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SPRING COLLEGE IN CONDENSED MATTER ON 'PHYSICS OF LOW-DIMENSIONAL STRUCTURES'

(23 April - 15 June 1990)

BACKGROUND MATERIAL

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These are preliminary lecture notes, intended only for distribution to participants.

TWO-DIMENSIONAL ELECTRON GAS IN GaAs

FACT SHEET

Units:	Carrier Density	N_s	$10^{11} cm^{-2}$
	Mobility	μ	$10^6 cm^2/V{\cdot}sec$
	Magnetic Field	B	Tesla, T
	Energy		Kelvin, K or meV

Effective Mass: $m^* = 0.067 m_0$

g-factor: $g = -0.44$

Fermi Wavevector: $k_F = 7.9 \times 10^5 \times N_s^{1/2} \ (cm^{-1})$

Fermi Energy: $E_F = 3.6 \times N_s \ (meV)$

Mobility Lifetime: $\tau = 40 \times \mu \ (psec)$

Mean Free Path: $\lambda = 5.4 \times \mu \times N_s^{1/2} \ (\mu m)$

Magnetic Length: $l_0 = 25.7 \times B^{-1/2} \ (nm)$

Landau Level Degeneracy: $D = eB/h = 2.42 \times 10^{10} \times B \ \ cm^{-2}$

Cyclotron Energy: $\hbar\omega_c = 20 \times B \ (K)$

Spin Splitting: $g\mu_B B = 0.29 \times B \ (K)$

Coulomb Energy Scale: $e^2/4\pi\epsilon l_0 = 50 \times B^{1/2} \ (K)$

The Quantized Hall Effect

H. L. Stormer and D. C. Tsui

Summary. Quantization of the Hall effect is one of the most surprising discoveries in recent experimental solid-state research. At low temperatures and high magnetic fields the ratio of the Hall voltage to the electric current in a two-dimensional system is quantized in units of h/e^2, where h is Planck's constant and e is the electronic charge. Concomitantly, the electrical resistance of the specimen drops to values far below the resistances of the best normal metals.

The Hall effect is one of the better understood physical phenomena and is widely used in semiconductor materials laboratories to determine the carrier concentration of a given specimen. A magnetic field of moderate strength, an electric current supply, and a voltmeter are sufficient to perform combined Hall and resistivity measurements, which can yield direct information on the basic electrical properties of a new material. From an effect seemingly so well understood and a measurement as routinely performed as this, one hardly expects any surprises. And yet, less than 3 years ago, a startling observation created a new interest in the physical principles underlying the Hall effect. Von Klitzing *et al.* (*1*) discovered that under certain conditions the Hall resistance of their specimen was surprisingly constant, and its magnitude coincided with the ratio of two fundamental physical constants to any accuracy to which they were able to measure the effect (see Fig. 1). The Hall resistance R_H (see Fig. 2) was found to be quantized to

$$R_H = \frac{h}{ie^2} \qquad (1)$$

where h is Planck's constant, e is the electronic charge, and the quantum number i (= 1,2,3, . . .) is the number of fully occupied quantum energy levels (the Landau levels). This result not only attracted the attention of solid-state physicists, experimentalists as well as theorists, but also stirred much interest in disciplines as distant from solid-state science as elementary particle physics. It presents the possibility of a quantum resistance standard in terms of fundamental physical constants and also a new method for determining the fine structure constant, which is a measure of the coupling between elementary particles and the electromagnetic field. The fine structure constant, α, can be related to the quantized Hall resistance by

$$\alpha = \frac{\mu_0 c}{2}\frac{e^2}{h} = \frac{\mu_0 c}{2}(R_H)^{-1} \qquad (2)$$

where μ_0 is the permeability of the vacuum and by definition equals $4\pi \times 10^{-7}$ H/m. Since the light velocity, c, is known very precisely, the quantized Hall effect immediately spurred speculations that it could provide a new solid-state determination of α with an accuracy higher than that of previous determinations. At this time an accuracy of 1.7 parts in 10^7 has already been achieved (*2*). This is comparable to the accuracy of earlier measurements based on different physical phenomena, and further improvement is expected.

The quantized Hall effect is observed under conditions that are uncommon compared to those of standard Hall measurements. Magnetic fields of approximately 100 kilogauss and temperatures close to absolute zero are required. The specimen, too, is exceptional. It contains a so-called two-dimensional electron gas, which is ultimately responsible for the occurrence of this new quantum effect. The active region of all metal oxide semiconductor field-effect transistors (MOSFET's) consists of such a two-dimensional electron gas. In these systems the carriers are confined to a very narrow region at the interface between two different materials; they are able to move freely along the plane of the interface but lack any degree of freedom normal to it. Being confined to a narrow well of approximately 10^{-6} cm, they are quantum mechanically bound to the interface. In Si-MOSFET's, the two-dimensional electron gas exists at the interface between a slab of crystalline silicon and a thin (~ 10^{-5} cm) amorphous silicon dioxide top layer. The carriers are kept at the interface by a strong electric field established by an external voltage (gate voltage) applied to a metal electrode (gate) which covers the oxide. The Si-MOSFET is the most common physical realization of the concept of a two-dimensional electron system, and its fundamental properties have been studied extensively during the past two decades. The quantized Hall effect was first observed in such a device.

More recently, a new structure became feasible which, in various respects, has proved to be a superior host material for two-dimensional electrons (*3*). The structure is called a modulation-doped GaAs-(AlGa)As heterojunction, and it is prepared by a highly sophisticated crystal growth technique termed molecular beam epitaxy (*4*). It resembles the metal oxide semiconductor structures, but in this case the electron gas exists at the highly perfect interface between two crystalline semiconductors. The GaAs-(AlGa)As interface provides a much smoother background for the in-plane motion of the electrons. Furthermore, the binding electric field is not established by an external voltage, as in the MOSFET but is generated internally through positively charged centers within the (AlGa)As. Shortly after the discovery of the quantized Hall effect in Si-MOSFET, the same phenomenon was observed in GaAs-(AlGa)As structures by Tsui and Gossard (*5*) (see Fig. 3). The effect could be observed at higher temperatures and lower magnetic fields, making the experimental requirements less stringent than in the case of the Si-MOSFET. Hence a good fraction of present studies are performed on GaAs-(AlGa)As materials.

Apart from high-precision measurements of R_H, a considerable amount of experimental and theoretical work (*6–14*) has been devoted to unraveling the physical principles underlying the phenomenon. The most startling recent observation concerns the resistivity of the specimen under study. It is found that, under conditions where the Hall resistance is quantized to any of its values $R_H = h/ie^2$, the resistivity ρ_{xx} (see Fig. 2) of the two-dimensional electron gas appears to

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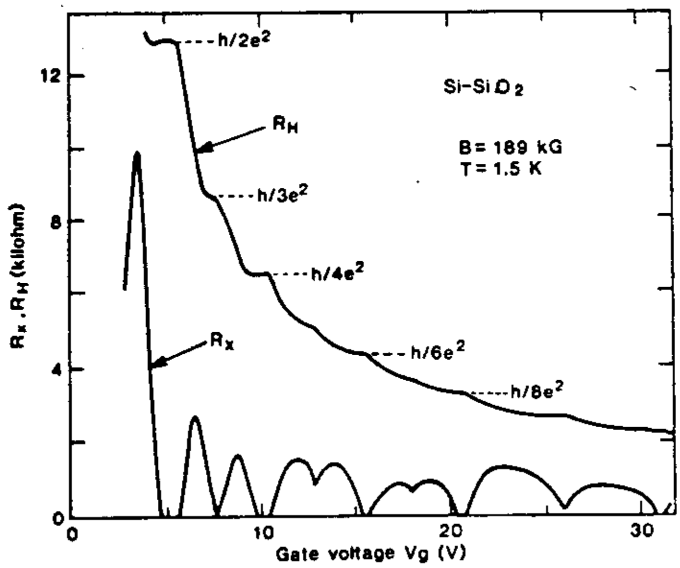
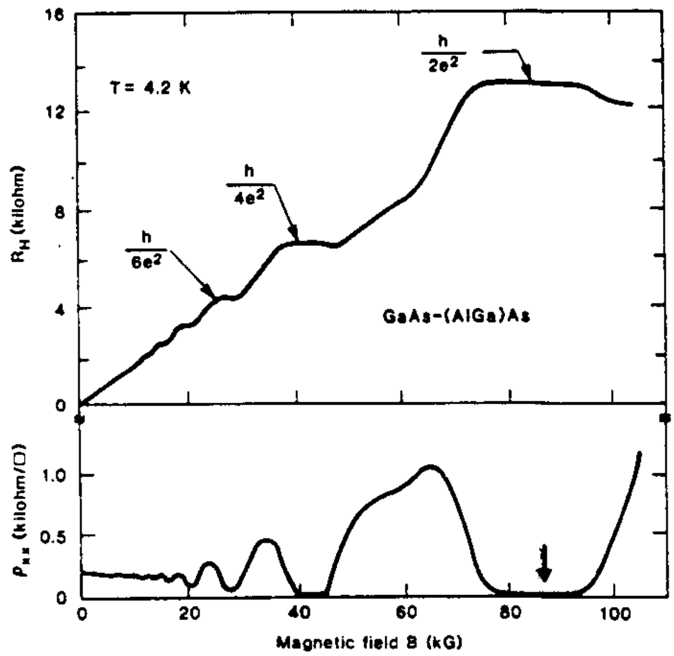
vanish as the temperature is lowered. Two-dimensional resistivities as low as $< 5 \times 10^{-7}$ ohm per square, equivalent to three-dimensional resistivities of $< 5 \times 10^{-13}$ ohm-centimeter have been reported at 1.23 K (7). This value is almost ten times lower than the resistivity of any nonsuperconducting material at any temperature, and it drops further when the temperature is reduced. Extrapolation to zero temperature indicates that a two-dimensional electron gas in a suitably high magnetic field is resistanceless; it represents an ideal electrical conductor, very similar to a superconductor, yet the phenomenon is caused by a completely different mechanism.

The beauty of the quantized Hall effect is that it represents the observation of such a fundamental relation in a field of physics seemingly so well understood. With a superficial glance at the system and a minimal understanding of two-dimensional transport, one may at first think the effect falls right into place. But there are many hidden complications that turn the quantized Hall effect into a scientific puzzle. Indeed, the recent discovery of fractional quantization with $i = 1/3$ and $2/3$ defies theoretical explanation. In order to transmit some of the flavor of this puzzle, we start with a classical description to clarify some aspects of the problem and then discuss the effect in the case of integral quantization.

The Classical Two-Dimensional Electron Gas

The motion of electrons in crossed magnetic and electric fields evades intuition. In order to illustrate some of their properties, we consider the geometry shown in Fig. 4. The motion of the electrons is restricted to the x-y plane without friction or scattering. In the absence of an electric or magnetic field, all electrons move in straight lines in the plane, and since the direction and the speed are completely random, there is no net electrical current. This monotonous situation changes the moment an electric field E is applied to the system. If E is pointed in the negative x direction in the plane, each electron will accelerate in the positive x direction and, in the ideal frictionless case, will speed up indefinitely. In real systems, electrons are scattered by imperfections or vibrations of the atoms, leading to a motion analogous to that of a particle in a viscous fluid. After a very short initial acceleration time, the system approaches a steady state with a constant drift velocity, v_D, which for small E is proportional to E. The resulting current density is given by $j = env_D$, where n is the average number of electrons per unit area. In macroscopic terms, the relation between E and j is characterized by either the





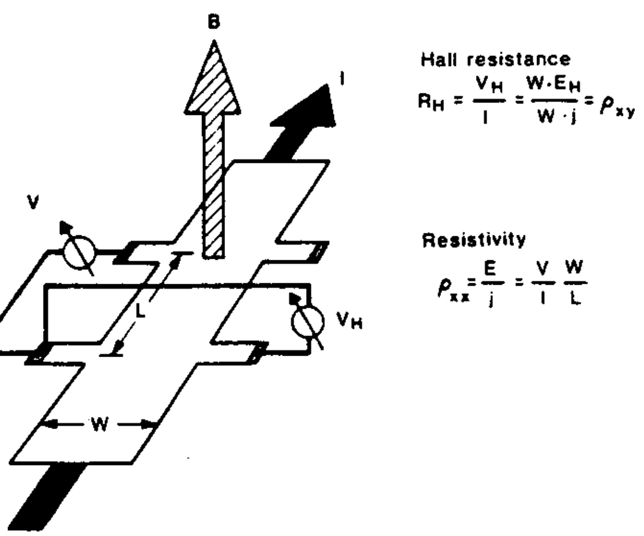


Fig. 1 (top left). The quantized Hall effect in a Si-MOSFET (1) in which the electron density is varied by a gate voltage V_g. Instead of being a smooth curve, the Hall resistance R_H develops plateaus having values h/ie^2, where i is an integer, and the resistance R_x of the specimen drops to very low values. Fig. 2 (bottom left). Schematic representation of a Hall experiment. The magnetic field B is perpendicular to the plane of the specimen and to the current I. The Hall resistance R_H and the resistivity ρ_{xx} are determined through the equations shown in the figure. Fig. 3 (above). The quantized Hall effect in GaAs-(AlGa)As heterojunctions (5). The electron density is fixed and the magnetic field is swept to exhibit the effect. At the arrow at 84 kG, ρ_{xx} is $< 5 \times 10^{-7}$ ohm/□.

conductivity σ or the resistivity ρ of the system through $j = \sigma E$ and $\rho = 1/\sigma$. The transport coefficient σ is given by $\sigma = env_D/E = en\mu$, where the mobility, $\mu = v_D/E$, describes the degree to which the carriers are able to move through the system.

Adding a magnetic field B to the system changes the situation considerably. In the absence of E, a B field in the z direction—that is, perpendicular to the electron plane—exerts a constant force $F = evB$ on an electron traveling with speed v. The direction of this Lorentz force is perpendicular to the direction of motion of the electron and perpendicular to the direction of B. As a result, the electron executes a rotating motion in the plane on a circle with a radius $r = mv/eB$, where m is its mass, and with a frequency (the cyclotron frequency) $\omega_c = v/r = eB/m$. Since the B field does not change the speed of the electrons, their energy remains independent of B and can be expressed as $\epsilon = 1/2 m\omega_c^2 r^2$. Thus, an ideal two-dimensional electron system in a magnetic field can be visualized as a system of electrons rotating with a constant frequency ω_c around the field lines on circles having radii proportional to the speed of the electrons (Fig. 5).

The addition of an electric field E affects the electron system quite differently than in the absence of B. Instead of drifting along the x direction, the carriers move in the direction that is perpendicular to the E and B fields—the y direction (Fig. 6). Each electron keeps rotating while the center of its rotation is drifting aside. This, again, is a result of the Lorentz force: the E field accelerates the electron in the x direction while the B field deflects the motion into the y direction. In contrast to the ideal frictionless case in the absence of B, the carriers are not accelerated indefinitely. The centers of their orbits move with a constant velocity $v_B = E/B$ parallel to the y direction. Therefore, the entire electron system drifts aside with a constant velocity in the direction perpendicular to E and B, representing a constant current in the y direction. The current density is given by $j = env_B = enE/B$. However, the current and the electric field are not parallel, as in the absence of a magnetic field, but perpendicular to each other. The current parallel to the E field is zero.

A description of this behavior in macroscopic terms, which requires two independent transport coefficients, leads to some surprising results. The conductivity, σ_{xx}, describing the current density along the electric field is zero. However,

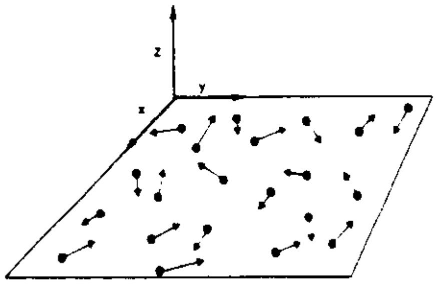


Fig. 4. Schematic of an ideal two-dimensional electron system where the electrons with random speed and direction are confined to move in the x-y plane.

the resistivity, ρ_{xx}, defining the electric field strength along the current path also vanishes, since there is no E-field component along the current. We encounter an exceptional situation where the conductivity and the resistivity vanish simultaneously. This striking result is induced by the magnetic field, which diverts the current from the direction of the applied electric field. In other words, the current and the electric field are mutually orthogonal and the conduction is free from dissipation. The Hall conductivity, σ_{xy}, and the Hall resistivity, ρ_{xy}, relating E and j through $j = \sigma_{xy}E$ or $E = \rho_{xy}j$, are given by

$$\sigma_{xy} = \frac{ne}{B} \tag{3}$$

and

$$\rho_{xy} = \frac{B}{ne} \tag{4}$$

We note that in two dimensions R_H is identical to ρ_{xy} and $R_H = \rho_{xy} = B/ne$ (see Fig. 2).

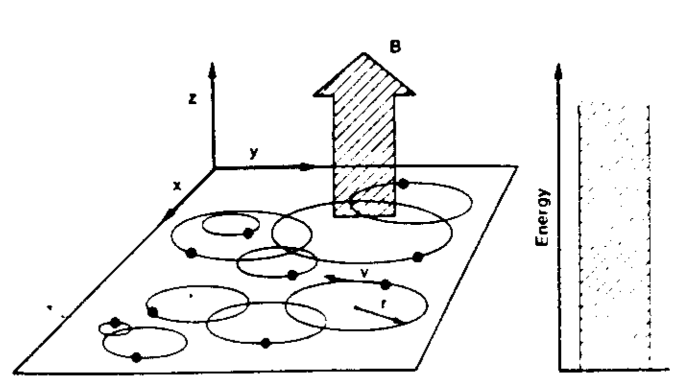
The Hall conductivity σ_{xy} and resistivity ρ_{xy} are unusual in that they relate current in one direction with an electric field pointing perpendicular to it. The usual parallel conductivity σ_{xx} and resistivity ρ_{xx} vanish completely. This last fact has important conceptual consequences. If we consider the x direction alone our system is an insulator since, in spite of the application of an electric field, there is no current flow along this direction. On the other hand, if we consider the y direction, the system should be termed an ideal conductor. Although a constant current is flowing, no electric field along this direction is necessary to support it and consequently, as in a superconductor, no dissipation of electric power accompanies the steady current. However, one cannot be too surprised about this result since the ideal model system excluded any kind of friction. Even in the absence of a magnetic field, such a system would appear to be without dissipation. In real systems, where electrons scatter at vibrating atoms or imperfections of the material, leading to a finite amount of friction, the values for σ_{xx} and ρ_{xx} generally deviate from zero. However, we will find that this ideal case can be realized in real two-dimensional systems under certain conditions.

Introduction of Some Quantum Mechanics

The inhibition of electron scattering in a real two-dimensional system in a high magnetic field is due to principles that are beyond our classical description. They require the introduction of some fundamental rules of quantum mechanics. The laws of quantum mechanics will not only bring about vanishing resistance of a real two-dimensional system in a magnetic field, but will also be responsible for the discontinuous behavior of the transport coefficients σ_{xy} and ρ_{xy} that led to the observation of the quantized Hall resistance and, in turn, to the high-precision determination of the fine structure constant. The following paragraphs introduce the quantum mechanical rules that are pertinent to the problem of electrons in a magnetic field.

The fundamental difference between a quantum mechanical and a classical treatment of an electron in a magnetic field is that only a discrete set of orbits is

Fig. 5. Classical motion of a two-dimensional electron system with a magnetic field (B) normal to the plane. The energy of the carriers is unaffected by the field and remains $\epsilon = 1/2\ mv^2$; hence all energies are possible.



accessible to the electron in the former case. Electrons can occupy only discrete states with well-defined discrete energies. The allowed radii for electron orbits in a magnetic field are the so-called Landau radii

$$r_l = \left[\frac{2\hbar}{eB}\,(l - 1/2)\right]^{1/2} \qquad (5)$$

where $\hbar$ is an abbreviation for $h/2\pi$ and the quantum number l can be any positive integer $l = 1,2,3 \ldots$ (see Fig. 7). Since their orbits are quantized, the energies of the electrons form a sequence of Landau levels, given by

$$\epsilon_l = 1/2 m\omega_c^2 r_l^2 = (l - 1/2)\hbar\omega_c \qquad (6)$$

where $\omega_c = eB/m$ is the cyclotron frequency.

Finally, electrons have to obey Pauli's exclusion principle that no two electrons can agree in all their quantum numbers. The exclusion principle in effect limits the number of electrons per unit area that can occupy each Landau level. This number is the degeneracy of each Landau level and is given by

$$s = \frac{eB}{h} \qquad (7)$$

At very low temperatures electrons will occupy the allowed states with the lowest energy. In a given magnetic field a system of two-dimensional electrons with density n will arrange itself in the following way. Of the n electrons per unit area, s will occupy the energetically lowest Landau level $l = 1$, each having an energy $\epsilon_1 = 1/2\hbar\omega_c$ and an orbit with a radius $r_1 = (\hbar/eB)^{1/2}$. The same number s will occupy the next higher level $l = 2$, having energies $\epsilon_2 = 3/2\hbar\omega_c$ and radius $r_2 = (3\hbar/eB)^{1/2}$. Loosely speaking, they form a second layer, although they actually reside within the same plane and "layer" is to be understood in energetic terms. All electrons can be accommodated by filling consecutive Landau levels. The last level generally will remain partially unoccupied since n generally is not an integral multiple of the degeneracy s. The Fermi energy (ϵ_F) is the energy of the last electron accommodated in the system at absolute zero temperature. It may be regarded as the energy that divides the filled and the empty levels of the system.

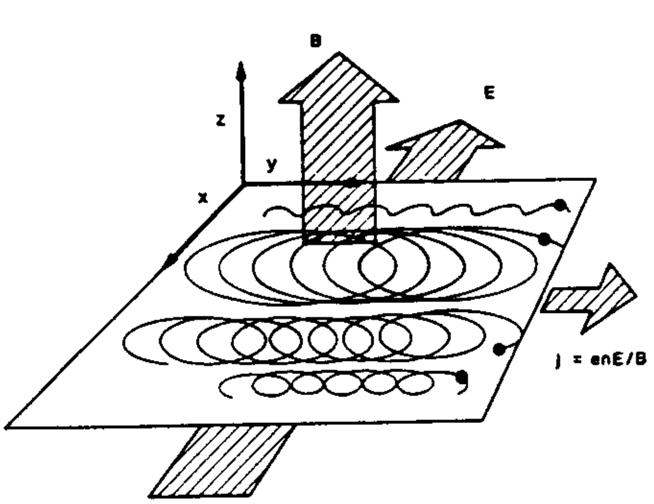
The important point to notice is that, in distributing the electrons over the levels, an abrupt break occurs whenever one Landau level is completely filled. This is due to the fact that an additional amount $\hbar\omega_c$ of energy is required to accommodate each electron in the next higher level. In the region where an integral number l of Landau levels is just filled ($n = l \cdot s = l\ eB/h$) a slight variation of n (or B) will drastically change the energy of the system. These jumps in energy, which do not occur in a classical treatment, have important consequences for the scattering of electrons in a real two-dimensional system, where a finite amount of scatterers is always present.

Electron Scattering in a Quantized System

An electron encountering any kind of a defect center will be scattered out of its orbit (initial state) into a new orbit (final state) and may lose or gain energy in the process. Such a scattering event can occur only if empty orbits are available for the electron to be scattered into. In our quantized two-dimensional system, since electrons can only assume discrete energies $\epsilon_l = (l - 1/2)\hbar\omega_c$, energetic exchange between scatterers and electrons is limited to multiples of $\hbar\omega_c$ (for inelastic events) or 0 (for elastic events). At low temperatures and high magnetic fields, when the Landau level splitting $\hbar\omega_c$ vastly exceeds all thermal energies, only elastic events, scattering of electrons among orbits within the same Landau level, are feasible. The scattering is therefore limited by the number of empty orbits within the same Landau level.

Total suppression of scattering occurs when all orbits of the occupied Landau levels are completely filled and all higher Landau levels are completely empty—that is, when the Fermi energy resides somewhere within the gap between two subsequent Landau levels. In this case no scattering can take place, since the empty orbits in the higher Landau levels are inaccessible to electrons in the completely filled Landau levels. Therefore, the complete occupation of an integral number of Landau levels leads to vanishing electrical resistance. We emphasize that the realization of this zero resistance does not require the absence of scatterers within the two-dimensional system, it requires the absence of possibilities for the electrons to scatter. In this way, the real two-dimensional system mimics the ideal model system, creating a state with vanishing resistance, $\rho_{xx} = 0$, in spite of the existence of scatterers.

In certain respects this zero-resistance state is similar to superconductivity. In both cases it is the existence of a finite gap in the energy spectrum, with all states below the gap occupied and all states above the gap unoccupied, which leads to vanishing electrical resistance. Nevertheless, the gaps are of very different origin, and various properties, like magnetic field exclusion in superconductors and the existence of the quantized Hall effect in two-dimensional systems, are not common to both phenomena.



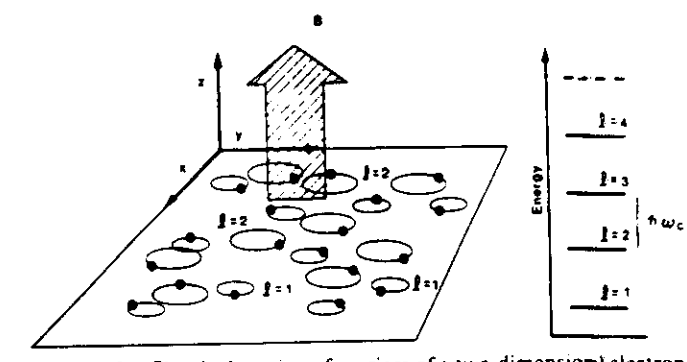


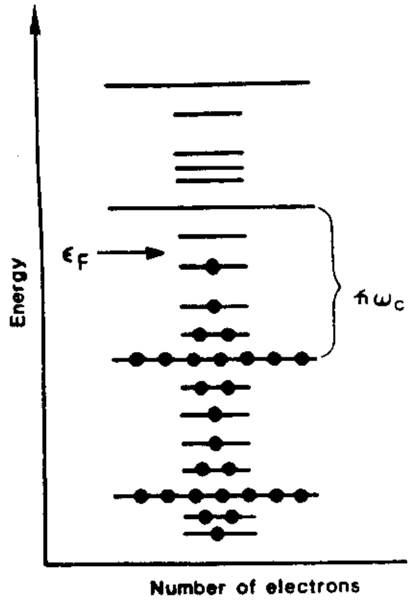
Fig. 6 (left). Classical motion of carriers of a two-dimensional electron system in crossed magnetic (B) and electric (E) fields. While rotating, the carriers drift into the direction normal to B and normal to E, giving rise to a current density $j = enE/B$ in the y direction. Fig. 7 (right). Quantum mechanical motion of carriers of a two-dimensional electron system in a perpendicular magnetic field B. Only certain orbits and their energies are allowed, as given in Eqs. 5 and 6 in the text. (Radii with $l > 2$ are omitted for clarity.)

Integral Quantization of the Hall Resistance

We have seen that, because of the discrete nature of its quantum mechanical energy spectrum, a real two-dimensional electron system can behave as our ideal classical model system. It can carry an electrical current without dissipation (that is, $\rho_{xx} = 0$) when an integral number, i, of Landau levels are completely filled. Under this condition, the total density of electrons has to remain $n = is = i\ eB/h$, where $s = eB/h$ is the degeneracy of each Landau level. The Hall resistance R_H, which in two dimensions is the same as the Hall resistivity ρ_{xy}, is then given by $R_H = \rho_{xy} = h/ie^2$, exactly as observed experimentally by Von Klitzing *et al.* (*1*) and Tsui *et al.* (*2*).

However, our discussion, which is based on a perfect two-dimensional system, also precludes the experimental observation of this quantum phenomenon. It does not provide the means to keep the filled Landau levels completely occupied for an extended range of either the electron density or the magnetic field, which is necessary for an experimental observation in the form of Hall plateaus (see Figs. 1 and 3). Interestingly, the existence of imperfections in the samples is essential for the observation of the quantized Hall effect.

Imperfections in the two-dimensional system give rise to the states that can trap electrons. The trapped electrons do not contribute to the electrical current and are referred to as localized electrons, setting them apart from the current-carrying delocalized electrons. Depending on the strength of the localizing potential, the energies of localized electrons deviate more or less from the quantized energies of the delocalized electrons and consequently are found somewhere within the gap region between the Landau levels. The modified energy spectrum of a real two-dimensional system therefore consists of Landau levels representing the delocalized orbits and a broad distribution of localized orbits filling the gaps in between (see Fig. 8). The existence of localized orbits buffers the abrupt jumps of the Fermi level from one Landau level to the next which would occur in the absence of these gap states. As long as a variation in density or in magnetic field adds electrons to or subtracts electrons from consecutive localized orbits, the Fermi energy resides within the gap region between Landau levels and the number of delocalized orbits remains unaltered for an extended range of electron density or magnetic field. Since only delocalized orbits contribute to the electric current, the transport properties of the system remain constant as long as E_F resides in the gap, leading to the occurrence of plateaus in ρ_{xy} and zero in ρ_{xx}. In this sense it is the imperfection of a real two-dimensional system which brings about the plateaus as a signature of the quantized Hall effect.



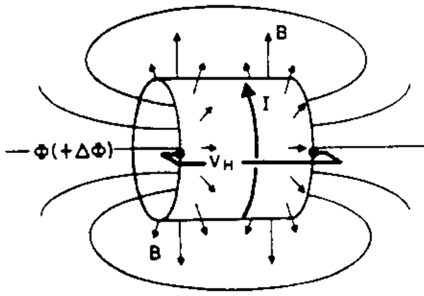


Fig. 8 (left). Energy spectrum of a two-dimensional electron system in a perpendicular magnetic field including electron localization. The energies of localized orbits (on short lines) are found in the gaps between Landau levels (long lines). The Fermi energy ϵ_F resides between Landau levels. Fig. 9 (right). Geometry for Laughlin's gedankenexperiment. The two-dimensional electron system forms a cylinder. A strong magnetic field B pierces it everywhere normal to its surface. A current I circles the loop, giving rise to a Hall voltage V_H and a small magnetic flux Φ along the axis of the cylinder.

However, this scenario is unable to account for the high accuracy to which the values of the plateaus are quantized. Since a given fraction of the carriers are localized, the density of delocalized electrons is diminished accordingly and the number of current-carrying electrons in each Landau level deviates considerably from its ideal value of $s = eB/h$. Hence ρ_{xy} is expected to deviate accordingly from its quantized value $\rho_{xy} = h/ie^2$.

Some light was shed on this puzzling situation by Prange (*8*) and Aoki and Ando (*9*), who calculated the current of an ideal two-dimensional system in a magnetic field containing an isolated scatterer which traps one electron, thereby removing it from the current-carrying electrons. They obtained the startling result that the remaining electrons make up in current for the localized electron, which they skirt, by increasing their own velocity. The situation is analogous to the flow of an incompressible fluid circumventing an obstacle and increasing its speed at the position of the bottleneck in order to keep the current constant (*15*).

Laughlin's Explanation

A very elegant gedankenexperiment by Laughlin (*10*), which was extended by Halperin (*11*), treats the quantized Hall effect from a very general point of view and arrives at the correct answer, independent of the physical details of the system. Their gedankenexperiment requires the notion of gauge invariance, a physical symmetry beyond common intuition, and we will outline the basic ideas underlying their arguments.

Laughlin based his consideration on an unusual, but feasible, geometry (see Fig. 9). The two-dimensional electron system is bent to form a cylinder whose surface is pierced everywhere by a strong magnetic field B normal to the surface. An arbitrary current I is assumed to circle the loop. As described earlier, the action of the magnetic field on the charged carriers gives rise to a voltage V_H perpendicular to the current—that is, from one edge of the cylinder to the other. As a result of this circulating current, a small magnetic field threads the current loop, giving rise to a magnetic flux Φ through the cylinder. The aim of the gedankenexperiment is to establish the relation between I and V_H.

To determine I, we use an electromagnetic equation

$$I = \frac{\delta U}{\delta \Phi} \qquad (8)$$

which relates I to the total energy, U, of the electronic system, which is free of dissipation, and the magnetic flux, Φ, piercing the current loop. The value of I can then be established by a slight variation, $\delta\Phi$, of the magnetic flux and simultaneous determination of the change in the total electronic energy, δU, of this system. The carriers are separated into two distinct classes: localized electrons, which are excluded from the transport of current, and delocalized electrons, which encompass the loop. The two groups react quite differently to $\delta\Phi$. Localized electrons remain totally unaffected, as one would expect, since there is no change in magnetic field at their positions and they do not enclose any fraction of Φ. Delocalized electrons, which enclose Φ, experience the flux change and generally do change their energy.

Since $\delta\Phi$ is too small to transfer electrons between Landau levels, its only effect is to move the electron orbits of the same Landau level within the surface of the cylinder. Any motion in the direction of the external electric field E established by the potential drop V_H will modify the electron energy by some amount δU. To determine the actual value of δU, Laughlin noticed that after the magnetic flux Φ is varied by a finite, though exceedingly small, amount of a flux quantum, $\Delta\Phi = h/e$, all electron orbits of the system are identical to those before the flux quantum is added. The distribution of electrons among the orbits might have changed during the process—for instance, electrons might have moved into other orbits, leaving empty orbits behind, or several electrons might have exchanged positions. Nevertheless, the orbits available to the carriers before the flux change are identical. For the general case of an arbitrary magnetic field, the change in orbit occupation is unknown and the evaluation of ΔU infeasible.

However, an exceptional situation develops when the Fermi level, ϵ_F, resides within the unaffected localized states. In this case, all delocalized orbits of all Landau levels below ϵ_F are completely filled, and excitation into a next higher Landau level is impossible because of the large amount of energy, $\hbar\omega_c$, required for such a transition. Since all accessible delocalized orbits were occupied before the addition of $\Delta\Phi$, all accessible delocalized orbits are occupied after the addition of $\Delta\Phi$, and all orbits before and after the change coincide, the total energy U of the system has to remain unchanged and $\Delta U = 0$. However, since one is unable to trace the motion of the electrons during the flux increase, one has to allow for the possibility that an integral number of electrons were transferred through the system during the flux change, entering the cylinder at one edge and leaving it at the opposite edge, without knowing their actual path. This electron transfer is the only way in which the highly degenerate two-dimensional electron system can vary its electronic energy. Moving from one edge of the cylinder to the other through the electrostatic potential V_H, an electron changes its energy by eV_H. If i electrons are transferred, the total change of the electronic energy is $\Delta U = ieV_H$, $i = 0,1,2,3, \ldots$. Returning to Eq. 5 and replacing the infinitesimal quantities by their finite equivalents, we find the current to be

$$I = \frac{\Delta U}{\Delta\Phi} = \frac{ieV_H}{e/h} = \frac{ie^2}{h}V_H \qquad (9)$$

and the Hall resistance, $R_H = V_H/I$, given by Eq. 1. Halperin (*11*) later identified the value of i as the number of occupied Landau levels.

The preceding discussion represents the present understanding of the origin of the quantized Hall effect. It shows that the existence of localized states is essential for the experimental observation. It is remarkable that a high-precision measurement should require the physical system to be imperfect, that the accuracy of quantum electrodynamics can be tested by an experiment resting on the localized states in a disordered system, and that the absence of electrical resistivity can be a consequence of the existence of imperfections.

Fractional Quantization

Very recently, investigations of GaAs-(AlGa)As heterostructures in magnetic fields as high as 200 kG and temperatures as low as 0.5 K revealed new surprises. In the so-called extreme quantum limit, when only the lowest Landau level is partially occupied, the quantum phenomena discussed above should not be present. Nevertheless, it has been discovered (*16*) that ρ_{xx} vanishes and ρ_{xy} is quantized in units of h/e^2 when occupation of the lowest Landau level is 1/3 and 2/3. This fractional quantization of the Hall resistance—that is, $R_H = h/ie^2$ with i = 1/3 and 2/3—differs from the integral quantization in that it is observable at lower temperatures and higher magnetic fields and is more pronounced in samples with higher electron mobility. These features suggest that the effect is more fundamental, and the search for an explanation of it is currently an active area of solid-state research. In short, the puzzle of the quantized Hall effect has not yet been entirely put together.

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THE FRACTIONAL QUANTUM HALL EFFECT

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Recent research has uncovered a fascinating quantum liquid made up solely of electrons confined to a plane surface. Found only at temperatures near absolute zero and in extremely strong magnetic fields, this liquid can flow without friction. The excited states of this liquid consist of peculiar particle-like objects which carry an exact fraction of an electron charge. Called quasiparticles, these excitations can themselves condense into new liquid states. Each such liquid is characterized by a fractional quantum number which is directly observable in a simple electrical measurement. This article attempts to convey the qualitative essence of this still-unfolding phenomenon, known as the Fractional Quantum Hall Effect.

I. INTRODUCTION

The collective behavior of the many atoms or molecules in a macroscopic system is a fundamental issue in modern physics. The periodic solid and the shapeless liquid are *condensed* forms of matter, distinguished from the gaseous state by the strong collective interactions of their constituent atoms. Of particular interest are those condensed systems whose macroscopic behavior is dominated by the laws of quantum mechanics. Such systems, in which the quantum uncertainty in the positions of the constituent particles exceeds their separation, often exhibit bizarre properties. Superconductors are notable examples; these materials can carry electrical current without any dissipation of energy. Less well known are the superfluids which exhibit frictionless flow and other peculiar properties like quantum whirlpools. These unusual effects are examples of *macroscopic* quantum phenomena, belying the notion that quantum mechanics concerns only the atomic world.

Physical systems of reduced dimensionality, in which the particles are confined to a plane or line rather than occupying three-dimensional space, have recently become subjects of intense scrutiny. Most often these systems are artificially fabricated from semiconductor crystals. While their great interest lies partly in real and potential electronics applications, they are equally fascinating from the pure physics point of view. Beyond providing an ideal testing ground for modern theories of condensed systems, these man-made structures have revealed totally new physical phenomena. Preeminent among these is the fractional quantum Hall effect; a macroscopic quantum phenomenon due to the condensation of a collection of electrons into a bizarre fluid state.

A two-dimensional system of electrons is surely one of the simplest many-particle systems imaginable. Add a magnetic field to it and a fascinating microcosm unfolds. An electron ***quantum liquid,*** unlike any other existing liquid, is created. Near absolute zero this liquid flows without dissipation, circumventing obstacles in the plane. Simple electrical measurements reveal the so-called Hall resistance to be *quantized* to exact rational multiples of a universal constant. A slight increase in temperature creates peculiar particle-like objects in the liquid that carry a precise fraction of the charge of an electron. These *quasiparticles* themselves can condense into liquid states, leading to a hierarchy of parent and daughter fluids. Very recently the *spin* of the electron has been found to further enrich the spectrum of phenomena. On the horizon lies the possibility that these strange liquids can somehow *freeze* into electron solids with new properties as yet unseen.

The first glimpse of this intriguing microscopic world was provided by the discovery[1] of the fractional quantum Hall effect in 1982. Since then much progress has been made toward a theoretical understanding of the underlying physics and many new experimental observations have been made.[2] Our article attempts to convey the qualitative essence of this new many-particle phenomenon and to highlight those aspects which remain enigmatic.

II. PRELIMINARIES

There are no truly two-dimensional systems in nature. Like a game of billiards however, some are well approximated by a 2D model. In the same way we can construct only approximately 2D systems of electrons. Nowadays the best such construction confines a pool of electrons to the interface between two ideally matched semiconductor crystals. It is a fascinating reality, due to quantum mechanics, that if a perfect crystal could be grown, without impurities or defects, an electron could move through it without resistance at zero temperature. Its wave-like nature allows it to flow through the crystal lattice of atoms without collisions. The same is true at the interface between two crystals, provided they are perfectly matched. The best such systems are fabricated from the semiconductors gallium arsenide (GaAs) and gallium aluminum arsenide (GaAlAs) which are grown in thin layers atop a suitable substrate. Excess electrons, donated by remote impurities, find their way to the interface and are bound there by the different chemical nature of the two semiconductors. After donating their electrons, the impurity atoms are left positively charged, the net charge of the sample thus being zero. Typical samples contain some 10^{11} electrons per cm^2, corresponding to a mean spacing of a few hundred angstroms. The interfacial binding does not restrict the electrons from moving in the plane. In fact, at present the best such samples[3] allow electrons to move the huge distance of about 0.1 mm in the 2D plane, passing some 400,000 atoms without suffering a severe collision. Such freedom is only obtained at temperatures near absolute zero where the crystalline vibrations - really a type of "imperfection" - are minimized.

The simplest way to probe the properties of any system of freely moving electrons is to measure their electrical properties. These so-called transport measurements have provided essentially all that we know about the fractional quantum Hall effect. To make such measurements a small "chip" of the layered semiconductor sample, typically a few millimeters on a side, is processed so that the region containing the 2D electrons has a well-defined geometry. The frequently used "Hall bar" geometry is depicted in Fig. 1. A tiny electrical current is driven along the central section of the bar, while the various side arms serve as probes to measure the induced voltages. Two probe configurations are important: the longitudinal voltage difference V between two probes on the *same* side of the central bar, and the so-called Hall voltage V_H between probes situated opposite one another *across* the bar. We usually convert these voltages into resistances by dividing by the current running down the bar. The longitudinal resistance R has the same significance as one's conventional notion of the electrical resistance of an ordinary material. Its magnitude is a measure of the frequency of collisions suffered by the electrons. The Hall resistance R_H however, is different.

Discovered 120 years ago, the Hall resistance is one of the most frequently measured quantities in solid state physics. R_H is zero in the absence of a magnetic field. When a field is applied perpendicular to the 2D plane the magnetic force causes the moving charges to accumulate at one side of the bar. This continues until the electric field that results from the charge separation exactly cancels the magnetic force. A classical analysis yields the simple result:

$$R_H=B/Ne \qquad (1)$$

where B is the magnetic field, N the number of charges per unit area in the plane, and e the charge of an electron. Thus, a Hall measurement establishes N, the carrier concentration. Only a decade ago this simple result was expected to remain valid in very high magnetic fields and at the lowest temperatures. Figure 2 shows both the Hall resistance R_H and the longitudinal resistance R in a 2D electron sample as functions of magnetic field. Obtaining such data is difficult; not only does the sample represent state-of-the-art crystal growth, but the magnetic fields (up to 30 Tesla = 300,000 Gauss) and temperatures (often as low as 0.02 Kelvin = 20mK) are extreme. The diagonal dotted line represents the simple result expected from Eq. 1. Obviously, 2D electrons in high magnetic fields were not at all understood 10 years ago.

There are two astonishing aspects to Fig. 2. While oscillations in the longitudinal resistance R were anticipated, that it would fall essentially to *zero* over wide ranges of magnetic field was totally unexpected. The second aspect, perhaps even more amazing than the first, are the *plateaus* in the Hall resistance R_H. Close examination of the values of R_H at these plateaus reveals that all can be described by a *universal* formula:

$$R_H=(h/e^2)/(p/q) \tag{2}$$

This expression depends only on the ratio of fundamental constants; the Planck constant h and the electronic charge e. The numbers p and q are simply integers. These *quantized* values are totally independent of the sample specifics. The plateaus in R_H and zeros in R, known collectively as the quantum Hall effect, are clear signatures of hitherto unappreciated aspects of 2D electron systems.

The subset of plateaus for which the ratio p/q = *1,2,3...* is an integer were discovered[4] before the first fractional value p/q =1/3 was found. We now know that the two cases reflect very different physics. The integer case can be understood solely in terms of *individual* electrons in a magnetic field. The fractional p/q values are far more subtle, reflecting entirely new physics arising from the *collective* behavior of all the electrons.

The essential ingredient for understanding the integer quantum Hall effect (IQHE) has been known for more than 50 years. It is the quantization of the circular motion of a charged particle in the presence of a magnetic field. Classically, an electron moves in a circular orbit perpendicular to the magnetic field. Any radius is allowed, only the period of revolution is fixed by the magnetic field strength, B. Quantum mechanics however, demands discrete values of the radius, in the same way as it enforces discrete Bohr orbits on an atom. Like the Bohr atom, these discrete orbits correspond to discrete energy levels, here called Landau levels. The Landau levels are spaced equally by an amount called the cyclotron energy which is proportional to the magnetic field B. Thus, for a system of electrons confined to a 2D plane, the entire energy spectrum consists of a ladder of discrete Landau levels with wide *energy gaps* in between. For neither 3D nor 1D systems do similar gaps exist. These gaps are at the heart of the integer quantum Hall effect.

Each Landau level can accommodate a large number of electrons, all at the same energy, because it is possible to place the center of each orbit at many equivalent places in the 2D plane. Since the size of the orbits decrease with increasing magnetic field (radius about 80Å at 10 Tesla), this so-called "degeneracy" of the Landau levels increases with field. In fact, every Landau level can accommodate $D=eB/h$ electrons per unit sample area; about 2.4×10^{11} per cm^2 at 10 Tesla. This is already a remarkable result since it is independent of all sample parameters.

Figure 3 illustrates these concepts. Dividing the number of electrons per unit sample area, N, by the degeneracy D of the Landau levels, defines the "filling factor" $\nu=N/D$. This quantity tells how many Landau levels are occupied. At very high magnetic field the degeneracy D exceeds N, all electrons lie in the lowest Landau level and $\nu<1$. On reducing the field two things happen: The spacing between Landau levels, as well as their degeneracy D, decreases. A magnetic field B_1 is reached for which the lowest level is exactly filled, i.e $D=eB_1/h=N$ and the filling factor is $\nu=1$. Further reduction of the field forces some electrons up into the second Landau level. Eventually a field B_2 is reached where the two lowest levels are exactly filled, $D=eB_2/h=N/2$ and the filling factor is $\nu=2$. For any integer j there is a field $B_j=Nh/je$ at which the j lowest Landau levels are exactly filled and all higher levels are empty. Let us evaluate the Hall resistance R_H at these special magnetic fields. First, using Eq. 1 and the definition of D, we can express R_H in terms of the filling factor:

$$R_H=B/Ne=(hD/e)/Ne=(h/e^{2)}/\nu \qquad (3)$$

At the special fields B_j the filling factor ν equals the integer j giving:

$$R_H=h/je^2 \qquad (4)$$

These are exactly the integer quantum Hall plateaus! We can even understand the vanishing of the longitudinal resistance R at these fields. Zero resistance implies no energy dissipation. Dissipation only occurs if electrons can easily scatter into empty energy levels. At the special fields B_j the nearest empty states are at much higher energy across the Landau gap. At low temperatures these states can not be reached and thus dissipation can not occur.

Is this all there is to the IQHE? A moment's thought reveals a serious problem with this simple picture. Our solution only works for the precise field values B_j. How can R_H remain flat over wide stretches of magnetic field? This is a formidable question and its solution[5] represents the second fundamental ingredient of our understanding the IQHE. The missing element is the residual imperfection inherent in any real sample. There are always some impurities or defects remaining in the sample despite one's best efforts. These imperfections can trap some of the 2D electrons and prevent them from participating in the current flow. Slight departures of the magnetic field from the special values B_j merely changes the number of these trapped electrons but not the number of occupied Landau levels. This causes no change in the resistances R and R_H which reflect only the non-trapped, current-carrying electrons. Larger magnetic field shifts overwhelm the capacity of the traps and thereby change the number of occupied Landau levels and thus the resistances. One is led to a paradoxical truth: the existence of the plateaus requires imperfections in the sample while the *value* of R_H on the plateau is a universal constant. Were the sample truly perfect the plateaus would be absent and R_H would return to the straight classical line!

What about the fractional plateaus, which actually dominate Fig. 2? Are they explained by some simple extension of the above argument? The answer is an unequivocal "no". We have argued that the integer plateaus are the result of gaps in the energy spectrum. Since the *phenomena* of the fractional effect appear the same as the integer case, we are led to search for additional energy gaps. Considering each electron individually leads only to the Landau gaps associated with integer values of the filling factor ν; there are no gaps at *fractional* values of ν. The fractional quantum

Hall effect (FQHE) must result from some new collective state in which all electrons participate.

III. THE STANDARD MODEL

Any description of the collective motion of many particles has to take into account the forces acting between them. In the case of electrons, this is the familiar coulomb repulsion of like charges. The motion of each electron depends, through this force, on the motion of all other electrons, especially those nearby. Furthermore, as we are dealing with electrons interacting on a microscopic scale, the notions of classical physics are inadequate and the inherently probabilistic principles of quantum mechanics must be considered. The final result of applying these principles is a *wavefunction* ψ whose magnitude gives the probability for finding the electrons in any particular configuration. For the most prominent FQHE state, at filling factor $\nu=1/3$, a remarkably simple, and nearly exact wavefunction has been obtained. This ingenious result, due to R.B. Laughlin, provides the basis for the standard model of the FQHE. Our objective is to qualitatively illustrate this wavefunction and thereby the electronic configuration underlying the FQHE.

Even with Laughlin's wavefunction in hand, we are confronted with the difficulty of illustrating a function which depends on the positions of many particles with only a single picture. To accomplish this, we make the great simplification of imagining a "snapshot" in which all of the electrons in the sample, *save one*, are held in fixed positions. The remaining electron, which we henceforth call the "representative", will be described by a smooth landscape whose elevation denotes the probability for finding this electron at a given location. This picture is thus a mixture of classical and quantum concepts and is not strictly correct. In reality, all electrons should be treated equally. This means any electron could be chosen as the representative. It also must be kept in mind that the companion electrons are *not* fixed in position and our "snapshot" is merely one frame of a larger film. On average the total electron distribution is really completely uniform.

We begin our illustration by stepping back to the simplest situation, for which the system has only one electron. In this case our earlier description of quantized circular orbits should be valid, and we can assume the single electron lies in the lowest Landau energy level. Since we do not know where the electron is in the 2D plane, we cannot locate the center of its cyclotron orbit. The probability of finding the electron is then completely uniform across the 2D plane, just as it would be if there were no magnetic field at all. How then does the magnetic field influence the probability distribution?

Associated with the magnetic field are so-called "flux quanta". In some sense these are the quantum counterparts of the classical notion of magnetic flux lines. While classical physics insists these lines themselves have no reality, in the quantum world they are more tangible. In fact, the regular array of flux lines trapped in a superconductor has been observed by various techniques. As with electrons, quantum mechanics requires uncertainty in the position of the flux quanta. Thus, just as a uniform charge density can result from a collection of discrete electrons, so a uniform magnetic field derives from a collection of discrete flux quanta. The magnitude of the flux quantum $\Phi_0=h/e=4.1\times10^{-7}$ Gauss-cm^2 is tiny by ordinary standards. The earth's small magnetic field of 0.3 Gauss corresponds to almost 10^6 flux quanta per square centimeter. Far higher flux densities than this are required for observation of the FQHE.

These flux quanta associated with the magnetic field create tiny vortex-like dimples in the probability distribution of our representative electron. As depicted in Fig. 4a, at the center of these vortices the probability of finding the electron is zero. How can this distribution be regarded as uniform? The answer to this lies in the huge degeneracy of the Landau level that we have already encountered. There are many equivalent ways to distribute the vortices around in the 2D plane, Fig. 4a represents just a specific choice. On the average, the probability for finding the electron is again completely uniform. Only when additional electrons are added to the system is this indeterminacy in the vortex positions tempered. As we will see, the FQHE arises from an unusually strong correlation between the positions of the electrons and the vortices.

1) The Ground State at $\nu=1/3$

We now would like to add electrons to our system. These additional electrons will be placed in fixed positions and our probability distribution will be that of the original "representative" electron. Again, any electron could be chosen as the representative and our illustration can only be thought of as a snapshot which belies the continual state of motion of all the electrons. On adding the first of the "companion" electrons we immediately confront a basic tenet in quantum mechanics: the Pauli exclusion principle. This requires that no two electrons may reside at the same position. Thus, we must put this second electron in a position avoided by the representative. From Fig. 4a, we see the obvious place is directly on one of the vortices associated with a flux quantum. All subsequent companion electrons must be placed onto unoccupied vortices. We can keep adding electrons until all available vortices are occupied. This situation, shown in Fig. 4b is clearly special; it corresponds to complete filling, ν=1, of the Landau level. Attempts to add more electrons requires placing them in higher Landau levels, at enormous energetic cost. We have stressed the indeterminacy in the position of any given electron or vortex. We now have a special case, the fully-filled Landau level, in which every electron has a single vortex attached to it. This association is due entirely to the Pauli exclusion principle. For the FQHE the Landau level is only partially filled and there are more vortices than electrons. The Pauli principle does not require any specific distribution of the "extra" vortices. It is the repulsive interactions between the electrons, the heart of the FQHE, that creates a new, *correlated*, arrangement between all the vortices and the electrons.

To see these new correlations, we now decrease the number of electrons below the ν=1 condition. As depicted in Fig. 4c, there is now an excess of vortices over electrons. While the companion electrons must sit on vortices, due to the Pauli principle, there are many equivalent permutations of the electrons among the vortices. The unoccupied vortices represent random positions that the representative electron avoids, to no energetic advantage. A far preferrable arrangement is to place these empty vortices *onto the existing electrons*. Multiple vortices are larger than single ones and are therefore more strongly avoided by the representative. Since a companion electron sits at the center of each multiple vortex, the repulsive interactions with the representative are reduced, and along with it the total energy of the system.

A particularly favorable state is created when the number of flux quanta is a multiple of the number of electrons. Such a situation arises at filling factor $\nu=1/3$ where there are three flux quanta for each electron. In this commensurate state, each electron sits in a large dimple and the total energy is significantly reduced. The situation is illustrated in Fig. 4d for the representative electron. Such a representation continues to hold true in the actual many-particle state in which all electrons create 3-fold vortices about all companion electrons. Similarly favorable situations should exist at filling factor $\nu=1/5$, $1/7$, etc. As we will see, states in which an *even* number of quanta are bound to each electron are quantum mechanically not allowed.

All these $\nu=1/m$ FQHE states have a beautifully simple mathematical representation first proposed by R. B. Laughlin.[6] We denote the position (x_j,y_j) of each electron j in the 2D plane by a complex number $z_j=(x_j-iy_j)$. Then, aside from an unimportant factor, the many-particle wave function for n electrons can be written as a simple product over all differences between particle positions (z_j-z_k)

$$\psi(z_1,z_2,z_3...z_n) = (z_1-z_2)^m\times(z_1-z_3)^m\times(z_2-z_3)^m\times....\times(z_j-z_k)^m\times\cdots(z_{n-1}-z_n)^m. \quad (5)$$

The square of the wave function $|\psi|^2$ represents the probability of finding a configuration in which there is one electron at position z_1, another electron is at position z_2, a third electron at position z_3 and so forth.

This mathematical representation automatically obeys the Pauli principle. The probability of finding two electrons at the same site is zero since one of the factors on the right-hand side becomes zero. A more subtle property of ψ is that if any two electrons are interchanged (e.g. $z_2 \leftrightarrow z_3$), ψ will change its *sign* if m is an odd integer.

It will *not* change its sign if m is even. Quantum theory insists that if ψ is to describe electrons, then it must change its sign under particle exchange. Thus Laughlin's wave function can hold only for filling factors $\nu=1/m$ with m–*odd*. For the Laughlin ground states the distribution of electrons is optimally correlated, reducing the repulsive Coulomb interaction to a minimum. Addition or subtraction of a single electron or flux quantum disturbs this inherent order at a considerable energetic cost. For this reason states at $\nu=1/m$ are referred to as *condensed many-particle ground states*. Since the mutual electronic positions are not fixed as in a solid, but rather free like in a liquid, and since this freedom is of a quantum-mechanical, rather than a classical nature, the term *condensed quantum liquid* applies.

2) Quasiparticles

The Laughlin ground state is an accurate description of the FQHE state only at absolute zero temperature and at the exact magnetic field for $\nu=1/m$ filling. Departure from either condition results in the creation of defects, called quasiparticles, in the liquid state. Theory asserts these defects carry fractional charge.

The charge $-e$ of an electron is the fundamental quantum of electric charge. No particle carrying a fraction of $-e$, has ever been directly observed. Even the famous quarks of high-energy physics, which are held to carry fractional charge, have not been found in isolation. The notion of quasiparticles charged to an exact rational fraction of e is, at first sight, a puzzling implication of the theory of the FQHE.

What are these quasiparticles? To be certain, our electrons do not dissociate into 3, 5, or 7... identical pieces. Fractionally charged quasiparticles are a convenient theoretical concept. They describe the fact that this many-electron system is able to harbor defects which act as though they carry fractional charge. Removal and addition of charges to the *total* system, can only be performed in units of e. With the framework of illustrations developed in the last section, these quasiparticles can, in fact, be intuitively described.

Using again the $\nu=1/3$ state as a concrete example, we recall that at exactly 1/3 filling all particles are condensed into a highly correlated many-particle ground state. This ground state is a uniformly charged 2D electron liquid in which the negative charge of each electron exactly compensates for the charge depletion caused by the surrounding threefold vortex. A minute change in filling factor, slightly off $\nu=1/3$, is not expected to destroy this condensed phase. The quantum fluid instead tries to remain condensed by creating a few defects in its fabric. To visualize these defects, imagine the removal of an electron from the 1/3 state in Fig. 4d. This leaves behind a threefold vortex effectively carrying a charge of $+e$. In the absence of the electron, the three surplus flux quanta are no longer bound together and, therefore, are able to drift apart, each one of them dragging with it a vortex in the electron distribution. The charge deficiency in each vortex amounts then to exactly $+e/3$. These local depressions in the charge density are called quasi-holes. Similarly, one can imagine the absence of one flux quantum. This situation, while harder to visualize, corresponds to a negatively charged defect ($-e/3$) called a quasi-electron. A number of recent experiments[7] have suggested that these fractionally-charged quasiparticles may, in fact, be observable.

Existence of such quasiparticle defects in the parent quantum liquid disturbs the correlated motion of the condensed carriers. The introduction of each quasiparticle raises the energy of the system by a fixed amount. This finite energy threshold for the creation of quasiparticles represents the sought after gap in the energy spectrum of the quantum liquid.

The existence of mobile charged particles, a gap in the energy spectrum, and the presence of a small degree of imperfection, provides all the ingredients for the observation of a quantization in the Hall effect and vanishing longitudinal resistance, R. From the point of view of electrical transport, the condensed quantum liquid at exactly $\nu=1/m$ filling, separated by a gap from its excited states, resembles the completely filled Landau level. There the nearest excited states are across the large Landau gap. The inaccessibility of these states at low temperature explains the vanishing resistance R in the IQHE. By exact analogy, we now expect R to vanish at $\nu=1/m$ in the FQHE. The only difference is that much lower temperatures are required since the FQHE gaps are much smaller than the Landau gaps.

Slight variation of the filling factor from exactly $\nu=1/m$ creates quasiparticles. Again in analogy to the IQHE, these initial excitations are trapped by imperfections. Hence, we expect R to remain zero and the Hall resistance to remain at its $\nu=1/m$ value $R_H=h/\nu e^2=mh/e^2$. Thus, the *transport* features of the FQHE are analogous to the IQHE. The fundamental new physics in the FQHE is the creation of a many-particle ground state separated from its excitations by an energy gap. Without this no analogy could be made.

The magnitude of the energy gap is characteristic of each FQHE state. Apart from the condensation energy of the ground state, it is the single most important parameter of the quantum liquid and has been determined theoretically by a variety of different analytical and numerical schemes. This gap energy is also the quantity most accessible to experiment.[8] Raising the temperature at exact fractional filling creates equal numbers of quasi-electrons and quasi-holes. These thermally created quasiparticles enhance the electrical conductivity of the system. The temperature dependence of the conductivity provides a measure of the energy gap. For the $\nu=1/3$ liquid, the strongest and best understood of the FQHE states, the experimentally determined energy gap approaches the theoretically calculated value to within 20%. Considering the tremendous computational difficulties in deriving the theoretical gap value, this represents an astonishingly good agreement and a great success for the standard theoretical model of the FQHE.

3) The Hierarchy

Laughlin's wave function, together with fractionally charged quasiparticles, provides an explanation for the FQHE at filling factor $\nu=1/m$ with m an odd integer. A case can also be made for $\nu=(1-1/m)=2/3, 4/5, 6/7$... arguing that at such filling factors the Landau level is depleted by 1/3, 1/5, 1/7... and condensed states develop among the *holes* in the distribution. However, many of the pronounced FQHE states, such as $\nu=2/5, 3/5, 3/7, 4/7$... are not included. The prevailing theoretical model regards these states as *daughter states* derived from the fluids at $1/m$. How does this come about? As the filling factor deviates considerably from exactly $1/m$, a large number of free quasiparticles are created in the quantum-liquid. Being charged, these quasiparticles correlate their relative positions and try to stay optimally apart. At a critical density they themselves can condense into a correlated quantum liquid of *quasiparticles*. As an example, the FQHE at filling factor $\nu=2/5$ is the many-particle daughter state condensed from $-e/3$-charged quasi-electrons of the $\nu=1/3$ primitive state. The equivalent daughter state condensed from quasi-holes emerges at $\nu=2/7$. Since daughter liquids develop quasiparticles of their own, the theoretical argument can be continued *ad infinitum* if not terminated by the formation of a yet unobserved quantum crystal.

Haldane showed how to arrange the resulting quantum-fluids into a hierarchy[9] of exclusively odd-denominator fractions which defines their line of descent. Fig. 5 shows the first daughter states of the primitive $\nu=1/3$ Laughlin liquid, several of which are visible in the experimental data of Fig. 2. The hierarchical scheme of daughter states provides a rationale for the existence of FQHE features at filling factor $\nu=p/q$ and orders the sequence of their appearance. However, compared to the Laughlin liquids at filling factor $\nu=1/m$, very little is known about these higher-order many-particle states. The theoretical calculations rapidly become intractable as one progresses down the hierarchy and experimental data on the energy gaps of daughter liquids have begun to emerge only recently.

The standard model seems to have established a satisfactory interpretation for the origin of the FQHE. Condensed quantum liquids at fractional filling factors with excitation gaps for fractionally charged quasiparticles provide all the necessary ingredients for an explanation of the experimentally observed transport features. For the most prominent and best studied of the FQHE states, at $\nu=1/3$, good quantitative agreement has been reached between theory and experiment. It appears that the FQHE has basically been understood.

IV. EVENS AND SPIN

Perhaps the most obvious feature of the hierarchy is the odd-denominator rule. Stemming from the grand Pauli exclusion principle applied to the primitive Laughlin states at $\nu=1/3$, $1/5$..., this "rule" seemed almost a "law", which it is not. Despite rumblings about possible fractional states at $\nu=3/4$, $11/4$, $5/2$, and $9/4$, the widespread view was that these "bad actors" would evaporate under closer scrutiny with better samples. To most everyone's surprise and excitement, one of these fractions has survived the critical test: a plateau has recently[10] been clearly identified with Hall resistance $R_H=(h/e^2)/(5/2)$. Figure 6 shows the first solid evidence for the 5/2-state. These data were obtained at the very low temperature of 25mK, attesting to the fragility of the new liquid state. Since these data were published, better samples have shown the state much more clearly though it remains delicate.

This first even-denominator state not only represents an egregious failure of the hierarchical model, but goes to the very root of our picture of the FQHE. Since two levels are completely filled and one is 1/2-filled at $\nu=5/2$, this state is really a 1/2-state and we are led back to Laughlin's family of $\nu=1/m$ wavefunctions. All such wavefunctions with m an even integer were discarded for not changing sign under interchange of two electrons. While no one seriously doubted this fundamental law of physics, it was clear that fitting $\nu=5/2$ into the picture required major revision of the standard model. Despite great effort, we still lack a conclusive theoretical understanding of this new surprise from the 2D electron system.

Suggestions of how one might *in principle* construct a 1/2-state had been around for several years. The most obvious way was to imagine particles which, under interchange, required their wavefunction to *not* change sign. Such particles exist in nature, they are called bosons, a helium atom being a notable example. With such particles the Laughlin wavefunction would be valid only for m=2,4,..., leading to states at filling factors $\nu=1/2$, 1/4.... Electrons are not bosons however, and so this approach is not of much help. Another early suggestion[11] for creating even-denominator states turned on a property of electrons which we have so far ignored: the electron spin. Each electron behaves like a tiny bar magnet, which can point either "up" or "down". This intrinsic magnetism of the electron is called spin. In a magnetic field the spin prefers one of the two orientations, which we will call "up". It requires energy, called the Zeeman energy, to force the electron to point "down" against the magnetic field. This energy increases linearly with field. The spin acts to split each Landau level in two, with the Zeeman gap in between. While not nearly as large as the Landau gap, the spin gap doubles the number of integer Hall plateaus. Although not mentioned above, all the odd-integer plateaus (ν=1,3,5...) are due to the spin gap, while the even-integer plateaus (ν=2,4,6...) are due to the Landau gaps. But how does this help to explain an even-denominator fractional state which occurs in one or the other of the spin sublevels? Halperin[11] pointed out that if pairs of electrons with opposite spin could form, one could regard the composite objects as bosons, and even-denominators would follow.

The problem is that it costs energy to flip spins, and at high magnetic fields this was considered prohibitive. In Laughlin's original work the spins were all simply assumed to be "up", an excellent assumption for explaining an effect occurring at enormous magnetic fields. But today's samples are so pure that fractional states can be seen at very low magnetic fields; the $\nu=5/2$ discovery was made at only 5 Tesla, much lower than the old $\nu=1/3$ state. At such fields spin flips may be relevant. If reversed spins are important in forming the condensed state at 5/2-filling then the application of a second magnetic field, this one *parallel* to the 2D plane, should destroy the state. Why is this so? To good approximation, only the spin-flip energy is affected by such an in-plane field, and it is increased. Loosely speaking, in order for a liquid to form the electrons had to expend some condensation energy on flipping spins. Increasing that expense may eventually prevent the state from forming at all. Recent experiments[12] have established just such an effect, lending strong support to the spin reversal hypothesis.

There is no theoretical agreement on the electronic structure of the even-denominator state. One very elegant model[13] has been proposed, for which spin reversal is crucial, but it is not clear that it is a viable description of realistic 2D electron systems.[14] Some workers[15] have even argued that the spins are not reversed at all, which is hard to square with experiment. At present we are far from understanding this obvious inadequacy of the standard model.

One may also fairly ask: If spin is important at the top of the hierarchical pyramid, then what about further below? The answer to this is simply not known yet. Interesting effects have already been observed. Certain fractions, for example $\nu=8/5$, occur in two distinct hierarchical schemes. In one the 8/5-state has all its spins aligned with the magnetic field, while in the other scheme half the spins are reversed, and the net spin of the state is zero. Which state is lowest in energy? This depends on the magnetic field at which $\nu=8/5$ occurs. If the spin-reversed variant is lower in energy, then adding a parallel magnetic field will destabilize it, just as with the 5/2 state. Adding as large enough parallel field can result in the spin-aligned 8/5-state becoming lowest in energy. The system thus undergoes a *phase transition* between the two ground states. The latest experiments[16] have uncovered just such phenomena. While most believe the standard model to apply at the highest magnetic fields, at lower fields, where spin becomes important, the subject is far less settled. Still more suprises may be in the offing.

V. CONCLUSION

The present picture of the dynamics of 2D electrons in high magnetic fields is an intricate web of distinct quantum liquid states connected by strange quasiparticle excitations carrying fractional charge. While the dominance of the coulomb interaction was recognized early on, only a small subset of the observed FQHE states are understood in any detail. This has been highlighted by the recent discovery of an even-denominator state and its likely connection to the electron spin. As a consequence, considerable reworking of the hierarchical model is underway.

Essentially all we now know of the FQHE has been determined through one type of experiment: simple electrical conduction. While many other potential probes exist, they are only just beginning to be employed. Optical investigations, microwave absorption studies, tunnelling experiments, and thermodynamic measurements will all add significantly to our understanding of the collective states underlying the FQHE.

Having appreciated the dominance of correlations in many-electron systems in high magnetic-fields, we expect further manifestations of this phenomenon. In two dimensions, the hierarchy of liquid states should eventually terminate with the electrons *freezing* into a solid. Much interest surrounds this predicted transition but conclusive experiments have yet to be done. Multilayer 2D electron systems in which electrons are allowed to interact between planes will allow for novel electron configurations as yet unobserved. Even "old-fashioned" 3D electron systems are expected to reveal new classes of condensed states. Novel crystal growth techniques are beginning to achieve the dramatic reductions in impurity levels required for the observation of such states. Intense interest has been generated quite recently in *one–dimensional* electron systems. A fascinating quantization of the resistance, akin to the integer quantum Hall effect, has already been observed.[17] In retrospect, the diversity of phenomena observed or expected from a system of only electrons seems astonishing. The fractional quantum Hall effect is perhaps only one spectacular example.

It is a pleasure to thank our many colleagues for their frequent and extensive help to us in this subject: K. W. Baldwin, G. S. Boebinger, A. M. Chang, S. M. Girvin, A. C. Gossard, F. D. M. Haldane, B. I. Halperin, T. Kovacs, R. B. Laughlin, P. B. Littlewood, A. H. MacDonald, L. N. Pfeiffer, D. C. Tsui, K. W. West, and R. L.

Willett. From active experimental collaboration to clear explanation of theory, these friends helped make this paper possible.

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FIGURE CAPTIONS

Figure 1. A typical Hall bar sample. The structure is formed by chemically etching away unwanted material. The dotted line indicates the two-dimensional electron gas at the interface between gallium arsenide (GaAs) and aluminum gallium arsenide (AlGaAs). The magnetic field B and electrical current I are shown, as are the longitudinal and Hall voltages, V and V_H, respectively. The shaded regions at the ends of each arm of the bar are where electrical contact is made to the 2D electron gas.

Figure 2. Composite view showing the Hall resistance R_H and longitudinal resistance R of a 2D electron gas *vs*. magnetic field. The diagonal dashed line passing through the R_H trace represents the classically expected Hall resistance for this sample. For each of the plateaus in R_H there is an associated minimum in R. The numbers give the value of p/q determined from the value of R_H on the plateaus. While some of the p/q values are integers, the great majority are fractions. Note in particular the "1/3-state" at the far right. This most prominent example of the fractional quantum Hall effect exhibits a Hall plateau at $R_H=(h/e^2)/(1/3)=3h/e^2$.

Figure 3. Three lowest Landau levels, j=1,2,3, in a five-electron system. Each panel corresponds to a specific magnetic field, B. The number of available states within each level is indicated. In the right-hand panel the magnetic field is high enough so that all five electrons may reside in the lowest level. In the middle panel the field has been reduced to the value B_1 where the lowest level is completely occupied and all higher levels are empty. This corresponds to the filling fraction ν=1. In the left panel the field has been further reduced, forcing some electrons into the j=2 Landau level.

Figure 4. Illustration of the wave function for the fractional quantum Hall effect (FQHE).

a. "Snapshot" of the probability of finding a single representative electron in a two-dimensional (2D) system pierced by a magnetic field. The flux-quanta (arrows) create tiny vortex-like dimples. At their center the probability of finding the electron vanishes.

b. Additional electrons (spheres) can only be placed in the vortex centers. Only they are avoided by the representative electron and the Pauli Principle requires that no two electrons ever reside at the same position. When all vortices are populated, the Landau level is completely filled characterized by a filling factor $\nu=1$.

c. At lower electron density only a fraction of the vortices is populated and there are many equivalent permutations. The placement of the companion electrons on to the vortices is necessitated by the Pauli principle. The avoidance of arbitrary positions in the 2D plane not associated with a companion electron is an energetic waste. An energetically preferable state is obtained by placing the extra vortices onto existing electrons (see Fig. 4d).

d. Probability distribution for the representative electron in the $\nu=1/3$ Laughlin state. In this commensurate state each electron "binds" exactly three flux quanta. The resulting wide dimples formed in the distribution of the representative electron around each fixed companion reduces the repulsive interaction and lowers the total energy. In the real system in which *all* electrons are delocalized, each electron develops a threefold vortex around each of its companions.

Figure 5. The hierarchy of fractional states deriving from the primitive 1/3-state. The 2/5 and 2/7-states are the first *daughters* of the 1/3-state being formed from its quasi-electrons and quasi-holes respectively.

Figure 6. First observation of a fractional quantum Hall effect at an even-denominator fraction, $\nu = p/q = 5/2$. A weak plateau is just beginning to form at $R_H = (h/e^2)/(5/2)$ and a strong minimum is seen in the longitudinal resistance, R. This data was obtained at a temperature of only 25mK. The straight diagonal line gives the classically expected Hall resistance. The nearby integer quantum Hall states at $\nu=2$ and 3 are also shown. Since this observation, better samples have shown a much stronger 5/2-state.

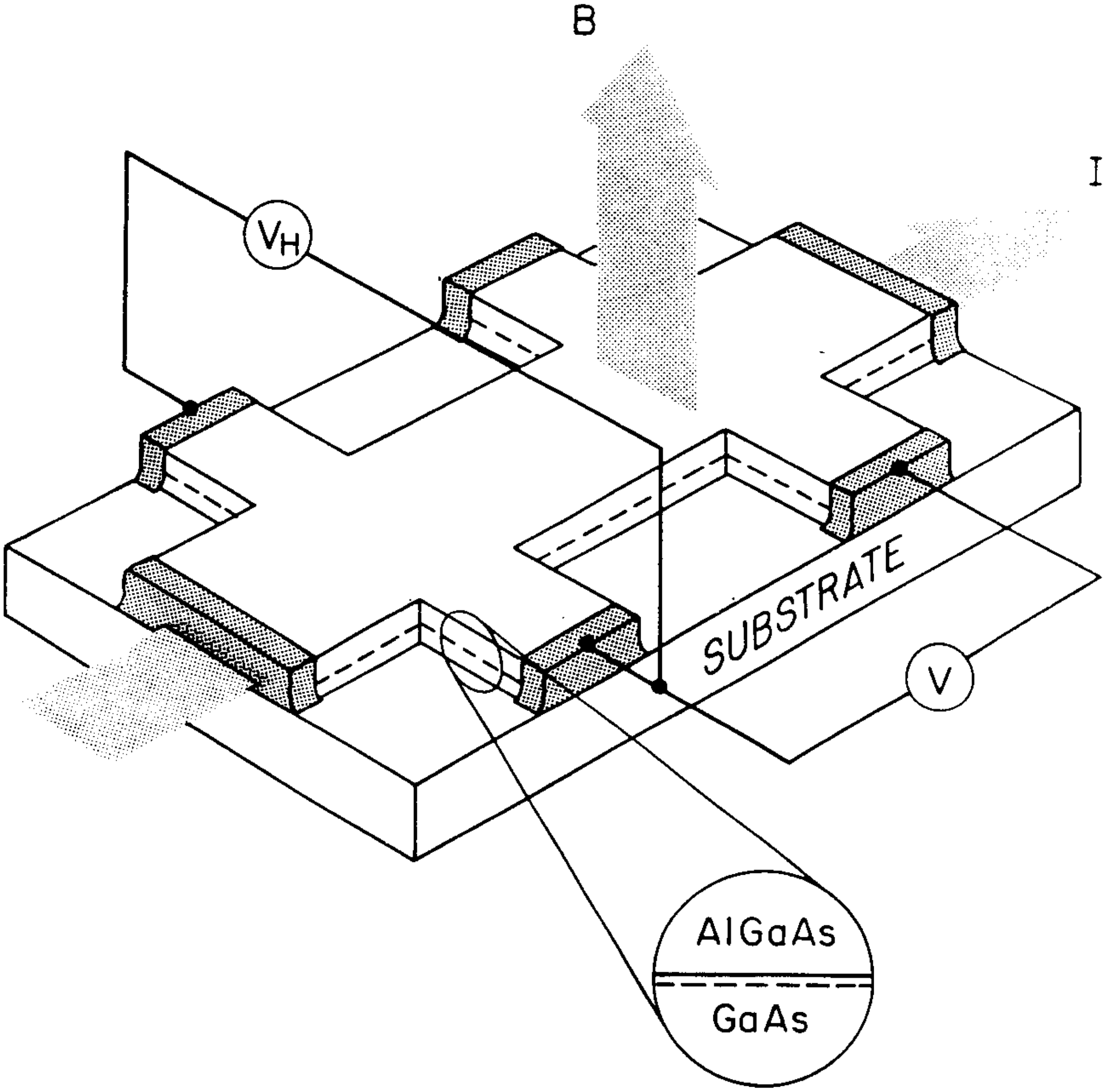
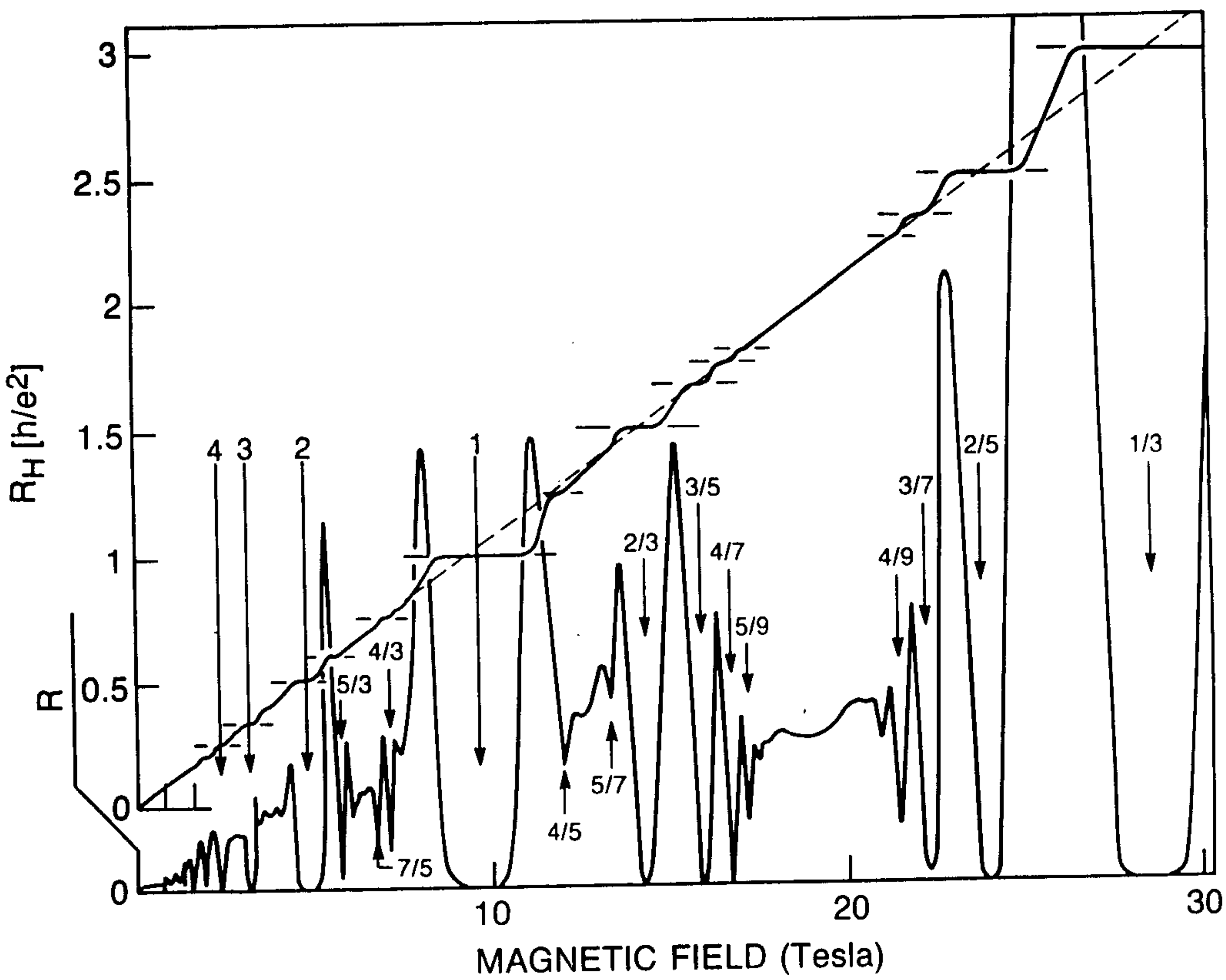


Fig. 1



R_H [h/e^2]
R
3
2.5
2
1.5
1
0.5
0
0
10
20
30
MAGNETIC FIELD (Tesla)
4
3
2
1
5/3
4/3
7/5
4/5
5/7
2/3
3/5
4/7
5/9
4/9
3/7
2/5
1/3

Fig. 2

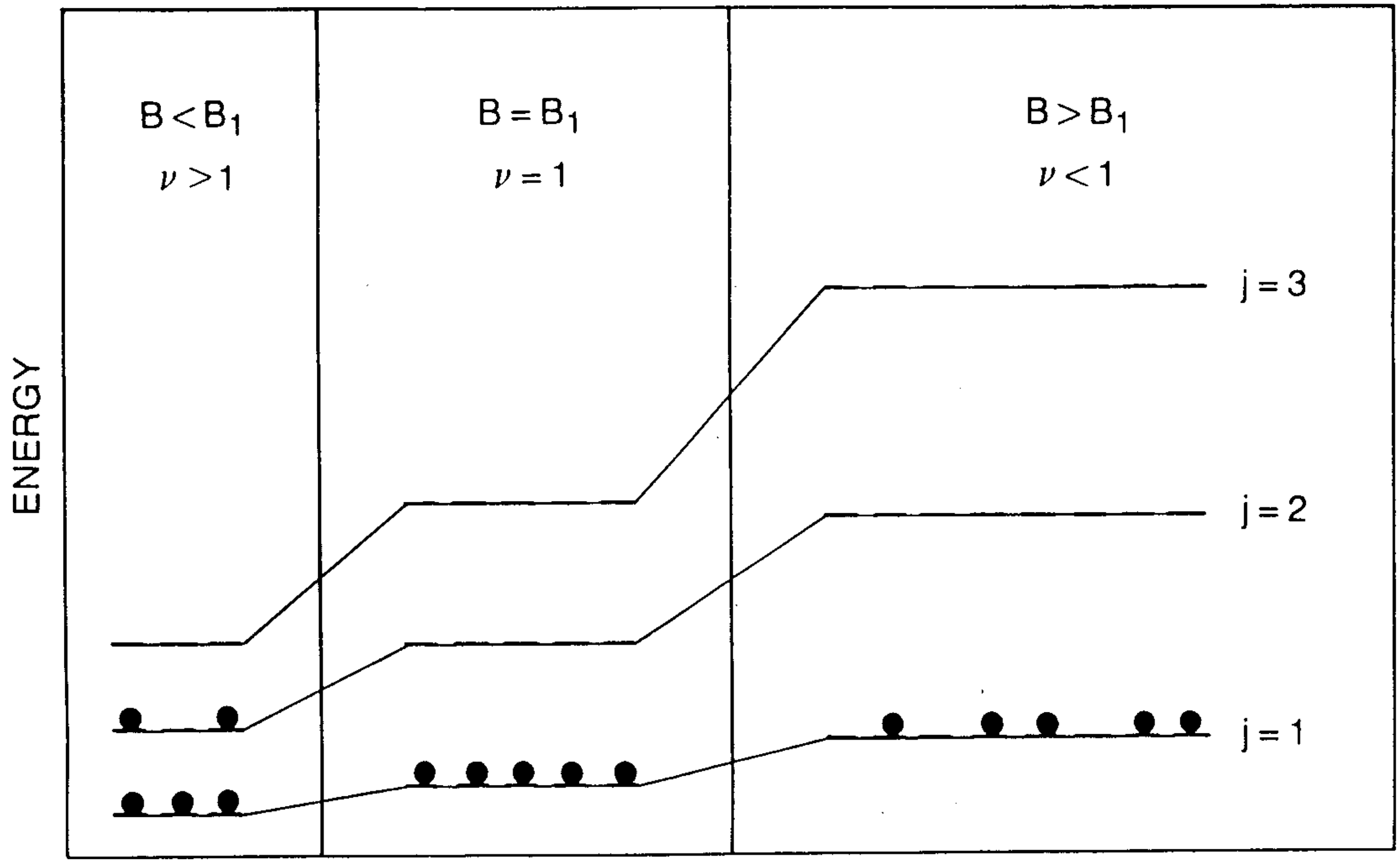


Fig. 3

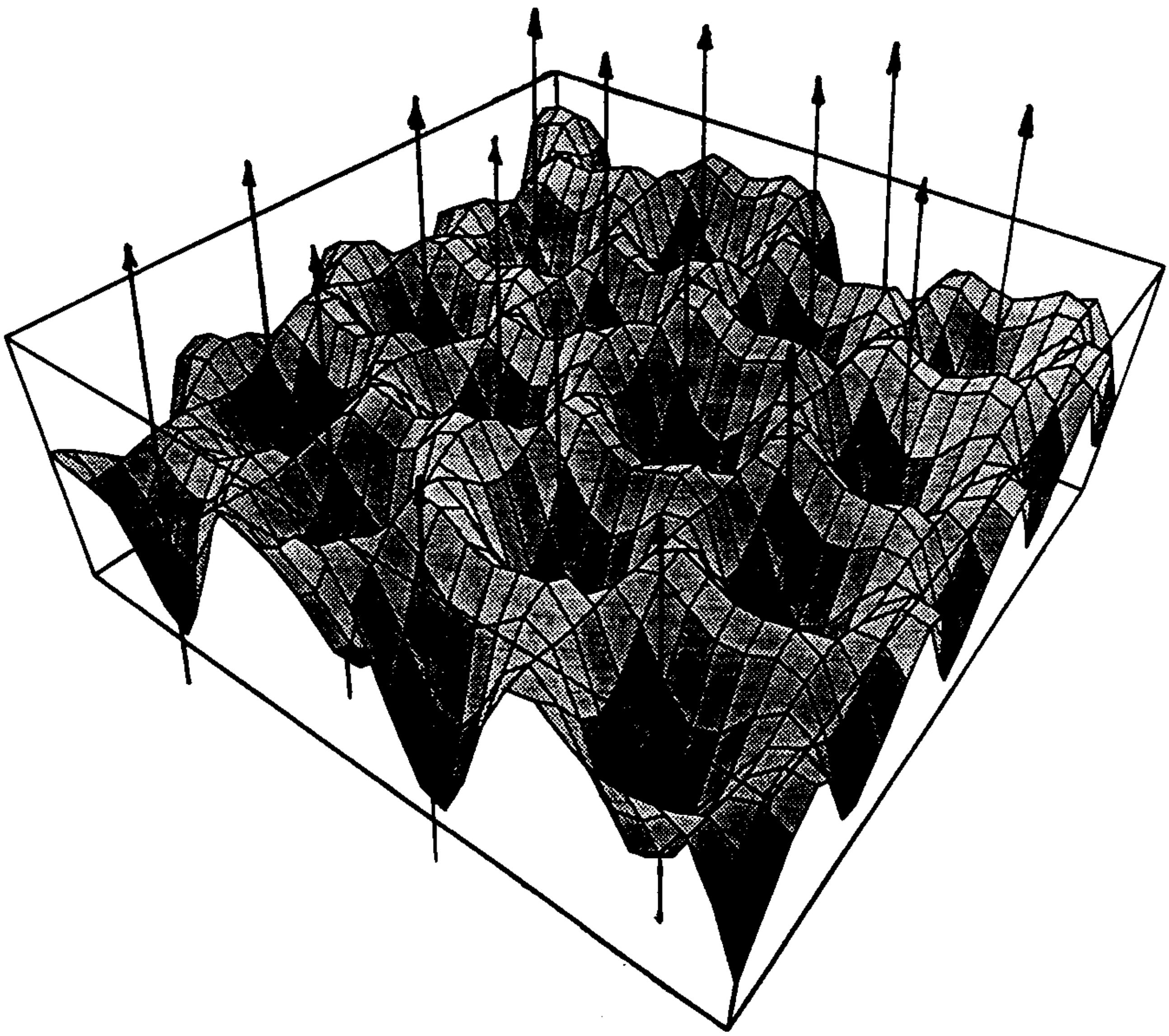


Fig. 4a

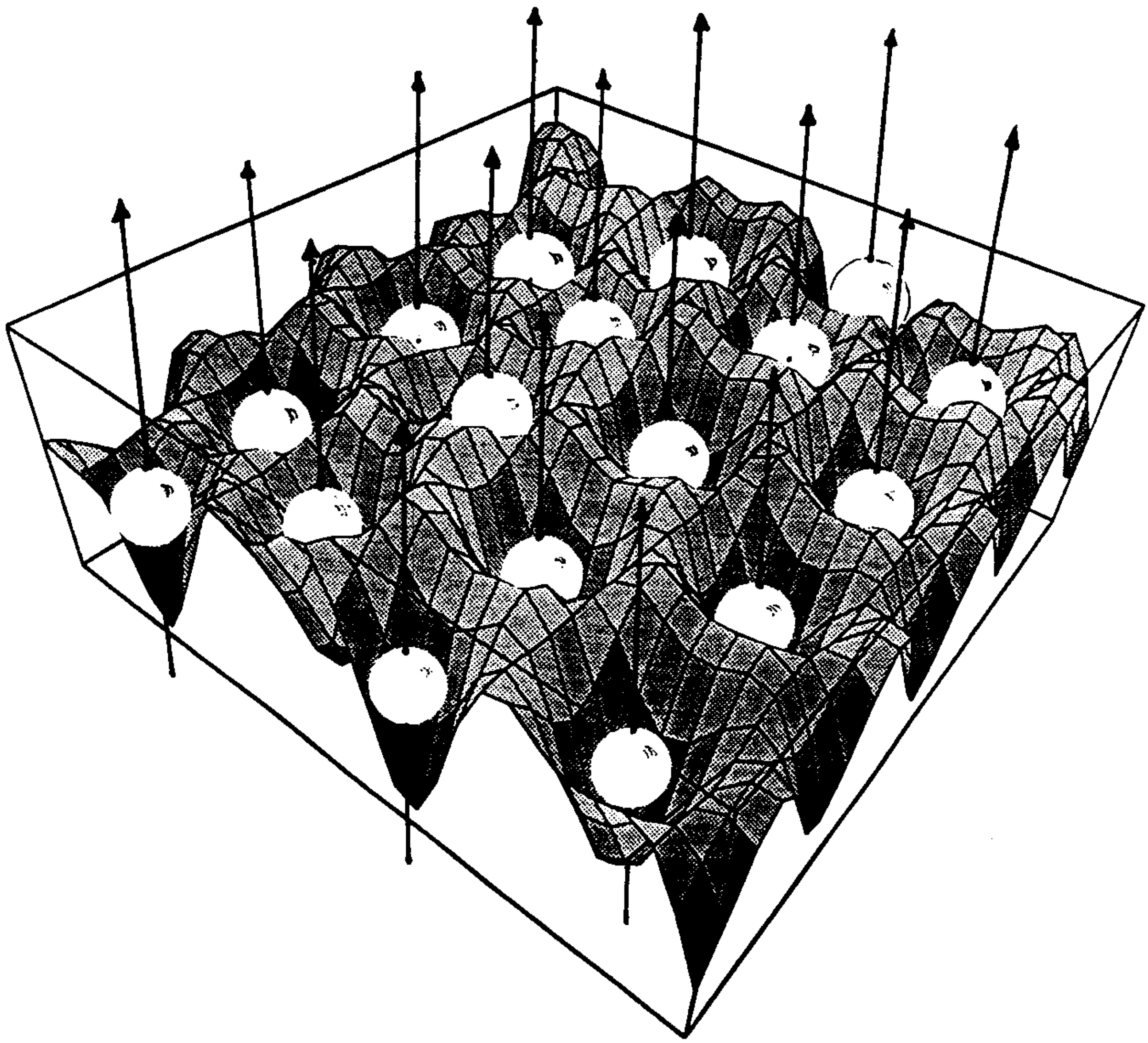


Fig. 4b

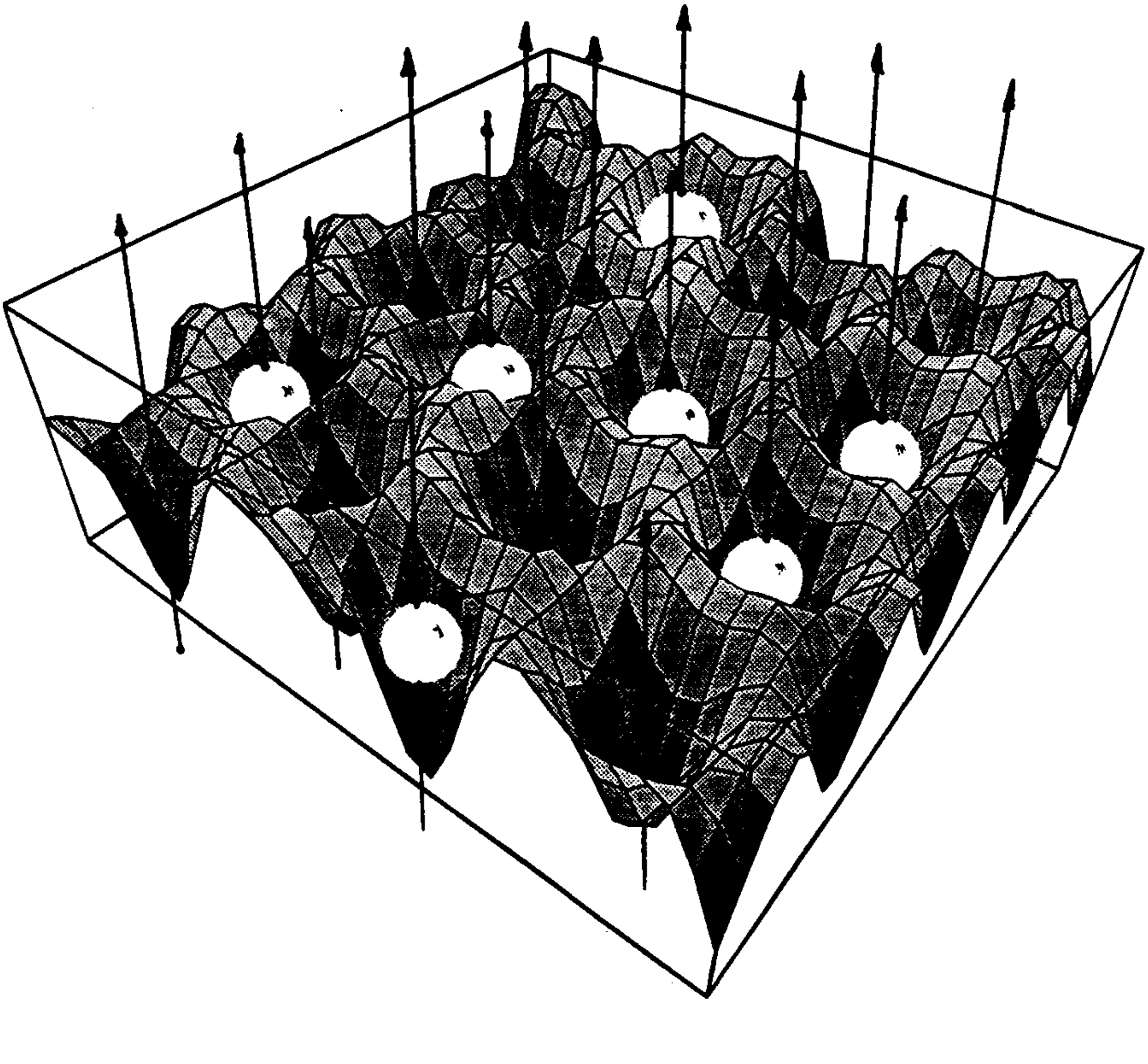


Fig. 4c

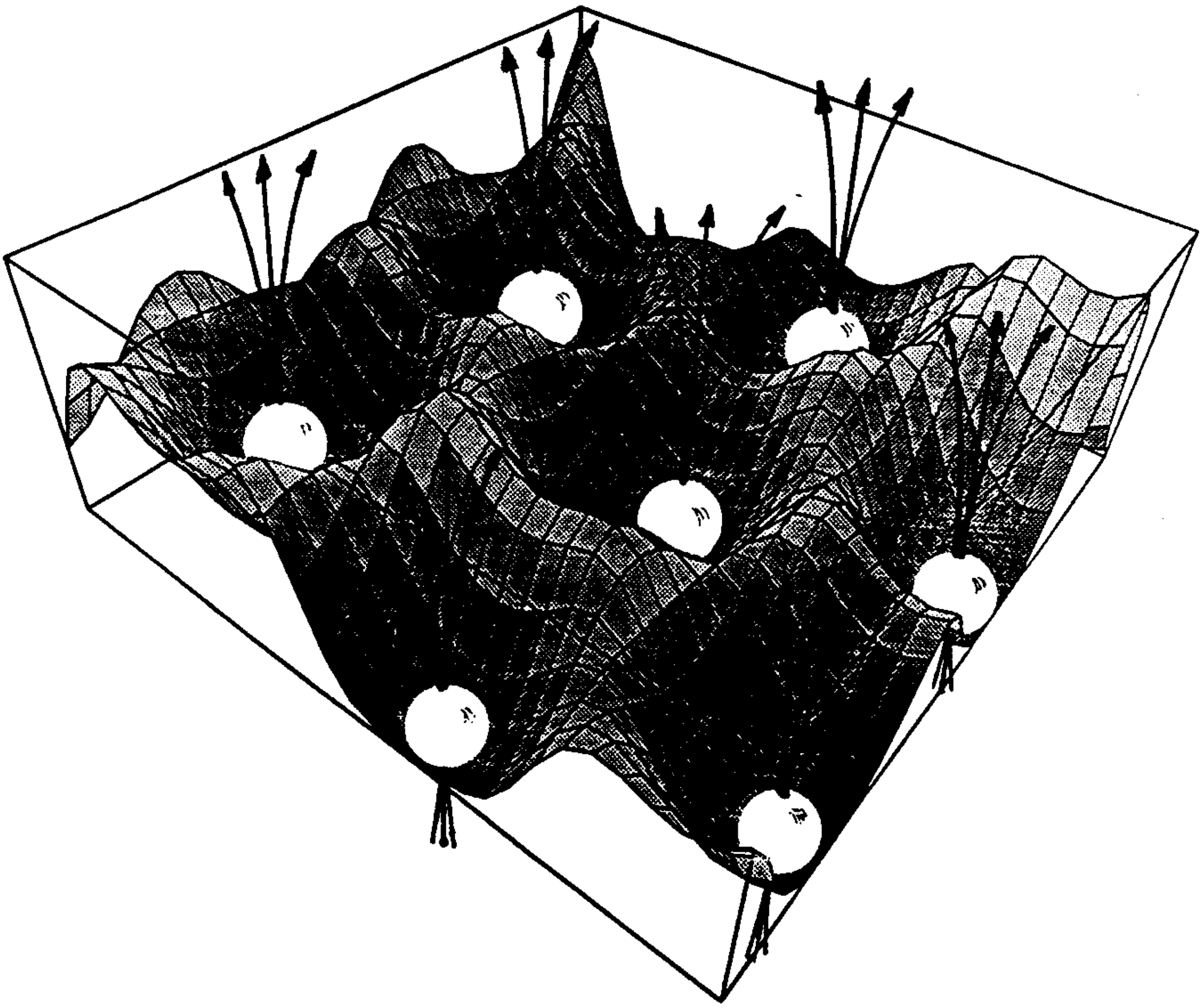


Fig. 4d

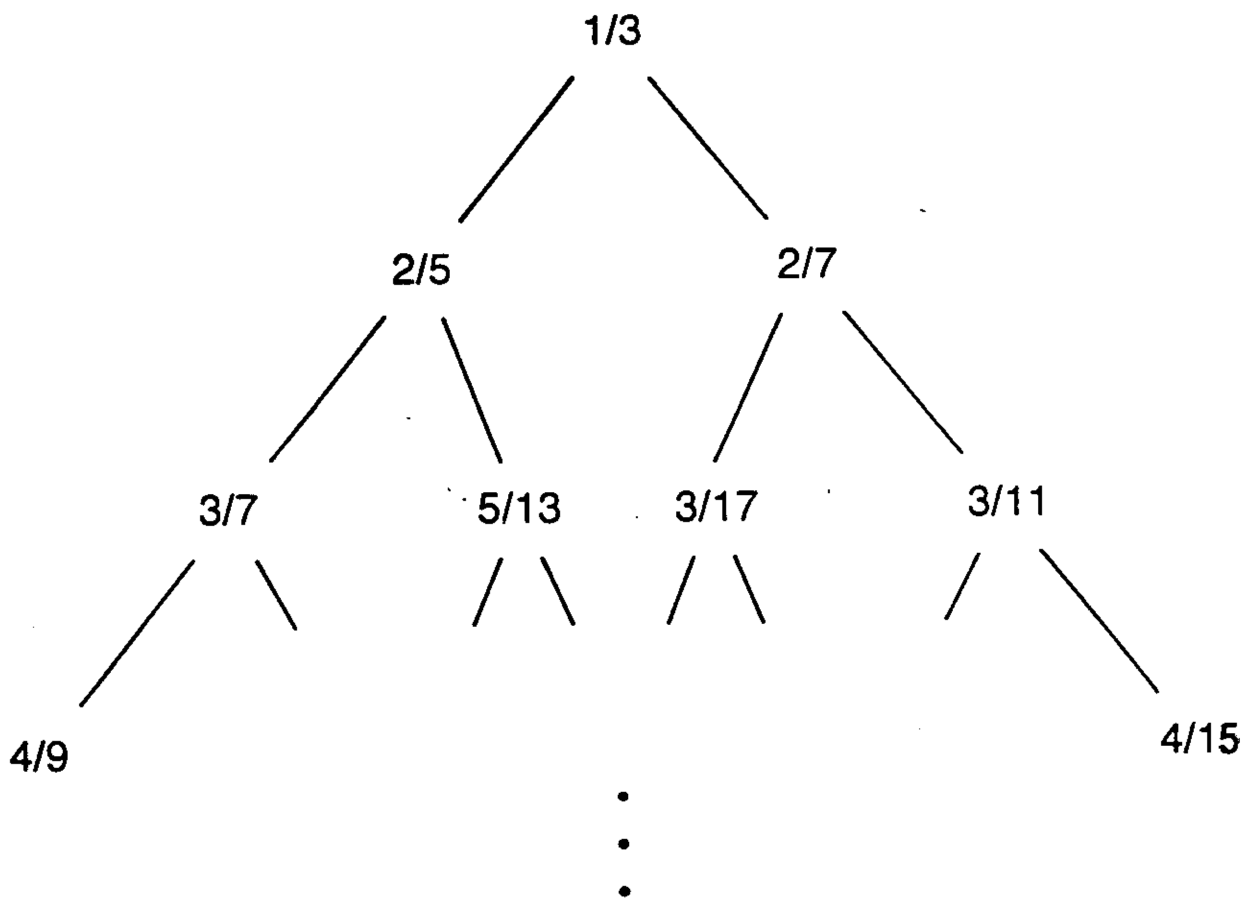


Fig. 5

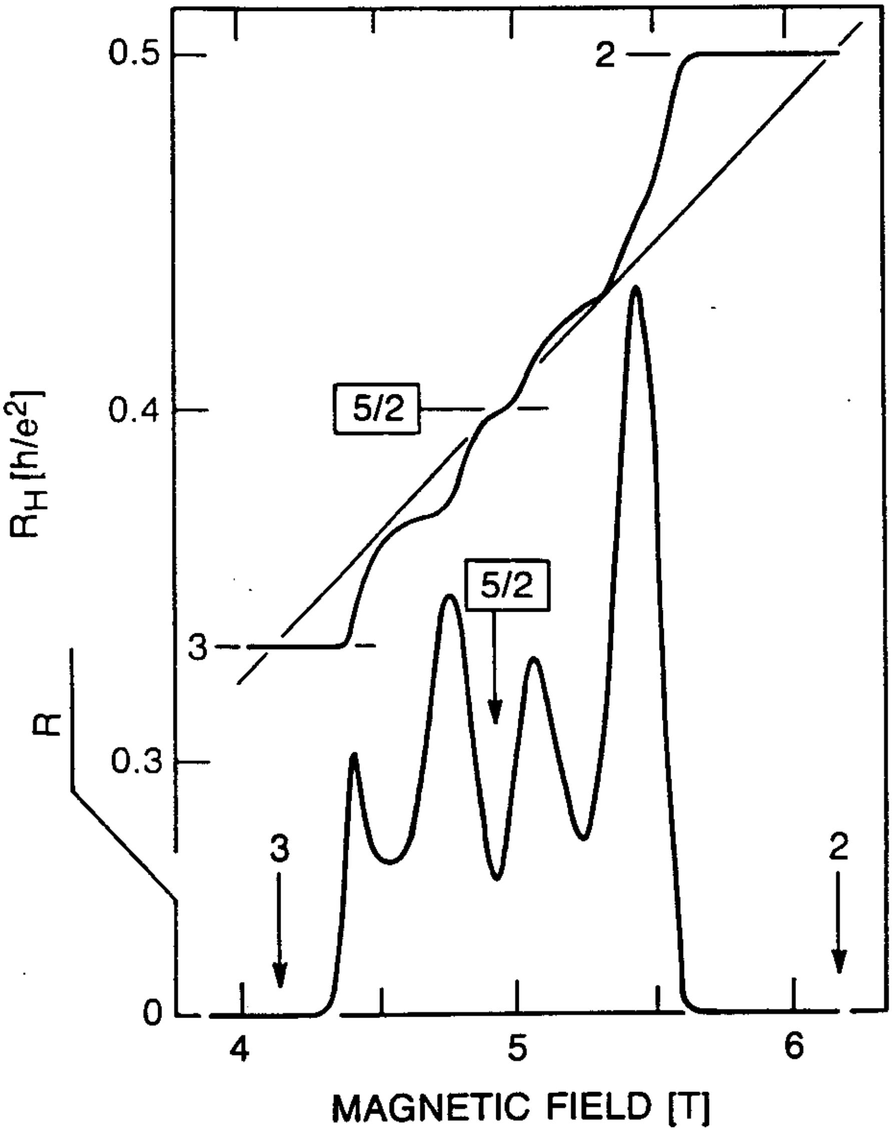


Fig. 6

QUANTIZED HALL EFFECT AND ZERO RESISTANCE STATE IN A THREE-DIMENSIONAL ELECTRON SYSTEM

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Quantization of the Hall effect and vanishing diagonal resistivity are observed in a GaAs-(AlGa)As superlattice which, in the absence of a magnetic field, conducts in all three spatial dimensions. In the quantized state, the conductivity parallel to the magnetic field tends toward zero. These findings suggest that rather than two-dimensionality of the electronic system, it is the absence of conductivity along the magnetic field which is a necessary condition for the observation of the quantized Hall effect.

Since the discovery of the integral quantized Hall effect (IQHE) in a Si-MOSFET[1], its confirmation in a variety of III-V heterojunctions[2] and its interpretation in terms of the singular density-of-states of a two-dimensional system in a high magnetic field[3], there has developed a prejudice that two-dimensionality of the electronic system is a prerequisite for its existence. Indeed, by now the IQHE has been observed in a variety of structures[4,5] that deviate from the simple one-layer, one-subband systems of the early work. Nevertheless, all those structures consist of electronically strictly two-dimensional systems lacking any dispersion and, therefore, conduction in the direction normal to the layers. Hence, they must be regarded as a stack of independent quantized Hall resistors connected in parallel. The question arises whether strict two-dimensionality is indeed a prerequisite or whether a generalization has to be adopted. In fact, Azbel has considered this question for highly anisotropic materials.[6] While he argues for vanishing diagonal resistivity, he does not predict quantization of the Hall effect.

In order to test this case we studied a system which is electrically three-dimensional by virtue of being a good, although anisotropic, conductor in all spatial dimensions. In spite of its three-dimensionality, this system exhibits the IQHE, $\rho_{xy} = h/ie^2$ and a pronounced zero-resistance state, $\rho_{xx} \rightarrow 0$.[7] While the conductivity, σ_{zz}, along the magnetic field (B) direction in general is non-zero, it vanishes ($\sigma_{zz} \rightarrow 0$) at values of B at which the system assumes the quantized state. This result suggests that rather than strict two-dimensionality, it is the absence of conductivity along the magnetic field direction which is a necessary condition for the occurrence of a IQHE and zero-resistance state.

The structure used in the experiment is a GaAs/AlGaAs superlattice (SL) with highly penetrable barriers. Two identical samples were grown via MBE on a semi-insulating substrate for in-plane transport and on a n^+-substrate for normal transport. The dimensions of the SL are illustrated in Fig. 1(a). Measurements of the Hall density and Hall mobility at 4.2K yield $n_H = 2.1 \times 10^{17}\ cm^{-3}$ and $\mu_H = 6400\ cm^2/V\text{-sec}$, assuming a thickness of $30 \times (188Å + 38Å)$.

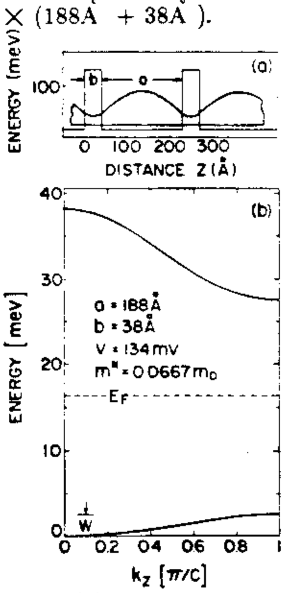

Fig. 1 (a) Kronig-Penney model and wave function for $k_z = 0$ for the GaAs/(AlGa)As superlattice employed. (b) Dispersion relation in the z direction.

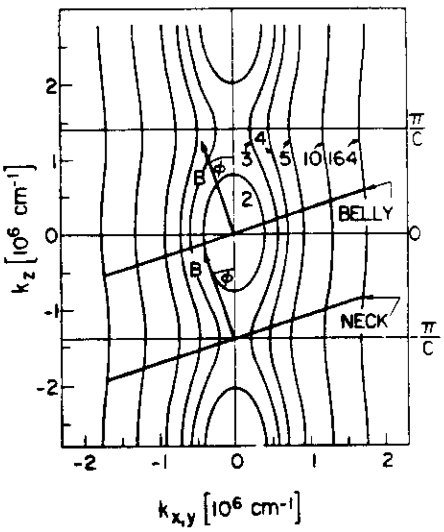

Fig. 2 Contours of constant energy (E = 1,2,3,..., 16.4 meV) in the k_{xy}-k_z plane for the band structure of Fig. 1(b). Belly and neck represent the two extremal orbits of the Fermi surface.

Heterostructures fabricated from GaAs/(AlGa)As are well understood and extremely well represented by a simple square-well potential.[8] We calculate the miniband structure and wave function of the electronic system using a Kronig-Penney with the parameters listed in Fig. 1(b). Nonparabolicity of the conduction band, a small difference in mass between GaAs and (AlGa)As, and a slight distortion of the potential well are neglected. From the calculations we obtain the dispersion relation shown in Fig. 1(b). Fig. 1(a) shows the z dependence of the wave function for $k_z = 0$. The variation from maximum in the well to minimum in the barrier is less than a factor of 4, demonstrating the high degree of transparency of the barriers.

Fig. 2 shows contours of constant energy in the k_z, k_{xy} plane for various energies up to $E_F = 16.4$meV. The three-dimensionality of the electronic system is evident. In comparison, a strictly two-dimensional system is represented in such a plot as a set of straight lines parallel to the k_z axis, indicating the lack of dispersion in the z direction. For lateral transport the samples were fabricated into Hall bars, while the specimen for normal transport were etched into mesas with 100μm diameter, see insert Fig. 5. The in-plane conduction at zero-field is $\sigma_{xx} = 210(\Omega\mathrm{cm})^{-1}$. The perpendicular resistance of the mesa amounts to $R = 1.2\Omega$ which translates into $\sigma_{zz} = 0.12(\Omega\mathrm{cm})^{-1}$, suggesting a rather large anisotropy of $\alpha \sim 10^3$. This value of α represents an upper limit of the anisotropy since a major fraction of the perpendicular resistance is due to contact and substrate resistance.

Experiments on the in-plane transport allows to determine the actual shape of the Fermi surface. The magneto-oscillations shown in Fig. 3(a) indicate a beating pattern between two different oscillations close in frequency. A node is clearly visible around B $\sim$ 2.3 which establishes the difference between belly and neck orbit to be $\Delta A = 1.1 \times 10^{11}$ cm^{-2}. The calculated value is $\Delta A = (9.0 - 7.6) \times 10^{12}$ cm^{-2} $= 1.4 \times 10^{11}$ cm^{-2}. Tilting the magnetic field away from $\Phi = 0$ deg shifts the node to lower fields (see Fig. 4) i.e. smaller differences between belly and neck orbits as expected from the Fig. 2. The non-vanishing conductivity in the z-direction at zero-field and the observation of separated belly and neck orbits establish convincingly the three-dimensionality of the electronic system under study.

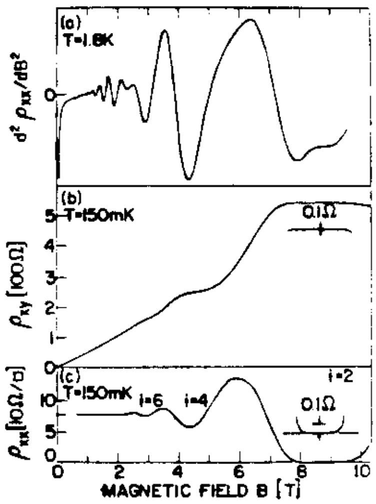

Fig. 3 (a) Second derivative of magnetoresistance ρ_{zz} vs magnetic field for $\Phi = 0$. (b) Hall resistance ρ_{zy} (c) Magneto resistance ρ_{zz} vs magnetic field.

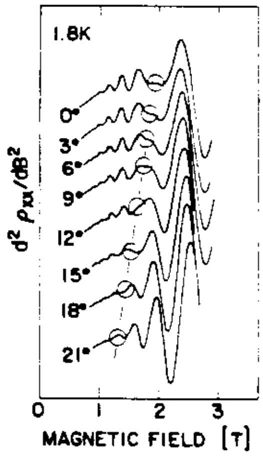

Fig. 4 Angular dependence of $d^2\rho_{zz}/dB^2$. The node of the beating pattern shifts to lower fields as expected from Fig. 2

Fig. 3(b) and 3(c) show our lowest-temperature data on ρ_{zy} and ρ_{zz}. At $B \sim 9$ T, ρ_{zz} develops a clear zero-resistance state with $\rho_{zz} < 0.01\ \Omega/\Box$. For $0.15\ K < T < 4.2\ K$ ρ_{zz} is activated over more than two decades with an activation energy of $\Delta/2 = 0.13$ meV (Fig. 6). Concomitant with the minimum in ρ_{zz}, a plateau appears in ρ_{zy} which is defined to about 0.01 Ω over a range of 1.5 T. The normal conductivity σ_{zz} (Fig. 5) oscillates in phase with ρ_{zz} and tends towards $\sigma_{zz} \to 0$ at a field position where ρ_{zy} approaches a plateau and ρ_{zz} vanishes. With decreasing temperature, the Hall resistance converges upon a constant value $\rho_{zy} = 537.73 \pm 0.03\ \Omega$ or $\rho_{zy} = h/48e^2$ to 5 parts in 10^5, well within the advertised accuracy of 1 part in 10^4 of the decade resistor. These findings suggest that in the quantized state the resistivity and the conductivity tensor assume the form

$$\rho = \begin{pmatrix} 0 & \frac{h}{ie^2} & 0 \\ -\frac{h}{ie^2} & 0 & 0 \\ 0 & 0 & \rho_{zz}\to\infty \end{pmatrix} \qquad \sigma = \begin{pmatrix} 0 & -\frac{ie^2}{h} & 0 \\ \frac{ie^2}{h} & 0 & 0 \\ 0 & 0 & \sigma_{zz}\to 0 \end{pmatrix}$$

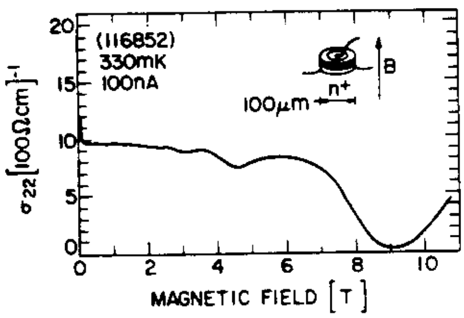

Fig. 5 Conductivity σ_{zz} normal to the layers of the superlattice as a function of magnetic field. This sample was grown on a conducting substrate.

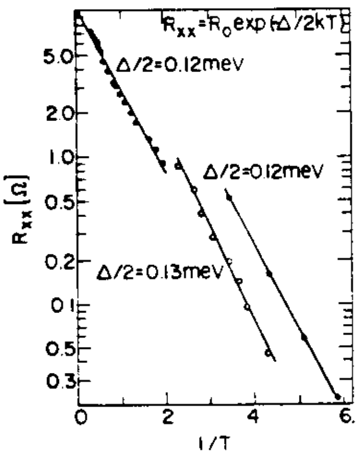

Fig. 6 Magnetoresistance in the i = 2 minimum of ρ_{zz} versus inverse temperature. Data are taken on 3 different specimen in 2 different systems.

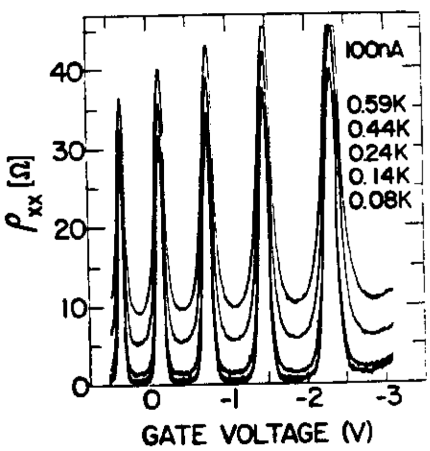

Fig. 7 Variation of ρ_{zz} at i = 2 minimum as a function of gate voltage.

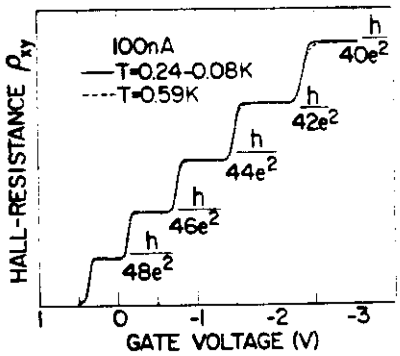

Fig. 8 Variation of ρ_{zy} at i = 2 plateau as a function of gate voltage.

One can develop an intuitive picture which is able to explain some of the experimental facts. In an ideal two-dimensional system in a high magnetic field along z, the Landau levels consist of δ functions separated by the cyclotron energy $\hbar\omega_c$. In a three-dimensional system, each quantized state in the plane is associated with a range of k_z, each having a slightly different energy. Therefore, each Landau level develops into a band. Since the z motion is not affected by the magnetic field, the shape of each band is field independent and reflects the one-dimensional density of states of the k_z

dispersion in the absence of a field. Its width reproduces the zero-field miniband width W. In high magnetic fields B, when $\hbar\omega_c$ exceeds W, the density of states again exhibits gaps, as in the ideal two-dimensional case, and the condition for the observation of a IQHE seems to be fulfilled.

With a total thickness L = jc there exist j different k_z states for each state in the plane. With a level degeneracy of d = eB/h in the x-y plane, a total of jeB/h states exist in each Landau band, and arguments used for the strictly two-dimensional case yield $\rho_{xy} = h/jie^2$. Therefore, with i = 2 and j = 30, one expects $\rho_{xy} = h/60e^2$ which, however, differs from the experimental result of $\rho_{xy} = h/48e^2$. This discrepancy is associated with the depletion of several top and bottom layers of the superlattice. In order to investigate this aspect, we fabricated a gate electrode on top of the superlattice allowing us to vary the depletion depth. Figs. 7 and 8 show the variation of ρ_{xx} and ρ_{xy} in the i = 2 state versus the gate voltage. Large oscillations in ρ_{xx} and concomitant step-like transitions from one quantized value to the next are being observed when the sample is depleted to increasing depth. A similar pattern results when a backside bias is applied to the substrate side. Since the repetitive pattern is assumed to arise from the periodicity of the superlattice, we expect the peaks in ρ_{xy} (the best defined structures) to appear at gate voltages.

$$V_k + \overline{V} = \frac{e}{2\epsilon\epsilon_o} Nc^2 \, [k - k_o + \phi]^2, \quad k,\ k_o = 0,1,2,..;\ 0 \leq \phi \leq 1$$

Here, $\overline{V}$ (a negative voltage) is the built-in surface potential measured from E_F, N is the 3D density, c is the period of the superlattice, k_o is the unknown number of initially depleted layers and ϕ is a phase factor close to 1/2. The voltage differences then are linear in k.

$$V_{k+1} - V_k = \frac{e}{2\epsilon\epsilon_o} Nc^2 \, [2(k - k_o + \phi) + 1]$$

Fig. 9 shows a plot of this quantity from which c = 207Å and $k_o = 4$ are deduced. The periodicity is in good agreement with the period determined by TEM. In the absence of a gate voltage there are $k_o = 4$ layers from the top and from $\rho_{xy} = h/48e^2$ we conclude that only two layers are depleted from the substrate side. It is striking that the transitions between subsequent states are so well defined and that a model assuming sequential depletion of individual layers seems to account for the observations. Yet, in a superlattice such a distinction between individual layers is no longer possible due to strong interlayer tunneling. Indeed, the transition between subsequent quantized states in this superlattice are not due to sequential depletion of layers but are transitions at which the whole bulk participates.

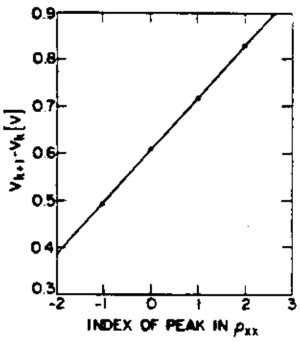

Fig. 9 Voltage difference between neighboring peaks of Fig. 7 versus peak index.

In a periodic one-dimensional system with finite boundaries, the variation of the uppermost state of each subband varies rather abruptly with decreasing distance between boundaries. These state which initially extend all across the superlattice are sequentially peeled off the continum and move rapidly to higher energy crossing the Fermi level whenever the confinement is reduced by one period. It is this periodicity which is responsible for Fig. 7 and 8.

A remaining puzzle is the small activation energy observed. At B $\sim$ 9T and with W $\sim$ 2.5meV, we arrive at a gap energy of $\Delta = \hbar\omega_c$ - W $\sim$ 12meV, while experimentally, we find only $\Delta \sim$ 0.26meV. In general, broadening of the Landau level (here Landau band) is assumed to be responsible for such a reduction. With a mobility of μ = 6400 cm^2/Vsec one deduces a lifetime broadening of $\Delta E \sim \hbar/\tau \sim$ 2.8meV, which accounts in part for the small activation energy. A better understanding of the magnitude of Δ will require a better knowledge of the occurrence and position of a mobility edge and its relation to the IQHE in such an anisotropic three-dimensional system. This problem should be at least as challenging as the much simpler, strictly two-dimensional case where a conclusion has still not been reached.

In summary, our findings suggest a generalization of the conditions under which the IQHE can be observed. Two-dimensionality of the electronic system is not essential. Rather the absence of conductivity along the magnetic field, either pre-existing (as in the two-dimensional case) or field-induced (as in the above case) seems to be the required criterion for the existence of a IQHE.

At present the high scattering rate of the heterojunction superlattice structures prevent observations of the fractional quantized Hall effect (FQHE).[10] Yet other material combinations might ultimately provide such a possibility. Since the FQHE results from strong correlations among the carriers, we expect dramatic consequences for this phenomenon from such a strong interlayer coupling.

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Density of States and de Haas–van Alphen Effect in Two-Dimensional Electron Systems

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The density of states of two-dimensional electron systems in GaAs/AlGaAs single-layer and multilayer heterostructures has been determined through measurements of the high-field magnetization. Our results reveal a substantial density of states between Landau levels, even in high-mobility single quantum wells. There is no existing theoretical explanation for this anomaly.

PACS numbers: 71.20.+c, 71.25.Hc, 73.40.Lq

After more than a decade of intense study of two-dimensional electron systems[1] (2DES) the density of states (DOS) in high magnetic fields has only recently come under direct scrutiny.[2,3] Current theoretical understanding of the quantum Hall effect (QHE) makes incisive assumptions about the DOS, the distinction between localized and extended electronic states being essential.[4] The remarkable phenomena of the QHE depend directly on the different topologies of these states, vividly demonstrating the unsuitability of transport measurements for DOS determinations. The DOS at the Fermi level can, however, be obtained by measurement of a thermodynamic quantity such as magnetization or heat capacity,[3] extended and localized states contributing equally in equilibrium. Early attempts[2,5] to measure the magnetization of the 2DES were hindered by insufficient sensitivity, by low-quality samples, and, in the case of ac measurements,[5] by spurious signals arising from nonequilibrium eddy currents. In this Letter we report high-precision dc measurements of the oscillatory magnetization (de Haas–van Alphen effect) of the 2DES in homogeneous, high-mobility GaAs/AlGaAs single-layer and multilayer heterostructures. From the data we have extracted important new information about the DOS, including the Landau-level widths and their magnetic field and mobility dependences. In spite of the high quality of the samples studied, we find a significant density of states between Landau levels, in agreement with recent heat-capacity measurements but far in excess of theoretical estimates.

Ideally, the DOS of a 2DES in a large perpendicular magnetic field B consists of a sequence of sharp Landau levels separated by gaps void of electronic states. If we ignore the small spin splitting of these levels, the degeneracy of each is $2eB/h$ per unit area. At absolute zero, both the Fermi level and the magnetization exhibit a saw-tooth oscillation periodic in inverse field, with discontinuities when an integral number of Landau levels are exactly filled.[6] The magnetization oscillations are of constant amplitude $M_0 = NA\mu_B^*$, where N is the fixed 2D carrier density, A the sample area, and μ_B^* the effective Bohr magneton obtained by substitution of the carrier effective mass ($m^* = 0.0665m_0$ for GaAs). In a real 2DES, with Landau levels broadened by various mechanisms, the oscillations will be attenuated and smoothed out.[7] Measurements of the amplitude and shape of the magnetization oscillations can be used, when compared to those calculated from a model DOS, to obtain information on the underlying electronic spectrum.

Our samples are modulation-doped GaAs/AlGaAs heterostructures, grown by molecular-beam epitaxy[8] on GaAs ⟨100⟩ substrates, rotated during growth to ensure homogeneity. While each single interface contains but one 2D electron layer, the multiquantum wells consist of many, well separated, layers stacked upon one another. The carrier densities, mobilities, and other structural parameters for the samples studied are listed in Table I. The last column of the table

TABLE I. Structural and two-dimensional parameters for all three samples.

Sample	GaAs well (Å)	AlGaAs Si-doped layer[a] (Å)	Periods	Low-T mobility ($m^2/V\cdot s$)	2D density (cm^{-2})	Total 2D area (cm^2)	rms $\Delta N/N$ (%) In plane	rms $\Delta N/N$ (%) Layer to layer
1	140	400	50	8.0	5.4×10^{11}	9.9	1.1	< 1.8
2	. . .	1000	1	28.5	3.7×10^{11}	1.5	1.0	*n/a*
3	175	450	51	3.9	5.5×10^{11}	12.6	0.8	< 2.0

[a]Including undoped spacer layers.

gives the measured rms variations in the carrier density of the samples, both within the 2D plane and perpendicular to it. To determine the transverse variation of density more than a dozen independent Shubnikov–de Haas (SdH) measurements were made on small segments (0.1×1-mm^2 bars) of the molecular-beam epitaxial wafer near each magnetization sample. An upper limit on the layer-to-layer variation of the density follows from the observed width of the low-temperature (~0.3 K) SdH peaks. For a single 2D layer the width of these peaks, whose magnetic field positions are proportional to carrier density, is determined by the fraction of extended states. Therefore, in a multilayer sample the peak width sets an upper limit on the density variations between layers. All three samples show a well-developed QHE at low temperature.

The magnetization measurements are performed with a recently developed torsional technique.[9] The samples are mounted on a thin fiber held perpendicular to the applied magnetic field and are oriented so that the normal to the 2D plane, along which the orbital magnetic moments must lie, is tilted away from the field direction by a small angle; this geometry is depicted in the inset to Fig. 1. The measurement consists of our slowly sweeping the field and recording, by a capacitive method, the torque on the sample. These measurements are quasi dc, limited only by the sweep rate of the field (~1 T in 5 min). Typical angular excursions of the sample are less than 10^{-4} and the resolution is about 5×10^{-13} J/T at 10 T.

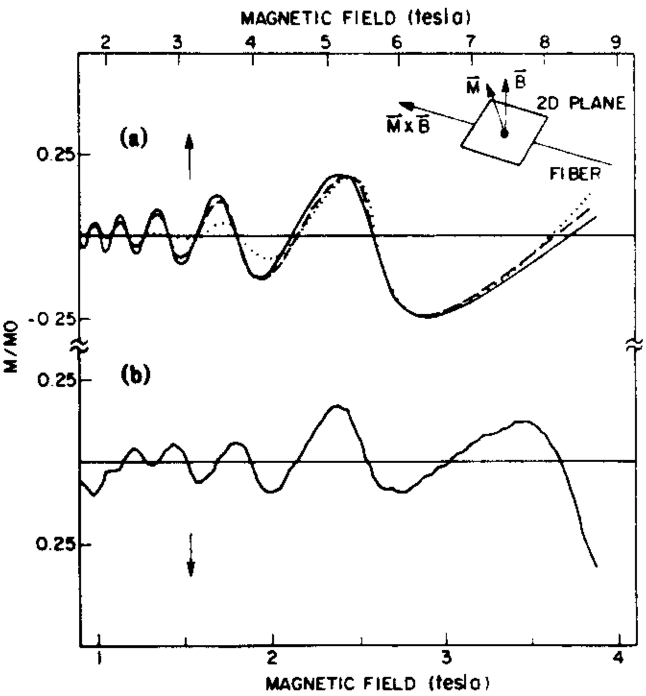

FIG. 1. Normalized magnetization for (a) sample 1 and (b) sample 2. Note different field scales. Dotted and dashed lines are fits; $\Gamma=2.4$ meV and $\Gamma=(1\ \mathrm{meV/T^{1/2}})\sqrt{B}$, respectively. The basic geometry of the magnetization measurements is depicted at top right.

Figure 1(a) shows normalized magnetization data from sample 1, along with theoretical curves which are described below. A small, smooth background has been subtracted from the magnetometer output. In a narrow region around 5.8 T, the data represent an average of sweeps up and down in field. This was done to eliminate the effect of eddy currents[10] associated with the deep zero-resistance state in this field range ($\rho_{xx}\sim10^{-4}\ \Omega/\Box$ at 0.4 K). While the magnetization varies smoothly over the entire field range the resistivity (not shown) undergoes order-of-magnitude fluctuations as the Fermi level passes between extended and localized states. The magnetization oscillations have the correct phase and periodicity to be unambiguously identified with the de Haas–van Alphen (dHvA) effect but the amplitude and general shape indicate significantly broadened Landau levels. The lack of discontinuities suggests the absence of gaps in the DOS. Our data show no evidence of the spin contribution to the magnetization; this is not surprising given the relatively small size of the spin splittings in n-type GaAs (more than 10 times smaller than the Landau splitting, even after including g-factor enhancement effects[11]).

Figure 1(b) represents the first observation of the dHvA effect in a single layer of electrons (sample 2) by a true dc technique. A large anisotropic background magnetization, nearly linear in magnetic field, has been subtracted from the data. The magnitude and temperature dependence of this background prevent reliable determination of the 2D magnetization below 4.2 K and above 4 T. Although uncertainties in the background subtraction preclude analysis of the shape of the oscillations in Fig. 1(b), the amplitudes are well determined and can be used to gain significant information about the DOS.

Figure 2 presents a synopsis of the magnetization data on the samples listed in Table I. Here the amplitudes of the magneto-oscillations, normalized by the ideal amplitude M_0, are plotted versus magnetic field; these points provide a basis for comparison to numerical calculations. The solid lines represent theoretical envelopes for the dHvA oscillations resulting from a DOS consisting of Gaussian Landau levels:

$$D(\epsilon)=\frac{2eB}{h}\sum_{j=0}^{\infty}\frac{1}{(2\pi)^{1/2}\Gamma}\exp\left[-\frac{(\epsilon-\epsilon_j)^2}{2\Gamma^2}\right]. \quad (1)$$

In this definition Γ is the rms half-width of the levels and $\epsilon_j=(j+\frac{1}{2})e\hbar B/m^*$ is the Landau-level energy. It is clear from the figure that the observed oscillations from all three samples are considerably smaller than the ideal-gas result and require Landau-level

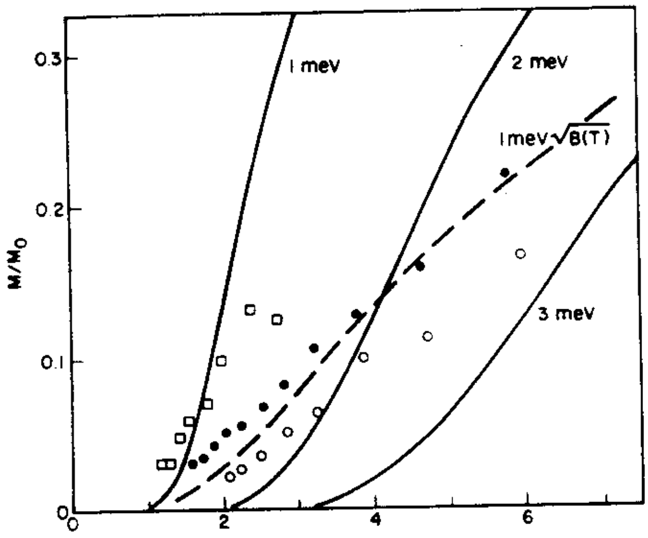

FIG. 2. Normalized dHvA oscillation amplitude vs magnetic field for all three samples: filled circles, sample 1; squares, sample 2; open circles, sample 3. The solid lines are theoretical envelopes described in text; the labels give the rms half-width of Landau levels. The dashed line is the result of assumption of a linewidth proportional to $\sqrt{B}$.

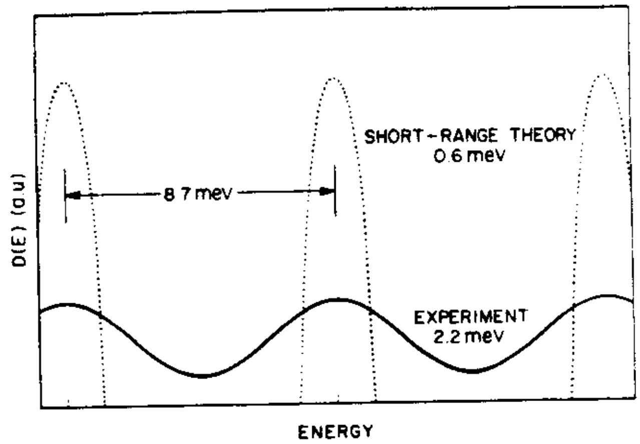

FIG. 3. Comparison of model DOS (solid line), used to fit the data from sample 1 at 5 T, with the short-range theory. At this field 8.7 meV is the Landau-level spacing. The rms half-widths of the levels are also shown.

half-widths in the 1–3-meV range. It is also apparent that the model DOS with a constant Γ does not give the correct magnetic field dependence for the oscillation amplitudes. The dashed line in the figure represents a calculation in which the width Γ is assumed to vary as $\sqrt{B}$. With a prefactor of 1 meV/$T^{1/2}$ the calculated magnetization oscillations are in considerably better agreement with the data for sample 1 than are any of the constant-linewidth calculations. Under the assumption of a width that is proportional to B, the resulting oscillation amplitude would be field independent and therefore give a horizontal line in Fig. 2. The dotted and dashed lines in Fig. 1(a) give the calculated results for the above DOS, for constant and $\sqrt{B}$ linewidths, respectively. The widths are adjusted to fit the oscillation amplitude around 6 T for sample 1. For the $\sqrt{B}$ fit, both the amplitude and general shape of the oscillations are reasonably approximated. Better fits to the low-field oscillations can be obtained with different line-shape functions, but discussion of such higher-order features of the DOS is not our purpose here.

For sample 2, unlike samples 1 and 3, the measurement temperature (4.2 K) contributes noticeably to the attenuation of the dHvA oscillations. The effect can be approximated by a mere increase of the effective Landau-level widths. At 2 T the observed rms half-width is 1.1 meV; removing the temperature broadening reveals a residual linewidth of about 0.9 meV, approximately 25% smaller than that observed in sample 1 at the same field. This is a modest narrowing given the almost factor-of-4 mobility difference between these two samples. It should be emphasized, however, that the link between mobility and high-field Landau-level width is highly uncertain.[12-14]

Small-scale sample inhomogeneities may present a plausible explanation for the apparently broad Landau levels. While our SdH studies rule out significant attenuation of the dHvA oscillation arising from variations of the 2D density on a gross lateral scale (~ 1 mm), as well as from layer to layer, we cannot assess smaller scale variations. Under the assumption that a local 2D density can be defined, the effect of inhomogeneities can be calculated by a simple averaging technique, the result being attenuated dHvA oscillations. We have not succeeded in generating the correct field dependence for the oscillation amplitude with such a simple model. The local-density assumption itself may be incompatible with the effect on the Landau levels of long-range potential fluctuations.[15]

At present there does not exist a coherent theoretical picture of the DOS of a 2DES in a high magnetic field. Theoretical calculations date back to times before the discovery of the QHE and are not necessarily applicable to our system. Ando and Uemura[16] derived, under various approximations, an expression for the rms width of the Landau levels assuming short-range scatterers and, while their result does give a $\sqrt{B}$ dependence for the width, the calculated magnitude is approximately 4 times smaller than our experimental results. The model DOS used to fit the data from sample 1 (at 5 T) is compared to the short-range result, appropriate to this sample, in Fig. 3. The absence of gaps in the observed DOS is striking. Given the large discrepancy between theory and experiment, it is not possible to ascertain the origin of the observed rough $\sqrt{B}$ dependence reported here.

Comparison of our results with the recent heat-capacity studies[3] shows agreement on the basic issue of the residual DOS between Landau levels but we do not find evidence for the relatively narrow structures atop a constant background cited by Gornik *et al.*[3] This discrepancy is not understood at present. The most current theoretical picture[15,17,18] reveals the DOS depending nontrivially on magnetic field and strongly upon the Landau-level filling. Neither in this work nor in the heat-capacity study were such complications included in the model DOS used for analysis. Without inclusion of such complex dependences there is no obvious way to compare the two experiments.

In summary, we have used the de Haas–van Alphen effect to determine the high-magnetic-field DOS of 2D electrons in GaAs/AlGaAs heterostructures, for both single-layer and multilayer samples. Our results give Landau-level widths which are magnetic field dependent, varying roughly as $\sqrt{B}$, but whose magnitude is about a factor of 4 larger than theoretical estimates. These widths imply a significant density of states between Landau levels, even in high-mobility ($\sim 300\,000$ cm^2/V·s) single-interface structures.

It is a pleasure to thank W. Wiegmann for growing some of the samples and K. Baldwin and A. Savage for their excellent technical assistance. We also thank T. Haavasoja for his help in the early stages of this work.

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Observation of an Even-Denominator Quantum Number in the Fractional Quantum Hall Effect

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An even-denominator rational quantum number has been observed in the Hall resistance of a two-dimensional electron system. At partial filling of the second Landau level $\nu = 2 + \frac{1}{2} = \frac{5}{2}$ and at temperatures below 100 mK, a fractional Hall plateau develops at $\rho_{xy} = (h/e^2)/\frac{5}{2}$ defined to better than 0.5%. Equivalent even-denominator quantization is absent in the lowest Landau level under comparable conditions.

PACS numbers: 73.40.Kp, 73.20.Dx, 73.50.Jt

The observation of exclusively *odd-denominator* rational quantum numbers in the fractional quantum Hall effect[1-3] (FQHE) represents a surprising experimental fact. This transport phenomenon manifests itself in two-dimensional electron systems at low temperatures and in high magnetic fields as minima in the diagonal resistivity ρ_{xx} and concurrent plateaus in the Hall resistance ρ_{xy} quantized to $(h/e^2)/(p/q)$. These characteristic features occur at fractional Landau-level filling $\nu = p/q$, where q is always odd ($\nu = nh/eB$, n is the areal density, and eB/h is the Landau-level degeneracy). The FQHE is presently accepted as being the consequence of the formation of an incompressible quantum fluid.[4] The ground state is well described by Laughlin's many-particle wave function[5] which, because of the requirement of antisymmetry under particle exchange, applies exclusively to odd-denominator fractional Landau-level filling. This odd-denominator restriction propagates to the hierarchical model[2] of daughter states which embraces all odd-denominator rational fractions and is well supported by numerical few-particle calculations.[4,6,7] While at present there exists ample evidence in theory and experiment alike for the absence of even-denominator quantum numbers, no physical symmetry has been found to exclude them *a priori*. In fact, recent work[8,9] has pointed to the possibility of condensation at $\nu = \frac{1}{2}$ although perhaps without display of the FQHE. Under these circumstances firm experimental evidence for an even-denominator rational quantum number will require a reevaluation of our understanding of two-dimensional electrons in the quantum limit.

The possibility of observing the FQHE at even-denominator filling factors has been suggested by some experimental findings. A minimum in ρ_{xx} has been noted by Ebert *et al.*[10] at $\nu = \frac{3}{4}$ in the lowest Landau level. More recently, Clark and co-workers[11,12] have conjectured that a family of even-denominator fractions may exist in the second Landau level at $\nu = \frac{9}{4}, \frac{5}{2}, \frac{11}{4}$, as displayed by weak minima in ρ_{xx}. Since minima in ρ_{xx} at such high filling factors are notoriously wide and invariably shift significantly with temperature, their association with a particular fractional filling is problematic. Only quantization of ρ_{xy} to the correct fractional value provides firm evidence for the existence of a given fractional state. Such crucial evidence has been lacking.

In this Letter we present experimental evidence for the appearance of the characteristic features of the FQHE at an even-denominator filling factor. This unexpected phenomenon occurs in the first excited Landau level $4 < \nu < 2$ at a filling factor $\nu = 2 + \frac{1}{2} = \frac{5}{2}$. Transport experiments show a plateau developing in ρ_{xy} centered at $(h/e^2)/\frac{5}{2}$ to within 0.5% concomitant with a deep minimum in ρ_{xx}. An equivalent quantization is not observed in the lowest Landau level $\nu < 2$ at similar temperatures. While all of the data reported here were obtained from a molecular-beam-epitaxy–grown single-interface GaAs/AlGaAs heterostructure of mobility 1.3×10^6 cm^2/V s and areal density 3.0×10^{11} cm^{-2}, similar but somewhat weaker structures were observed in two other samples. Low-temperature illumination for several minutes with a light-emitting diode is necessary to produce the persistent carrier concentration and mobility given above. These parameters are found to depend slightly on the precise illumination conditions.

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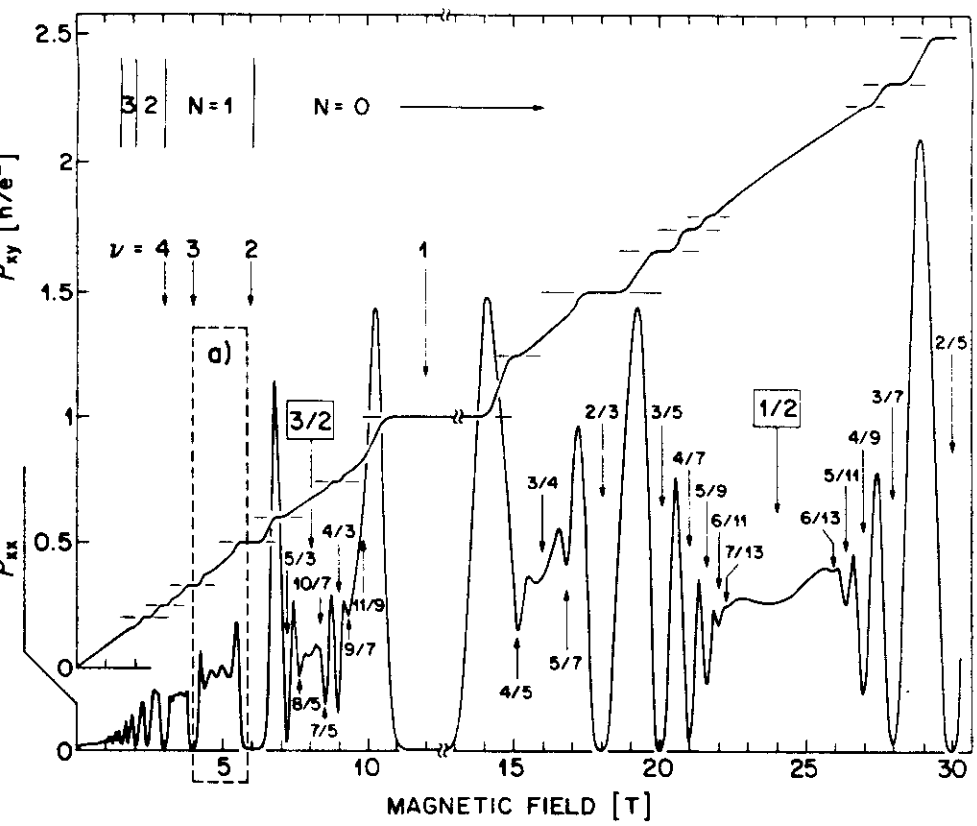

FIG. 1. Overview of diagonal resistivity ρ_{xx} and Hall resistance ρ_{xy} of sample described in text. The use of a hybrid magnet with fixed base field required composition of this figure from four different traces (breaks at $\simeq 12$ T). Temperatures were ≈ 150 mK except for the high-field Hall trace at $T=85$ mK. The high-field ρ_{xx} trace is reduced in amplitude by a factor 2.5 for clarity. Filling factor ν and Landau levels N are indicated.

Transport measurements were performed at magnetic fields up to 30 T and at temperatures down to 20 mK with two different dilution-refrigerator–magnet systems. Great care has been exercised in order to assure thermal equilibrium between the 2D electrons and the crystal lattice. Since large changes in resistivity were observed upon cooling of the crystal lattice from 40 to 25 mK (as measured with a nearby carbon resistance thermometer) a gross electron-lattice disequilibrium seems unlikely.

Figure 1 displays the low-temperature diagonal and Hall resistivities over a wide range of magnetic field and filling factor. In Fig. 2, the interval $3>\nu>2$ is expanded, revealing our most startling result. The ρ_{xy} data at 25 mK show a plateau forming at the field corresponding to $\nu=\frac{5}{2}$, intersected by the classical Hall line determined from the measured 2D density. More importantly, this plateau is centered at $\rho_{xy}=(h/e^2)/\frac{5}{2}$ to within 0.5%. Simultaneously a deep relative minimum is found in ρ_{xx}. While not yet fully developed, these features emerge in a manner analogous to conventional odd-denominator FQHE states. Taken together, these data provide striking evidence for an even-denominator FQHE.

To highlight further the ρ_{xy} data contained in Fig. 2, the positions of the high-order odd-denominator fractions $\frac{32}{13}$ and $\frac{33}{13}$ are indicated ($\frac{5}{2}\pm 1.5\%$). No features are found in ρ_{xy} at these fractions which lie well clear of the observed $\frac{5}{2}$ plateau. From this it can be assumed that the $\frac{5}{2}$ plateau is not likely the consequence of two high-order odd-denominator plateaus blending together to form an apparent, but spurious, plateau at $\nu=\frac{5}{2}$.

Figure 2 also shows that the strong temperature dependence of the $\frac{5}{2}$ minimum in ρ_{xx} commences below 100 mK, indicating a very small associated energy scale. Although not shown in the figure, the plateau in ρ_{xy} at $\nu=\frac{5}{2}$ exhibits the same temperature dependence as the minimum in ρ_{xx}. Above about 100 mK the plateau disappears and the Hall resistance follows the classical line. The development of the resistivity feature is noteworthy. Instead of forming a zero in ρ_{xx}, the minimum itself remains roughly constant while the adjacent flanks rise steeply as the temperature is reduced. The same phenomenon has been observed at odd-denominator fractions as well.[13] Such behavior results from the competition between the tendency for the ρ_{xx} background to rise as the temperature falls and the development of the resistivity minimum.

In addition to the plateau at $\nu=\frac{5}{2}$ there is other evidence of the FQHE in the first excited Landau level, $4>\nu>2$. As shown in Fig. 2, there are broad minima

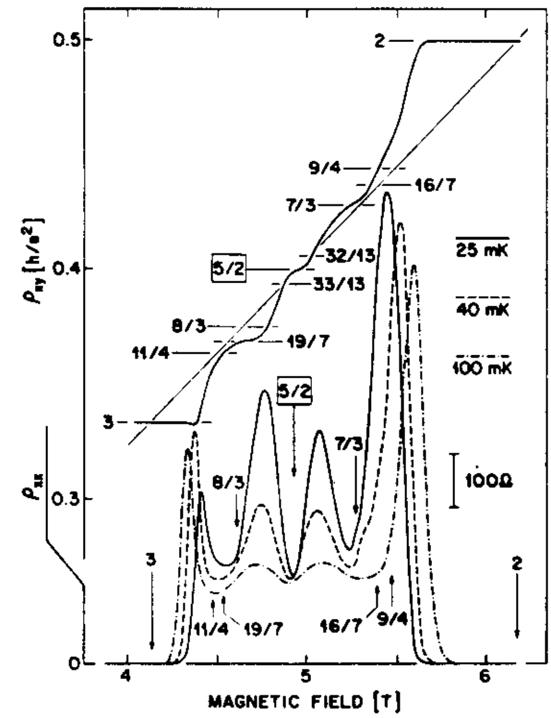

FIG. 2. Diagonal resistivity ρ_{xx} and Hall resistance ρ_{xy} [enlarged section (*a*) of Fig. 1] at $T=100$ to 25 mK. Filling factors ν are indicated in ρ_{xx} while quantum numbers p/q are shown in ρ_{xy}.

near $\nu=\frac{9}{4}$ and $\frac{11}{4}$ which shift considerably with temperature. By the lowest temperatures a plateau, off the classical line, has formed at $\frac{19}{7}$ corroborating the earlier work of Clark *et al.*[12] and a much weaker one is appearing near $\nu=\frac{7}{3}$. Thus, aside from $\nu=\frac{5}{2}$, we have no evidence for an even-denominator FQHE in the range $3>\nu>2$.

At high temperature ($\simeq 100$ mK) ρ_{xx} data from the higher spin state of the first excited Landau level, $4>\nu>3$, are broadly similar to the range $3>\nu>2$. A minimum is found at $\nu=\frac{7}{2}$ as well as in the vicinity of $\nu=\frac{13}{4}$ and $\frac{15}{4}$. Lowering the temperatures causes an overall increase in resistivity over the entire range without significant enhancement of the fractional features. Only weak structure in ρ_{xy} is found at $\nu=\frac{7}{2}$. Potential observation of the FQHE at $\nu=\frac{7}{2}$ awaits samples of higher quality.

Having evidence for an even-denominator fraction within the first excited Landau level, we reexamined the lowest Landau level for equivalent features. Using the same specimen, we focused on $\nu<1$. As Fig. 1 shows there exist a broad basin in ρ_{xx} around $\nu=\frac{1}{2}$, but no inflection occurs in ρ_{xy}. In fact, in this field range, ρ_{xy} follows the classical Hall line. Furthermore, the broad feature around $\nu=\frac{1}{2}$ is in stark contrast to the much sharper neighboring odd-denominator minima which have now been observed with denominators up to $q=13$ (Fig. 1). The absence of a quantized plateau in ρ_{xy} and the uncharacteristically wide depression in ρ_{xx}, in spite of the fact that higher magnetic fields vastly amplify FQHE features,[14] suggests a characteristic difference between electron correlation in the lowest and first excited Landau levels. A similar observation can be made around $\nu=\frac{3}{2}$ which was closely investigated at temperatures as low as 25 mK without showing evidence for even-denominator quantization.

With the resolution of increasingly higher-order odd-denominator fractional states of the sequences[3] $\nu=(m+1)/(2m+1)$ and $\nu=m/(2m+1)$ $(m=1,2,3,\ldots)$, which converge toward $\nu=\frac{1}{2}$, the broad basin in its vicinity may actually be caused by even higher-order, yet unresolved members of the same sequences. Such a conjecture is supported by distinct features now observed around $\nu\simeq\frac{3}{4}$. With our high-mobility sample, we discovered representatives of both odd-denominator sequences converging towards $\nu=\frac{3}{4}$. Distinct minima are observed at $\nu=\frac{4}{5}$ and $\nu=\frac{5}{7}$ associated with plateaus (not shown in Fig. 1) quantized to the appropriate values to better than 1%.

To summarize our results, in the first excited Landau level we have firm evidence for fractional quantization of the Hall effect to an even-denominator fraction, $\nu=\frac{5}{2}$, with no other even-denominator fraction apparent at $\nu=p/4$ for temperatures as low as $\simeq 20$ mK. In spite of our resolving several new fractions in the lowest Landau level, no evidence for even-denominator quantization exists presently for $\nu<2$.

Although no physical principle has been found excluding the observation of even-denominator fractions in the FQHE, there exists presently no theoretical model describing such states. Theory has been very successful in developing an understanding of odd-denominator fractions in terms of a highly correlated quantum fluid existing specifically at primitive odd-denominator filling $(\nu=\frac{1}{3},\frac{1}{5},\ldots,1-\frac{1}{3},1-\frac{1}{5},\ldots)$.[5] Laughlin's wave function fulfills the requirement for antisymmetry of the wave function only for odd-denominator rational filling. The same restriction applies to the hierarchy of fractional daughter states $(\nu=\frac{2}{5},\frac{3}{7},\frac{2}{7},\frac{4}{9},\ldots)$ derived from those primitive parental ground states.

Generalization of this theoretical model to include even-denominator quantum numbers requires the particles to be bosons rather than fermions. Such possibilities have been discussed previously by Halperin[15] who proposes bound-electron pairs as such candidate bosons. Given the low field at which we find the $\nu=\frac{5}{2}$ FQHE and the consequent small spin Zeeman energies, already predicted to influence the FQHE state,[16] potential pairing mechanisms involving spin-reversed electrons cannot be rejected *a priori*. Numerical few-particle calculations[4] have led to considerable progress in quantifying the properties of the fractional states. None of these elaborate techniques has hinted towards the existence of even denominators. A recent cooperative ring exchange theory[17] may allow for quarter fraction, but makes no mention of $p/2$. However, most of these calculations have focused on the lowest Landau level, where even-denominator quantization indeed remains unobserved. Studies for higher Landau levels rest largely on a generalization[18] of Laughlin's quantum fluid. Numerical calculations for the second Landau level[19] find condensed ground states at filling factors $\nu = \frac{7}{3}, \frac{8}{3}, \frac{11}{5}, \frac{14}{5}, \frac{16}{7}$, and $\frac{19}{7}$. However, there is no evidence for the existence of even-denominator quantum numbers found in these numerical results.

Our observation of the first even-denominator quantum number, $p/q = \frac{5}{2}$, shows that fractional quantization of the Hall effect is not limited to odd-denominator fractions. If the odd-denominator FQHE is any guide, we must expect to find more and possibly different even denominators in the future. It remains to be seen whether a common theoretical description can be found or whether one is dealing with two distinctively different "new states of matter."[5]

We would like to express our thanks to the staff of the Francis Bitter National Magnet Laboratory where the high-field experiments were performed. We would also like to thank P. A. Wolff for generous support and K. W. Baldwin for excellent technical assistance. The Francis Bitter National Magnet Laboratory is supported by the National Science Foundation. The work at Princeton University is supported by the National Science Foundation through Grant No. DMR-8212167 and by the U.S. Office of Naval Research through Contract No. 00014-82-K-0450.

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Evidence for a Phase Transition in the Fractional Quantum Hall Effect

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We observe a novel transition between distinct fractional quantum Hall states sharing the *same* filling fraction $\nu = \frac{8}{5}$. The transition is driven by tilting the two-dimensional electron-gas sample relative to the external magnetic field and is manifested by a sharp change in the dependence of the measured activation energy on tilt angle. After an initial decline, the activation energy abruptly begins to increase as the tilt angle exceeds about 30°. A plausible model for these results implies a transition from a spin-unpolarized quantum fluid at small angles to a polarized one at higher angles.

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The earliest ideas concerning the fractional quantum Hall effect (FQHE) in two-dimensional electron systems (2D ES) held the spin Zeeman energy to be so large that all fractional states could be safely assumed to be fully spin polarized. It was also generally thought that only one incompressible quantum liquid existed at any given filling fraction that displayed the FQHE. Halperin[1] was the first to point out that the small g factor ($g \sim 0.5$) in GaAs made the usual assumption of full spin polarization worth reexamining. He proposed various candidate ground states containing reversed spins. In particular, the *unpolarized* ground-state wave function he suggested for the $\nu = \frac{2}{5}$ FQHE was later shown[2] to have a lower energy, in the absence of the Zeeman term, than the usual polarized state thought to be a "daughter" of the primitive $\nu = \frac{1}{3}$ fluid. While at high magnetic fields the Zeeman energy will stabilize the polarized state, the possibility remains for a transition to an unpolarized fluid at lower fields. The purpose of this Letter is to present evidence consistent with just such a spin transition in the FQHE ground state at $\nu = \frac{8}{5}$.

It is becoming apparent that the spin degree of freedom may in fact play an important role in forming both the condensed ground state[1-8] and its quasiparticle excitations,[9,10] at least at sufficiently low magnetic field B. The energy gap for creating spin-reversed quasiparticles above the $\nu = \frac{1}{3}$ state has been found[9,10] to be less than that for polarized quasiparticles, at sufficiently low magnetic field. This has been suggested as a way to explain the magnetic field dependence of the observed energy gaps[11] in the FQHE. Recent tilted-field studies[12,13] on the FQHE have also been cited as suggestive of the influence of spin.

The recent discovery[14] of a Hall plateau in the FQHE at the *even-denominator* filling fraction $\nu = \frac{5}{2}$ has generated renewed interest in the possibility of spin-unpolarized ground states. A plausible way to overcome the odd-denominator restriction inherent in Laughlin's many-body wave function[15] describing the primitive FQHE ground states at $\nu = \frac{1}{3}$, $\frac{1}{5}$, etc., is to form pairs of electrons with opposing spins. This was made concrete by Haldane and Rezayi[6] who proposed an unpolarized spin-singlet wave function for the $\nu = \frac{5}{2}$ FQHE. Eisenstein *et al.*[16] have presented experimental evidence that the underlying ground state at $\nu = \frac{5}{2}$ may, in fact, be unpolarized. Their data showed a rapid collapse of the $\frac{5}{2}$ state as the magnetic field was tilted away from the normal to the 2D plane, while nearby odd-denominator states remained largely unaffected. Since the predominant effect of the tilt is the enhancement of the spin-flip energy,[17] the collapse of the $\frac{5}{2}$ state with increasing tilt angle was cited as evidence for a significantly reduced spin polarization.

In the present paper we describe a transition between two distinct FQHE states at the same odd-denominator filling factor $\nu = \frac{8}{5}$. The transition is driven by tilting the magnetic field and the data are consistent with a change from a spin-unpolarized fluid to a polarized one. We have so far found no similar transition in the FQHE states at $\nu = \frac{5}{3}$, $\frac{4}{3}$, $\frac{7}{5}$, or $\frac{11}{7}$.

The sample employed in this study is a GaAs/AlGaAs heterostructure grown by molecular-beam epitaxy. With a 2D carrier concentration $N_s = 2.3 \times 10^{11}$ cm^{-2} and mobility of about 7×10^6 cm^2/V s, both established by brief low-temperature illumination with a red light-emitting diode, this sample is of extremely high quality. This is evidenced by the substantially enhanced strength of the delicate $\nu = \frac{5}{2}$ FQHE in comparison to earlier observations.[14,16,18] The sample has allowed for a quantitative study of the even-denominator state, the results of which will be published separately.

The sample is mounted upon an *in situ* rotation device attached to the mixing chamber of a dilution refrigerator. Magnetotransport measurements are typically performed using 10-nA, 5-Hz excitation. We have reliably obtained electron temperatures as low as 16 mK with this arrangement. As in our earlier work,[16] the tilt angle is determined by observing the orderly $\cos\theta$ shift of strong features in the diagonal resistivity ρ_{xx}. Details of our techniques have been published earlier.[14,16,18] The use of *in situ* rotation at low temperatures (<100 mK) is a prerequisite for obtaining reproducibility of delicate

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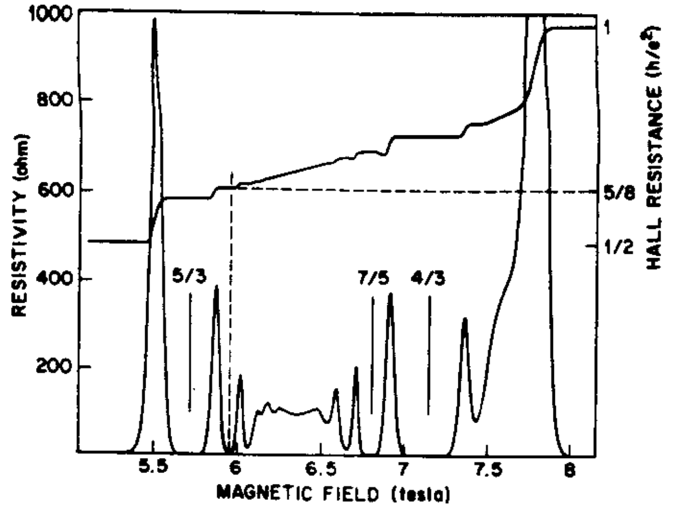

FIG. 1. Overview of diagonal resistivity ρ_{xx} and Hall resistance ρ_{xy} at 25 mK, with the magnetic field perpendicular to the 2D plane. Dashed lines indicate location of the $\frac{5}{3}$ FQHE. Other important FQHE states are indicated.

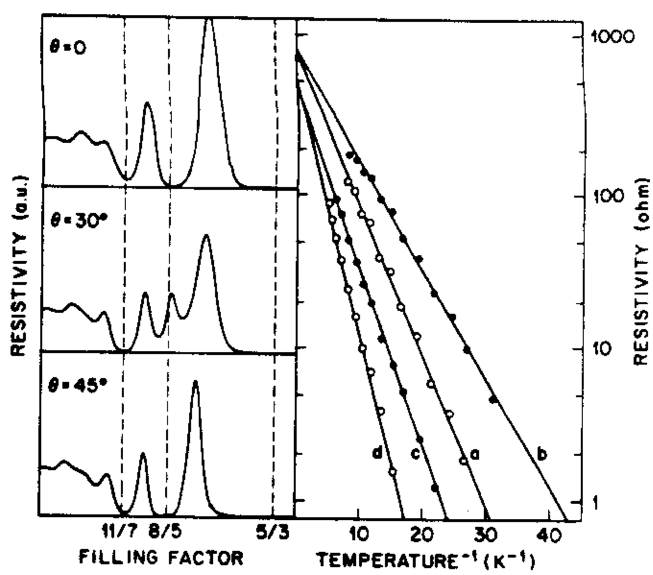

FIG. 2. Left three panels: Expanded views of ρ_{xx} vs *filling factor* in a narrow range around $\nu=\frac{5}{3}$. Note the splitting of $\frac{5}{3}$ minima at $\theta=30°$. Data shown were taken at about 30 mK. Right panel: Arrhenius plots for $\frac{5}{3}$ minimum at various angles. Plot *a*, $\theta=0°$; *b*, $\theta=18.6°$; *c*, $\theta=42.4°$; and *d*, $\theta=49.5°$.

FQHE features. Since the bulk of our data is comprised of activation energy determinations, reliable thermometry in the millikelvin regime is necessary. For this we have employed a ^{3}He melting-curve thermometer similar to that described by Greywall.[19]

Figure 1 shows an overall view of both ρ_{xx} and the Hall resistance ρ_{xy} at 25 mK with the magnetic field perpendicular to the 2D plane, i.e., $\theta=0$. Only the field range corresponding to filling factors $2>\nu>1$ is shown. The filling factor is defined as $\nu=N_s/(eB/h)$, where eB/h is the degeneracy of the individual spin subbands of each Landau level. Thus, Fig. 1 displays electron correlation effects in the upper spin subband of the lowest Landau level. While numerous fractional quantum Hall states are present, as evidenced by minima in ρ_{xx} and plateaus in ρ_{xy}, we will be primarily interested in the $\nu=\frac{5}{3}$ state.

Upon tilting the sample relative to the applied magnetic field an interesting reentrant behavior obtains at $\nu=\frac{5}{3}$. With the field perpendicular to the 2D plane a well-developed FQHE minimum is observed at $\nu=\frac{5}{3}$ (see Fig. 2). Qualitatively, as the angle θ is increased from zero, the $\frac{5}{3}$ FQHE gradually *weakens*. At about 25° a weak satellite minimum appears about 1% higher in field (i.e., lower in filling factor) than the main $\frac{5}{3}$ minimum. Increasing θ further, to about 30°, results in two weak minima of about equal strength whose field positions straddle the location of the $\nu=\frac{5}{3}$ filling factor. The typical splitting of the doublet is only 1% in filling factor. Further tilting reverses all these trends. The high-field component of the doublet becomes dominant and gradually centers on $\nu=\frac{5}{3}$. Beyond about 37° a single, well-developed $\frac{5}{3}$ minima dominates, steadily *strengthening* as θ is increased. For both the low- and high-angle regimes, the Hall resistance exhibits the expected plateau at $\rho_{xy}=5h/8e^2$.

To obtain a quantitative picture of this phenomenon we have carefully measured the activation energy Δ versus tilt angle for the $\frac{5}{3}$ minimum in ρ_{xx}. We define Δ so that $\rho_{xx}=\rho_0\exp(-\Delta/2T)$. Complete temperature dependences are important since we found it easy to be deceived when assigning relative magnitudes to activation energies based solely upon the depth of ρ_{xx} minima at a single temperature. Figure 2 shows typical Arrhenius plots for the $\frac{5}{3}$ state at several tilt angles. For $\theta<25°$ and $\theta>40°$ the ρ_{xx} data display activated behavior over almost two decades in resistivity. On the other hand, around 30°, where the $\frac{5}{3}$ minimum is split, the temperature dependence is complicated. Figure 3 shows the angular dependence of the observed activation energy. As θ increases from zero, Δ smoothly declines. Beyond about 30° Δ begins to rise again, eventually exceeding its value at $\theta=0$. In the range where the doublet is resolved, assignment of an activation energy is somewhat questionable. This is due to the complex interdependence of the two minima, especially at the lowest temperatures. For this narrow range of angles and where it appeared sensible from the Arrhenius plot, we used the high-temperature portion of the data to assign activation energies to one or both members of the doublet. This slight complication is rendered moot since the reentrant behavior of the activation energy is apparent from the data outside the doublet regime.

The data presented in Fig. 3 are all obtained at a fixed filling factor $\nu=\frac{5}{3}$ and hence a fixed perpendicular magnetic field $B_\perp\sim5.95$ T. They are plotted against *total* magnetic field, $B_{tot}=B_\perp/\cos\theta$. This is the natural choice

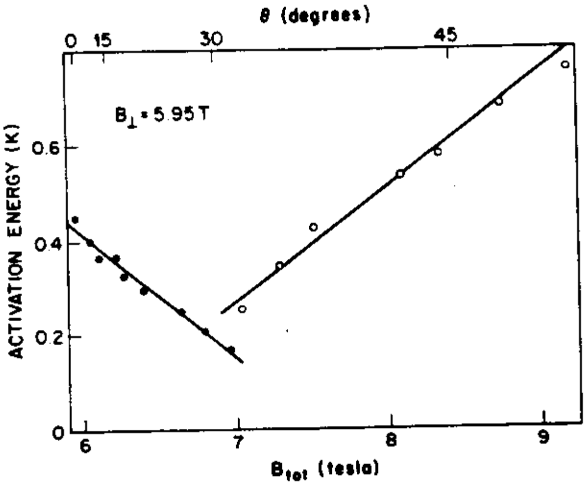

FIG. 3. Activation energy for $\frac{8}{5}$ FQHE vs B_{tot}. Difference between symbols is relevant only in doublet regime around 30°. Solid and open symbols refer to low-field and high-field components, respectively.

if the spin Zeeman energy, which is proportional to B_{tot}, dominates the tilt dependence of the activation energy. It is clear that a sharp transition occurs in the slope $d\Delta/dB_{tot}$ at around 30°. The transition occurs in coincidence with the splitting of the ρ_{xx} minimum.

This behavior suggests a possible phase transition between two distinct quantum-liquid states $\nu = \frac{8}{5}$. However, the data in Fig. 3 cannot be directly interpreted as a crossover of two separate liquid ground-state energies. The activation energy Δ reflects the energy gap for exciting quasiparticles out of the ground state. Thus, a transition between two species of quasiparticles arising from the same ground state cannot be ruled out. Nevertheless, a plausible model for our results can be proposed by assuming the ground state at small angles (i.e., small Zeeman energy) is an unpolarized state analogous to Halperin's $\frac{2}{5}$ state. The $\frac{8}{5}$ and $\frac{2}{5}$ states are presumably closely related, at least in the absence of inter-Landau-level coupling, since $\nu = 2 = \frac{2}{5} + \frac{8}{5}$ represents the fully filled lowest Landau level. Halperin's unpolarized state is expected[2] to be lower in energy than the polarized state at sufficiently low magnetic field. As the Zeeman energy at $\nu = \frac{8}{5}$ is increased by tilting, the relative energy advantage of the unpolarized state decreases and eventually the polarized state will become the new ground state. Other contributions to the ground-state energy, such as exchange and correlation effects, depend to first approximation on $B_\perp$ alone and this is constant at fixed ν.

For the unpolarized fluid at small angles, spin-$\frac{1}{2}$ quasielectron and quasihole excited states must each be spin split. Thus, the energy cost for creating an unbound pair of such excitations is smallest when each aligns itself appropriately with the applied field. This implies an activation energy of the form $\Delta = \Delta_0 - g\mu_B B_{tot}$, where Δ_0 is the energy gap in the absence of a Zeeman term. We will assume both excitations have the same g factor. In addition, we will ignore any residual angular dependence of Δ_0 due, for example, to wave-function "squashing" effects.[13] From the slope $d\Delta/dB_{tot}$ at small angles in Fig. 3 we find $g \sim 0.4$ in remarkable agreement with recent spin-resonance measurements[20] on 2D electrons in GaAs. Interestingly, Furneaux, Syphers, and Swanson[13] obtain a similar result from tilted-field studies of the $\nu = \frac{2}{3}$ FQHE. Whether the g factor for the fractionally charged quasiparticles should be so close to that of uncorrelated electrons is not known.

In the high-angle, polarized state the excitation energies may also depend on the spin of the quasiparticles. At sufficiently low magnetic field, excitation of a spin-reversed quasielectron and spin-polarized quasihole out of the $\nu = \frac{1}{3}$ FQHE ground state has been predicted[9,10] to provide the lowest-energy gap, with the net Zeeman contribution being $+g\mu_B B_{tot}$. Hence, we now write $\Delta = \Delta_0' + g\mu_B B_{tot}$ with Δ_0' the gap in the absence of the Zeeman energy. The high-angle data in Fig. 3 yield a slope $d\Delta/dB_{tot}$ which is nearly the same, but of opposite sign, as that of the low-angle unpolarized state. Hence, the same g factor obtains. At still higher total fields, excitation of polarized quasielectrons should be favored, yielding a roughly angle-independent gap. Such measurements require a larger magnet than is currently fitted on our apparatus.

There are aspects of our data which are not understood at present. In particular, the splitting of the ρ_{xx} minumum around the critical angle is intriguing. It may result from small density variations in the sample that lead to spatial separation of the polarized and unpolarized phases. The splitting is small, requiring a density variation of not more than ~2% across the entire 5-mm sample. Such a level of inhomogeneity is not at all unusual. On the other hand, phase separation might occur even in the absence of inhomogeneities, perhaps in analogy with domain formation. A consistent picture describing the nature of the phase separation would be most interesting. A puzzling feature of our data concerns the lack of a discontinuity in the magnitude of Δ at the transition. Although the ground-state energies are becoming degenerate at this point, there is no obvious reason why the excitation gaps should also coincide.

We have also studied the $\nu = \frac{4}{3}$, $\frac{5}{3}$, and $\frac{7}{5}$ FQHE states in tilted fields. All show some variation in activation energy with tilt. They do not, however, display the qualitative reentrant behavior we find at $\nu = \frac{8}{5}$, at least within the same range of angle, field, and temperature. Further measurements on these states are underway.

In summary, we have presented evidence for a phase transition in the FQHE at a filling factor $\nu = \frac{8}{5}$. The transition is driven by tilting the magnetic field away from normal to the 2D plane. We propose a model for our observations in which the FQHE ground state at $\nu = \frac{8}{5}$ undergoes a transition from being spin unpolarized at small angles to spin polarized at larger angles. The angular dependence of the activation energies yields a g factor for quasiparticles of the FQHE that coincides with that found for uncorrelated 2D electrons in GaAs.

It is a pleasure to thank R. Willett, B. I. Halperin, A. H. MacDonald, and P. B. Littlewood for discussions and Kirk Baldwin for his fine technical help. S. M. Girvin also deserves thanks.

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ACTIVATION ENERGIES FOR THE EVEN-DENOMINATOR FRACTIONAL QUANTUM HALL EFFECT

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Quantitative activation energy data on the $\nu=5/2$ even-denominator fractional quantum Hall effect are reported. The energy gap is found to drop linearly with total magnetic field in tilted fields with a slope roughly consistent with the bare g-factor for electrons in GaAs. These data further support the suggestion that the $\nu=5/2$ ground state is spin unpolarized.

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The discovery[1] of a plateau in the Hall resistance ρ_{xy} of two-dimensional electron systems (2DES) at the even-denominator Landau level filling fraction $\nu=5/2$ has generated a renewed interest in the general theoretical picture of the fractional quantum Hall effect (FQHE). Prior to the work of Willett, et al.[1] the FQHE appeared restricted, in theory and experiment, to solely odd-denominator filling fractions. In the standard picture, this restriction arose from the requirement of exchange antisymmetry of the variational wavefunction constructed by Laughlin[2] to describe the primitive FQHE correlated states at $\nu=1/3$, $1/5$, $1/7$, etc. and was found to persist through the higher order fractions $\nu=p/q$, $q-odd$, via the hierarchical model[3].

In the original picture of the FQHE, the electron spin was generally assumed fully polarized. The formation of spin-reversed pairs[4] of electrons, while energetically expensive, would obviously alter the effect of exchange antisymmetry and provides a plausible route to even-denominator quantization. Haldane and Rezayi[5] constructed an explicit spin-singlet wavefunction which displayed the FQHE at 1/2-filling. They further argued their wavefunction would be favored in the *second* Landau level and thereby account for the observation of the FQHE at $\nu=2+1/2=5/2$ and its apparent absence[1] at $\nu=1/2$ or $3/2$ in the lowest Landau level. The tilted-field studies of Eisenstein, et al.[6] showed a rapid collapse of the 5/2-FQHE as the angle between the normal to the 2D plane and the magnetic field was increased. Since the primary effect of such tilting is an enhancement of the spin Zeeman energy while the coulomb energy stays fixed, this result was cited as evidence that the ground state at $\nu=5/2$ contained a large number of reversed spins, in agreement with the model of Haldane and Rezayi[5].

The degree to which the so-called "hollow-core model" proposed by Haldane and Rezayi approximates real coulomb interactions between 2D electrons has been criticized[7]. Recent numerical calculations[8] on few-electron systems suggest the ground state at $\nu=5/2$ filling may be spin-polarized after all. It is the purpose of this contribution to present new data on the 5/2-FQHE from which the activation energy associated with the quasiparticle energy gap has been determined. The tilted-field dependence of the activation energy has also been measured, and lends further support to models in which the liquid ground state has a reduced, or even zero, net spin polarization.

The data presented here have been obtained using a GaAs/AlGaAs heterostructure with mobility $\mu=7x10^6 cm^2/Vs$ and 2D carrier concentration $N_s=2.3x10^{11} cm^{-2}$. Details of our experimental method have been published previously[6]. Figure 1 shows ρ_{xx} data at three temperatures over the magnetic field range spanning the filling factors $3>\nu>2$. At low temperatures the dominant feature is the strong FQHE minimum at $\nu=5/2$ filling. In contrast to our earlier data[1], there are additional FQHE features observable in this filling factor range. A strong minimum is observed at $\nu=7/3$ and a weaker one at 8/3.

There is no evidence of states at 9/4 and 11/4 as suggested earlier[9], although additional weak minima are evident in Fig. 1. Still higher sample quality and lower temperatures may be required to observe any hierarchical descendants of the 5/2-state.

Figure 2 shows the Hall resistance ρ_{xy} at 20mK for this sample. A fully formed plateau at $\rho_{xy}=2h/5e^2$ is observed. In addition, we find a plateau at $\rho_{xy}=3h/7e^2$ establishing the existence of the 7/3-FQHE in the second Landau level. It must be noted that as our samples are not etched into Hall bar geometries, the data contained in Fig. 2 may contain an admixture of the magnetoresistance ρ_{xx}. This has no significance for the well-established FQHE state at 5/2 for which ρ_{xx} is very small. We have verified this by employing several different contact configurations; in all cases the correct plateau at 5/2 is found.

In common with our earlier findings[1] the flanks adjacent to the 5/2-minimum rise up steeply as the temperature is reduced. In this sample however, the minimum at $\nu=5/2$ *falls* with temperature, rather than remaining roughly fixed as it did earlier. This obvious sign of higher sample quality allows for a determination of an activation energy Δ as defined by $\rho_{xx}=const.\times exp(-\Delta/2T)$. Figure 3 shows Arrhenius plots of ρ_{xx} at the 5/2-minimum *vs.* $1/T$. For the data taken in a perpendicular field, $\theta=0$, the plot is linear over a decade in resistivity with a slope giving $\Delta(\theta=0)=105mK$.

The activation energy measured for a perpendicular field, $\Delta(0)$, contains coulomb, exchange and Zeeman contributions as well as some effect of the disorder present in the sample. As such, it is hard to interpret its magnitude. On the other hand, the use of the tilted-field method allows isolation of the Zeeman contribution. This is because the correlation energies and the effect of disorder depend, to first approximation at least, on the perpendicular component of the magnetic field $B_{\perp}$ alone and this is held fixed in a tilt experiment. The spin Zeeman energy, by contrast, depends on the *total* magnetic field $B_{tot}=B_{\perp}/cos\theta$.

In Fig. 4 the activation energy Δ is plotted *vs.* B_{tot}. This is the obvious choice if the tilt dependence of the energy gap is dominated by the spin flip energy. It is apparent that the data exhibits a roughly linear dependence upon B_{tot}, supporting the idea that spin flips are involved. A simple model for the energy gap above an unpolarized ground state contains spin-1/2 quasi-electron and quasi-hole states each of which are Zeeman split. This leads to a gap of the form $\Delta=\Delta_0(B_{\perp})-g\mu_B B_{tot}$. From the slope of the straight line drawn in Fig. 4 we infer a g-factor of 0.56. The most recent ESR measurements[10] on uncorrelated 2D electrons in GaAs/AlGaAs heterostructures obtain $g\sim0.4$. This approximate agreement, while supporting the general picture outlined above, should not be over-emphasized. Without a definitive theoretical model for the 5/2-state we do not know whether such an agreement is to be expected. Secondly, the uncertainty in Δ becomes larger as θ increases since the span of ρ_{xx} values used to determine Δ is narrowing.

An important question is whether the data in Fig. 4 could result from a *polarized* ground state with spin-reversed excitations. This seems unlikely since creation of such a quasiparticle will reduce the net spin of the total system leading to an increase in the net Zeeman energy. It has been predicted[11,12] that spin-reversed excitations above the fully polarized $\nu=1/3$ FQHE ground state do lead to a positive Zeeman component to the energy gap. Thus, observing a gap that falls with increasing Zeeman energy (via tilting) seems a sign of spin-reversal in the ground state. Whether the ground state is completely unpolarized, or is only partially so cannot be determined from our data alone in the absence of a theoretical model.

There have recently been additional indications of unpolarized FQHE states within the 'conventional' odd-denominator FQHE[13,14]. At $\nu=8/5$ for example, Eisenstein, et al[14] have observed a re-entrant behavior strongly suggesting a phase transition between an unpolarized ground state at small tilt angles and a polarized one at higher angles. The collapse of the unpolarized 8/5-state at small angles is qualitatively the same as seen for the 5/2-FQHE. The interesting difference is that a polarized 8/5-state is available at higher angles (i.e. higher B_{tot}), whereas no such polarized version of the 5/2-state has so far been found.

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Figure Captions

Figure 1. Magneto-resistance ρ_{xx} in filling factor range $3>\nu>2$ showing deep minima at $\nu=5/2$ and $7/3$.

Figure 2. Hall resistance ρ_{xy} at 20mK in the region around $\nu=5/2$.

Figure 3. Arrhenius plots of ρ_{xx} at $\nu=5/2$ FQHE minimum for selected tilt angles θ.

Figure 4. Energy gap Δ *vs.* total magnetic field B_{tot}. Straight line gives g-factor $g=0.56$.

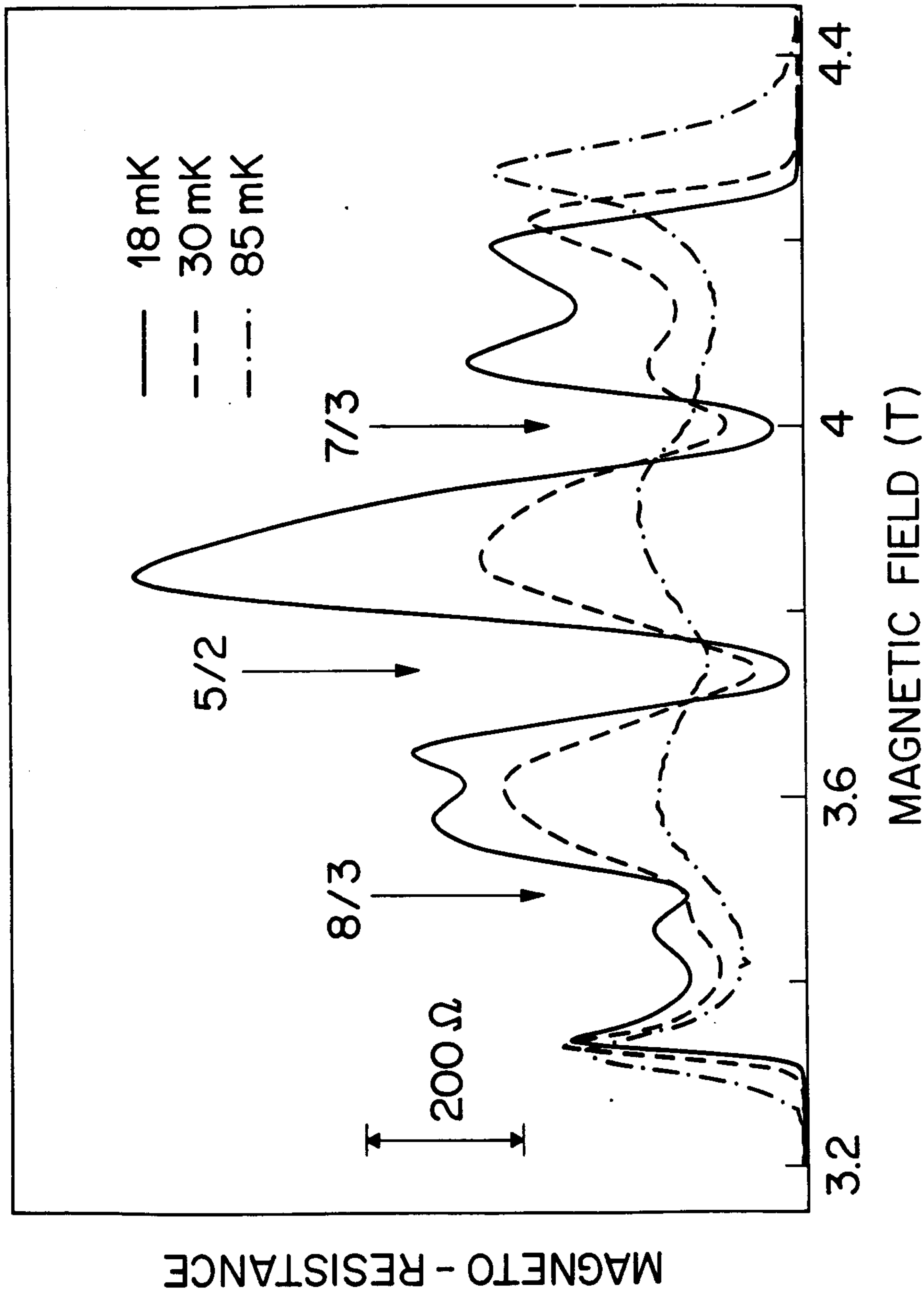
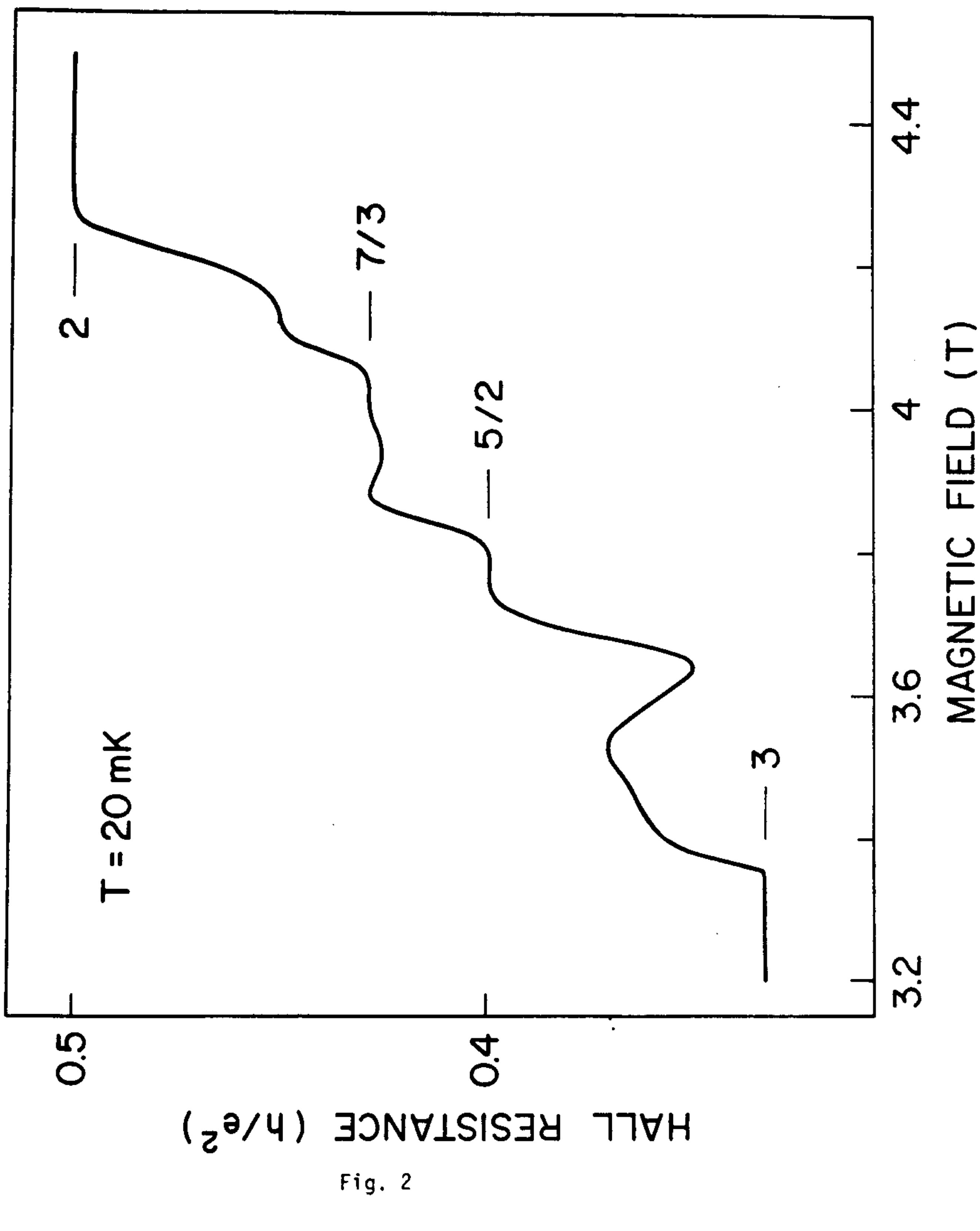

18 mK
30 mK
85 mK
7/3
5/2
8/3
200 Ω
3.2
3.6
4
4.4
MAGNETIC FIELD (T)
MAGNETO - RESISTANCE

Fig. 1



T = 20 mK
3
5/2
7/3
2
3.2
3.6
4
4.4
MAGNETIC FIELD (T)
0.5
0.4
HALL RESISTANCE (h/e^2)

Fig. 2

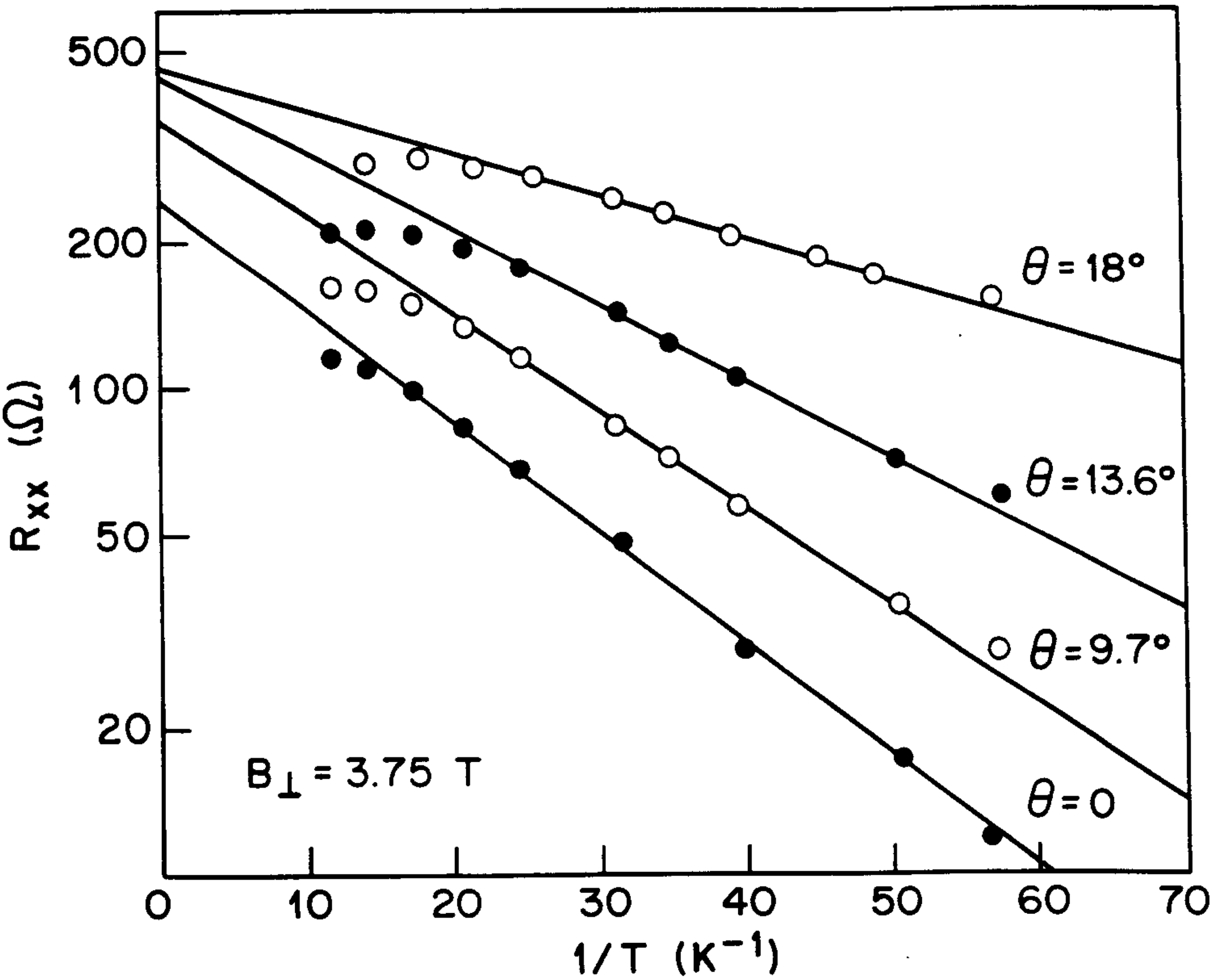

500
200
100
50
20
R_{xx} (Ω)
0
10
20
30
40
50
60
70
1/T (K^{-1})
θ = 18°
θ = 13.6°
θ = 9.7°
θ = 0
$B_{\perp}$ = 3.75 T

Fig. 3

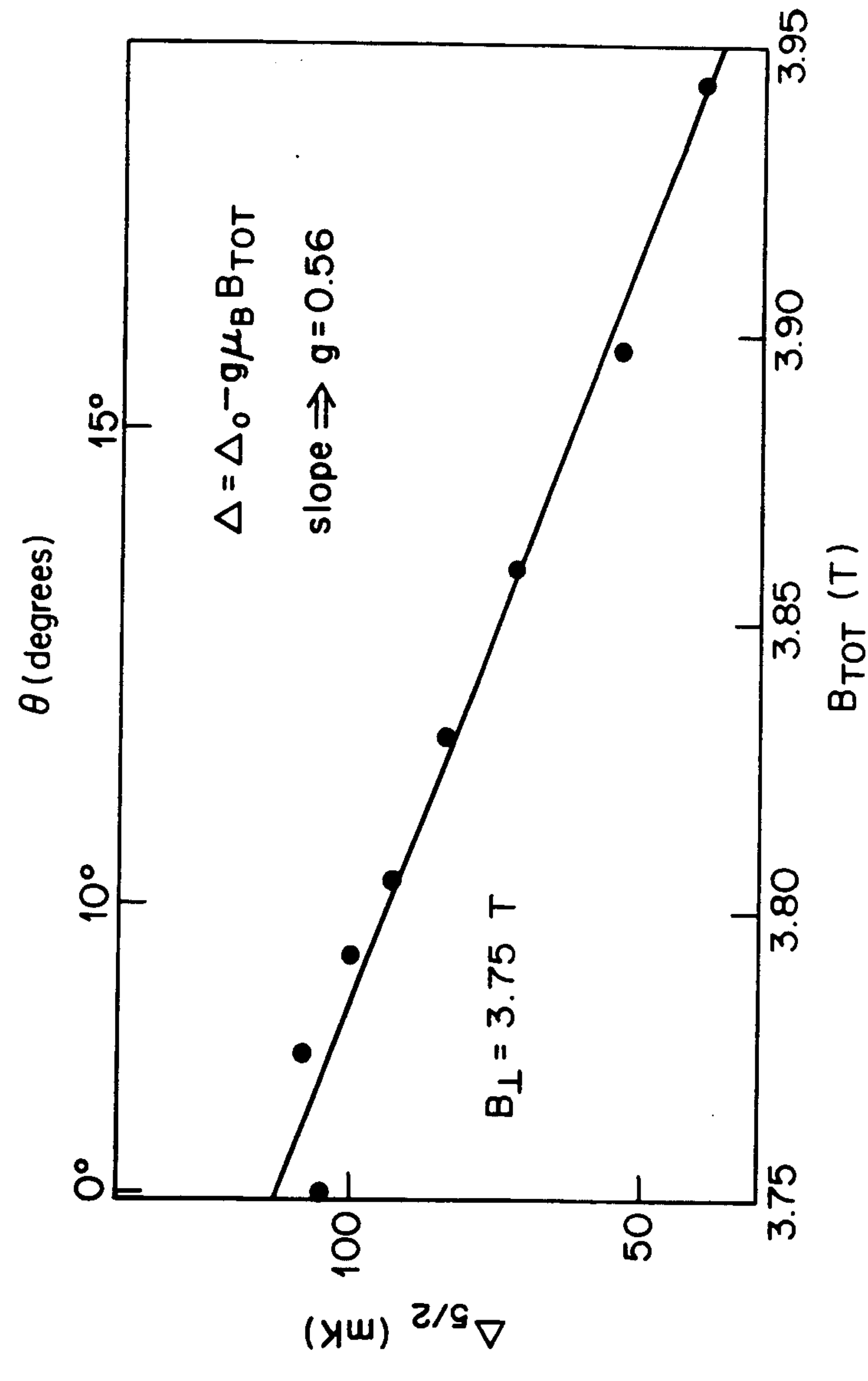

θ (degrees)
0°
10°
15°
Δ = Δo − gμB BTOT
slope ⇒ g = 0.56
B⊥ = 3.75 T
Δ5/2 (mK)
100
50
3.75
3.80
3.85
3.90
3.95
BTOT (T)

Fig. 4