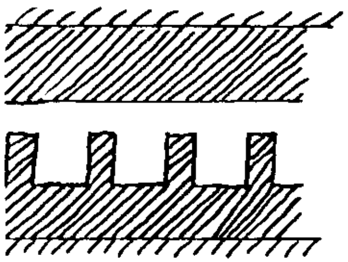
- quantized conductance

quantum dot



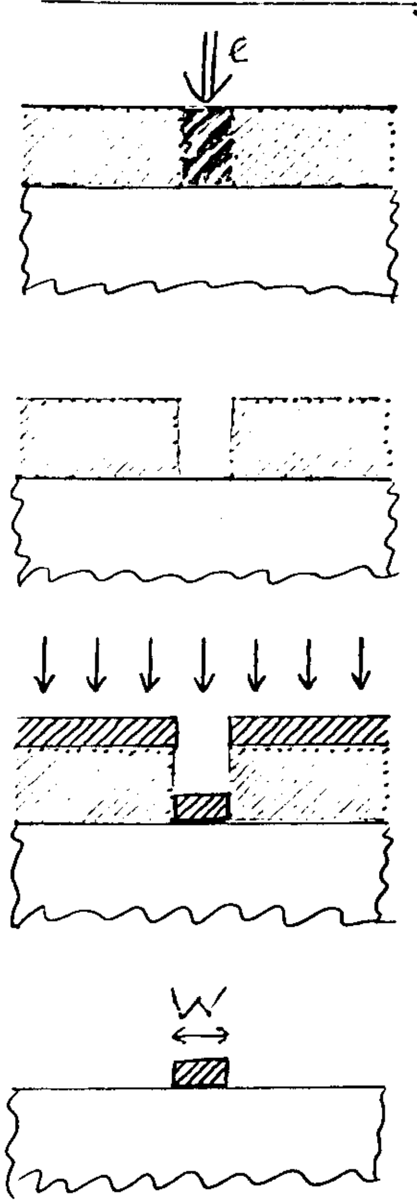
- two QPCs in series
→ adiabatic transport (edge channels)
- 0D - states

coupled quantum dots



- finite 1D crystal
→ mini-band structure

NANO-LITHOGRAPHY



electron - beam

~100 nm organic resist [illegible]

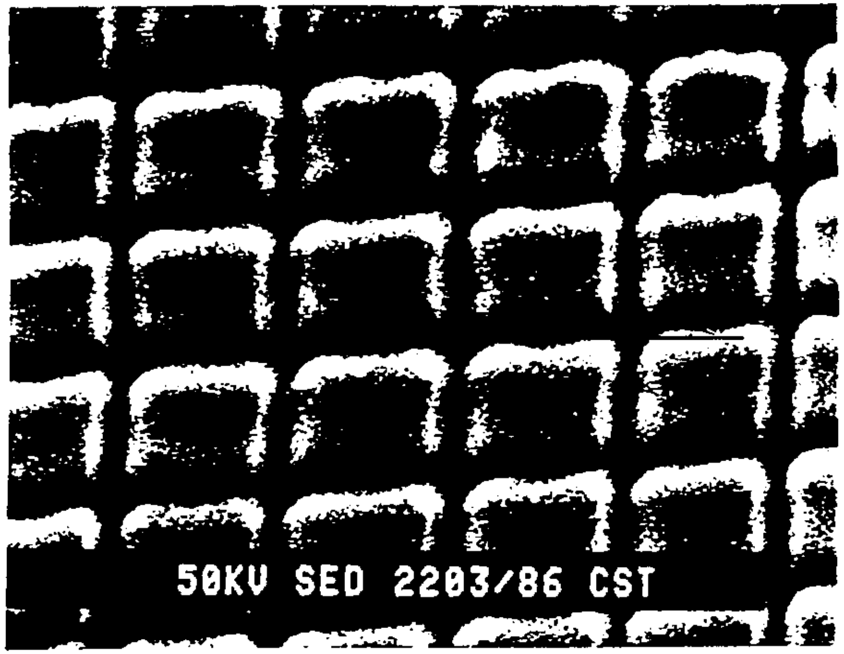
substrate

after development
→ mask

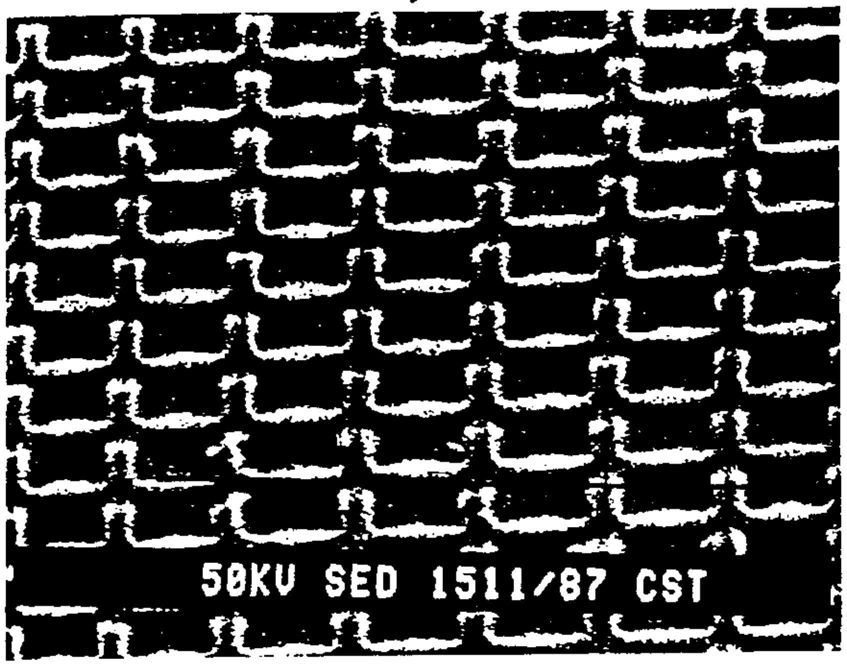
evaporation (Au)

after lift-off

- resolution : $W \approx 10$ nm

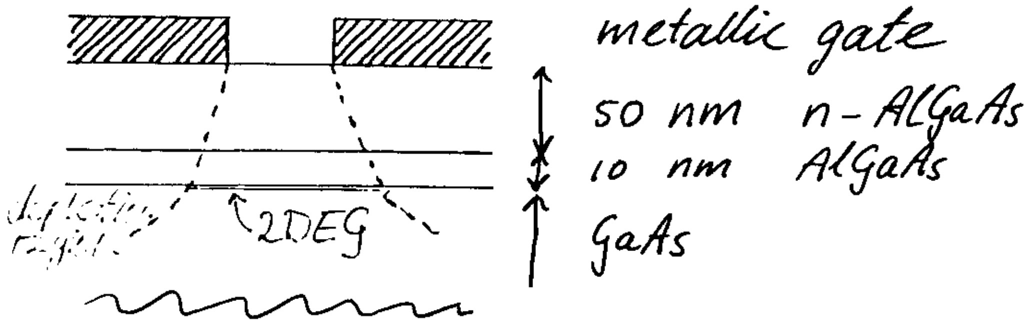


period = 200 nm ; width ≈ 30 nm

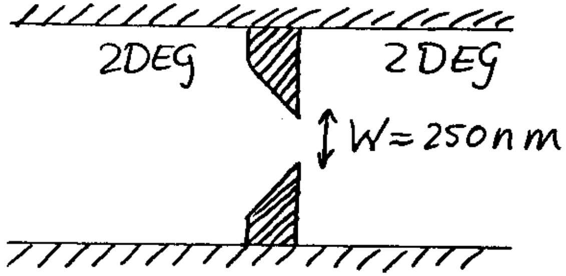


SPLIT GATE HETERO STRUCTURE

Cross-section



Top-view



$n_s = 3.5 \cdot 10^{15}\ m^{-2}$

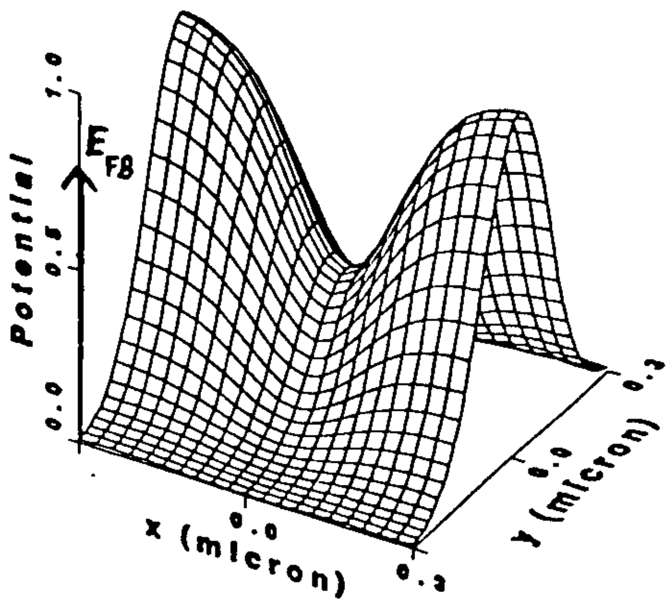
$\lambda_F = \left(\frac{2\pi}{n_s}\right)^{1/2} = 42\ nm$

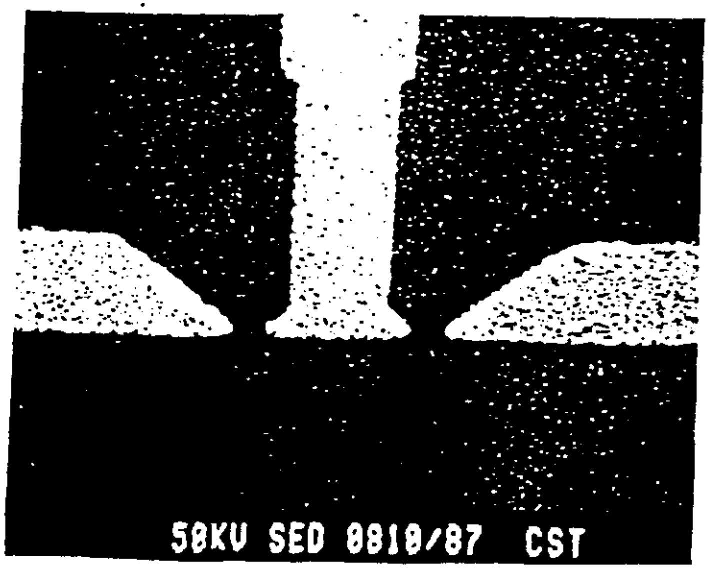
$E_F = 13\ meV$

$\mu = 100\ m^2/Vs$

$l_e = 10\ \mu m \rightarrow$ ballistic regime

Quantum point contact





Sharvin Resistance of a 2D-point contact

1/ Ballistic Regime

$l_e \gg W, L$

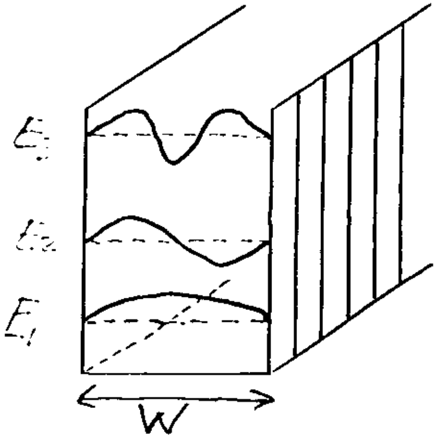
$R_S = \frac{h}{2e^2} \cdot \frac{\lambda_F}{2W} \qquad G_S = \frac{2e^2}{h} \cdot \frac{2W}{\lambda_F}$

2/ Quantum Ballistic Regime

$W \sim \lambda_F \qquad \left(\lambda_F = \left(\frac{2\pi}{n_s}\right)^{1/2} \right)$

2D d.o.s. → splits into 1D d.o.s

particle in a 1D box



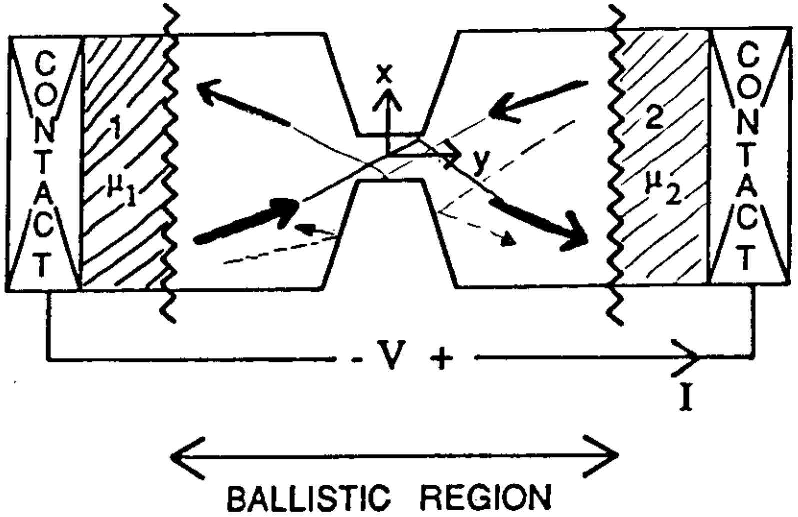
$W = \frac{1}{2} n \lambda_F$

if $W = \frac{1}{2} N_c \lambda_F$

$\rightarrow G = \frac{2e^2}{h} \cdot N_c$

CONSTRICTION CONDUCTANCE G_C

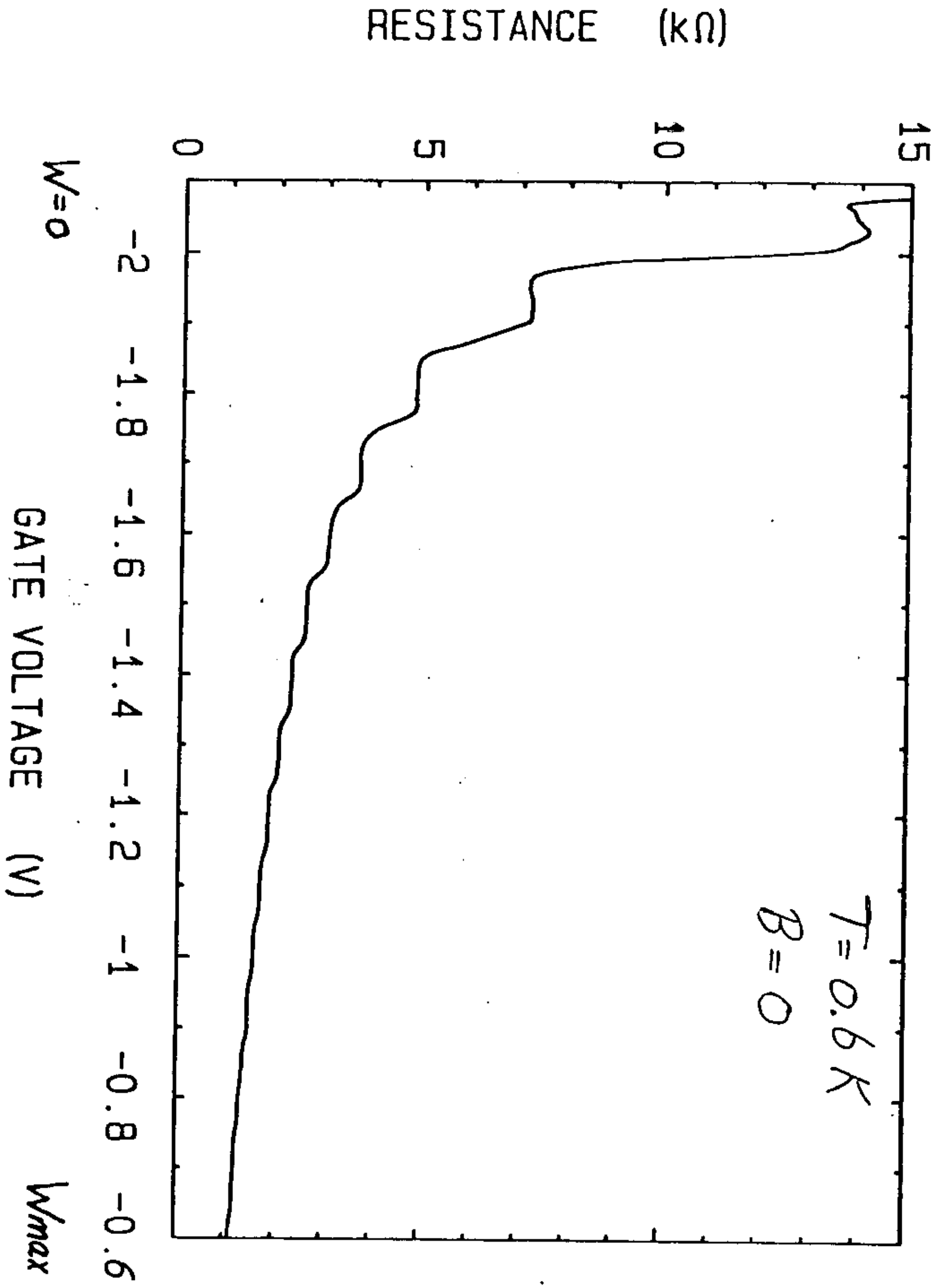
Sharvin point contact



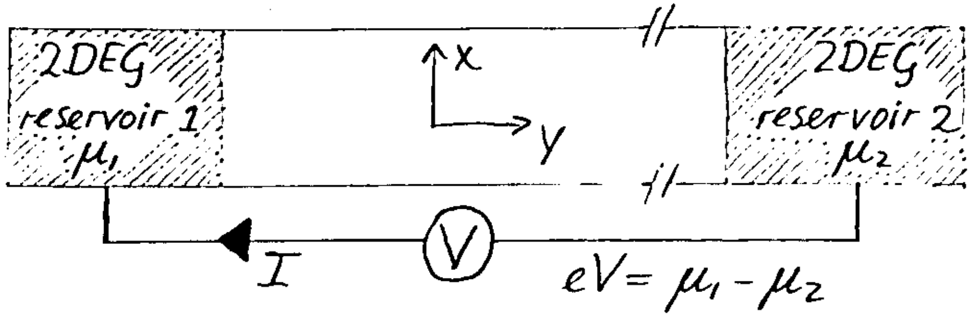
$$eV = \mu_2 - \mu_1$$

$$\boxed{G_C = \frac{e\,I}{\mu_2 - \mu_1} = \frac{I}{V}}$$

2-terminal conductance



1D - Transport

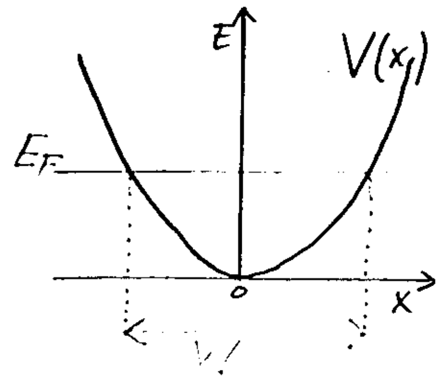


$eV = \mu_1 - \mu_2$

$$H = \frac{P_x^2}{2m^*} + V(x) + \frac{P_y^2}{2m^*}$$

model for boundary potential:

$$V(x) = \tfrac{1}{2} m^* \omega_0^2 x^2$$



width W at classical turning points:

$$W = \left\{ \frac{8 E_F}{m^* \omega_0^2} \right\}^{1/2}$$

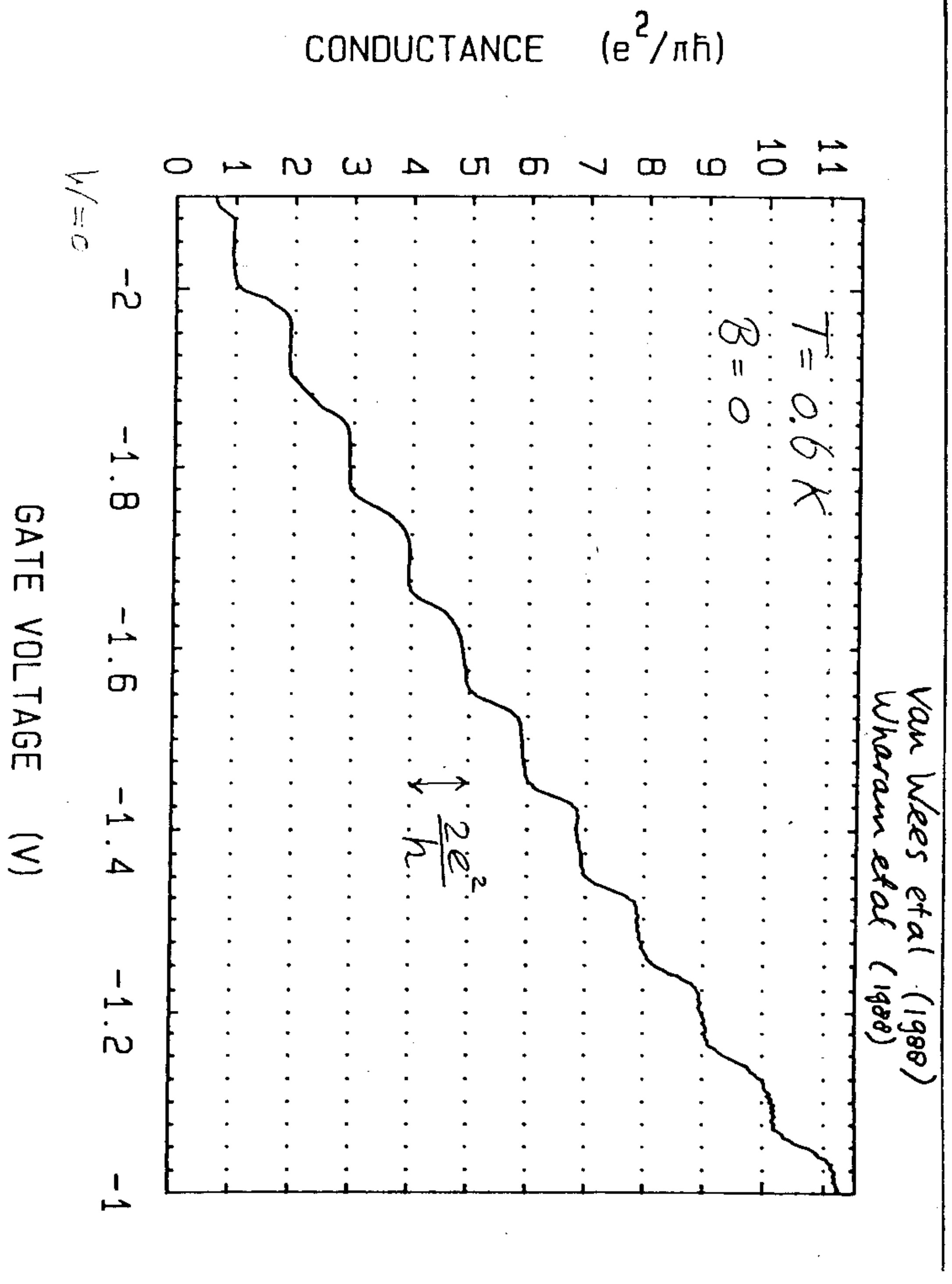
E_F = Fermi energy in channel

solutions of Schrödinger equation

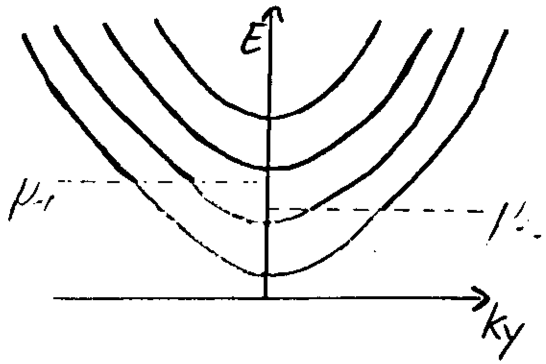
→ plane waves in longitudinal direction

→ harmonic oscillator in transverse direc

$$E_n = (n - \tfrac{1}{2})\hbar\omega_0 + \frac{\hbar^2 k_y^2}{2m^*}$$



1 D Subbands



empty states
occupied states

$$eV = \mu_1 - \mu_2$$

$$I = neV = e \sum_{n=1}^{N_c} \int_{\mu_2}^{\mu_1} dE \cdot \frac{1}{2} \frac{dN_n}{dE} \cdot V_n \cdot T_n$$

$$\left.\begin{array}{ll} \text{1D d.o.s :} & \frac{dN_n}{dE} = \frac{2}{\pi}\left(\frac{dE_n}{dk_y}\right)^{-1} \\ \text{velocity :} & V_n = \frac{1}{\hbar}\left(\frac{dE_n}{dk_y}\right) \end{array}\right\} \frac{dN_n}{dE} \cdot V_n = \frac{2}{\pi\hbar}$$

$$\rightarrow I = \frac{2e}{h}(\mu_1 - \mu_2) \sum_{n=1}^{N_c} T_n$$

$$\rightarrow G = \frac{I}{V} = \frac{2e^2}{h} \sum_{n=1}^{N_c} T_n$$

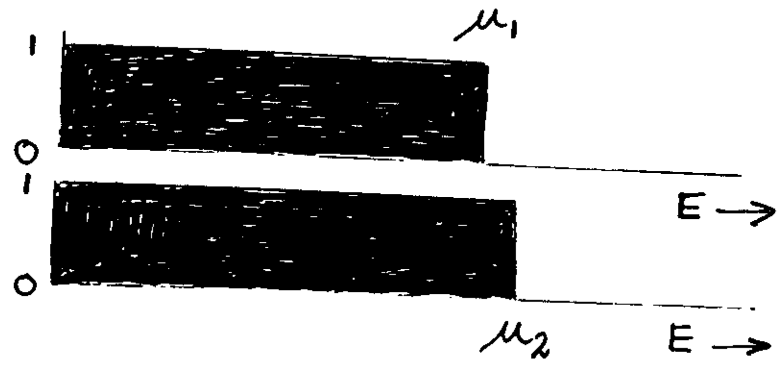
$$\text{if } T_n = 1 \Rightarrow G = N_c \frac{2e^2}{h}$$

(Landauer - Bütticker formalism)

$$\delta V_g \rightarrow \delta E_F \rightarrow \begin{cases} \Delta N_c = 0 \rightarrow G \text{ at plateau} \\ \Delta N_c = \pm 1 \rightarrow \Delta G = \frac{2e^2}{h} \end{cases}$$

Effect of finite temperature

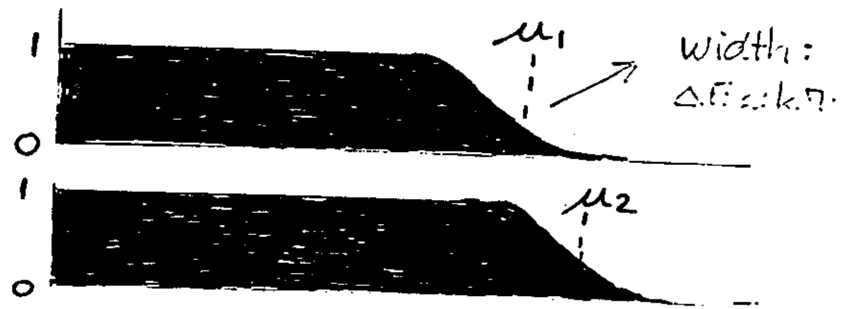
T = 0



$$\mu_1 - \mu_2 = eV$$

$$eV \rightarrow 0 \rightarrow \boxed{G = \frac{2e^2}{h} T(E_F)}$$

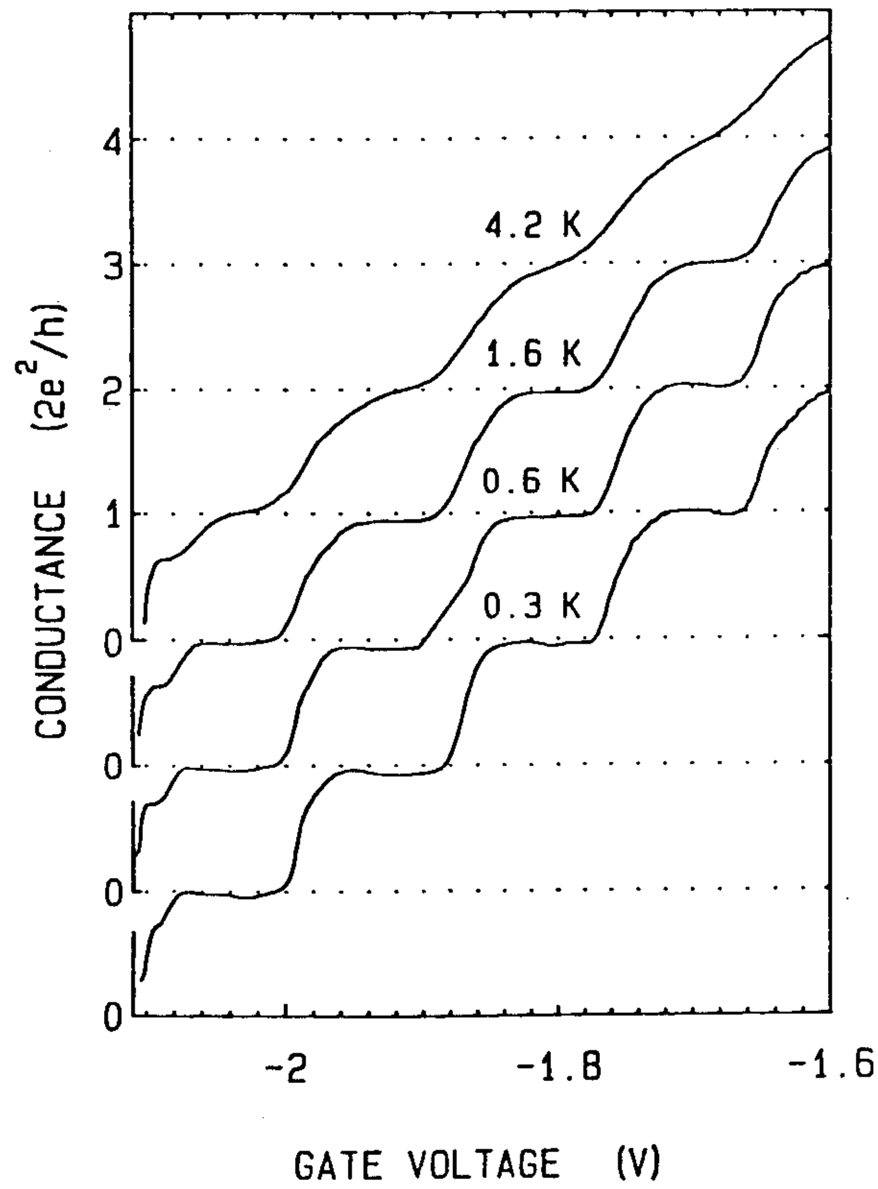
T > 0



FERMI-DIRAC distribution: $f(E) = \frac{1}{1 + \exp\left\{\frac{E - E_F}{kT}\right\}}$

$$\boxed{G = \frac{2e^2}{h} \int \frac{\partial f}{\ldots} T(E) dE}$$

$3.5 * k_B T(4.2K) = 1.3\ meV$

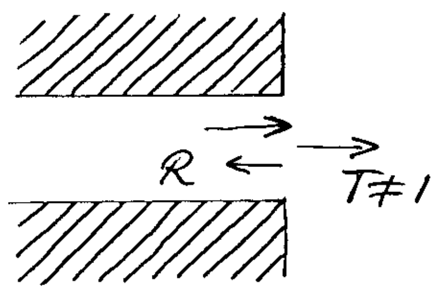


Breakdown of quantization

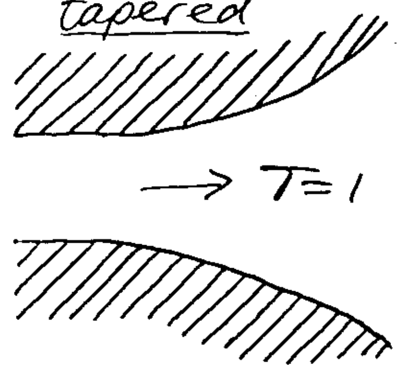
- temperature $k_B T \sim \Delta E_n$
  $\{k_B T(1K) = 0.1\ meV\}$

- Transmission $T \neq 1$ (backscattering)
  → impurities
  → reflection at entrance or exit of channel

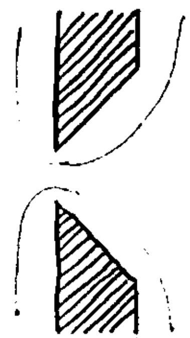
abrupt



tapered



→ smooth boundary potential in quantum point contact



numerical calculations show that a smooth

Magnetic field perpendicular to 2DEG

$\vec{p} \to \vec{p} + e\vec{A}$ Landau gauge: $\vec{A} = A_y = B\cdot x$
$(\vec{B} = B_z)$

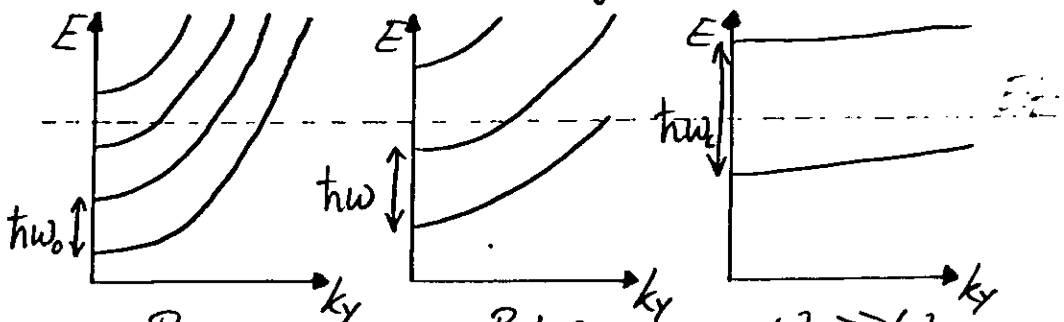
Schrödinger eq. → 1D harmonic oscillator

$$E_n = (n-\tfrac{1}{2})\hbar\omega + \frac{\hbar^2 k_y^2}{2m_B^*}$$

with: $\omega^2 = \omega_0^2 + \omega_c^2$; cyclotron freq.: $\omega_c = \frac{eB}{m^*}$

$m_B^* = m^* \frac{\omega^2}{\omega_0^2}$ → larger B ⇒ larger effective mass

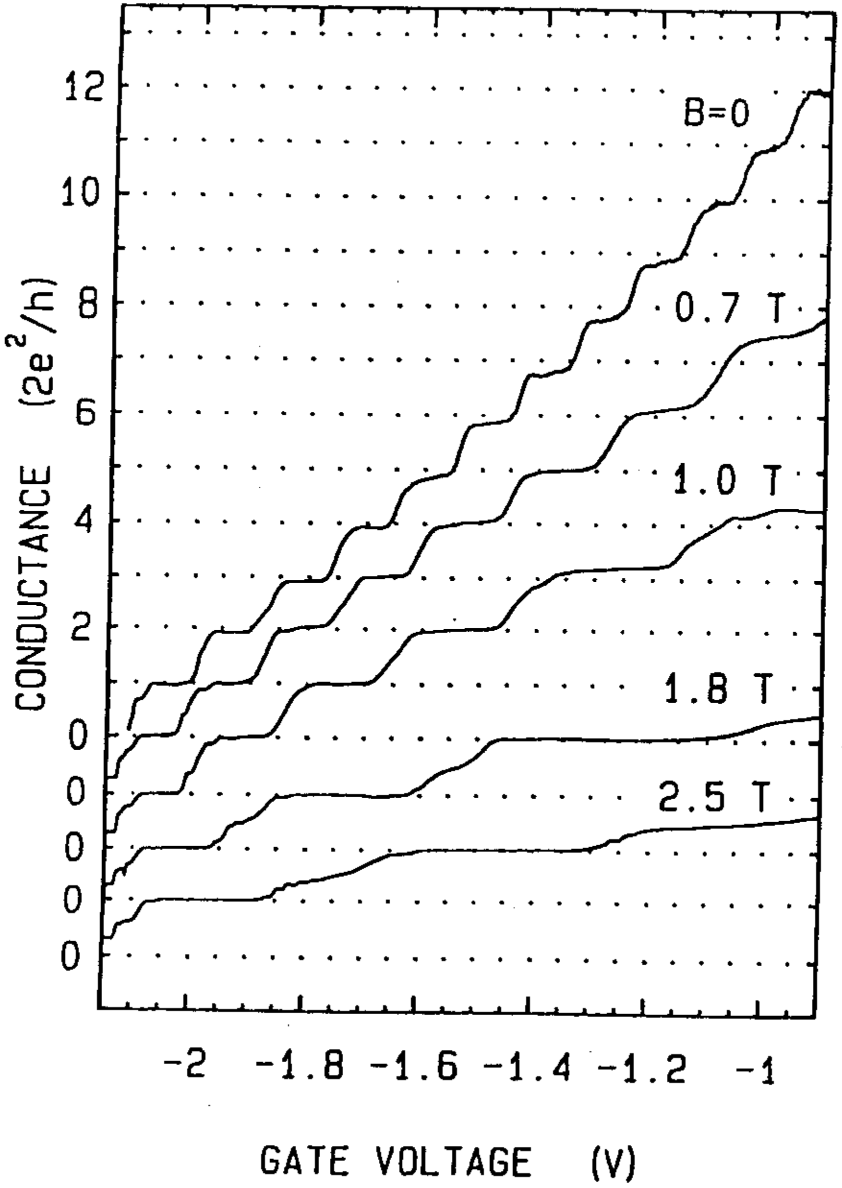
→ Hybrid electric-magneto subbands



$B=0$ electric | $B\neq 0$ hybrid | $\omega_c \gg \omega_0$ magneto (Landau levels)

$G = \frac{2e^2}{h} N_c$ with $N_c = \text{trunc}\left[\frac{E_F}{\hbar\omega} + \frac{1}{2}\right]$
(spin degenerate)

⇒ Transition from quantized conductance to quantum Hall effect.



Conclusions

- 1D transport → quantized conductance
  (quantization accuracy ≈ 0.1 %)

- device : Quantum Point Contact
  → short, narrow and smooth
  constriction in a 2DEG

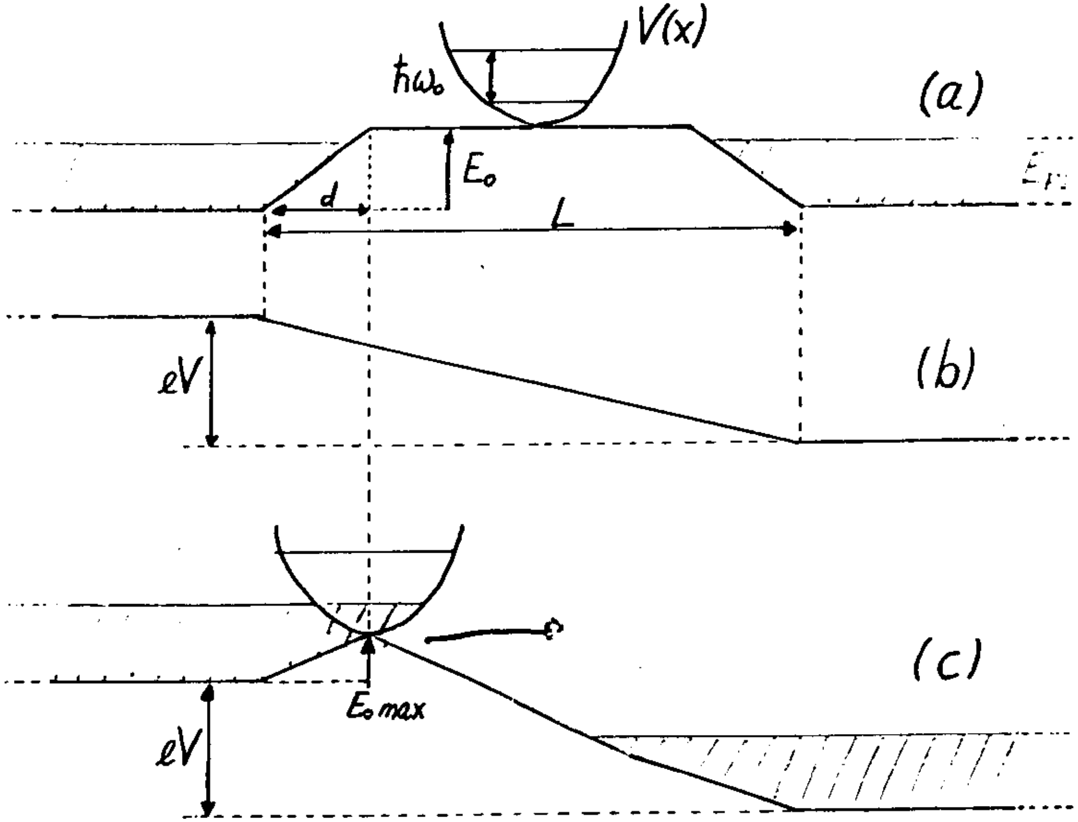
- gradual transition from 'quantized
  conductance at $B=0$ to
  quantum Hall effect at high B.

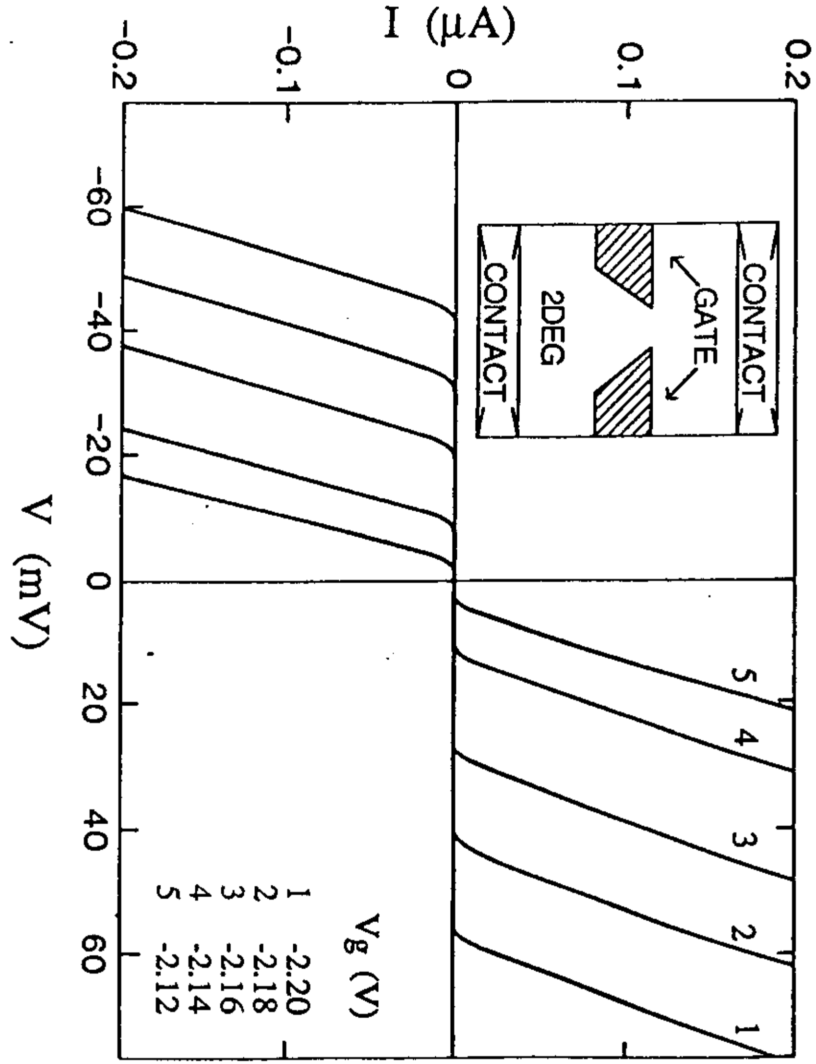
NON-LINEAR CONDUCTANCE OF QUANTUM POINT CONTACTS

- model non-linear 1D transport
- current-voltage characteristics
  ($B=0$, $B \neq 0$)
- additional plateaus

Non-linear Transport

a/ $G=0$ for small V (pinch-off)





$B=0$ $T=0.6K$

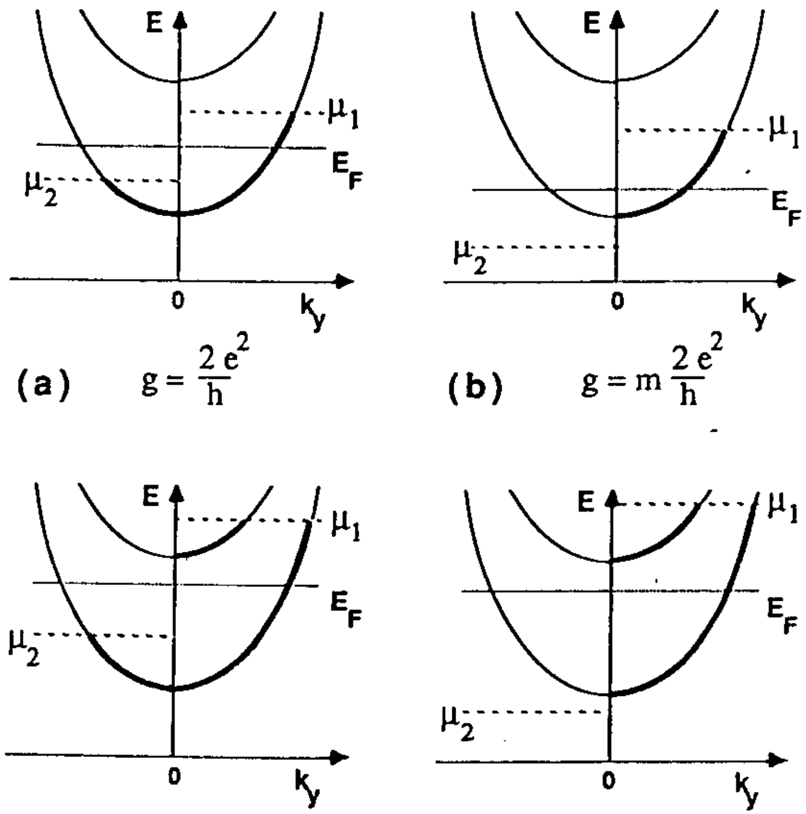
$a+b=c \longrightarrow E_{0max} = E_0 - \frac{d}{L}\cdot eV$

$E_{FB} - (E_{0max} + \frac{1}{2}\hbar\omega_0) < 0 \longrightarrow I=0$

$I>0 \longrightarrow \frac{dI}{dV} = \frac{2e^2}{h}\cdot\frac{d}{L}$

$$eV = \mu_1 - \mu_2$$

$$\mu_1 = E_F + meV$$
$$\mu_2 = E_F - (1-m)eV$$
$$m \approx 0.5$$



(a) $g = \frac{2e^2}{h}$

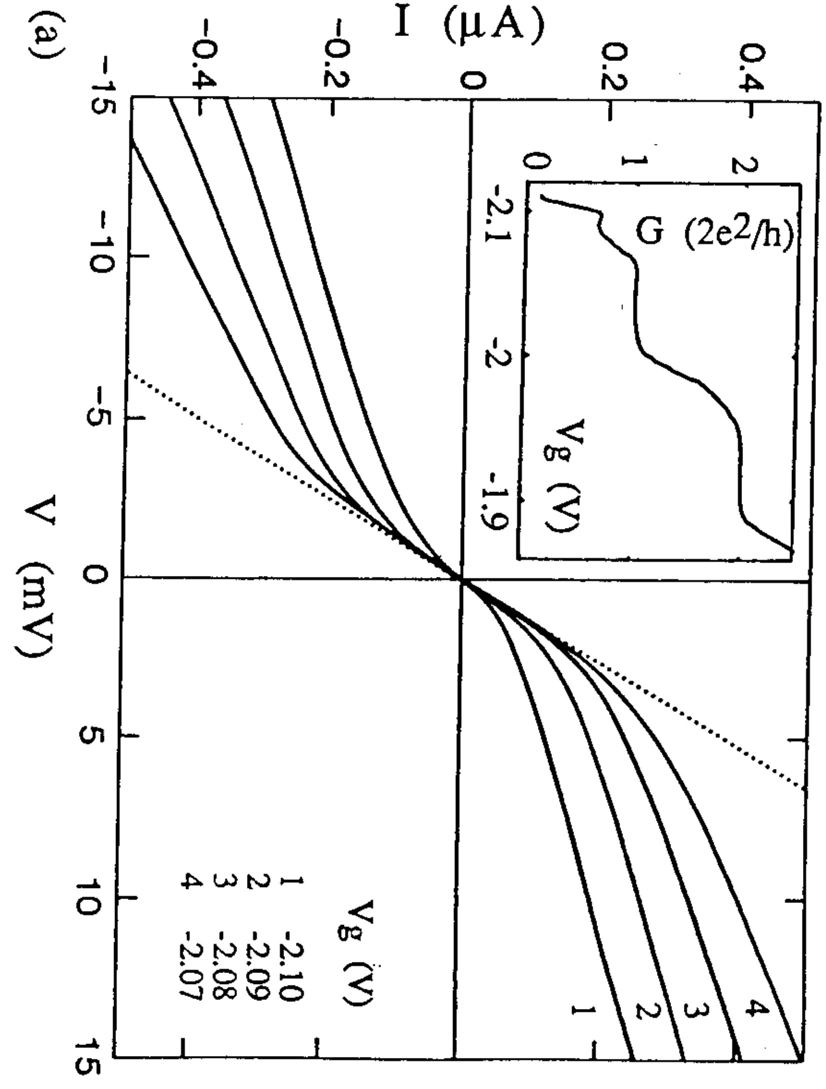
(b) $g = m\frac{2e^2}{h}$

(c) $g = (1+m)\frac{2e^2}{h}$

(d) $g = 2m\frac{2e^2}{h}$

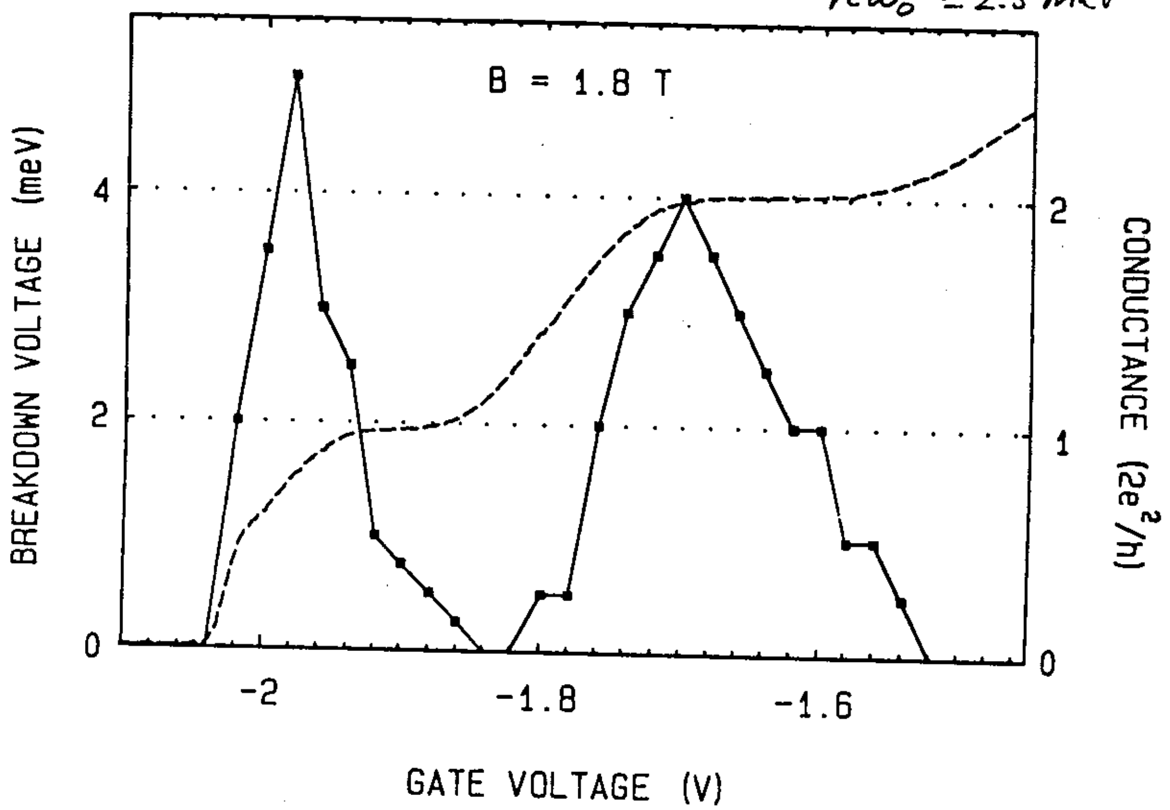
$$g = \frac{\partial I}{\partial V}$$

$$eV_{BR,max} = \Delta E$$

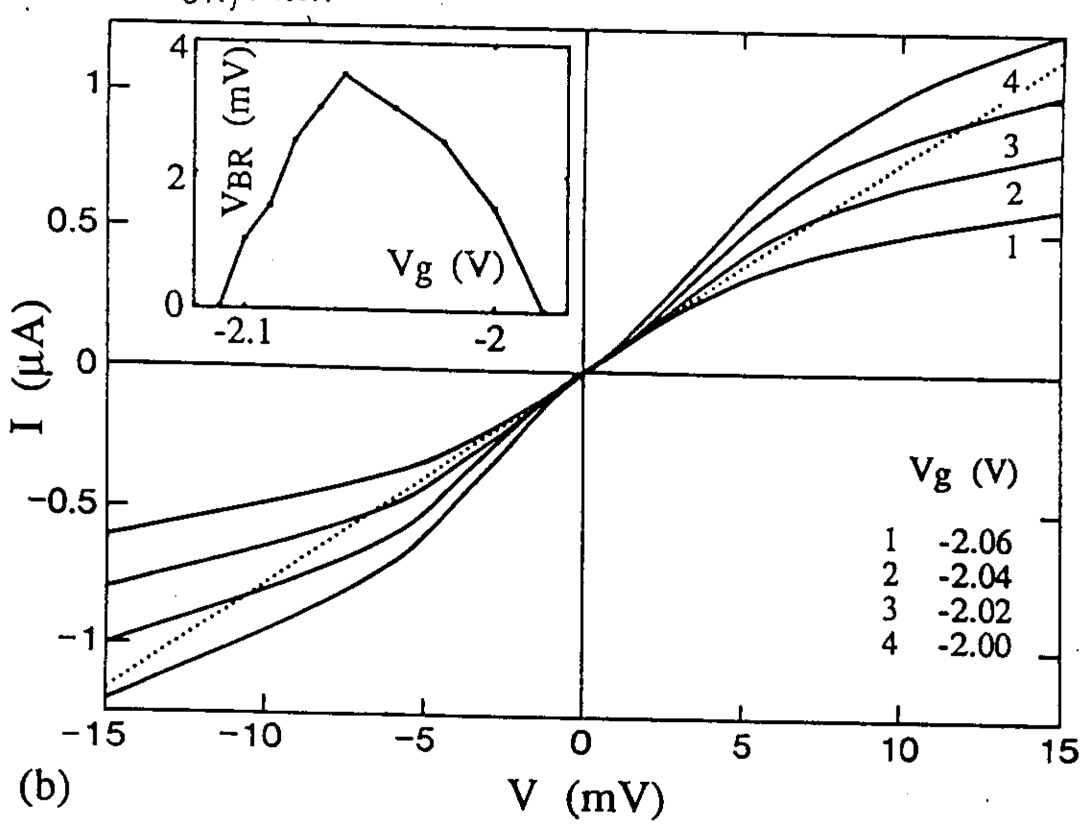


$$\hbar\omega = \hbar\sqrt{\omega_0^2 + \omega_c^2} \quad ; \quad \omega_c = \frac{eB}{m^*}$$

$$B = 1.8T \longrightarrow \hbar\omega_c = 3.1\,meV \longrightarrow \hbar\omega_0^1 = 3.9\,meV$$
$$\longrightarrow \hbar\omega_0^2 = 2.5\,meV$$



$V_{BR,max} = 3.5\,mV$



(b)

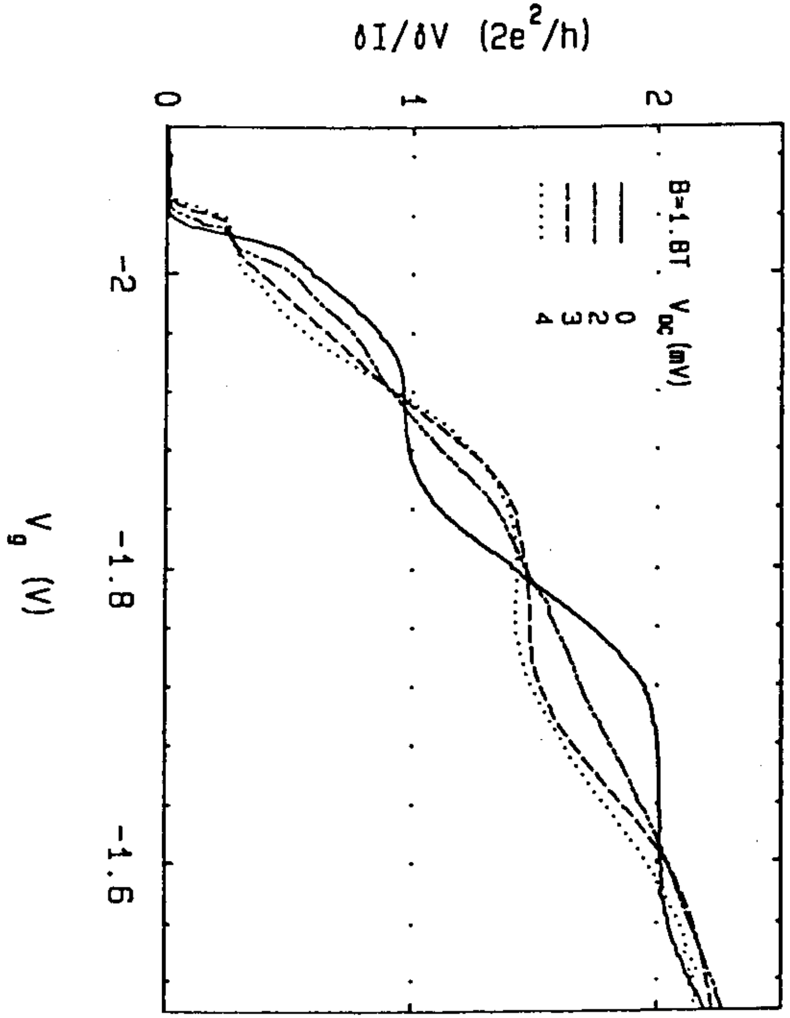
CONCLUSIONS

- I-V char. reveal the occupation of the 1D-subbands for the individual velocity-directions.

- Breakdown of the quantization, whenever the number of occupied subbands becomes different for the two velocity directions.

- Method to determine energy separation between subbands from breakdown-triangle-maximum.

- Ballistic transport is maintained up to an energy difference of about E_F.



Glazman & Khaetskii Jetp. Lett. 48, 591 (1988)

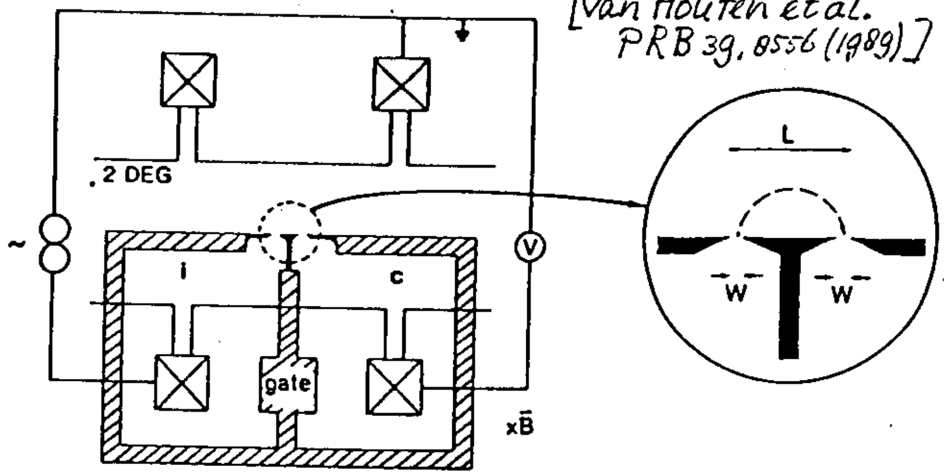
Electron motion in a B-field

- cyclotron orbits
- skipping orbits
→ electron focussing

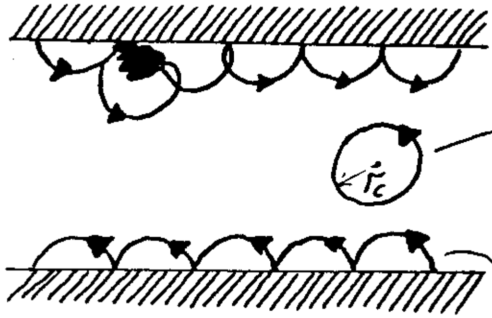
- edge channels
→ Quantum Hall effect.

Skipping Orbits - Electron Focussing

[van Houten et al. PRB 39, 8556 (1989)]



- electron motion in a perpendicular B-field.



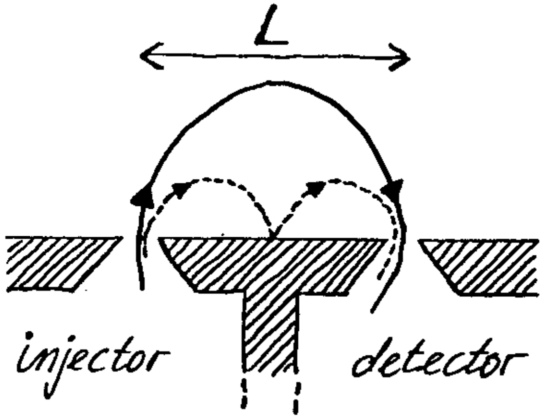
$F_L = e v \times B = \frac{m^* v^2}{r}$

cyclotron orbit

$\frac{v}{r_c} = \omega_c = \frac{eB}{m^*}$

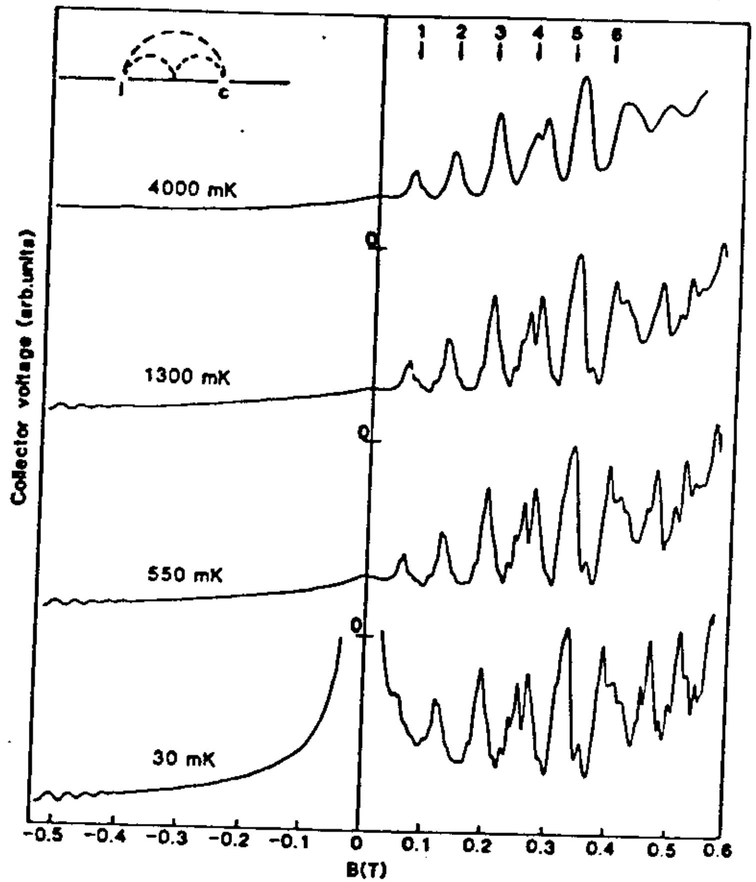
radius   frequency

skipping orbit

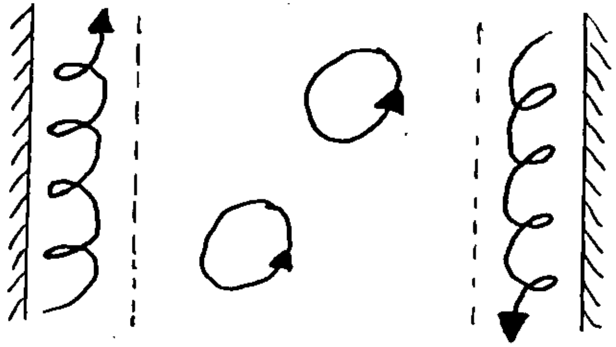


$n \cdot 2 r_c = L$

$B_f = \frac{2 m^* v_F}{eL}$

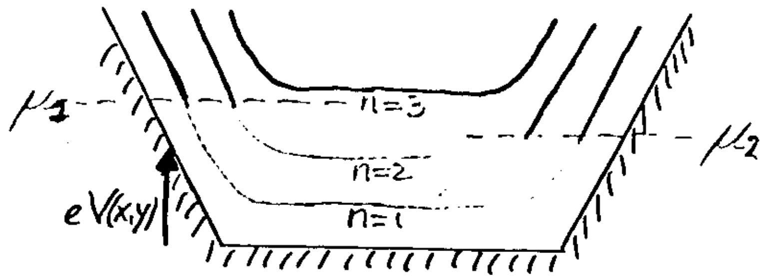


EDGE CHANNELS



$r_c = \frac{m^* v_F}{eB}$, $\omega_c = \frac{eB}{m^*}$, $v_D = \frac{E}{B}$

→ Landau levels: $E_n = (n-\frac{1}{2})\hbar\omega_c \pm \frac{1}{2} g \mu_B B + eV(x,y)$

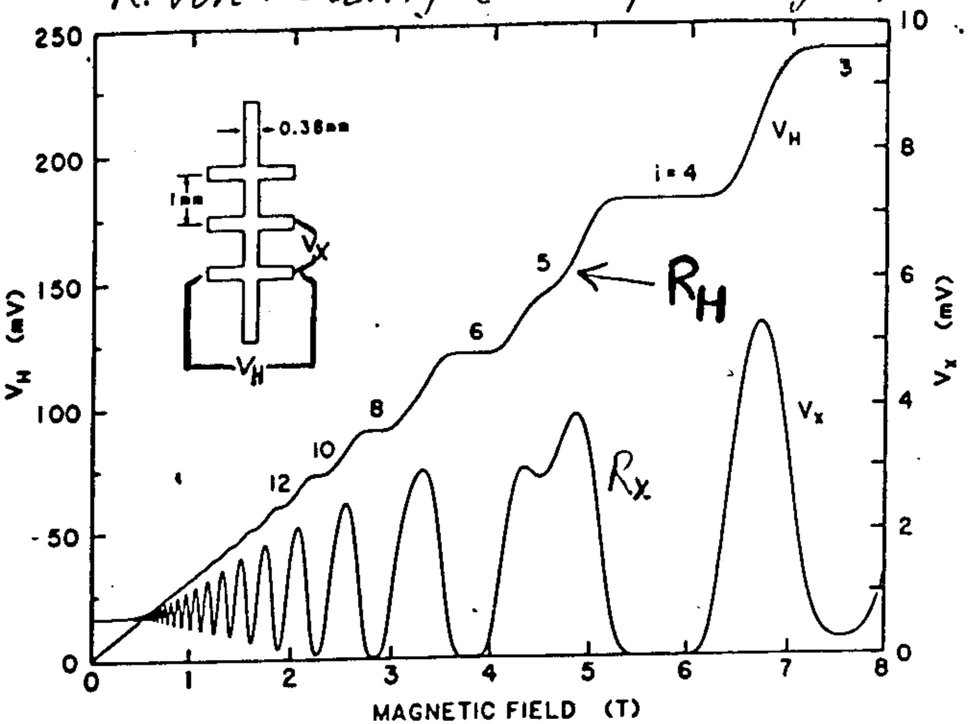


$eV_H = \mu_1 - \mu_2$; $I = \frac{e}{h}(\mu_1 - \mu_2)\cdot N_L$

- Transport in edge channels is 1D.
- Absence of Backscattering. (M. Büttiker)
- Adiabatic transport: conservation of n.

Quantum Hall Effect

K. von Klitzing (nobelprize 1985)



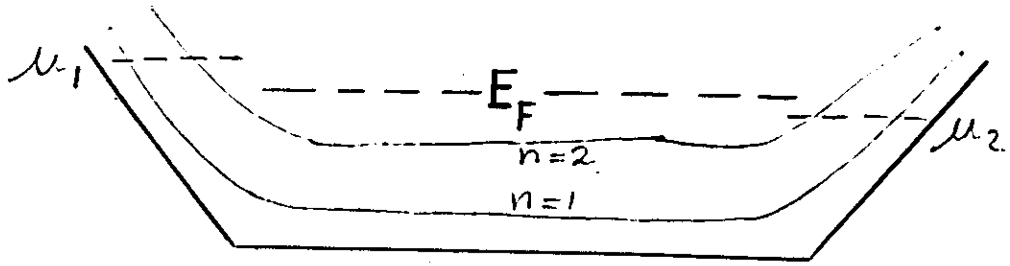
Quantized values:

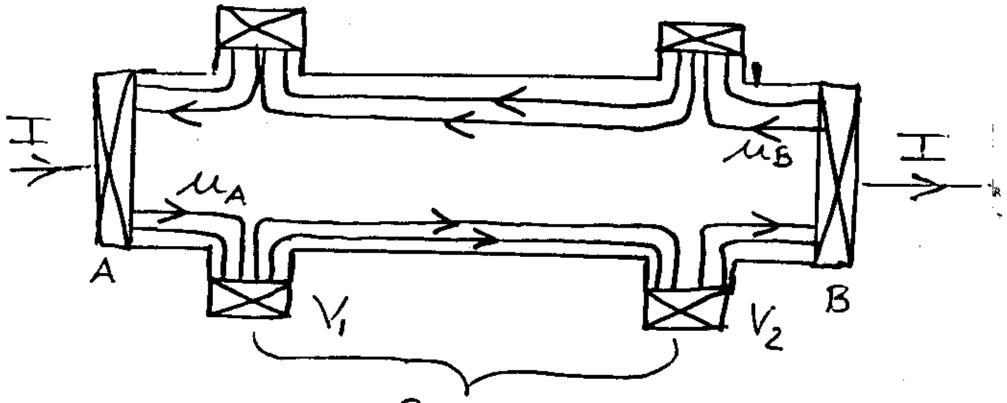
$$R_H = \frac{h}{e^2} \cdot \frac{1}{N_L} \qquad G_H = \frac{1}{R_H} = \frac{e^2}{h} N_L$$

(N_L = number of occupied Landau levels)

R_x = Shubnikov-de Haas oscillations

QUANTUM HALL PLATEAU



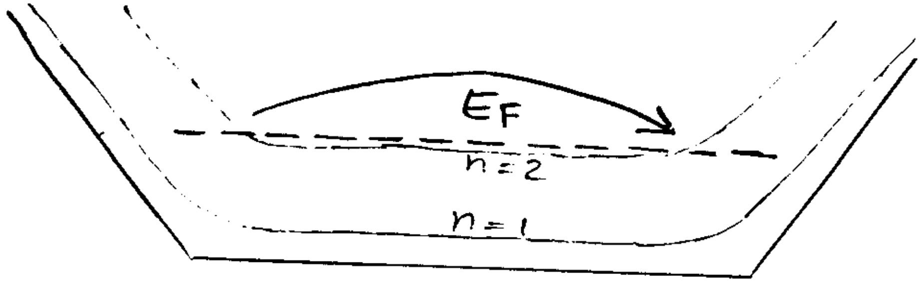


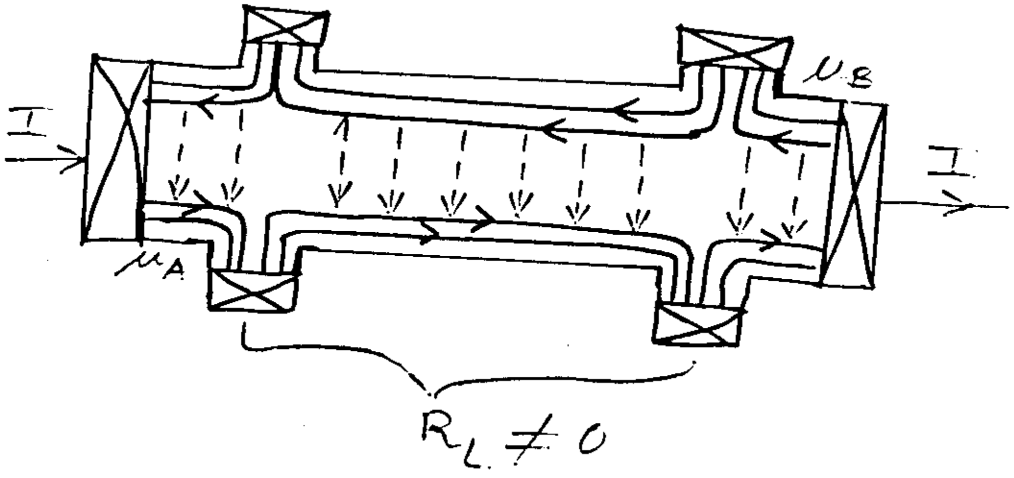
$$R_L = \frac{V_1 - V_2}{I} = 0$$

ABSENCE OF BACKSCATTERING

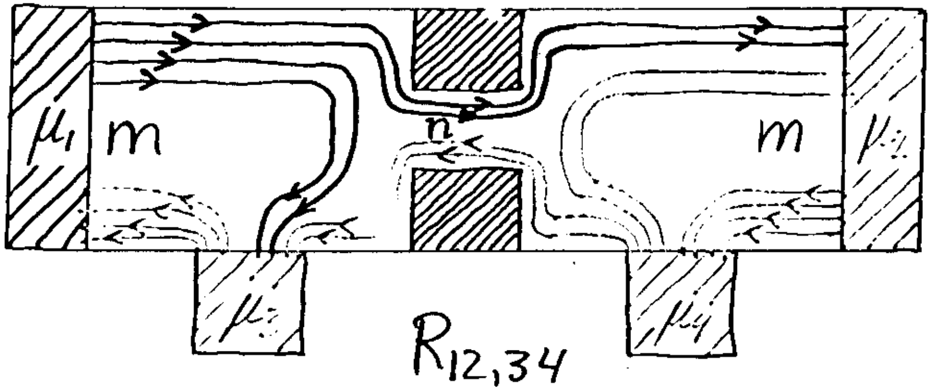
(M. BÜTTIKER)

NON-QUANTIZED HALL RESISTANCE





Controlled Backscattering



$$I = \frac{e}{h} \cdot n\,(\mu_1 - \mu_2) \quad ; \quad \mu_4 = \mu_2$$

$$I_{3,in} = -I_{3,out} \rightarrow \mu_3 = \frac{n\mu_2 + (m-n)\mu_1}{m}$$

$$eV_{34} = \mu_3 - \mu_4 = \frac{m-n}{m}(\mu_1 - \mu_2)$$

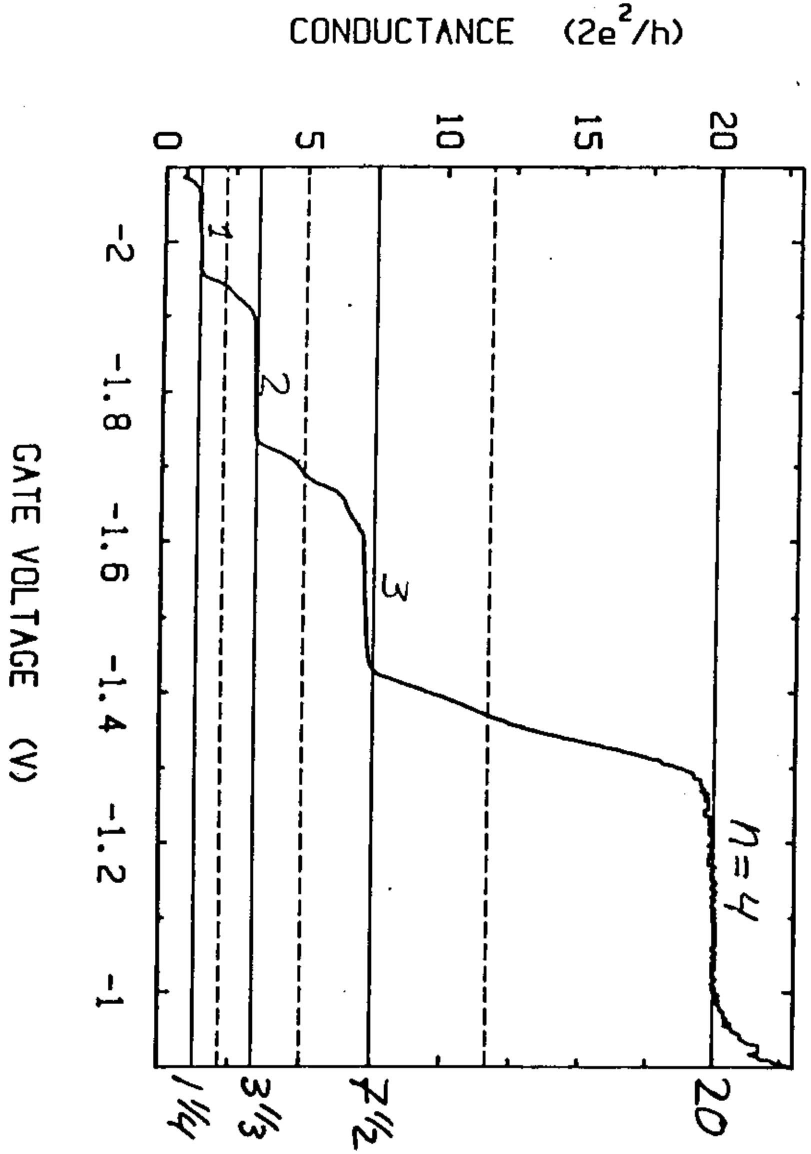
$$\frac{V_{34}}{I} = R_{12,34} = \frac{h}{e^2}\left(\frac{1}{n} - \frac{1}{m}\right)$$

$$G_{12,34} = \frac{e^2}{h}\left(\frac{n \cdot m}{m-n}\right) = \frac{e^2}{h} \cdot m \cdot \left(\frac{(n/m)}{1-(n/m)}\right)$$

with $T = \frac{n}{m} \rightarrow G_{12,34} = \frac{e^2}{h} \cdot m \cdot \left(\frac{T}{1-T}\right)$

4-terminal Landauer-Büttiker formula

2-terminal: $G_{12,12} = \frac{e^2}{h} n = \frac{e^2}{h} \cdot m \cdot T$

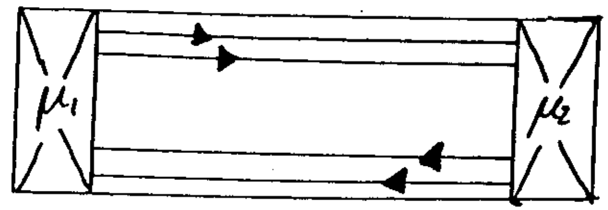


$B = 1.43\,T \to m = 5$ (spin-degenerate)

$G = \frac{2e^2}{h}\cdot\left(\frac{m\cdot n}{m-n}\right)$

absence of backscattering

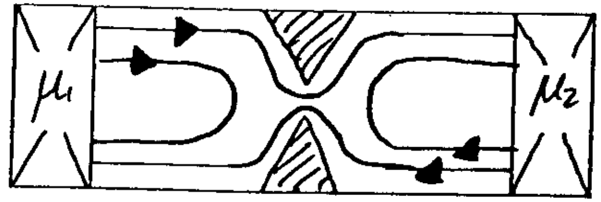
(Büttiker)



$G = 2\cdot\frac{e^2}{h}$

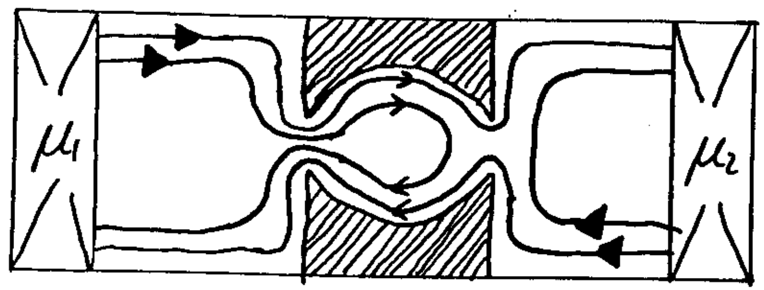
Controlled backscattering

[Haug et. al. (1988) and Washburn et.al (1988)]



$G = 1\cdot\frac{e^2}{h}$

adiabatic transport



$G = 1\cdot\frac{e^2}{h}$