INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONE: 2240-1
CABLE: CENTRATOM - TELEX 460392-I

SMR/459 - 36

SPRING COLLEGE IN CONDENSED MATTER
ON
'PHYSICS OF LOW-DIMENSIONAL SEMICONDUCTOR STRUCTURES'
(23 April - 15 June 1990)

COLLECTIVE EXCITATIONS IN 2D ELECTRONIC SYSTEMS

Yu.A. KOSEVICH
All-Union Surface and Vacuum Research Centre
Andreyevskaya Nab.2
Moscow 117334
U.S.S.R.

Collective excitations in 2D electronic systems

Introduction

Plasmons – resonant collective excitations of an electron or hole plasma.

Plasmon-polariton type of excitations exist in widely different systems, e.g. surface plasmons in thin films and 2D-plasmons in semiconductor heterostructures and metal-oxide-semiconductor devices. The study of these excitations allows a detailed investigation of many different electronic interactions.

Some general aspects for plasmon-polariton excitations in 2D and quasi 2D-electronic systems are discussed.

(I) 3D electron gas:

$$\frac{\partial n}{\partial t} + \operatorname{div}\vec{J} = 0,$$

$$\vec{J} = -ne\vec{V},$$

$$m^*\frac{\partial \vec{V}}{\partial t} = -e\vec{E} - \frac{m\vec{V}}{\tau},$$

$$\varepsilon \equiv 1 + 4\pi\chi = 1 - \frac{\omega_p^2}{\omega^2 + i\omega/\tau}$$

where

$$i\vec{q}\times\vec{H} = \vec{J} - i\omega\vec{E} \equiv -i\omega\varepsilon(\omega)\vec{E};$$

Bulk excitations:

$$\operatorname{div}\vec{D} \equiv \varepsilon(\omega)\operatorname{div}\vec{E} = 0 \Rightarrow \varepsilon(\omega) = 0$$

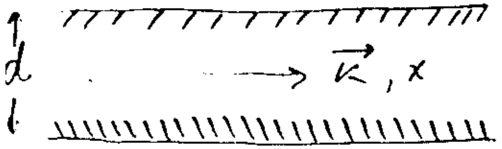
$$\omega = \omega_p - \frac{i}{2\tau} \quad \text{– 3D plasmon;}$$

$$\omega_p^2 = \frac{4\pi ne^2}{m^*}, \quad \omega_p\tau \gg 1 \text{ – low damping}$$

$\omega_p \sim 10^{15}\,s^{-1}$ – for 3D metals

$\omega_p \sim (10^{13} \div 10^{14})\,s^{-1}$ – for semiconductors

) Thin metal film:



$\rightarrow \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$,

Neglecting retardation: $\vec{E} = -\nabla\varphi$, $\Delta\varphi = 0$

$$\omega_\pm^2(k) = \frac{\omega_p^2}{2}\left(1 \pm e^{-kd}\right);$$ Ritchie ('57)

а) $kd \gg 1$, $\omega_\pm(k) \rightarrow \frac{\omega_p}{\sqrt{2}}$;

б) $kd \ll 1$, $\omega_+^2(k) \approx \omega_p^2$; $\omega_-^2(k) = \frac{\omega_p^2}{2} kd$. } (1)

Account for retardation:

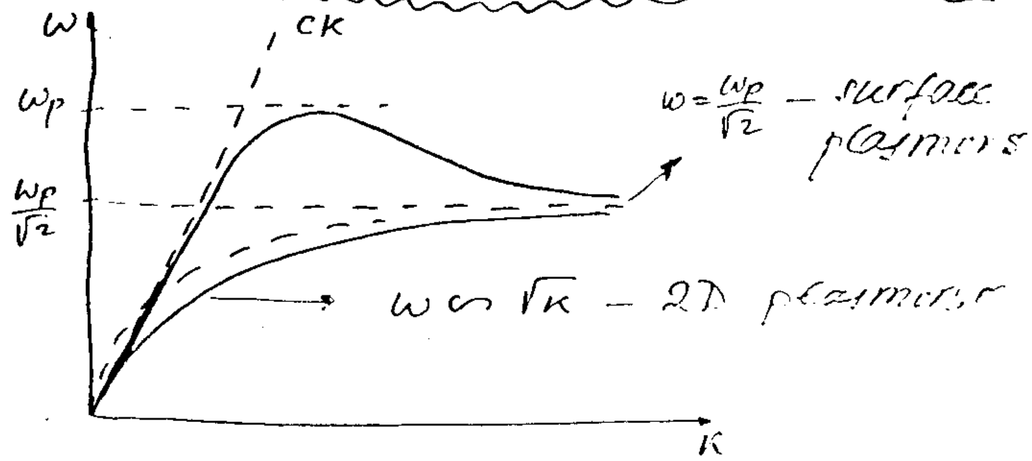
2) $\text{æ} = \pm \frac{p_e}{\varepsilon_{xx}(\omega)}\left[\operatorname{tg}\frac{p_e d}{2}\right]^{\pm 1}$ – TM-modes, Y.K. ('89)

where $\text{æ} = \left(k^2 - \frac{\omega^2}{c^2}\right)^{1/2}$;

$$p_e = \left(\varepsilon_{xx}\frac{\omega^2}{c^2} - k^2\frac{\varepsilon_{xx}}{\varepsilon_{zz}}\right)^{1/2}.$$

The (1) is available only when

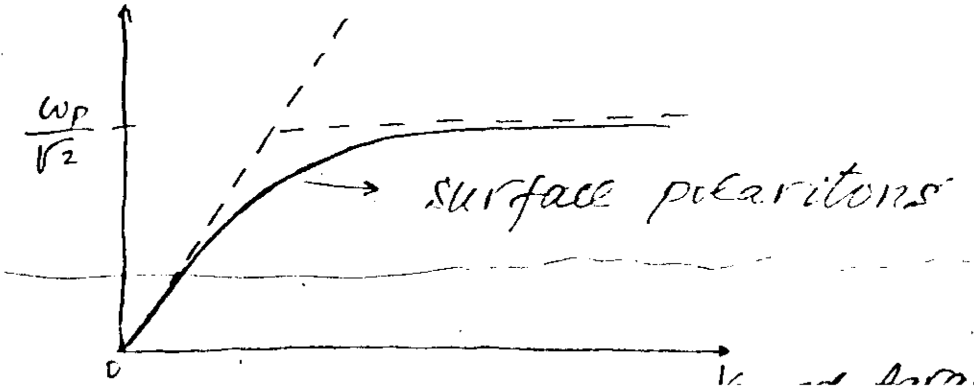
1) For $d \ll \frac{c}{\omega_p} \sim 10^{-5}$ cm – splitting:



Theory: Ritchie ('57); Ferell ('58); Stern ('67); Kliever, Fuchs ('67); Dahl, Sham ('74)

Experiment: Kroage ('68); Vincent, Silcox ('73).

2) For $d \gg \frac{c}{\omega_p}$ – no splitting



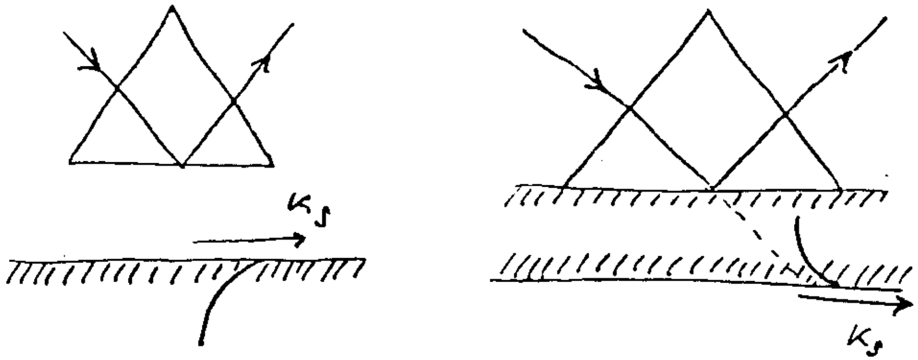
Agranovich

$\varepsilon(\omega) = -1$ – for surface plasmons;

$\varepsilon(\omega) = 0$ – for bulk 3D plasmons.

Experimental methods:

1. Attenuated Total Reflection (ATR)



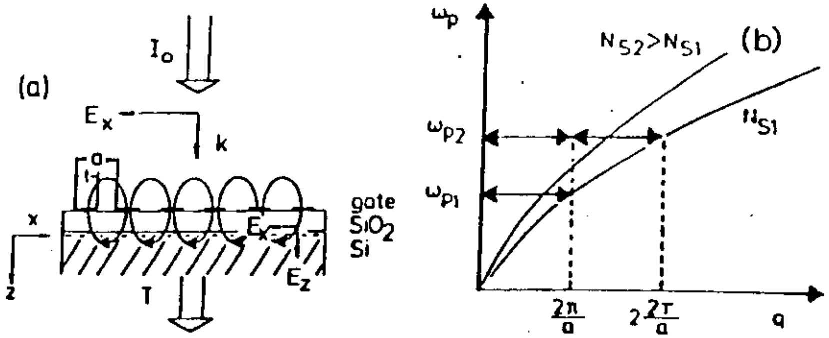
Otto ('68) Kretschman ('71)

Surface- and 2D plasmons – nonradiative modes: $K_s > \frac{\omega}{c}$ ⇒

⇒ no direct coupling with light.

2. Grating coupler.

$$\vec{E}(x) \sim e^{ikx} \sum_m e^{imQx} \Rightarrow k \to k + mQ$$



(a) Grating coupler effect of a periodical structure. For normally incident FIR radiation spatially modulated parallel $(e_x(\omega,q))$ and perpendicular $(e_z(\omega,q))$ electric field components are induced. (b) Coupling of FIR radiation to 2D plasmons of wavevectors $q_1=2\pi/a$ and $q_2=2\cdot2\pi/a$

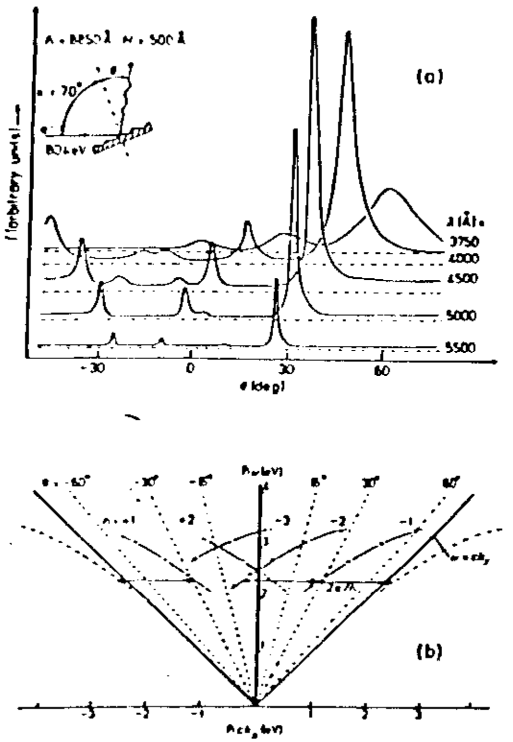


Fig. 1. (a) Angular dependence of the light emission caused by the radiative decay of surface plasmons on a sinusoidally modulated Ag surface with grating constant Λ = 8850 Å and a modulation height of about 500 Å for various wavelengths. The surface plasmons are excited by 80 keV electrons which are incident with an angle of incidence α = 70° onto the sample. The zero intensity of each curve (shown as horizontal broken lines) has been shifted vertically for clarity. (b) $\omega - k_x$ diagram of the peak positions. The broken lines indicated by angles θ are related to the angles of observation. The left- and right-hand branches of the surface-splamon dispersion (chain curves) are derived by adding reciprocal grating vectors 2π/Λ to the peak dispersion. The integer numbers *n* indicate the number of scattering processes [ ]

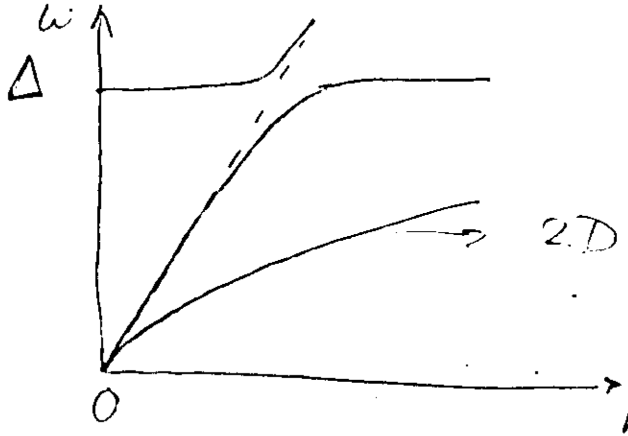
What is 2D plasmon?

$\omega \ll \omega_p, \Delta_{is}$;

a) $\tilde{\varepsilon}_{xx} = 1 - \frac{\omega_p^2}{\omega^2} f(K,d) \Rightarrow \tilde{\varepsilon}_{xx} = 0$ Stern ('67)

b) $\varepsilon_{zz} = 1$ (in Eq. (2))

Then



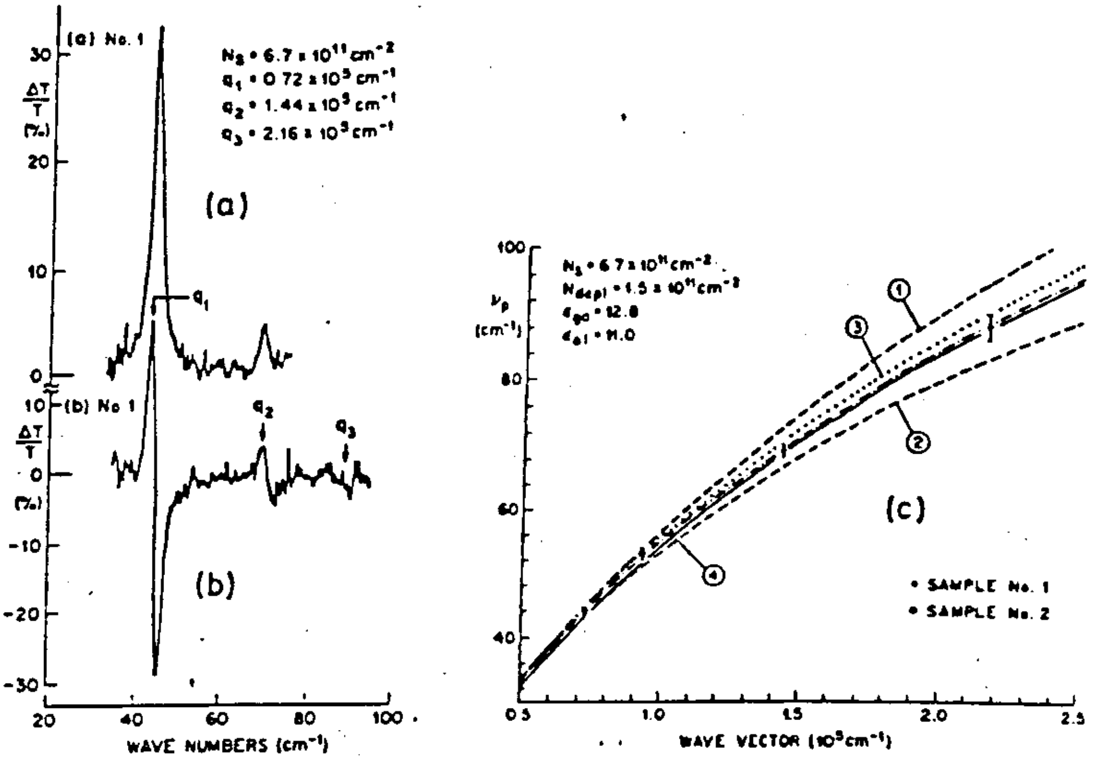
$\Delta^2 = \Delta_{is}^2 + \omega_p^2$ Dahl, Sham ('77)

Corrections to the spectrum:

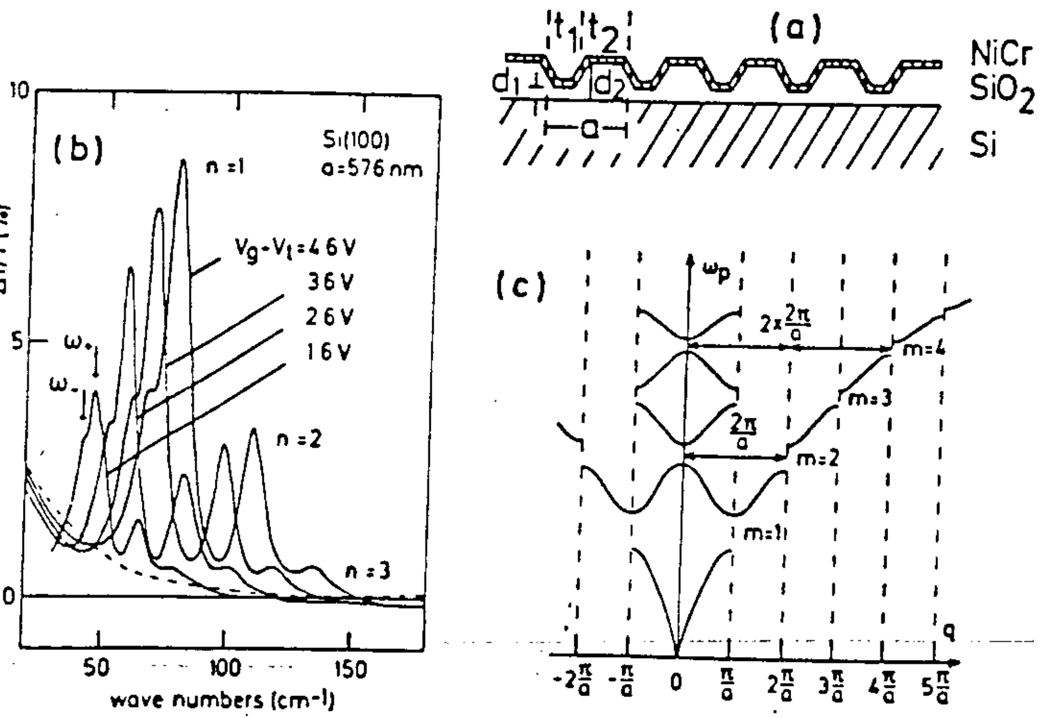
nonlocal

finite-thickness effect

intersubband coupling



Experimental plasmon resonances for a 2DES in GaAs with $N_s = 6.7 \times 10^{11} cm^{-2}$ (a). The arrows mark plasmon resonance positions. The lower trace (b), qualitatively the derivate of the upper trace, is obtained by changing N_s slightly via the persistent photoeffect. (c) Theoretical and experimental plasmon dispersion. The solid line is the classical local plasmon dispersion. The curves marked 1–4 are defined as follows: curve 1, plasmon dispersion including nonlocal correction; curve 2, plasmon dispersion including finite-thickness effect; curve 3, plasmon dispersion including nonlocal and finite-thickness corrections combined; and curve 4, plasmon dispersion including all correction terms, i.e., intersubband coupling, see Ref. [28].



Schematical geometry of a MOS sample with modulated oxide thickness (a), excitation of 2D plasmons with split resonances (b) due to the superlattice effect of the charge density modulation on the 2D plasmon dispersion (c) (from Refs. [9] and [10]).

References

1. Ritchie R.H., Phys. Rev., 1957, 106, 874.
2. Ferrell R.A., Phys. Rev., 1958, 111, 1214.
3. Stern F., Phys. Rev. Lett., 1967, 18, 546.
4. Kliever K.L., Fuchs R., Phys. Rev., 1967, 153, 498.
5. Dahl D.A., Sham L.J., Phys. Rev. B, 1977, 16, 651.
6. Vincent R., Silcox J., Phys. Rev. Lett., 1973, 25, 1487.
7. Kosevich Yu. A., Sov. Phys. JETP, 1989, 69(1), 200.
8. Agranovich V.M., Mills D.L. (eds.), Surface Polaritons, North Holland, 1982.

IC/89/132

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

MAGNETOPLASMA OSCILLATIONS OF THE WAVE-GUIDED TYPE IN SEMICONDUCTOR SUPERLATTICES

J.C. Granada

A.M. Kosevich

and

Yu.A. Kosevich

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1989 MIRAMARE - TRIESTE

IC/89/132

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

MAGNETOPLASMA OSCILLATIONS OF THE WAVE–GUIDED TYPE IN SEMICONDUCTOR SUPERLATTICES *

J.C. Granada
International Centre for Theoretical Physics, Trieste, Italy,

A.M. Kosevich
Institute for Low Temperature Physics and Engineering,
310164 Kharkov, USSR

and

Yu.A. Kosevich
All–Union Surface and Vacuum Research Centre,
117334 Moscow, USSR.

MIRAMARE – TRIESTE
June 1989

* Submitted for publication.

ABSTRACT

For electromagnetic wave-guided oscillations of two-dimensional (2D) electron layer periodic array in a transversal magnetic field, the dispersion relation is analized. Oscillation frequencies much lower than the electron cyclotron frequency exist under certain conditions. Additionally, the perpendicular propagation to the layers is investigated. It is demonstrated, that a strong magnetic field causes a frequency shift and splitting, which are in inverse relation to the external magnetic field and the period of the layered electron system.

1. Recently much attention has been focused on elementary excitations of semiconductor superlattices in an external magnetic field [1-7]. The dispersion relation of the cyclotron waves for a layered electron system in the Voigt configuration with a static magnetic field perpendicular to the planes was derived in [1]. On the basis of experimental studies of the magnetic field dependence of far infrared transmission through highly doped InAs/GaSb superlattices, it was suggested the existence of helicon modes in these systems [2]. Tselis et al.[3] and Kushwaha [4] obtained the dispersion relation for the frequency of the collective excitations in a superlattice as a function of the components of the wave vector parallel and perpendicular to the layers, and the existence of helicon waves propagating along the superlattice axis was demonstrated. Wendler and Kaganov [5-7] found, that these low-frequency helicon waves under the conditions of the Quantum Hall Effect behave like helicon modes propagating in a homogeneous medium, but showing undamping.

In experimental investigations of the high frequency absortion by a two-dimensional (2D) electron gas, trapped on the liquid ^{4}He surface, electromagnetic resonance frequencies much lower than the electron resonance frequency were observed [8]. A model of "edge" magnetoplasmons was proposed for the theoretical explanation of these resonances [9, 10]. In Ref. [11] the behaviour of "edge" magnetoplasmons in a semiconductor superlattice bounded by a lateral surface perpendicular to the layers was studied. It was shown, that in systems containing inhomogeneous 2D electron gas resonance modes much lower than the electron cyclotron frequency and depending

inversely on the external magnetic field can propagate.

In previous papers [12, 13] the spectrum of magnetoplasma waveguided oscillations of a singlu two-dimensional electron layer in a bounded system was described. It was shown that a strong magnetic field causes a frequency shift and splitting, depending inversely on the external magnetic field. In the present communication the magnetoplasma oscillations of the wave-guided type in semiconductor superlattices are studied (in the case of a zero magnetic field these modes were described by Korzh and Kosevich for a periodic array of 2D conducting layers [14]). We shall demonstrate that oscillation frequencies much lower than the electron cyclotron frequency and depending inversely on the external magnetic field can exist under certain conditions in a periodic array of homogeneous 2D electron layers.

2. The model we have adopted to describe the superlattice structure consists of a periodic array of strictly 2D electron layers at the positions $z=nd$, where $n=0, \pm1, \pm2, \ldots$ is the layer index, and d is the distance between adjacent layers. The array is imbedded in a homogeneous dielectric medium with dielectric constant ε. A static magnetic field H_o is applied along the direction perpendicular to the planes. The vibration spectrum of the described system can be obtained by writing the general solution of the wave equation in the regions between the electron layers, assuming that the electric field of the collective excitations forms a Bloch wave and imposing the standard electromagnetic boundary conditions at each of the 2D electron layers. In the local limit $k \ll k_f$, (k_f being the Fermi wave

vector of the 2D electron gas, and k being the component of the wave vector of the collective excitations parallel to the layers) the resulting dispersion relation is given by

$$\left[\omega^2 + \frac{\Omega c\alpha}{2\varepsilon} S(\alpha,q) \right] \left[1 + \frac{\Omega}{2c\alpha} S(\alpha,q) \right] = \omega_H^2 , \qquad (1)$$

where $\alpha=(\omega^2\varepsilon/c^2-k^2)^{1/2}$, $\Omega=4\pi e^2\eta_o/m^*c$, $\omega_H=eH_o/m^*c$, $m^*=m(1+i/\omega\tau)$, e, m, η_e and τ are respectively the charge, the effective mass, the surface density and the relaxation time of the 2D- carriers. The structure factor of the superlattice $S(\alpha,q)$ is given by the expression $S(\alpha,q)=\sin(\alpha d)/[\cos(\alpha d)-\cos(qd)]$ (here $\hbar q$ is the quasi-impulse of the collective excitations along the axis of the superlattice [15]). We have written the dispersion relation (1) in a form, which permits us to describe the electromagnetic waves of the wave-guided type interacting with the periodic array of two-dimensional electron layers. For these modes $\omega^2>c^2k^2/\varepsilon$, i.e. $\alpha^2>0$. In the following discussion we assume, that the frequencies under consideration are much higher than the inverse relaxation time of the 2D carriers ($\omega\tau>>1$).

3. First of all we analyse the relation (1) in the close vicinity of the line $\omega=ck/\varepsilon^{1/2}$. In this case the coupling between layers is strong ($\alpha d<<1$) and Eq. (1) can be written in the form

$$\omega^2=\omega_o^2+\left[\left(\frac{\alpha^2d^2}{2} - 2\sin^2(qd/2) \right)^{-1} - \frac{\omega_H^2 d^2}{2}\varepsilon \left(\frac{s_o^2}{c^2}\varepsilon + 2\sin^2(qd/2) \right)^{-2} \right] \omega_{pl}^2 \frac{\alpha^2d^2}{2} . \qquad (2)$$

Here $\omega_{pl}=(\Omega c/\varepsilon d)^{1/2}$ is the frequency of the 2D plasmon with effective bulk density $n_B=\eta_o/d$, $s_o=(\Omega cd/\varepsilon)^{1/2}$ is the characteristic phase velocity of the "acoustic" 2d plasmon [16], and the squared frequency ω_o^2 is given by

$$\omega_o^2 = \left[2\sin^2(qd/2)/[(s_o/c)^2\varepsilon + 2\sin^2(qd/2) \right] \omega_H^2 . \qquad (3)$$

For qd=0 Eq. (2) takes the following form

$$\omega^2 = \omega_{pl}^2 - (\omega_H/\omega_{pl})^2 (c\alpha)^2/\varepsilon . \qquad (4)$$

If we put here $\alpha=0$, we can find the frequency at the point, where the curve $\omega=\omega(k,q)$ intersects the line $\omega=ck/\varepsilon^{1/2}$. At this point the frequency of the collective excitations is $\omega=\omega_{pl}$, i.e. the qd=0 mode in the strong coupling limit is just like a 3D plasmon, which propagates with the group velocity given by

$$v_o=(\partial\omega/\partial k)_{\alpha=0}= \left[\omega_H^2 / (\omega_{pl}^2+\omega_H^2) \right] c/\varepsilon^{1/2} . \qquad (5)$$

We can see that under the condition $\omega_H<<\omega_{pl}$ this group velocity depends on the external magnetic field: in this case $v_o=\omega_H^2 d/\Omega$ ($v_o<<c$). If $\omega_H>>\omega_{pl}$ the group velocity of the qd=0 mode is the physical velocity of the wave propagating in a homogeneous medium with dielectric permittivity ε ($v_o=c/\varepsilon^{1/2}$). As a result of the above consideration we can state that in the close vicinity of the point $\alpha=0,\omega=\omega_{pl}$ a slowly increasing oscillation frequency is observed when the electron cyclotron frequency is much lower than ω_{pl}. Let us remark that this oscillation frequency depends on the squared external magnetic field.

For qd≠0 Eq.(2) takes the form

$$\omega^2 = \omega_o^2 - s^2\alpha^2 . \qquad (6)$$

Here s is a characteristic magnitude with dimension of velocity. It is given by the following expression:

$$s^2 = \left([2 \sin^2(qd/2)]^{-1} + [2 \sin^2(qd/2)+(s_o/c)^2\epsilon]^{-2} \frac{\omega_H^2 d^2}{2 c^2} \epsilon \right) s_o^2 \quad . \qquad (7)$$

We see that in the $qd\neq o$ case ω_o represents the frequency at the point where the curve $\omega=\omega(k,q)$ intersects the line $\alpha=0$. If the condition $\Omega d>>2c$ is satisfied, then ω_o is much less than the electron cyclotron frequency ω_H. However, if $\Omega d<<2c$ and $\Omega<<cq$, the frequency at the intersection point is $\omega_o \approx \omega_H$.

It is easy to show that the group velocity of the $qd\neq 0$ mode at the point $\alpha=0, \omega=\omega_o$ is

$$v_o = \left(\frac{2 \sin^2(qd/2)}{[(s/c)^2\epsilon + 2 \sin^2(qd/2)]} \right) \left(\frac{s}{c} \right) \frac{s}{\epsilon^{1/2}} \quad . \qquad (8)$$

If $qd>>s_o/c$ and $\Omega d/2c<<1$, then the group velocity v_o is much less than $c/\epsilon^{1/2}$. For these parameter relations the character of v_o depends on the dimensionless quantity $\zeta_H=\omega_H\epsilon^{1/2}d/c$: if $\zeta_H<<1$ then the group velocity does not depend on the external magnetic field; if $\zeta_H>>1$, then v_o is proportional to the external magnetic field. Therefore, we can state that in a layered system, for $qd\neq o$, a linear in k increasing oscillation frequency above the electron cyclotron resonance frequency can be observed.

6. Let us now consider the frequency splitting and the shift of homogeneous ($k=0$) wave-guided magnetoplasma oscillations of a layered electron system. In this case Eq. (1) can be transformed to the form

$$\sin(\zeta)/[\cos(\zeta)-\cos(qd)]=-(2c/\Omega d)\ (\zeta \pm \zeta_H) \qquad (9)$$

where $\zeta=\omega\epsilon^{1/2}d/c$. In the particular case $qd=\pi/2$ the relation (9) coincides with the formula which describes the shift and the splitting of homogeneous magnetoplasma oscillations of a <u>single</u> 2D electron layer in a symmetric screened system [12,13]. This coincidence is due to the fact, that the electrodynamics of a single 2D electron layer in a symmmetric screened system is equivalent to the electrodynamics of an array of 2D electron layers with surfaces currents oscillating with a difference of phase of $\pi/2$ between adjacent layers.

In order to understand the distribution of the starting points of the dispersion curves of the wave-guided type let us analyze the graphycal solutions of Eq. (9) with the aid of the intersection points of the functions $F_1(\zeta)=\sin(\zeta)/[\cos(\zeta)-\cos(qd)]$ and $F_2=-(2c/\Omega d)\ (\zeta\pm\zeta_H)$ (Fig.1). We see that the interaction between wave-guided electromagnetic modes and collective excitations of the layered system leads to the appearance of a shift and splitting of the frequencies corresponding to the asymptotics of the function $F_1(\zeta)$ ($1/F_1(\zeta_n)=0$ for $\omega_n=(2\pi n+q)c/d\epsilon^{1/2}$; $n=0, 1, 2, \ldots$). The behaviour of the indicated shift and splitting depends on the positions of the frequencies ω_n with respect to the characteristic frequency ω_H: (i) If $\omega_n<\omega_H$, then one of the splitted frequencies is localized above ω_n and the second splitted frequency is localized below ω_n. (ii) If $\omega_n>\omega_H$ then both splitted frequencies are localized above ω_n. In this case the gap existing between the splitted frequencies decreases with the increasing of the number n of the wave-guided mode.

We see also, that as a result of the interaction of the electromagnetic waves with the cyclotron oscillations of the layered electron gas, the electron cyclotron resonance frequency ω_H is shifted, but it does not undergo splitting. It is seen that the shift of ω_H is positive (negative) if ω_H lies below (above) the frequency ω_e corresponding to the nearest zero of the function $F_1(\xi)$. In the case when $\omega_H=\omega_e$ the electron cyclotron frequency is not shifted.

In the case of $qd\ll1$, and assuming that the layers are close together (in the sense that $\xi\ll1$) the dispersion relation of the collective modes takes the form [3,6]

$$\omega=\omega_H[cq]^2/(\omega_{pl}^2+[cq]^2) \qquad (10)$$

which is just the dispersion relation for the helicon propagating along the superlattice axis.

If $\omega_H\gg cq/\varepsilon^{1/2}$ and $\Omega\ll\omega_H$, then Eq.(9) has solutions corresponding to the frequencies much lower than the electron cyclotron frequency ($\omega\ll\omega_H$). These frequencies can be calculated with the aid of the relations

$$\omega=(2\pi n-qd)c/d\varepsilon^{1/2} \pm (2\pi\sigma_H/\varepsilon d), \qquad n=1,2,\ldots \qquad (11)$$

$$\omega=(2\pi n+qd)c/d\varepsilon^{1/2} \pm (2\pi\sigma_H/\varepsilon d), \qquad n=0,1,2,\ldots \qquad (11a)$$

Here $n\ll\omega_H\varepsilon^{1/2}d/2\pi c$ and $\sigma_H=ecn_o/H_o$. Let us anote that σ_H is the Hall component of the two-dimensional electron gas. It is well known (see for example [17]) that in high magnetic fields at low temperatures the Hall conductivity of the degenerate 2D electron gas is quantized. Therefore the splitting and the shift of the oscillation frequencies of the wave-guided type interacting with the layered electron gas in a strong magnetic field will be quantized as well under such conditions.

We see that for $\omega_H\gg cq/\varepsilon^{1/2}$ and $\Omega\ll\omega_H$ the frequency splitting is small and proportional to the small dimensionless parameter Ω/ω_H. In other words, we can state that a strong magnetic field causes a frequency shift and splitting of the starting points of the wave-guided modes in a periodic layered system, depending inversely on the external magnetic field H_o and the period of the array of two-dimensional electron layers d. However, in the case of resonance interaction of the homogeneous (k=0) electromagnetic wave-guided modes and the electron cyclotron resonances the frequency splitting is proportional to the square root of the indicated small parameter: $\Delta\omega/\omega \sim (\Omega/\omega_H)^{1/2}$. A similar effect of splitting increase was discussed in [18] for the resonance of excitons in thin dielectric films with surface polaritons and was discussed also in [12,13] for the resonance interation between wave-guided oscillations and cyclotron waves in a single 2D electron layer.

5. The deformation (due to the action of a strong magnetic field) of the lowest wave-guided modes interacting with the layered system is shown in Fig. 2. In the inset it is shown the splitting of the lowest mode in the long-wavelength limit. For values of the wave vector corresponding to the limit $ka\ll1$ the dependence of the splitted frequencies on the wave vector k can be described with the aid of the following expression:

$$\omega_\pm = \frac{cq}{\varepsilon^{1/2}}\left[1+2(kd/\pi)^2\right] \pm \frac{\Omega c}{2\,\omega_H\varepsilon d} \qquad (12)$$

We see that the splitted frequencies increase with the square of the component of the wave vector k perpendicular to the external magnetic field.

The vicinity of intersept of straight lines $\omega=\omega_e$ and $\omega=ck/\varepsilon^{1/2}$ is shown in Fig. 3. We see that with the increasing of the wave vector k the dispersion curve describing the lowest splitted branch shows a kink when it approximates the frequency ω_o. At $\omega=\omega_o$ this curve intersects the line $\omega=ck/\epsilon^{1/2}$, enters the region $\alpha^2>o$ and continuously transforms into the dispersion law for the plasma oscillations of the layered electron gas. The highest of the splitted branches approximates, with the increasing of k, to the line $\omega=ck/\epsilon^{1/2}$, but does not intersect it and does not show kink when ω is near the frequency ω_o. We see also, that the curve describing the coupled cyclotron - wave-guided modes shows a small increasing on k when $k<\omega_M\epsilon^{1/2}/c$, but it shows a kink when k reaches the vicinity of the line $\omega=ck/\epsilon^{1/2}$. Therefore, we can state that in a system with small concentration of the 2D carriers (in the sense that $\Omega d/c<<1$) the behaviour of the dispersion curves can be explained as a result of the intersection of the wave-guided dispersion curves with the dispersion curve describing the cyclotron oscillations of the layered electron gas.

6. The dispersion relation for magnetoplasma oscillations of the wave-guided type in semiconductor superlattices has been presented. The behaviour of these modes in the close vicinity of the line $\omega=ck/\epsilon^{1/2}$ has been discussed. The frequency shift and splitting caused by a strong magnetic field has been studied.

We have demonstrated that resonance modes whose frequencies are smaller than ω_M and decrease with increasing H_o are not only observed in systems with inhomogeneous 2D electron gas, but they can exist under certain conditions in a layered system of unbounded extended along the planes electron layers.

We should mention an important difference between "edge" modes and described by Eq. (9) electromagnetic excitations. For the description of the splitting of the starting points of the wave-guided modes (9) it is necessary to take the retarded effects of the electromagnetic waves into account. On the other hand, the existence of "edge"" magnetoplasmons is connected essentially with the loss of translation invariance of the system. It is clear that the behaviour of "edge" modes does not essentially depend on the account of retarded effects.

The frequency splitting indicated in Eqs. (11, 11a) is associated with the Faraday effect for electromagnetic waves propagating along the direction of the magnetic field perpendicular to the planes with 2D current carriers. Let us remark that the damping of the magnetoplasma oscillations for $\omega<<\omega_M$, $\omega_M\tau>>1$ is determinated principally by the dissipative conductivity of the 2D electron gas: $\sigma_{xx}\sim\sigma_{xy}/\omega_M\tau<<\sigma_{xy}$, i.e. the splitting of the resonance lines exceeds their width.

ACKNOWLEDGMENTS

One of the authors (J.C.G.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

REFERENCES

[1]. J. Mizuno, M. Kobayashi, and I. Yokota, J. Phys. Soc. Japan, **39**, 18 (1975).

[2]. J.C. Maan, M. Altarelli, H. Sigg, P. Wyder, L.L Chang and L.Esaki, Surf. Sci., **113**, 347 (1982).

[3]. A. Tselis, G. Gonzales de la Cruz and J.J. Quinn, Solid State Communs., **47**, 43 (1983).

[4]. M.S. Kushwaha, Phys. Status Solidi, **136 b**, 757 (1986)

[5]. L. Wendler and M.I. Kaganov, Pis'ma Zh. Eksp. Teor. Fiz., **44**, 345 (1986).

[6]. L. Wendler and M.I. Kaganov, Phys. Status Solidi, **138 b**, K33 (1986).

[7]. L. Wendler and M.I. Kaganov, Phys. Status Solidi, **142 b**, K63 (1987).

[8]. D.C. Glattli, E.Y. Andrei, G. Deville, J. Poitrenaud and F.I.B. Williams, Phys. Rev. Lett., **54**, 1710 (1985).

[9]. P.B. Mast, A.J. Dahm and A.L. Fetter, Phys. Rev. Lett. **54**, 1706 (1985).

[10]. V.A. Volkov and S.A. Mikhailov, Sov. Phys. JETP, **67**, 1639 (1988).

[11]. J.W. Wu, P. Hawrylak and J.J. Quinn, Phys. Rev. Lett. **55**, 879 ((1985).

[12]. Yu.A. Kosevich, A.M. Kosevich and J.C. Granada, Phys. Lett., **127 a**, 52 (1988).

[13]. A.M. Kosevich, Yu.A. Kosevich and J.C. Granada, Fiz. Nizk. Temp., **14**, 926 (1988).

[14]. S.A. Korzh and A.M. Kosevich, Fiz. Nizk. Temp., **7**, 1382 (1982).

[15]. A.L. Fetter, Ann. Phys. (N.Y.), **88**, 1 (1974).

[16]. Yu.P. Monarkha, Fiz. Nizk. Temp., **3**, 1459 (1977).

[17]. R.E. Prange and S.M. Girvin (eds.), The Quantum Hall Effect (Springer, N.Y. 1987) p.419.

[18]. V.M. Agranovich and V.L. Ginzburg, Crystal Optics with Spatial Dispersion, and Excitons (Springer, Berlin, 1984) p.290.

FIGURE CAPTIONS

FIG. 1: Graphical solution of Eq. (9). The thick (thin) continuous curves represent the defined in the text function $F_1(\zeta)$ $(F_2(\zeta))$. The dashed- dotted lines represent the asymptotics of $F_1(\zeta)$.

FIG. 2: Dispersion curves of the lowest branches of the magnetoplasma oscillations in a periodic array of 2D electron layers: $\Omega d/c=3\cdot10^{-8}$, $\Omega/\omega_H=1\cdot10^{-3}$, $qd=\pi/3$, $n_e=1\cdot10^{11}$ cm^{-2}, $m=1x10^{-28}$ gr., $\epsilon=13$, $d=5\cdot10^{-4}$ cm. The dashed- dotted line represents the dispersion law $\omega=ck/\epsilon^{1/2}$. Inset: Splitting of lowest wave-guided mode.

FIG. 3: Same as Fig.2: details of the oscillation branches in the vicinity of intercept of straight lines $\sigma=0$ and $\omega=\omega_e$.

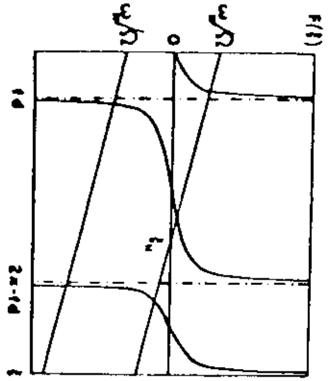


Fig. 1

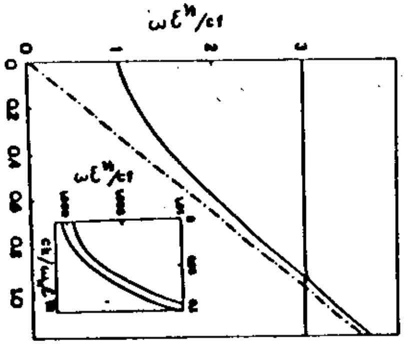


Fig. 2

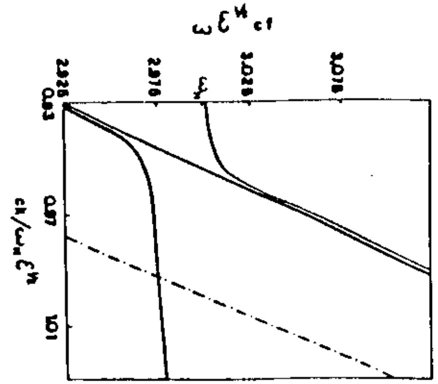


Fig. 3

MAGNETOPLASMA OSCILLATIONS OF A TWO-DIMENSIONAL ELECTRON LAYER IN A BOUNDED SYSTEM

Yu.A. KOSEVICH
All-Union Surface and Vacuum Research Centre, 117334 Moscow, USSR

A.M. KOSEVICH
Institute for Low Temperature Physics and Engineering, 310164 Kharkov, USSR

and

J.C. GRANADA
Kharkov State University, 310077 Kharkov, USSR

Received 1 November 1987; accepted for publication 30 November 1987
Communicated by V.M. Agranovich

The spectrum of magnetoplasma oscillations of a two-dimensional electron layer in a transversal magnetic field is studied under the condition that the electron system is unbounded along the layer plane and screened in the perpendicular direction. It is shown that under certain conditions oscillation frequencies much lower than the electron cyclotron frequency exist. Also the electromagnetic wave-guided oscillations in the system are described. It is shown that a strong magnetic field causes a frequency shift and splitting, depending inversely on the external magnetic field and the transversal specific dimension.

Recently the interest in the high-frequency dynamics of a two-dimensional electron layer in a strong magnetic field has increased. At the present time the two-dimensional electron systems are realized in the following three variants: (i) in the layered metal–dielectric–semiconductor system (MDS structure), (ii) in semiconductor heterojunctions (such as GaAs–(AlGa)As), (iii) in the electrons, trapped on the surface of liquid ^{4}He by a pressing static electric field. All such systems are characterized by a spatial restriction along the direction perpendicular to the layer plane. This restriction modifies the spectrum of the electromagnetic oscillations of such systems.

In recent experimental investigations of the high-frequency field absorption by a two-dimensional electron gas, trapped on the liquid ^{4}He surface, electromagnetic resonance frequencies much lower than the electron cyclotron resonance frequency were observed [1,2]. This observation does not correspond to the previous calculation of the frequency shift of the two-dimensional plasma oscillations in a magnetic field [3,4]. A model of "edge magnetoplasmons" was proposed for the theoretical explanation of these resonances [5,6].

We shall demonstrate that oscillation frequencies much lower than the cyclotron frequency can exist under certain conditions in a two-dimensional electron system which is unbounded along the layer plane. In order to derive the dispersion relations for these frequencies it is necessary to take the retardation effects into account in the equations of motion.

A two-dimensional electron layer, located between two dielectric slabs with dielectric constants ϵ_1 and ϵ_2, and thicknesses d_1 and d_2, is investigated. The slabs are boundlessly extended along the electron layer plane. The external surfaces of the slabs are metallized (they coincide with metal surfaces). This configuration is usual for various systems, such as the MDS contact, and electrons trapped on the liq-

0375-9601/88/$ 03.50

uid helium surface [7]. A static magnetic field $\boldsymbol{H}$ is applied along the direction perpendicular to the electron layer plane.

In order to investigate the vibration spectrum of a bounded system, it is necessary to solve the Maxwell equations in the medium with the relevant boundary conditions. If a two-dimensional current with density $\boldsymbol{j}_s$ exists in the electron layer, then the boundary conditions take the form

$$(\boldsymbol{H}_1-\boldsymbol{H}_2)\times\boldsymbol{n}=\frac{4\pi}{c}\boldsymbol{j}_s,\quad(\boldsymbol{E}_1-\boldsymbol{E}_2)\times\boldsymbol{n}=0\,,\tag{1}$$

where $\boldsymbol{n}$ is the normal unit vector directed from medium 1 to medium 2.

The surface current can be written as $\boldsymbol{j}_s=e\eta_0\boldsymbol{v}$, where η_0 is the surface charge density and $\boldsymbol{v}$ is the two-dimensional electron velocity. The velocity $\boldsymbol{v}$ satisfies the equation of motion

$$m\frac{\partial\boldsymbol{v}}{\partial t}=e\boldsymbol{E}_t+\frac{e}{c}\boldsymbol{v}\times\boldsymbol{H}-\frac{m}{\tau}\boldsymbol{v}\,,\tag{2}$$

where τ is the characteristic relaxation time of the charged carriers and $\boldsymbol{E}_t$ is the tangential electric field in the electron layer. Eq. (2) is written in the time-relaxation approximation in the local limit and permits one to present a semiphenomenological description of the phenomena under consideration.

Solving Maxwell's equations in the dielectric medium with the boundary conditions (1) and (2), and also with the condition $E_t=0$ at the external surfaces, we obtain the dispersion equation for the magnetoplasma waves,

$$\left(\omega^2-\frac{c\Omega^*}{(\epsilon_1/q_1)\mathrm{cth}(q_1d_1)+(\epsilon_2/q_2)\mathrm{cth}(q_2d_2)}\right)\times\left(1+\frac{\Omega^*/c}{q_1\,\mathrm{cth}(q_1d_1)+q_2\,\mathrm{cth}(q_2d_2)}\right)=\omega_H^{*2}\,,\tag{3}$$

where

$$q_{1,2}^2=k^2-(\omega/c)^2\epsilon_{1,2},\quad m^*=m(1+\mathrm{i}/\omega\tau)\,,$$
$$\omega_H^*=eH/m^*c,\quad\Omega^*=4\pi e^2\eta_0/m^*c\,.\tag{4}$$

Formula (3) contains all the known results as limiting cases.

In the case $\omega_H^*=0$ ($H=0$) we obtain from (3) and (4) the dispersion equation for the surface polaritons in a bounded medium:

TH mode

$$\frac{\epsilon_1}{q_1}\mathrm{cth}(q_1d_1)+\frac{\epsilon_2}{q_2}\mathrm{cth}(q_2d_2)=\frac{\Omega^*c}{\omega^2}\,,\tag{5}$$

TE mode

$$q_1\,\mathrm{cth}(q_1d_1)+q_2\,\mathrm{cth}(q_2d_2)=-\frac{\Omega^*}{c}\,.\tag{6}$$

If we put $d_2=\infty$, $\Omega^*=0$ then we can derive a dispersion relation, that is similar to that obtained in ref. [8]. The right-hand sides of eqs. (5) and (6) permit one to find the frequency changes of the TH and TE modes, due to the conductivity of the two-dimensional electron gas, when the external magnetic field is absent. In the limit $d_1=d_2=\infty$ similar expressions were derived in ref. [9] for the case of $\omega\tau\gg1$ and in ref. [10] for the case of $\omega\tau\ll1$.

Let us consider the collisionless limit and neglect the retardation of electromagnetic waves ($1/\tau\ll\omega\ll(k/\sqrt{\epsilon_{1,2}})$. Then the following relation holds

$$\omega^2=\frac{ck\omega_H^2}{ck+\Omega[\mathrm{cth}(kd_1)+\mathrm{cth}(kd_2)]^{-1}}+\frac{ck\Omega}{\epsilon_1\,\mathrm{cth}(kd_1)+\epsilon_2\,\mathrm{cth}(kd_2)}\,,\tag{7}$$

where

$$\Omega=4\pi e^2\eta_0/mc,\quad\omega_H=eH/mc\,.$$

We see that the usual dispersion relation

$$\omega^2=\omega_H^2+\frac{\Omega ck}{\epsilon_1+\epsilon_2}\tag{8}$$

is valid in the limiting case $kd\gg1$ and $ck\gg\max\{\Omega,\omega_H\}$.

Note first of all that there are two characteristic values of k for $kd\gg1$: $k_1=\omega_H/c$ and $k_2=\omega_H^2/c\Omega$. If

$$c\left(\frac{1}{d_1}+\frac{1}{d_2}\right)\ll\Omega\ll\omega_H\,,\tag{9}$$

then all the values of k can be divided into three intervals:

(1) In the range $k\gg k_2$, when $\omega\gg\omega_H$, we have

$$\omega^2=\frac{\Omega ck}{\epsilon_1+\epsilon_2}\,.\tag{10}$$

-23-

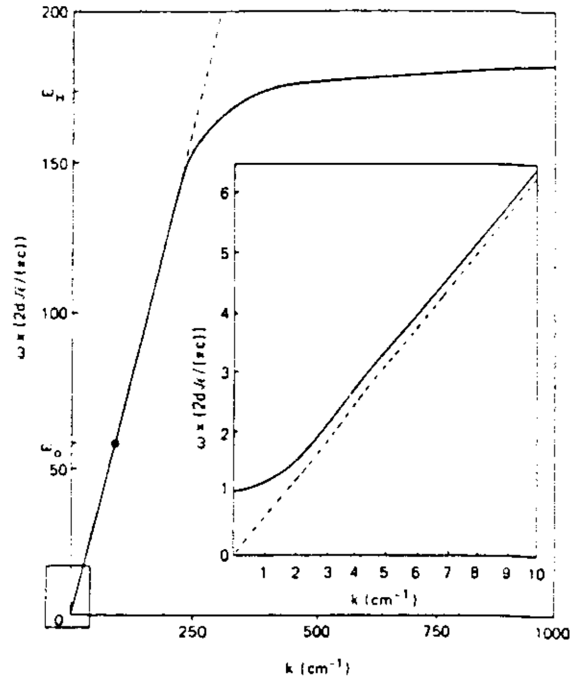


Fig. 1. The dispersion law $\omega=\omega(k)$ of the lowest branches of the magnetoplasma oscillations of a two-dimensional electron layer in a bounded system. $\Omega d/c=35$, $\Omega/\omega_H=0.13$, $\eta_0=10^{12}$ cm^{-2}, $H=5\times10^4$ G, $m=10^{-28}$ g, $d_1=d_2=1$ cm, $\epsilon_1=\epsilon_2=1$.

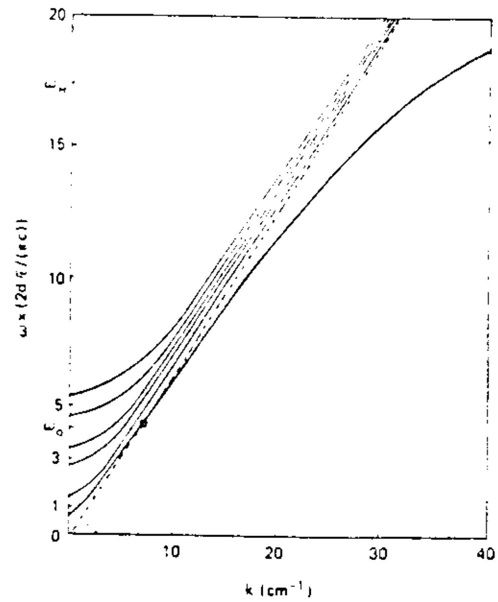


Fig. 2. Same as fig. 1, but with parameters $\Omega d/c=35$, $\Omega/\omega_H=1.3$, $\eta_0=10^{12}$ cm^{-2}, $H=5\times10^3$ G, $m=10^{-28}$ g, $d_1=d_2=1$ cm, $\epsilon_1=\epsilon_2=1$.

Eq. (10) is the usual dispersion relation for the two-dimensional plasma oscillations in the absence of external magnetic field.

(2) In the interval $k_1 \ll k \ll k_2$ we have

$$\omega=\omega_H\frac{1+\frac{1}{2}k/k_2}{\epsilon_1+\epsilon_2},$$

i.e. a slowly, linear in k, increasing oscillation frequency above the electron cyclotron resonance frequency. We see a kink on the dispersion curve close to the frequency $\omega=\omega_H$ (fig. 1).

If the inequality

$$c\left(\frac{1}{d_1}+\frac{1}{d_2}\right)\ll\omega_H\ll\Omega \qquad (11)$$

holds instead of eq. (9), then the dispersion relation (10) holds for $kd\gg1$ under the condition $ck\gg\Omega$ (the lowest curve in fig. 2). Thus, the kink on the lowest curve in fig. 1 in the vicinity of the frequency $\omega=\omega_H$ disappears, if $\omega_H\sim\Omega\gg c/d$.

(3) In the range $ck\lesssim\max\{\Omega,\omega_H\}$ the frequency $\omega\approx ck/\sqrt{\epsilon}$, and it is necessary to take into consideration retardation effects. In order to analyse the dispersion relation in the close vicinity of the line $\omega=ck/\sqrt{\epsilon}$, take the limit $qd\ll1$ in the case $\epsilon_1=\epsilon_2=\epsilon$, when eq. (3) yields the relation

$$\omega^2=\frac{\omega_H^2}{1+\Omega d/2c}+s^2q^2, \qquad (12)$$

where

$$\frac{2}{d}=\frac{1}{d_1}+\frac{1}{d_2},\quad s^2=\frac{c\Omega d}{2\epsilon},\quad q^2=k^2-\frac{\omega^2}{c^2}\epsilon.$$

If we put $q=0$, we can find a point where the curve $\omega=\omega(k)$ intersects the line $\omega=ck/\sqrt{\epsilon}$. At this point the frequency squared,

$$\omega^2=\omega_0^2\equiv\omega_H^2(1+\Omega d/2c)^{-1}, \qquad (13)$$

is much less than ω_H^2 under the condition

$$\Omega d\gg c. \qquad (14)$$

It is easy to show that the derivative $d\omega/dk$ at the point $q=0$, $\omega=\omega_0$, is

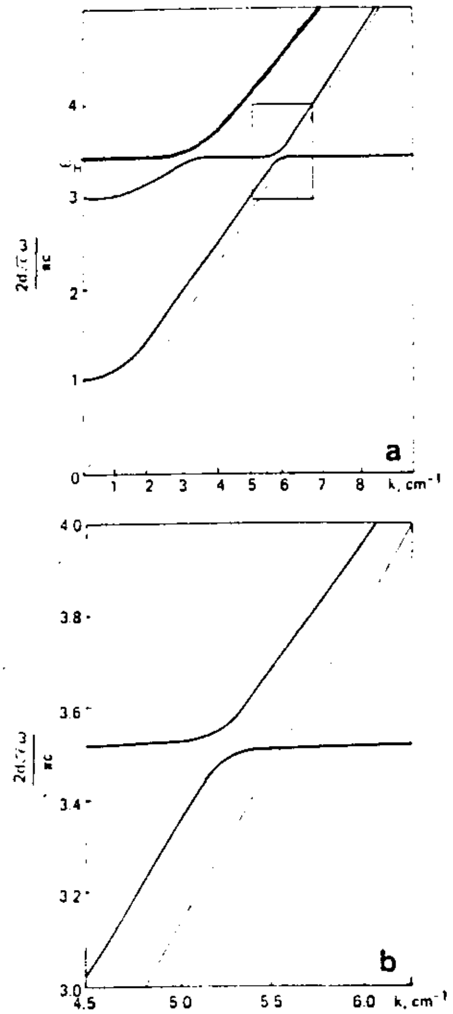


Fig. 3. (a) Same as fig. 1, but with parameters $\Omega d/c=0.0035$, $\Omega/\omega_H=0.00063$, $n_0=10^8$ cm^{-2}, $H=1\times10^4$ G, $m=10^{-27}$ g, $d=1$ cm, $\epsilon=1$. (b) Details of the lowest oscillation branches for $\omega\approx\omega_H$.

$$v_0=\left.\frac{d\omega}{dk}\right|_{q=0}=\frac{c}{\sqrt{\epsilon}}\frac{(s/c)^2\epsilon}{1+(s/c)^2\epsilon}. \quad (15)$$

It follows from eq. (15) that (1) $v_0\ll c/\sqrt{\epsilon}$ under the condition $\Omega d\ll c$, and (2) $v_0=c/\sqrt{\epsilon}$ under the condition (14). The former case may be illustrated with fig. 3 and the latter is represented by figs. 1 and 2 for different ratios Ω/ω_H.

As a result of the above consideration, we can state that in a bounded system under the condition (14) there are magnetoplasma oscillation frequencies in the range $\omega<ck/\sqrt{\epsilon}$, which are much lower than the electron cyclotron resonance frequency.

The above results are related with $ck\geqslant\omega\sqrt{\epsilon}$ and $\omega\geqslant\omega_0$ where ω_0 is defined by eq. (13). Let us now describe wave guided magnetoplasma oscillations in the screened system which correspond to the condition $\omega\geqslant ck/\sqrt{\epsilon}$. We are interested in a shift and in a splitting of such oscillation frequencies, caused by a magnetic field.

We illustrate this phenomenon in the symmetric system when $\epsilon_1=\epsilon_2=\epsilon$ and $d_1=d_2=d$. For frequencies which satisfy the relation $\omega^2\geqslant c^2k^2/\epsilon$, the dispersion equation (3) reduces to

$$\sin\kappa d=0\,, \quad (16)$$

$$[2\kappa(\omega^2-\omega_H^2)\,\mathrm{ctg}(\kappa d)+\omega^2\,\Omega/c]\times[(2\epsilon/\kappa)(\omega^2-\omega_H^2)\,\mathrm{ctg}(\kappa d)+\Omega c]=(\Omega\omega_H)^2\,, \quad (17)$$

where $\kappa^2=\omega^2\epsilon/c^2-k^2$.

Eq. (16) corresponds to oscillations of the vector $E_\parallel$, antisymmetric with respect to the electron layer plane. These oscillations do not interact with the two-dimensional electron system.

The solutions of eq. (17) define the frequencies of the long-wavelength oscillations of the wave guided type, interacting with the electron system. First of all, we consider the frequency splitting for $k=0$ and $\kappa=\omega\sqrt{\epsilon}/c$. In this case eq. (17) can be transformed to the form

$$\mathrm{tg}\left(\frac{\omega d}{c}\sqrt{\epsilon}\right)=-\frac{2\sqrt{\epsilon}}{\Omega}(\omega\pm\omega_H)\,. \quad (18)$$

If $c/d\ll\omega_H$ and $\Omega\ll\omega_H$, then eq. (18) has solutions, corresponding to the frequencies $\omega\ll\omega_H$. These frequencies can be derived from the following equation,

$$\mathrm{ctg}\left(\frac{\omega d}{c}\sqrt{\epsilon}\right)=\pm\frac{\Omega}{2\omega_H\sqrt{\epsilon}}, \quad (19)$$

and they can be calculated with the relation

$$\frac{\omega d}{c}\sqrt{\epsilon}=\tfrac{1}{2}\pi(2n+1)\pm\frac{\Omega}{2\omega_H\sqrt{\epsilon}},$$

$$n=0, 1, 2\ldots\,. \quad (20)$$

-25-

Anomalous damping of low-frequency edge magnetoplasma oscillations in case of quantum Hall effect

Yu. A. Kosevich

Institute of Physical Problems, Academy of Sciences of the USSR

(Submitted 1 November 1989)

Pis'ma Zh. Eksp. Teor. Fiz. 50, No. 11, 471-475 (10 December 1989)

The diffusion component of the current near a boundary gives rise to an additional damping of low-frequency edge magnetoplasma oscillations in inhomogeneous 2D electron systems (including superlattices). This damping increases with decreasing value of the dissipative conductivity σ_{xx}. The possibility of experimentally observing the anomalous damping of edge magnetoplasma oscillations in the case of an integer quantization of the Hall effect is discussed.

In the absence of an external magnetic field, plasmons in 2D electron systems[1] are damped only in the collisionless limit $\omega\tau \gg 1$, where τ is the momentum relaxation time. In a strong magnetic field, such that the condition $\omega_c\tau \gg 1$ holds (ω_c is the cyclotron frequency), and Hall currents outweigh the dissipative currents, weakly damped low-frequency magnetoplasma oscillations can exist in both the collisionless limit and the local hydrodynamic limit $\omega\tau \ll 1$. Weakly damped magnetoplasma oscillations with frequencies below ω_c and $1/\tau$ have recently been observed experimentally and are presently the subject of active research. These are edge magnetoplasma oscillations (EMOs) in bounded 2D electron systems under conditions corresponding to the quantum Hall effect (see, for example, Refs. 2-5 and the bibliographies there). The EMO frequencies are inversely proportional to the magnetic field and to the transverse dimension of the 2D system. It follows from recent experiments[6] that the damping of low-frequency ($\omega\tau \ll 1$) EMOs is not described at all by the existing theories. In the present letter we show that if the ratio of the dissipative and Hall conductivities of inhomogeneous 2D systems (including superlattices) is sufficiently small, $\sigma_{xx}/\sigma_{xy} \ll 1$, the diffusion component of the electric field near a boundary leads to a damping of EMOs which increases with decreasing value of the ratio σ_{xx}/σ_{xy}. Under conditions corresponding to the quantum Hall effect in strong magnetic fields, this mechanism may lead to an anomalous damping and to the disappearance of the EMOs. This property of EMOs stems from the vanishing of the normal component of the current at the boundary of the system; in the case of a vanishingly low dissipative conductivity, while the Hall conductivity is nonzero, the result will be a pronounced increase in the gradients of the nonequilibrium-carrier density near a boundary and an increase in the dissipation. In an ideal Hall sample ($\sigma_{xx} = 0$, $\sigma_{xy} \neq 0$), the boundaries of the sample are equipotentials, so edge magnetoplasma waves accompanied by oscillations of the electron density and the boundary potential cannot propagate.

1. We consider a semi-infinite ($y < 0$) conducting 3D medium in an external magentic field $H_0 \| Z$. Effectively qualifying as a medium of this sort might be, for example, a superlattice of 2D electron layers separated by thin insulating interlayers of thickness d in the limit $kd \ll 1$ (Refs. 7 and 8). This effect can be described correctly only if the drift and diffusion components of the current are taken into account simultaneously in the magnetized conducting medium, so the system of equations for low-frequency oscillations ($\omega\tau \ll 1$) consists of the Poisson equation, the equations of electrostatics, and the charge conservation law, along with the constitutive equations for the electric displacement $\mathbf{D} = \epsilon\mathbf{E}$ and the current density $\mathbf{j} = \hat{\sigma}(\mathbf{E} - \beta\nabla\rho)$, where ϵ, ρ, and $\hat{\sigma}$ are respectively the dielectric constant, volume charge density, and conductivity tensor of the 3D medium, and $\beta > 0$ is the Einstein coefficient. At the $y = 0$ interface between a conducting medium and an insulating external medium (with a dielectric constant ϵ_0), boundary conditions are imposed: The electostatic potential φ and the normal component of the electric displacement, D_y, are continuous, and the normal component of the current, j_y, vanishes. For the frequency $\omega(k)$ of a surface magnetoplasmon which is propagating along this boundary, in the direction across the external magnetic field (Refs. 9 and 10, for example), we find the dispersion relation

$$(\sigma_{xx} + i\sigma_{xy})(\epsilon\varkappa_D + \epsilon_0 k) = \frac{i\omega\epsilon}{4\pi\sigma_{xx}}(\epsilon + \epsilon_0)(\sigma_{xx}\varkappa_D + ik\sigma_{xy}), \qquad (1)$$

where the parameter $\varkappa_D = [k^2 + (4\pi\sigma_{xx}/\epsilon - i\omega)/\beta\sigma_{xx}]^{1/2}$ is the reciprocal of the depth (δ) to which oscillations in the carrier density penetrate into the conducting medium ($1/\delta = \mathrm{Re}\,\varkappa_D$). In the case of a small but nonzero σ_{xx}, corresponding to the conditions $\sigma_{xx} \ll \sigma_{xy}$, $k\delta \ll \sigma_{xx}/\sigma_{xy}$, in which case we have $\delta = (2\beta\sigma_{xx}/\omega)^{1/2}$, the spectrum ω' and the damping ω'' of a surface magnetoplasmon take the form

$$\omega' = \frac{4\pi\sigma_{xy}}{\epsilon + \epsilon_0}, \qquad \omega'' = \frac{4\pi}{\epsilon + \epsilon_0}\left(\sigma_{xx} + \frac{\sigma_{xy}^2}{\sigma_{xx}}k\delta\right), \qquad (2)$$

In other words, in the case $(\sigma_{xx}/\sigma_{xy})^2 \ll k\delta \ll \sigma_{xx}/\sigma_{xy}$ the damping of a surface magnetoplasmon is determined completely by the diffusion processes near the boundary, and it does indeed increase with decreasing value of the ratio σ_{xx}/σ_{xy}, as $\omega'' \propto (\sigma_{xy}/\sigma_{xx})^{1/2}$. In the case of vanishingly small values of σ_{xx} $(\sigma_{xx}/\sigma_{xy} \ll k\delta \ll 1)$, on the other hand, the quality factor of surface magnetoplasma oscillations becomes less than unity, and these weakly damped surface magnetoplasmons cease to propagate.

The spectrum and damping of surface magnetoplasmons of the type in (2) in the region of their weak absorption can be found not only from dispersion relation (1) but also directly through the use of the surface-charge conservation law $\dot{\rho}_S = -i\omega'\rho_S = -j_y = -ik\varphi_S\sigma_{xy}$, $\pi\rho_S = k\varphi_S(\epsilon + \epsilon_0)$, the dissipation function Ψ $(\omega'' = \Psi/2\mathcal{E})$,

$$\Psi = \int_V dV \sigma'_{xx} [\,|E_x - \beta \frac{\partial\rho}{\partial x}|^2 + |E_y - \beta\frac{\partial\rho}{\partial y}|^2\,] \qquad (3)$$

and the electrostatic energy of the oscillations, $\mathcal{E}$,

$$\mathcal{E} = \int_V dV \frac{\epsilon}{8\pi} (|E_x|^2 + |E_y|^2) + \int_{V_0} dV_0 \frac{\epsilon_0}{8\pi} (|E_x|^2 + |E_y|^2)\,. \qquad (4)$$

The vanishing of the total current inside the diffusion layer near the interface is taken into account here: $ik\varphi_S\sigma_{xy} - \sigma_{xx}\beta\rho/\delta \simeq 0$. In expressions (3) and (4), V and V_0 are the regions of the conducting and external media. The integral outside (inside) the diffusion surface layer determines the first (second) term in expression (2) for the damping of the surface magnetoplasmons.

2. A similar qualitative method can be used to find the spectrum and damping of EMOs (in the region of their weak absorption), with allowance for the inhomogeneity of the carriers near the boundary in a bounded 2D electron system (see also Ref. 4). Working from the law of conservation of the edge charge Q, which sets up a potential $\varphi = 2Q\ln(1/kr)\exp(ikx - i\omega t)$ in vacuum (r is the distance from the edge of the 2D system, k is the wave number along it, and $kr \ll 1$), and the expressions for the dissipation function and the electrostatic energy, as in expressions (3) and (4), we find the following logarithmic-accuracy estimate of the spectrum and damping of the EMOs in the case $k\delta \ll \sigma_{xx}/\sigma_{xy}$, $k\delta \ll 1$:

$$\omega'_p = 2\sigma_{xy} k \ln(1/k\delta),$$

$$\omega''_p = 2\sigma'_{xx} [\frac{1}{\delta\ln(1/k\delta)} + \frac{k}{\ln(1/k\delta)} + |\frac{\sigma_{xy}}{\sigma_{xx}}|^2 k^2\delta\ln(1/k\delta)]\,, \qquad (5)$$

$$\omega'_p \gg \omega''_p\,.$$

For the depth (δ) of the localization of the nonequilibrium carriers near the edge of the 2D system, the following expression holds in our diffusion approximation:

$$\frac{1}{\delta} = \mathrm{Im}\,\{[(\frac{\pi}{\beta})^2 + \frac{i\omega}{\beta\sigma_{xx}}]^{1/2} - \frac{\pi}{\beta}\}\,. \qquad (6)$$

The first and second terms in (5) for ω'' of the EMOs are determined by the dissipation outside layer δ, while the third is determined by the dissipation in the layer itself. At frequencies $\omega \ll |\sigma_{xx}|/\beta$, we find from (5) and (6)

$$\delta = 2\pi\,|\sigma_{xx}|^2/(\omega\sigma'_{xx}), \quad \omega'' \approx 2\pi\sigma_{xy}k + 2\sigma'_{xx}k/\ln(1/k\delta). \qquad (7)$$

Expression (7) for ω'' of the EMOs differs from the corresponding expression which has been derived by Volkov and Mikhaĭlov[4] in that it contains a term proportional to $\sigma'_{xx}k$. This term can contribute significantly to the damping of EMOs in magnetic fields (or of the value of ϵ_F of the 2D electrons) which do not correspond to the middle of the plateau of quantized values of σ_{xy}, in which case σ'_{xx} depends strongly on the external parameters and increases significantly.[8] At frequencies $\omega \gg |\sigma_{xx}|/\beta$, the depth δ is equal to the diffusion length $[\delta = (2\beta\sigma_{xx}/\omega)^{1/2} \gg 2\pi\sigma_{xx}/\omega]$, and it follows from (5) that under the condition $\sigma_{xx}/[\sigma_{xy}\ln(1/k\delta)] \ll k\delta \ll \sigma_{xx}/\sigma_{xy}$ the damping of the EMOs is determined completely by the diffusion currents near the boundary. It increases with decreasing value of the ratio σ_{xx}/σ_{xy}, as $\omega'' \propto (\sigma_{xy}/\sigma_{xx})^{1/2}$ [if we ignore $\sigma''_{xx}(\omega)$ in the limit $\omega\tau \ll 1$; cf. expression (2) for the damping of surface magnetoplasmons]. In the case $k\delta \gtrsim \sigma_{xx}/\sigma_{xy}$, in contrast, the EMOs become strongly damped (with a quality factor less than unity). The transition from weak damping to anomalous damping of EMOs thus occurs at $\omega \sim \sigma_{xx}/\beta$, $\delta \sim \beta$, i.e., under the condition

$$\sigma_{xx}/\sigma_{xy} \lesssim k\beta\ln(1/k\beta), \qquad (8)$$

where we have $\omega''/\omega' \sim \sigma_{xy}k\delta/\sigma_{xx} \sim 1/\ln(1/k\beta) \ll 1$.

Working from experimental data on the density of states near the Fermi level, $D(\epsilon_F)$, for energies ϵ_F between Landau levels,[11] we find the estimate $\beta = 1/e^2D(\epsilon_F) \sim 10^{-5}$ cm. In strong magnetic fields $H_0 \simeq 10$T we thus have $\delta \sim \beta \gg r_c \sim v_F/\omega_c \sim 10^{-6}$ cm and $\delta \gg a_H = (\hbar c/eH_0)^{1/2} \sim 10^{-6}$ cm, justifying our use of the local hydrodynamic approximation under these conditions (r_c is the radius of a cyclotron orbit, and a_H is the magnetic length).[12,13] For the values $k \sim 10$ cm^{-1} in the experiments of Ref. 6, we find from (8) a lower limit on the values of the dynamic dissipative conductivity $\sigma_{xx}(\omega_p)$ of a sample at which there is an anomalous damping of EMOs:

$$\sigma_{xx}/\sigma_{xy} \sim 10^{-3} - 10^{-4}.$$

A dynamic conductivity of this magnitude could apparently be achieved in high-quality 2D conducting channels with a high carrier mobility in strong magnetic fields (for small integer values of the filling factor, $\nu = 1, 2$).

I wish to thank N. N. Beletskiĭ, V. A. Volkov, I. M. Grodnenskiĭ, M. I. Kaganov, A. Yu. Kamaev, S. A. Mikhaĭlov, L. P. Pitaevskiĭ, V. L. Pokrovskiĭ, V. M. Pudalov, and M. I. Reznikov for useful discussions.

[1] T. Ando, A. B. Fowler, and R. Stern, Rev. Mod. Phys. 54, 437 (1982).
[2] V. A. Volkov et al., Pis'ma Zh. Eksp. Teor. Fiz. 44, 510 (1986) [JETP Lett. 44, 655 (1986)].
[3] S. A. Govorkov et al., Pis'ma Zh. Eksp. Teor. Fiz. 44, 380 (1986) [JETP Lett. 44, 487 (1986)].

[4] V. A. Volkov and S. A. Mikhailov, Zh. Eksp. Teor. Fiz. **94**(8), 217 (1988) [Sov. Phys. JETP **67**, 1639 (1988)].
[5] E. Y. Andrei *et al.*, Surf. Sci. **196**, 501 (1988).
[6] V. I. Tal'yanskii, Pis'ma Zh. Eksp. Teor. Fiz. **50**, 196 (1989) [JETP Lett. **50**, 221 (1989)].
[7] V. I. Tal'yanskii, Zh. Eksp. Teor. Fiz. **92**, 1845 (1987) [Sov. Phys. JETP **65**, 1036 (1987)].
[8] L. Vendler and M. I. Kaganov, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 345 (1986) [JETP Lett. **44**, 445 (1986)].
[9] J. W. Wu, Phys. Rev. B **33**, 7091 (1986).
[10] N. N. Beletskii *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 589 (1987) [JETP Lett. **345**, 751 (1987)].
[11] I. V. Kukushkin *et al.*, Usp. Fiz. Nauk **155**, 219 (1988) [Sov. Phys. Usp. **31**, 511 (1988)].
[12] R. F. Kazarinov and S. Luryi, Phys. Rev. B **25**, 7626 (1982).
[13] V. B. Shikin, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 471 (1988) [JETP Lett. **47**, 555 (1988)].

Translated by Dave Parsons