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SMR.478 - 8

THIRD AUTUMN COURSE ON MATHEMATICAL ECOLOGY

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"Matrix Population Models - Part II"

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These are preliminary lecture notes, intended only for distribution to participants.

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Analysis (cont'd.)

$$\lim_{t\to\infty} \frac{\underline{n}(t)}{\lambda_1^t} = c_1 \underline{w}_1$$

λ_1 = asymptotic rate of increase

$\underline{w}_1$ = stable stage distribution

c_1 ?

$$\underline{n}(0) = \sum c_i \underline{w}_i \qquad \text{let } \underline{c} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_s \end{pmatrix}$$

$$= \underline{W}\,\underline{c}$$

$$\Rightarrow \underline{c} = \underline{W}^{-1}\underline{n}(0) \qquad \text{let } \underline{W}^{-1} = \underline{V} = \begin{pmatrix} \underline{v}_1^* \\ \underline{v}_2^* \\ \vdots \\ \underline{v}_s^* \end{pmatrix}$$

(left eigenvectors)

$$\Rightarrow c_1 = \underline{v}_1^*\,\underline{n}(0)$$

So the elements of $\underline{v}_1$ measure the value of the different stages of $\underline{n}(0)$ to asymptotic population size

$\Rightarrow$ $\boxed{\underline{v}_1 = \text{reproductive value distribution}}$

Interpretation

What justification for calculating λ, $\underline{w}$, $\underline{v}$ based on constant vital rates?

prediction (forecast) = statement about what <u>will</u> happen to the population

projection = statement about what <u>would</u> happen to the population, if current conditions were maintained

environment $\Rightarrow$ vital rates $\Rightarrow$ $\underline{A}$ $\Rightarrow$ $\left\{ \begin{array}{c} \lambda \\ \underline{w} \\ \underline{v} \end{array} \right\}$ $\rightsquigarrow$ future population

<u>projection</u> provides information about current conditions, not future dynamics.

speedometer

importance of comparative studies

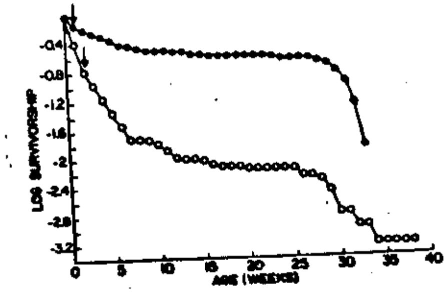
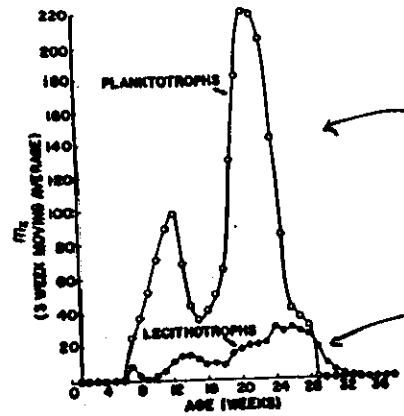


FIG. 1. Survivorship of *Streblospio benedicti* with planktotrophic and lecithotrophic development, reared in the laboratory at 20°C and salinity 34‰. (O) planktotrophic strain; (●) lecithotrophic strain. Age 0 corresponds to the time of release of larvae from the mother. ↓ Indicates the time of settlement.



$\lambda^{(p)} = 1.205$

$\lambda^{(l)} = 1.319$

FIG. 2. Age-specific fecundity (m_x) of *Streblospio benedicti* with planktotrophic and lecithotrophic development, reared in the laboratory at 20°C and salinity 34‰.

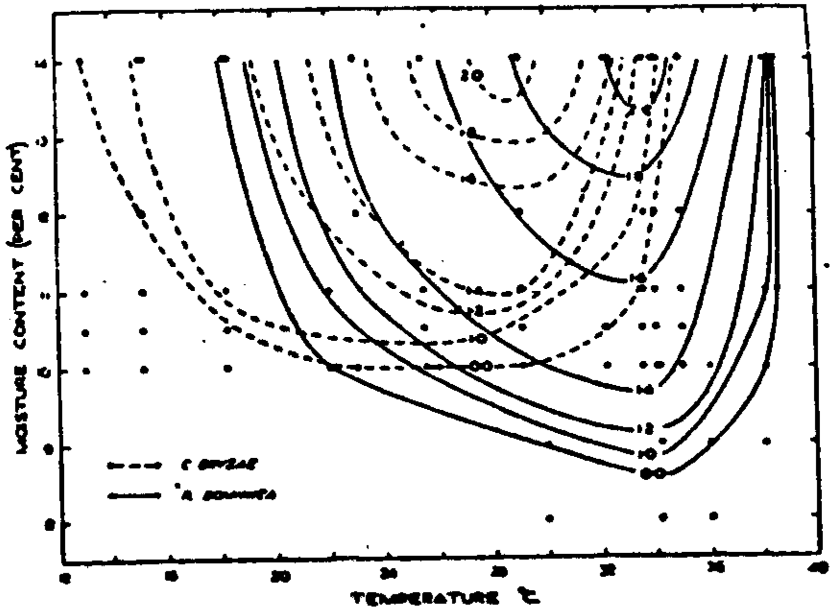


FIG. 8. The finite rate of increase (λ) of *Calandra oryzae* (small strain) and *Rhizopertha dominica* living in wheat of different moisutre contents and at different temperatures. (See final columns of Tables III and IV.)

Perturbation analysis

- how sensitive is λ (or other indices) to changes in the vital rates within the life cycle
  - evaluation of management strategies
  - understanding impact of environmental factors (e.g. toxicant exposure)
  - evolution

genetic variation → variation in vital rates → variation in fitness (λ)

→ selection → evolution

- measurement errors

- Basic formula

$$\underline{A}\,\underline{w} = \lambda\,\underline{w}$$

$$\underline{A}\,d\underline{w} + d\underline{A}\,\underline{w} = \lambda\,d\underline{w} + d\lambda\,\underline{w} \quad \text{(differential)}$$

$$\langle \underline{A}\,d\underline{w}, \underline{v}\rangle + \langle d\underline{A}\,\underline{w}, \underline{v}\rangle = \langle \lambda\,d\underline{w}, \underline{v}\rangle + \langle d\lambda\,\underline{w}, \underline{v}\rangle$$

(scalar product with $\underline{v}$)

$$= \lambda\,\underline{v}^* d\underline{w} + \underline{v}^* d\underline{A}\,\underline{w} = \lambda\,\underline{v}^* d\underline{w} + d\lambda \langle \underline{w}, \underline{v}\rangle$$

$$d\lambda = \frac{\underline{v}^* d\underline{A}\,\underline{w}}{\langle \underline{w}, \underline{v}\rangle} \implies \frac{\partial \lambda}{\partial a_{ij}} = \frac{v_i\,w_j}{\langle \underline{w}, \underline{v}\rangle}$$

Perturbation analysis (cont'd.)

$$\frac{\partial \lambda}{\partial a_{ij}} = \frac{v_i w_j}{\langle \underline{w}, \underline{v} \rangle}$$

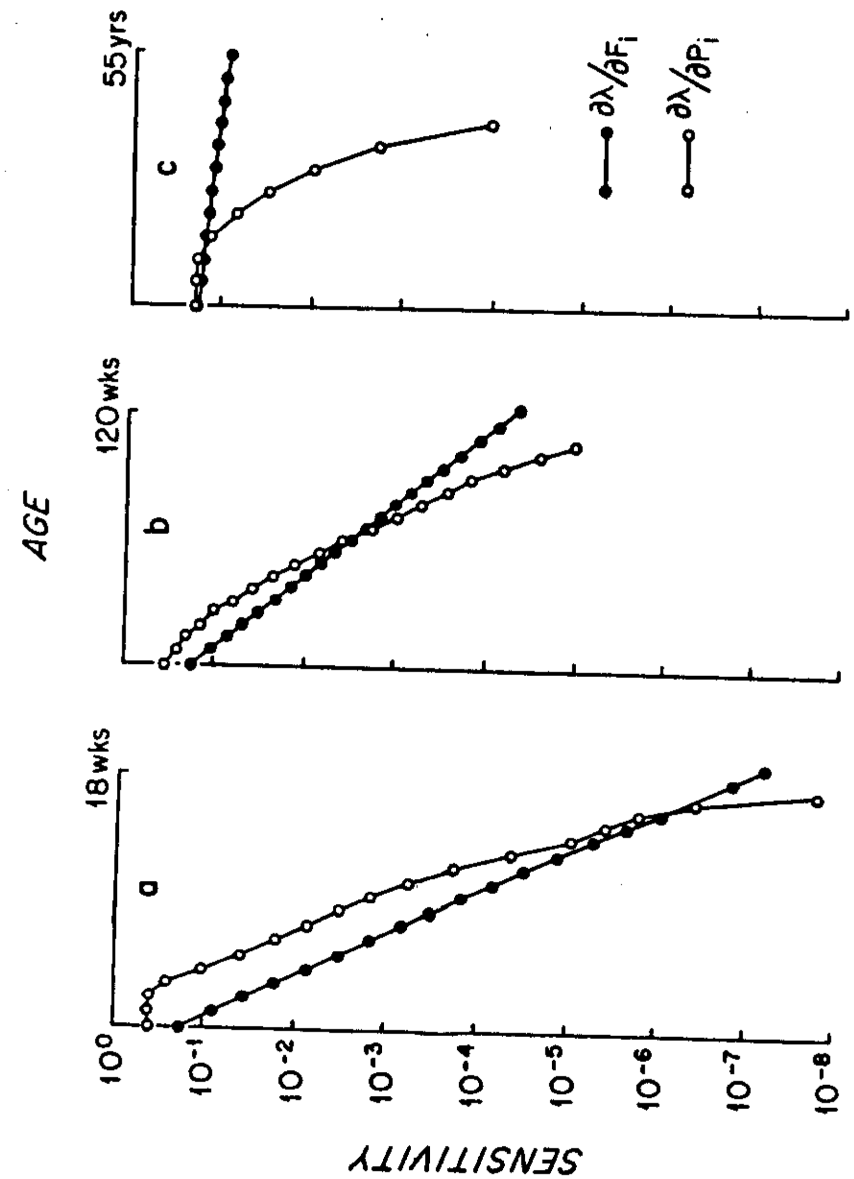
$j \xrightarrow{a_{ij}} i$

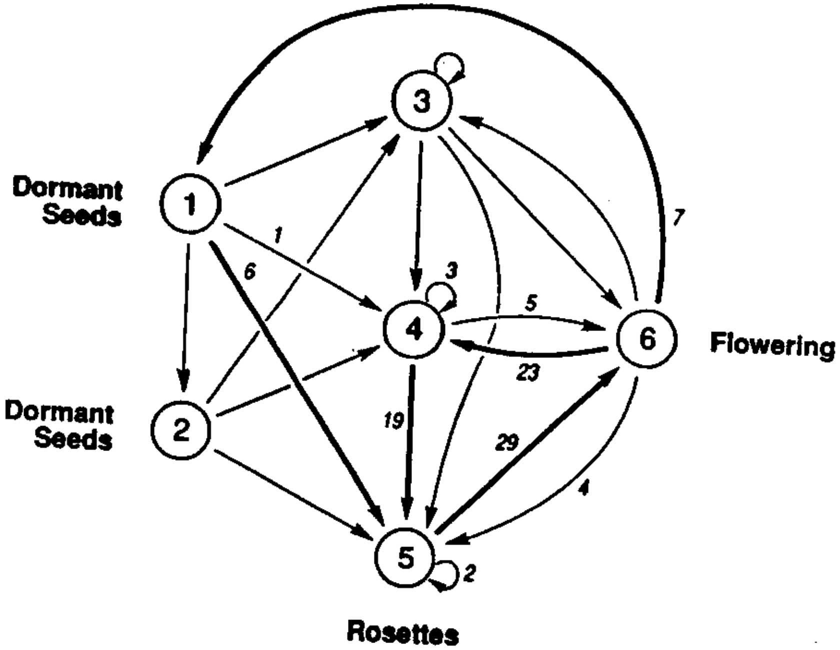
$$= s_{ij} = \underline{\text{sensitivity}}$$

$$\frac{a_{ij}}{\lambda}\frac{\partial \lambda}{\partial a_{ij}} = e_{ij} = \underline{\text{elasticity}} \quad \text{(proportional sensitivity)}$$

$$\sum_i \sum_j e_{ij} = 1$$

e_{ij} = proportional contribution of a_{ij} to λ [more or less]





Dipsacus sylvestris

Applications of sensitivity analysis

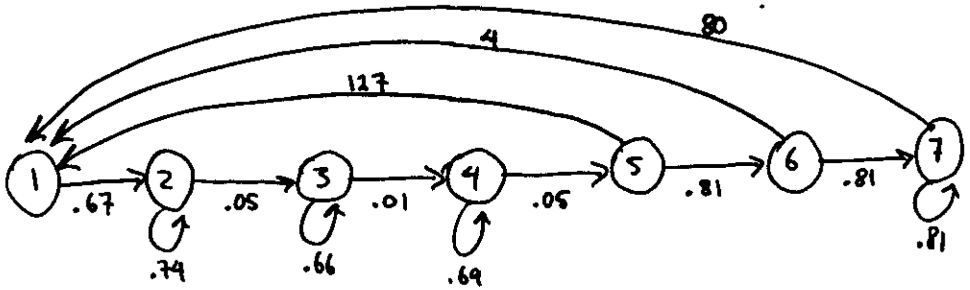
1. Conservation biology and management

Loggerhead sea turtle
(*Caretta caretta*)

D.T. Crouse, L.B. Crowder, and H. Caswell. 1987. *Ecology* 68: 1412–1423

Stage	size	age (approx.)
1. hatchlings	< 10 cm	< 1 yr.
2. small juveniles	10–58	1–7
3. large juveniles	58–80	8–15
4. subadults	80–87	16–21
5. novice breeders	>87	22
6. first year remigrants	>87	23
7 mature breeders	>87	24–54

data quality?



$$\lambda_1 = 0.9450$$

$$r = -0.056$$

Management strategies

Goal – protection of population

⇒ increase value of λ

other criteria?

extinction probability

abundance / yield

Tactics

– protection of beaches, nests, eggs, hatchlings

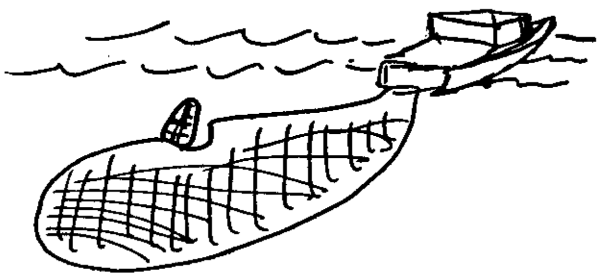
⇓

increases P_1, F

– reducing mortality due to fishing nets (Turtle Exclusion Device) (TED)

⇓

increases survival of juveniles, subadults, adults



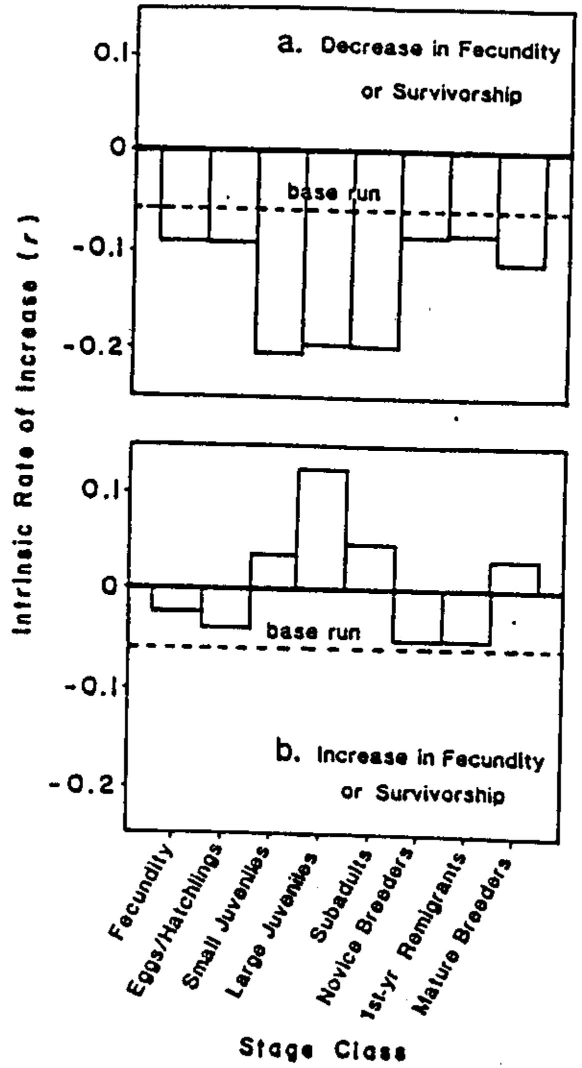


FIG. 1. Changes in rate of increase r resulting from simulated changes in fecundity and survival of individual life history stages in the loggerhead population matrix (remaining components held constant). The dashed line represents the r determined in the baseline run on the initial matrix. (a.) Simulations represent 50% decreases in fecundity or survivorship. (b.) Simulations represent a 50% increase in fecundity or an increase in survivorship to 1.0. Stages 2–4 (juveniles and subadults) show the strongest response to these simulated changes. (Specific calculations are presented in Crouse 1985.)

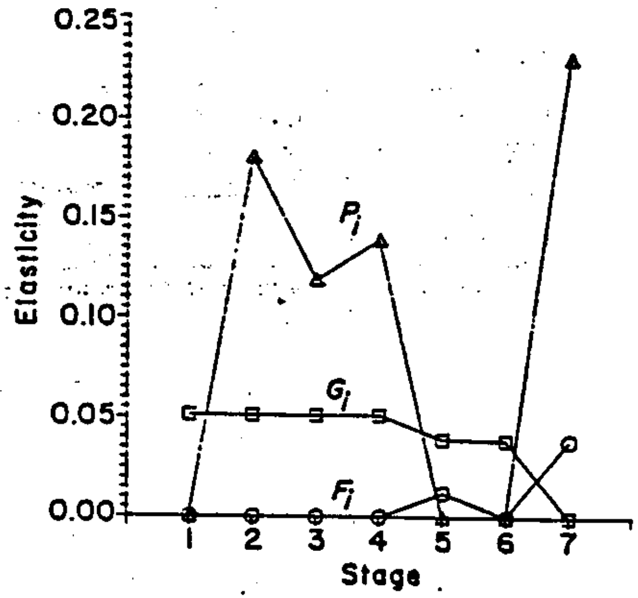


FIG. 3. The elasticity, or proportional sensitivity, of λ_m to changes in fecundity F_i (○), survival while remaining in the same stage P_i (△), and survival with growth G_i (□). Because the elasticities of these matrix elements sum to 1, they can be compared directly in terms of their contribution to the population growth rate r.

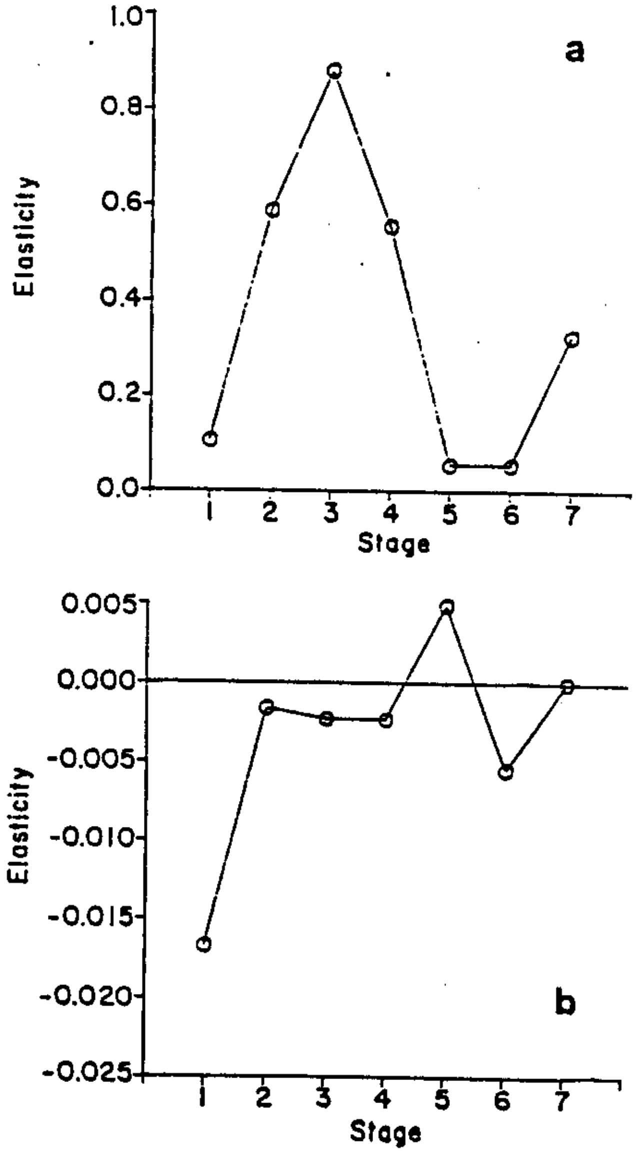


FIG. 4. (a) The elasticity, or proportional sensitivity, of λ_m to changes in annual stage-specific survival probability p_i. (b) The elasticity of λ_m to changes in stage duration d_i. Elasticity in stage duration is negative because stage duration and population growth rates r are inversely related. Overall, proportional sensitivity is much higher to survival than to stage duration.