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"A Consideration on Species Abundance Relations"

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These are preliminary lecture notes, intended only for distribution to participants.

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A Consideration on Species Abundance Relations

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1. Introduction

The characterization of population size distribution of the species in naturally preserving biological communities has been studied as an interesting subject of the community ecology (R.A.Fisher, A.S.Corbet & C.B.Williams 1943; F.W.Preston 1948; T.W.Anscombe 1950; R.H.MacArthur 1957; D.E.Barton and F.N.David 1959; E.C.Pielou 1969; R.M.May 1975; etc.).

As we know that Maxwell-Boltzmann and Fermi distribution played the fundamental role of the bridge between microscopic (classical or quantum dynamics) and macroscopic (thermodynamics) views in the world of physical sciences, the common character of population structure or species-abundance relations found in biological communities possibly give a effective insight into the fundamental nature of architectural design of field communities, including the thermodynamical restrictions imposed as a part of the functional system (food-webb, ecosystem).

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Thus, really a number of distribution functions and rank-size relations have been proposed to characterize the observed and sampling data of various field communities.

logseries distribution (Fisher, Corbet & Williams)
lognormal distribution (Preston, MacArthur, Williams)
geometric series distribution (Motomura, Odum)
negative binomical distribation (Anscombe)
broken stick distribution (MacArthur)
hypergeometric series distribution (Yule)

Regarding some of these distributions, theoretically argumentative discussions based on respectively specialized reasoning have been presented. However, within my knowledge, there are no theoretical argument on the population structure of multispecies community based on the stability analyses of population dynamics. Today I like to show you just a result obtained by population dynamical analyses of a special type of ~~comm~~ competitive species community.

2. Broken stick distribution as a model of random resource allocation.

Before I enter the main topic, I like to add a short comment concerning the so-called broken stick distribution, ~~the~~

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which was proposed by MacArthur as the niche nonoverlapping model.

He supposed that if S species randomly occupy the limited niche space without overlapping and they can maintain the population of proportional size to occupied niche area, the problem can be reduced to the probability distribution of fragment sizes of randomly broken bar. When we devide a bar into S fragments by randomly locating S-1 points on a line of unit length, the probability that a fragment has the length in $(x, x+dx)$ is given by

$$P_S(x)dx = (S-1)(1-x)^{S-2}, \qquad (1)$$

and if we assign the ranks to S fragments in the descending order of their sizes, the average size of k-th fragment is given by

$$\langle x_k \rangle = \frac{1}{S}\sum_{j=k}^{S}\frac{1}{j} \quad . \qquad (2)$$

This is usually called as broken-stick distribution and refered in many text books, sometimes with the criticism concerning the one-dimensional mapping of niche or resource space.

However we can show that it is possible to give another interpretation to this distribution function, considering random allocation of resources. Consider that N packs of

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resources are randomly allocated to S species, then combinatorial calculation gives the probability that a species obtains n packs as

$$P_S(n) = \binom{N-n+S-2}{S-2} \Big/ \binom{N+S-1}{S-1}. \qquad (3)$$

The probability that the numbers of allocated packs up to j-th least numbers are $n_1 < n_2 < \cdots < n_j$ $(n_1 + n_2 + \cdots + n_j = N_j)$ is

$$P_S(n_1, n_2, \ldots, n_j) = (S)_j \binom{A_j + S - j - 1}{S-j-1} \Big/ \binom{N-1}{S-1} \qquad (4)$$

$$A_j = N - N_j - (S-j)n_j .$$

Here, if we take the limit $N \to \infty$ and $n \to \infty$ with fixed value $n/N = x$, we can obtain the distribution function (1) directly from (3) and rank-size relation (2) can be also derived from (4) through some integral calculations.

3. Population dynamical analyses of one-sided competition model.

Here we consider Lotka-Volterra competition model

$$dn_k/dt = \left(\varepsilon_k - \sum_{j=1}^{k} \alpha_{kj} n_j\right) n_k \qquad k = 1, 2, \ldots, S \qquad (5)$$

where we assumed that S species are labeled $(1, 2, \ldots, S)$ in the order of competitive dominancy, and each species suffers competitive interferences only from the species of higher ranks. Furthermore we assume that the coefficients of mutual interference α_{kj} can be factorized into offensive and defensive factors as

$$\begin{aligned} a_{kj} &= \sigma_k \alpha_j \qquad \text{for } j = k \\ &= \sigma_k \beta_j \qquad \text{for } j \neq k . \end{aligned} \qquad (6)$$

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Then we have

$$dx_k/dt = \sigma_k(r_k - x_k - \sum_{j=1}^{k-1}\xi_j x_j)x_k \qquad (7)$$

where $x_k = \alpha_k n_k$, $r_k = \varepsilon_k/\sigma_k$ and $\xi_k = \beta_k/\alpha_k$. In this case, it can be shown that under the condition

$$r_i \geq r_{i-1} \quad \text{and} \quad \xi_i = \beta_i/\alpha_i < 1 \quad \text{for all } i \qquad (8)$$

there is a globally stable solution

$$x_k^* = (r_k - r_{k-1}) + \sum_{j=1}^{k-1}\Omega_{kj}(r_j - r_{j-1}), \quad \Omega_{kj} = \prod_{i=j}^{k-1}(1-\xi_i). \qquad (9)$$

In some special cases the solution (9) gives the following simple forms.

1) intraspecific interference $\alpha_i = \alpha$, defense factors $\sigma_i = \sigma$ offence factors $\xi_i = \beta_i/\alpha = \xi < 1$ and growth rate $r_i = \varepsilon_i/\sigma = r$ for all i, population size of k-th species $n_k^* = x_k^*/\alpha$ is

$$n_k^* = (r/\alpha)(1-\xi)^{k-1} \quad \text{geometric series distribution}$$

2) species of lower ranks has higher growth rate as $r_k - r_{k-1} = d > 0$, with common values of other parameter as case (1),

$$n_k^* = \frac{r_1}{\alpha\xi}(\xi - \frac{d}{r_1})(1-\xi)^{k-1} + \frac{d}{\alpha\xi}$$

$\xi r_1 > d$: elebated geometric series distribution

$\xi r_1 < d$: ascending

3) ~~offense factor~~ species of lower rank has smaller offense factor as $\xi_i = 1 - \frac{i}{c+i}$

$$n_k^* = \frac{r}{\alpha}\,\frac{1\cdot 2\cdots(k-1)}{(c+1)(c+2)\cdots(c+k-1)} \quad \text{hypergeometric series distribution.}$$