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PHYSICS OF RADIOLOGICAL IMAGING

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ARGUMENTS DISCUSSED IN THESE TWO LESSENS

- *BASIC EXPERIMENTAL X RAY PHYSICS:SOURCES, DETECTORS*
- *RADIOGRAPHIC IMAGING*
- *ABSORPTION COEFFICIENTS*
- *STATISTICAL CONSIDERATIONS*
- *GAMMA (SENSITIVITY), BACKGROUND*
- *SPATIAL RESOLUTION, CONTRAST AND CONTRAST RATIO*
- *RESPONSE FUNCTION OF THE ABSORPTIOMETRY*
- *DUAL ENERGY ABSORPTIOMETRY AND K-EDGE FLUOROSCOPY*
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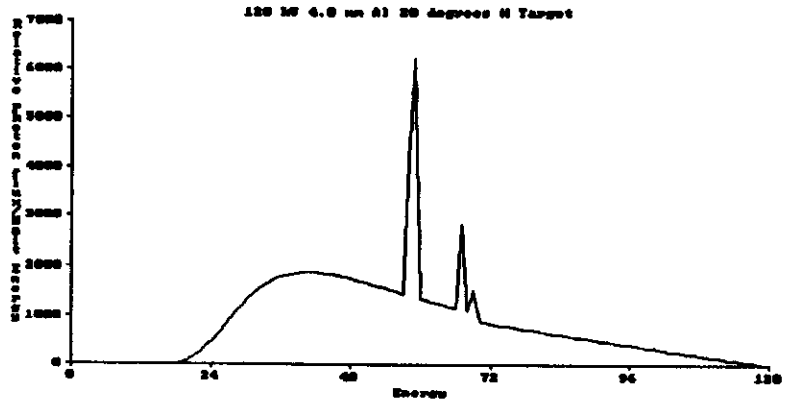
ARGUMENTS THAT MUST BE DISCUSSED

- *THE PROCESS OF PRODUCTION OF A RADIOGRAPHIC IMAGE*
- *IMAGING USING FILM AND FILM-SCREENS*
- *NOISE, AND SIGNAL TO NOISE RATIO*
- *IMAGE INTENSIFIERS AND THE NEW POSITION SENSITIVE DETECTORS*
- *MODULATION TRANSFER FUNCTION AND WINER SPECTRA*



1.INTRODUCTION

One of the early applications of the x rays, in the first months after their discover by Roentgen, was the production of radiographic images. The most interesting feature of these image resides in the possibility to see part of the imaged objects it is impossible to see directly. Many of us remember the image of the hand of the Roentgen's wife with the ring.

At that time the instrumentation in use consisted in a Crookes tube (the father of the modern x ray tube) and in a photographic film. These two devices are still in use in the modern radiography.

An x ray tube is an high voltage vacuum tube fig.1 with an anode and a cathode; the electron are strongly accelerated from the cathode to the anode impinging on the latter with a kinetic energy of thousand electron-volt (eV), being accelerated in the vacuum by a strong electric field produced by a ΔV of tenths of KV.

$$\Delta V = \frac{1}{2} m v^2 \qquad (1)$$

Where m is the electron mass; the electron impinging on the anode can be considered as monochromatic being their velocity at the start from the cathode negligible.

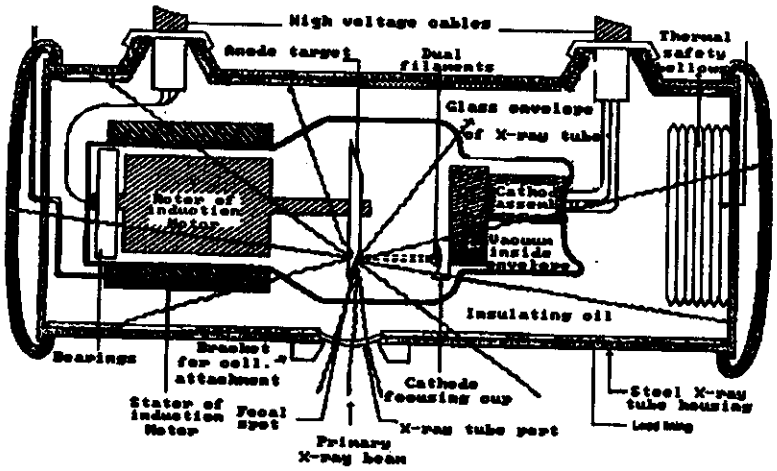
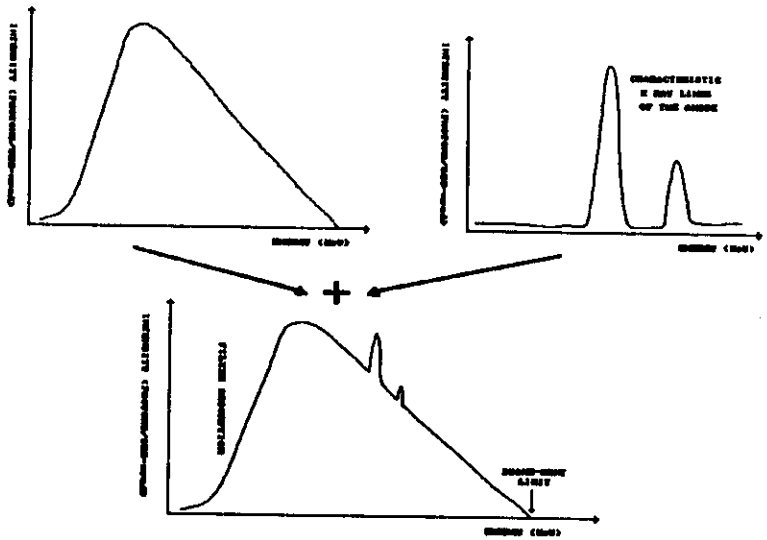


Diagram of an X-ray tube assembly that depicts the major components



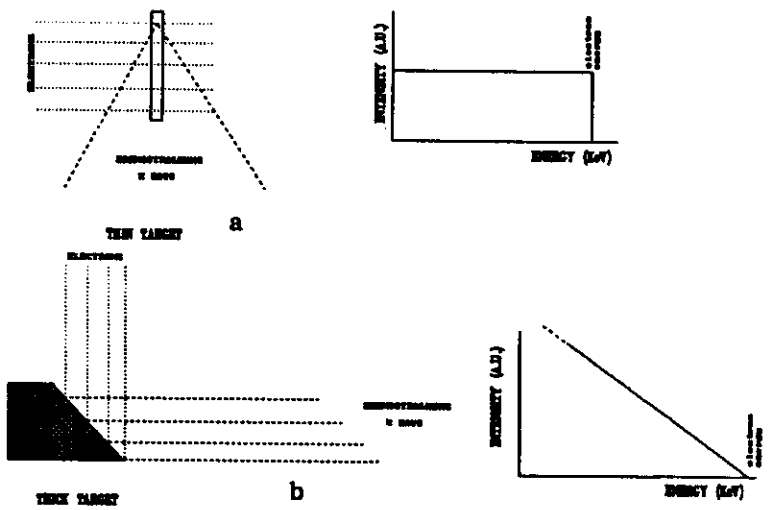
Entering in the anode the electrons give rice mainly to two types of interaction the bremsstralhung and the ionization. There are other different interactions but their probabilities to happen are for an electron (i.e. a charged particles characterized by a very large charge over mass ratio) very small.

Focusing our attention on the bremsstralhung it is easy to note that the beam emitted in the anode area in which the accelerated electrons impinge, is constituted mainly by the bremsstralhung photons emitted during the progressive deceleration of the electron in the target. The typical spectrum emitted is shown in fig.2; the principal characteristics of this spectrum are the linear decrease of the intensity for unit energy toward the higher energies and the maximum photon energy (Duane-Hunt limit):

$$I(h\nu)\, dE = K\,(h\nu_0 - h\nu) + B$$

$$h\nu_0 = \Delta V\,(KV)\ ; \qquad (2)$$

The bremsstralhung spectrum produced by an electron beam traversing a thin layer of material is continuous with equal intensity for unit energy (fig.3 a). However, the fact that the electron, enetring in the anode, traverse a thickness that is much more larger than their mean free path in the material, cause the linear decrease of the intensity with the photon energy (fig.3 b). In fact, the bremsstralhung produced in the layer immediately below the surface have proportionally less photon of high energy than the first one because some electrons have been partially decelerated and the mean energy of the electron beam is proportionally reduced. It is easy to image the process and the linear decrease of intensity, being constant the decrease of the electron energy for unit path-length. So in the case of an



electron beam traversing a thick target one can observe a maximum of intensity in the foreword direction. In particular, the position of this maximum depends from some parameters, such as the energy of the electrons and the material. The manufacturer give the angle of the target, that is usually calculated taking as zero an angle of 90° The decrease of the intensity toward the low energies is due to the greater absorption of these photons by the window of the vacuum tube. In many cases it is useful to place a thin sheet of an absorbing material after the window. The absorption of the photon by the filter depends by the material constituting the filter; the higher is its atomic number the higher is the energy of the maximum intensity in the tube spectrum. Sometimes it is useful to use a K-edge filter, i.e. a filter made with one or two element with a K-edge energies which allow the selection of one energy interval. In this case the spectrum emitted by the x ray tube encompass a small energy band.

The second kind of interaction, i.e. the ionization process, give rise to the production in the anode of the characteristic x ray lines of the elements present in the anode. For example in the very common case of a tungsten anode the following x ray line will appear in the spectrum emitted by the tube:

CHARACTERISTIC X RAY LINES OF THE TUGNSTEN

$K_{\beta 2}$	$K_{\beta 1}$	$K_{\alpha 1}$	$K_{\alpha 2}$	$L_{\gamma 1}$	$L_{\beta 2}$	$L_{\beta 1}$	$L_{\alpha 1}$	$L_{\alpha 2}$
69.09	67.20	59.31	57.97	11.28	9.96	9.67	8.40	8.333

Obviously, if the electron energy is less than the energy needed in order to extract one electron from its orbit the corresponding characteristic

x ray line will not appear in the spectrum. In order to see the characteristic K-line of the tungsten an high voltage of more than 70 KV is necessary.

A radiographic film can be considered as a continuous x ray detector, the working principle of which is basicly the same as for photography, i.e. the formation of a latent image through, in this case, the ionization of the silver atoms. In the following the development of the film, that is characterized by an amplification of the primary signal, will produce a continuous image. The efficiency of the detector is proportional to the linear absorption coefficient of the film and to the size of the silver bromine grains. A fast film is more efficient, i.e. with grains of larger size; in the development this kind of film give rice to large islands of silver that is possible to see with a microscope. This film is a low resolution one. The efficiency of a film is very small if compared with that typical of other x ray detectors, scintillator and solid state detectors, that is in the keV energy interval very high. The real advantage in using films is in the possibility to image a large area, in fact a film should be considered an "area detector". Until the last decade very few alternatives it could be found, and in general characterized by poor resolutions if compared to that typical of an x ray film. The storage capacity of a film is immense. Resolution for very fine grained film is about 4 μm^2; therefore, a 14"x17" film contains about $4x\ 10^8$ pixels; a figure that must be compared with the actual capacity of a digital system that is about 10^6 pixels. In this sense we are far to the possibility to substitute the film in the high resolution x ray imaging. Obviously if a resolution of 100 μm^2 is enough (or we want image a smaller area) we can find excellent alternative to the film.

The process with which a density pattern is formed by exposing the film to the radiation coming from the source and partially absorbed by the object can be schemetized as follow: (i) a x ray interacting with a silver atom reduce it, so the number of silver grain that is reduced for unit film area is proportional to the photon flux (photons/sec) impinging on this area; (ii) during the development a reduced silver grain will form a small cluster of silver that consisting of many silver atoms (amplification factor). The optical density (OD), that is a measure of the physical sensation of luminosity, it is related to the luminous flux emerging from one point of the film illuminated with an uniform light source by the relation:

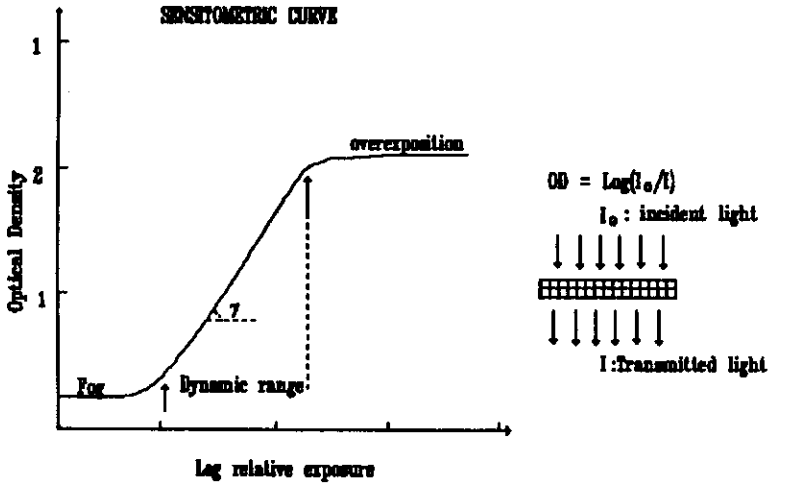
$$OD = \mathrm{Log}\,[\Phi/\Phi_0] \qquad (3)$$

where Φ and Φ_0 are the transmitted and incident flux respectively. The optical density is related to the number of silver atoms by a logarithmic relation:

$$OD \sim \ln (N) \qquad (3')$$

In the case of a polychromatic photon beam we must consider that the number of silver grains that is reduced in the passage of a single photon depends by the energy of this latter. Therefore the OD is more precisely proportional to the log of the exposure (Ex). Unfortunately, the amount of silver fixed depends from the exposure level, it means that the at very high or very low exposures the amount of silver fixed after the development is

smaller than in the optimal range of exposure. Therefore, the OP vs log(Ex) relation is linear only in this interval that is fortunately very large. The relationship between exposure vs optical density is commonly called an H&D curve; in fig.4 a typical H&D curve, describing the sensitometric response of a film-screen, is shown. This response function is characterized by several factors (i) the gamma (γ), (ii) the "fog", (iii) the amplitude of the exposure range in which the function is linear (latitude). The gamma is the value of $d(OP)/d(Log(E))$, it is constant in a range of two-three order of magnitude of exposure ΔEx. The fog is the shift toward higher OP values of sensitometric curve that can't be avoided, in fact in each film area some silver atoms are randomly reduced for many reasons, for example an unwanted pre-exposition of the film to light give rice to an higher value of the fog. One important consideration that it worth to do is that the film work as a counter working



in a integral mode, it means that it sum the exposures in the time. Therefore a pre or post- exposition of the film limits its performances, because some silver grains are already reduced.

The dynamic range of an x ray film is the exposure range in which the sensitometric curve is linear ΔEx, changing the film characterisitcs, size of the silver grains, superficial density of the grains etc., it is possible to extend and move the ΔEx. The corresponding ΔOD range is almost fixed by the readability of the film, it is usually in the range 0.8-2.5 OD units. Therefore the γ assume the meaning of a real measure of the performances of a film, in fact a smaller γ indicates a larger dynamic range. This range is usually very large in the modern x ray films, there are in addition many tricks to exend the γs, for example using fluorescent screens. In order to digitalize a film without lost of information we should consider that the ΔEx is usually of two-three order of magnitude and the precision should be of few percen. Therefore you should convert the signal with at least a 12-bit ADC. For example using different exposure conditions you can have in the some film

OD values in the range 0.8-3 OD, how many different gray levels you can differentiate in this range of OD values? It depends from the noise, in fact this is the some question of this. If you can measure the radius of a cell with a precision of 1 μm and the cell radius can be in the range between 5 μm and 105 μm how many cell size classes you can differentiate. The answer is 100. In the case of a photographic emulsion the fluctuations of the optical density for unit film area are normally distributed with a variance proportional to the inverse of the smallest area observed (A). It means that, if we sample the film with a microdensitometer having an aperture A, we can observe fluctuations of the OD in an area of the film uniformly exposed (see fig.5). If we increase this sampling area these fluctuations become less

A microdensitometer scan of a uniformly exposed processed photographic film. The x ray film exposed to light scanned by a 30 x 30 μm aperture. The density scale in OD units, the mean density being ~ 1.0

evident. This is exactly what happen with the eye which sample an image with a characteristic aperture, if the typical aperture of the eye is to small it will observe fluctuations in the number of photons arriving from each unit area and the sensation would be of a noisy object. In the case of a film all will depend from the number of silver grains for unit film area; in a high definition film this number will be very high and, if the exposure will be optimal, for the some sampling aperture we will observe a number of silver atom fixed larger with fluctuations that will be smaller (in other wards we will have a better statistic). If the amount of silver for unit film area can't be increased, in order to have not a noise sensation we must increase the sampling area, and consequently the resolution will be worse. In any case in a photographic film we can observe, using the eye, several OD levels but we can discriminate using a densitometer a much more larger number of levels. If we speak in terms of OD with a noise level of 0.02 in a range of 3 we can discriminate 150 gray levels. It is better to speak of number of counts, like in tomography or in terms of optical density? The answer is, if we consider the ability of our eye to distinguish different gray levels in general it is assumed that 256 (8 bits) levels are much more than enough to produce a graphic image in a display. In addition our eye is characterized by a logaritmic response function, therefore the use of the optical density units match this characteristic. More in general is better to reason in terms of

number of counts for unit area and time, because is more easy to make comparisons between different techniques.

If we acquire an x ray image with a 8 bits ADC we can observe that it is noisy and poor. Now if we observe the some image acquired with a system characterized by a wider dynamic range we are able to note that the image is less noisy. This is exactly what happens with a radiographic image, the some image taken with a low and high definition film and optimal exposure conditions appear of different quality. You should acquire with a wide dynamic in order to reduce the influence of noise in the image that you show to the eye. A film can achieve a dynamic of 10^5 that is very wide, if we speak in terms of exposure. The some wide range of exposure cannot be measured with an ordinary x-ray area detector, because it will probably saturate. In this sense a film is a much more powerful area detector. To give an example the ordinary CCD cameras are currently limited to 8 bits at video rates (30 frames/sec) only for particular applications is possible to use specialized cameras with wider dynamic (12 bits). These particular devices are necessary if you attempt to digitalized a film.

There are several sources of noise in the radiographic process of formation of the image; the primary source is the photon statistic, that in the tomographic systems is the most important one. In fact, for a given exposure level, the number of photon impinging on a small area of a film fluctuate following a Poisson distribution. In this case, being the variance equal to the number of photons, the relative fluctuations will decrease $\frac{\Delta N}{N} = \frac{1}{\sqrt{N}}$. The number of silver grains reduced will depend from the number of impinging photons and from the number of silver grains. The non uniform distribution of the number of grains for unit area, will amplified the fluctuation of the OD, such as the fluctuation in the number of silver atom fixed in the development process for each reduced silver grain. Unfortunately, these noise sources convolute between them, i.e. are in cascade, therefore the final noise will be the product of all these noises. This happen also in an electronic chain in which the noise generated in the detector is enlarged by the amplifier gain.

IMAGING WITH X RAYS

The basic assumption that one make in production of an image using the x ray is that the photon cross the imaged object following rectilinear lines. This means that the refractive index for the x rays is one, that is reasonably true in a first approximation. We assume in addition that traversing the imaged object only one part of the original number of photons are absorbed and that the removal of the photons from the beam traversing the object is due only to the interactions with the atoms belonging to this latter. With this hypothesis we can assume that the probability of interaction for unit length (the linear absorption coefficient, μ is constant along the target. In this case we can write the well known equation of absorption:

$$\frac{dN}{N} = \mu\, dx = d\mu\, x;$$

$$N = N_0 e^{-\mu x} \tag{4}$$

μ dx is the probability of an interaction in the path dx, such as $d\mu$ x is the increment of probability due to the μ change; μ depends by the sample density ρ and elemental composition:

$$\mu x = \frac{\mu}{\rho}\rho x \tag{4'}$$

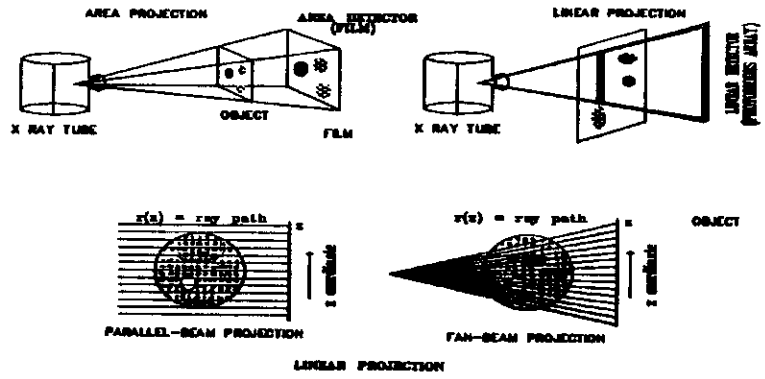
where μ/ρ is the mass absorption coefficient and m=ρx the mass for unit surface of the sample.

One useful concept to use is that of mean free path of the radiation in a target. If we know the composition (the matrix) and the density of the object being imaged we can calculate the mass absorption coefficient and also linear absorption coefficient. It is easy to deduct that the inverse of linear absorption coefficient is the mean free path of the radiation in the object. In fact:

$$\bar{x} = \frac{\int_0^\infty x\, e^{-\mu x}\, dx}{\int_0^\infty e^{-\mu x}\, dx} = \frac{1}{\mu} \tag{5}$$

being the probability that the path of one photon would be x equal to $\frac{N}{N_0}$. It is useful to note that the thickness $1/\mu$ is also that at which the intensity of the beam is $1/e$ of the original intensity.

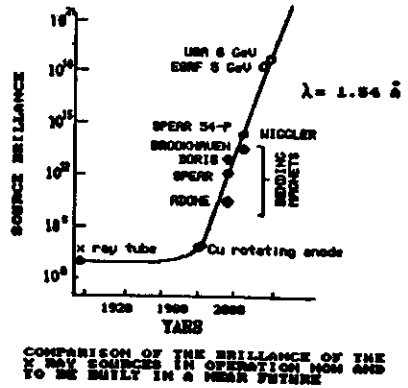
A very simple way to produce an x ray image is to perform a single projection obtaining a linear projection or an area projection (fig.6). Geometrical considerations, used also in the optic, lead to the conclusion that the source can be model as a point source if the distance between the source and the detector is much more larger than the diameter of the source. If this approximation can't be applied the source is extended. When we consider a point source approximation we speak about a fan beam, when the source is extended we speak of parallel beam (see fig.6). An x ray tube can be considered in general a point source, being its emitting area $\approx 1\,mm^2$ for an ordinary x ray tube and less the 0.5 mm^2 for a microfocus. An interesting problem to consider is the divergence of the beam and its brightness. The brightness of a source is the intensity emitted by one small area dS of it in the unit solid angle. A small divergence source, like a laser, is in general characterized by an high brilliance, because the beam is concentrated in a small solid angle. This occur also for the Synchrotron Light (SL) that is characterized by an high brightness and a small divergence of the beam. An high brightness low divergence source can be considered, as in the case of SL a parallel source more then a point source. In fig. 7 the brightness of a x ray tube is compared with that of actual and future SL sources; several orders of magnitude in brightness have been gained. The use of such x ray sources would be restricted to that problems in which an high brightness



and small divergence of the beam are mandatory, as in the microanalyses or in microtomography in which a small object is analyzed with a very high spatial resolution. Using these source in the imaging of big objects is not in general useful.

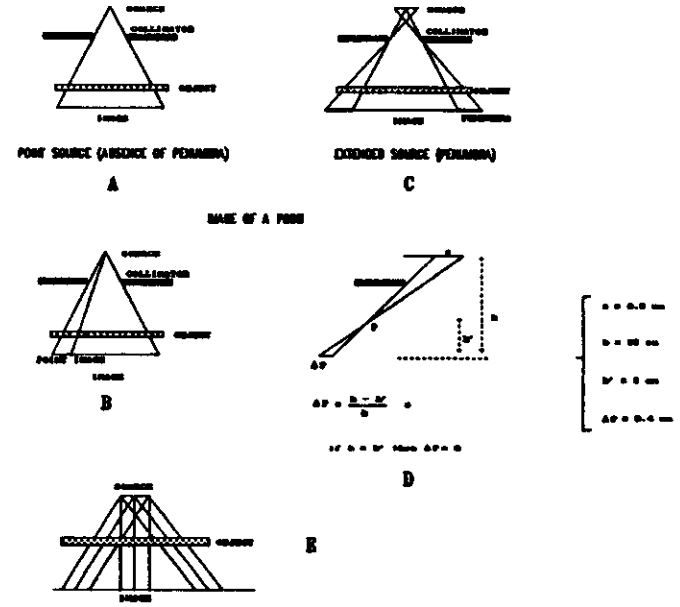
One problem with the image obtained with a single projection is the formation of the penumbra when the source is extended; in fact, as clearly shown in fig. 8 d, the projection of a point placed at a distance h' between the source and the detector (film) has a diameter ΔP given by:

$$\Delta P = (h - h')/h \qquad (6)$$



COMPARISON OF THE BRILLANCE OF THE X RAY SOURCES IN OPERATION NOW AND TO BE BUILT IN A NEAR FUTURE

Obviously the penumbra disappear when the detector is attached to the object and the latter is very thin; unfortunately this approximation seldom can be applied in the case of medical radiography. An object imaged with a fan beam geometry is magnified h'/h times, this occur also in the tomographic techniques using fan beam. The use of a parallel beam can be done only using well collimated position sensitive detectors, in fact, as shown in fig. 8 e the image produced with a

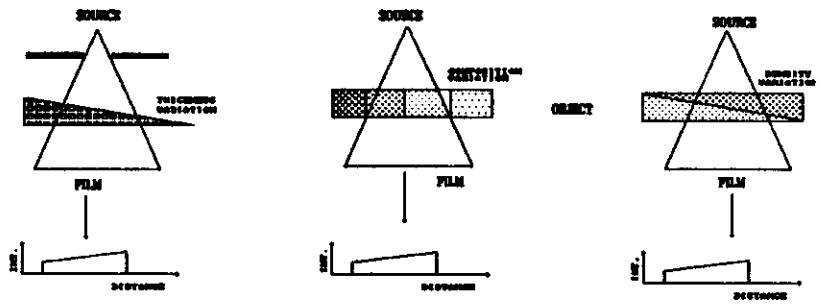


parallel beam without collimator is completely degraded by the presence of the transverse rays (an extended source can be seen, obviously, as the sum of an infinite number of point sources). In tomography parallel beam geometry are in use (with a narrow collimation of the detectors), whereas in radiography this case is very rare.

In a single projection (see fig.6) the process of imaging of a three dimensional object can be schemetized as follow: a) a three dimensional object can be represented by its ability to absorb the photons (μ(x,y,z/E) where E is the energy of the photon; b) the photons impinging in one element of the area detector (e.g., a small area of the film) are given by

$$N(x,y) = N_0 \int_r e^{-\mu(x,y,z)l}\, dl \qquad (7)$$

In the above equation the integral is along the x ray path (r), that in the case of a fan beam is a transverse line. At the end of this process the image we obtained is a two dimensional representation (N(x,y)) of a three dimensional object, it means that we have lost something in the process. In particular, the integral along the line imply that we lose the information about the changes of μ along the path. In this simple image formation model we can



consider that one x ray path (r(x,y) correspond to one point in the area detector. Therefore the response function of a radiographic system can be written in a first approximation as:

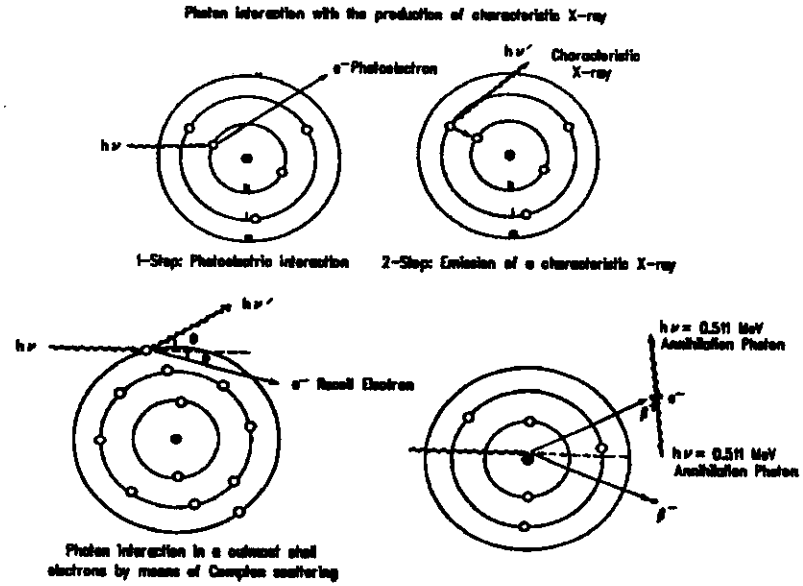
$$N(x,y) = N_0 e^{-\mu r(x,y)} \tag{8}$$

In the above equation we make the assumption that r is the path length inside the object and μ is a mean absorption coefficient along theray path.

Unfortunately with a single projection we are not able to discriminate a variation due to a change in thickness or in density from another one due to change of mass absorption coefficient. In fact a variation of density can mask a variation in thickness and composition and vice versa (fig. 9). This is a severe restriction for the single projection radiography, only the advent of the tomographic techniques have partially removed these limitations.

The mass absorption coefficient μ/ρ depends only from the elemental composition of examined sample and from the energy of the incident photons. This coefficient can be divided in three parts which are related to the major interactions that take place during the passage of a x ray photon beam of energy less then 100 KeV through a sample (see fig.10, in these energy range the annihilation processes don't exist). That interactions are the photoelectric, Compton (or incoherent) and Rayleigh (or coherent) scattering. The dependence of these three parts of the mass absorption coefficient from the atomic number of the sample and the energy of the photons can be approximately written as follow:

$$\frac{\mu}{\rho} = \frac{\mu}{\rho}_{ph} \frac{\mu}{\rho}_{incoh} \frac{\mu}{\rho}_{coh} = 20.64\, E^{-3.28} (Z^*)^{4.62} + \sigma(E) Z^* + 2.8\, E^{-2.02} (Z^*)^{2.86} \tag{9}$$



where σ(E) is the Klein-Nishina cross section and Z^* is the effective atomic number of the sample (Rutherford et al. 1975). These formula shows as the dependence from energy and atomic number of the photoelectric part of the coefficient is stronger (3.24 and 4.62 respectively) that of coherent and incoherent. This mean that the decrease of this contribution is proportionally more rapid that of the other two terms. In this sense we can speak about one energy range in which the photoelectric scattering is dominant and an energy range, at higher energy, in which the incoherent scattering is dominant.

When we study the absorption by a sample of a polychromatic photon beam we should take into account the changes with energy of the mass absorption coefficients. The response function of an absorptiometric technique in this case change as follow:

$$N(x,y) = \int_0^{\infty} N_0(E)\, e^{-\mu r(x,y)}\, dE \tag{10}$$

Where the integration will carried out on the whole source spectrum ($N_0(E)$). It is in general cumbersome to study the response function of absorptiometric technique considering the effect of polychromaticity of primary radiation,

therefore it is considered an acceptable simplification to represent the primary beam with an effective energy, for example the mean energy:

$$E = \frac{\int_0 E\, N_0(E)\, dE}{\int_0 N_0(E)\, dE} \qquad (11)$$

This simplify the formalism, but don't underline adequately effects that are in general related to the fact that the primary spectrum (e.g., the spectrum of x ray tube) and the detected spectrum (i.e., that impinging on the area detector) are different. This effect is referred to as beam hardening, in fact the detected spectrum is more rich of high energy photons than the primary one. In order to eliminate the influence of this effect we should use an energy dispersive detector (such as a solid state detector), but this is in practice impossible. It is useful to remark that the influence of this effect is very strong if we attempt to correlate absorption measurements made on samples with large changes in composition. This effect can be in general neglected in medical radiography because in this case the variation of μ are very small.

THE RESPONSE FUNCTION OF ABSORPTIOMETRY

When we look at the distribution in the plane x,y of the optical densities we can think about some question as: (i) how I can define the minimum detectable variation in OD, (ii) what is the minimum detectable OD, (iii) how is possible measure the spatial resolution, etc.

To start with the answers we should define what we measure and which is the most useful approach to handle this argument. If we look at a radiographic film the measured quantity is the optical density, but it is self evident that its usefulness concern more the characteristic of our vision than its physical meaning. In fact, to approach the not easy argument of the detectabiliy of one defect in an image we should fix the borders of our considerations to a particular aspect. In this case we are interested to the detectability in a physical sense, therefore we will not speak about the ability of our eye to detect a particular defect in an image. To stress this we will use the number of counts detected for unit film (detector) area (N), that is a figure having a more direct physical meaning that the OD, with which is strictly correlated.

One very useful approach is to study the response function of a theoretical absorptiometric device, i.e. a single projection, as a function of photon energy sample thickness and composition, in order to fix some basic concepts. We can limit our considerations to a parallel beam, in order to simply the formalism, and we start with the case of an uniform sample or a sample surrounding a defect (i.e., a zone with a different linear absorption coefficient). If we can model our object as an uniform sample, it is possible to measure its linear absorption coefficient knowing its thickness (L) (or knowing its mass for unit surface, we can calculated its mass absorption coefficient). What is the smallest variation of detected counts (ΔN) that we can appreciate? The answer is immediate if we follow a simple statistical

approach. It is well know that the detected counts follow a Poisson density distribution, therefore a good estimation of the variance is N (if we can repeat the measure n-times we can use the well known formula to estimate the variance). Therefore, we can consider that the difference between two counts N' and N is statistically significant if:

$$N' > N + b\,\{\sigma^2(N') + \sigma^2(N)\}^{1/2}$$

$$\text{if } \sigma^2(N') \approx \sigma^2(N) \text{ then } N' > N + b\sqrt{2\,\sigma^2(N)}$$

$$\frac{dN}{d\mu} = -L\,N$$

$$\Delta\mu = \frac{b\sqrt{2}}{L\sqrt{N}} = \frac{b\sqrt{2}\,e^{\mu L}}{L\sqrt{N_0}} \qquad (12)$$

where we can choose b (≈ 1,2,3) according to the uncertainty level satisfactory for our application.

It worth to discuss a concrete case as that of a small cube of side dl inside of an uniform sample (see fig. 11). If we take a single linear projection of this object we will obtain a result very similar to that shown in fig. 11. We know that the $\Delta N = N - N'$ is the contrast and N_0 are the impinging counts and $\frac{\Delta N}{N}$ is the contrast ratio. Observing the detected profile you can note that the counts for unit film area don't reproduce exactly the edge, this is due to the resolution of our measuring system. It is easy to image that with a poor contrast it is impossible to detect this edge.

With the equation 12 we have answered to the question of the minimum contrast that we can detect in a uniform sample of lenght L. In the case of a defect immerse in a uniform object the variances that we must consider are that of N' and N. Looking at fig. 11 we can observe that we can calculate these variancess ampling the function N(x) at left and/or at right of the defect and inside the defect respectively. In a completely uniform area of the sample we can assume that the variance is statistical, therefore we can approximate $\sigma(N) \approx \sqrt{N}$. Under this hypothesis, assuming the thickness of the defect (dl) and of the object (L) as constants, we can affirm that $\Delta N = N - N'$ $\Delta\mu$ can be written as:

$$\Delta N = N - N' = N_0\, e^{-\mu' L}\,[e^{-\Delta\mu dl} - 1] \text{ and}$$

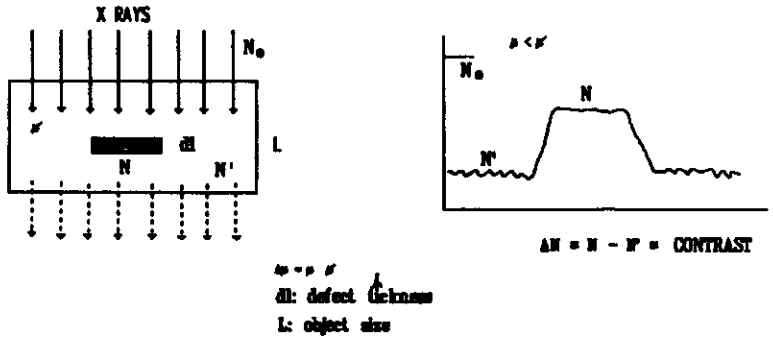
$$\frac{N}{N'} = N_0\, e^{-\Delta\mu dl}$$

where $\Delta\mu = \mu - \mu'$

and the sensitivity to μ (or to $\Delta\mu$) is given by:

$$\frac{dN}{d\mu} = -N' \, dl \, e^{-\Delta\mu dl} = -N \, dl \qquad (13)$$

Therefore the sensitivity depends directly from the defect size dl and it decreases with the object thickness (L).



It is useful note that the minimum detectable ΔN can be also written as follow:

$$\Delta N = -N \, dl \, \Delta\mu = b\sqrt{2N}$$

so the Δμ can be written as:

$$\Delta\mu = \frac{b\sqrt{2}}{dl\sqrt{N}} \qquad (12')$$

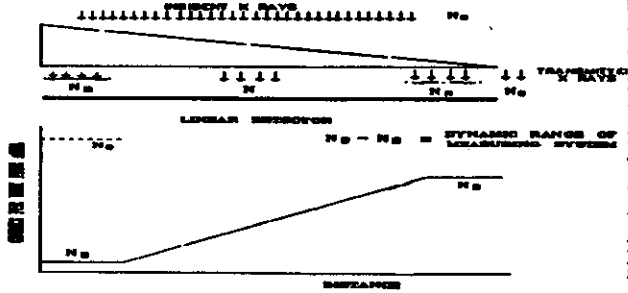
It worth note that either the sensitivity as the minimum detectable ΔN or Δμ depend by L. It mean in particular that they worsen if the some defect is inside a bigger object. This severe limitation con be overcome only using the tomography; if we take L=0 in the equation 13 and 12' we obtain two relation valid for tomography and we can immediately note that the contrast ΔN is greater. In tomography the quantity that is reconstructed is the map of linear absorption coefficients μ(x,y) the equation 12' is still valid if we consider that each ray sum can be decomposed in m pixel of length dl (L = m dl), therefore

the average detected counts are $N_D = m\,N$, substituting N_D in equation 12' we obtain the following relation:

$$\sigma(\mu) = b\frac{\sqrt{2\,m}}{dl\sqrt{N_D}} \qquad (14)$$

This relation is very similar, apart the terms depending from the reconstruction algorithm, to that deducted by many authors. Our deduction clearly show that one of the advantages of tomography is in the decrement of the minimum detectable Δμ or in other words an increment of the contrast. Another advantage is that the dl in the tomography is chosen "a priori", consequently we can more easily optimize the contrast, increasing or decreasing the exposure (and the dose) at the desired level.

A deduction that one can do from the above equations is that we can decrease the minimum detectable Δμ at will increasing the exposure time, i.e. the detected counts for unit area. Can this be true? obviously not. What is inexact in our model to lead to these wrong conclusion? nothing of really unrecoverable, we have forget that our system, as all real measuring systems, is characterized by a dynamic range and by a background (N_B). In particular, as clearly shown in fig. 12 there are two limits of total counts that you can detect for unit detector area and unit time, the background N_B counts and the N_S saturation counts. Our measuring system is in fact characterized by a background counting rate, that is the minimum amount of counts per unit time that are in any case detected, which origin can be scattering in the sample or noise in the detector, and by a saturation counting rate, which is the maximum number of count per unit time that the film or the detector can detect. If an higher flux of photons reach the detector the detected events fluctuate around the N_S value. The equation 12' is therefore valid only in the range $N_B < N < N_S$. The dynamic range of our measuring system is $N_S - N_B$, one advantage in using x ray films is that the dynamic range coincides with the OD interval in which the response function OD vs log(Ex) is linear, at the margins of this interval the OD still increases or decreases with a smaller γ.



In this sense the film is more reliable detector that other that at the margins of the dynamic range suddenly stop to work. Is frequent to see radiographic films that are overexposed or underexposed but that can be read, even with some difficulties. When we speak about the need of a wide dynamic range in theray radiography, we implicitly refer to the fact that is not easy to match the dynamic of X-rays impinging on the detector with the detector dynamic range.

Another interesting deduction that we can do from equation 12' is, considering only statistical fluctuation, the minimum detectable $\Delta\mu$ and the minimum detectable defect size (dl) are inversely proportional. To detect smaller defects we should increase the contrast, for example increasing the statistic or increasing the $\Delta\mu$, choosing an energy in which it is maximum. Around these considerations turn the optimization of a radiographic exam.

Let us suppose we can choose the object thickness or its μ (changing for example the energy of primary beam), we can know for example the optimal thickness of the sample? The answer can be easily deduct minimizing the $\Delta\mu$ as a function of the thickness (L), this can easily done deriving equation 12 and putting this derivative equal to zero, the result is veri simple and attractive:

$$\mu = \frac{2}{L} \tag{15}$$

The some things can be done for the equation 12' deriving for the defect size dl and arriving to the conclusion that the $\Delta\mu=2/dl$, fixing thus an optimum $\Delta\mu$ for a defect size.

Anyway it is possible to ask to the response function of an absorptiometric system which is the optimal set of parameters in order to minimize for example the minimum detectable contrast. It is necessary underline that some of these parameters can't be changed in regular medical systems, such as the path length of the beam inside the body that is fixed by the body size. This is not true in general and the new revival of microradiographic and microtomographic techniques show how are the potentialities of the absorptiometric techniques when you can optimize simultaneously all the parameters. The procedure to find the optimal parameters set it is very easy in the case of a single energy system but can be more cumbersome for dual energy absorptiometer or for CT systems. The most interesting part of these studies are usually the optimization of the primary(-ies) beam energy. In order to correctly place the question we should remember there are many problems that you can solve using an absorptiometric system; in this sense you should define before the nature of the wanted information. Using an absorptiometric system it is possible to determine the density and the elemental composition of the examined sample. In particular, it is possible also to measure thicknesses (or better thickness variations), assuming that the sample is homogenous, this kind of measurement, frequent in industrial use, is quite infrequent in the medical applications. How it is possible to determine the optimal conditions in order to determine such quantities? The answer in essentially in the dependence of the μ from the quantity we want measure. In particular, it worth immediately recall that the mass absorption

coefficient of a particular element $((\mu/\rho)_d$ is the sum of the three partial coefficients for photoelectric $(\mu/\rho)_{ph,d}$ incoherent (Compton) $((\mu/\rho)_{incoh,d}$ and coherent $((\mu/\rho)_{coh,d}$ scattering respectively. The dependence from Z of these coefficients is strongly different, such as their dependence from energy. In particular, the photoelectric coefficient is dominant in the low energy range, whereas the Compton one is dominant at higher energies. The dependence of the Compton coefficient from Z is about linear, therefore in the energy range in which this interaction is dominant the dependence of the μ from Z is more weak than in the energy range in which the photoelectric interaction is dominant. All these statements are well known to the people working withrays. At first glance we can conclude that in order to optimize the energy we should find that energy of the primary beam that maximize the $\Delta\mu$ between the two parts of our object we want discriminate. This task is very simple because if we know roughly the composition of the particular we want observe and that of the surrounding tissue we can choose that energy for which the $\Delta\mu$ is maximum. It worth stress that we should maximize the $\Delta\mu$ and not the $\Delta(\mu/\rho)$ because several times the major contribution to the change is due to density variations more than Z ones. Unfortunately, working with radiography we can't do this simple procedure because we must take into account also other significant parameters, such as the object size. A statement that it is easy to do is the following: if you want measure Z-variations you should use lower energy in order to increase the contribution of photoelectric part of mass absorption coefficient, if you want measure density variations is better to use higher energy in order to increase the Compton contribution. In practice you should found an useful compromise between the energy that optimize the counting rate and the energy that maximize the $\Delta\mu$, taking the object size as fixed.

DUAL ENERGY ABSORPTIOMETRY

The observed sample can be model in many cases as a two component mixture, such as in the case of measurement of Bone Mineral Content (BMC). In this case we can suggest a simple model(see fig. 13) in which a central region of the object is composed of a mixture of one component having the Z of a completely mineralized tissue and the other of a typical soft tissue. This central region is surrounded by soft tissue. The problem is more or less the some as before in which a small defect was surrounded by an uniform object. In this case the problem is to find the weight fraction of bone in the central region. That is a typical case in the absorptiometric measurements, it is convenient to approximate the μ of the central region as follow.

$$\mu = \mu_{st}\, w_{st} + \mu_{bm}\, w_{bm}$$

$$w_{st} + w_{bm} = 1 \tag{16}$$

Now we can use two independent absorptiometric measurements, for example at two energies in which dominate the photoelectric and Compton contribution respectively. It is possible to write two absorptiometeric relations:

$$\log\left(\frac{I_B}{I_T}\right) = (\mu_{bm}(E_1)\, w_{bm} + \mu_{st}(E_1)\, w_{st})\, dl$$

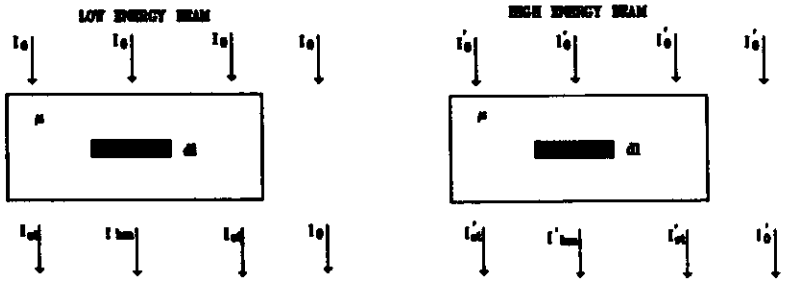
$$\log\left(\frac{I'_B}{I'_T}\right) = (\mu_{bm}(E_2) w_{bm} + \mu_{st}(E_2) w_{st})\, dl \qquad (17)$$

Where I_T, I_B, I'_T and I'_B are the intensities of the beams at the two energies E_1 and E_2 respectively. Solving the system of two equations considering the two unknowns the thickness of the bone and the w_{bm} we obtain:

$$w_{bm} = \frac{\mu_{st}(E_2) - k\,\mu_{bm}(E_1)}{k\,\Delta\mu - \Delta\mu'} \qquad (18)$$

where $k = \frac{\log(I_B/I_T)}{\log(I'_B/I'_T)}$ and $\Delta\mu = \mu_{bm}(E_1) - \mu_{st}(E_1)$ and $\Delta\mu' = \mu_{bm}(E_2) - \mu_{st}(E_2)$

It is evident that the precision with which it is possible to measure w_{bm} depends from the statistical error on the measure of the intensities at the two energy. It is easy to conclude that using two energies, such in the case of Bone Mineral measure, at the higher energy we have an higher counting rate, the μ's are lower, but the difference between $I_0(E_2)$ and $I(E_2)$ are smaller, whereas at the lower energy happen the opposite. In practice, we detect the Bone Mineral with the lower energy and the higher energy measurement is used in order to correct for the size of the bone region. Therefore, we can optimize the lower energy in order to measure the w_{bm} and the higher energy to measure the bone size. This is what it is done in practice, for example to measure the Bone Mineral of a small size bone, as that of the ulna in the leg, we can use an energy of about 30 KeV that is not convenient for the



measure on the spine, which is placed inside the trunk with an optical path of about 20 cm and an absorption that is very high. In this case we need an higher energy in order to optimize the measurement.

It is possible to make the some consideration in the case of two white spectra obtained with two different KV and filtration of the beam. This is done for dual energy tomography, in which are use two mean energy, one that fall in the energy interval in which the photoelectric contribution is dominant and the other in an interval dominated by the Compton. This is actually an little more than an attractive prospective because the construction of the dual energy tomographs is a not easy job.

K-EDGE FLOROSCOPY.

Another very interesting possibility, that is used in practice in the radiographic exams, is the use of a contrast medium, i.e. the injection in the observed object of a substance having an higher absorption than the object matrix. Assuming that the injected substance is concentrated in some parts (substructures) of the object we can evidentiate better the shape and the size of these substructure. This is done in particular for the arteria, injecting in the blood circulus an iodinated medium, this technique is commonly know as angiography. There are several problems with angiography especially if we attempt to perform dynamic studies. In fact, if we want detect the perfusion of the iodinated medium in the circle we must inject the medium instantly, also in this case, if we use only one energy it is not easy to determine the size of the vassels and of the obstructions. Now there are several attempt to perform angiographic studies using two energies, one above the K- abserption edge of the iodine and one below. In particular the use in this case of a synchrotron source is very attractive because you can produce two intense monochromatic beams very near the K-absorption edge, so we can completely observe advantage of the discontinuity of the iodine linear absorption coefficient.

The K-edge floroscopy is the technique with which we can quantitatively determine the concentration of the contrast mediun in a region of the observed object or the size of this region. In fact, if $\mu_{(E1)}$ is the the linear absorption coefficient below the K-edge and $\mu(E_2)$ that above we can model these coefficients as follow:

$$\mu(E_1) = \mu_T(E_1)\, w_T + \mu_I(E_1)\, w_I$$

$$\mu(E_2) = \mu_T(E_2)\, w_I + \mu_I(E_2)\, w_I \qquad (19)$$

with $w_T + w_I = 1$

The two parts of the linear absorption coefficients are referred to the tissue (T) and to the contrast medium (I) respectively. Being the two energies very near we can assume that:

$$\mu_T(E_1) \approx \mu_T(E_2) \text{ and}$$

$$\mu_I(E_2) \approx \mu_I(E_1) + \Delta\mu_K$$

so $\mu(E_2) = \mu(E_1) + \Delta\mu_K w_I$

where the $\Delta\mu_K$ is the jump of the linear absorption coefficient at the K-edge.

We can so write the two following absorptiometric relations:

$$\log\left(\frac{I_0}{I}\right) = \mu(E_1)\, dl$$

$$\log\left(\frac{I'_0}{I'}\right) = (\mu(E_1) + \Delta\mu_K w_I)\, dl \qquad (20)$$

solving the system for the w_I we obtain:

$$w_I = \frac{\mu(E_1)}{\Delta\mu_K}\left(\frac{\log(I_0/I)}{\log(I'_0/I')} - 1\right) \qquad (21)$$

The I_0, as in the case of the dual energy, are the intensity detected passing trough a section contenting only tissue. In this case the first absorptiometric relation, that at an energy below the K-edge, is used in order to correct for the thickness of the substructure with inside the contrast medium, whereas the wieght fraction of the iodine is determined through the second relation.

Appendix I

The concept of resolving power of a measuring system. If x is the quantity we want esteem and f(x) the measured quantity, the resolving power Δx is related to the response function f(x) of the device and to the estimated standard error of the measure $\sigma(f(x))$ by the following relation:

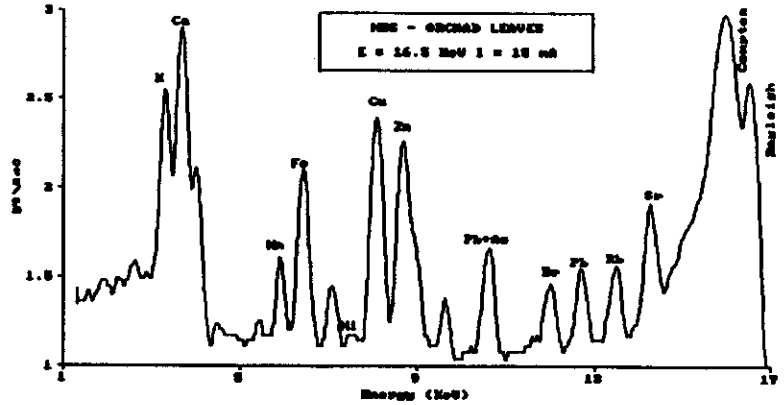
$$\Delta x = \frac{b\,[\sigma^2(f(x_1)) + \sigma^2(f(x_0))]^{1/2}}{\frac{df(x)}{dx}} \qquad (1a)$$

Where the $df(x)/dx$ is the sensitivity and $\sigma^2(f(x))$ the variance of the measured quantity. In the above equation if we assume that measured quantities $f(x_{1,0})$ have the same variance we can simply the relation as follow:

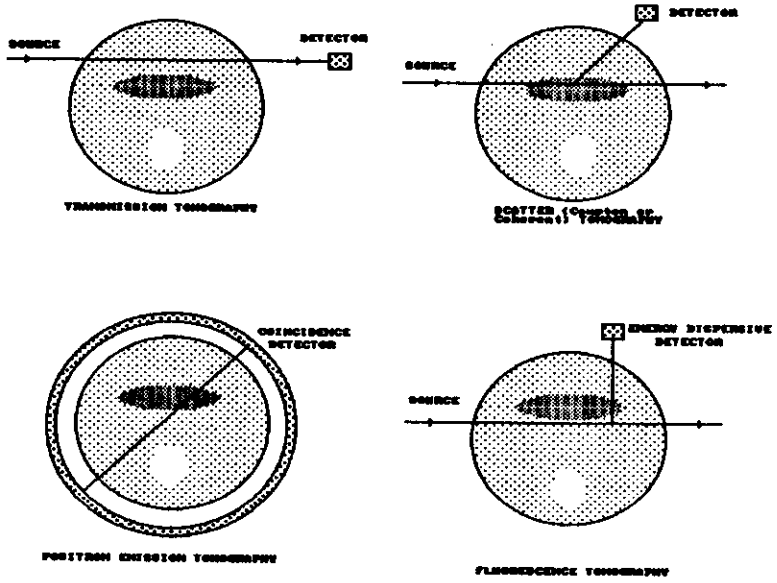
$$\Delta x = \frac{b\,[2\,\sigma^2(f(x))]^{1/2}}{\frac{df(x)}{dx}} \qquad (1'a)$$

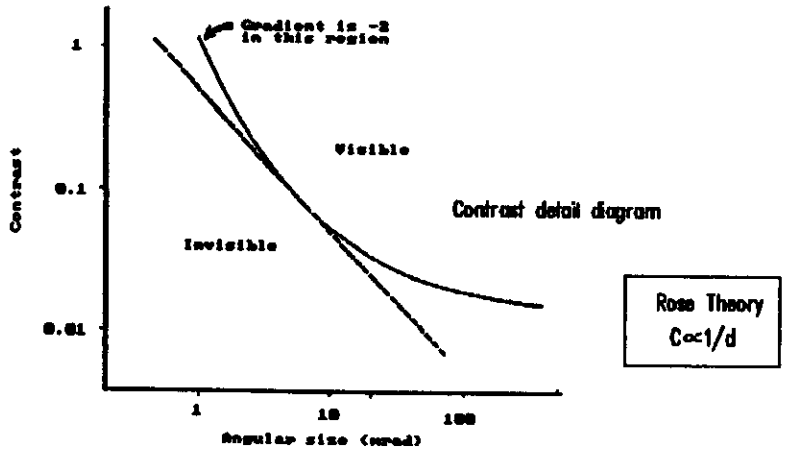
The parameter b can assume values (1-3) that are related to the level of uncertainty of the estimation the we can accept. In the case in which we measure independently n time this variation the $\sigma 2$, is the calculated variance. If we measure only one time the value f(x) we should make assumptions on the statistical distribution of this quantity. This can be easily done in the case of the measurement of total counts in a certain time interval

(or of the counting rate), because in this case we can assume a Poisson density distribution. In order to look inside the model we are now proposing for the study of a measuring system, let us speak one moment about the response function of x and/or γ) ray spectrometric system. It consists of a detector with an intrinsic ability to discriminate photons of different energies, an electronic chain and a multichannel analyzer. In fig.14 a typical spectrum as measured with this system is presented. It consist of discrete structure, the peaks, and a continuous background which originate, with different mechanism, either in the spectrometer either outside it. In any case this background cant be suppressed, as can't be suppressed the noise that produces the fluctuation in the total number of counts in a particular peak.



Let me consider now the measure of an unknown activity of one particular radionuclide. In this case we can have two different possibilities (i) the activity is very low and in this case we wish know if some activity is present in our sample or (ii) the case in which the presence of the radionuclide is evident and we want measure it or variation of it. In the fist case the variance that we must consider in the above equation is that of the background counts, whereas in the second case the variance to be used is that of the detected counts. In particular we can stress that in the first case we are measuring the minimum detectable activity, whereas in the second case we are estimating the minimum detectable variation. These concepts can be extrapolated usefully to the absorptiometric measurements.





Contrast detail diagram

Rose Theory
C∝1/d

----- Expected relation between contrast and angular size according to Rose's theory

—— Experimental results (Hay and Chesler,1976)

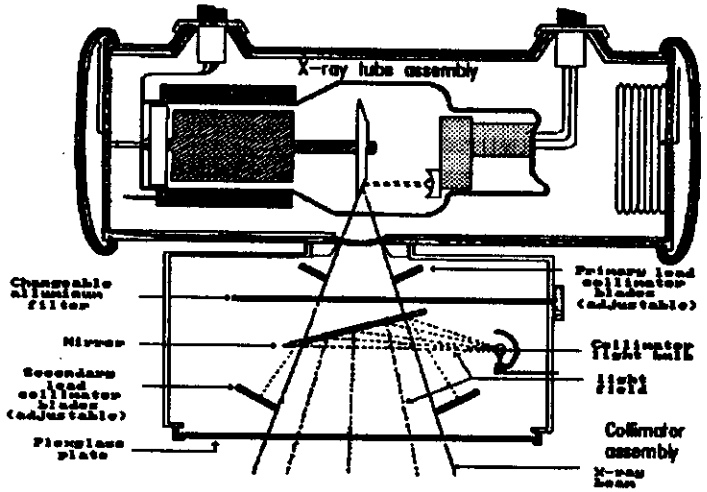
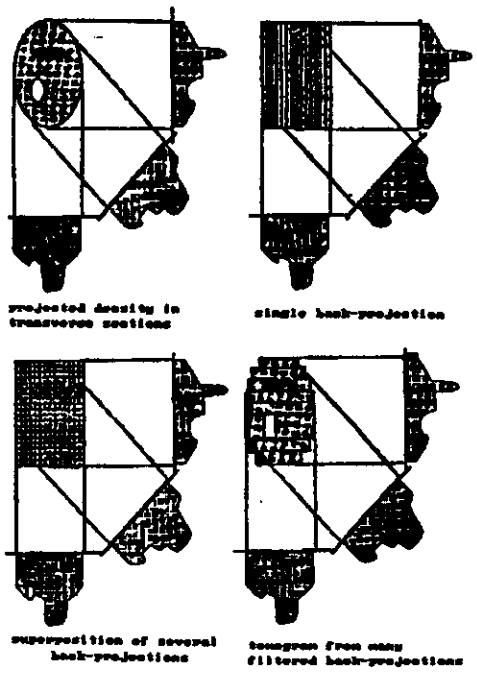


Diagram of an X-ray collimator assembly that depicts major components



BACK PROJECTION SCHEMATIC DIAGRAM