INTERNATIONAL ATOMIC ENERGY AGENCY
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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR.459 - 29

SPRING COLLEGE IN CONDENSED MATTER ON:
"PHYSICS OF LOW-DIMENSIONAL STRUCTURES"

(23 APRIL - 15 JUNE 1990)

QUANTUM ADIABATIC ELECTRON TRANSPORT IN BALLISTIC CONDUCTORS

(PART II)

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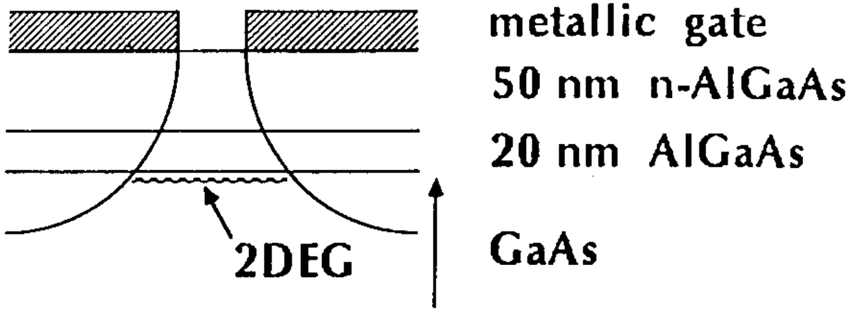
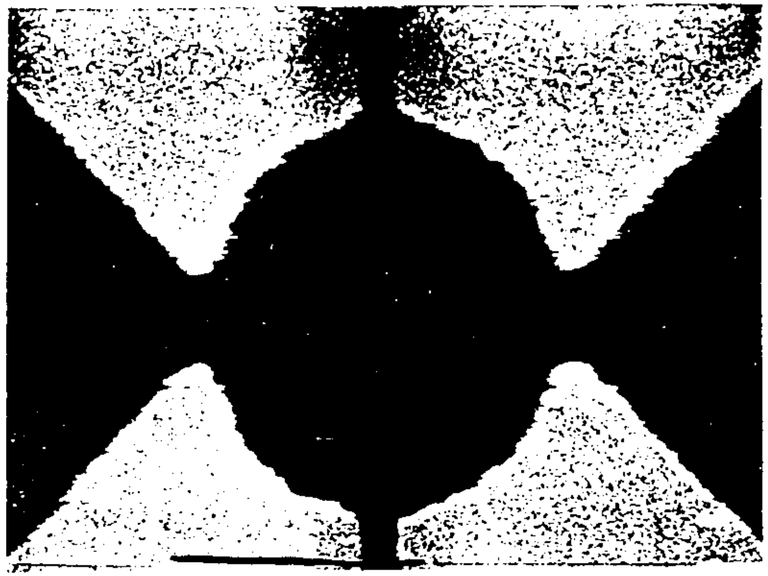
DEVICE GEOMETRY

GaAs–AlGaAs hetero structure
$n_s = 2.3 \cdot 10^{15}\ m^{-2}$, $l_e = 9\ \mu m$

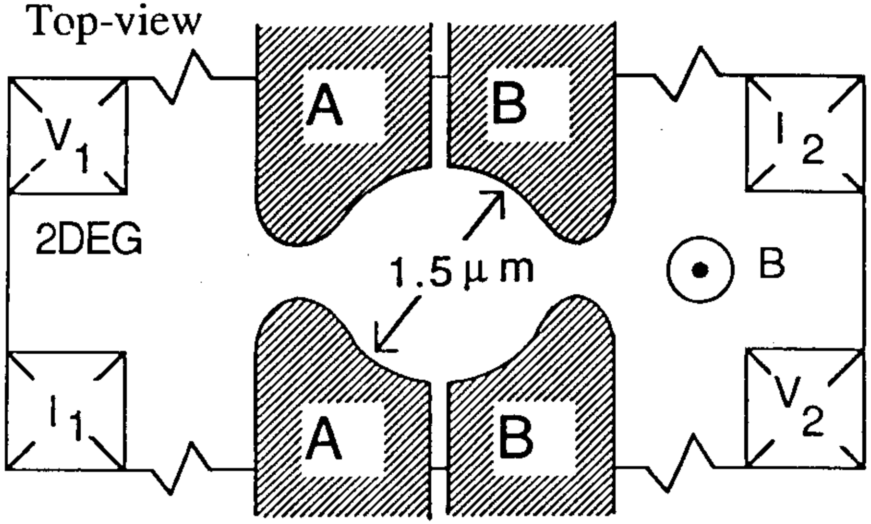
Cross-section





1 μm

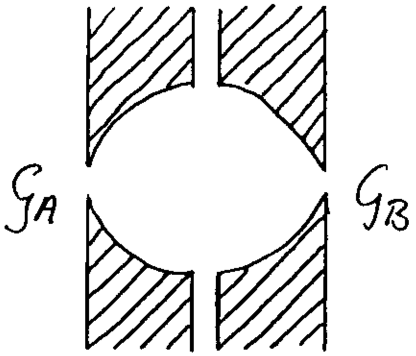
Top-view



$l_e >> 1.5\ \mu m \rightarrow$ ballistic transport

OUTLINE

1/ 2 QPCs in series connected by a ballistic cavity

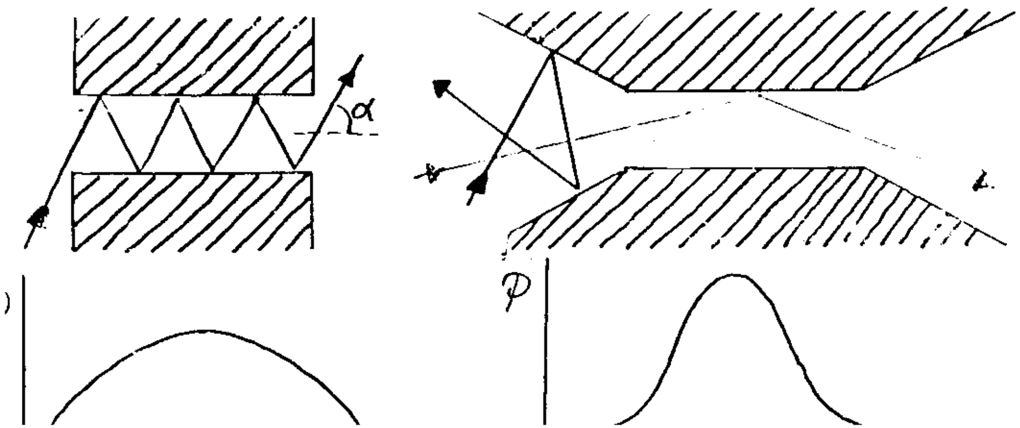


G_{ser} ?

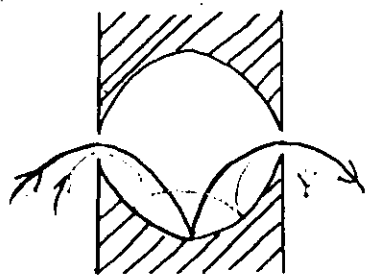
Ohmic transport → Adiabatic transport
$(B = 0)$ $(B \neq 0)$

$G_{ser}^{-1} = G_A^{-1} + G_B^{-1}$ $G_{ser} = \min(G_A, G_B)$

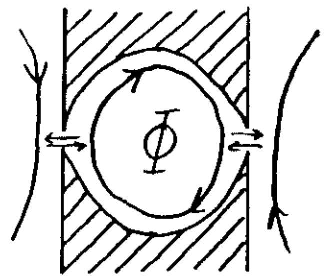
2/ Electron beam collimation by a hornlike constriction



3/ Electron focussing in a cavity



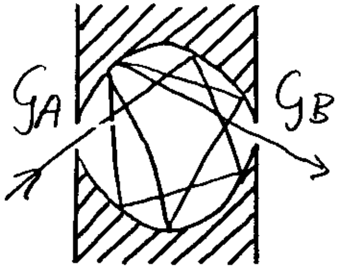
4/ 0D-states in a 1D edge channel loop



Resonant transmission when $\Phi = n\Phi_0$

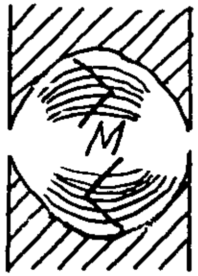
TRANSPORT THROUGH 2 QPCs IN SERIES

(Wharam et al. '88; Beenakker & van Houten '89; [illegible] '89)



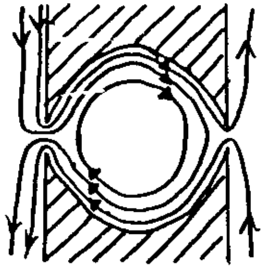
Ohmic addition

$$\frac{1}{G_{ser}} = \frac{1}{G_A} + \frac{1}{G_B}$$



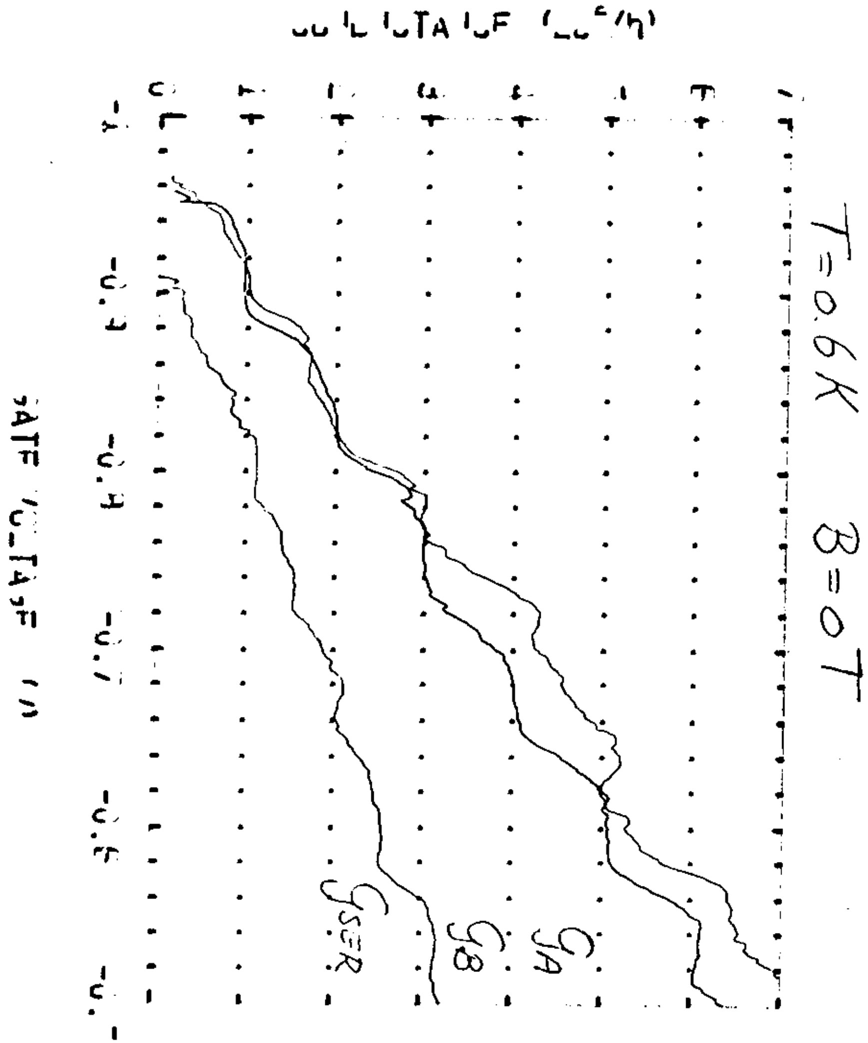
Equilibration among finite number of subbands M in cavity

$$\frac{1}{G_{ser}} = \frac{1}{G_A} + \frac{1}{G_B} - \frac{1}{\frac{2e^2}{h} M}$$



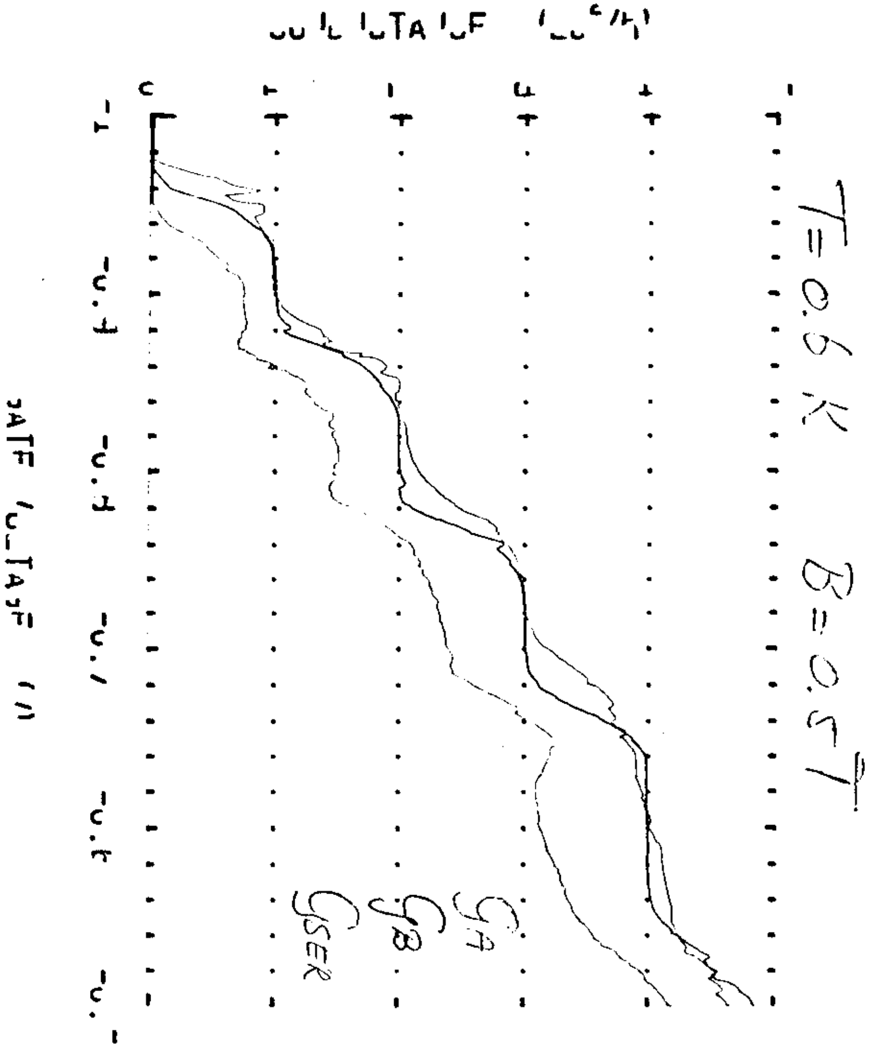
Adiabatic transport (conservation of subband number)

$$G_{ser} = \min(G_A, G_B)$$

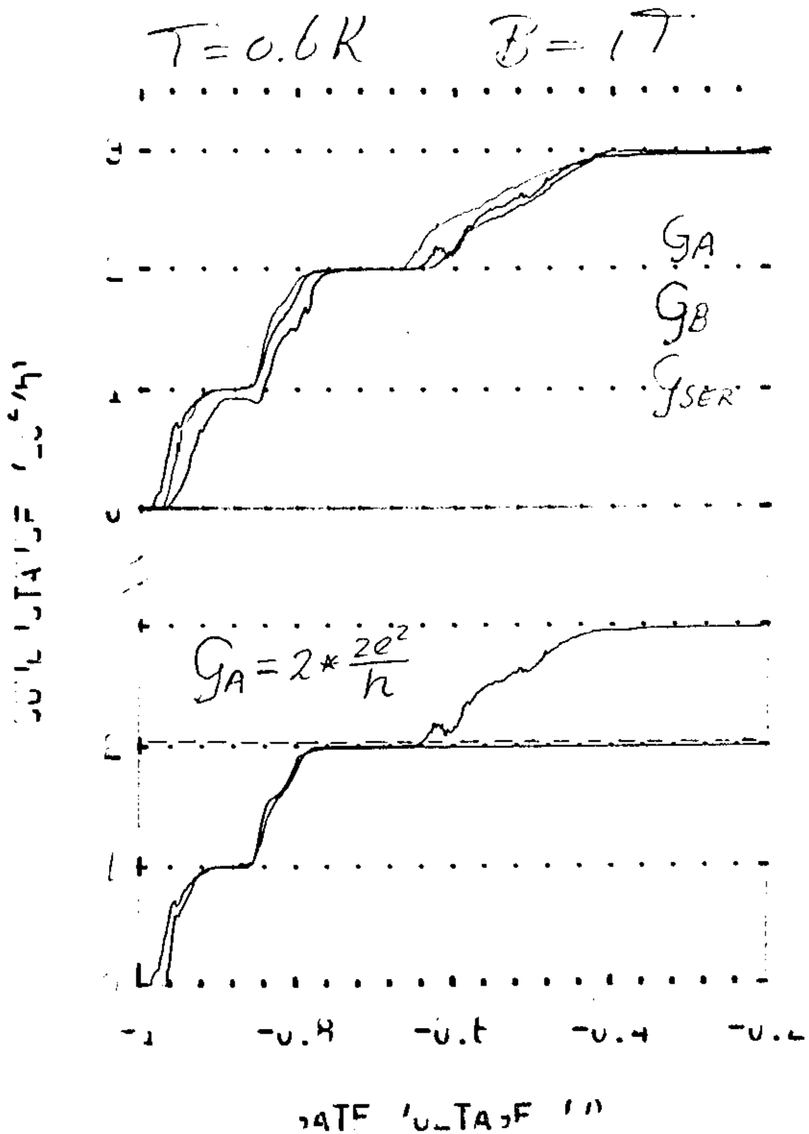


Transition from:

Ohmic →	finite-M →	adiabatic regime
$B = 0$	$B \neq 0$	QHE regime
$M = \infty$	$M = N$	~~[illegible]~~



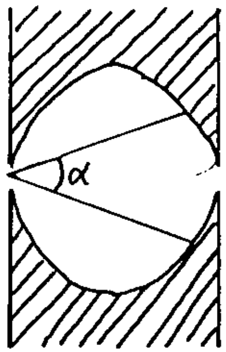
T=0.6K B=0.5T
GA
GB
GSER



T=0.6K B=1T
GA
GB
GSER
GA = 2 * 2e²/h

ELECTRON BEAM COLLIMATION

(v Houten and Beenakker (1989); Molenkamp et al. '89



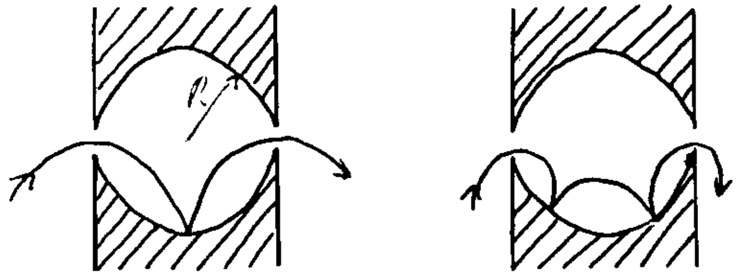
Collimation → direct transmission
→ enhanced G_{ser} above Ohmic addition

Deflection of beam by a magnetic field
→ decreasing direct transmission

complete deflection if:

$$B > B_{min} = \frac{2\hbar k_F}{eL} \sin(\tfrac{1}{2}\alpha)$$

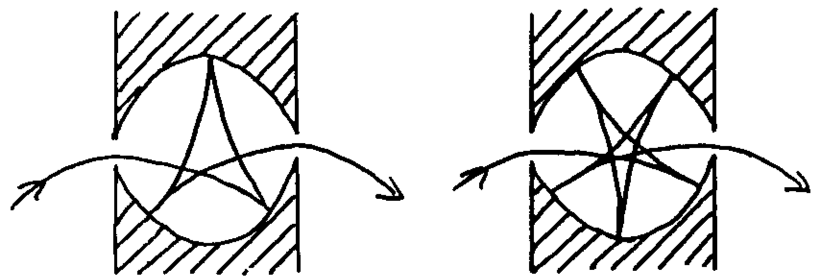
ELECTRON FOCUSSING IN A DOT



$n=1$ $\qquad$ $n=2$

$$B_{focus} = \frac{\hbar k_F}{eR} \tan^{-1}\left(\frac{\pi/2}{n+1}\right)$$

Note: Chaotic electron focussing
(geometrical resonances)



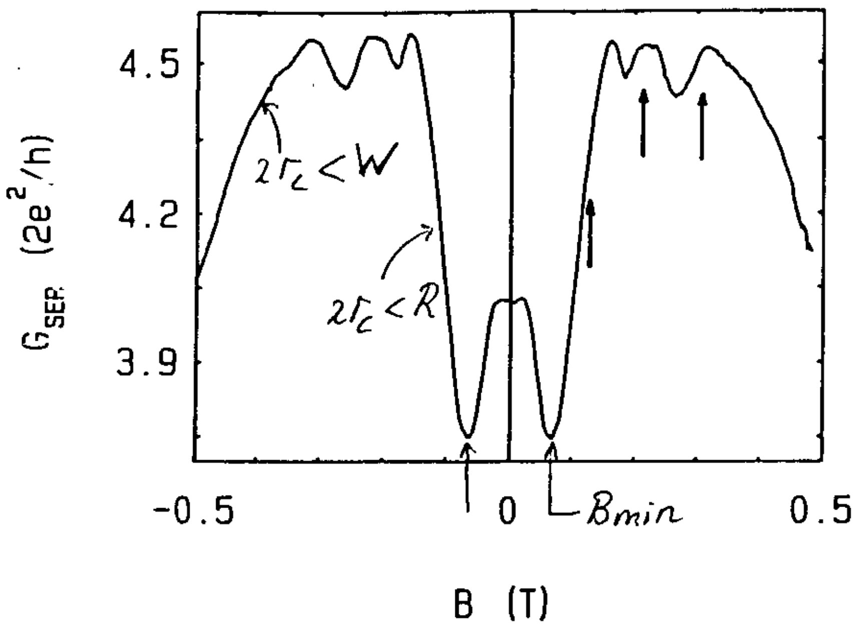
$n=3$, $m=1$ $\qquad$ $n=5$, $m=2$

resonance condition:

$$r_c = R \tan\left(\frac{2m+1}{n+1} \cdot \frac{\pi}{2}\right) \qquad r_c = \frac{\hbar k_F}{eB}$$

$k_B T \to \Delta k_F \to \Delta r_c \Rightarrow$ thermal smearing of $m > 0$ resonances

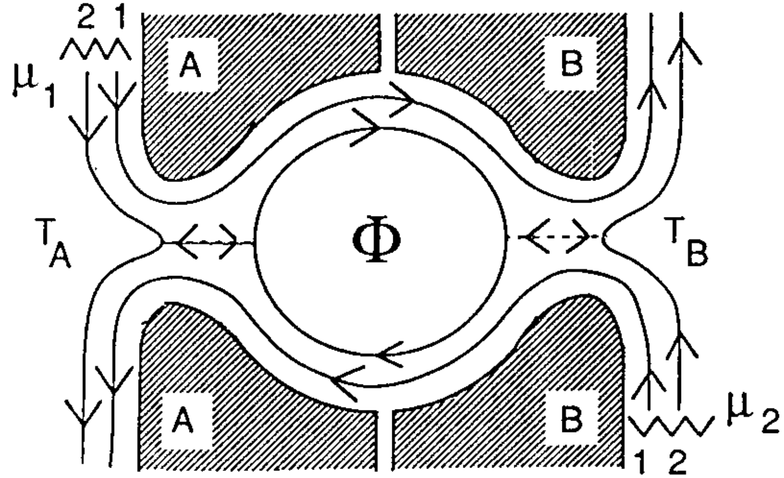
$$\frac{1}{G_{ser}} = \frac{1}{G_A} + \frac{1}{G_B} - \frac{1}{\frac{2e^2}{h} N_L}$$



$B_{min} = 0.07\,T \rightarrow \alpha = 70°$

$T_D \approx 5 - 10\%$

TRANSMISSION PROBABILITY OF A QUANTUM DOT



$$G_D = \frac{e^2}{h}(N + T_D)$$

$$T_D = T_D\,(\,T_A, T_B, \Phi\,)$$

- 1D transport when $T_D = 0$
  then $G_D = \frac{e^2}{h} N$ = quantized

- transport through 0D-states
  when $T_D \neq 0$
  then G_D oscillates with period $\Delta\Phi = \Phi_0$

OUTLINE

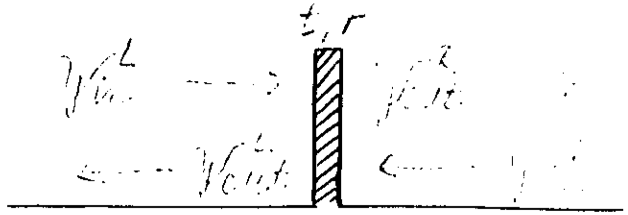
- Transmission through a 1D system with 1- [redacted] 2- and N- barriers
  → formation of a mini-band structure

- possible experiments - $B=0$
  - $B \neq 0$ (edge channels)

- Experiments ($B \neq 0$)
  → 0D-states in a single quantum dot.
  → mini-band structure in a series of coupled quantum dots.

- Discussion.

1D SYSTEM WITH 1 BARRIER



time reversal symmetry : $t = t'$
assume symmetric barrier → $r = r'$

$$\begin{pmatrix} \psi_{out}^R \\ \psi_{out}^L \end{pmatrix} = \begin{pmatrix} t & r \\ r & t \end{pmatrix} \begin{pmatrix} \psi_{in}^L \\ \psi_{in}^R \end{pmatrix}$$

↳ scattering matrix S

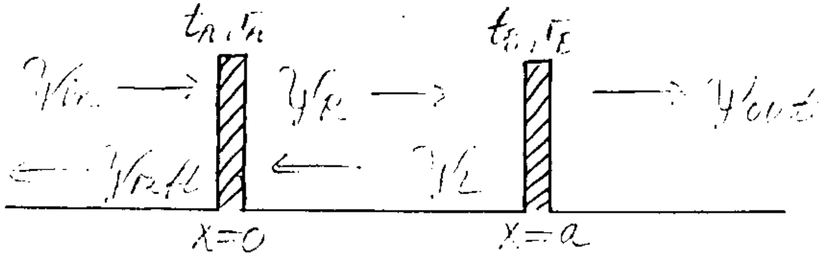
conservation of probability (or current)
→ S = unitary , $S^\dagger S = I$

⇒ $tt^* + rr^* = 1$ and $\frac{t}{t^*} = -\frac{r}{r^*}$

t = complex transmission amplitude
r = " reflection "
(imaginary parts include the phase shifts in the barrier)

$T = tt^*$ = transmission probability
$R = rr^*$ = reflection "

1D SYSTEM WITH 2 BARRIERS

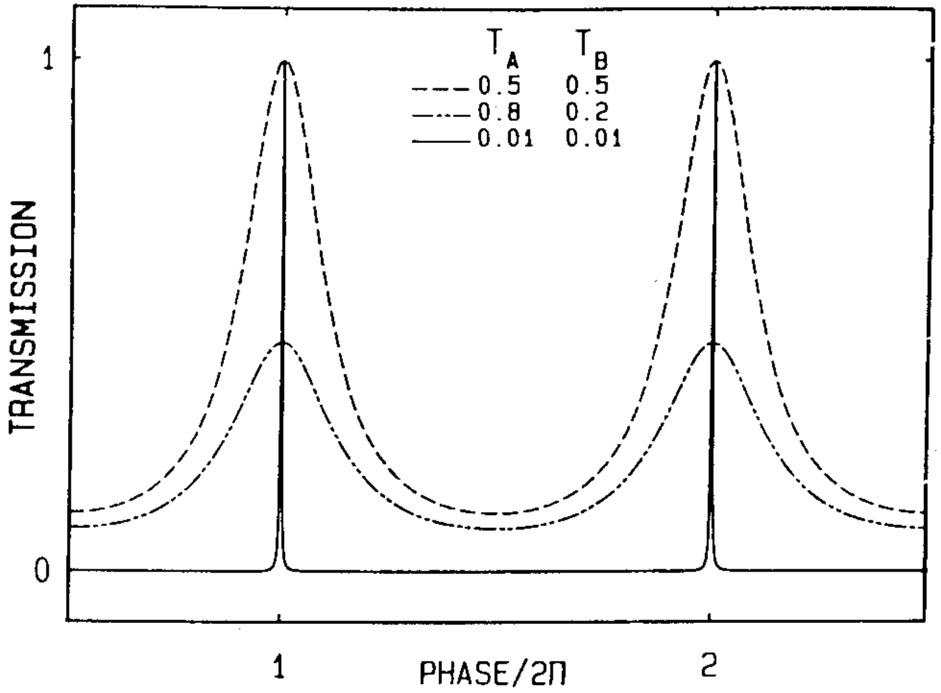


$$\psi_R(x=0) = t_A\, \psi_{in}(x=0) + r_A\, \psi_L(x=0)$$
$$\psi_L(x=0) = r_B\, \psi_R\, \exp(i\theta)$$

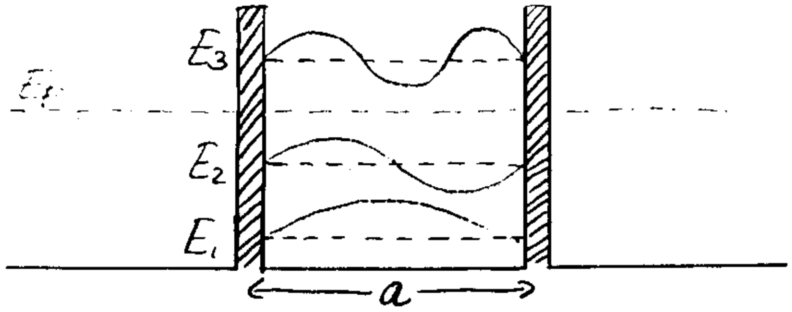
$$\psi_{out}(x=a) = t_B\, \psi_R(x=0) \exp(i\theta/2)$$
$$= \underbrace{\frac{t_A t_B \exp(i\theta/2)}{1 - r_A r_B \exp(i\theta)}}_{t_2}\, \psi_{in}(x=0)$$

$$T_2 = \frac{|\psi_{out}|^2}{|\psi_{in}|^2} = t_2 t_2^*$$
$$= \frac{T_A T_B}{1 - 2\sqrt{R_A R_B}\cos(\theta + \theta_r) + R_A R_B}$$

θ_r = total phase shift after reflection at barrier A and B.



0D-STATES IN A 1D-CAVITY



Energy levels: $n \frac{1}{2} \lambda = a$

$$E_n = \frac{h^2 n^2}{8 m a^2}$$ (particle-in-a-box states)

resonant transmission when $E_F = E_n$

Transport

$$T_2 = \frac{T_A T_Z}{1 - 2\sqrt{R_A R_Z}\cos(\theta + \theta_r) + R_A R_Z}$$

phase $\theta = 2\pi \cdot \frac{2a}{\lambda}$

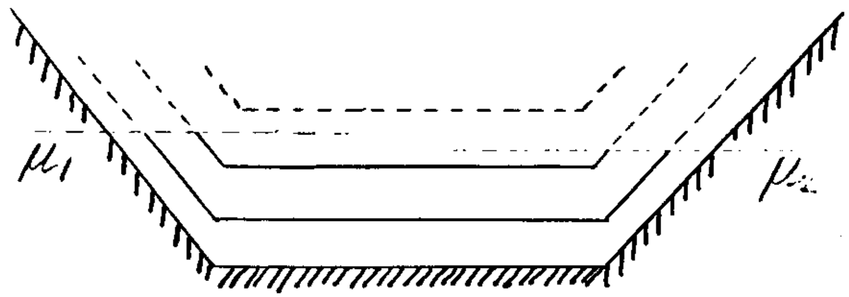
→ oscillating T_2 when a or λ are varied

(Smith et al., 1989)

0D-STATES IN A 1D EDGE CHANNEL LOO

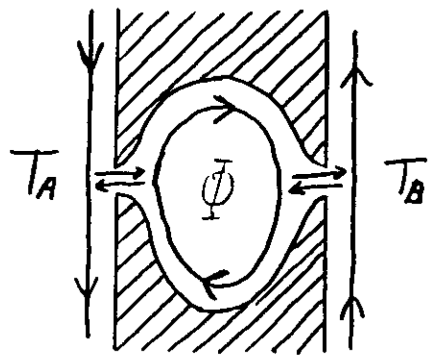
- high magnetic field perpendicular to a 2D

→ formation of edge channels



$eV_H = \mu_1 - \mu_2$; $I = \frac{e}{h}(\mu_1 - \mu_2) \cdot N_L$

- transport in edge channels is 1D and adiabatic on distances of order 1 μm.



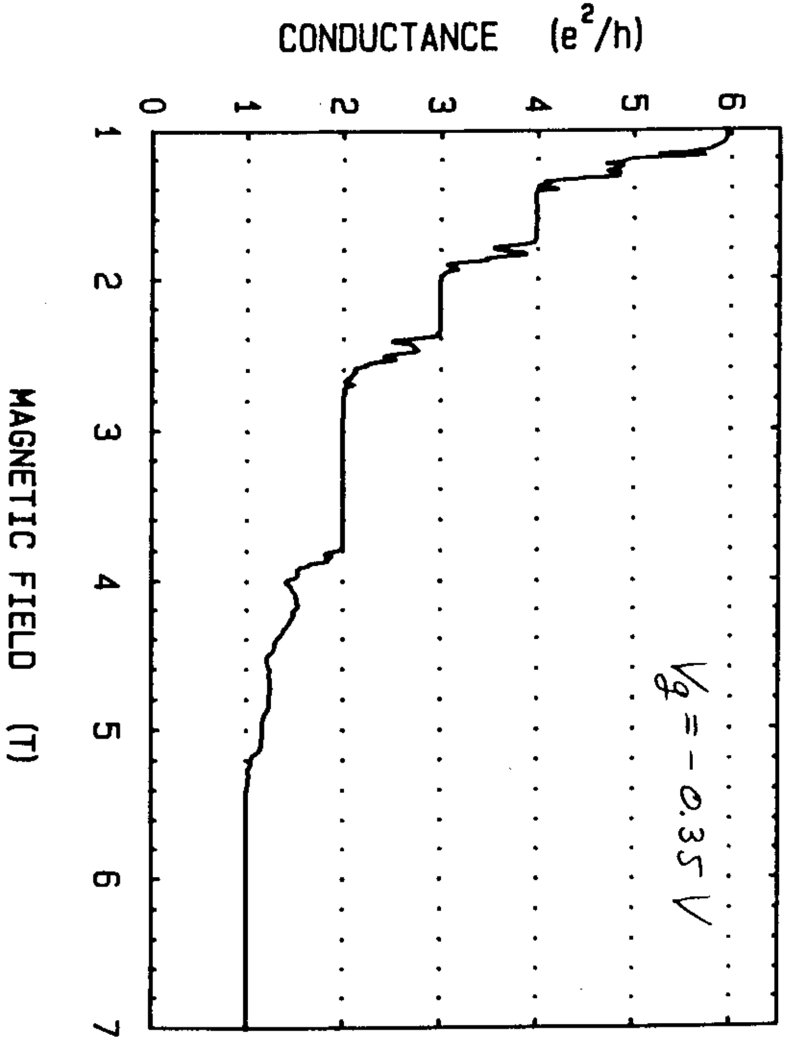
Aharonov-Bohm pha

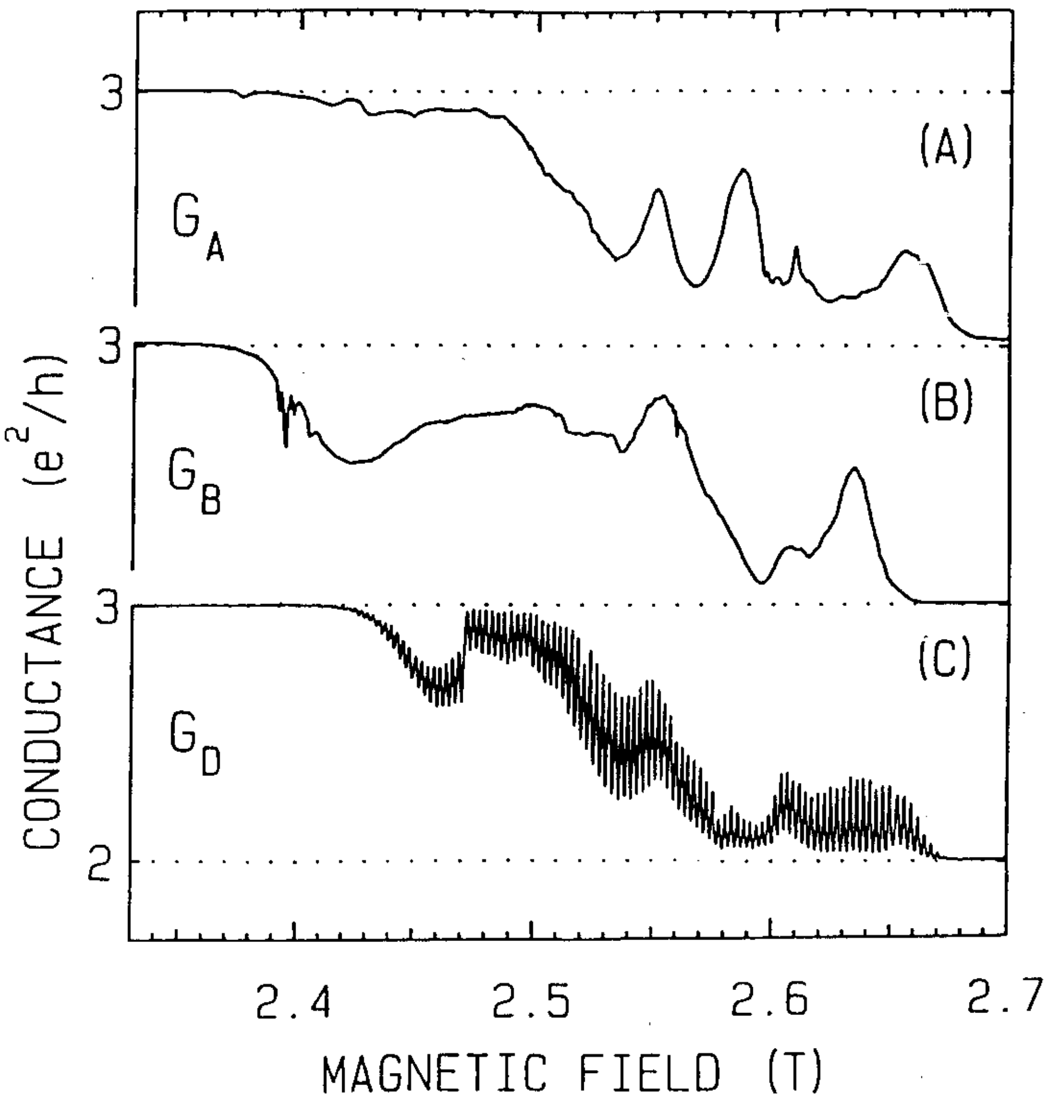
$\theta = 2\pi \frac{\Phi}{\Phi_0}$

$\Phi = B \cdot A$

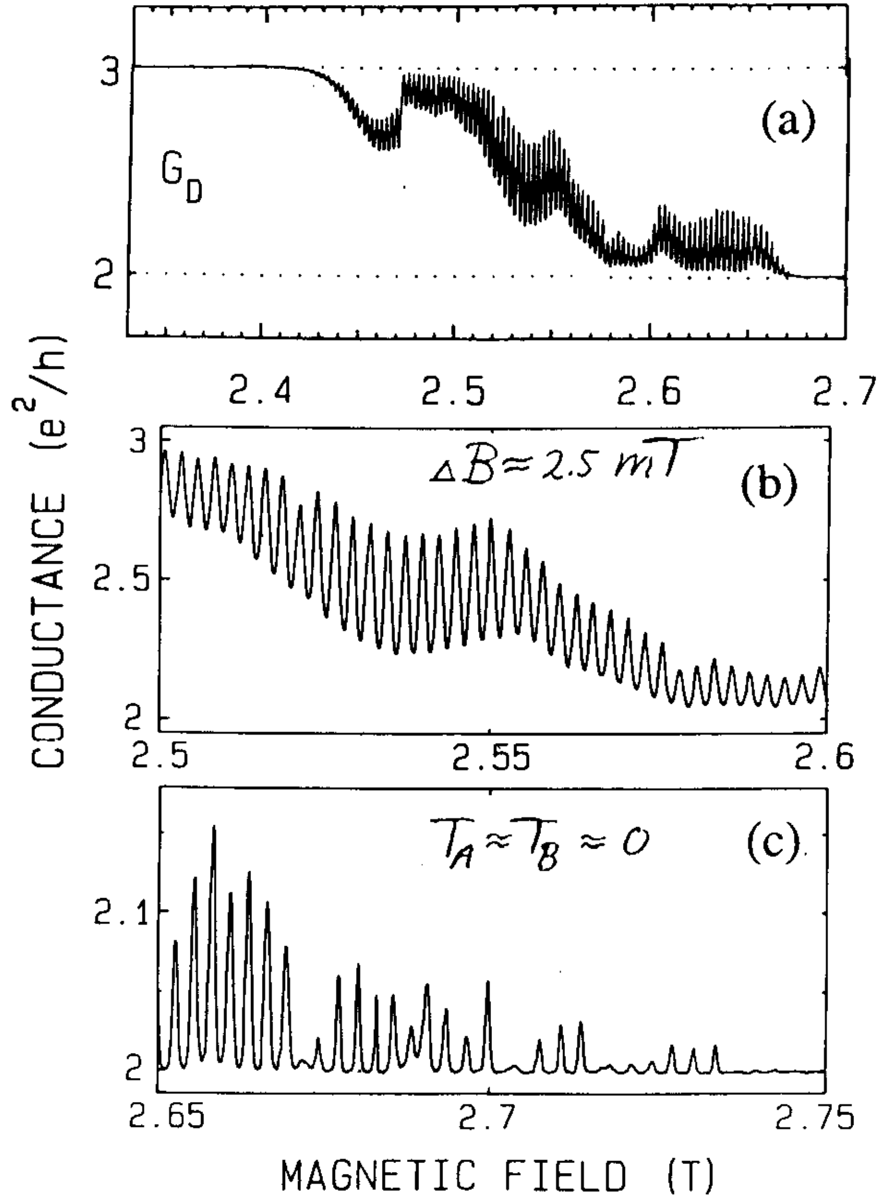
$\Phi_0 = \frac{h}{e}$ fluxquantu

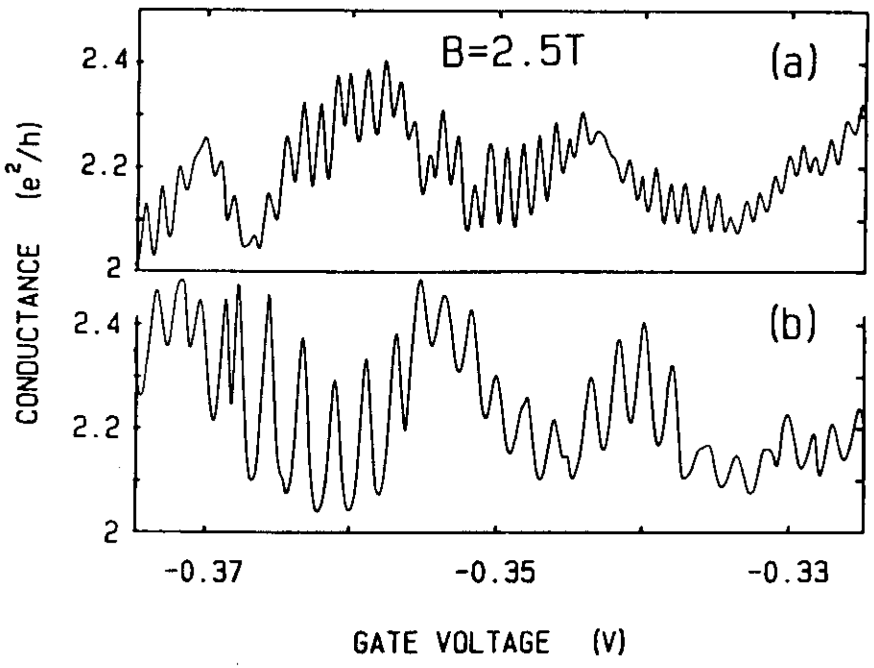
0D-states when $\frac{\Phi}{\Phi_0}$ is an integer.


CONDUCTANCE (e²/h)
MAGNETIC FIELD (T)
Vg = -0.35 V


(A)
G_A
(B)
G_B
(C)
G_D
CONDUCTANCE (e²/h)
MAGNETIC FIELD (T)

$T = 10\,mK$, $V_g = -0.35\,V$





(a) ΔV_g on both gates $\rightarrow$ period $= 1\,mV$

(b) ΔV_g on one gate $\rightarrow$ period $= 2\,mV$

Discussion

$$T_D = \frac{T_A\, T_E}{1 - 2\sqrt{R_A R_E}\,\cos\nu + R_A R_E}$$

$T_A = T_E \approx 0 \;\rightarrow$ modulation $\approx e^2/h$

envelope function:

- constructive interference $\rightarrow \cos\nu = 1$
- destructive interference $\rightarrow \cos\nu = -1$

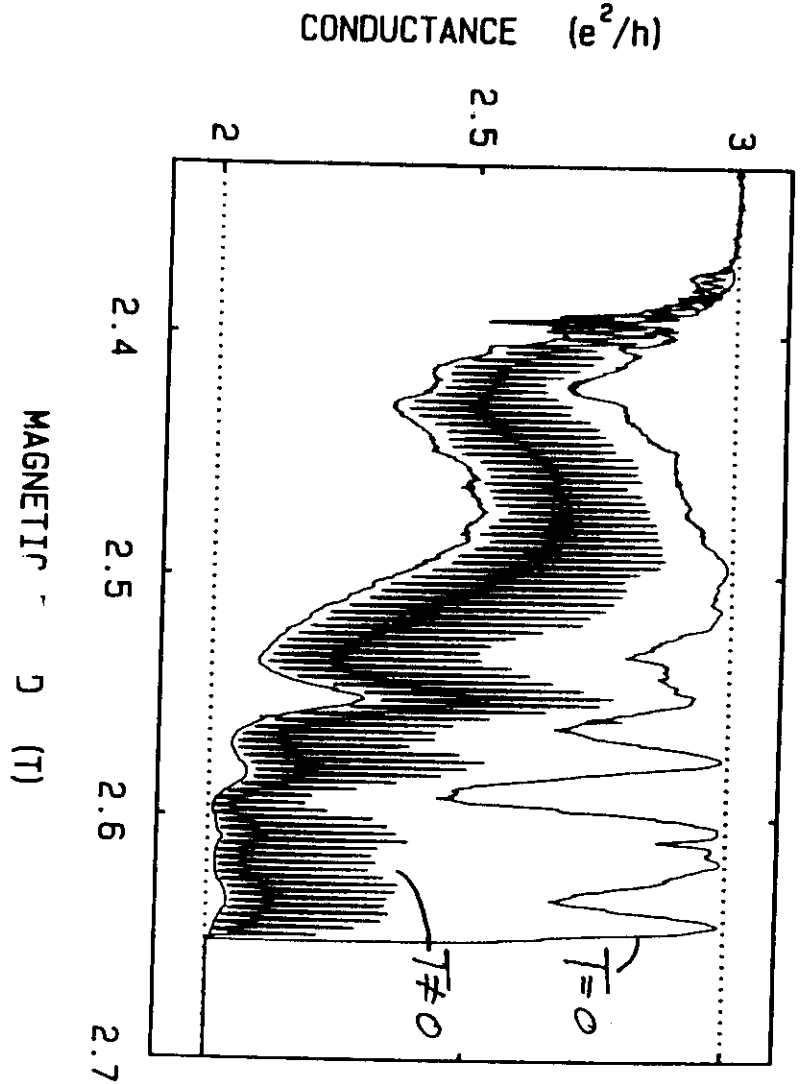
energy averaging:

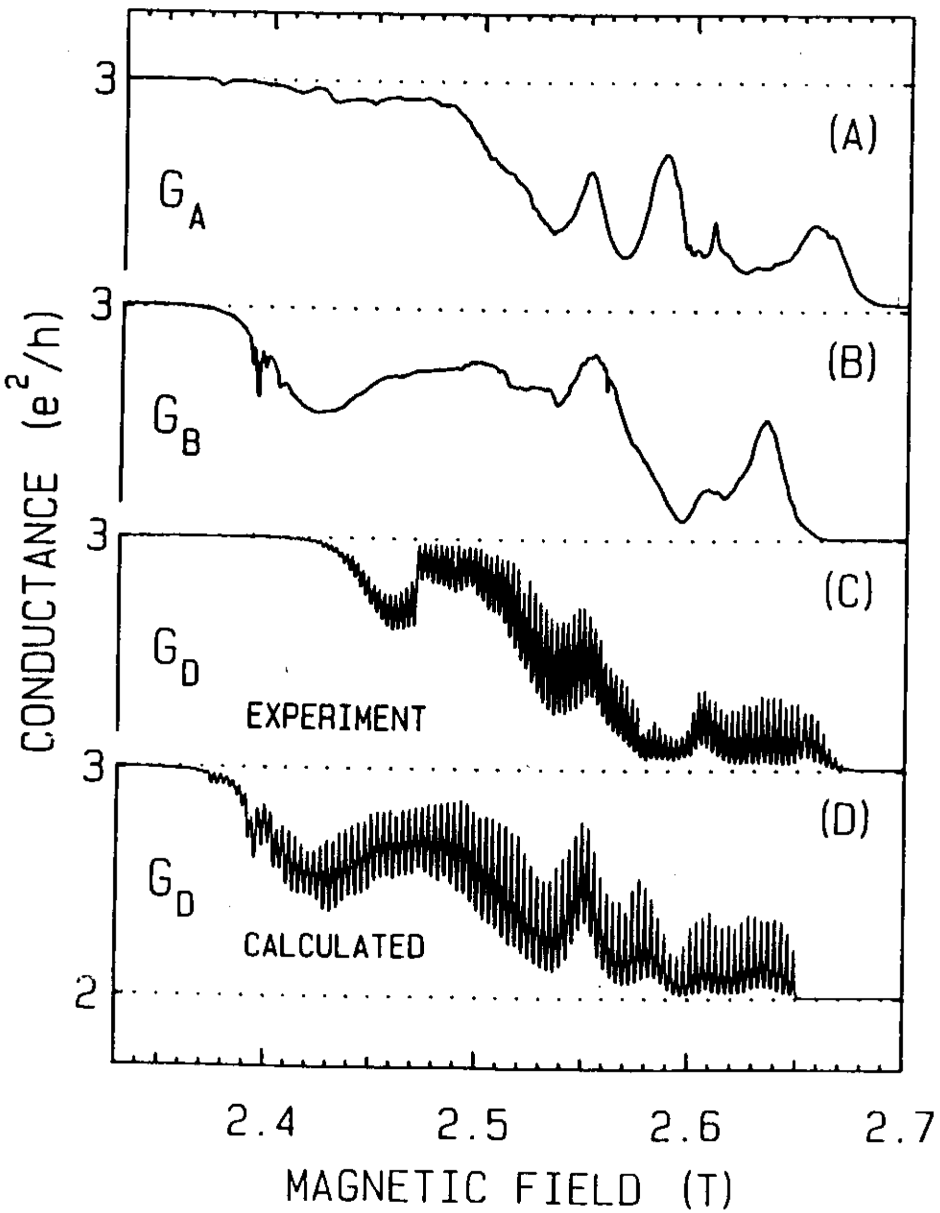
- experimental: oscillations disappear if $T > 200\,mK$ and $V > 40\,\mu V$

  $\rightarrow \Delta E = 40\,\mu eV$

- $G_D(T) = \int dE \cdot G_D(E) \cdot \left(\frac{-\partial f}{\partial E}\right)$

- temperature averaging over 30 mK
- fixed chosen period = 3 mT





edge channel index	period (mT)
2	5.3
3	2.5
4	2.1
5	1.4

period $B_c = \Delta B \frac{\phi_0}{\Delta \phi}$

$\Delta \phi = \Delta(B \pi r^2) = \pi r^2 \Delta B + B 2\pi r \Delta r$

(estimation of Δr with $E_g = -eV(x,y)$; $E = -\frac{\partial V}{\partial r}$

- observation of a single, well defined perio[d] for each transition indicates that the conductance modulation originates from a single edge channel.

⇒ In the ballistic regime is the net curren[t] completely carried by edge channels

Conclusions

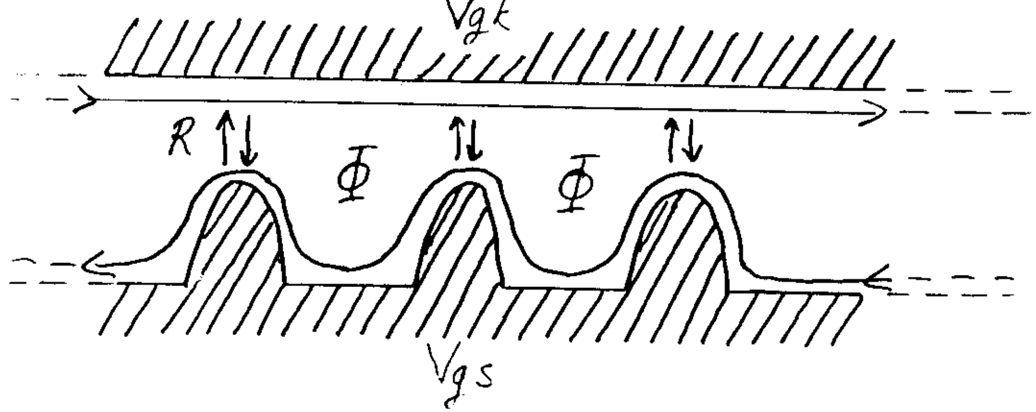
- In a high magnetic field → adiabatic transport through edge channels over distances of a few μm.
  (Müller et al (1990): adiabatic transport over 250 μm)

⟹ edge channels can be viewed as independent 1D current channels

- edge channels } 1D electron interferometer
  QPCs }
  → magnetically induced 0D-states
  → conductance oscillations (40% of e^2/h)
  → good agreement with Landauer-Büttiker formalism.

TRANSPORT THROUGH COUPLED QUANTUM DOTS.

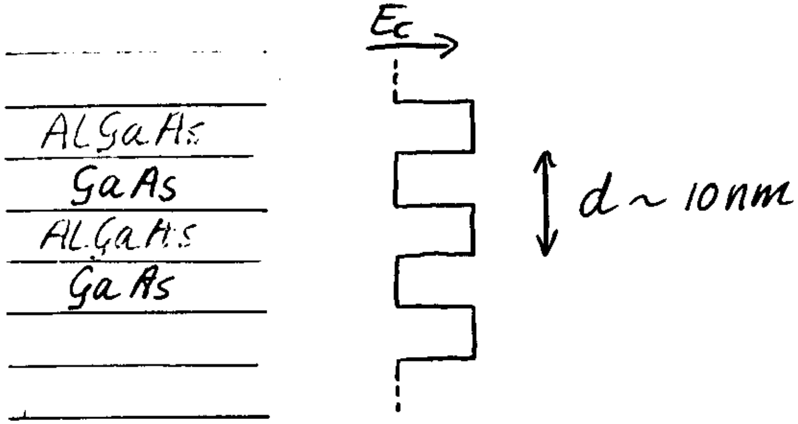
(mini-band structure)

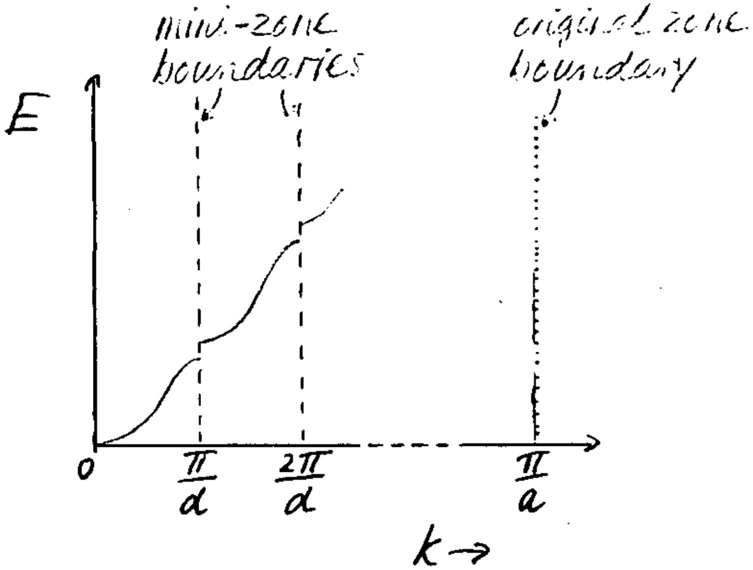


$\Phi = BA$ $\quad$ $A = A(V_{gk})$

$R = R(V_{gk})$

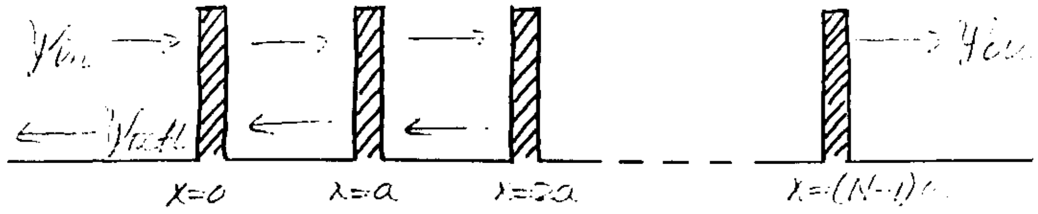
MINI-BAND STRUCTURE IN A 1D SUPERLATTICE
(Esaki & Tsu, 1970)





- after growing process, a final mini-band structure and Fermi energy

1D SYSTEM WITH N BARRIERS
(Tsu & Esaki, 1973 and Ulloa et al., 1990)

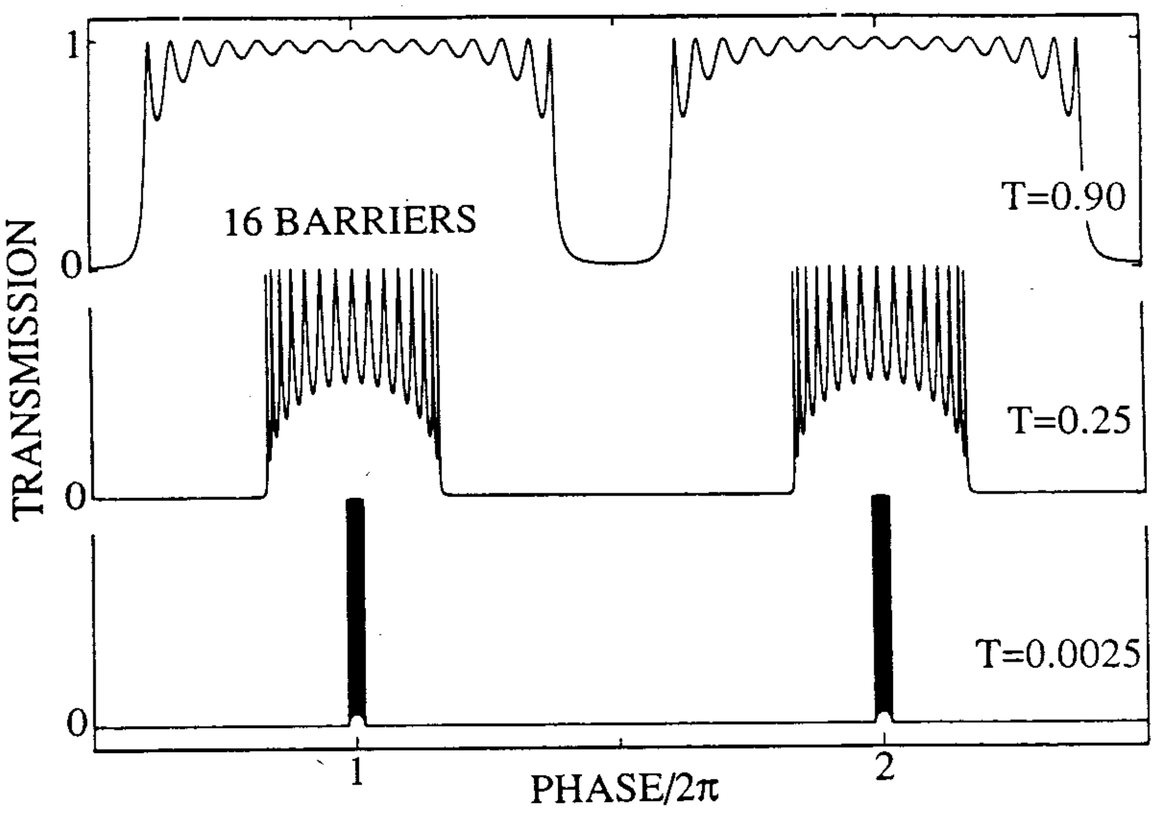


$$t_N = \frac{t\, t_{N-1}}{1 - r\, \bar{r}_{N-1} \exp(i\theta)}$$

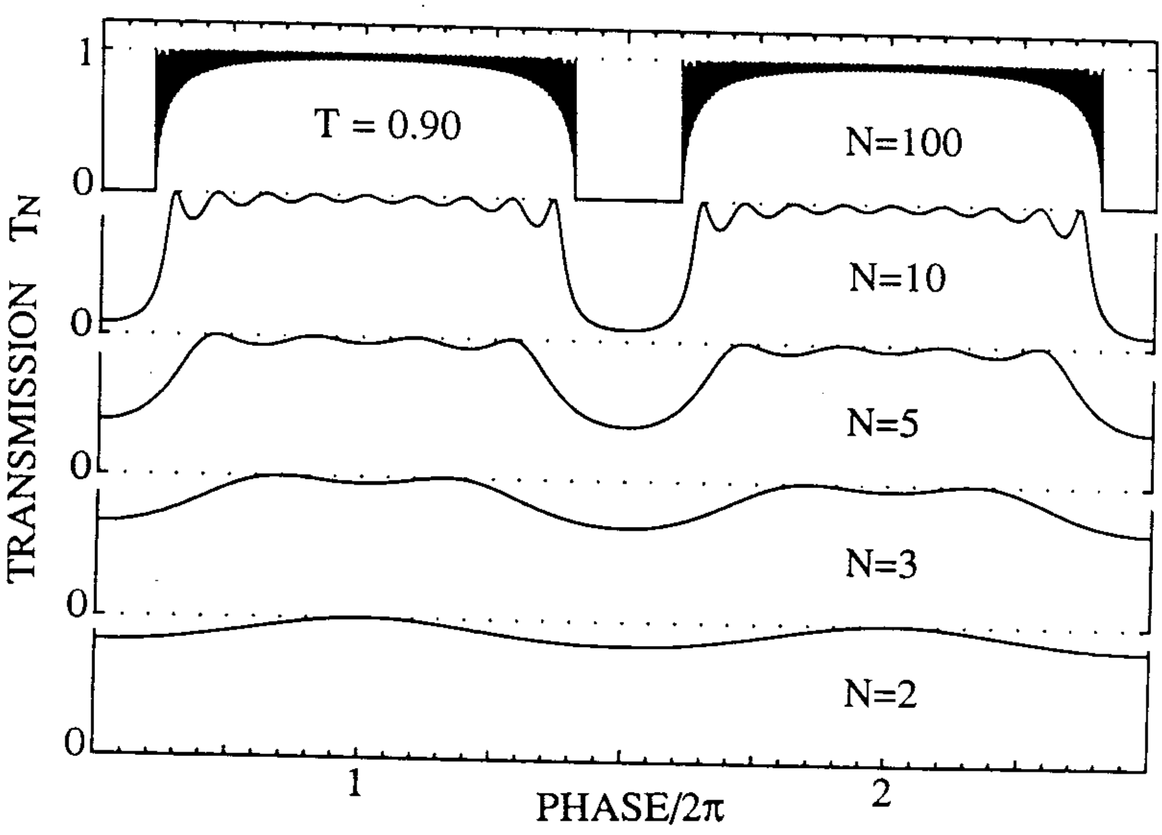
current conservation: $t_N t_N^* + r_N r_N^* = 1$

$$\frac{t_N}{t_N^*} = -\frac{r_N}{r_N^*}$$

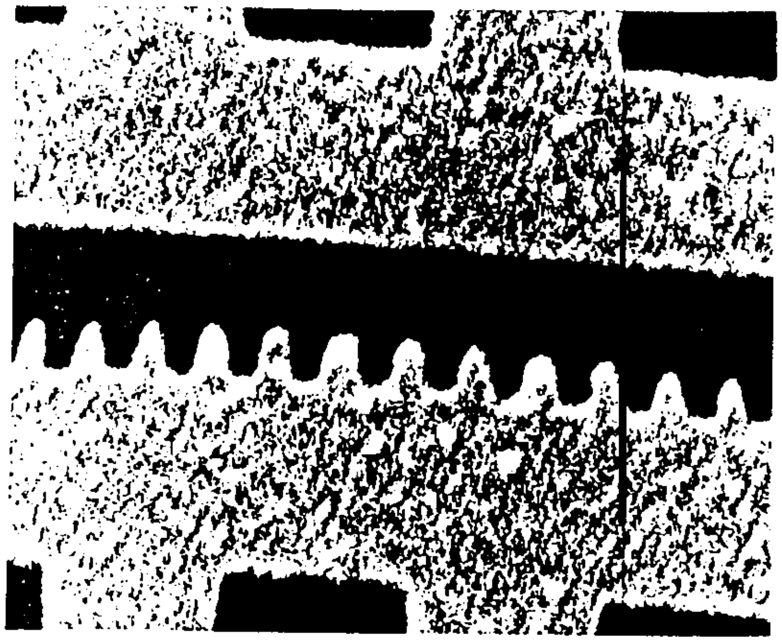
numerical solution → $T_N = t_N t_N^*$

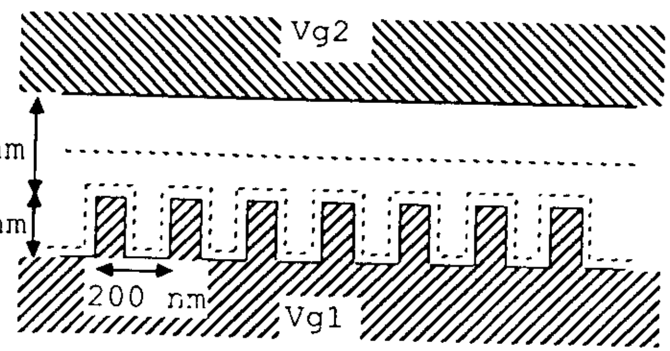


1
0
0
0
TRANSMISSION
16 BARRIERS
T=0.90
T=0.25
T=0.0025
1
2
PHASE/2π

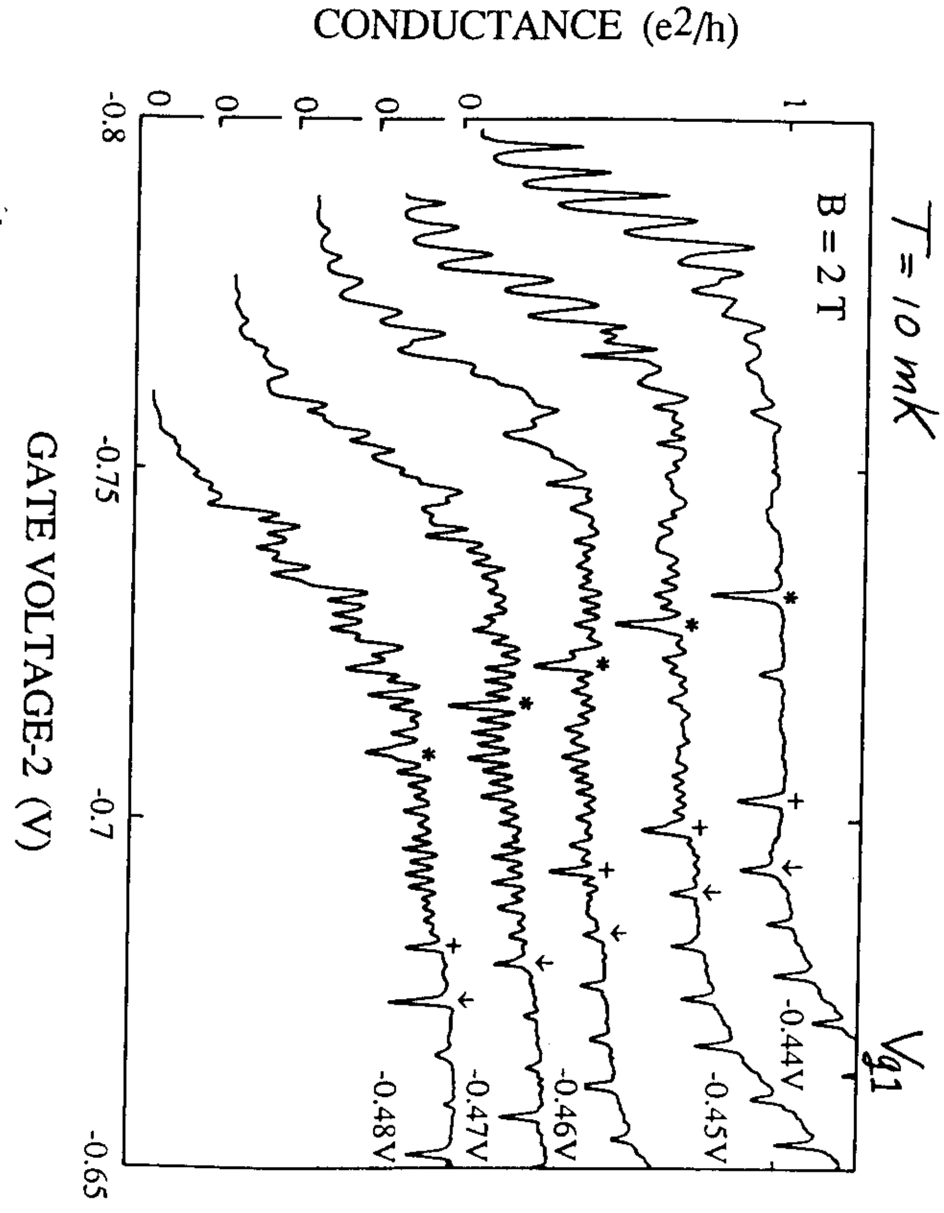


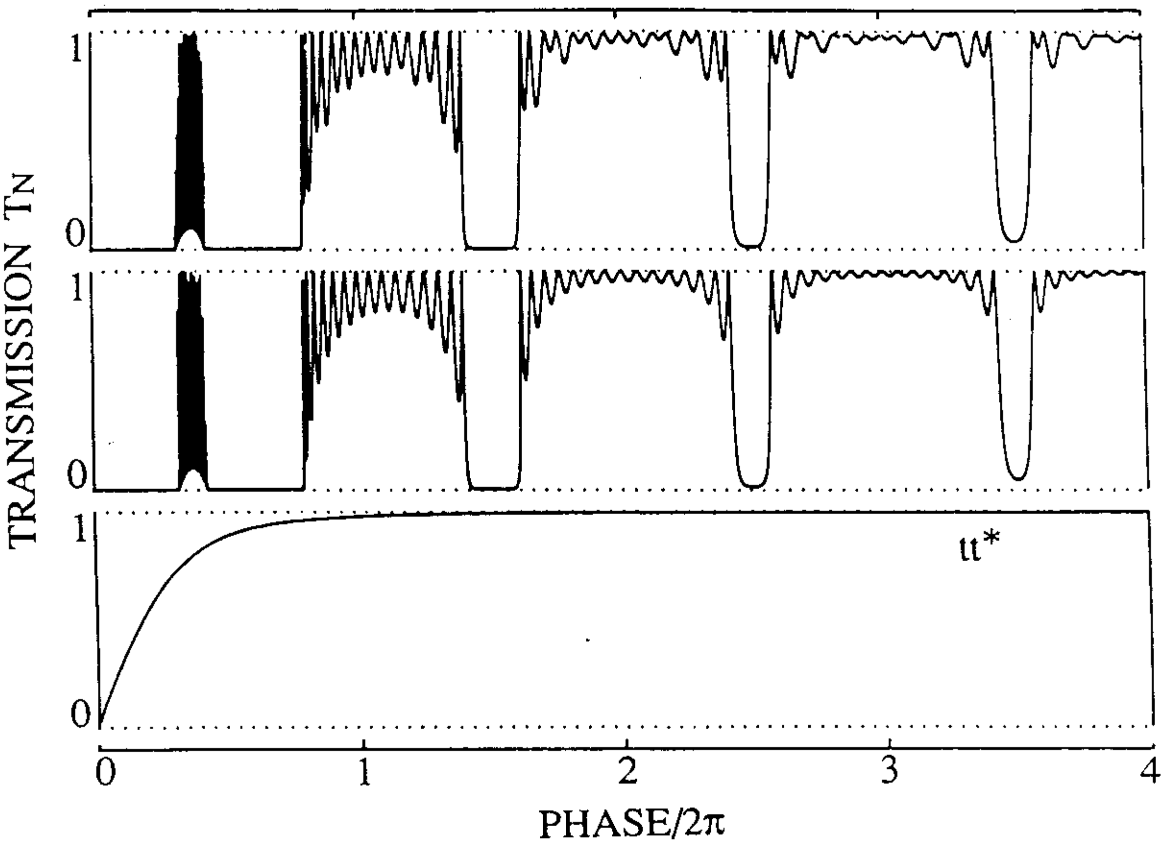
1
0
0
0
0
0
TRANSMISSION T_N
T = 0.90
N=100
N=10
N=5
N=3
N=2
1
2
PHASE/2π

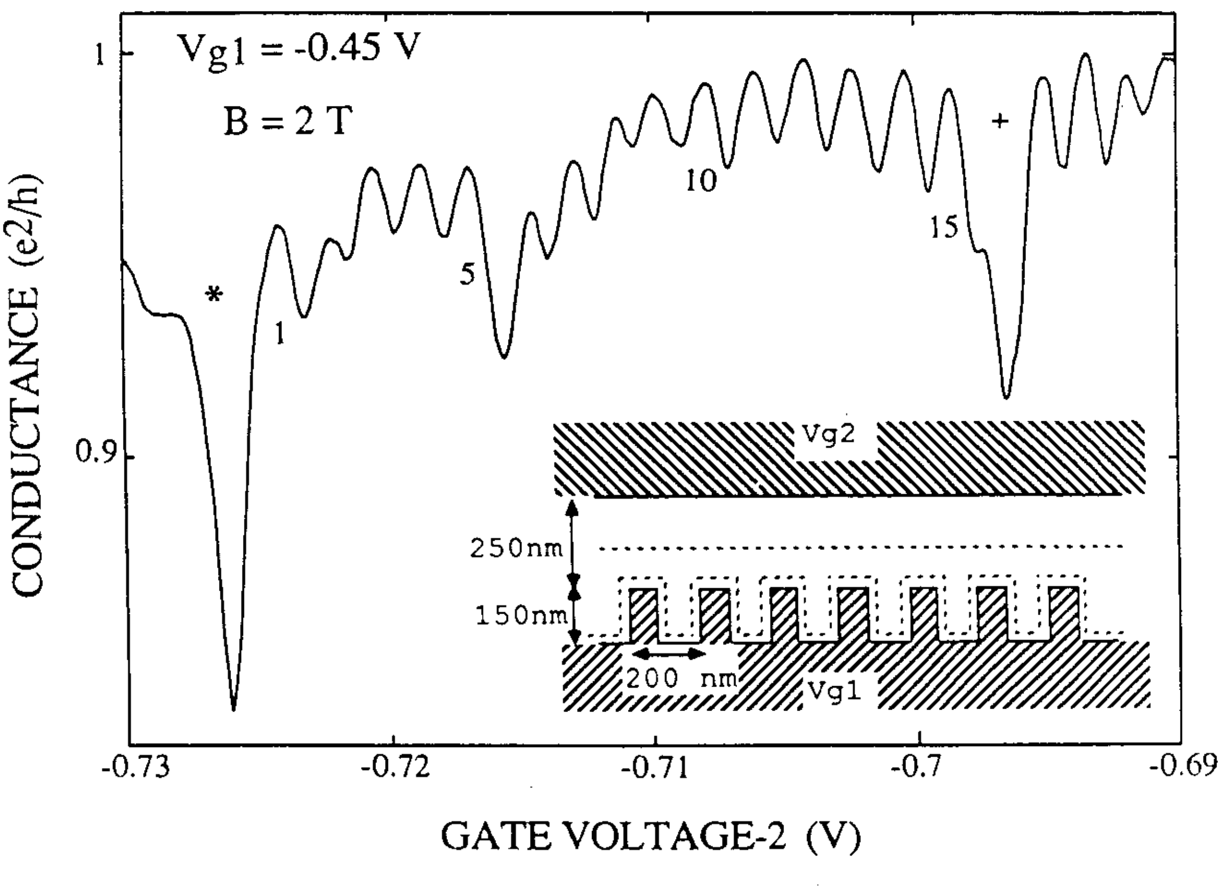




in total 16 fingers
(or 15 quantum dots)




TRANSMISSION T_N
1
0
tt*
0
1
2
3
4
PHASE/2π


CONDUCTANCE (e^2/h)
Vg1 = -0.45 V
B = 2 T
1
0.9
*
1
5
10
15
+
Vg2
250nm
150nm
200 nm
Vg1
-0.73
-0.72
-0.71
-0.7
-0.69
GATE VOLTAGE-2 (V)

SUMMARY and CONCLUSIONS

- Basic solid state properties as the formation of a band structure can now be studied from the linear transport properties of periodic sub-micron structures

- gated samples allows one to tune the Fermi energy through the discrete electronic states which are formed in the confined regions.

→ magnetically induced 0D-states in a single quantum dot (1D electron interferometer)

→ mini-band structure in a series of coupled quantum dots (finite 1D crystal)

⇒ 1D-adiabatic-phase coherent-electron transport in edge channels can be realized on a distance of a few μm.

→ This allows one to describe the conductance with simple 1D models to calculate the transmission and the Landauer-Büttiker formulas.

REFERENCES (reviews)

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