INTERNATIONAL ATOMIC ENERGY AGENCY
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SPRING COLLEGE IN CONDENSED MATTER
ON
'PHYSICS OF LOW-DIMENSIONAL SEMICONDUCTOR STRUCTURES'
(23 April - 15 June 1990)

TWO-DIMENSIONAL ELECTRON CRYSTALS

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2- Dimensional
Wigner Crystals

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1. The phase diagram (n, T, B)

2. Experimental realizations of 2-D electron layers

3. R.F. detection techniques

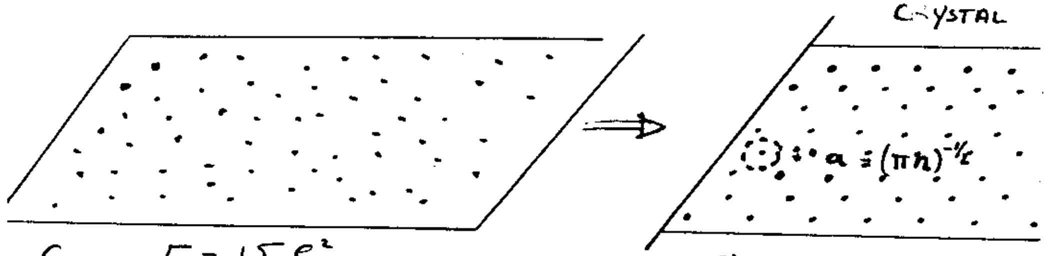
4. Propagation of shear in the 2-D electron crystal in the classical regime.

5. Specific and latent heat of the classical 2-D electron crystal

6. The Quantum Solid - Observation of the magnetically induced Wigner crystal.

Collaborators: F.I.B. Williams, D.C. Glattli, G. Deville, A. Valdez - SACLAY
B. Etiene, E. Paris - Bagneux
J.J. Foxon, C.T. Harris - Philips
B. Clark P. Wright - Oxford
M. Heiblum - I.B.M.

T=0, POINT PARTICLES OF CHARGE e, DENSITY n(

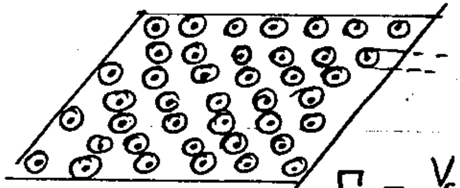


ϵ, $E = \frac{1}{\epsilon}\sum_{i,j} \frac{e^2}{|r_{ij}|}$

TRIANGULAR LATTICE

$V_c = \frac{e^2}{\epsilon a}$ ENERGY PER PARTICLE

$T \neq 0$ THERMAL KINETIC ENERGY $K = kT$



$\langle u^2 \rangle \simeq \frac{\epsilon a^3}{e^2} kT$

$$\Gamma \equiv \frac{V_c}{K} = \frac{e^2 \pi^{1/2}}{\epsilon kT} n^{1/2} \sim \frac{a^2}{\langle u^2 \rangle}$$

CRYSTAL MELTS WHEN LONG RANGE ORDER IS LOST

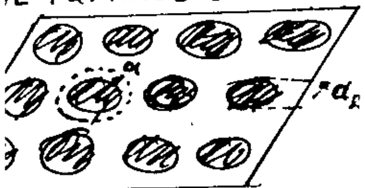
$\Gamma^m = 127 \pm 3$ (EXPERIMENTS ON ELECTRONS ON He

FOR $n = 10^8\ cm^{-2}$, $\epsilon = 1$ $T_m = 250\ m°K$

REAL PARTICLES

1 particle $\psi(r) \sim e^{i\vec{k}\vec{r}}$

n particles (low density)

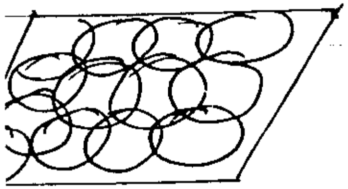


$K_a = \frac{\hbar^2 \pi n}{m^*}$ ZERO POINT ENERGY

$V_c = \frac{e^2}{\epsilon}(\pi n)^{1/2}$

$$r_s = \frac{V_c}{K_a} = \frac{e^2 m^*}{\epsilon \hbar^2} \cdot \frac{1}{(\pi n)^{1/2}} = \frac{a}{a_B}$$

h density $\Rightarrow$ MELTING $r_s \leq 33$ (CEPERLEY 78)



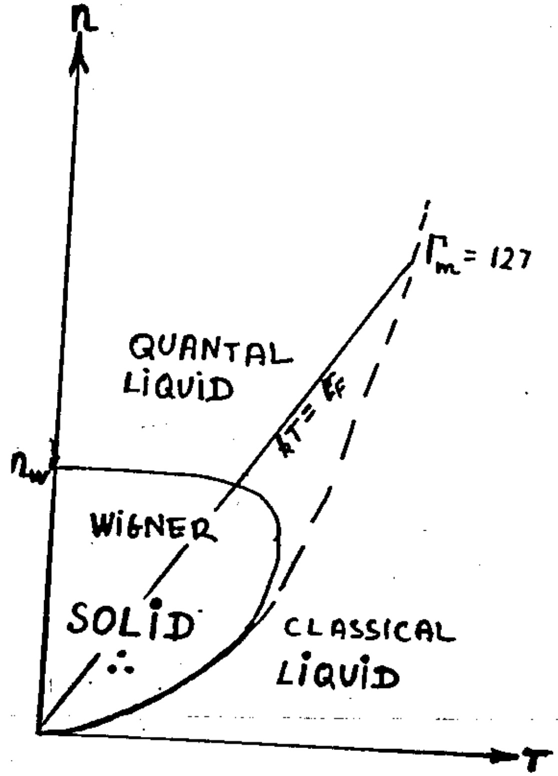
MELTING UPON COMPRESSION

WIGNER TRANSITION (E. WIGNER 1930)

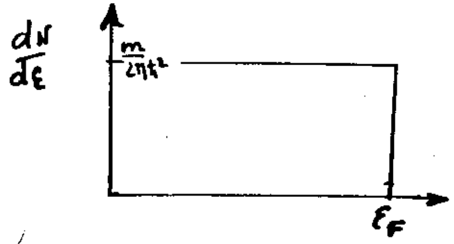
$$n_W = \frac{1}{(33\pi a_B)^2} = \left(\frac{e^2 m^*}{33\pi \epsilon \hbar^2}\right)^2$$

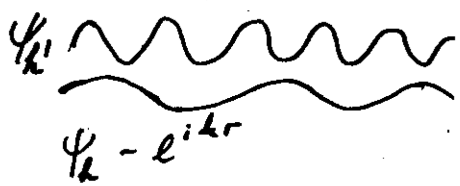
DRIVEN BY QUANTUM FLUCTUATIONS

PHASE DIAGRAM

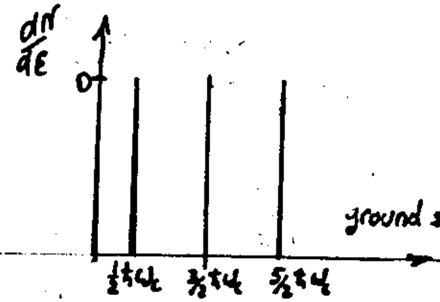


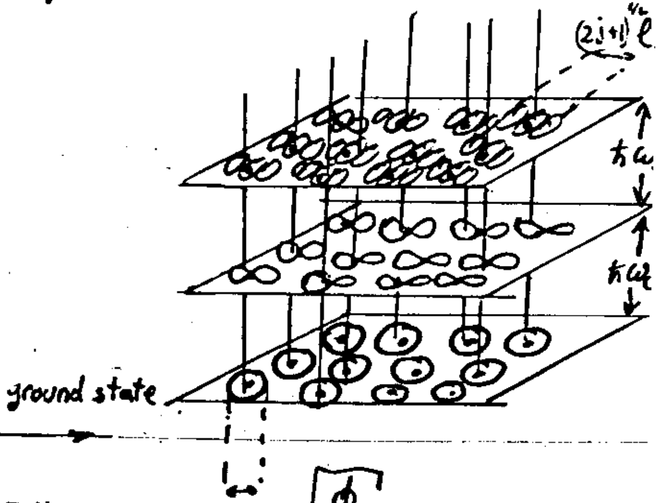
$r_s \gg 1$ DEEP QUANTUM REGIME : $a \ll a_B$, $\langle u^2 \rangle^{1/2}$





$B_\perp \neq 0$





MAGNETIC LENGTH : $\ell_c = \sqrt{\dfrac{\phi_0}{2\pi B}}$

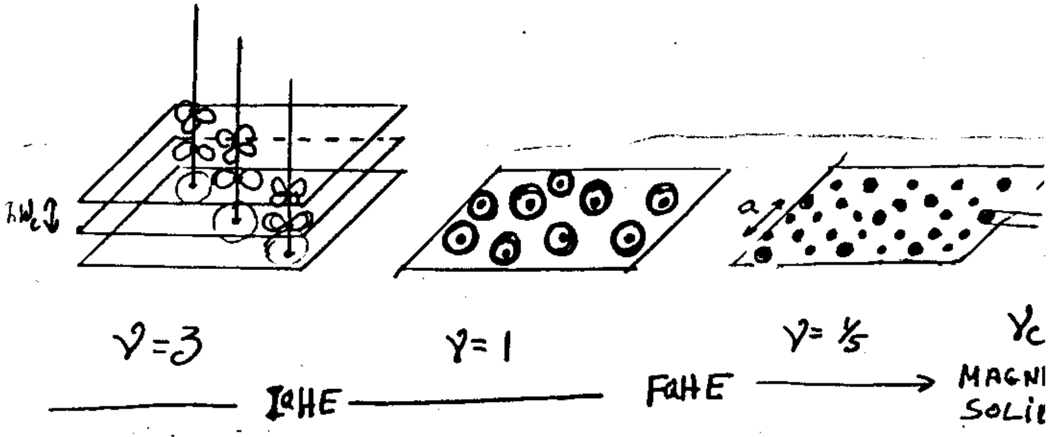
FUNDAMENTAL FLUX UNIT $\phi_0 = \dfrac{hc}{e}$

DEGENERACY $D = \dfrac{B}{\phi_0}$

LANDAU LEVEL $E_j = \left(\frac{1}{2} + j\right)\hbar\omega_c$, $\omega_c = \dfrac{eB}{mc}$.

FILLING FACTOR : $\nu \equiv \dfrac{n}{D} = \phi_0 \dfrac{n}{B}$: # OF ELECTRONS PER FLUX LINE

$2\pi\omega_c = 360\ GHz/Tesla$, $\phi_0 = 10^{-8}\ Gauss \times cm^2$ $\ell_c = 40\,Å/\sqrt{Tesla}$

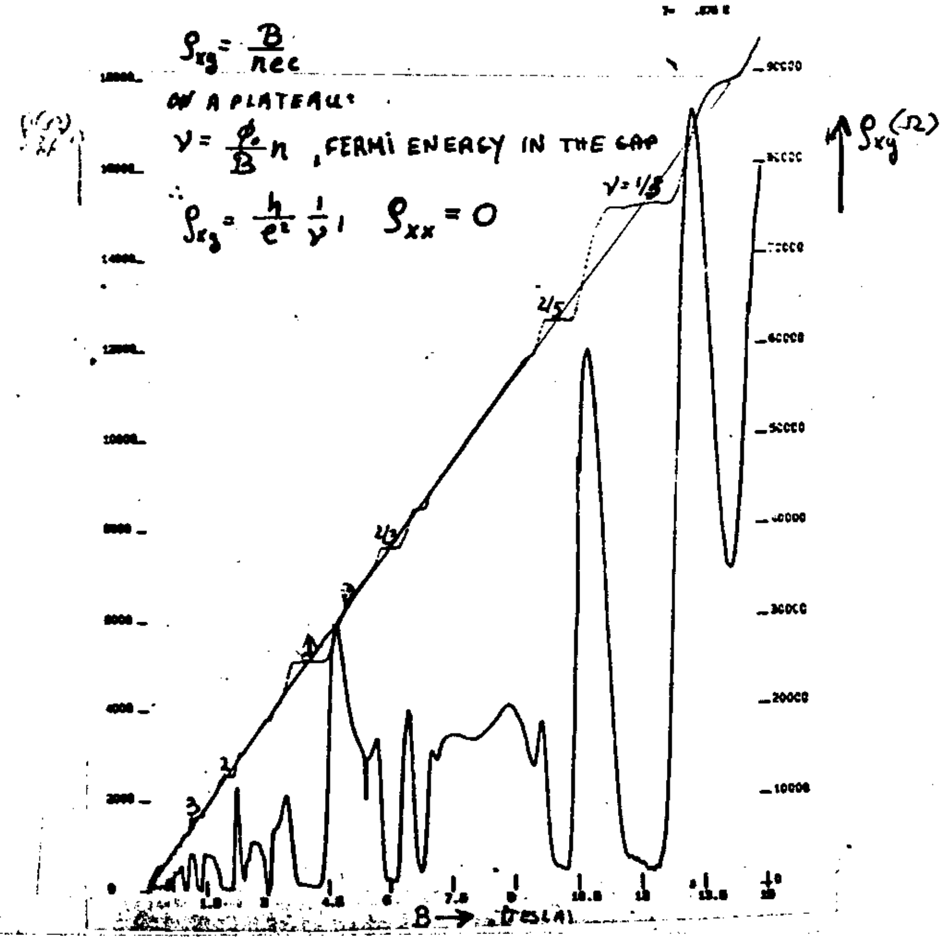


$\rho_{xy} = \dfrac{B}{nec}$

AT A PLATEAU:

$\nu = \dfrac{\phi_0}{B} n$, FERMI ENERGY IN THE GAP

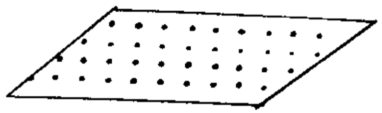
$\therefore\ \rho_{xy} = \dfrac{h}{e^2}\dfrac{1}{\nu}$, $\rho_{xx} = 0$



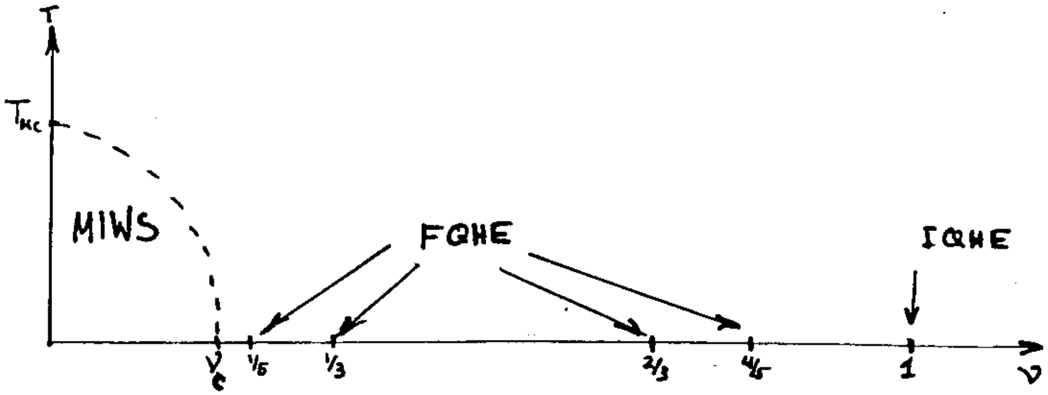
THE MAGNETIC FIELD LOCALIZES THE ELECTRON INTO A CYCLOTRON ORBIT

$$\xrightarrow[\nu \to 0]{B \to \infty} \quad \ell_c \ll a$$ NO MORE OVERLAP → CLASSICAL

$T \le T_{MC}$ MAGNETICALLY INDUCED WIGNER SOLID

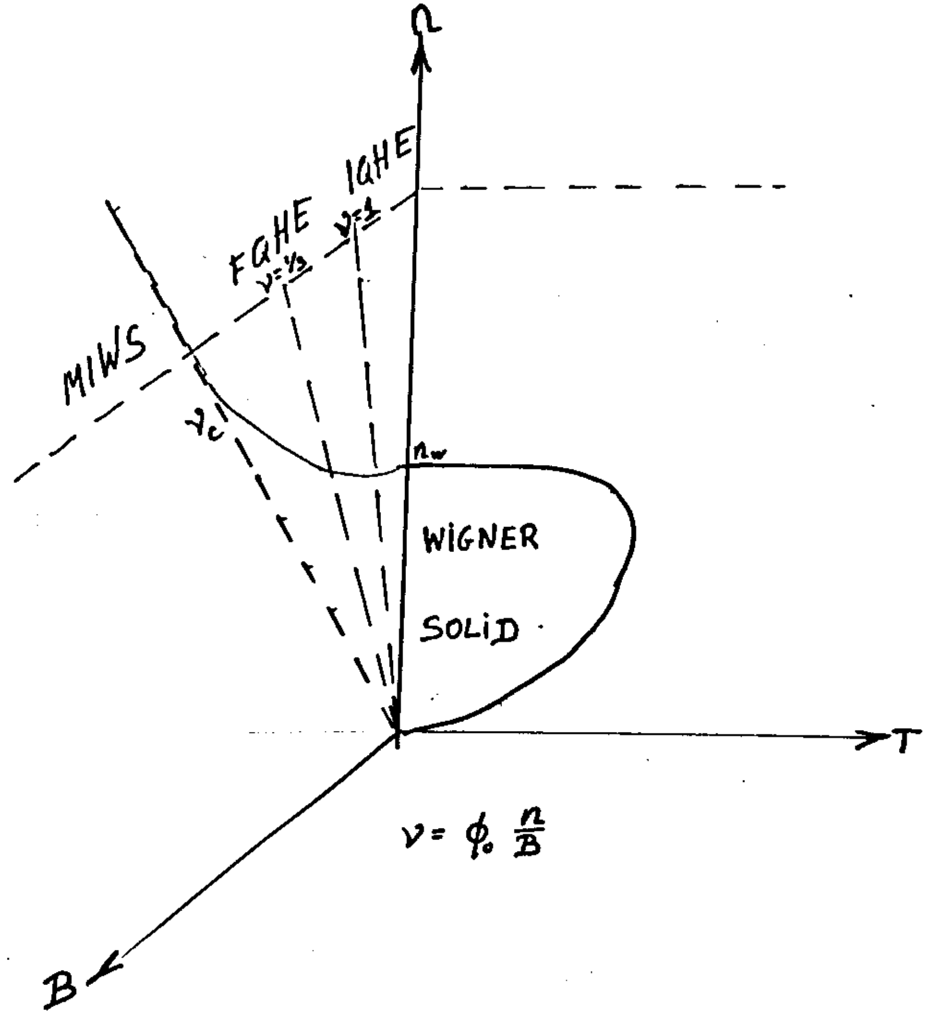


(DURKHAN, ELIOT, MARCH 68)



(Levesque, Weiss, McDonald 84) $0.11 \le \nu_c^{th} \le .25$ (MAKI, ZOTOS 83)

PHASE DIAGRAM IN n, T, B



$$\nu = \phi_0 \frac{n}{B}$$

WHAT IS A SOLID? (3-D)

1. MACROSCOPIC DEF.:

   A SYSTEM THAT RETURNS TO ITS ORIGINAL SHAPE UPON THE REMOVAL OF SHEAR STRESS.

2. MICROSCOPIC DEF.:

   PERIODIC STRUCTURE => LONG RANGE ORDER

$$\langle \rho_c(0)\, \rho_c^*(r) \rangle \equiv C_{\rho_c}(r) \xrightarrow[r\to\infty]{} \text{CONST.}$$

LIQUID

1. MACROSCOPIC:

   VISCOUS RESPONSE TO SHEAR STRESS

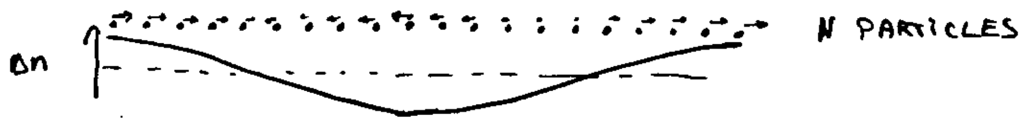
2. MICROSCOPIC: NO LONG RANGE ORDER

$$C_\rho(r) \sim e^{-r/\xi}$$

MELTING:

IS MEDIATED BY FLUCTUATIONS

FLUCTUATIONS



FOR A HARMONIC INTERACTION BETWEEN NEAREST NEIGHBOURS AND SMALL DISPLACEMENTS THE ENERGY OF THIS DENSITY FLUCTATION IS:

	1-D	2-D	3-D
$\propto$	$N\left(\frac{2\pi}{N}\right)^2$	$N^2\left(\frac{2\pi}{N}\right)^2$	$N^3\left(\frac{2\pi}{N}\right)^2$
$N\to\infty$	$\to 0$	$\to$ CONST	$\to\infty$
$T\to 0$ LONG RANGE ORDER?	NO	?	YES

1923 – 1937 PEIRELS, LANDAU, 1966 MERMIN & WAGNER:

$$\langle u^2 \rangle \sim kT \log N$$

IN A 2-D SYSTEM (of continuous symmetry and n.n. interaction) THE LONG RANGE ORDER IS DESTROYED BY FLUCTUATIONS AT ANY $T>0$. (solids, superfluid films, superconducting films, xy mod)

CAN A 2D SOLID EXIST?

1968 - MERMIN, WAGNER

$C_g(R) \xrightarrow[R\to\infty]{} R^{-\alpha}$

$\downarrow$

QUASI LONG RANGE POSITIONAL ORDER

$C_g(\theta) \xrightarrow[R\to\infty]{} \text{Const}$

$\downarrow$

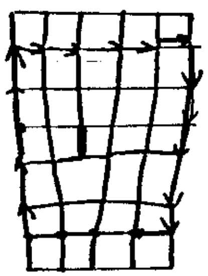
LONG RANGE ORIENTATIONAL ORDER

$C_g(\theta)$ = BOND ANGLE CORRELETION FUNCTION

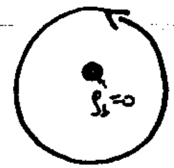
1972 - KOSTERLITZ, THOULESS   1978 - HALPERIN NELSON YOUNG

THE 2·D SYSTEM AT $T>0$ IS VIEWED AS AN ORDERED SYSTEM $\oplus$ THERMALLY EXCITED TOPOLOGICAL DEFECTS.

CRYSTAL	SUPERFLUID	SUPERCONDUCTOR
$\oplus$	$\oplus$	$\oplus$
DISLOCATIONS	VORTICES	FLUX LINES



$\vec{b}$ → BURGESS VECTOR



$\Delta\varphi = 2\pi$

$\psi(r) \to \psi(r)e^{i\varphi}$

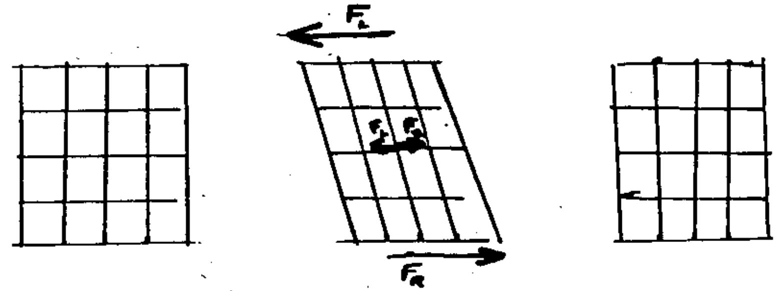
$\Delta\varphi = 2\pi$

$\psi(r) \to \psi(r)e^{i\varphi}$

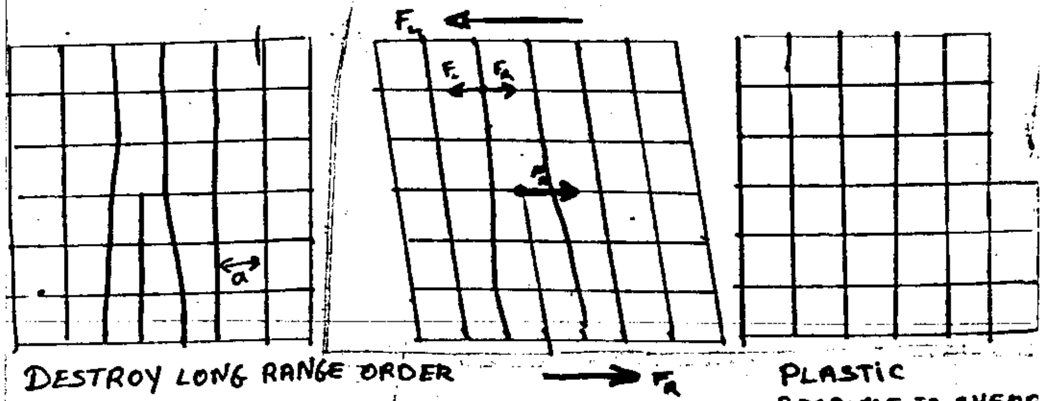
PERFECT CRYSTAL

• LONG RANGE ORDER (MICROSCOPIC)

• ELASTIC RESPONSE TO SHEAR STRESS (MACROSCOPIC)



TOPOLOGICAL DEFECTS: DISLOCATIONS



DESTROY LONG RANGE ORDER

PLASTIC RESPONSE TO SHEAR (LIQUID)

∴ UNPAIRED DISLOCATIONS ⇒ LIQUID PHASE.

FREE ENERGY OF DISLOCATION IN SOLID

$$F = E_c + \frac{a^2\mu}{2\pi}\ln N - 2k_B T \ln N$$

μ = shear modulus

$$\boxed{T_m = \frac{a^2\mu_m}{4\pi k_B}}$$

AT $T \gtrsim T_m$ LATTICE BECOMES UNSTABLE TO CREATION OF FREE DISLOCATIONS ⇒ MELTING

CONSIDER A PAIR OF DISLOCATIONS OF EQUAL BUT OPPOSITE $\vec{b}$ SEPARATED BY A DISTANCE d:

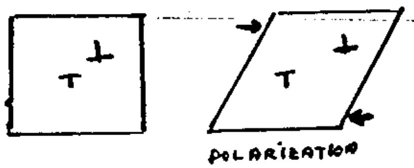
THE FREE ENERGY OF SUCH A PAIR:

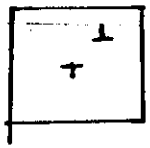
$$F \simeq 2E_c + \frac{b^2\mu}{2\pi}\log\frac{d}{a} - 4k_BT\log\frac{L}{a}$$

- $0 < T < T_{KT}$: FINITE CONCENTRATION OF THERMALLY EXCITED DISLOCATIONS. (VORTICES, FLUX LINES)

  - CONSERVE THE LONG RANGE ORIENTATIONAL ORDER / ALGEBRICALLY DECAYING ORIENTATIONAL ORDER

  - MACROSCOPIC PROPERTIS OF THE ORDERED PHASE PERSIST.





ELASTIC RESPONSE TO SHEAR
⇓
SYSTEM IS SOLID

- $T > T_{KT}$ FREE TOPOLOGICAL DEFECTS ARE PRESENT

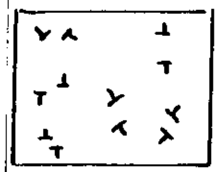
  - $C_g(r) \xrightarrow[r\to\infty]{} e^{-r/\xi}$, $C_g(\theta) \longrightarrow r^{-\eta}$

  - MACROSCOPIC PROPERTIES OF ORDERED PHASE ARE LOST

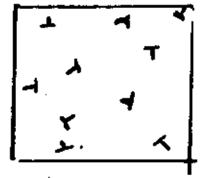
$\mu = 0$ PLASTIC RESPONSE TO SHEAR; $\rho_s = 0$ NO SUPERFLUIDITY

THE KOSTERLITZ-THOULESS TRANSITION

MEDIATED BY UNBINDING OF PAIRED DISLOCATIONS



$T < T_{KT}$

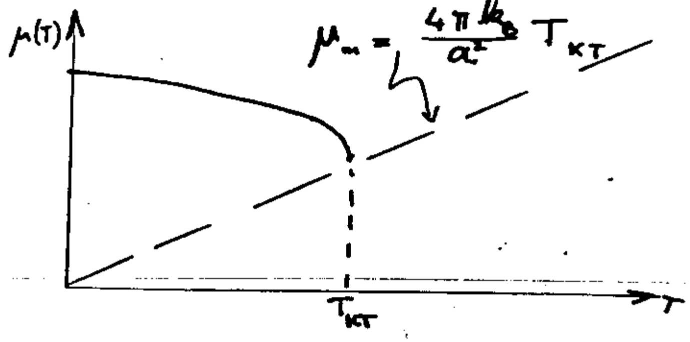


$T > T_{KT}$

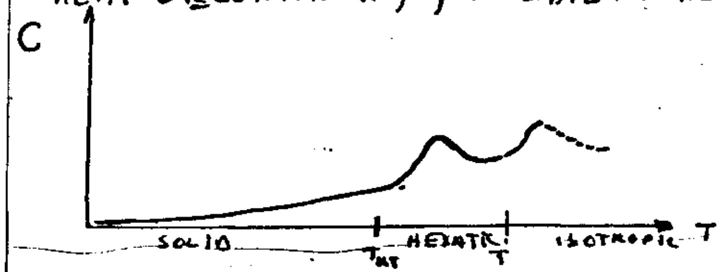
$$T_{KT} = \frac{a^2\mu_m}{4\pi k_B}$$

AT MELTING - UNIVERSAL JUMP IN SHEAR MODULUS

$$\mu_m = \frac{4\pi k_B}{a^2}T_{KT}$$



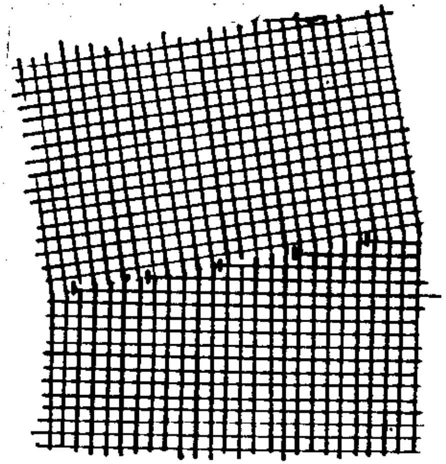
TRANSITION IS OF CONTINUOUS ORDER - NO SPECIFIC HEAT DISCONTINUITY, NO LATENT HEAT.



TRANSITION TO ISOTROPIC PHASE PROCEEDS VIA INTERMEDIATE HEXATIC PHASE → UNBINDING OF PAIRED DISCLINATIONS → LOSS OF ORIENTATIONAL ORDER

ALTERNATIVE MELTING MECHANISM: GRAIN BOUNDARY MELTIN[G]

· CHUI, SAITOH 78; MORF 79; KALIA, VASHISHTA, DE LEEUW 81



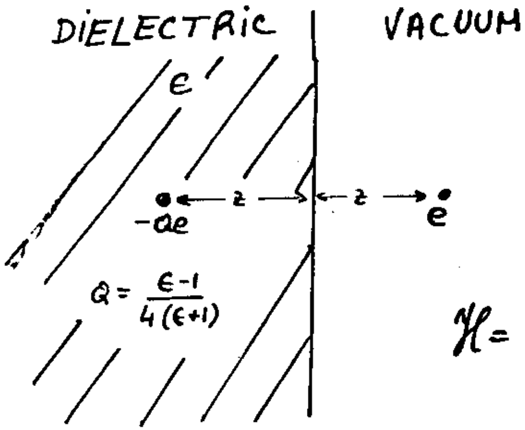
[S]YSTEM BECOMES UNSTABLE [T]O FORMATION OF SMALL [D]OMAINS SEPARATED BY [G]RAIN BOUNDARIES

$T_M \leq T_M^{KT}$

[T]RANSITION IS FIRST [O]RDER

[N]UMERICAL ESTIMATES OF ENTROPY JUMP AT MELTING $\Delta S_{GB} \geq .3\, k_B$.

REALIZATIONS OF 2-D ELECTRON SYSTEMS



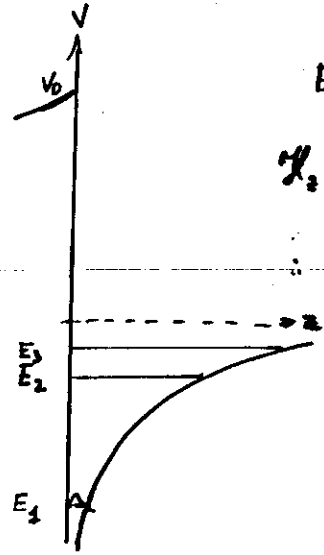
FOR $z \gg a \Rightarrow$ TRANSLATIONAL INVARIANCE IN XY PLANE

$$\mathcal{H} = \begin{cases} \underbrace{\frac{P_x^2}{2m} + \frac{P_y^2}{2m}}_{\mathcal{H}_{xy}} + \underbrace{\frac{P_z^2}{2m} - \frac{e^2 Q}{z}}_{\mathcal{H}_z} & z > 0 \\ \frac{P_x^2}{2m} + \frac{P_y^2}{2m} + V_0 & z \leq 0 \end{cases}$$

$$E_{lk} = E^{\parallel}(k) + E_l^{\perp} = \frac{\hbar^2 k^2}{2m} + E_l^{\perp}$$

$\mathcal{H}_z$: 1-D HYDROGEN ATOM WITH PROTON OF CHARGE Q

$$E_l^{\perp} = -\frac{Q^2}{n^2} R_{yd} \qquad l = 1, 2, \ldots$$



$\psi_l^{\perp}(z)$ = Hydrogenic radial wave functi[on]

$$\psi_1^{\perp}(z) = 2\gamma^{3/2}\, z\, e^{-\gamma z}$$

$$\gamma = \frac{mQe^2}{\hbar^2} = \frac{Q}{a_B}$$

$$\langle z \rangle_1 = \frac{3}{2}\frac{a_B}{Q} = \frac{.75}{Q} \text{ Å}$$

HO-T DIELECTRICS $\epsilon \gtrsim 4 \Rightarrow \langle z \rangle_1 \sim 5\text{Å} \simeq a$

$\Rightarrow$ ELECTRON is SENSITIVE TO LOCAL STRUCTURE OF SURFACE

$\Rightarrow$ NO TRANSLATIONAL INVARIANCE

$\rightarrow$ TRAPPING

WHEN $Q \ll 1$, $\epsilon - 1 \ll 0 \Rightarrow \langle z \rangle_1 \gg a \Rightarrow$ 2D ELECTRON SYSTEM

WEAKLY POLARIZABLE MATERIALS

	ϵ	Q	E_1 (meV)	$\langle z \rangle_1$ (Å)
^3He	1.026	5.2×10^{-3}	0.39 ~ 4°K	150
^4He	1.057	6.9×10^{-3}	0.79 ~ 8°K	114
Ne	1.11	22×10^{-3}	11.5 ~ 110°K	36
H_2	1.13	26×10^{-3}	18 ~ 180°K	28

ELECTRONS ON HELIUM:

- $T < 4.2°K$ Helium is liquid $\rightarrow$ surface is smooth : All fluctuations time average to zero.

$\langle z \rangle_1 \gg a \sim 2°\text{A} \Rightarrow$ Translational invariance

$V_0 \sim \frac{\hbar^2}{m r_0^2} \sim 1\,ev$ Pauli exclusion principle

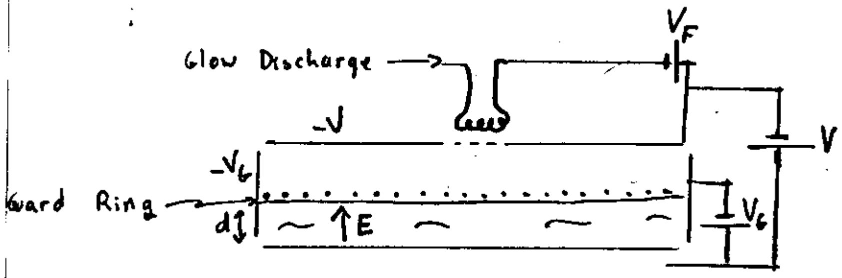
$\Rightarrow V_0 \gg E_1$ infinite barrier approximation justified

$E_1 \sim 8°K \gg$ Energy of motion in plane $\leq 1°K$

$\Rightarrow$ TWO DIMENSIONAL BEHAVIOUR

ELECTRONS ON LIQUID HELIUM : $\epsilon = 1.057$, $Q = 7 \times 10^{-3}$

$z_1 = 80\text{Å}$, $E_1 = 8°K$



$E = \frac{V}{d} = 4\pi e n$ FULLY CHARGED SURFACE

TEMPERATURE $20\,mK \leq T \leq 1°K$

MOBILITY	$\mu \gtrsim$	$10^7\ cm^2/v \cdot sec$
SCATTERING TIME	$\tau \leq$	6 nsec
MEAN FREE PATH	$\leq$	3 μm

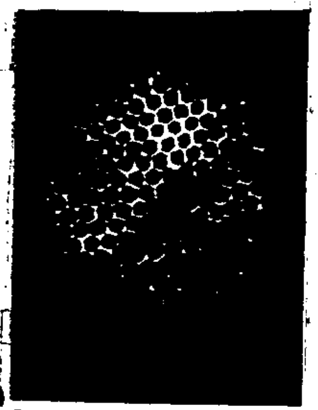
HELIUM SURFACE INSTABILITY:

THE RIPPLON DISPERSION RELATION

$$\omega_k^2 = gk + \frac{\alpha}{\rho} k^3 - \left(E_\perp^2 + 4\pi^2 n^2 e^2\right) \frac{k^2}{2\pi \rho}$$

BECOMES UNSTABLE AT $n \gtrsim 2 \times 10^9\,cm^{-2}$

$\Rightarrow$ MANY ELECTRON DIMPLE INSTABILITY: (P. LEIDERER)



Periodic array of charged dimples on a liquid helium surface. Each dimple is about a millimeter wide, a tenth of a millimeter deep and contains about 10^7 electrons. (Photograph taken by Paul Leiderer at University of Mainz.)

$n < n_w = 10^{13}$ NO WIGNER TRANSITION

SYSTEM IS CLASSICAL : $T_F \leq 0.02°K$

IS IT REALLY TWO-DIMENSIONAL?

IN 3D

MAGNETIC FIELD ⇒ LORENZ FORCE

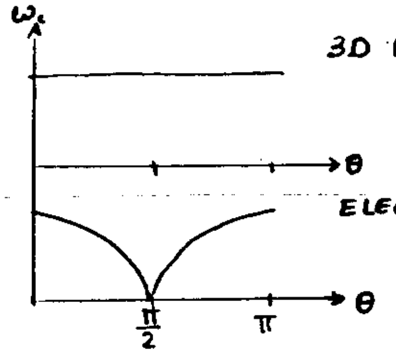
⇒ CYCLOTRON RESONANCE

$$\omega_c = \frac{eH}{m^* c}$$

N 2D

ONLY IN-PLANE MOTION is POSSIBl

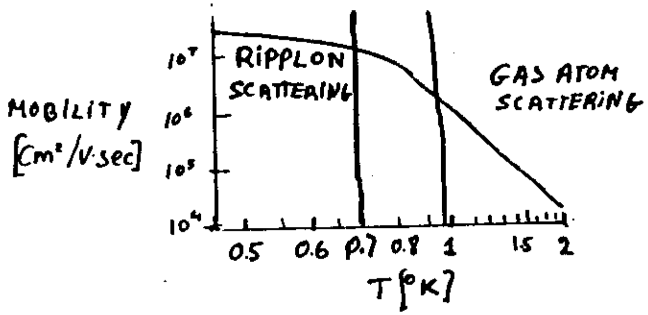
$$\omega_c = \frac{eH\cos\theta}{m^* c}$$



3D PLASMA

ELECTRONS ON HELIUM T ≪ 8°K

INTERACTIONS WITH THE 3rd DIMENSION:

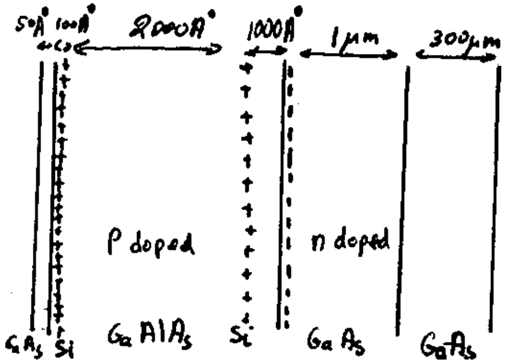


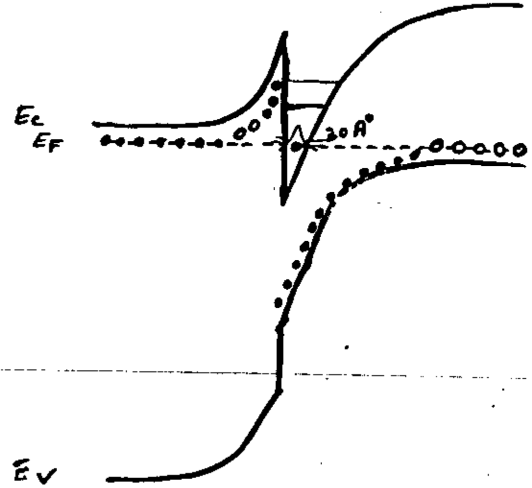
∴

MEAN FREE PATH $\lesssim 3\mu$

SCATTERING TIME $\lesssim$ 6 nsec

2-DIMENSIONAL ELECTRON GAS IN A GaAs/GaAlAs heterojunction





$E_B \sim 500K$

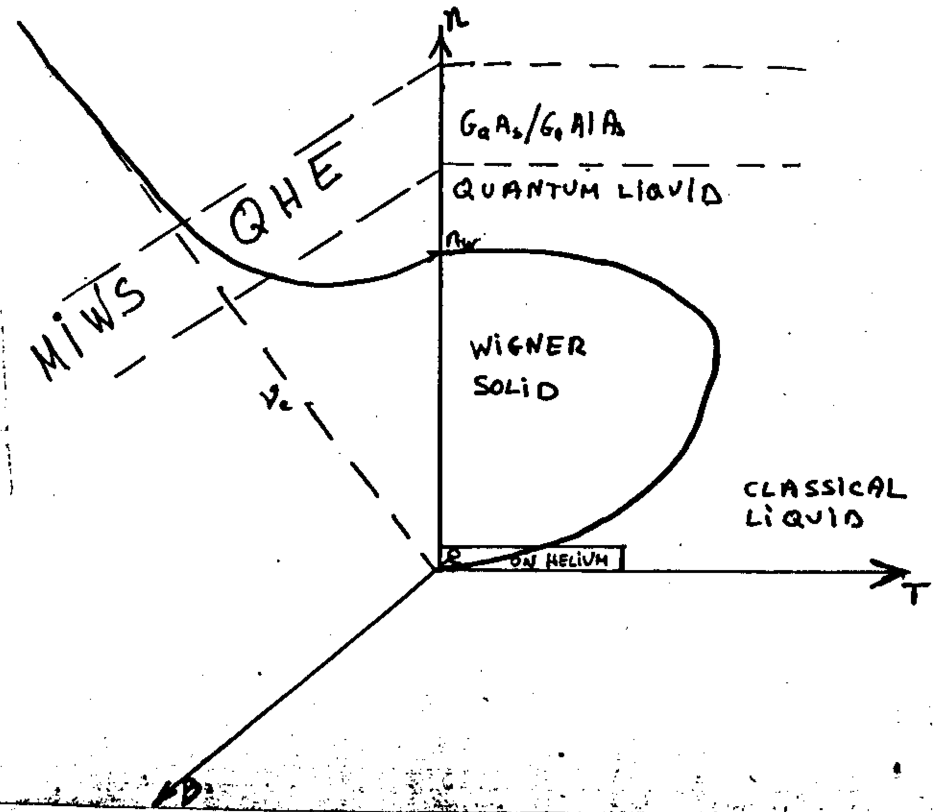
$n \gtrsim 3\times10^{10}\,cm^{-2}$

$\mu \leq 10^6\,cm^2/Vsec$

$\ell_{mfp} \lesssim .2\,\mu m$

$\tau_{sc} \leq 2\,nsec$

		ELECTRONS ON HELIUM	GaAs/GaAlAs HETEROJUNCTION
MASS	m^*	m_e	$0.07\,m_e$
DIELECTRIC CONSTANT	ϵ	1.057	~13
MOBILITY	μ	$\leq 10^7\,cm^2/Vsec$	$\leq 2\times10^6$
SCATTERING TIME	τ	≤ 6 nsec	$\leq .08$ nsec
MEAN-FREE PATH	ℓ	$\leq 3\,\mu m$	$\leq .4\,\mu$
DENSITY	n	$\leq 2\times10^9\,cm^{-2}$	$\geq 5\times10^{10}\,cm^{-2}$
CRITICAL DENSITY	n_W	$4\times10^{13}\,cm^{-2}$	$10^9\,cm^{-2}$
BINDING ENERGY	E_1	~8°K	~500°K
FERMI TEMPERATURE	T_F	≤ 0.02°K	≥ 8°K

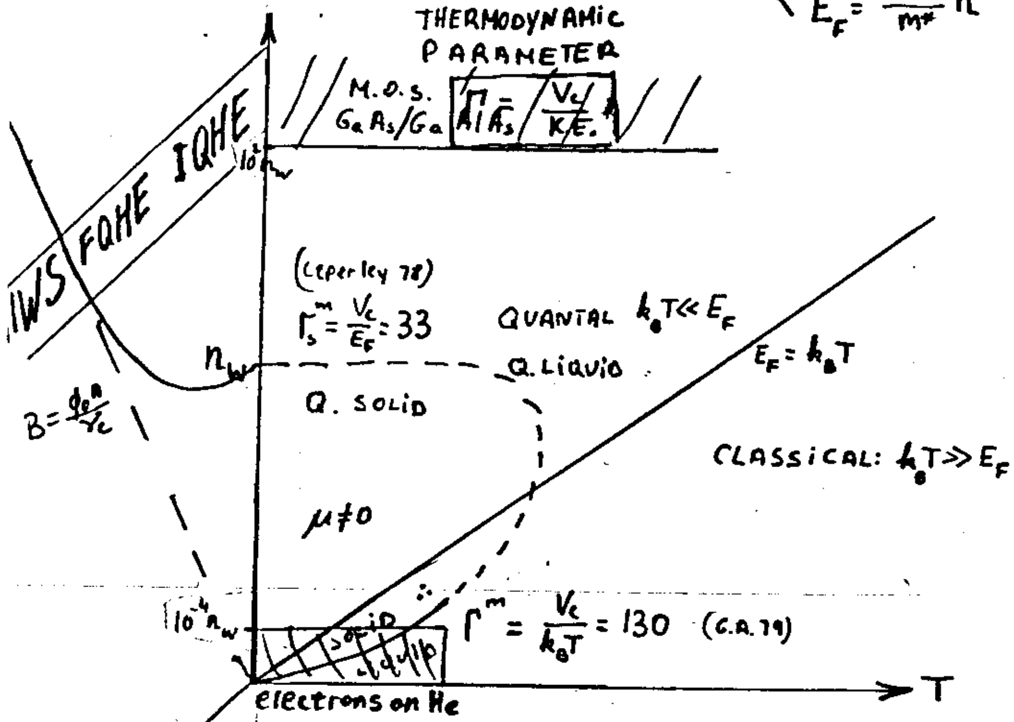


2-D ELECTRON PLASMA

electron density: $n = (\pi a^2)^{-1}$

Coulomb correlation energy $V_c = \frac{e^2}{\epsilon}(\pi n)^{1/2}$

$$K.E. \left\{ \begin{array}{l} k_B T \\ E_F = \frac{\hbar^2 \pi}{m^*} n \end{array} \right.$$



$\nu = \frac{a^2}{\ell_c^2} = \phi_0 \frac{n}{B}$

	m^*/m_0	ϵ	n_W	n	r_s
e on He	1	1.06	1.5×10^{13}	$\leq 2 \times 10^{9}$	≥ 2000
GaAs	0.07	13	5×10^{8}	$\geq 3 \times 10^{10}$	≤ 3

HOW TO DETECT A CRYSTAL?

DISTINGUISHING PROPERTIES:

1. MICROSCOPIC: LONG RANGE ORDER ⟹ STRUCTURE FACTOR

⟹ SCATTERING EXPERIMENTS: X-RAYS, LIGHT, ELECTRONS

2D ELECTRON LAYER ≲ 10^{10} PARTICLES

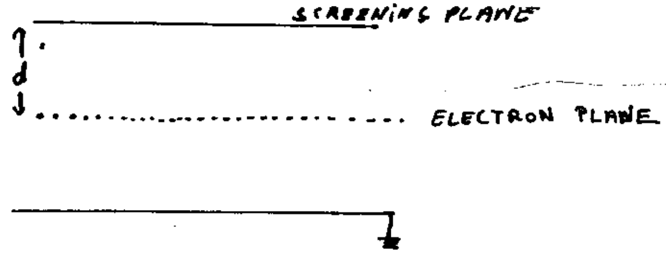
SCATTERING EXPERIMENTS DON'T WORK

2. MACROSCOPIC: ELASTIC RESPONSE TO SHEAR

⟹ TRANSVERSE MODE.

COLLECTIVE EXCITATIONS

DENSITY = n_0



1. ELECTRON LIQUID

$$n = n_0 + n_k e^{i(kx-\omega t)}$$

CALCULATE RESPONSE TO ELECTRON DENSITY MODULATION

LAPLACE: $\Delta\phi = -\frac{4\pi n e}{\epsilon}\delta(z)$

CONTINUITY $\frac{\partial n}{\partial t} + n\vec{\nabla}_2\vec{v} = 0$

MOTION $m\frac{\partial v}{\partial t} = e\vec{\nabla}_2\phi$

$$\omega_p^2 = \left\{\frac{2\pi n e^2 k}{m\epsilon}\right.$$

$$\Downarrow$$

$$\omega_p^2 = \begin{cases} \frac{2\pi n e^2}{m\epsilon}k & kd \gg 1 \quad \text{UNSCREENED} \\ \frac{2\pi n e^2 d}{m\epsilon}k^2 & kd \ll 1 \quad \text{SCREENED} \end{cases}$$

COMPARE TO 3-D : $\omega_p^2 = \frac{4\pi n e^2}{m}$ TOTAL SCREENING

CHARGE FLUCTUATIONS - LOCAL

$V_g = 0$ DO NOT PROPAGATE

2-D : SCREENING ONLY IN PLANE - 2D PLASMA

CANNOT SCREEN 3-D INTERACTION

2D PLASMON IS DISPERSIVE, DENSITY FLUCTUATIONS CAN PROPAGATE

2. 2D ELECTRON SOLID

- THE TRIANGULAR LATTICE IS THE MOST STABLE CONFIGURATION OF THE SOLID — L. BONSAL, A MARADUDIN Phys Rev. B 15 (1977) 1959

- THE NORMAL MODES OF THIS SOLID:

$$\begin{cases} \omega_L^2(k) = \omega_p^2(k) - 5\omega_t^2(k) & \text{LONGITUDINAL} \\ \omega_t^2(k) = C_t^2 k^2 & \text{TRANSVERSE} \end{cases}$$

$$C_t = \left(\frac{\mu}{mn}\right)^{1/2}, \quad \mu \doteq .21\,\frac{n^{3/2}e^2}{\epsilon} = \text{SHEAR MODULUS}$$

ORDERS OF MAGNITUDE: FOR $n = 10^8$, $k = 500\ \text{cm}^{-1}$, ω

$\omega_t = 4\times10^8$ Rad/sec

$\omega_p = 10^{10}$ Rad/sec $\gg \omega_t$ $\Longrightarrow$ $\omega_L(k) \simeq \omega_p(k)$

LIQUID PHASE

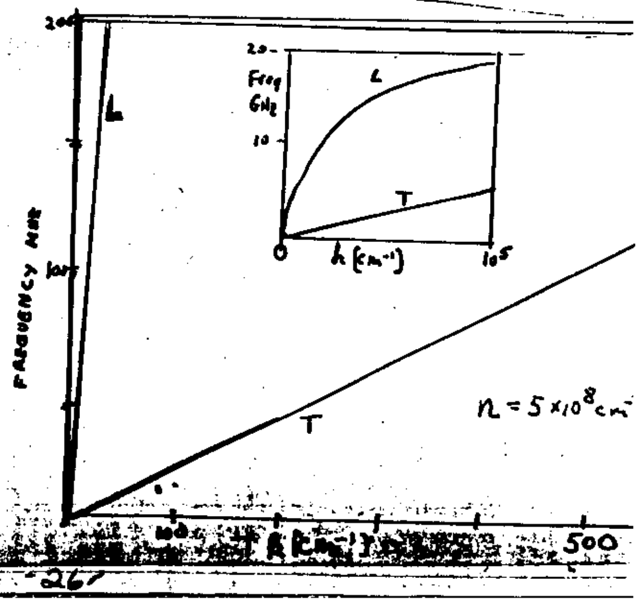
$\omega_p^2(k) = \frac{2\pi n e^2}{m\epsilon}k$ Longitudinal

SOLID PHASE

$\omega_L^2(k) = \omega_p^2(k)$ Longitudinal

$\omega_t^2(k) = \frac{\mu}{mn}k^2$ Transverse

$\therefore$ $\mu \neq 0 \Leftrightarrow$ SOLID



How can we measure μ?

to $\Rightarrow$ Elastic response to shear stress ⊕ inertia $\Rightarrow$ propagating shear wave

APPLY SHEAR FORCE $F_s = F_t e^{i(kx-\omega t)}$

⇓

MEASURE RESPONSE OF SYSTEM u_t

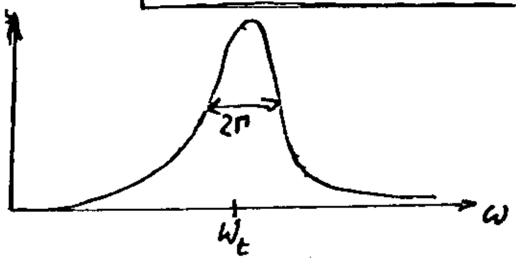
$$\underbrace{m\ddot{u}_t}_{\text{inertia}} = -\underbrace{m c_t^2 k^2 u_t}_{\text{restoring shear force}} + \underbrace{m/\tau_t \, \dot{u}}_{\text{Damping}} + \underbrace{F_t}_{\text{External force.}}$$

real system : $\mu = \mu' + i\omega\underbrace{\eta}_{\text{dissipation}}$ & $\underset{\text{scattering time}}{\tau_t^{-1}} = \frac{\omega\eta}{mn} k^2$

$$u_t = \frac{F_t/m}{\omega^2 - \omega_t^2 + i\omega\Gamma_t} \equiv F_t \underset{\text{SUSCEPTIBILITY}}{\chi} = F_t(\chi' + i\chi'')$$

$$P_{abs}^t = Re[n \langle \dot{u} \rangle F_t] = \frac{n\omega}{2}\chi'' F_t^2$$

$$\boxed{P_{abs}^t = \frac{n}{2m}F_t^2 \frac{\omega^2\Gamma_t}{(\omega^2-\omega_t^2)^2 + \omega^2\Gamma_t^2}}$$



$\omega_t = c_t k \Rightarrow \left\{ \mu = mnc_t^2 \right.$

$\Delta\omega_t = 2\Gamma \Rightarrow \left\{ \eta = \frac{mn}{k^2}\Gamma \right.$

TO EXCITE A LONGITUDINAL MODE (PLASMON) APPLY F_e

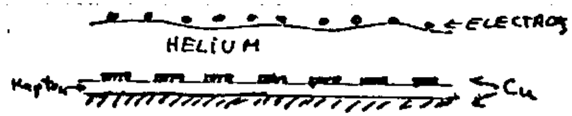
$$P_{abs}^{\ell} = \frac{nF_e^2}{2m}\frac{\omega^2\Gamma_e}{(\omega^2-\omega_p^2)^2 + \omega^2\Gamma_e^2}$$

TO MEASURE μ WE NEED 1. FINITE k $\omega_t \propto \mu^{1/2} k$

2. APPLY TRANSVERSE FORCE.

USE A 50Ω MEANDER STRIPLINE TO IMPOSE FINITE k

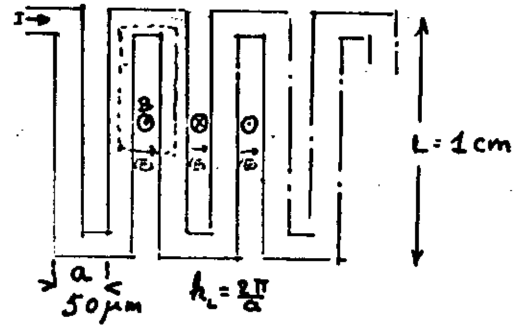
MODULATION OF ELECTRIC FIELD IN ELECTRON PLANE:



$\langle E \rangle = \frac{1}{2} i\omega L \langle B \rangle \sim e^{i(\frac{k_L L}{a} - \omega t)}$

Long wavelength LONGITUDINAL electric field - inductive coupling to electrons

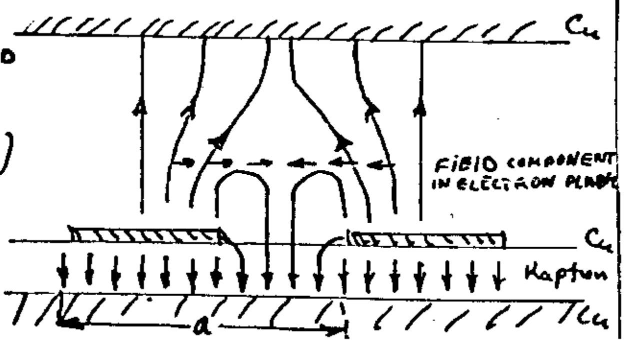
$V_L = c\frac{a}{L}$ SLOW PROPAGATION



SHORT WAVELENGTH LONGITUDINAL ELECTRIC FIELD CAPACITIVE COUPLING

$E_c \sim \hat{x}\, k_1 e^{-k_1 z} e^{i(k_1 x - \omega t)}$

$k_1 = \frac{\omega}{V_L} + k_L$

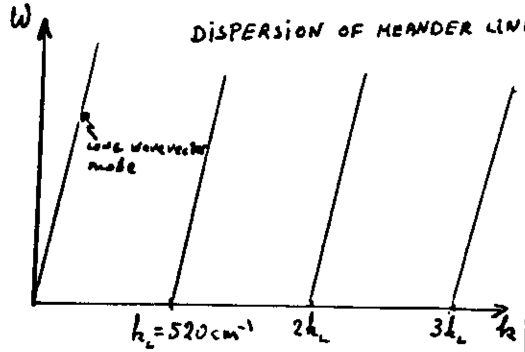


THE STRIPLINE CREATES A PLANE WAVE LONGITUDINAL ELECTRIC FIELD IN THE PLANE OF THE ELECTRONS WITH SPATIAL MODULATION CHARACTERIZED BY

$$k_s = \frac{\omega}{V_L} + s k_L \qquad s = 0, 1, 2, \ldots$$

$$\vec{E} \simeq \hat{x} \sum_s k_s e^{-k_s z} e^{i(k_s x - \omega t)}$$

EXCITES ONLY PLASMONS!



. How to PRODUCE A TRANSVERSE FIELD ?

USE LORENTZ FORCE : $F_t = \frac{e}{c} \vec{V}_L \times \vec{B} \propto \vec{F}_\ell \times \vec{B}$

$\Rightarrow$ APPLY MAGNETIC FIELD B_z $\Rightarrow$ COUPLE U_ℓ and U_t

$$\begin{cases} m\ddot{u}_\ell + m\omega_p^2 u_\ell - m/\tau_\ell\, \dot{u}_\ell + \frac{e}{c}\dot{u}_t B_z = eE_\ell \\ m\ddot{u}_t + m\omega_t^2 u_t - m/\tau_t\, \dot{u}_t - \frac{e}{c}\dot{u}_\ell B_z = 0 \end{cases}$$

COUPLED EQUATIONS

$$\Downarrow$$

$$\underbrace{\begin{pmatrix} \omega^2 - \omega_p^2 - i\omega\Gamma & i\omega\omega_c \\ -i\omega\omega_c & \omega^2 - \omega_t^2 - i\omega\Gamma \end{pmatrix}}_{\text{DYNAMICAL MATRIX}} \begin{pmatrix} u_\ell \\ u_t \end{pmatrix} = \begin{pmatrix} \frac{e}{m}E_\ell \\ 0 \end{pmatrix}$$

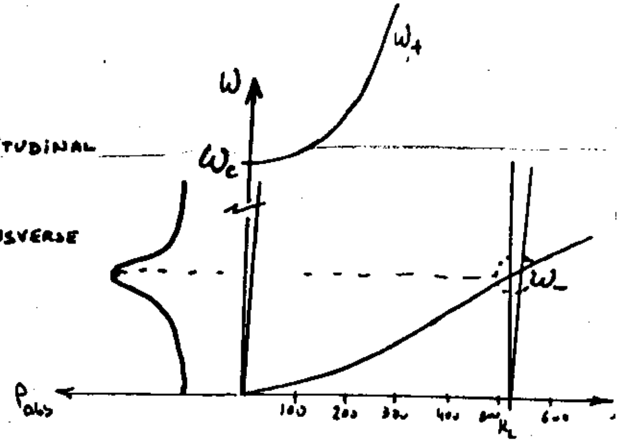
$\omega_c = \frac{eB_z}{mc}$; CYCLOTRON FREQUENCY ~ 2.7 MHz/GAUSS

$$U_t \sim \omega\omega_c E_\ell$$

THE NORMAL MODES

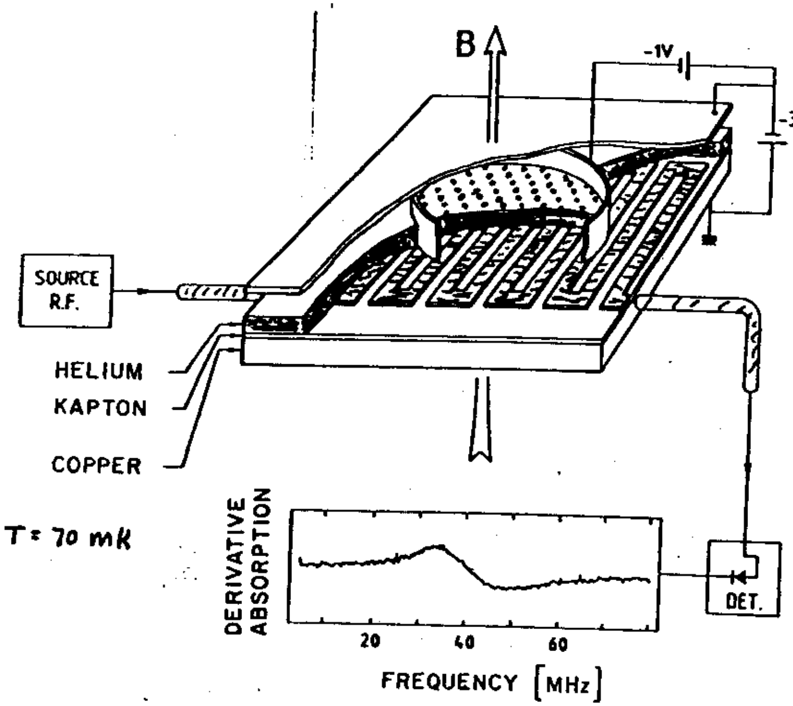
$\omega_+^2 = \omega_c^2 + \omega_p^2$ MOSTLY LONGITUDINAL

$\omega_-^2 \simeq \frac{\omega_p^2}{\omega_p^2 + \omega_c^2} \cdot \omega_t^2$ MOSTLY TRANSVERSE



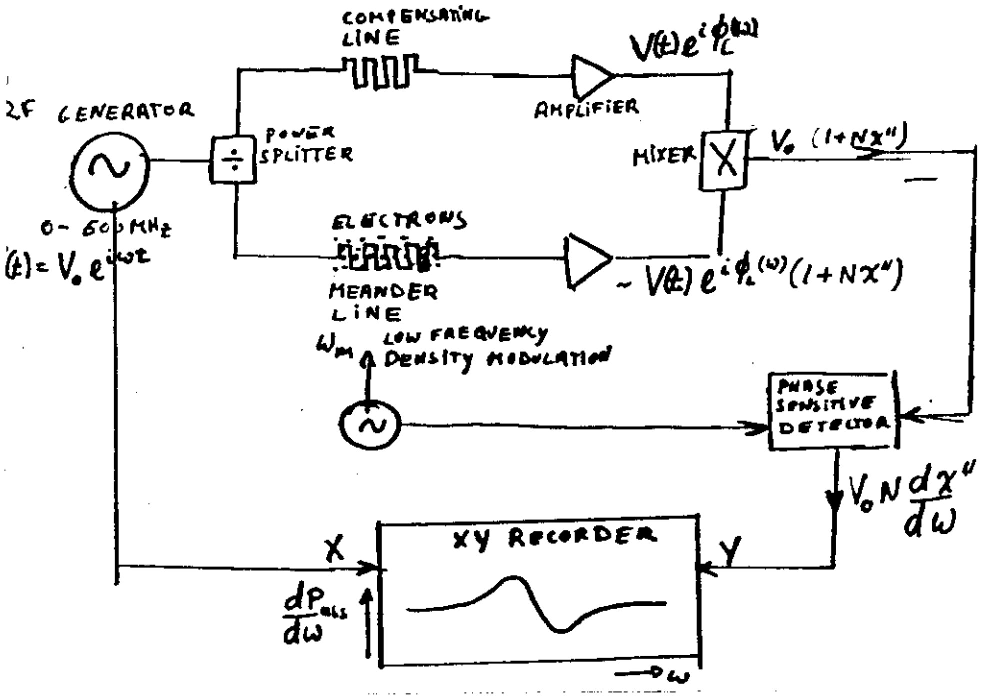
THE EXPERIMENT:

P.R.L. 53, (1984) 588

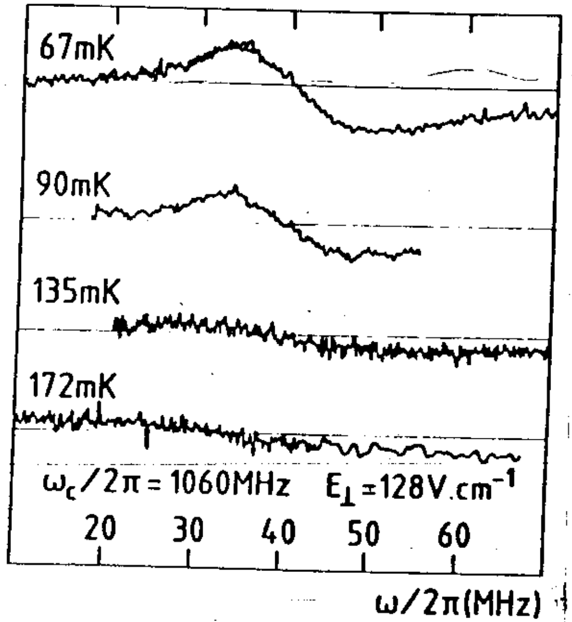


$n = 6 \times 10^7\,cm^{-2}$; $T = 70$ mK

DETECTION SCHEME



DISAPPEARENCE OF SHEAR MODE AT T_m.



$n = 6\times10^{7}\,cm^{-2}$

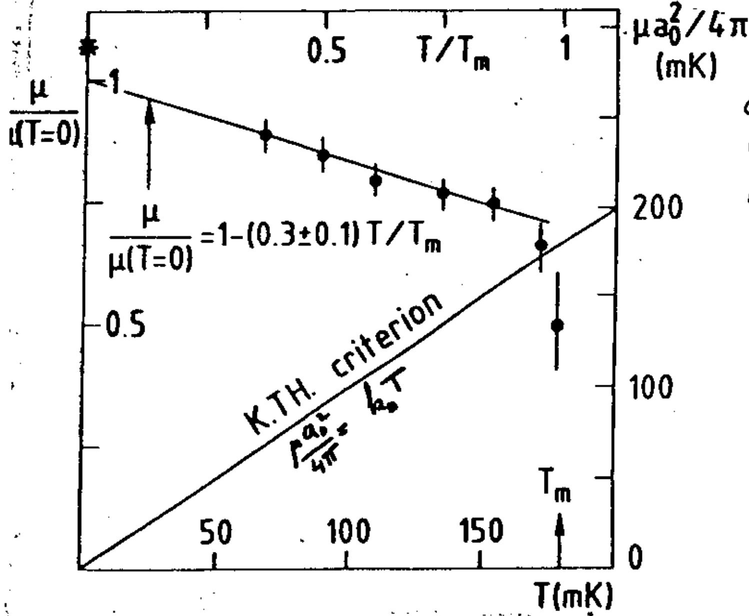
$k_t = 520\ cm^{-1}$

$\frac{\omega_t}{k} = V_t = \sqrt{\frac{\mu}{mn}}$

$\omega_c = 1060\ MHz$

$\omega_p(k_t = 520\,cm^{-1}) = 1.1\ GHz$

EXPERIMENT DEMONSTRATES THAT SHEAR PROPAGATES IN 2-D SOLID.



CONSISTENT WITH KOSTERLITZ THOULESS MELTING MECHANISM

$T_m = T_{KT}(1\pm 10\%)$

FOR $T < 0.7 T_M$,

$\mu(T)$ AND $\eta(T)$ ARE LINEAR IN

- $\mu(T) = \mu(T=0)\,(1 + \alpha\, T/T_m)$

  $\alpha = 0.3 \pm 0.1$

- THEORETICAL ESTIMATES OF α

  → RENORMALIZATION OF SHEAR DUE TO PHONON-PHONON INTERACTIONS: $\alpha = 0.18$

  D. FISHER Phys Rev B 26, 5009 (1982)

  CHANG, MAKI Phys Rev B 27, 1646 (1983)

  → MOLECULAR DYNAMICS SIMULATIONS

  $\alpha = 0.24$

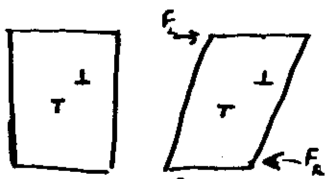
  R MORF Phys. Rev. Lett 43, 931 (1979)

$T > 0.7 T_M$

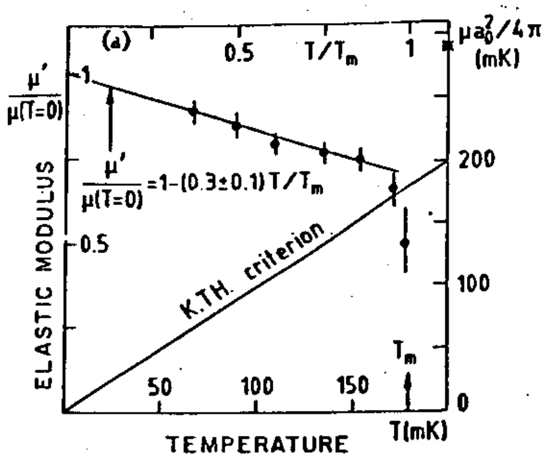
$\mu(T)$, $\eta(T)$ VARY MORE RAPIDLY WITH TEMPERATURE

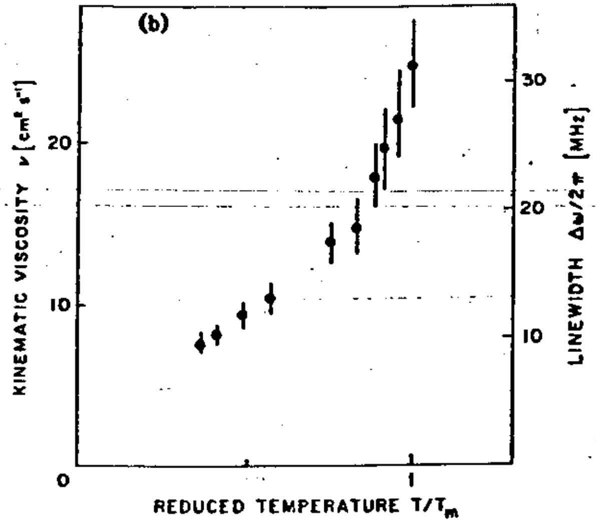
POSSIBLY DUE TO ANOTHER TYPE OF MECHANISM: DISLOCATION PAIRS

⇒ SCREEN SHEAR BY INCREASING THEIR DIPOLE MOMENT

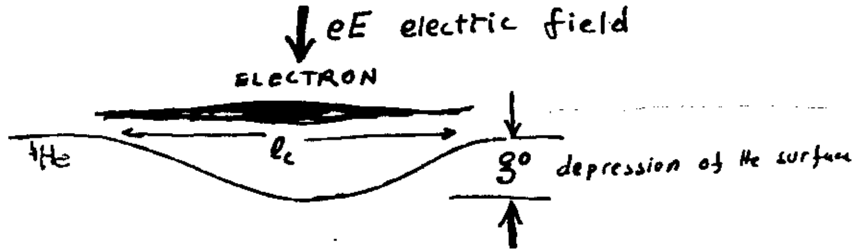


POLARIZED DISLOCATION CAN SCREEN SHEAR.





BUT THE HELIUM IS A DEFORMABLE SUBSTRATE!



$$\ell_c = \left(\frac{\alpha}{\rho g}\right)^{1/2} \quad \text{capillary length} \simeq 0.5\ \text{mm}$$

ξ_0 IS CALCULATED BY MINIMIZING THE ENERGY OF THE ELECTRON⊕ HELIUM DIMPLE COMPLEX (Shikin, Monarkha JETP 38, (1974) 373)

$$W_T = \underset{\text{ELECTROSTATIC}}{W_e} + \underset{\text{CAPILLARY}}{W_c} + \underset{\text{GRAVITATIONAL}}{W_g}$$

$$\left.\begin{aligned} W_e &= eE \int \psi^2(r)\, \xi(r)\, d^2r \\ W_c &= -\underset{\text{SURFACE TENSION}}{\alpha}\ \underset{\text{SURFACE INCREMENT}}{\Delta S} \\ W_g &= -\tfrac{1}{2} \rho g \int \xi^2\, d^2 r \end{aligned}\right\} \Rightarrow \xi^0 \sim \frac{eE}{4\pi\alpha}$$

FOR A TYPICAL EXPERIMENT $E \le 1000\ \text{V/cm}$ $\quad 50_{mK} < T < 1.2\text{K}$

$$\xi^0 \gtrsim 0.1\ \text{Å}$$

$$W_T \lesssim 3 \times 10^{-18}\ \text{erg} = 30\ \text{mK} < T_{exp.}$$

∴ THE ELECTRON⊕ DIMPLE COMPLEX IS UNSTABLE FOR AN ELECTRON GAS OR LIQUID. THE HELIUM SURFACE IS FLAT UNDER THE ELECTRON LIQUID [illegible] AFFECT ITS DYNAMICS.

IN THE SOLID PHASE THE ELECTRON MOTION IS CORRELATED

$$W_T = N e E \xi_0 \quad \text{ENERGY GAINED BY TRAPPING } N \text{ ELECTRONS}$$

$$E_T \simeq k_B T \quad \text{THERMAL ENERGY OF A 2-D ELECTRON CRYSTAL}$$

⇒ FOR A CRYSTAL WITH CORRELATION LENGTH $\gtrsim 10a$ THE ELECTRON⊕ DIMPLE COMPLEX IS STABLE.

∴ THE ELECTRON SOLID ON LIQUID HELIUM "STAMPS" A LATTICE OF DIMPLES ON THE HELIUM WHICH MODIFIES THE DYNAMICS OF THE ELECTRON LATTICE.

HOW DOES THE HELIUM SUBSTRATE MODIFY THE PHONON SPECTRUM OF THE 2-D ELECTRON CRYSTAL.

$$P(r) = e E_\perp \sum_{i=1}^{N} \delta(\vec r - \vec r_i(t)) \quad \text{ELECTRON PRESSURE ON He}$$

$$r_i(t) = R_i + \underset{\substack{\text{SLOW MOTION} \\ \to \text{HELIUM CAN FOLLOW} \\ \omega < 1/\tau_{He}}}{u_i^s(t)} + \underset{\substack{\text{FAST MOTION} \\ \omega > 1/\tau_{He}}}{u_i^f(t)}$$

τ_{He} characteristic response time of Helium surface.

$$P_G = -e E_\perp \frac{1}{S} \sum_i e^{i \vec G \cdot (u_i^s(t) + u_i^f(t))}$$

$$\bar P_G \simeq -e E \langle n_G \rangle \left(1 + i \vec G \cdot \vec u^s\right)$$

$$\langle n_G \rangle = n\, e^{-\frac{1}{4} G^2 \langle u^2 \rangle} \qquad \langle u^2 \rangle = \frac{\hbar}{[illegible]} \int^{\omega_M} \frac{d^2 k}{[illegible]} \coth \frac{\hbar\omega}{[illegible]}$$

THE HELIUM SURFACE RESPONDS TO EXTERNAL FORCE WITH SURFACE WAVE EXCITATIONS – RIPPLONS

$$\sim\sim\sim \quad \xi_q e^{i\bar q \bar r}$$

OF FREQUENCY $\Omega_q = \left(\frac{\alpha}{\rho}\right)^{1/2} q^{3/2}$

AND EFFECTIVE MASS ρ/q

FOR UNIFORM TRANSLATIONS $u(t)$

THE ELECTRON LATTICE CAN ONLY EXCITE RIPPLONS AT $\bar q = \bar G$

$\therefore$

$$\underbrace{\rho/G\left(\ddot\xi_G + \Omega_G^2\,\xi\right)}_{\text{RIPPLON OSCILATOR}} = \underbrace{-eE_\perp \langle n_G\rangle \left(1 + i\bar G\cdot\bar u\right)}_{\text{ELECTRONIC PRESSURE}},$$

$$\therefore\ \xi_G = -\frac{eE_\perp\langle n_G\rangle}{\alpha G^2}\left(1 + i\,\bar G\cdot\bar u\cdot\frac{\Omega_G^2}{\Omega_G^2-\omega^2}\right) \quad \text{SURFACE RESPONSE}$$

THE SURFACE DEFORMATION ξ MODIFIES THE TOTAL ENERGY VIA $H_{int} = eE\sum_{i=1}^{N}\xi(r_i) \Rightarrow$ RESULTING IN AN EFFECTIVE FORCE $-\nabla H_{int}$ WHICH DRIVES THE ELECTRONS:

$$m\Big[\omega^2 - \omega_\lambda^2 - \sum_G e^{-2W_G}\,\frac{n_0(eE_\perp)^2}{2m\alpha}\,\underbrace{\frac{\omega^2}{\omega^2-\Omega_G^2}}_{\omega=\Omega_G \text{ MAXIMUM COUPLING}}\Big] = 0$$

$\omega_\lambda = \begin{cases}\omega_p & \text{plasmon}\\ \omega_t & \text{transverse mode.}\end{cases}$

$W_G = \frac{1}{4}G^2\langle u^2\rangle_t \sim T$ DEBYE WALLER FACTOR

$$W_G \simeq \frac{1}{8\pi}\,\frac{k_B T G^2}{m n}\sum_\lambda\int_{\omega_m}^{\omega_M}\left(\frac{dk}{d\omega_\lambda}\right)\frac{k}{\omega_\lambda^2}\,d\omega \quad \text{CLASSICAL LIMIT}$$

THE INTERACTION WITH THE HELIUM SUBSTRATE OPENS GAPS IN THE NORMAL MODE SPECTRUM OF THE ELECTRON LATTICE AT FREQUENCIES CORRESPONDING TO

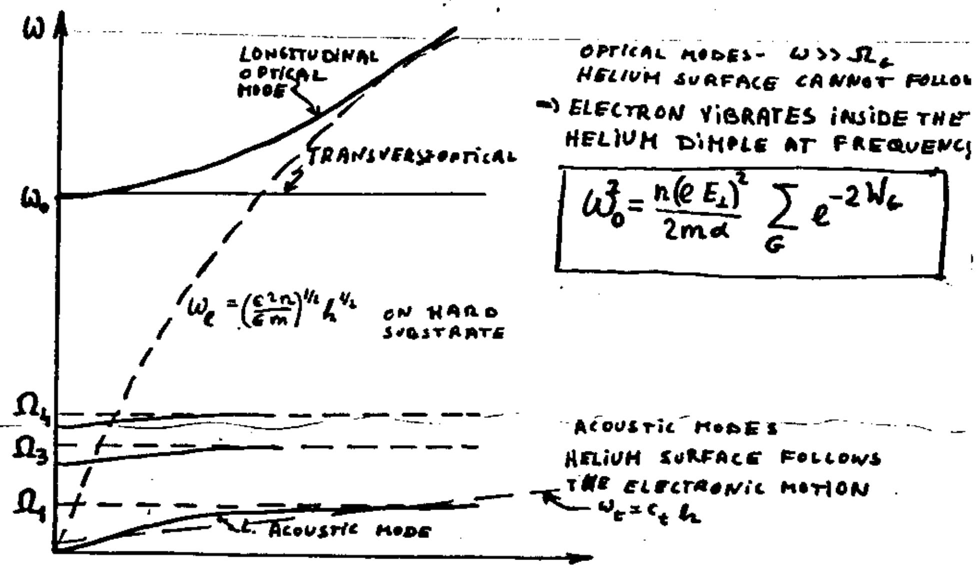
$$\omega = \Omega_{G_h}, \quad \Omega_{G_h} = \left(\frac{\alpha}{\rho}\right)^{1/2} G_h^{3/2}$$

G_h ARE THE RECIPROCAL VECTORS OF THE TRIANGULAR LATTICE:

$$G_h = h^{1/2} G_1, \quad h = 1, 3; 4, 7, 9\ldots$$

$$G_1 = 2\pi\left(\frac{2}{\sqrt{3}}\,n\right)^{1/2}$$

- DUE TO THE FACTOR $e^{-\frac{1}{4}G^2\langle u^2\rangle}$ ONLY RIPPLONS WITH LOW G_h WILL BE EXCITED BY THE ELECTRONS.



ORDERS OF MAGNITUDE: FOR $n = 10^8\,cm^{-2}$, $E_\perp = 100\,V/cm$

$\Omega_{G_1} = 4.5$ MHz , $\omega_0 \approx 10$ MHz

W_0 DECREASES WITH INCREASING TEMPERATURE.

AT $T \geq T_m$ $\omega_0 = 0$

THE SPECTRUM CHANGES ABRUPTLY AT $T = T_m$

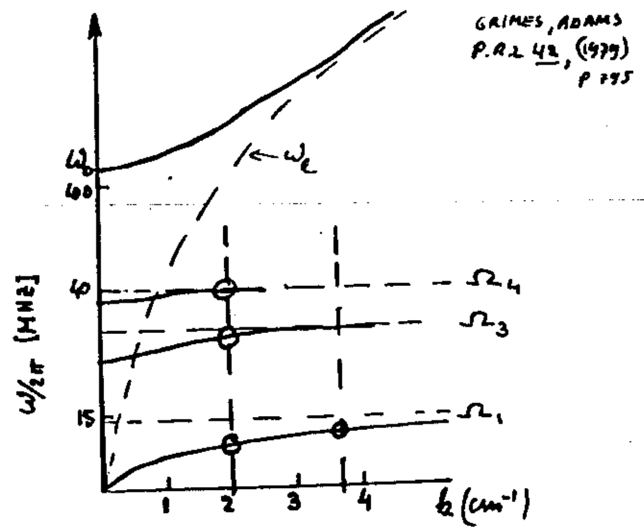
ALLOWING A PRECISE DETERMINATION OF T_M

THE DEPTH OF THE HELIUM DIMPLE

$$\xi^0 \ll 10^{-2}\,A^\circ \ll kT$$

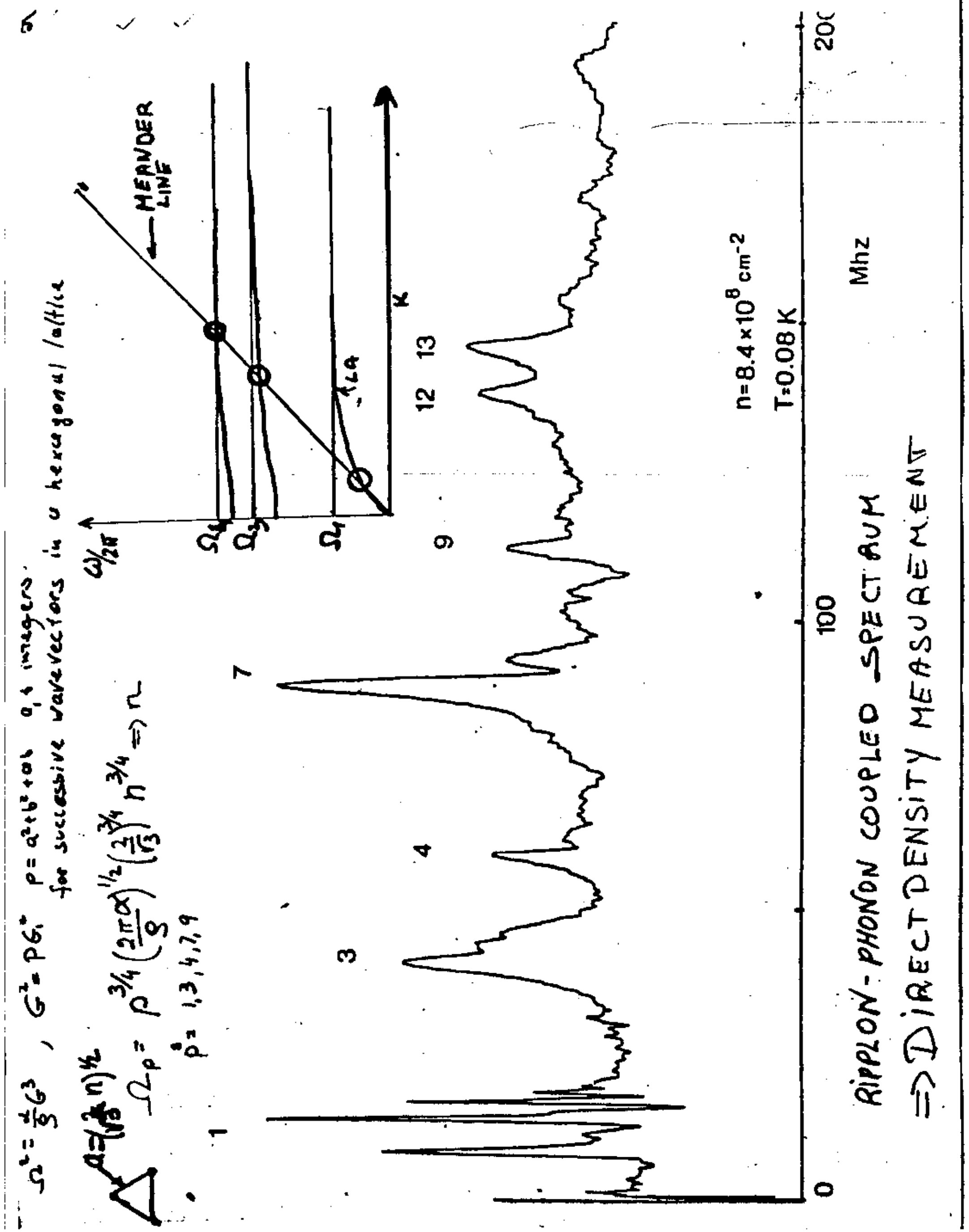
=> THE TRAPPING ENERGY DOES NOT MODIFY THE THERMODYNAMICS OF THE ELECTRON LATTICE.

THE COUPLING WITH THE HELIUM SURFACE ALLOWS THE DETERMINATION OF THE LATTICE STRUCTURE:



$\therefore\ \Omega_1, \Omega_3, \Omega_4 \Rightarrow$

TRIANGULAR LATTICE

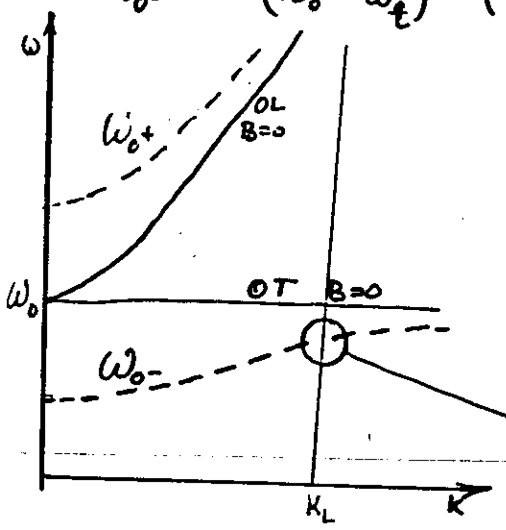


2-D ELECTRON SOLID ⊕ HELIUM SUBSTRATE ⊕ B

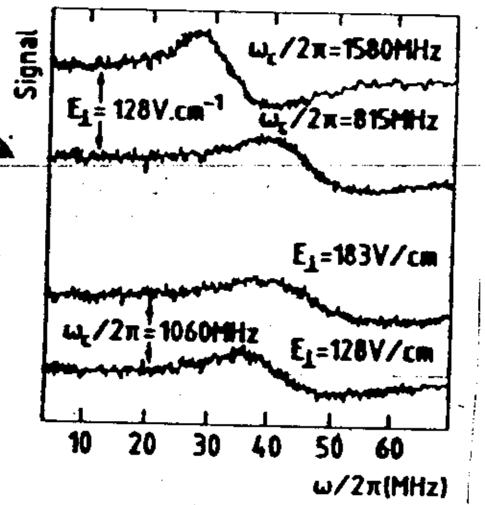
DISPERSION RELATION FOR OPTICAL MODES:

$$\omega_{o+} = \left(\omega_o^2 + \omega_p^2\right)^{1/2}\left(1 + \frac{1}{2}\,\frac{\omega_c^2}{\omega_p^2 - \omega_t^2}\right)$$

$$\omega_{o-} \sim \left(\omega_o^2 + \omega_t^2\right)^{1/2}\left(1 - \frac{1}{2}\,\frac{\omega_c^2}{\omega_p^2 - \omega_t^2}\right)$$



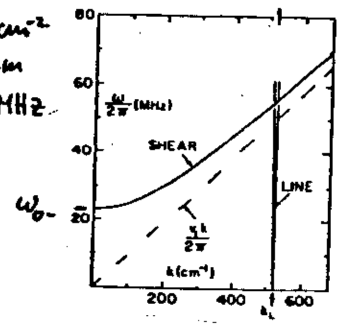
$$\omega_o^2 = \frac{n(eE_\perp)^2}{2\alpha m}\sum_G e^{-2W_G}$$

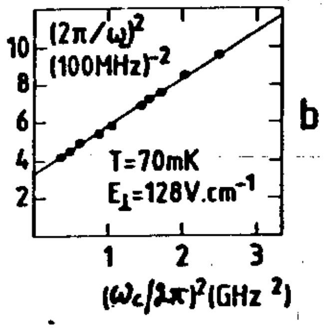


EXTRAPOLATION SCHEME - TO DETERMINE 'PURE' ω_t

EXPERIMENT:

$n = 6\times10^{10}\,cm^{-2}$
$E_\perp = 115$ V/cm
$\omega_c = 1060$ MHz





1. AT FIXED $E_\perp$ MEASURE ω_- FOR A SERIES OF MAGNETIC FIELD VALUES · EXTRAPOLATE TO $B \to 0$

$$\omega_{o+}(k_L, E_\perp, B) \xrightarrow[B\to 0]{} \omega_{oT}(k_L, E_\perp)$$

ELIMINATED MAGNETIC FIELD EFFECTS

2. REPEAT PROCEDURE FOR A SERIES OF PRESSING FIELD VALUES EXTRAPOLATE

$$\omega_{oT}(k_L, E_\perp) \xrightarrow[E_\perp\to 0]{} \omega_t = v_t k_L^2$$

ELIMINATED SUBSTRATE EFFECTS.