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Formal Description of a Seismic Source

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FORMAL DESCRIPTION OF A SEISMIC SOURCE

The description of a seismic source we will consider is based on a formalism developed by Backus and Mulcahy in 1976.

We will start from *motion equation*

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i , \quad i,j = 1,2,3 \tag{1}$$

Here u_i - i-component of displacements; $\ddot{u}_i$ - 2-nd derivative of u_i with respect to the time; σ_{ij} - elements of simmetric stress tensor; $\sigma_{ij,j} = \sum_{j=1}^{3} \partial\sigma_{ij}/\partial x_j$ (the summation convention for repeated subscripts is used); ρ - density; f_i - components of external force.

The stresses and dispplacements are connected by *Hooke's law*

$$\sigma_{ij} = \lambda\delta_{ij}e_{kk} + 2\mu e_{ij} \quad \text{(in isotropic case)}, \tag{2}$$

where $e_{ij} = 0.5(u_{i,j} + u_{j,i})$ - elements of strain tensor.

We will assume that before t=0 there was not any motion, so *initial conditions* are following

$$\dot{u} = u = 0 \quad \text{for } t < 0 . \tag{3}$$

Elastic body under consideration is bounded by free surface S_0. It meens that homogeneous *boundary conditions* have to be satisfied:

$$\sigma_{ij} n_j \Big|_{S_0} = 0 , \tag{4}$$

where n_j - components of the normal to the S_0.

The solution of the problem (1)-(4) can be expressed by formula

$$u_i(\mathbf{x},t) = \int_0^t d\tau \int_\Omega G_{ij}(\mathbf{x};\mathbf{y};t-\tau) f_j(\mathbf{y},\tau)\, dV_y \tag{5}$$

or

$$u_i(\mathbf{x},t) = \int_0^{t_e} d\tau \int_\Omega H_{ij}(\mathbf{x};\mathbf{y};t-\tau) \dot{f}_j(\mathbf{y},\tau)\, dV_y . \tag{6}$$

Here G_{ij} - Green function, $H_{ij}(\mathbf{x};\mathbf{y};t) = \int_0^t G_{ij}(\mathbf{x};\mathbf{y};\tau)\, d\tau$,

and $0 < t < t_e$ - time interval when $\dot{\mathbf{f}}$ is not identically zero.

Sources of seismic disturbance.

Seismic disturbances most frequently arise from the action of internal sources (earthquakes or explosions) in absence of any external body forces. One must then set $f_j = 0$ in (1), so that the only solution that satisfies the homogeneous initial

(3) and boundary (4) conditions, as well as Hooke's law (2), will be $u_i = 0$. Non-zero displacements cannot arise in the medium, unless at least one of the above conditions is not true. Following Backus and Mulcahy (1976), we assume seismic motion to be caused by a departure from Hooke's law within some volume of the medium at some time interval $t_e > t > 0$.

Let $u_i(\mathbf{x},t)$ describe the displacements and $\sigma_{ij}(\mathbf{x},t)$ the stresses that would have existed in the medium had Hooke's law (2) been true everywhere in it. Let $s_{ij}(\mathbf{x},t)$ be the actual stresses. The difference

$$\Gamma_{ij}(\mathbf{x},t) = \sigma_{ij}(\mathbf{x},t) - s_{ij}(\mathbf{x},t), \tag{7}$$

called the *stress glut tensor,* is not identically zero within the three-dimensional region Ω. That region we define as source region. Within, and only within, that region, the tensor $\dot{\Gamma}_{ij}(\mathbf{x},t)$ too is not identically zero.

We shall assume that Ω lies wholly within the medium (does not come out to the surface) and that, since some instant of time $t_e > 0$, $\dot{\Gamma}_{ij}(\mathbf{x},t) = 0$ everywhere in the medium. The integral of Γ_{ij} over Ω is called the seismic moment tensor (Kostrov, 1970; Aki and Richards, 1980). As the true motion obeys the equation $s_{ij,j} = \rho\ddot{u}_i$, in accordance with (1) ($\mathbf{f}$=0), one derives from (7)

$$\sigma_{ij,j} + g_i = \rho\ddot{u}_i , \tag{8}$$

$$g_i = -\Gamma_{ij,j} , \tag{9}$$

where $g_i(\mathbf{x},t)$ we will define as equivalent force.

Then the resulting displacements are given by the same formulas, (5) and (6), with f_i replaced by g_i. Using relation (9) for g_i and the Gauss-Ostrogradsky theorem, we finally get

$$u_i(\mathbf{x},t) = \int_0^t dt \int_\Omega G_{ij,k}(\mathbf{x};\mathbf{y};t-\tau)\Gamma_{jk}(\mathbf{y},\tau)\, dV_y \tag{10}$$

or

$$u_i(\mathbf{x},t) = \int_0^{t_e} dt \int_\Omega H_{ij,k}(\mathbf{x};\mathbf{y};t-\tau)\dot{\Gamma}_{jk}(\mathbf{y},\tau)\, dV_y \tag{11}$$

The G_{ij}, H_{ij} are here differentiated with respect to y_k.

If the departure from perfect elasticity is confined to some arbitrary finite area at the inner surface Σ, the stress glut tensor becomes $\Gamma_{jk}(\mathbf{x},t)=m_{jk}(\mathbf{x},t)\delta_\Sigma(\mathbf{x})$, where $\delta_\Sigma(\mathbf{x})$ is a distribution that satisfies

$$\int_V \delta_\Sigma(\mathbf{x})\varphi(\mathbf{x})\,dV_x = \int_\Sigma \varphi(\mathbf{x})\,d\Sigma_x$$

for any function $\varphi(\mathbf{x})$. Integration over the volume V_y in (10), (11) will then reduce to that over the surface Σ:

$$u_i(\mathbf{x},t)=\int_0^t dt\int_\Sigma G_{ij,k}(\mathbf{x};\mathbf{y};t-\tau)\,m_{jk}(\mathbf{y},\tau)\,d\Sigma_y$$

where the points $\mathbf{y}$ belong to Σ. If the departure from perfect elasticity is defined as a discontinuity in displacement $\mathbf{u}$ at Σ without a stress discontinuity, then we have

$$m_{jk}(\mathbf{x},t)=n_q(\mathbf{x})\left[u_p(\mathbf{x},t)\right]c_{jkpq}(\mathbf{x}),$$

where $\mathbf{n}$ is the normal to Σ, $\left[u_p\right]$ - components of the vector of discontinuity. For an isotropic medium we shall have

$$m_{jk}=\lambda\left[u_p\right]n_p\delta_{jk} + \mu(n_j\left[u_k\right] + n_k\left[u_j\right]) \;;$$

in the case of tangential (shear) dislocation we have

$$n_p\left[u_p\right] = 0 \qquad \text{and}$$

$$m_{jk}=\mu(n_j\left[u_k\right] + n_k\left[u_j\right]) \quad . \tag{12}$$

If the departure from perfect elasticity is confined to a small vicinity of $\mathbf{x}_0$ (the region Ω shrinks to a point), then

$$\Gamma_{jk}(\mathbf{x},t)=m_{jk}(t)\,\delta(\mathbf{x}-\mathbf{x}_0)$$

and the equivalent forces g_i take the dipole form

$$g_j = -m_{jk}(t)\frac{\partial\delta(\mathbf{x}-\mathbf{x}_0)}{\partial x_k} \tag{13}$$

Such a source excites a field of the form

$$u_i = \int_0^t m_{jk}(t)\,G_{ij,k}(\mathbf{x};\mathbf{x}_0;t-\tau)\,dt, \tag{14}$$

or

$$u_i = \int_0^{t_e} \dot m_{jk}(t)\,H_{ij,k}(\mathbf{x};\mathbf{x}_0;t-\tau)\,dt, \tag{15}$$

where the G_{ij}, H_{ij} are differentiated with respect to y_k at the point $\mathbf{y} = \mathbf{x}_0$.

A point center of expansion (an ideally concentrated explosion) in an isotropic medium will produce (Aki and Richards, 1980)

$$m_{jk} = m(t)\,\delta_{jk} \quad , \tag{16}$$

while for a point source of slip we shall have

$$m_{jk} = m(t)\,(\varkappa_j n_k + \varkappa_k n_j), \tag{17}$$

where the $\varkappa_j$ are unit vector components in the direction of the discontinuity $\left[\mathbf{u}\right]$ (slip vector) and $m(t) = \mu\left|\left[\mathbf{u}\right]\right|$. The quantity $m_0 = \lim_{t\to\infty} m(t)$ is called the seismic moment.

Relation between the temporal and geometrical parameters of aseismic source and the spatio-temporal stress glut moments.

The region of an earthquake source was defined above as a region at each point of which the tensor $\dot\Gamma_{ij}$ is not identically zero. The source duration is the time during which anelastic motion occurs at various points within the source region, i.e., $\dot\Gamma_{ij}$ is different from zero. Along with these concepts, we shall deal with an instantaneous source, i.e.,a volume involved in anelastic motion at some instrant t, and the local source duration, i.e., the time during which such motion occurs at some point $\mathbf{x}$.

This section contations formulas for estimating the geometry of a source region and the time-averaged geometry of an instantaneous source, as well as formulas for estimating the source duration and the space-averaged local source duration. Following Backus (1977), Bukchin (1989) all these parameters are expressed in terms of spatio-temporal moments of $\dot\Gamma_{ij}(\mathbf{x},t)$ of total degree (both in space and time) 0, 1, and 2. In turn, these moments can, as shown below, be estimated by using long-period records of the displacement at a number of sites at the free surface.

The moment $\dot\Gamma^{(m,n)}(\mathbf{q},\tau)$ of spatial degree m and temporal degree n with respect to point $\mathbf{q}$ and instant of time τ is a tensor of order $m + 2$ and is given by

$$\dot\Gamma^{(m,n)}_{ij;k_1\ldots k_m}(\mathbf{q},\tau) = \int_\Omega dV_x \int_0^\infty \dot\Gamma_{ij}(\mathbf{x},t)(x_{k_1} - q_{k_1})\ldots(x_{k_m} - q_{k_m})(t-\tau)^n dt\,, \tag{18}$$

$$i,j,k_1,\ldots,k_m = 1,2,3.$$

where Ω is a volume outside of which we have $\dot\Gamma(\mathbf{x},t) = 0$. Later we will consider the problem of extracting the moments

(18) from long-period displacement records; for the present we discuss the information on earthquake source involved in the moments of total degree $m + n$, equal to 0, 1 and 2.

When i and j are fixed, the moments (18) characterize the spatio-temporal configuration of the scalar field $\dot{\Gamma}_{ij}(\mathbf{x}, t)$. A seismic source is described by six (because $\dot{\Gamma}_{ij}$ is symmetric) different scalar functions, making an interpretation of its moments difficult. Following Backus (1977), we set up a correspondence between an earthquake source and the scalar field

$$c(\mathbf{x}, t) = \dot{\Gamma}_{ij}^{(0,0)}\dot{\Gamma}_{ij}(\mathbf{x}, t) \tag{19}$$

regarding the geometrical and tempolar parameters of this field as estimates of the respective source parameters.

Note that $\dot{\Gamma}_{ij}^{(0,0)}$ is a limiting value (as $t \to \infty$) of the seismic moment tensor $\int_{\Omega} \Gamma_{ij}(\mathbf{x}, t) dV_x$ (see Kostrov, 1970). The moments of $c(\mathbf{x}, t)$ are given by a formula similar to (18):

$$c^{(m,n)}_{k_1 \ldots k_m}(\mathbf{q}, \tau) = \tag{20}$$

$$= \int_{\Omega} dV_x \int_0^{\infty} c(\mathbf{x}, t)(x_{k_1} - q_{k_1}) \ldots (x_{k_m} - q_{k_m})(t-\tau)^n dt$$

If the moments of $\dot{\Gamma}_{ij}(\mathbf{x}, t)$ are known, the moments of $c^{(m,n)}$ can be got from

$$c^{(m,n)}_{k_1 \ldots k_m}(\mathbf{q}, \tau) = \dot{\Gamma}_{ij}^{(0,0)}\dot{\Gamma}^{(m,n)}_{ij;k_1 \ldots k_m} \tag{21}$$

Estimation of temporal and geometrical parameters of an earthquake source. Suppose we know the moments (20) of $c(\mathbf{x}, t)$ as given by (19). The inequality $c(\mathbf{x}, t) \geq 0$ is assumed to be true. According to Backus (1977), many types of seismic sources are consistent with it. Thus, it is readily shown that, when the source is a plain ideal fault (the displacement discontinuity vector lies in the rupture plane) the inequality is equivalent to the requirement of the slip velocity vector being within $\pi/2$ of some fixed direction. Backus (1977) puts forward some estimates of

the temporal and geometrical parameters of an earthquake source based on the assumption of $c(\mathbf{x}, t)$ being nonnegative. Source location is estimated by the spatial centroid $\mathbf{q}_c$ of the field $c(\mathbf{x}, t)$ defined as

$$\mathbf{q}_c = \int_{\Omega} dV_x \int_0^{\infty} c(\mathbf{x}, t)\mathbf{x} dt \Big/ \int_{\Omega} dV_x \int_0^{\infty} c(\mathbf{x}, t) dt , \tag{22}$$

which can be written in the form

$$c^{(0,0)}\mathbf{q}_c = \mathbf{c}^{(1,0)}(0) . \tag{23}$$

If $m(\mathbf{x}) = \int_0^{\infty} c(\mathbf{x}, t) dt$ is regarded as a distribution of mass in space, its center of mass is identical with the centroid $\mathbf{q}_c$. In a similar fashion, the temporal centroid τ_c is estimated by

$$c^{(0,0)}\tau_c = \mathbf{c}^{(0,1)}(0) . \tag{24}$$

The values of $\mathbf{q}_c$ and τ_c combined define the spatio-temporal centroid of a source. The source duration is estimated by $2\Delta\tau$, where

$$(\Delta\tau)^2 = \int_{\Omega} dV_x \int_0^{\infty} c(\mathbf{x}, t)(t-\tau_c)^2 dt \Big/ \int_{\Omega} dV_x \int_0^{\infty} c(\mathbf{x}, t) dt \tag{25}$$

or, in our notation,

$$(\Delta\tau)^2 = c^{(0,2)}(\tau_c)/c^{(0,0)}. \tag{26}$$

Let $\mathbf{r}$ be a unit vector. The mean source size along $\mathbf{r}$ is estimated by $2l_r$, where

$$l_r^2 = \int_{\Omega} dV_x \int_0^{\infty} c(\mathbf{x}, t)\left[(\mathbf{x}-\mathbf{q}_c)\mathbf{r}^2\right] dt \Big/ \int_{\Omega} dV_x \int_0^{\infty} c(\mathbf{x}, t) dt \tag{27}$$

We define the matrix

$$\mathbf{W} = c^{(2,0)}(\mathbf{q}_c)/c^{(0,0)}. \tag{28}$$

Then l_r^2 can be written as a quadratic form:

$$l_r^2 = \mathbf{r}^T \mathbf{W} \mathbf{r} \tag{29}$$

where $\mathbf{r}^T$ is the transpose of $\mathbf{r}$.

From (29) it follows that a source region has the least size along that eigenvector of $\mathbf{W}$ corresponding to the least eigenvalue and the greatest size along that eigenvector of the same matrix corresponding to the greatest eigenvalue.

To interpret the moment $c^{(1,1)}$, the last of the moments of $c(\mathbf{x},t)$ of total degree 0,1, and 2, we consider the following problem. Among points $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}t$ that move at uniform velocity, find the one around which $c(\mathbf{x},t)$ is concentrated in the best manner in the sense of minimizing the function

$$\Phi(\mathbf{x}_0,\mathbf{v}) = \int_\Omega dV_x \int_0^\infty c(\mathbf{x},t)(\mathbf{x}-\mathbf{x}_0-\mathbf{v}t)^T(\mathbf{x}-\mathbf{x}_0-\mathbf{v}t)\,dt \ . \tag{30}$$

As shown by Backus (1977), the solution is

$$\mathbf{x}_0 = \mathbf{q}_c - \mathbf{v}\tau_c \ , \qquad \mathbf{v} = \mathbf{w} \ / \ (\Delta\tau)^2 \tag{31}$$

where

$$\mathbf{w} = c^{(1,1)}(\mathbf{q}_c,\tau_c) \ / \ c^{(0,0)}.$$

Instantaneous source and estimation of its mean geometrical parameters. Unelastic motion involves various domains of the space as a result of source action. The source region discussed above is the union of all these domains . An instantaneous source is understood to be a time-variable volume which at each instant of time includes points where anelastic motion occurs, i.e., the glut stresses are functions of time. In our opinion, even approximate estimates of time-averaged geometrical parameters for such a region may be useful in studying the processes taking place at the earthquake source.

Thus, a point $\mathbf{x}$ at time t_0 belongs to an instantaneous source, if $\dot\Gamma_{ij}(\mathbf{x},t)$ is not identically zero within any vicinity of t_0. In a similar fashion to the above, we consider a scalar field $c(\mathbf{x},t)$ given by (29) instead of a tensor field $\dot\Gamma_{ij}(\mathbf{x},t)$: at each instant of time the instantaneous source is a region at each point of which $c(\mathbf{x},t)$ is not identically zero within any vicinity of t_0.

An instantaneous centroid is understood to be $\mathbf{q}(t)$ as given by formula

$$\mathbf{q}(t) = \int_\Omega c(\mathbf{x},t)\mathbf{x}\,dV_x \ / \int_\Omega c(\mathbf{x},t)\,dV_x \tag{32}$$

As $c(\mathbf{x},t) \ge 0$, (32) defines the function for which the following functional attains its minimum

$$F\big[\mathbf{z}(t)\big] = \int_0^\infty dt \int_\Omega c(\mathbf{x},t)\big[\mathbf{x}-\mathbf{z}(t)\big]^T\big[\mathbf{x}-\mathbf{z}(t)\big]dV_x \ . \tag{33}$$

As to the times t at which $c(\mathbf{x},t) \equiv 0$ over $\mathbf{x}$, $\mathbf{q}(t)$ may be defined in any manner for these.

Similarly to (27), the mean size of an instantaneous source along $\mathbf{r}$ is estimated by $2d_r$, where

$$d_r^2 = \int_\Omega dV_x \int_0^\infty c(\mathbf{x},t)\Big\{\big[\mathbf{x}-\mathbf{q}(t)\big]\mathbf{r}\Big\}^2 dt \ / \int_\Omega dV_x \int_0^\infty c(\mathbf{x},t)\,dt \ . \tag{34}$$

An estimate of d_r is thus obtained from knowledge of $\mathbf{q}(t)$ that minimizes (33) . The solution (32) is expressed in terms of the spatial moments of $c(\mathbf{x},t)$ which we do not know. We wish to express d_r in terms of the spatio-temporal moments (20) of $c(\mathbf{x},t)$. In the general case this can be done only approximately. Namely, the $\mathbf{q}(t)$ in (34) will be fitted by a function $\mathbf{y}(t)$ that minimizes (33) among polynomials of degree n. The existence and uniqueness of this minimum was proved by Bukchin (1989).

We are going to derive explicit formulas for a linear approximation of $\mathbf{q}(t)$, i.e., when n=1. The minimizing function is then identical with (30) discussed in Backus (1977) and

$$\mathbf{y}(t) = \mathbf{q}_c + (t-\tau_c)\mathbf{w}/(\Delta\tau)^2 \ , \tag{35}$$

i.e., $\mathbf{y}(t)$ is the radius-vector of a point moving at a constant velocity $\mathbf{w}/(\Delta\tau)^2$, and which is at $\mathbf{q}_c$ at time τ_c.
Substituting $\mathbf{y}(t)$ from (35) into (34) and denoting the approximate value of d_r by $\hat d_r$, we get

$$\hat d_r^2 = \mathbf{r}^T\left(\mathbf{W} - \frac{\mathbf{w}\mathbf{w}^T}{(\Delta\tau)^2}\right)\mathbf{r} \ . \tag{36}$$

Note that the estimate $\hat d_r$ is an upper bound on d_r, i.e., the inequality $d_r \le \hat d_r$ holds.

Within the approximation considered, an instantaneous source has the least mean size along the eigenvector of $W - \frac{w\,w}{(\Delta\tau)^2}$ corresponding to the least eigenvalue while the greatest size is along the eigenvector corresponding greatest eigenvalue of the same matrix.

Estimation of averaged local source duration. Anelastic motion is excited at different points of the source region during different time intervals that make up the source duration whose estimation was discussed above. The local source duration at point x_0 is understood to be the time interval during which anelastic motion occurs at the point, i.e., $c(x_0, t)$ is different from zero.

The local time centroid $\tau(x)$ is defined to be

$$\tau(x) = \int_0^\infty c(x,t)\,t\,dt \Big/ \int_0^\infty c(x,t)\,dt \quad . \tag{37}$$

Here x belongs to the source region, so that $\int_0^\infty c(x,t)\,dt \neq 0$.

Since $c(x,t) \geq 0$, (37) minimizes the functional

$$Q\left[\varphi(x)\right] = \int_\Omega dV_x \int_0^\infty c(x,t)\left[t-\varphi(x)\right]^2 dt \ , \tag{38}$$

i.e., $\varphi(x) = \tau(x)$ is the point where Q is a minimum.

The space-averaged local source duration will be estimated by $2\Delta\theta$ where, similarly to (25),

$$(\Delta\theta)^2 = \int_\Omega dV_x \int_0^\infty c(x,t)\left[t-\tau(x)\right]^2 dt \Big/ \int_\Omega dV_x \int_0^\infty c(x,t)\,dt \ . \tag{39}$$

From (38), (39) it follows that

$$(\Delta\theta)^2 = Q\left[\tau(x)\right]/c^{(0,0)} \ . \tag{40}$$

The estimate $\Delta\theta$ is this obtained by minimizing Q. In a similar fashion to the estimation of the mean instantaneous sourse sizes, $\Delta\theta$ can only approximately be expressed in terms of the spatio-temporal moments of $c(x,t)$. $\tau(x)$ in (40) can be fitted by a function $\theta(x)$ that minimizes Q among polynomials of degree n in x. The existence and uniqueness of this minimum was proved by Bukchin (1989).

Let the polynomial $\theta(x)$ is such that $Q\left[\theta(x)\right]$ is the minimum

of Q. Then $(\Delta\theta)^2$ will be approximated by

$$(\Delta\hat{\theta})^2 = Q\left[\theta(x)\right]/c^{(0,0)} \ . \tag{41}$$

One can easily see that $\Delta\theta \leq \Delta\hat{\theta}$ must be true.

We'll consider the linear approximation of $\tau(x)$, i.e., $n=1$, in some detail. The expression for $\theta(x)$ can then be written as

$$\theta(x) = \tau_0 + u^T(x-q_c) \ , \tag{42}$$

From (42) one can see that $\theta(x)$ is the arrival time at point x for a plane with the normal $u/|u|$ propagating at velocity $u/(uu)$, so that $\theta(q_c) = \tau_0$. We denote this plane by Σ_θ. here u is the slowness vector.

It is easy to get relations

$$\tau_c = \tau_0 \ , \quad w = W\,u \ . \tag{43}$$

Whence we obtain

$$\theta(x) = \tau_c + w^T W^{-1}(x-q_c) \ , \tag{44}$$

$$(\Delta\hat{\theta})^2 = (\Delta\tau)^2 - w^T W^{-1} w \ . \tag{45}$$

Formulas (26) and (29) provide estimates for overall source duration and the mean source sizes, formulas (45) and (36) estimate space-averaged local source duration and the time-averaged sizes of an instantaneous source. We now derive a relation connecting these quantities. Let $2l_u$ estimate the source size along the normal to Σ_θ, and let $2\hat{d}_u$ be time-averaged size of an instantaneous source in the same direction. Then we put $r = u/\sqrt{u\,u}$ in (29) and (36) and, remembering that $u = W^{-1}w$, derive from (29), (36) and (45) :

$$\hat{d}_u / l_u = \Delta\hat{\theta}/\Delta\tau \ . \tag{46}$$

Provided the local source duration is small compared with the overall source duration, i.e., $\Delta\theta/\Delta\tau \ll 1$, it follows from (29), (36) and (45) that the approximate relation $\Delta\theta \approx |u|\,l_u$ holds, so from (46) we get

$$\hat{d}_u/\Delta\hat{\theta} \approx 1/|u| \tag{47}$$

(one should remember that $1/|u|$ is the velocity of the plane Σ_θ).

Relations (46) and (47) demonstrate consistency between the estimates of spatial (29), (36) and temporal (25), (45) parameters of an earthquake source.

Relation between the moments of a stress glut tensor and the moments of an equivalent force. The displacement field excited by a source with the stress glut tensor $\Gamma_{ij}(x,t)$ is the solution to the boundary value problem (1) through (4) (the force f_i should be replaced by the equivalent force g_i given by (9)). Since the problem is correctly formulated, the equivalent force $\mathbf{g}(\mathbf{x},t)$ is uniquely determined by the displacement field $\mathbf{u}(\mathbf{x},t)$ due to it, while the stress glut tensor $\Gamma_{ij}(\mathbf{x},t)$, as can be deduced from (9) , is determined by the displacements (through $\mathbf{g}(\mathbf{x},t)$), apart from the tensor $\tilde{\Gamma}_{ij}(\mathbf{x},t)$ for which the following equality is true

$$\tilde{\Gamma}_{ij,j} = 0 \ .$$

Determination of $g_i(\mathbf{x},t)$ based on (1) through (2) requires knowledge of the displacement $\mathbf{u}(\mathbf{x},t)$ at any point within the medium. If we are interested in the moments of the equivalent force $g^{(m,n)}_{i;k_1\dots k_m}$ or in those of its time derivative $\dot{g}^{(m,n)}_{i;k_1\dots k_m}$, these can, as will be shown below, be expressed in terms of the displacement or its spectrum at some finite number of points at the free surface (the moments of $\mathbf{g}^{(m,n)}$and $\dot{\mathbf{g}}^{(m,n)}$ are given by (18) with Γ_{ij} replaced by g_i or $\dot{g}_i$, respectively).

We wish to find out whether the moments of $\dot{\Gamma}_{ij}$ can be expressed in terms of those of an equivalent force. From the definition of moments of $\dot{g}_i$ and $\dot{\Gamma}_{ij}$ (18),formula (9) , and the Gauss-Ostrogradsky theorem we have

$$g_i^{(0,n)}(\tau) = \dot{g}_i^{(0,n)}(\tau) = 0 \quad , \tag{48}$$

$$\dot{g}^{(m,n)}_{i;k_1\dots k_m}(\mathbf{q},\tau) = \dot{\Gamma}^{(m-1,n)}_{ik_1;k_2\dots k_m}(\mathbf{q},\tau) +$$

$$+ \dot{\Gamma}^{(m-1,n)}_{ik_2;k_3\dots k_m k_1}(\mathbf{q},\tau) + \dots + \dot{\Gamma}^{(m-1,n)}_{ik_m;k_1\dots k_{m-1}}(\mathbf{q},\tau) \ , \tag{49}$$

$$n \ge 0 \ , \ m \ge 1 \ ; \ i,k_1,\dots,k_m = 1,2,3 \ .$$

Relation (48) is natural, since the zero (spatially) moment of $\mathbf{g}$ is the resultant of internal forces. The summation on the right-hand side of (49) involves all elements of the moment $\dot{\Gamma}^{(m-1,n)}$ obtained by cyclic permutation of $k_1,\dots,k_m$.

Fixing m and n, and assuming some values of $i,k_1,\dots,k_m$, one can obtain from (49) a set of equations for the elements of $\dot{\Gamma}^{(m-1,n)}_{ij;k_1\dots k_{m-1}}(\mathbf{q},\tau)$. When $m = 1$, (49) becomes

$$\dot{g}^{(1,n)}_{i;k}(\mathbf{q},\tau) = \dot{\Gamma}^{(0,n)}_{ik}(\tau) \ . \tag{50}$$

From (18) and the symmetry of $\dot{\Gamma}_{ij}$ one deduces the symmetry of $\dot{\Gamma}^{(m,n)}_{ij,k_1\dots k_m}$ with respect to i,j and $k_1,\dots,k_m$, the moment $\dot{g}^{(m,n)}_{i;k_1\dots k_m}$ being symmetric with respect to $k_1,\dots,k_m$. Formula (50) gives the result that $\dot{g}^{(1,n)}_{i;k}$ is symmetric with respect to i and k. In the general case the number of different elements of $\dot{\Gamma}^{(m-1,n)}_{ij;k_1\dots k_{m-1}}$ which has the above symmetries is given by

$$N_\Gamma = 3m\ (m + 1)$$

To sum up, the number of unknowns in (49) is N_Γ . The number of equations is determined by the number of different elements of $\dot{g}^{(m,n)}_{i;k_1\dots k_m}$; taking its symmetry for $m \ge 2$ into account, this number is

$$N_g = 3(m+1)(m+2)/2 \ .$$

The discussion in Baskus and Mulcahy (1976) shows that the equations are linearly independent. Thus, when $m \ge 2$, formula (49) defines a set of N_g equations in N_Γ unknowns, and

$$N = N_\Gamma - N_g = 3(m + 1)(m - 2)/2 \ . \tag{51}$$

It is easy to see that we have $N = 0$ when $m = 2$, i.e., the number of equations equals the number of unknowns. Formula (49) then becomes

$$\dot{g}^{(2,n)}_{i;k_1k_2}(\mathbf{q},\tau) = \dot{\Gamma}^{(1,n)}_{ik_1;k_2}(\mathbf{q},\tau) + \dot{\Gamma}^{(1,n)}_{ik_2;k_1}(\mathbf{q},\tau) \ . \tag{52}$$

A cyclic permutation of i,k_1,k_2 on the left-hand side of (52) yields

$$\dot{g}^{(2,n)}_{k_1;k_2 i}(\mathbf{q},\tau) = \dot{\Gamma}^{(1,n)}_{k_1k_2;i}(\mathbf{q},\tau) + \dot{\Gamma}^{(1,n)}_{k_1 i;k_2}(\mathbf{q},\tau) \tag{53}$$

$$\dot g^{(2,n)}_{k_2;ik_1}(\mathbf{q},\tau) = \dot\Gamma^{(1,n)}_{k_2i;k_1}(\mathbf{q},\tau) + \dot\Gamma^{(1,n)}_{k_2k_1;i}(\mathbf{q},\tau) \qquad (54)$$

The single solution of (52) through (54) is

$$\dot\Gamma^{(1,n)}_{ik_1;k_2}(\mathbf{q},\tau) =$$
$$= 0,5\left[\, \dot g^{(2,n)}_{i;k_1k_2}(\mathbf{q},\tau) + \dot g^{(2,n)}_{k_1;k_2i}(\mathbf{q},\tau)_i - \dot g^{(2,n)}_{k_2;ik_1}(\mathbf{q},\tau)\right]. \qquad (55)$$

To sum up, the moments of $\dot\Gamma^{(m-1,n)}$ for $m \le 2$ are uniquely expressed in terms of $\dot g^{(m,n)}$ by formulas (50) and (55) . When $m \ge 2$, the number of unknowns in (49) exceeds the number of equations (see (51)) and moments of the stress glut tensor cannot be uniquely expressed in terms of those of the equivalent force. This can however be done by using some prior information on the source. Assumption that the source is a plane ideal fault, for example, will provide such a uniqness.

Relation between the displacement field and the moments of the equivalent force.

We are going to discuss relations that connect observed displacements with the moments of the equivalent force and can be used to estimate the moments.

Replacing f_i by $\dot g_i$ in (6) , we get a formula that expresses the displacement u_i in terms of the derivative of the equivalent force $\dot g_j$:

$$u_i(\mathbf{x},t) = \int_0^{t_e} d\tau \int_\Omega H_{ij}(\mathbf{x},\mathbf{y},t-\tau)\,\dot g_j(\mathbf{y},\tau)\,dV_y \ .$$

Replacing in the expression the function $H_{ij}(\mathbf{x},\mathbf{y},t-\tau)$ by its Taylor series in powers of $\mathbf{y}$ and in powers of τ, we get

$$u_i(\mathbf{x},t) = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{m!\,n!}\dot g^{(m,n)}_{j;k_1\ldots k_m}(0,0)\ \times$$
$$\times\ \frac{\partial^n}{\partial t^n}\frac{\partial}{\partial y_{k_1}}\cdots\frac{\partial}{\partial y_{k_m}}H_{ij}(\mathbf{x},\mathbf{y},t)\big|_{y=0} \ . \qquad (56)$$

Expanding H_{ij} in powers of $\mathbf{y}$, we assume the elastic parameters

to be sufficiently smooth. We have for the Fourier transforms $\hat u_i(\mathbf{x},\omega)$ and $\hat H_{ij}(\mathbf{x},\mathbf{y},\omega)$ from (56) :

$$\hat u_i(\mathbf{x},\omega) = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{m!\,n!}\dot g^{(m,n)}_{j;k_1\ldots k_m}(0,0)\ \times$$
$$\times\ (i\omega)^n\frac{\partial}{\partial y_{k_1}}\cdots\frac{\partial}{\partial y_{k_m}}\hat H_{ij}(\mathbf{x},\mathbf{y},\omega)\big|_{y=0} \ . \qquad (57)$$

Since (56) and (57) involve infinite series, these relations cannot be used to compute the moments $\dot g^{(m,n)}$. However, when the displacement function $u_i(\mathbf{x},t)$ and the integral of Green function $H_{ij}(\mathbf{x},\mathbf{y},t)$ have been lowpass filtered, the terms in (56) and (57) start to decrease with m and n increasing and one might then restrict oneself to considering finite sums only.

Most of the low-frequency displacement energy is usually transported by surface waves. The long-period displacements or their spectrum in a surface wave can be represented by (56) and (57) , respectively. H_{ij} is to be replaced by the integral of the appropriate long-period surface wave Green function for the model and wave type in hand, and $\hat H_{ij}$ by the spectrum $(i\omega)^{-1}\hat G_{ij}$.

Let $\hat u_i(\mathbf{x},\omega)$ in (57) be the low-frequency spectrum of such a wave. From formulas for $\hat H_{ij}$ one can deduce that the quantity $\partial/\partial y_{k_1}\ldots\partial/\partial y_{k_m}\hat H_{ij}$ is proportional to ω^m. We choose the time origin so that the source starts at an instant close to t=0, while the origin of coordinates is chosen at a point that is close to the region Ω or belongs to it. Let $L=max_{\mathbf{x}\in\Omega}|\mathbf{x}|$. Then the moment $\dot g^{(m,n)}(0,0)$ does not exceed a value proportional to $L^m t_e^n$ (t_e is the source duration). Assuming L and t_e to be proportional quantities for a seismic source, we conclude that, when $\omega t_e < 1$, the terms in (57) decay at least as rapidly as $(\omega t_e)^{m+n}$. Thus, the infinite sums in (56) and (57) can, with adequate accuracy for low enough frequencies ω, be replaced by sums involving a few first terms such that $m + n \le M$. Representing in this form displacements (or their spectrum) in surface waves recorded at a number of sites at the free surface, we can derive a set of equations for determining the moments $\dot{g}^{(m,n)}$ of total degree $m + n \le M$. Different types of surface waves and different modes yield independent equations for $\dot{g}^{(m,n)}$, as they contain different kinds of information on the source processes. The moments can be similarly estimated using body wave records.

We wish to note that the spatio-temporal parameters of an earthquake source cannot be estimated directly from the moments of the equivalent force (without using the moments of the stress glut tensor), because the zero spatial moment of $\dot{g}_i$ is identically zero(see formula (48)).

SOURCE PARAMETERS' ESTIMATION FROM LONG PERIOD SURFACE WAVE DATA

Relation between displacement field and seismic moment tensor

Let us consider a low frequency part of spectrum of the k-th component of displacements carried by some Love or Rayleigh mode $u_k(\mathbf{r},\omega)$. If frequency ω is small (time duration of the source is much smaller than period, and size of the source region is much smaller than wave length), then we can approximate the source by an instant point source and express $u_k(\mathbf{r},\omega)$ by formula

$$u_k(\mathbf{r},\omega) = -(i/\omega) M_{mn} G_{km,n}(\mathbf{r},\mathbf{s},\omega) \tag{58}$$

(we take into account only the first term for $m = 1$ and $n=0$ in formula (57), $M_{mn} = \dot{\Gamma}^{(0,0)}_{mn} = \dot{g}^{(1,0)}_{m;n}$).

Here M_{mn} - elements of seismic moment tensor; $\mathbf{r}$ - point of registration; $\mathbf{s}$ - radius-vector of the source; $G_{km}(\mathbf{r},\mathbf{s},\omega)$ - Fourier transform of surface wave part of Green function, which corresponds to a given Love or Rayleigh mode;

$$G_{km,n}(\mathbf{r},\mathbf{s},\omega) = \partial G_{km}(\mathbf{r},\mathbf{s},\omega)/\partial s_n .$$

The summation convention for repeated subscripts is used (m,n = 1,2,3; 1 corresponds to vertical coordinate z, 2 and 3 - to horizontal coordinates x and y).

We'll consider only media with smooth horizontal inhomogeneity. It means that variation of properties is small along any horizontal direction, in comparison with wave length.

For this assumption surface waves spectral parameters are locally determined (they depend on horizontal coordinates x and y) and are the same as in horizontally homogeneous medium with the same structure as under the surface point (x,y).

In this case function G_{km} can be described by formula

$$G_{km}(\mathbf{r},\mathbf{s},\omega) = (-1)^{m(m-1)/2} A W^{(k)}(\omega,0,\varphi)\big|_{M_r} W^{(m)}(\omega,h,\varphi)\big|_{M_s} \times \\ \times \exp(-i\psi) , \tag{59}$$

where $A = 1/\sqrt{8\pi\omega}\, \exp(-i\pi/4)/\sqrt{(vcI)\big|_{M_s} (cI)\big|_{M_r} J(\omega,\mathbf{r})}$; $\psi = \omega L/\hat{v}$;

v - phase velocity; c - group velocity; M_s marks the medium at the source region and M_r - the medium near the station;

$I = \int_0^\infty \rho \left[V^{(3)}(\omega,z)\right]^2 dz$ for Love wave (ρ - density),

$I = \int_0^\infty \rho \left\{\left[V^{(1)}(\omega,z)\right]^2 + \left[V^{(2)}(\omega,z)\right]^2\right\} dz$ for Rayleigh wave;

$V^{(1)}$, $V^{(2)}$ - vertical and radial components of vector eigenfunction of Rayleigh differential operator; $V^{(3)}$ - eigenfunction of Love differential operator; $\hat{v}=L/\int_{\mathbf{L}} \frac{dL}{v}$ - average phase velocity along the path; $\mathbf{L}$ - the ray from **s** to **r**; $\mathbf{s} = (h,0,0)$; **r** - a point on the free surface; L - the length of the ray $\mathbf{L}$; J - geometrical spreading;

for Love wave

$$W^{(1)}(\omega,z,\varphi) = 0, \quad W^{(2)}(\omega,z,\varphi) = -i \sin\varphi \; V^{(3)}(\omega,z),$$
$$W^{(3)}(\omega,z,\varphi) = i \cos\varphi \; V^{(3)}(\omega,z);$$

for Rayleigh wave

$$W^{(1)}(\omega,z,\varphi) = V^{(1)}(\omega,z), \quad W^{(2)}(\omega,z,\varphi) = -i \cos\varphi \; V^{(2)}(\omega,z),$$
$$W^{(3)}(\omega,z,\varphi) = -i \sin\varphi \; V^{(2)}(\omega,z);$$

φ - initial azimuth of the ray $\mathbf{L}$.

Derivatives $G_{km,n}$ are determined by equations

$$G_{km,1} = (-1)^{m(m-1)/2} A W^{(k)}(\omega,0,\varphi)\big|_{M_r} \partial W^{(m)}(\omega,h,\varphi)/\partial z\big|_{M_s} \times$$
$$\times \exp(-i\varphi) \;, \qquad (60)$$

$$G_{km,2} = i\xi \cos\varphi \; G_{km} \;, \quad G_{km,3} = i\xi \sin\varphi \; G_{km} \;,$$

where $\xi=\omega/v$.

This same formulas (58)-(60) describe generation of surface waves by instant point source in the case, when there are sharp boundaries in the medium, but the scattering of energy at these boundaries is small. This assumption can be made for the wide class of models of media (Levshin, 1985). It is clear from formula (59) that the Green function depends on the structure of medium in the source region and in the region of station, on the average phase velocity along the path and on the geometrical spreading $J(\omega,\mathbf{r})$.

If all this characteristics of medium are known the representation (58) of the spectrum of displacements of surface waves recorded at several points on the free surface for a set of frequencies ω gives us possibility to obtain a system of linear equations for elements of seismic moment tensor. Different types of waves give independent equations.

Main features of moment tensor correspondent to a plain ideal fault

Let we assume that the source is a plain ideal fault - a shear

dislocation. Let Σ be the plane of displacement discontinuity with the unit normal **n**. Then the stress glut tensor $\Gamma_{jk}(\mathbf{x},t)$ is given by (12). For the moment tensor $M_{jk} = \Gamma^{(0,0)}_{jk}$ we have in this case

$$M_{jk} = n_j a_k + n_k a_j \qquad (61)$$

where

$$a_k = \int_0^{\infty} dt \int_{\Sigma} \mu(\mathbf{x}) \left[\dot{u}_k(\mathbf{x},t)\right] d\Sigma = \int_{\Sigma} \mu(\mathbf{x}) \left[u_k(\mathbf{x},\infty)\right] d\Sigma \;.$$

Since $\left[u_k(\mathbf{x},t)\right]$ lies in Σ, the vector **a** lies in the same plane and is orthogonal to **n**, i.e., $a_k n_k = 0$ is true. Let $\mathbf{b} = \mathbf{a} \times \mathbf{n}$ be a vector that is orthogonal both to **a** and **n**. We then have $n_k b_k = a_k b_k = 0$. Multiplying both sides of (61) by b_k and summing over k, we get $M_{jk} b_k = 0$, i.e., the matrix **M** has the eigenvector **b** corresponding to the zero eigenvalue. It is also easy to see that **M** has a zero trace, for from (61) we have

$$\mathrm{Tr}\; \mathbf{M} = 2 n_k a_k = 0 \;.$$

However since the sum of the eigenvalues of **M** equals the trace, while one of the eigenvalues is zero, it follows that the sum of the other two must be zero. Hence, denoting the eigenvalues of **M** as λ_1, λ_2, λ_3, we have $\lambda_1=\lambda$, $\lambda_2=-\lambda$, $\lambda_3=0$. Since **M** is symmetric, the eigenvectors are mutually orthogonal and form a basis on which M_{jk} is

$$M_{jk} = \lambda \delta_{j1} \delta_{k1} - \lambda \delta_{j2} \delta_{k2} \;, \qquad (62)$$

i.e., all elements M_{jk} are zero, except $M_{11} = \lambda$ and $M_{11} = -\lambda$. Multiplying both sides of (61) by n_k and summing over k, we get

$$M_{jk} \, n_k = a_j \;. \qquad (63)$$

On the other hand, from (62) it follows that the following relation must hold on the basis composed of the eigenvectors of **M** :

$$M_{jk} \, n_k = \lambda (n_1 \delta_{j1} - n_2 \delta_{j2}) \;. \qquad (64)$$

On the same basis, from (63) and (64) we have

$$a_j = \lambda (n_1 \delta_{j1} - n_2 \delta_{j2}) \;,$$

Multiplying this by n_j, we get $\lambda(n_1^2 - n_2^2) = 0$ or $n_1^2=n_2^2$ ($a_3 = n_3 = 0$, $\mathbf{a} \perp \mathbf{b}$, $\mathbf{n} \perp \mathbf{b}$). Hence **n** bisects the vertical angles made by those eigenvectors of **M** corresponding to nonzero eigenvalues. The vector **a** bisects the other pair of angles made by the same vectors. So if moment tensor corresponding to an ideal fault is determined then a pair of vectors **a** and **n** are determined too. But one cannot distinguish between the two vectors based on knowledge of the moment M alone.

Nonlinear invertion for moment tensor and source depth

As it was mentioned above if all characteristics of medium are known the representation (58) of the spectrum of displacements in surface waves gives us a system of linear equations for elements of seismic moment tensor. But usualy the average phase velocities along the waves' paths are very poorly known. In this case the seismic moment tensor, depth of the source and corrections for zero approximation of phase velocities of surface waves averaged along paths can be determined by iterative procedure.

Let $u_k(r,\omega)$ - observed spectrum and $u_k^{(i)}(r,\omega)$ - theoretical spectrum determined at i-th iteration by formulas (58)-(60) for

$$M_{mn} = M_{mn}^{(i)},\ h = h^{(i)},\ \psi = \psi^{(i)}.\ \text{Here}\quad M_{mn}^{(i)} = M_{mn}^{(i-1)} + \Delta M_{mn}^{(i)},$$
$$h^{(i)} = h^{(i-1)} + \Delta h^{(i)},\ \psi^{(i)} = \psi^{(i-1)} + \Delta\psi^{(i)},\ h^{(0)} = h_0,\ \psi^{(0)} = \psi_0,$$

$M_{mn}^{(0)}$- solution of the system (58) for $h = h_0$ and $\psi = \psi_0 = \omega L/v_0$, where h_0 and v_0- zero approximation of depth of the source and phase velocity.

Using a linear approximation of the dependence of deviation $u_k(r,\omega)-u_k^{(i-1)}(r,\omega)$ on corrections $\Delta M_{mn}^{(i)}$, $\Delta h^{(i)}$ and $\Delta\psi^{(i)}$ we can get an equation for these corrections

$$u_k(r,\omega)-u_k^{(i-1)}(r,\omega) = -(i/\omega)\Big[M_{mn}^{(i-1)}\partial/\partial h G_{km,n}^{(i-1)}(r,s,\omega)\Delta h^{(i)} + G_{km,n}^{(i-1)}(r,s,\omega)\Delta M_{mn}^{(i)}\Big] - iu_k^{(i-1)}(r,\omega)\Delta\psi^{(i)}. \qquad (65)$$

Here $G_{km,n}^{(i-1)}(r,s,\omega)$ correspondent to $h = h^{(i-1)}$ and $\psi = \psi^{(i-1)}$. The summation convention for repeated subscripts is used. Complex equation (65) can be replaced by a pair of independent real equations. A pair of such equations for $\Delta M_{mn}^{(i)}$, $\Delta h^{(i)}$ and $\Delta\psi^{(i)}$ corresponds to every station for each mode and frequency. If we measured spectra of displacements in surface waves at a few points at the free surface for a set of frequencies we can get a system of linear equations for chosen model of medium. The least square solution of such a system can be obtained at every iteration.

After corrections $\Delta\psi$ are found corrections of average

phase velocities (under condition that the origin time of the source and epicenter are known) can be calculated using relation

$$\Delta(1/v) = \Delta\psi/(\omega L)$$

In conclusion of this section we'll show that solution obtained above is not unique.

We have from relations (58)-(60):

$$u_k(r,\omega) = Q_k(\omega,\varphi)P(\omega,h,\varphi)\exp\big[-i\psi(r,\omega)\big] \qquad (66)$$

where $Q_k(\omega,\varphi) = -(i/\omega)AW^{(k)}(\omega,0,\varphi)$;

for Love wave

$$P(\omega,h,\varphi) = \varepsilon V^{(3)}(\omega,h)\Big[0.5\sin2\varphi(M_{33}-M_{22}) + \cos2\varphi\, M_{23}\Big] + i\,\frac{\partial V^{(3)}(\omega,h)}{\partial z}(\sin\varphi\, M_{12} - \cos\varphi\, M_{13})\ ;$$

for Rayleigh wave

$$P(\omega,h,\varphi) = M_{11}\frac{\partial V^{(1)}(\omega,h)}{\partial z} - (M_{22}\cos^2\varphi + M_{33}\sin^2\varphi + M_{23}\sin2\varphi)\varepsilon V^{(2)}(\omega,h) + i\,(M_{12}\cos\varphi + M_{13}\sin\varphi)\Big[\varepsilon V^{(1)}(\omega,h) + \frac{\partial V^{(2)}(\omega,h)}{\partial z}\Big]\ .$$

It is clear from appearance of the multiplier $P(\omega,h,\varphi)$ in formula (66) that unique values of elements M_{ij} and phases $\psi(r,\omega)$ can not be determined simultaneously. If elements M_{12} and M_{13} will be multiplied by -1, or all other elements of moment tensor will change their signs, magnitude of the right part of relation (66) will not change for all values of frequency ω and azimuth φ and variation of its phase can be compensated by correcting propagation phases $\psi(r,\omega)$. As a result there are four different moment tensor solutions.

Direct trial of possible focal mechanisms

Assuming that the source is a plane ideal fault let us consider a rough grid in space of angles (dip, strike, and rake) determining a focal mechanism of the source.

Let models of media in the source region and in the stations regions be given and coordinates and initial time of the source be fixed.

Using formula (58) we can calculate theoretical values of surface waves spectra for every possible focal mechanism. Comparison of calculated and observed spectra gives us scalar moment and normalized residual. Results of this comparison can be represented by maps of residuals corresponding to all possible directions of compression and tension axes (in stereographic projection). We can obtain amplitudes residuals map as well as amplitude-phase residuals map. Let $u^{(i)}(r,\omega)$ be any observed value of displacement spectrum, $i=1,\dots,N$, $\varepsilon^{(i)}_{amp}$ - corresponding residual of $|u^{(i)}(r,\omega)|$, $\varepsilon^{(j)}_{ph}$ - residual of $\arg(u^{(j)}(r,\omega))$, $j=1,\dots,n$. We define normalized amplitude residual by formula

$$\varepsilon_{amp} = \left[\left(\sum_{i=1}^{N} \varepsilon^{(i)2}_{amp} \right) / \left(\sum_{i=1}^{N} |u^{(i)}(r,\omega)|^2 \right) \right]^{1/2},$$

normalized phase residual - by formula

$$\varepsilon_{ph} = \frac{1}{\pi} \left[\left(\sum_{j=1}^{n} \varepsilon^{(j)2}_{ph} \right) / n \right]^{1/2},$$

and amplitude-phase residual ε - by formula

$$\varepsilon = 1 - (1-\varepsilon_{amp})(1-\varepsilon_{ph}).$$

We may use phase information not for all observed data, but for any part of it ($n < N$). Say, for more or less homogeneous paths, or for long periods only.

In the case of amplitudes residuals tension and compression axes can not be distinguished and map is symmetric with respect to the center (as it was shown , every solution has three equivalent solutions). By use of any apriory phase information (values of phase velocities of any surface wave under consideration for chosen frequencies) it is possible to obtain a map of residuals distribution for tension axis as well as for compression axis. The best solution is unique in this case.

Results of direct trial can be used as zero approximation for iterative procedure described above.

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