THIRD AUTUMN COURSE ON MATHEMATICAL ECOLOGY

(29 October - 16 November 1990)

"Fur Seal and Blue Whale: The Bioeconomics of Extinction"

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FUR SEAL AND BLUE WHALE:
THE BIOECONOMICS OF EXTINCTION

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Abstract.   Common property exploitation repeatedly has been implicated in instances of mismanagement of marine biological resources, resulting in depletion, even exhaustion, of stocks, and in dissipation of the economic benefits that the harvest might entail. Here I shall re-examine historical patterns in the competitive exploitation of two marine mammals, examining the interplay between common property harvest practices and inertial effects that result, among other things, from irreversible capital investment ("sunk capital") in the harvesting industry. I find that common property exploitation tends to exaggerate the swings and overshoots that inertial features introduce into the temporal pattern of harvesting, leading to an excessive buildup of capital capacity, followed by an excessive depletion of the resource stock. Under some conditions, these exaggerated swings can result in stock extinctions which

optimal management might have avoided. The formal model is set up as a differential game, and analyzed by control theory methods.

§1. Bioeconomic Predator-Prey Models

The obvious parallel, between predator-prey interactions in natural ecosystems and competitive harvesting of a wild biological resource, has inspired considerable theoretical analysis and model-building.

In his seminal article, "The economic theory of a common-property resource: The fishery," Gordon (1954) argued that, in harvesting a previously unexploited fish stock, "man's intrusion would have the effect of any other predator; and that can only mean that the species population would reach a new equilibrium at a lower level of abundance." More decisively, Gordon enunciated the principle that the human predator population size, governed by the interaction between biological and economic forces, would come into a certain "bioeconomic equilibrium" with the prey population. This would occur at a level of harvest effort which, having driven down the reproducing stock of fish, would yield a rate of income from harvest only just balancing the costs (including "opportunity costs") of exerting the effort.

A full predator-prey dynamic model of this process was proposed by Smith (1969). It incorporates the standard "surplus production" harvest model for the prey dynamics

[1] $dx/dt = F(x) - h(t)$,

h $x(t)$ being stock size, $F(x)$ intrinsic growth rate, and $h(t)$ harvest rate. rvest is assumed to be proportional to both stock size and fishing effort level; is $h = qxK$, where $K(t)$ is a measure of the size of the active fishing fleet.

Smith's model couples the harvest equation [1] with a second equation ich describes the evolution in the size of the fleet. The simple behavioral rule that the rate of entry of vessels into the fleet (or of exit from the fleet) is portional to the current per-unit net profit (or loss) rate

$$\pi / K = pqx - c .$$

nce

$$dK / dt = \beta [pqx - c] .$$

A voluminous theoretical literature has developed around this dynamic dator-prey formulation and its variants (for a partial account, see Berck 1979). m the general structure of the equations, it is clear that--assuming downward-oing per-capita growth rate--there is a locally-stable equilibrium at Gordon's economic point $x_\infty = c / pq$, $K_\infty = F(x_\infty) / qx_\infty$. Trajectories sufficiently close to equilibrium will spiral in toward it. But the dynamics of motion distant from the ilibrium would seem to be entirely ad hoc , reflecting the arbitrary nature of behavioral entry rule in [2].

Nevertheless, considerable attention has been given to theoretical study he dynamics of this system, to determine whether trajectories sufficiently far n equilibrium might in principle lead to stock extinction. With the special linear cture of [2], the conclusion will depend very much on the prey growth function nd the effect of stock size on the effort-harvest relation.

The Smith model has been shown to exhibit considerable power in empirical analysis. Of particular interest to us here is a study by Wilen (1976) in which he re-examined the historical data from the late 19th century pelagic harvest of the Northern Pacific fur seal. At issue is whether that uncontrolled common property harvest would, if continued unchecked, have driven the fur seal stock to extinction. Wilen's study seems to suggest it would not have, but one is entitled to wonder to what extent these indications are model specific.

It is natural to wish to refine Smith's predator-prey model to incorporate non-equilibrium dynamics which, as in Gordon's original analysis of equilibrium, would be based more explicitly on the individual harvester's presumed profit-maximizing behavior. Such a refinement might, for one thing, provide a better insight into a bioeconomic system's "self-regulating" mechanisms. The key requirement is a proper characterization of entry into, and exit from, the active "predator" population.

Now, models of the **optimal** management of a harvested biological resource are well-established in the resource economics literature, and incorporate explicit profit-maximizing behavior on the part of the monopolistic sole owner or social manager (see, for example, Clark 1976). The appropriate generalization, to analyze competitive exploitation by a number of independent firms, particularly of a common property resource, is more difficult in principle, and has been studied much less (but see Levhari and Mirman 1980).

Of particular interest to us here is the model, due to Clark et al. (1978), of the sole owner exploitation of a marine population (fish or mammal), taking explicit account of "capital immalleability"; specifically, the irreversibility of major investment decisions. Capital immalleability is important in a modern, capital-

intensive resource harvesting industry since it imparts a certain inertia to the bioeconomic system, through the so-called sunk-capital effect. The immalleable capital model has been applied by Clark and Lamberson (1982) in a normative study of the mid-century Antarctic whaling industry, presuming optimal management.

In this article, I develop a dynamic model of competitive exploitation of a common property renewable resource, assuming profit-maximizing goals by the individual participants in the harvest, and formulating their interaction as a differential game. Like Clark et al. (1978), I assume that irreversible investment decisions are important. With such an assumption, a natural rule for capital entry can be derived. This model is a generalization to an oligopolistic resource industry of an earlier model of mine, which addressed perfect competition (McKelvey 1985).

After deriving the model and noting some of its properties, I turn to applications. I re-examine the Clark and Lamberson (1982) analysis of Antarctic whaling, taking note that in its later phases the industry became in effect an oligopoly of five nations which negotiated harvest shares through the medium of the International Whaling Commission (I.W.C.).

Finally, I return to Wilen's (1976) analysis of the bioeconomics of fur seal harvest, where investment seems to have been almost completely reversible, but where other, shorter-term inertial factors seem to have played a role.

§2. The Model

I shall consider a fishing or whaling industry consisting of N firms, each operating a vessel or fleet of vessels, and each exerting harvest effort $\epsilon_n(t)$ within its effort capacity $0 \le \epsilon_n \le \kappa_n$, for $n = 1, 2, ..., N$. Investment ι_n in capital capacity is irreversible, $0 \le \iota_n$; i.e., disinvestment is ruled out. Taking account of depreciation,

[1] $d\kappa_n / dt = \iota_n - \gamma\kappa_n$, $\quad 0 \le \iota_n$.

Total industry rate of investment is $I = \sum_{n=1}^{N} \iota_n$, total capacity is $K = \sum_{n=1}^{N} \kappa_n$, and total rate of effort is $E = \sum_{n=1}^{N} \epsilon_n$.

As is customary in modeling the harvesting of a wild marine stock, I shall assume that the individual firm's harvest rate $h_n(t)$ is proportional to its exerted effort, and also is proportional to the population size $x(t)$ of the harvested stock:

$h_n = qx\epsilon_n$.

Thus, the constraint on effort translates into a constraint on harvest:

$0 \le h_n \le qx_n$.

Total industry harvest rate is $H(t) = \sum_{n=1}^{N} h_n(t)$.

The objective of each firm is assumed to be to maximize the discounted stream of profits:

[2] $\quad \pi_n = \int_0^\infty e^{-\delta t} \left[p(H) \, h_n - w_n \, e_n - c_n(I) \, \iota_n \right] dt$

$\quad\quad = \int_0^\infty e^{-\delta t} \left\{ \left[p(H) - W_n(x) \right] h_n - c_n(I) \, \iota_n \right\} dt \ .$

Here $W_n(x) = w_n / q_n x$ is the harvest cost rate per unit of catch, dependent on the current stock size x. Both price $p(H)$ and investment cost $c_n(I)$ may depend on total demand.

It remains to specify stock growth rate and how the biological stock responds to harvest. The simplest density dependent model (surplus production model) specified that

[3] $\quad dx/dt = F(x) - H \ , \quad 0 \le H = qxE \le qxK \ .$

Here $F(x)$ is the intrinsic growth rate for the population, assumed to depend only on current stock size. This formulation is broad enough to allow for a variety of differing characteristic responses of the population to the harvest. We shall examine cases in which $F(x)$ is, respectively, compensatory, depensatory, or over-depensatory (e.g., Clark 1976): These alternatives are illustrated in Fig. 1. Prototypes for the three cases are, respectively,

$\quad F(x) = rx \left(1 - x/\bar{x} \right), \ \text{or}$

$\quad\quad = rx^{2+\alpha} \left(1 - x/\bar{x} \right), \ \alpha \ge 0, \ \text{or}$

$\quad\quad = rx \left(1 - x/\bar{x} \right) \left(x/\underline{x} - 1 \right) \ .$

The response to a **constant** harvest effort rate E by each of these model variants is well-known (e.g., Clark 1976). For the compensatory model, every stock level $x^\# < \bar{x}$ can be achieved as steady state, by maintaining effort $E = F(x^\#) / qx^\#$. This is a globally stable equilibrium. Only if an excessive effort is maintained ($E > \sup F(x) / qx$) will the population be driven to extinction. In the depensatory case, there is a second unstable equilibrium $x_\#(E) < x^\#(E)$. Populations initially above $x_\#$ approach $x^\#$ under constant E, but populations below $x_\#$ will be driven to extinction. Finally, with over-depensation, extinction results for a population initially below $\underline{x}$ even if harvest effort ceases altogether.

A question before us is whether economic rationality will lead, for each of the three kinds of populations, to a timely slackening of harvest effort, leading to a sustained harvest rather than driving the population below critical levels and eventually to extinction.

Economic theory suggests that individual rationality ought to bring about a Nash (competitive) equilibrium among the firms in the bioeconomic system [1]-[3] (Owen 1982). This means that firms' individual harvest strategies will mesh, so that each firm achieves the maximum net return π_n **compatible with the actions of all the others**. Thus, no firm can unilaterally alter its harvest pattern without suffering losses.

Let us focus, then, on the typical individual firm n, and its harvest and investment strategy in response to **specified** harvest and investment patterns $h_m(t)$, $\iota_m(t)$ for all the other firms $m \ne n$. We shall use the notation

$H = h_n + H_n, \ \text{with} \ H_n = \sum_{m \ne n} h_m \ ;$

$$I = \iota_n + I_n, \qquad \text{with } I_n = \sum_{m \neq n} \iota_m .$$

Stated formally, the problem for firm n is: **Given** $H_n(t)$ and $I_n(t)$, **to choose** $h_n(t)$ and $\iota_n(t)$ so as to maximize π_n, under the constraints [1] and [3].

The problem may be reduced, using Mangasarian's (1966) version of the maximum principle. Here we shall simplify the analysis by limiting attention to an industry with firms that are alike in their unit harvest costs $W(x)$, and which face a constant price p and a constant and identical unit investment cost c. However, we return to the more general formulation in §5.

The current value Hamiltonian for firm n is

$$H_n = [p - W(x)] h_n - c\iota_n + \mu_n [\iota_n - \gamma\kappa_n] + \lambda_n [F(x) - h_n - H_n]$$

and the Lagrangian is

$$L_n = H_n + \sigma_n [q\kappa_n - h_n] + \hat{\sigma}_n h_n + \rho_n \iota_n .$$

Here μ_n and λ_n are shadow prices for capital and biological stock, and the non-negative multipliers σ_n, $\hat{\sigma}_n$, and ρ_n are zero unless the corresponding constraint binds.

Differentiating L_n by κ_n and x yields the dual dynamic equations (in current value formulation):

[4] $\qquad d\mu_n / dt - \delta\mu_n = -\partial L_n / \partial\kappa_n = \gamma\mu_n - \sigma_n q ,$

and

[5] $\qquad d\lambda_n / dt - \delta\lambda_n = -\partial L_n / \partial x = -F'(x)\lambda_n + W'(x)h_n - \sigma_n q\kappa_n .$

The control variables h_n, ι_n are chosen so as to maximize H_n. In particular, setting $h_n{}^{max} = q\kappa_n$ we find that

[6] $\qquad h_n = \begin{cases} 0 & \text{if } p < W(x) + \lambda_n \\ h_n{}^{max} & \text{if } p > W(x) + \lambda_n . \end{cases}$

Furthermore, when $p > W(x) + \lambda_n$ then the capacity constraint binds and $\sigma_n = p - W(x) - \lambda_n$; otherwise $\sigma_n = 0$. Hence, in every case

[7] $\qquad \sigma_n = [p - W(x) - \lambda_n]^+ ,$

with $[y]^+ = max[0,y]$, indicating positive part.

Similarly, by maximizing H_n over ι_n one finds that

[8] $\qquad \iota_n = \begin{cases} 0 & \text{if } \mu_n < c \\ \infty & \text{if } \mu_n > c . \end{cases}$

The latter alternative indicates an instantaneous pulse investment. An equilibrium state, where state variables remain constant over time, requires a constant finite investment rate and constant harvesting at capacity. Hence, from [8] and [6],

[9] $\qquad$ (a) $\quad \mu_n = c,$ $\qquad$ (b) $\quad W(x) + \lambda_n + \sigma_n = p .$

Differentiating totally, using $dx / dt = 0$, yields

$$d\mu / dt \equiv 0, \qquad d\lambda_n / dt + d\sigma_n / dt \equiv 0.$$

Substituting from equations [4] and [5] yield

[10] (a) $\sigma_n = (\delta + \gamma) c / qx,$ (b) $\lambda_n = \dfrac{-W'_T(x)h_n}{\delta - F'(x)}$

where

$$h_n = qx\kappa_n = F(x) - H_n$$

and

$$W_T(x) = W(x) + (\delta + \gamma)c/qx = [w_0 + (\delta + \gamma)c] / qx$$

is the total cost (including amortized investment cost) for operating at full capacity. Eliminating λ_n between [9b] and [10b] yields a characterization of the equilibrium x:

$$\lambda_n = h_n \frac{[p - W_T(x)]'}{\delta - F'(x)} = p - W_T(x)$$

We now invoke symmetry: All firms react identically, hence have the same harvests and investments. Hence, $H = Nh_n$, $I = Ni_n$, $K = N\kappa_n$. Dropping subscripts on λ_n, μ_n, and σ_n, the dynamic equations become:

[11a] $dx / dt = F(x) - H$

[11b] $dK / dt = I - \gamma K$

[11c] $d\lambda / dt = (\delta - F')\lambda - \sigma qK / N + W'(x)H / N$

[11d] $d\mu / dt = (\delta + \gamma)\mu - \sigma qx$

where

[11e] $\sigma = [p - \lambda - W(x)]^+$

and

[11f] $H = \begin{cases} 0 & \text{when} \quad p < W(x) + \lambda \\ qxK & \text{when} \quad p > W(x) + \lambda \end{cases},$

[11g] $I = \begin{cases} 0 & \text{when} \quad \mu < c \\ \infty & \text{when} \quad \mu > c \end{cases}.$

The equilibrium stock level x^* and shadow price λ^* are characterized by

[12a] $\lambda^* = \dfrac{1}{N} F(x^*) \dfrac{[p - W_T(x^*)]'}{\delta - F'(x^*)} = p - W_T(x^*)$

while

[12b] $H^* = F(x^*),$ $I^* = K^*/\gamma = F(x^*)/qx^*,$ and $\mu^* = c.$

Note that the case $N = 1$ corresponds to monopoly. The limit as $N \to \infty$ corresponds to rent-dissipating bioeconomic equilibrium: One simply sets $1/N = 0$ and $\lambda(t) \equiv 0$ in the preceding formulas. A direct development of the bioeconomic open-access fishery is to be found in McKelvey (1985).

Note that, in the bioeconomic limit, even though firms put a zero shadow value λ on the resource, they **do** anticipate future positive returns to harvest effort $[p > W(x)]$. However these returns serve only to balance off apportioned costs of prior capital investment $[p = W(x) + \sigma(t)]$. New capital investment at any time $t \geq 0$ is in response to a net positive real return, $\mu - c \geq 0$, for that asset, and may be thought of as entry by new firms (recall that $N = \infty$). These firms, according to [11d], must take into account intertemporal tradeoffs, as reflected in the discount rate δ: They do **not** simply put $\delta = \infty$.

§3. Phase Plane Analysis

The polar cases of $N = 1$ and $N = \infty$ have been analyzed by, respectively, Clark et al. (1978) and McKelvey (1984). The behavior of the model's solution trajectories for $1 < N < \infty$ will be intermediate between these.

Phase plane trajectories for $N = 1$ and $N = \infty$ are sketched in Fig. 2, which assumes a compensatory growth function $F(x)$. In both cases, the trajectories spiral around the long-run equilibrium at (x^*, K^*). For a nascent industry there will be an initial pulse of investment to a relatively high level of capitalization. The larger the initial biological stock, the greater this capitalization level will be.

Harvesting then draws down the biological stock to well below the long-run equilibrium level. In fact, the stock level tends toward a middle-run "quasi-equilibrium" (x^o, K^o) which corresponds to free capital $(c = 0)$ in [12]. This is the "sunk-cost effect": Once investment has been committed irreversibly, vessels will freely utilize the available capacity, and will continue to operate so long as

variable costs (including shadow costs) at the margin do not exceed price p. This effect also implies that stock will not be drawn down **below** x^o, since that would entail a loss relative to variable costs, closing down the harvest (see equation [11f]).

By making the model non-linear in factor costs, one could replace pulse investment and the off-on switching of harvest effort by less abrupt transitions. Note also that the spread between x^* and x^o (and thus the width of the spirals) results from the relative size of $W(x) = w_0 / qx$ and $W_T(x) = [w_0 + (\delta + \gamma) c] / qx$; i.e., the proportion of costs that represent amortized investments.

A comparison of the diagrams in the polar cases $N = 1$ and $N = \infty$ reveals two primary differences. First, the short and long run equilibrium positions are shifted, with higher capital investment and lower biological stock levels when $N = \infty$. Both of these are aspects of the "over-capitalization" that characteristically is associated with common property exploitation of a renewable resource.

Figure 3 shows how the model's solution trajectories are affected by a depensatory or over-depensatory growth function $F(x)$. The key conclusion is that, with depensatory F, though stock recovery from low levels will be slowed down, the stock eventually will recover. Only with over-depensatory F can stocks be driven to extinction, and then only for those particular trajectories, arising from high initial stock level, that fall below the critical level $\underline{x}$. This behavior is related to the feature of the model that imposes a complete cut-off of harvest when marginal variable costs rise above unit price.

Plainly, the conclusions of the last paragraph could be altered by straight-forward changes in the model, reflecting changed assumptions about

nature and the economics of the harvest. Thus, for instance, the assumption that $H = qxE$; i.e., that harvest from a given effort drops linearly with the stock level, may be violated for certain species or environments, thereby making the stock more (or possibly less) vulnerable to harvest pressure, and thereby altering our conclusions about extinction (see Clark 1985).

§4. Application: The Great Whales

.. The effect of immalleability naturally is most significant in a capital intensive resource industry. This is the basis of the analysis of the mid-century Antarctic whaling industry by Clark and Lamberson (1982). Antarctic whale stocks, while heavily exploited beginning around 1925, were not harvested during World War II and had somewhat recovered by the end of the war. There then followed a 15-year period of rapid build-up of whaling fleets accompanied by an equally rapid decline of whale stocks. The fleet size peaked in 1961, with a total of 21 enormous factory vessels plus 261 catcher vessels, representing Japan, Norway, USSR, the United Kingdom, and the Netherlands. Thereafter, these countries succeeded in negotiating harvest quotas for each nation's fleet, through the International Whaling Commission (I.W.C.), and the following 20 years had been a period of relative stability of the size of the whale stock (though at a depressed level) and a steady decline in the size of the fleet.

Clark and Lamberson (1982) applied the Clark et al. (1979) model to examine what would have been the consequence of monopolistic control of the whaling industry. They assumed a logistic growth function

$$F(x) = rx(1 - x/\bar{x})$$

and made rough estimates of the biological and economic parameters. I have extended their analysis, assuming alternative industry structures: An open-access competitive whaling industry ($N = \infty$), and the Nash competitive equilibrium for common property exploitation by $N = 5$ nations. In our context, the monopolistic case ($N = 1$) may be presumed to forecast a cooperative solution achievable by the whaling nations, bargaining through the I.W.C.

The equilibrium stock level x^*_N for the multi-firm industry can be calculated by the following formula, which generalizes the standard formula for $N = 1$ with the logistic. In dimensionless form, let

$$Z_N^* = x_N^* / \bar{x} \qquad \text{and} \qquad \theta = \delta/\gamma$$

Then

$$Z_{\sim} = w_T/pq\bar{x} \qquad \text{and} \qquad Z_N = \frac{1}{4}\left[-B + \sqrt{B^2 + A} \right],$$

where

$$A = 8Z_{\sim}^*\left[\frac{1}{N} + \theta - 1\right], \qquad B = Z_{\sim}^*\left[\frac{1}{N} - 2\right] + \theta - 1.$$

Likewise, the "free capital" levels $Z^0_N = x^0_N/\bar{x}$ are given by parallel expression but with $w_T = w_0 + (\delta + \gamma)c$ replaced by w_0. Equilibrium capital levels are computed by $K_N^* = r(1 - Z_N^*)/q$.

Non-equilibrium characteristics are obtained by numerical integration of the differential system [11] working backwards from equilibrium at $t = \infty$. The results are shown graphically in phase-plane portraits, and key features, such

as initial pulse investment K_{max}, and lowest whale stock level x_{min}, read off the curves.

The predictions of each of the models can be compared to the actual historical record of fleet size and evolution, and the corresponding (estimated) whale stock size. The results are presented in Figures 4 5 and 6.

None of the models successfully simulates the initial 15-year capital build-up phase of the industry. This of course is an artifact of the model's linearity assumption concerning investment costs, which implies an instantaneous initial investment pulse. As I discuss in §5, one could do much better here by incorporating non-linear "adjustment costs". Indeed, the Smith (1969) phenomenological model simulates this build-up stage quite well, provided one "tunes" the *ad hoc* investment response factor β appropriately.

Possibly, the predicted size of the capital build-up is more significant than its phasing. In the cooperative model ($N = 1$) the predicted build-up is only about half of that actually observed. Both the common property ($N = 5$) and open-access ($N = \infty$) models double it.

It is during the subsequent period, of capital stock decline, that the models of immalleable investment ought to show their predictive superiority over the Smith (1969) phenomenological model. (For example, they respond to short-term losses by immediately shutting down excess harvest capacity.) The fact that the whale stock level actually observed (or estimated) is nearly constant during this period suggests that we are near the steady state equilibrium. The predicted cooperative equilibrium stock level x_1^* seems much too high, compared with the observed stock level, but once again the common property and open access values are reasonably close. Of course, parameter

values estimated in the models are very crude. As Clark has pointed out (1985), a lowered estimate of the whale stock's linear growth rate r would substantially lower predicted steady state stock levels.

The small difference between the results for $N = 5$ and $N = \infty$, as compared to $N = 1$, might be taken as indicating a very limited potential for achieving economic efficiency through limiting entry to 5 national fleets. But there was a substantial potential through I.W.C. negotiations. A comparison of the calculated cooperative solution ($N = 1$) with the realized historical trajectory suggests that this potential for cooperative advantage never was achieved.

Possibly the most striking feature of the model's behavior, and one which seems likely to be robust under model refinements, is the extent to which competitive common property practices serve to exaggerate the swings in the trajectory of resource utilization: Not only is the final equilibrium state shifted (to the high capital, low resource-stock bionomic path), but the initial build-up of investment and the degree of overshoot to low resource-stocks both are very greatly exaggerated. This interaction, between sunk capital and common property exploitation, is likely to prove to be a prevasive feature of renewable resource utilization, wherever it occurs (see for example McKelvey 1986). It is yet another pernicious aspect of the "tragedy of the common".

§5. Application: Pacific Fur Seal

In an imaginative study Wilen (1976), has investigated the economic history of the late 19th century open-access pelagic harvest of the Northern Pacific fur seal. A widespread fear at the time was that this intensive

uncontrolled harvest on the open seas might have been driving the fur seal to extinction. Eventually economic losses due to the combined effects of falling prices and a precipitate decline in harvest success led to a period of more restrained, monopolistic harvesting, and ultimately the pelagic harvest was banned altogether.

Wilen (1976) re-examined the available historical data on prices, fleet size, and harvests, in the context of Smith's (1969) predator-prey model. He began by plotting the historical values of fleet size and (inferred) stock size on an x, K-phase plane, and then, by adjusting parameters in the dynamic equations [1] and [2] obtained a trajectory which gives a remarkably good fit. The adjustment involved tuning the unknown opportunity cost (which determines the equilibrium x_∞) and the reaction parameter β (which determines the tightness of the spiral).

Wilen's study suggests that bioeconomic factors might have combined so as to temper harvest enough to avoid extinction, though the historical experiment was interrupted by exogenous effects. So far as the model is concerned, one can prove for the Smith's model that over-depensation or strong depensation ($\alpha > 0$) are necessary in order for any trajectory to lead to extinction (see for example Berck 1979, Goh 1980).

In a rough way, the phenomenological model of Smith and the more mechanistic irreversible investment model developed in §2 are qualitatively similar in their solution dynamics. However, irreversible investment was not a significant factor in the Pacific fur seal harvest: According to Wilen, vessels could easily and rapidly disengage from sealing, by switching to the halibut fishery instead. And of course, our assumption of constant unit investment costs,

and the resulting instantaneous capital build-up is totally unrealistic for Antarctic whaling, as well as for the pelagic seal harvest (this point is further discussed in §4).

Aside from irreversible investment effects, there are at least two other specific economic forces that we ought to consider, which can introduce inertia to a model of a bioeconomic system. The first of these is imperfect competition: where the industry faces a downward sloping demand curve for its product. The second is non-linear investment costs, the so-called adjustment costs, of rapid investment or disinvestment. Adjustment costs provide a less extreme form of capital immalleability. Also, for the pelagic seal harvest, an appropriate idealization may be that of the open access competitive limit, $N = \infty$. Hence, the modeling framework developed in my earlier paper (McKelvey 1985) applies. Here, I shall only sketch the modifications needed in equations [11].

With market price $p = p(H)$, a decreasing function of total harvest H, the competitive harvest level is set by $\lambda \equiv 0$ and

$$p(H) = W(x) ,$$

provided this equation has a solution in the range $0 \leq H \leq H_{max} = qxK$. Otherwise,

$$H = \begin{cases} 0 & \text{when} & p(0) < W(x) \\ H_{max} & \text{when} & p(H_{max}) > W(x) . \end{cases}$$

(This rule reduces to [11f] when applied to constant price p.) The function σ in [11e] becomes

$$\sigma = [p(H_{max}) - W(x)]^+ .$$

A caveat is required here. Wilen's (1979) data show that, in the pelagic seal harvest, prices and harvests rose and fell together around the peak period of the industry. However, it isn't clear that price was responding to harvest. In fact, on the way up, price led harvest, suggesting that the price rise was exogenous; i.e., $p = p(t)$.

To introduce adjustment costs we require that $C(I)$, the total cost of investment at rate I, be a monotone increasing and convex function: $C'(I) > 0$, $C''(I) > 0$ on $-\infty < I < \infty$, with $C'(0) = 0$. The slope $C'(I)$ might be discontinuous at $I = 0$, as one passes from disinvestment to investment.

The rate of investment is determined in the model to satisfy

$$\mu = C(I)/I = AC'(I) .$$

This is another manifestation of Gordon's (1954) bionomic principle: Open access investment rises to the level of break-even; there are no net profits.

The effect of adjustment costs will be to induce a gradual build-up of capital, and subsequent disinvestment as the biological stock level declines. As before, there will be some overshoot of the equilibrium. In a general way, this model's behavior parallels that in §3 and §4. The common property effects, on initial build-up and overstock, are the same.

Unfortunately, there seems to be no way to actually measure adjustment costs directly: All we can do is have recourse to curve-fitting, much in the way that Wilen has done. Still, the notion has a major conceptual advantage over Smith's (1969) phenomenological model in incorporating explicit profit-maximizing behavior of individual sealing vessels, and in allowing direct comparison of common property and monopolistic regimes.

In particular, the bioeconomic equilibrium level now is determined by

$$p(H) = W(x) + (\delta + \gamma) AC(I)/qx = W_T(x,I)$$

$$H = F(x) , \quad I/\gamma = K = F(x)/qx ,$$

and this might serve to define an improved version of Smith's empirical "flexible accelerator" rule (see Nickell 1978).

There remains yet one other modeling device, that I wish to mention, as a means of introducing inertia to our bioeconomic system. This is the direct use of lags in the dynamic equations. Lags may be motivated by biological or physical constraints on the systems' rate of adjustment to changing circumstances, or may be regarded simply as providing a phenomenological description of system dynamics. So far as I know, no study has been made of the precise effect of lags in a model of immalleable investment in a common property resource industry.

It may be, indeed that, aside from short term exogenous effects, what was occurring throughout this period was a straightforward driving-down of the stocks due to common property over-exploitation. In fact, the pelagic harvest did continue beyond the period modeled by Wilen. In the late 1890's a Japanese fleet entered the Bering Sea and Northwest Coast sealing grounds, and remained active until the 1911 Treaty stopped all pelagic sealing. A simple plot (Fig. 7) of catch-per-effort (an index of stock abundance) against time through that period shows a seemingly inexorable decline. In this representation the stock recovery observed in the late 1980's looks very much like a short-term statistical aberration.

Which interpretation is right? Probably the evidence isn't adequate to decide. Still, from a theoretical perspective, a turn around in seal stock levels has to be tied to some inertial effects present, and these have not been identified. In their absence one must conclude that the tragedy of the common surely was being played out yet again.

§6. Acknowledgments

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Figure 1. Growth curves $F(x)$: (a) compensatory, (b) depensatory, and (c) over-depensatory. Equilibrium under constant-effort harvest E is at intersection of the graph of $F(x)$ with the straight line xE. Arrows show non-equilibrium direction of motion.

Figure 2. Phase-plane portrait: Compensatory model, (a) $N = \infty$ and (b) $N = 1$. (After, respectively, McKelvey 1985, and Clark et al. 1979).
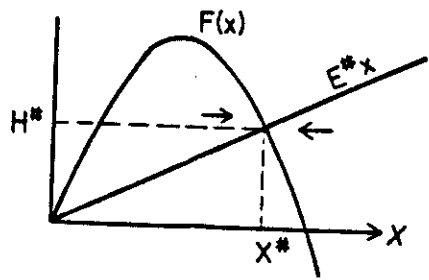
Figure 3. Phase-plane portrait: $N = \infty$; (a) depensation and (b) over-depensation.
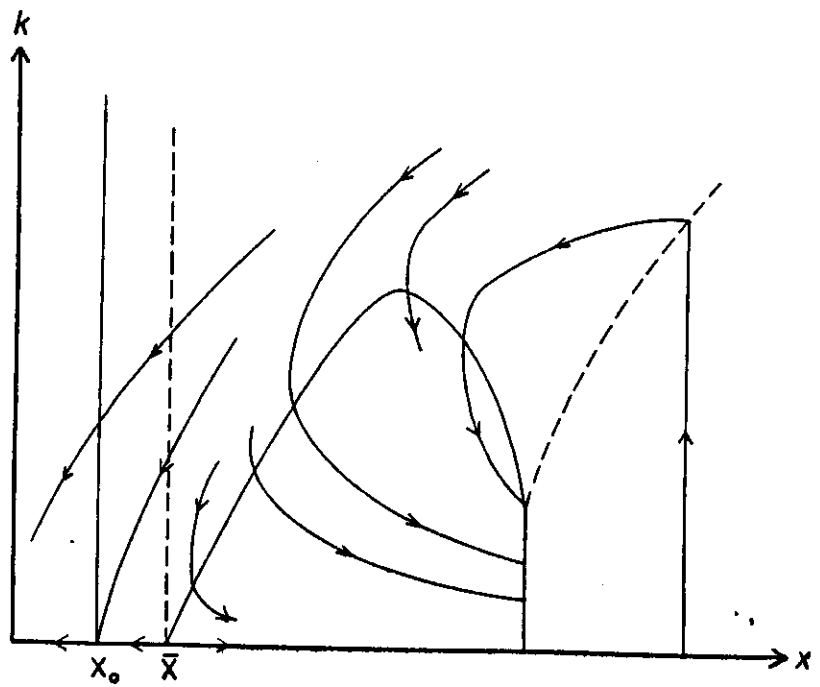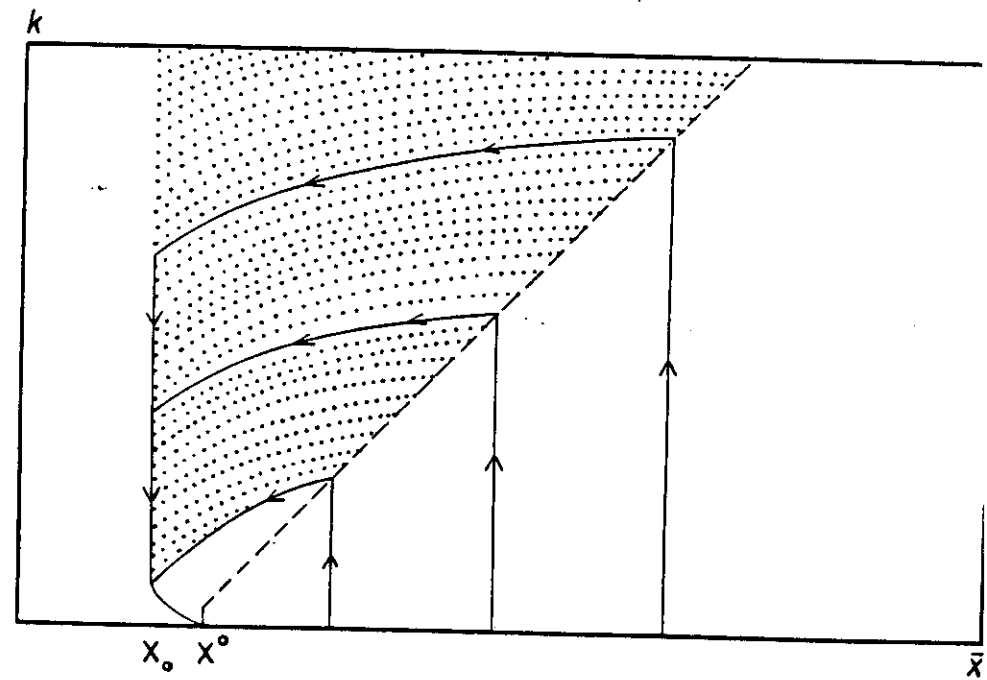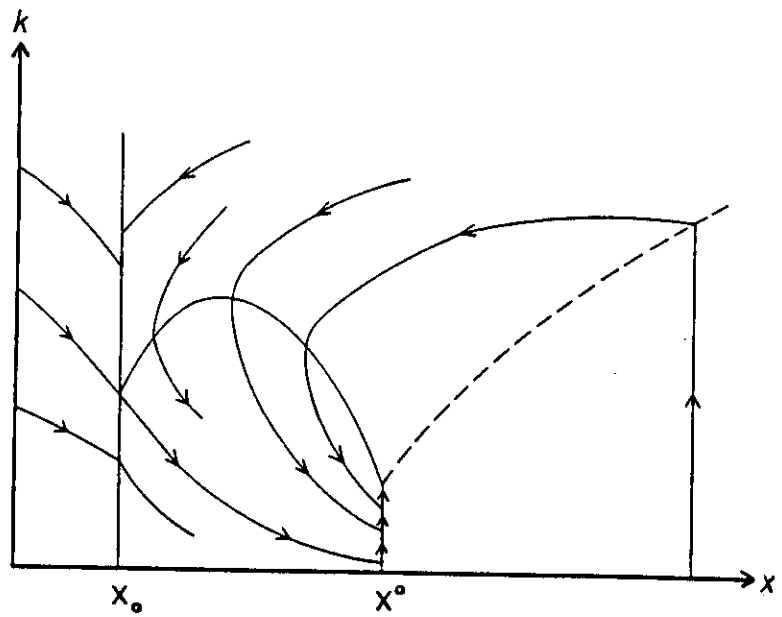
Figure 4. Open access whale harvest: $N = \infty$. Logistic model with parameter values from Clark and Lamberson (1982).
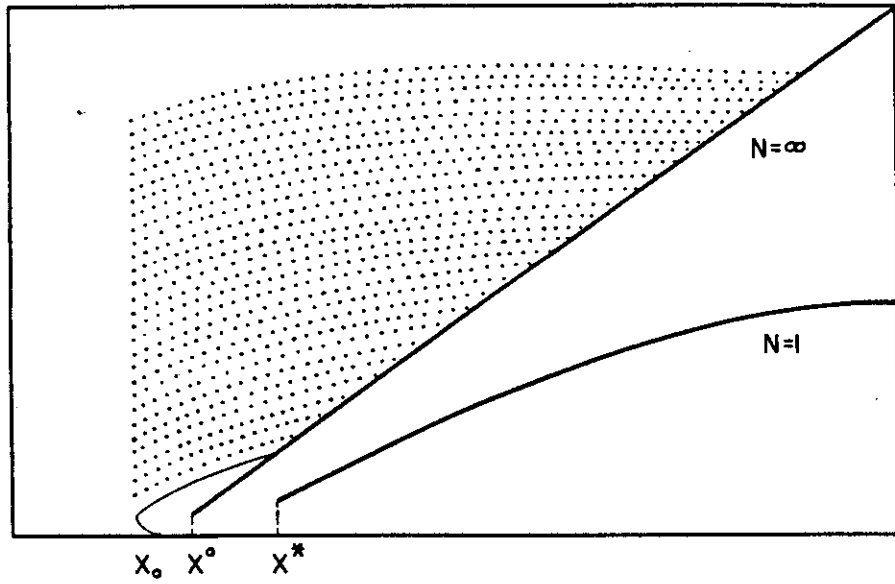
Figure 5. Threshold investment in a whale fleet as a function of initial whale stock level. $N = \infty$ vs $N = 1$. The logistic model is with parameter values from Clark and Lamberson (1982).

Figure 6. Rise and decline of the Antarctic Whaling Industry. Historical reconstruction is compared to model trajectories.

Figure 7. The Pelagic Seal Harvest. Harvest per vessel vs time: 1886-1909. (Calculations based on data quoted by Wilen 1976).
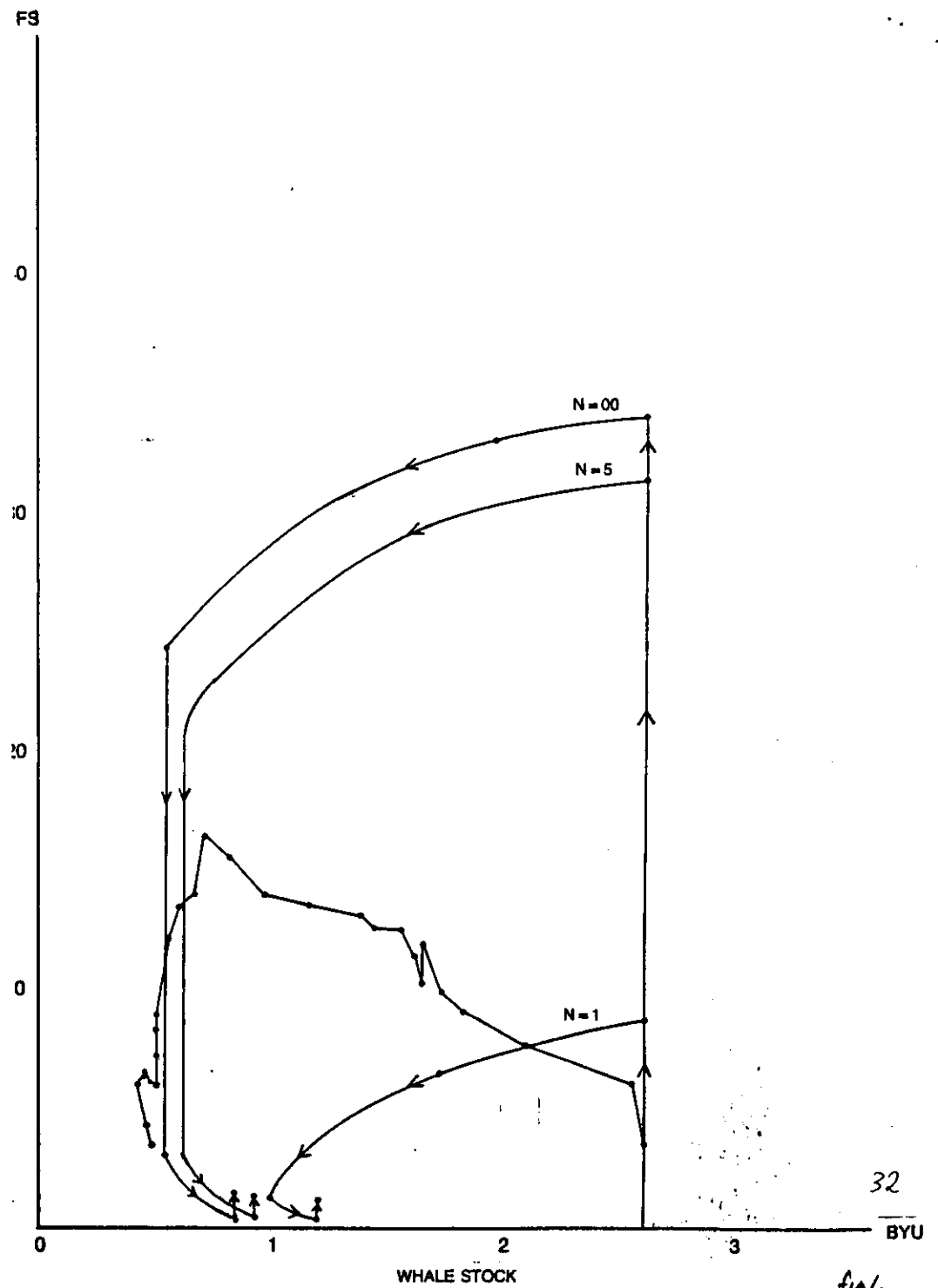
F(x)

H^*

$E^* x$

X^*

X

X_* $\bar{X}$ X^*

X

K_* K^*

X

$N=\infty$

k

K_0

K^0

X_0 X^0

x

E=0   E=K   I=∞
I=0   I=0

$N=1$

k

K_*

K^*

X_* X^*

x

E=0   E=K   I=∞
I=0   I=0

27

N=∞

N=1

X₀   Xº   X*

31

fig 5

FS

N=00

N=5

N=1

0   1   2   3   BYU

WHALE STOCK

32

fig 6

HARVEST PER VESSEL

33

fig 7