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"Mathematical Modelling in a Sewage Outfall Design"

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MATHEMATICAL MODELING IN A SEWAGE OUTFALL DESIGN

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ABSTRACT

Several studies have been conducted in the past to examine various aspects of environmental pollution in coastal and estuarine waters around Penang Island, especially in the eastern and northern shores. Recently the Municipal Council of Penang commissioned a study to determine the capability of the Western and Southern channels to transport and disperse domestic sewage out into the open sea for subsequent dilution and degradation.

This paper will present the use of simulation and modeling techniques to assist Municipal Council planners to decide on the most optimal option and combination of treating and piping the sewage for final discharge, with the eventual relocation of the present sewage outfall in Jelutong to a suitable site.

INTRODUCTION

Coastal seas around the northern and eastern shores of Penang Island serve multiple purposes which include commercial, social and aquacultural uses as well as serving to transport and disperse domestic and industrial sewage to the open sea for dilution. Various studies have been conducted to date to determine the physical, chemical and biological baseline quality of the sea waters around Penang (Environal Services, 1980 ; ERG, 1983 and Owen, 1978). In addition to these, other studies have been commissioned to characterise the hydrographic and hydraulic regimes of the area concerned (Christiani, 1972 ; DHI, 1982 and Koh and Zubir, 1987). Results of these and other studies have indicated the need to provide some degree of sewage treatment and the necessity to relocate the present outfall near Jelutong to a location where the ecological system is less sensitive to the impact of sewage disposal. It has been correctly pointed out that with a proper selection of the site of discharge, the degree of treatment can be dramatically reduced (Pearson, 1975).

THE STUDY AREA

Georgetown, the capital city of Penang, is located at Lat 5°25'N and Long 100°20'E, on the eastern shore of Penang Island, and has a population of 350,000. Domestic waste is conveyed by trunk sewer to a sewage outfall at Jelutong and discharged into the Western channel (Figure 1), the channel between Jelutong and Middle Bank. The Southern channel is the large channel to the east of the Middle Bank. The study area covers the estuary between Penang Island and Butterworth and stretches from the latitude at Tg. Tokong (point C) to that at Batu Maung (point D) for a total length of 22 km, and is called the Straits of Penang.

The tides and current regimes in the Straits of Penang are astronomical and predominantly semidiurnal. Typical daily variations in tidal levels measured at Kedah Pier near Georgetown for the month of July 1986 are indicated in Figure 2, while hourly measured current speed at depths of 10-12 m at location C on September 14, 1986 and at location D on August 13, 1986 are shown in Figures 3.a and 3.b respectively. A mean spring tide has a tidal range of 200 cm with a maximum current speed of 1.05 ms^{-1} slightly to the south of A. A mean neap tide, on the other hand, has a tidal range of 50 cm with a maximum current speed of 0.32 ms^{-1} at the same location.

Data on wind consist of hourly averages of wind speed and direction for the period 1968-1982. The dominant wind direction is from the north which occurs for 20% of the time while that from the north east accounts for some 14%. The southerly and south-westerly winds account for another 20%. Much of the wind falls within the speed of 5 ms^{-1}. On the annual basis, the period of calm accounts for 22%.

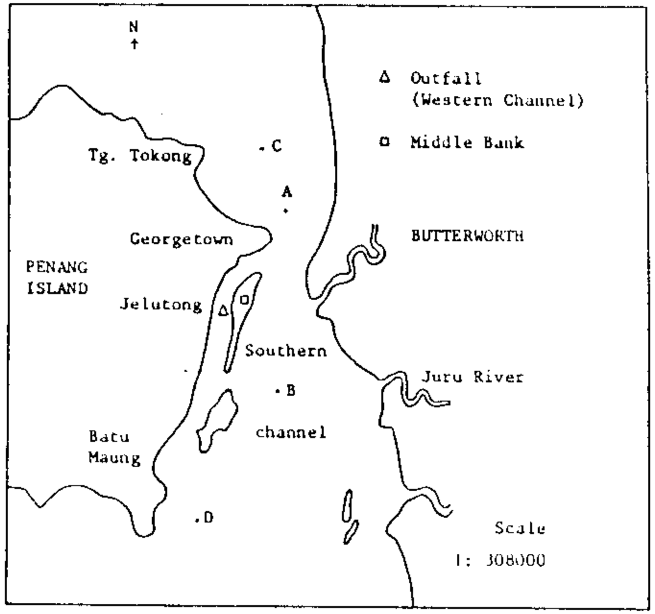


Figure 1. Map of the Straits of Penang

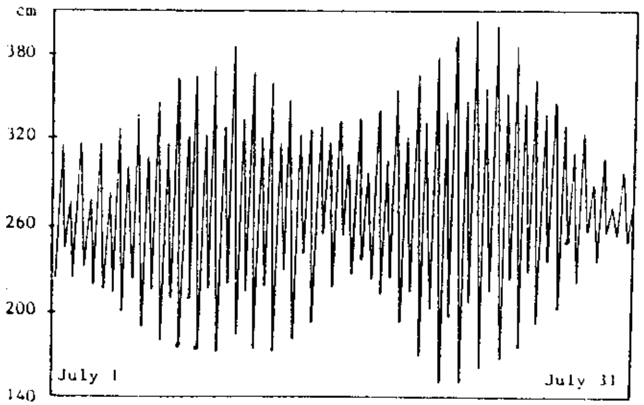
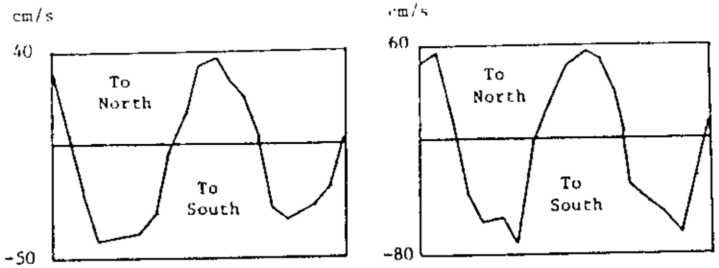


Figure 2. Daily Variation in Water Levels in cm at Kedah Pier for July 86. Period Between High and Low is About 6.5 Hours.



(a) September 14, 86 at C (b) August 13, 86 at D

Figure 3. Hourly current speeds in cm/s at depth 10-12 m

The studies conducted to date have determined the status of the following water quality parameters DO, PH, COD, BOD, suspended solids, temperature, total and faecal coliform counts, conductivity, amoniacal and organic nitrogen, as well as oil and greese in sea waters in Penang. Using coliform bacteria count as an indicator for human and animal waste contamination and including the findings contained in the reports mentioned earlier it can be concluded that all coastal areas around Penang contain faecal coliform count far exceeding the limit of 200 MPN/100 ml set by the United States Environmental Protection Agency (USEPA) for bathing waters (1979). A later PDC study (Yahya and Leong, 1987) indicates, however, that with respect to heavy metals, the Straits of Penang is practically unpolluted. It can therefore be surmised that the single most important source of pollution in the coastal waters around Penang is human and animal waste. Indeed, the Environal Report concluded-that the Jelutong outfall is inadequate for the disposal of current and future domestic waste in Georgetown, and that the North coast needs a proper marine outfall or adequate treatment facilities.

HYDRODYNAMIC MODEL

Data on temperature and salinity distributions in the area indicate that the water column is vartically well-mixed. Hence a vertically integrated two dimensional shallow water hydrodynamic model is

suitable. In this case, the pressure is hydrostatic and the two horizontal velocity components u and v are vertically integrated. The model consists, then, of a conservation of mass equation and two horizontal momentum equations.

$$\frac{\partial H}{\partial t} + \frac{\partial (Hu)}{\partial x} + \frac{\partial (Hv)}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial \zeta}{\partial x} - fv + \frac{gu\sqrt{u^2+v^2}}{C^2H} - \frac{\zeta^x}{H} = 0 \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial \zeta}{\partial y} + fu + \frac{gv\sqrt{u^2+v^2}}{C^2H} - \frac{\zeta^y}{H} = 0 \tag{3}$$

where x is positive eastward (m),
y is positive northward (m),
u is the eastward velocity (m/s),
v is the northward velocity (m/s),
g is the gravity (m/s^2),
h is h(x,y), the distance from mean sea level to the bottom of the estuary (m),
ζ is $\zeta(x,y,t)$, the distance from mean sea level to the water surface (m),
H is the total depth, $h+\zeta$ (m),
f is the Coriolis parameter (s^{-1}),
c is the chezy coefficient ($m^{\frac{1}{2}}/s$),

In the tropics, f may be set to zero. The second last term in equations (2) and (3) accounts for frictional losses at the bottom of the estuary. The last term refers to the wind stress conceived to act as a body force throughout the water column, with magnitude given by (Wu, 1969)

$$\zeta^x = \gamma\left(\frac{\rho_a}{\rho}\right) w^2 \cos\theta, \quad \zeta^y = \gamma\left(\frac{\rho_a}{\rho}\right) w^2 \sin\theta, \tag{4}$$

where w is the wind speed, θ the angle between the x axis and the direction of wind, ρ_a the density of air, ρ the density of water and γ is a dimensionless coefficient called the wind stress coefficient with value approcimately 0.0026. Turbulent eddy viscosity can be readily incorporated in the model. The boundary conditions which must be applied to solve the coupled set of equations are the specification of zero flow normal to a solid boundary, and the specification of the normal flow or of the elevation of the water surface at a boundary which is open to a river or sea.

The hydrodynamic equations can be solved by the finite difference method (Abbott, 1980 and Book, 1981). An excellent review of this method is given by (Liu and Leendertse, 1978). They have also been solved by the finite element methods (Kawahara et al., 1980). The Basic foundations of the finite element methods are available in many sources (Zienkiewicz, 1978 and Tan and Koh, 1978) while those features of the methods most relevant to the shallow water equations may be found elsewhere (Connor and Brebbia, 1976 and Pinder and Gray, 1977). We will therefore be brief. If the estuary indicates stratification, a multiple level formulation of (1)-(3) may be devised (Kawahara et al., 1983).

Finite Element Method

The present model uses quadrilateral isoparametric elements with quadratic basis functions for the two velocity components and linear basis functions for water surface elevation (Walters and Cheng, 1978). We use the conventional Galerkin method which requires that the weighted residual of each of (1)-(3) be zero. The weighting functions chosen here are the basis functions themselves.

The time derivatives are replaced by finite difference. Generally there are 3 broad classes of time differencing, namely the explicit, the implicit and the implicit-explicit methods. Each has its own merits. Thus the fully explicit method is direct and requires no matrix inversion but the time step is limited by the Courant stability criterion. On the other hand, the fully implicit method is not limited in the choice of time step, but requires non-linear matrix inversion at each time step. Here we adopt an implicit-explicit method which yields a linear system of algebraic equations with time-invariant coefficient matrix and which is not severely restricted in time step by the Courant criteria.

This method involves 3 time levels per iteration, namely $t-\Delta t$, t and $t+\Delta t$ which are represented by superscripts $k-1$, k, $k+1$ respectively. The terms that are treated implicitly are the divergence terms in the continuity equations and the pressure gradient, eddy viscosity and wind stress terms in the momentum equations. All other terms are treated explicitly. This method is therefore leap-frog with respect to the terms treated explicitly, while is trapezoidal with respect to the implicit terms. Thus if a differential equation is represented by

$$\frac{dg}{dt} = \alpha + \beta \qquad (5)$$

where α is to be treated explicitly and β implicitly, then the difference scheme for (5) becomes

$$\frac{g(t+\Delta t) - g(t-\Delta t)}{2\Delta t} = \alpha(t) + \tfrac{1}{2}(\beta(t+\Delta t) + \beta(t-\Delta t)) \qquad (6)$$

Numerical testings show that with this time difference scheme, the time steps used were 10 to 20 times larger than that allowed by the courant criterion

$$\Delta t \leq \frac{\Delta x}{\sqrt{2}\, c} \qquad (7)$$

where $c = \sqrt{gH}$ is the celerity and Δx the linear grid size.

The hydrodynamic model is used to simulate the current regimes in the Penang Straits under various astronomical and meteorological conditions. The model is caliberated with current data at locations A and B (Figure 1) for a mean spring tide available from Admiralty chart 1366 (1985) and validated by data at same locations during mean neap tide, and other data. A time step of Δt = 3 minutes was used.

The boundary condition imposed at the north entrance, point C of figure 1, is a $\sin(\omega t + \eta)$ with η = 0.45 while that at the south entrance point D, is a $\sin(\omega t)$. Values of a are 1 and 0.25m at spring and neap tide respectively and ω corresponds to the semidiurnal tide or 12.42 hrs.

Figures 4a and 4b show the flow regimes in the Western channel for a tide of [illegible]m range or a value of a = 0.55m.

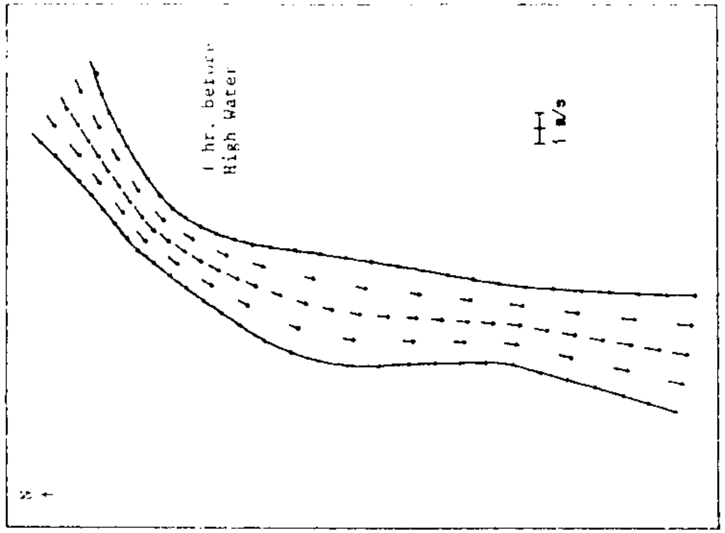


Figure 4b. Velocity Plot at Flood Tide

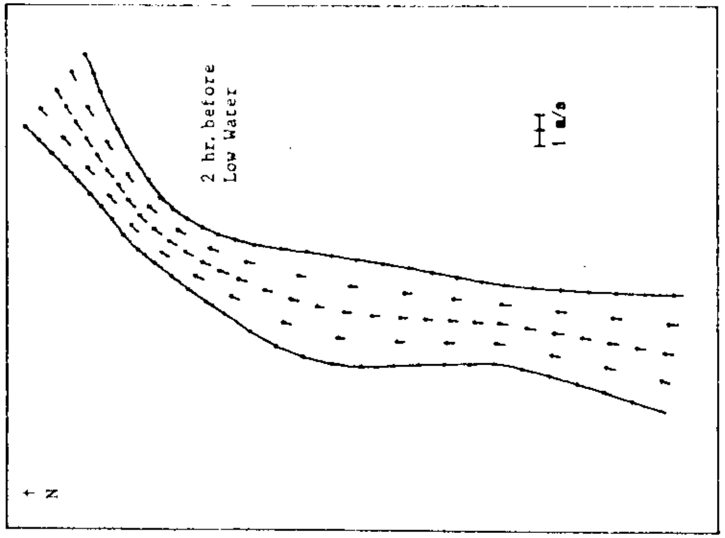


Figure 4a. Velocity Plot at Ebb Tide

ADVECTION-DIFFUSION MODELS

The simulation and modeling of the transport phenomena of pollutants released into the environment has become a useful technique in many environmental management and impact studies (Lam et al., 1984). Typically the phenomena are modeled by the advection-diffusion equations for a single substract or a group of interacting substracts. We consider a single neutrally buoyant substract and its transport in an estuarine or coastal environment.

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} - \frac{1}{H}\frac{\partial}{\partial x}(H.k_x\frac{\partial c}{\partial x}) - \frac{1}{H}\frac{\partial}{\partial y}(H.k_y\frac{\partial c}{\partial y}) + \delta c - R = 0 \qquad (8)$$

where k_x, k_y are diffusion coefficients in the x, y directions respectively, δ the decay rate and R is the source-sink term. Here c is the vertically averaged concentration.

The advection-diffusion equation has been solved by the finite difference method (Dasgupta et al., 1984 and Patel and Markatos, 1986) or the finite element method (Baker et al., 1978) or the less popular constant volume moving coordinat method (O'kane, 1980). Herein, we will consider the finite element method.

Using the Galerkin residual method weighted with the basis functions N_j, equation (8) may be convented to the following ordinary differential equations

$$A\frac{\partial c}{\partial t} + (M + D + E)C - S = 0 \qquad (9)$$

where C is the concentration vector, A is the matrix associated with the inner product $\int N_i N_j dA$, M, D, E are the matrices arising from the advection, diffusion and decay terms respectively and S the matrix due to the source term, see (ERG, 1983).

Time discretization of (9) poses some difficulty. In this regard we first consider the simplied 1-dimensional advection-diffusion equation.

$$\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial x} + k\frac{\partial^2 c}{\partial x^2}. \tag{10}$$

Consider now the explicit forward in time centred-in-space (FTCS) finite difference scheme.

$$\frac{c_i^{n+1} - c_i^n}{\Delta t} = -\frac{u}{2\Delta x}(c_{i+1}^n - c_{i-1}^n) + \frac{k}{\Delta x^2}(c_{i+1}^n - 2c_i^n + c_{i-1}^n). \tag{11}$$

Define the dimensionless parameters

$$a = \frac{u\Delta t}{\Delta x}, \quad d = \frac{k\Delta t}{\Delta x^2}, \quad R_c = \frac{a}{d} = \frac{u\Delta x}{k}, \tag{12}$$

which are called the Courant number, the diffusion number and the cell Reynold number respectively. It is well known that the Von Neumann stability criteria are (Roache, 1976)

$$a \leqslant 1, \quad d \leqslant \tfrac{1}{2}, \quad R_c \leqslant 2. \tag{13}$$

It has been pointed out (Leonard, 1980 and Su et al., 1983), that the third condition $R_c \leqslant 2$ is too restrictive, and should be replaced by

$$a\,R_c \leqslant 2, \quad \text{or} \quad a^2 \leqslant 2d. \tag{14}$$

Thus (13) is now replaced by the more appropriate criteria

$$a^2 \leqslant 2d \leqslant 1. \tag{15}$$

Now consider the following typical parameters used in a FTCS scheme for the Western channel: $u = 0.5\sin(wt)$, $k = 50\text{m}^2\text{s}^{-1}$, $\Delta x = 400$ m. It turns our that the limiting criterion is $aR_c \leqslant 2$, or $u^2\Delta t \leqslant 2k$, implying $\Delta t \leqslant 400$ seconds.

Since the velocity is time dependent, we discretize equation (9) with the advection term explicit and all other terms implicit (trapezoidal). This scheme results in a coefficient matrix that is time invariant, and hence need to be inverted once only. Implicitization of the other terms helps to stabilize the scheme to some degree.

A series of tracer studies using Rhodamine B were conducted for the Western channel, with the dye released near the outfall. The Western channel is approximately 1 km wide and 7 km long, see Figures 1 and 4. Tracer concentration was monitored for up to 3 hours after release by which time the tracer patch has moved by some 2 to 3 km. To calculate the dispersion coefficients, the concept of the linear length scale model (Okubo, 1976 and Dyke and Robertson, 1985) is used. For the purpose of this paper, however, equivalent constant Pickian coefficients are used as approximates. This approximation has been found to be adequately accurate (Lam et. al., 1984). After smoothing out the data, the following values were obtained: $k_x = 10\ \text{m}^2\text{s}^{-1}$, $k_y = 50\ \text{m}^2\text{s}^{-1}$, which appear to fit the diffusion diagrams given by Okubo (1971). For more discussion on the phenomena of turbulent diffusion, the reference (Csanady, 1973) is an excellent source of information. Stability and accuracy consideration limits $\Delta t = 3$ minutes. To simulate the transport and assimilation of sewage in the Western channel, we consider a typical average semidiurnal tide with a range of 1.1m, whose current velocity plots are illustrated in Figures 4.

Proper boundary conditions must be imposed on transport equation. On land boundary the usual no flux transport condition is used. On the sea boundary where the Western channel meets the larger Southern channel, the boundary condition depends on whether the tide flows out of the Western channel or into it. When the tide flows out of the channel across a sea boundary, we assume that the sewage is advected out without dispersion. On the other hand when sea water flows into the channel across a sea boundary, the condition to be imposed get complicated. The precise exchange of water across the boundary is not known. The condition used in this paper is an approximation that appears to work reasonably well under all test runs. We denote the concentration of pollutant outside the Western channel by c_o, assumed to be time independent. We denote the concentration at a boundary point at time t after the tide has reversed and entered the Western channel by $c(t)$. The boundary condition during the time when sea water enters the channel is taken to be

$$c(t + \Delta t) = c(t) \cdot e^{-kt} + c_0(1 - e^{-kt}), \; k > 0. \qquad (16)$$

The transport and decay of faecal coliform is being simulated. The source strength used is based upon the present uniform loading of 3.5 mgd with an average count of 2 x 10^7 MPN/100 ml, discharged as a point source at the outfall. A decay value of $\delta = 4$ per day is used, which corresponds to a value of $t_{90} = 0.57$ days.

Figure 5 gives the velocity variations over a semidiurnal tidal cycle at a point near the outfall, while Figure 6 provides the corresponding tidal variation in faecal coliform concentration, with the time axis aligned. It can be seen that the concentration builds up with the slowing of the current and peaks about 45 minutes after the tide reverses.

The spatial distributions of concentration along the centre line of the plume from the north entrance through the outfall to the south entrance at four evenlly spaced time instances in a tidal cycle are indicated in Figure 7. The numbers marked on the curves refer to time instances marked on the time axis in Figure 5.

CONCLUSION

The results presented in the previous section provide insight into the consequences of a particular loading of sewage discharge. Other variations in loading scenario have been and will be studied by means of the models discussed in the paper. Results and recommendations of this study will be communicated to our client in due course. The methodology presented herein has been proven to be useful in other studies conducted elsewhere (Conner et al., 1983 and Miller, 1984). While the basic idea about the simulations of hydrodynamic and transport phenomena in general is well established, as can be seen from the references cited earlier, as well as other variations of them (Donea, 1984 and Goussebaile et al., 1984), the applications to tidal estuaries have not appeared to be as popular.

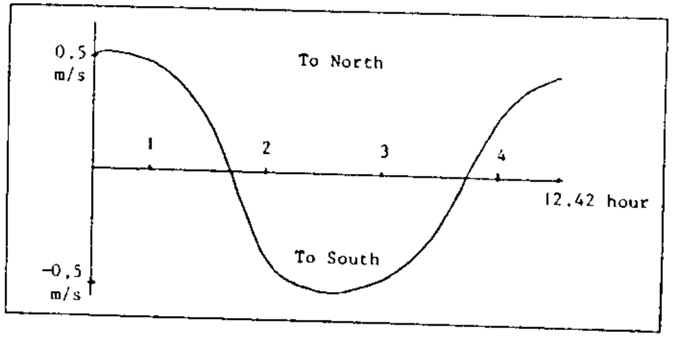


Figure 5. Current Speed Near Outfall

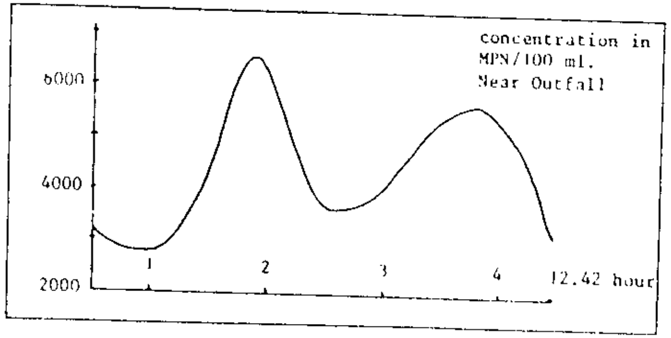


Figure 6. Coliform Concentration Variation with time

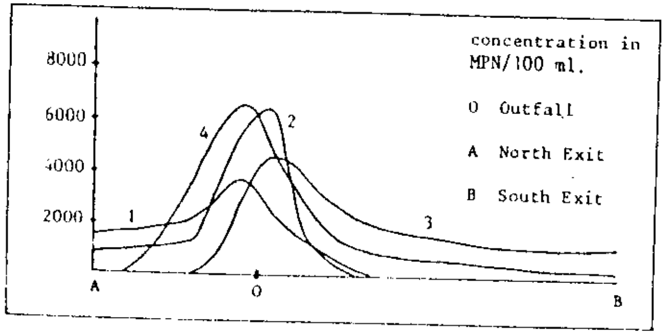


Figure 7. Coliform Distribution Along Plume Centre Line

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