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I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

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THE PROBLEM OF THE GREEN FUNCTIONS FOR THE FRACTIONAL QUANTUM HALL EFFECT

YU BYCHKOV
Academy of Sciences of the USSR
L.D. Landau Institute for Theoretical Physics
Kosygin St. 2
Moscow 117940
USSR

These are preliminary lecture notes, intended only for distribution to participants.

The problem of the Green functions for the fractional quantum Hall effect

[redacted] Much of the phenomenology of the fractional quantum Hall effect can be explained by a theoretical picture which was evolved from a paper by Laughlin. Central to this picture is the notion that the two-dimensional electron gas in a strong magnetic field perpendicular to 2D layer has a series of especially stable ground states at rational values of the Landau level filling factor

$$\nu = 2\pi l_H^2 \mathcal{N}$$

$\mathcal{N}$ - is the electron density

$l_H = \left(\frac{c\hbar}{eB}\right)^{1/2}$ - the magnetic length

When the interaction occurs only between electrons on the same Landau level than the Hamiltonian in occupation number representation and for Landau gauge eigenstates is

$$\hat{H}_{int} =$$

$$\frac{1}{2}\sum_{k_1, k_2} \int \frac{d^2 q}{(2\pi)^2} \tilde{V}(q) e^{i q_x (k_1 - k_2 - q_y)} \hat{a}^+_{k_1} \hat{a}^+_{k_2} \hat{a}_{k_2+q_y} a_{k_1-q_y}$$

k - is a wave number of electrons.

It is interesting to investigate the most general properties of this Hamiltonian. The part of the Hamiltonian is operator

$$\hat{A}_{k_1 k_2} = \int d^2 q \, \tilde{V}(q) e^{i q_x (k_1 - k_2 - q_y)} \hat{a}_{k_2+q_y} a_{k_1-q_y}$$

The next transformation is identity

$$\hat{A}_{k_1 k_2} \equiv$$

$$\iint d^2q' \, \delta(\vec{q}-\vec{q}\,') \, d^2q \, \tilde{V}(q') e^{iq_x(k_1-k_2-q_y)} \hat{a}_{k_2+q_y} \hat{a}_{k_1-q_y}$$

From the usual representation of δ- function it follows

$$\hat{A}_{k_1 k_2} \equiv \frac{1}{(2\pi)^2} \int \tilde{V}(q') e^{i\vec{q}\,'\vec{p}} d^2q' \times$$

$$\times \iint d^2p \, d^2q \, e^{-i\vec{q}\vec{p}+iq_x(k_1-k_2-q_y)} \hat{a}_{k_2+q_y} \hat{a}_{k_1-q_y}$$

After very simple integrations we can receive the resulting expression

$$\hat{A}_{k_1 k_2} \equiv \int d^2q \, \tilde{V}'(q) e^{iq_x(k_1-k_2-q_y)} \hat{a}_{k_1-q_y} \hat{a}_{k_2+q_y}$$

where the new potential

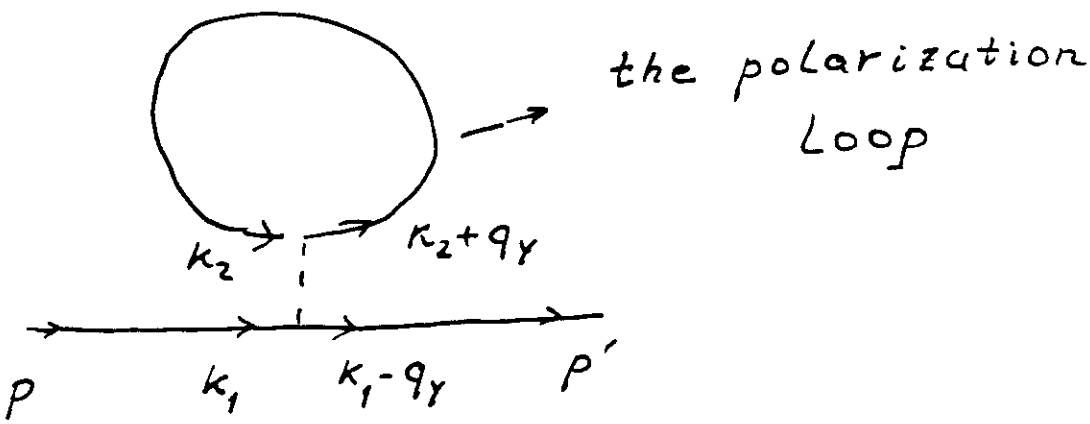
$$\tilde{V}'(q) = \frac{1}{2\pi} \int d^2p \, e^{i\vec{q}\vec{p}} \tilde{V}(p)$$

Due to this transformation:

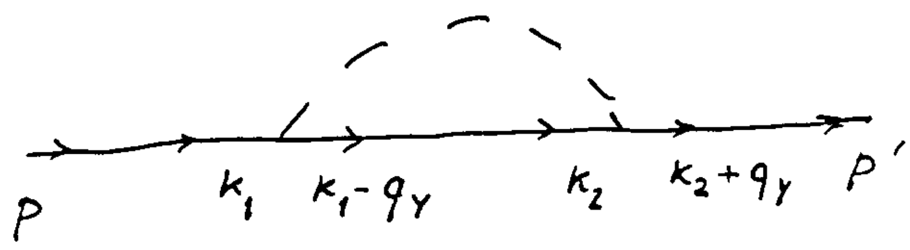
1. The indecies of the operators have changed their positions
2. The potential is renormalazed

There are very serious consequencies from this transformation. The first order diagrams for the usual Green functions are:

a)



b)



In the terms of the Green functions the diagram a) corresponds to

$$G_{p,k_1}\, G_{k_1-q_y,p'}\, G_{k_2+q_y,k_2}$$

After the transformation we have

$$k_1 - q_y \longleftrightarrow k_2 + q_y,$$

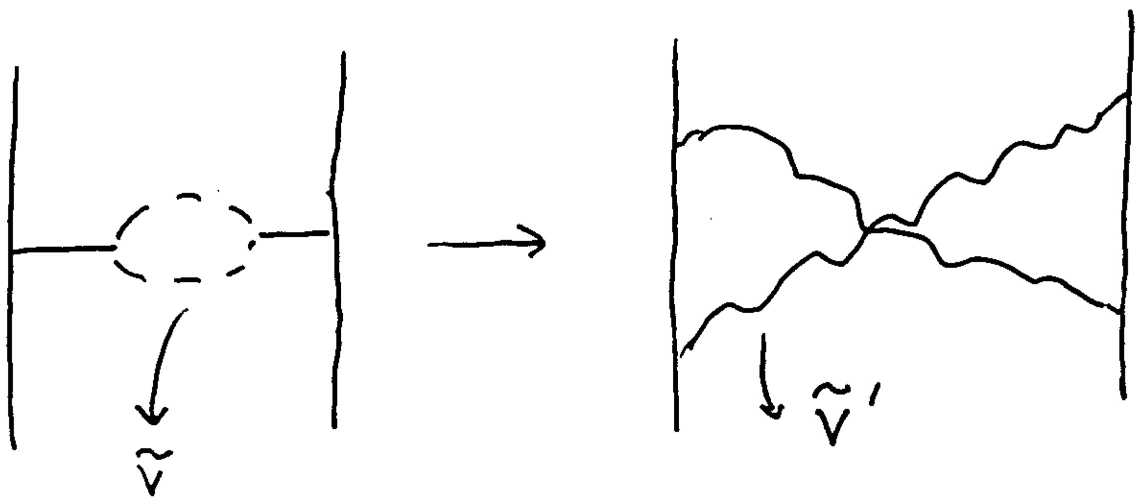
$$\tilde{V}(q) \longrightarrow \tilde{V}'(q)$$

so that we obtain

$$G_{p,k_1}\, G_{k_2+q_y,p'}\, G_{k_1-q_y,k_2}$$

and we get the diagram b) but with the new potential $\tilde{V}'(q)$

Another exemple of the topological transformation of the diagram:

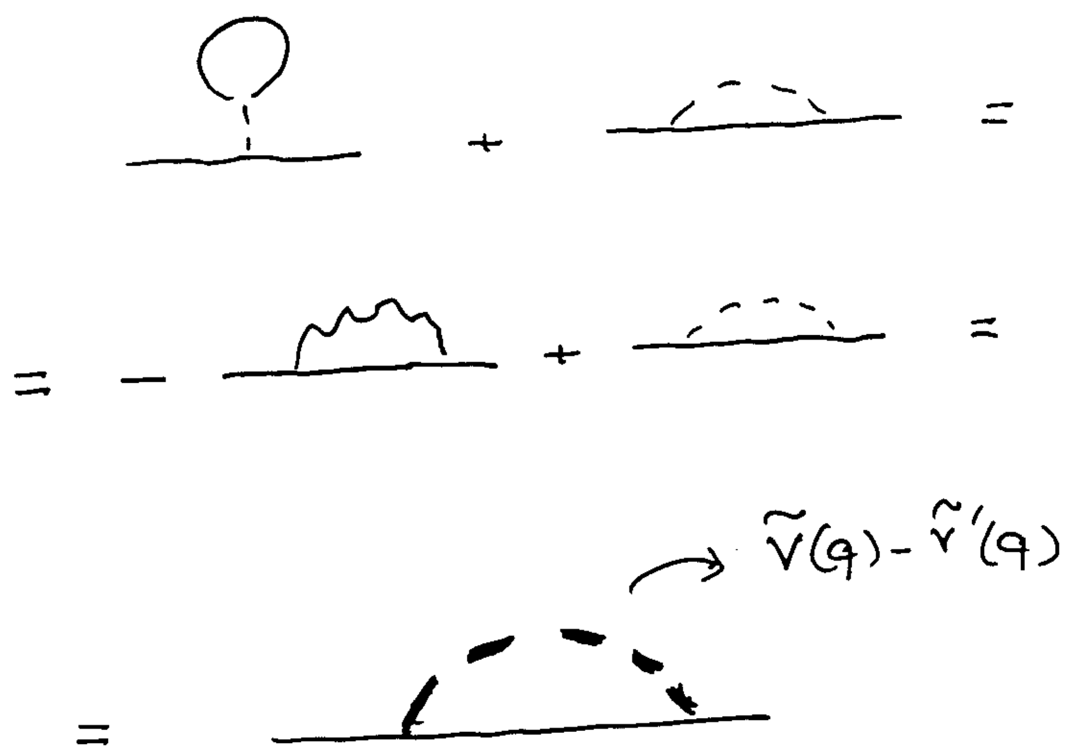


The general result is:

Any diagram with polarization loops
can be transformed into the diagram
without Loops but with the new
potential. The new diagrams must
be added with the old ones so that
resulting diagrams are without Loops
but correspond to the new effective
potential

$$\tilde{V}(q) - \tilde{V}'(q)$$

The simle example



So the problem of interecting 2D system is equivalent to system interacting with some external field.

The off-diagonal Green functions

To evaluate the quasiparticle state energies we need the two-point correlation function:

$$n^{(2)}(\vec{r}_1, \vec{r}_2) = \langle \hat{n}(\vec{r}_1)\, \hat{n}(\vec{r}_2) \rangle$$

where $\hat{n}(\vec{r})$ is the density operator.

The most interesting feature of Laughlin trial wave functions is the presence of high order zeroes.

In the symmetric gauge for the vector potential the correlator is:

$$n^{(2)}(\vec{r}_1, \vec{r}_2) = \sum \Psi^*_{m_1}(\vec{r}_1)\, \Psi^*_{m_2}(r_2)\, \Psi_{m_2'}(\vec{r}_2)\, \Psi_{m_1'}(\vec{r}_1)$$

$$\times \langle \hat{a}^+_{m_1} \hat{a}^+_{m_2} \hat{a}_{m_2'} \hat{a}_{m_1'} \rangle$$

$\Psi_m(\vec{r})$ - the wave function which corresponds to the eigenstate with the azimuthal quantum number m.

For Laughlin trial function the correlator $n^{(2)}(\vec{r}_1, \vec{r}_2)$ vanishes as

$$(\vec{r}_1 - \vec{r}_2)^{2k}$$

with the odd $k = 3, 5, \ldots$

S. Girvin proposed the following expression for the correlator:

$$\langle \hat{a}^+_{m_1} \hat{a}^+_{m_2} \hat{a}_{m_2'} \hat{a}_{m_1'} \rangle = \nu^2 (\delta_{m_1 m_1'} \delta_{m_2 m_2'} - \delta_{m_1 m_2'} \delta_{m_2 m_1'} - \gamma^{m_1 m_1'}_{m_2 m_2'}),$$

so that the function $n^{(2)}(\vec{r}_1, \vec{r}_2)$ has zeroes of any order. Due to Girvin the anomalous correlator $\gamma^{m_1 m_1'}_{m_2 m_2'}$ is of the form:

a) $$\gamma^{m_1 m_1'}_{m_2 m_2'} \sim \sum_\ell f_\ell(m_1, m_2) f_\ell(m_1', m_2') C_\ell$$

I can propose the correlator of the form:

b) $\langle \hat{a}^{+}_{m_1} \hat{a}^{+}_{m_2} \hat{a}_{m_2'} \hat{a}_{m_1'} \rangle \sim \varphi(m_1, m_1') \varphi(m_2, m_2')$

For the special form of the function $\varphi(m, m')$ the correlator $n^{(2)}(\vec{r}_1, \vec{r}_2)$ vanishes as $(\vec{r}_1 - \vec{r}_2)^{2k}$.

It is very temptative to introduce the anomalous Green functions:

a) $F_{mm'}(\tau, \tau') = - \langle T_\tau \hat{a}_m(\tau) a_{m'}(\tau') \rangle$

b) $G_{mm'}(\tau, \tau') = - \langle T_\tau \hat{a}_m(\tau) \hat{a}^{+}_{m'}(\tau') \rangle$

$m \neq m'$

T_τ - the operator of the time ordering.

F-function is analogous to the corresponding function in the theory of superconductivity.

Both Green functions are matrix elements between the states with different azimuthal quantum number.

In a magnetic field there are operator of the full azimuthal quantum number

$$\hat{M} = \sum m \hat{a}_m^+ \hat{a}_m$$

and the operators of the magnetic translations:

$$\hat{t}_+ = \sum_m \sqrt{2(m+1)}\, \hat{a}_{m+1}^+ \hat{a}_m$$

$$\hat{t}_- = \sum_m \sqrt{2m}\, \hat{a}_{m-1}^+ \hat{a}_m$$

with the commutators

$$[\hat{H}, \hat{M}] = 0, \quad [\hat{H}, \hat{t}_\pm] = 0$$

On the other hand the commutator

$$[\hat{M}, \hat{t}_\pm] = \pm \hat{t}_\pm$$

so that the operators of the magnetic translations change the azimuthal quantum number (but don't change the energy of the system.

The Green functions can exist as off-diagonal matrix elements between eigenstates with equil energies but different azimuthal quantum numbers.

We can introduce the generalized Bogolubov transformations

$$\hat{a}_m = \sum_{m_1} \left(U_{mm_1} \hat{b}_m + V_{mm_1} \hat{b}_m^+ \right).$$

$\hat{b}_m$ ($\hat{b}_m^+$) are the operators of the quasiparticles.

These new off-diagonal Green functions can be useful for the constraction of the Ginzburg-Landau approximation in the theory of fractional quantum Hall effect.