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I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

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WORKSHOP ON EARTHQUAKE SOURCES & REGIONAL LITHOSPHERIC STRUCTURES FROM SEISMIC WAVE DATA

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Surface Waves Fundamentals

A.L. Levshin

**International Inst. of Earthquake Prediction & Math. Geophysics of the USSR
Moscow, U.S.S.R.**

A.L.LEVSHIN

SURFACE WAVES FUNDAMENTALS

INTRODUCTION

Surface waves form the longest and strongest portion of a seismic record excited by explosions and shallow earthquakes. Traversing areas with diverse geologic structures,they "absorb" information on the properties of these areas which is best reflected in dispersion, the dependence of velocity on frequency. The other properties of these waves - polarization, frequency content, attenuation, azimuthal variation of the amplitude and phase - are also controlled by the medium between the source and the recording station; some of these are affected by the properties of the source itself and by the conditions around it.

In recent years surface wave seismology has become an indispensable part of seismological practice. The maximum amplitude in the surface wave train of virtually every earthquake or major explosion is being measured and used by all national and international seismological surveys in the determination of the most important energy parameter of a seismic source, namely, M_s magnitude. The relationship between M_s and the body wave magnitude m_b is routinely employed in identification of underground nuclear explosions. Surface waves of hundreds of earthquakes recorded every year are being analysed to estimate the seismic moment tensor of earthquake sources,to determine the periods of free oscillations of the Earth,to construct regional dispersion curves from which in turn crustal and upper mantle structure in various areas is derived,to evaluate the dissipative parameters of the mantle material. Work began on surface wave "tomography" of the Earth's mantle - indentification of lateral inhomogeneities and elastic anisotropy from worldwide seismic observations. Surface waves from explosions and earthquakes are being used to study hardly accessible areas, for instance, in a search for major sedimentary basins in the arctic shelf.

These spectacular successes of surface wave seismology have been made possible by recent developments in long-period seismic instrumentation, seismic arrays, the array-like use of

national and international networks of long-period stations.

On the other hand, computerization helped to develop methods for analysis of surface wave observations that could more accurately estimate surface wave dispersion and polarization. Techniques of quantitative data interpretation have been developed that can effectively determine velocity and density models consistent with the observations and evaluate the non-uniqueness and resolution of the data.

All these results were essentially based on an advanced theory and efficient techniques for calculating the spectral characteristics and wave fields of surface waves in vertically and radially varying structures. Further development extended some of these results to the media with lateral inhomogeneities. Two types of lateral inhomogeneity were treated in details. One of them is a sharp discontinuity between two vertically inhomogeneous quarter-spaces. The other one is characterized by very smooth variations of properties along horizontal directions.

These lectures will present a short introduction to fundamentals of surface wave propagation in the Earth. No proof of statements concerning surface wave properties will be given. For more detailed information see Aki & Richards,1980; Ben-Menahem & Singh,1981;Keilis-Borok (ed), 1989.

BASICS OF THE SURFACE WAVES PROPAGATION THEORY

a. What are surface waves?

Seismic waves radiated by a natural or artificial source propagate in all direction outward the point or zone of excitation. So the wave fronts or surfaces of equal phase are close to spherical ones. The energy which is transferred by such a *body* wave decays with a distance from the source as R^{-1}. Although in the course of further propagation through inhomogeneous medium the form of body wave fronts becomes distorted they penetrate at the all parts of the medium according to rules of geometrical seismics. So body wave propagation is essentially three-dimensional and in general there is no preferable direction or limitations for propagation except the Earth surface. As velocities as paths of body waves having different frequency content are basically the same.

Surface (and their close relatives *channel)* waves have essentially different manner of propagation. The necessary condition for their generation is existence of some discontinuity of elastic properties or a waveguide with velocities of elastic waves being lower than in surrounding medium. They concentrate near such a discontinuity or inside such a waveguide and propagate along them. As a result their propagation is essentially two-dimensional, i.e., they may be considered as propagating waves in (horizontal) x, y - directions and standing waves in the third (vertical) z -direction. Their surfaces of constant phase are cylindrical and their amplitudes decay with the horizontal distance from the source r as $r^{-1/2}$. So a stationary surface wave is decribed by expression

$$f(z)\ (r)^{-1/2}\ \exp(i\omega t - i\varphi(\omega, r)),$$

where t is a time, ω is a circular frequency, $f(z)$ is a distribution of an amplitude with depth; this distribution may be quite complicated but for big enough z it decays exponentialy. Phase $\varphi(\omega, r) = -\omega r/C(\omega)$ and $C(\omega)$ is a phase velocity.The volume which is illuminated by a propagating surface wave depends on its frequency and increases in direction perpendicular to the propagation direction as a frequency becomes lower. So the reason for higher intensity of surface waves lies in the two- dimensional character of their propagation.

The simplest well-known example of surface waves is a Rayleigh wave along the free boundary of homogeneous isotropic elastic halfspace. Another simple wave of this type is an *interface* Stoneley wave along the boundary of elastic and liquid halfspaces or two elastic halfspaces with special properties. Both types of waves can be described as superposition of inhomogeneous longitudinal and shear plane waves propagating with the same phase velocity C independent on frequency.

The phase velocity of Rayleigh waves C_R is in the range of $0.87b < C_R < 0.96b$, where b is the shear velocity in a halfspace, and depends on the Poisson ratio (see APPENDIX 2). The phase velocity of Stoneley wave C_{St} is less than minimal body wave velocity in the medium and depends on elastic parameters of both

halfspaces.

More general type of surface (or channel) waves is generated in presence of waveguides able to tunnel the wave energy. Typical examples of waveguides are shown at Fig.1. Such waves can be considered as a result of constructive interference of infinetely big number of elementary body waves. Each elementary wave has some phase delay depending on the wave path, presence of caustics and phase shifts in result of reflections at discontinuities.. The constructive interference may take place when these elementary waves have the same phase velocity and the same phase delay. It is possible only at some frequencies. As result their phase velocity depends on frequency. Many properties of these *interferentional* waves especially in high frequency range may be deduced from the ray theory. The real Earth models combine discontinuities of physical properties and waveguides of different types. It makes all phenomena connected with surface wave propagation more complicated and difficult for analysis. It is essential to use efficient computer codes for their studies.

Now we will give more accurate description of surface waves. At first we'll formulate basic elements of the problem.

b.Basic equations.

Surface waves are considered within the framework of linear elasticity theory, mainly for perfectly elastic bodies. Internal sources of disturbance are described by means of equivalent forces. Small departures from perfect elasticity in wave propagation theory can be treated by means of perturbation theory.

Equations of motion. Equations of motion for a point x with coordinates (x_1, x_2, x_3) acted on by body forces have the form (Aki and Richards, 1980)

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i . \qquad (1.1)$$

Here the σ_{ij} are components of the symmetrical stress tensor; the f_i's are components of body force per unit volume; ρ is the density; the u_i's, components of the displacement vector. The subscripts i,j take on the values 1,2,3. A symbol with a dot above it denotes a time derivative, a subscript j when preceded by a comma means differentiation with respect to the spatial coordinates x_j. Summation over double (mute) subscript is understood everywhere below, unless otherwise indicated.

The stress tensor is linearly related to small strains e_{ij} through Hooke's law

$$\sigma_{ij}=c_{ijpq}e_{pq} \qquad (1.2)$$

c_{ijpq} being a tensor of elasticity constants that depends on x. It has the symmetries $c_{ijpq}=c_{jipq}=c_{ijqp}=c_{pqij}$.

Subsequent discussion is mainly confined to isotropic bodies: $c_{ijpq} = \lambda\delta_{ij}\delta_{pq} + \mu(\delta_{ip}\delta_{jq} + \delta_{iq}\delta_{jp})$,

δ_{ij} being the Kronecker delta. The Lame constants λ and μ determine the velocities of elastic waves, a and b:

$a = [(\lambda + 2\mu)/\rho]^{1/2}$ for compressional (P) waves,

$b = [\mu/\rho]^{1/2}$ for shear (S) waves.

Hooke's law for isotropic media has the form

$$\sigma_{ij} = \lambda\delta_{ij}\theta + 2\mu e_{ij} \qquad (1.3)$$

where $\theta = e_{kk}$ is the dilatation.

Small strains and displacements are related through

$$e_{ij} = 1/2\ (u_{i,j} + u_{j,i}) \qquad (1.4)$$

Initial conditions. The medium is assumed to be at rest before the time t=0:

$$u_j = \dot{u}_j = 0 \qquad \text{when } t<0. \qquad (1.5)$$

Boundary conditions. Any elastic body that we shall consider below is bounded (wholly or partially) by a free surface S_0. This means that the following relation holds

$$T_i(\mathbf{n}) = \sigma_{ij}n_j\big|_{S_0} = 0 \qquad (1.6)$$

where the n_j are components of the outward normal to S_0, n. Here and below, the T_i(n) are components of traction T(n), i.e., the force on an element of area dS_0.

Conditions at interfaces. The functions a(x), b(x), ρ(x) are assumed to be positive and piecewise continuous. There is welded contact at any surface where these functions are discontinuous, i.e., the displacement vector u and stress tensor σ_{ij} are continuous across an interface.

Sources of disturbance. Elastic motion is excited by a vector field of body force f that is a function of spatial and

time coordinates (its physical meaning will be discussed later by Dr.Bukchin).

It will be assumed in what follows that the field is spatially finite, i.e. $f_j(\mathbf{x},t)=0$ outside some closed region Ω, which will be termed the source region.

The time derivative of f_j is assumed to be a finite function of time,i.e. $f_j(\mathbf{x},t)=\dot{f}_j(\mathbf{x},t)=0$ when $t<0$ and $\dot{f}_j(\mathbf{x},t)=0$ when $t>t_e$.

Green function. To solve a forward problem in elastic wave theory is to determine the displacement vector field $u_j(\mathbf{x},t)$ and the stress tensor field $\sigma_{ij}(\mathbf{x},t)$ that obey Eqs.(1.1),(1.3),(1.4) and the initial and boundary conditions (1.5),(1.6).

When the body force field is concentrated in space at $\mathbf{x}_0$ and in time at t_0, and points along the x_j-axis, i.e.,

$$f_i(\mathbf{x},t) = \delta_{ij}\delta(\mathbf{x}-\mathbf{x}_0)\delta(t-t_0)$$

δ being the Dirac delta function, the resulting displacement u(x,t) is called the fundamental solution, or Green function for the boundary value problem (1.1), (1.5), (1.6)(Aki and Richards, 1980). It will be denoted $\mathbf{G}^{(j)}(\mathbf{x};\mathbf{x}_0;t-t_0)$, its i-th component being $G_{ij}(\mathbf{x};\mathbf{x}_0;t-t_0)$.

The solution for an arbitrary force $f_j(\mathbf{x},t)$ that is zero when $t<0$ can be expressed in terms of the Green function as follows

$$u_i(\mathbf{x},t) = \int_0^t d\tau\int_\Omega G_{ij}(\mathbf{x};\mathbf{y};t-\tau)f_j(\mathbf{y},\tau)\,dV_y \qquad (1.7)$$

where dV is an element of the volume Ω (source region) within which $f_j(\mathbf{x},t)$ is not identically zero.

c. Surface waves due to a point source in a vertically varying half-space .

We shall now confine ourselves to considering a far more restricted class of models compared with those outlined above, viz.,ones in which the properties of the medium are functions of a single coordinate. The theory of surface waves for such media is fairly complete (Levshin, 1973; Aki and Richards, 1980, Ben-Menahem and Singh,1981); we shall briefly summarize it below.

We shall deal with a vertically varying half-space having the cartesian coordinates $x = x_1$ $(-\infty<x<\infty)$, $y = x_2$ $(-\infty<y<\infty)$, $z = x_3$ $(0<z<\infty)$ and bounded by a free surface S_0 (z=0). The velocities a and b, the density ρ are functions of z only. The medium is homogeneous below $z = Z$, that is, $a(z) = a(Z+0)$, $b(z) = b(Z+0)$, $\rho(z) = \rho(Z+0)$ when $Z<z<\infty$; also, $b(Z+0) = \max\, b(z)$, $a(Z+0)=\max\, a(z)$.

It is known that the full solution $\mathbf{u}(\mathbf{x},t)$ for such a medium can be represented in an integral form. Its principal part at large (compared with the wavelength) distances from the source region Ω to observation site is formed by surface waves. It is supposed that the depths of both source region and observation site are much smaller that the horizontal distance between them. The surface wave part of the solution separates into two independent fields $\mathbf{u}_R$ and $\mathbf{u}_L$ (Rayleigh and Love waves) with different polarizations. We shall frequently employ the symbol D ($D = R$ or L) in what follows to indicate the relevant wave type.

Green function; surface-wave part. The surface-wave part of the relevant Green function G^D can be represented as a sum of infinitely many terms (modes, normal waves)

$$G^D_{ij} = \sum_{k=1}^{\infty} G^{kD}_{ij}(\mathbf{x};\mathbf{x}_0;t), \qquad D=R,L$$

the mode with k=1 being usually called the fundamental and the others , higher modes (overtones). We shall denote Fourier transforms, spectral transforms of time functions, by the same letters as their originals, but with a ^ above. The contribution of each mode can then be represented as

$$G^{kD}_{ij} = \frac{1}{\pi}\,\mathrm{Re}\int_{\bar\omega_{kD}}^{\infty} \hat G^{kD}_{ij}(\mathbf{x};\mathbf{x}\ ;\omega)\, e^{i\omega t} d\omega$$

where $\bar\omega_{kD}$ is the lowest frequency (the mode does not exist at frequencies below ω_{kD}).

We shall employ two sets of coordinates whose origins are at the "epicenter" - a point at the free surface lying at the same vertical with the source (point $\mathbf{x}_0$): a cartesian set (x,y,z) and a cylindrical one (z,r,φ) in which the angle φ is measured clockwise from the x-axis (as viewed from above). A force applied at the source will be projected onto the cartesian unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$. We also introduce at in a receiver point $\mathbf{x}$ a local vector basis $\mathbf{e}_z$, $\mathbf{e}_r$, $\mathbf{e}_\varphi$ for which $\mathbf{e}_z$ is parallel to $\mathbf{e}_3$; $\mathbf{e}_r$ is directed along the radius vector of the cylindrical system and $\mathbf{e}_\varphi$ is in the direction of increasing φ. The corresponding components of the Green function in this basis will be denoted $\hat G^{kD}_{pq}$, where p stands for the z-, r- or φ-component, the index q taking on the values 1,2,3.

The components $\hat G^{kD}_{pq}$ are found from

$$\hat G^{kD}_{pq} = \frac{\exp(-i\pi/4)}{\sqrt{8\pi}}\,\frac{\exp(-i\xi_{kD}r)}{\sqrt{\xi_{kD}r}}\ \times$$
$$\times\ \frac{s_p V_k^{(i_p)}(\omega,z)}{\sqrt{C_{kD}U_{kD}I^{(0)}_{kD}}}\,\frac{s_q V_k^{(i_q)}(\omega,h)}{\sqrt{C_{kD}U_{kD}I^{(0)}_{kD}}} \tag{1.8}$$

Here the $V_k^{(i)}$ are eigenfunctions belonging to the discrete spectra of the one-dimensional problems discussed in details in APPENDIX 1; since an eigenfunction is determined apart from a constant, we shall assume that $V_k^{(1)}(\omega,0) = V_k^{(3)}(\omega,0) = 1$; $C_{kD}(\omega)=\omega/\xi_{kD}(\omega)$ is phase velocity, $\xi_{kD}(\omega)$ being the inverse function of $\omega_{kD}(\xi)$;

$$U_{kD}(\omega) = \left(\frac{d\xi_{kD}}{d\omega}\right)^{-1} = C_{kD}\left(1 - \frac{\omega}{C_{kD}}\,\frac{dC_{kD}}{d\omega}\right)^{-1} \tag{1.9}$$

is group velocity; $I^{(0)}_{kD}$ is proportional to the mean kinetic energy transported by a mode over a cycle of oscillation

$$I^{(0)}_{kR} = \int_0^\infty \rho(z)\,[(V_k^{(1)})^2 + (V_k^{(2)})^2]\,dz;$$
$$I^{(0)}_{kL} = \int_0^\infty \rho(z)\,[(V_k^{(3)})]^2\,dz \tag{1.10}$$

The values of s_p , s_q and i_p , i_q are given in Table 1.1.
Note that (1.11) is asymptotic, being true when $\xi_{kD}r \gg 1$, $r \gg h$, $r \gg z$. If these conditions are not satisfied, not only the asymptotic formula (1.11) is not true, but the very separation of the field into the "body" and "surface" wave parts may turn out to be somewhat forced.

Table 1.1

D	p	s_p	i_p	q	s_q	i_q
	z	1	1	1	$i\cos\varphi$	2
R	r	$-i$	2	2	$i\sin\varphi$	2
	φ	0	-	3	1	1
	z	0	-	1	$i\sin\varphi$	3
L	r	0	-	2	$-i\cos\varphi$	3
	φ	i	3	3	0	-

Displacement field due to spatially concentrated forces. When the source is a combination of point forces

$$f_q = K_q(t)\,\delta(\mathbf{x}-\mathbf{x}_0)$$

the resulting displacement becomes

$$\hat{u}_p^{kD}(\mathbf{x};\mathbf{x}_0;\omega) = \hat{G}_{pq}^{kD}\,\hat{K}_q(\omega).$$

This gives an asymptotic formula for $\hat{u}_{pq}^{kD}$

$$\hat{u}_p^{kD} = \frac{\exp(-i\pi/4)}{\sqrt{8\pi}}\,\frac{\exp(-i\xi_{kD}r)}{\sqrt{\xi_{kD}r}}\times \tag{1.11}$$

$$\times\frac{s_p V_k^{(i_p)}(\omega,z)}{\sqrt{C_{kD}U_{kD}I_{kD}^{(0)}}}\,\frac{W^{kD}(\omega,\varphi,h)}{\sqrt{C_{kD}U_{kD}I_{kD}^{(0)}}}$$

where

$$W^{kD} = (s_q\,V_k^{(i_q)})\,\hat{K}_q(\omega). \tag{1.12}$$

Displacement field due to spatially concentrated dipoles. When the source is a dipole of the form

$$f_q = -\,m_{qs}(t)\frac{\partial\,\delta(\mathbf{x}-\mathbf{x}o)}{\partial x_s} \qquad (q,s=1,2,3),$$

the resulting displacement is

$$\hat{u}_p^{kD} = \hat{G}_{pq,s}^{kD}\,\hat{m}_{qs}(\omega)$$

Making use of asymptotic estimates for $\hat{G}_{pq,s}^{kD}$ that do not involve terms that would fall off with distance faster than $r^{-1/2}$,

we get an expression for $\hat{u}_p^{kD}$ of the form (1.11), where

$$W^{kD} = B_{qs}^{kD}\hat{m}_{qs}\ . \tag{1.13}$$

Like $\hat{m}_{qs}$, B_{qs}^{kD} is symmetric in q and s. The formulas for B_{qs}^{kD} are listed in table 1.2.

Formulas (1.11) to (1.13) together with the complementary tables 1.1, 1.2 provide a complete asymptotic description of surface wave fields excited by point forces and dipoles in a vertically varying half-space. Extension to more complex point forces is straightforward.

Table 1.2

D	q	s	B^{kD}_{qs}
	1	1	$-\ell_{kR}\cos^2\varphi\ V_k^{(2)}(\omega,h)$
	2	2	$-\ell_{kR}\sin^2\varphi\ V_k^{(2)}(\omega,h)$
	3	3	$\left.\dfrac{dV_k^{(1)}(\omega,z)}{dz}\right\vert_{z=h}$
R	1	2	$-(1/2)\ell_{kR}\sin 2\varphi\ V_k^{(2)}(\omega,h)$
	1	3	$(i/2)\cos\varphi\left(\ell_{kR}V_k^{(1)}(\omega,h) + \left.\dfrac{dV_k^{(2)}(\omega,z)}{dz}\right\vert_{z=h}\right)$
	2	3	$(i/2)\sin\varphi\left(\ell_{kR}V_k^{(1)}(\omega,h) + \left.\dfrac{dV_k^{(2)}(\omega,z)}{dz}\right\vert_{z=h}\right)$
	1	1	$-(1/2)\ell_{kL}\sin 2\varphi\ V_k^{(3)}(\omega,h)$
	2	2	$(1/2)\ell_{kL}\sin 2\varphi\ V_k^{(3)}(\omega,h)$
	3	3	0
L	1	2	$(1/2)\ell_{kL}\cos 2\varphi\ V_k^{(3)}(\omega,h)$
	1	3	$(i/2)\sin\varphi\ \left.\dfrac{dV_k^{(3)}(\omega,z)}{dz}\right\vert_{z=h}$
	2	3	$-\ (i/2)\cos\varphi\ \left.\dfrac{dV_k^{(3)}(\omega,z)}{dz}\right\vert_{z=h}$

d. Physical interpretation

Formulas like (1.11), (1.13) for spectral displacement amplitudes in surface waves can be given a simple physical interpretation. Apart from the first factor, a complex constant, they involve three more factors, each of these being controlled by certain physical conditions and parameters of the observational procedure. The second factor, $(\ell_{kD}r)^{-1/2}\exp(-i\ell_{kD}r)$,

describes the effect of cylindrical geometrical spreading affecting the energy flux of a surface wave and its propagation-assosiated phase delay , $\ell_{kD}r=\omega r/C_{kD}$, which steadily increases with the distance and is a nonlinear function of frequency. The functions $C_{kD}(\omega)$, $U_{kD}(\omega)$, i.e., the dispersion curves, are determined by the properties of the medium only, namely, by the velocity and density distributions $a(z), b(z), \rho(z)$.

When weak dissipation is present, the resulting attenuation and the extra dispersion due to it (see below) will naturally be incorporated in the same factor.

The third factor, $s_p\ V_k^{(i_p)}(\omega,z)(C_{kD}U_{kD}I_{kD}^{(0)})^{-1/2}$, is controlled by the receiver depth z and the recorded component of displacement $p (p=z,r,\varphi)$. Actually in a seismic experiment, the horizontal seismometers are usually oriented east-west and north-south; knowing the epicenter coordinates, however, one can convert the N-S and E-W components of the seismogram to the r- and φ-components by means of a simple linear transformation.

It follows from (1.11)-(1.13) that a Rayleigh wave ($D=R$) is elliptically polarized in a vertical plane that contains the source at $\mathbf{x}_0$ and the receiver at $\mathbf{x}$, that is, its φ- component is equal to zero, while the z- and r- components have a phase difference of $\pi/2$. The direction of particle motion and the form of the ellipse are controlled by the ratio $V_k^{(2)}(\omega,z)/V_k^{(1)}(\omega,z)$ which depends on the frequency, receiver depth, and the properties of the medium. The quantity $\chi_k(\omega) = [V_k^{(2)}(\omega,0)/V_k^{(1)}(\omega,0)]$ is called ellipticity; it equals the ratio of horizontal and vertical axes of the ellipse along which particles of the free surface are moving in the process of R-wave propagation. It must be borne in mind that it is only for purely sinusoidal oscillations that one can speak of a strictly elliptic particle motion; particle paths in transient motion may be very unlike ellipses, even though remaining in the vertical plane. Love waves ($D=L$) have the φ - component of motion alone, i.e., are linearly polarized in a horizontal direction normal to the Rayleigh wave polarization plane.

The variation of displacement amplitude over depth is fully determined by the eigenfunctions $V_k^{(i_p)}(\omega, z)$, i.e. by the properties of the medium, receiver depth, and the frequency, being independent of epicentral distance r.

When the frequency response of the recording instrument is to be included, the relevant complex expression can conveniently be incorporated in the third factor.

The last, fourth factor is $W^{kD}(\omega,\varphi,h)(C_{kD}U_{kD}I_{kD}^{(0)})^{-1/2}$. This depends both on the medium and source parameters: source depth, the relative locations of source and receiver, and the source mechanism, i.e. the relation between components of the force vector $K_q(t)$ or of the seismic moment tensor $m_{qs}(t)$.

Expressions for W^{kD} are given below for simple point sources constructed to imitate explosions and earthquakes.

(1) Center of dilatation

$$W^{kD} = \left(\left. \frac{dV_k^{(1)}(\omega, z)}{dz} \right|_{z=h} - \varepsilon_{kR}(\omega) V_k^{(2)}(\omega, h) \right) \hat{m}(\omega) \tag{1.14}$$

$$W^{kL} = 0 \quad .$$

In this case the fourth factor depends on frequency, the medium, source depth and spectrum.

(2) Point shear dislocation along a direction $\varkappa$ which is tangent to an area with normal n. Denote

$n_x = \sin\gamma\ \cos\alpha \quad n_y = \sin\gamma\ \sin\alpha \quad n_z = \cos\gamma$

$\varkappa_x = \sin\beta\ \cos\delta \quad \varkappa_y = \sin\beta\ \sin\delta \quad \varkappa_z = \cos\beta$

(α and δ are azimuths of the horizontal projections of n and $\varkappa$ as measured from the x-axis; γ and β are angles which the two vectors make with the vertical z). W^{kD} can be expressed as follows:

$$W^{kR} = \left[2\cos\beta\cos\gamma \left. \frac{dV_k^{(1)}}{dz} \right|_{z=h} - 2\varepsilon_{kR}\sin\beta\ \sin\gamma\ \cos(\delta-\varphi)\cos(\alpha-\varphi) V_k^{(2)}(\omega,h) + \right.$$

$$\left. + i[\sin\beta\ \sin\gamma\ \cos(\alpha-\varphi)] \left(\varepsilon_{kR} V_k^{(1)}(\omega,h) + \left. \frac{dV_k^{(2)}(\omega,z)}{dz} \right|_{z=h} \right) \right] \hat{m}(\omega), \tag{1.15}$$

$$W^{kL} = \left[\varepsilon_{kL}\sin\gamma\ \sin\beta\ \sin(\alpha+\delta-\varphi) V_k^{(3)}(\omega,h) - i[\sin\beta\ \cos\gamma\ \sin(\delta-\varphi) + \right.$$

$$\left. + \sin\gamma\ \cos\beta\ \sin(\alpha-\varphi)] \left. \frac{dV_k^{0(3)}(\omega,z)}{dz} \right|_{z=h} \right] \hat{m}(\omega)$$

In this case the fourth factor is controlled by frequency, the medium, source depth and spectrum, as well as by source geometry, viz., orientation of fault plane and slip vector with respect to the source-receiver pair.

To illustrate the above expressions we present in APPENDIX 2 some simple examples of elastic media and surface waves arising in them.

SPHERICITY CORRECTIONS

Much of work in seismology uses surface waves that propagate to such great distances and penetrate to such great depths as to require corrections for the Earth's spherical shape. Indeed, large magnitude earthquakes excite surface waves of so large an amplitude that they can travel several times round the Earth before dying out because of dissipation in the medium. A seismic station then records two sequences of waves that travel in opposite directions.

We define a set of spherical coordinates with the origin at the center of a sphere of radius R_0, putting the receiver at the point $\mathbf{x}(R,\theta,\varphi)$ and the source at $\mathbf{x}_0(R_s,0,0)$ where $R_s = R_0 - h$, and shall consider the p-th components of displacement corresponding to the local vector basis $\mathbf{e}_p(p=R,\theta,\varphi)$. Let $\tilde{\theta}$ be the total distance travelled by the wave:

$$\tilde{\theta} = (-1)^g\theta + 2\pi(l+g),$$

where $l=0,1,2,...$ is the number of the Earth's great circles the wave has travelled; g = 0 or 1, depending on whether the wave comes from the epicentre or from the opposite direction. In that case formulas (1.11) become for $\theta \neq k\pi$

$$\hat{G}_{pq}^{kD}(\mathbf{x};\mathbf{x}_0;\omega) = \frac{\exp(-i\pi/4)}{\sqrt{8\pi}} \frac{\exp[-i\omega R_0\tilde{\theta}/C_{kD} + (i\pi/2)(2l+g)]}{\sqrt{\nu_{kD}\sin\theta}} \times$$

$$\times \frac{\varepsilon_p V_k^{(i)_p}(\omega, R)}{\sqrt{C_{kD} U_{kD} I_{kD}^{(0)}}} \; \frac{\varepsilon_q V_k^{(i)_q}(\omega, R_s)}{\sqrt{C_{kD} U_{kD} I_{kD}^{(0)}}} \;; \tag{1.16}$$

phase velocity is defined as

$$C_{kD}(\omega) = \frac{\omega R_0}{\nu_{kD} + 1/2} \tag{1.17}$$

and group velocity as

$$U_{kD}(\omega) = \left[\frac{d\nu_{kD}(\omega)}{d\omega}\right]^{-1} R \tag{1.18}$$

VARIATIONAL FORMULAS

a. Integral formulas for phase and group velocities. The theory of perturbations yields the following integral formulas for phase and group velocity in a vertically varying half-space:

$$C_{kR} = \left\{\left[\left[I_{kR}^{(1)} + I_{kR}^{(2)} + \frac{2}{\varepsilon_{kR}}\left(I_{kR}^{(3)} + I_{kR}^{(4)}\right) + \frac{1}{\varepsilon_{kR}^2}\left(I_{kR}^{(5)} + I_{kR}^{(6)}\right)\right]\right] / I_{kR}^{(0)}\right\}^{1/2} \tag{1.19}$$

$$C_{kL} = \left[\left(I_{kL}^{(1)} + \frac{1}{\varepsilon_{kL}^2} I_{kL}^{(2)}\right) / I_{kL}^{(0)}\right]^{1/2}$$

$$U_{kR} = \left[I_{kR}^{(1)} + I_{kR}^{(2)} + \frac{1}{\varepsilon_{kR}}\left(I_{kR}^{(3)} + I_{kR}^{(4)}\right)\right] \Big/ \left(C_{kR} I_{kR}^{(0)}\right) \tag{1.20}$$

$$U_{kL} = I_{kL}^{(1)} \Big/ \left(C_{kL} I_{kL}^{(0)}\right)$$

where the integrals $I_{kD}^{(j)}$ are

$$I_{kR}^{(1)} = \int_0^\infty b^2 \rho \left[V_k^{(1)}\right]^2 dz$$

$$I_{kR}^{(2)} = \int_0^\infty a^2 \rho \left[V_k^{(2)}\right]^2 dz$$

$$I_{kR}^{(3)} = \int_0^\infty b^2 \rho \left[\frac{dV_k^{(2)}}{dz} V_k^{(1)} + 2\frac{dV_k^{(1)}}{dz} V_k^{(2)}\right] dz$$

$$I_{kR}^{(4)} = -\int_0^\infty a^2 \rho \frac{dV_k^{(1)}}{dz} V_k^{(2)} \, dz \tag{1.21}$$

$$I_{kR}^{(5)} = \int_0^\infty b^2 \rho \left[\frac{dV_k^{(2)}}{dz}\right]^2 dz$$

$$I_{kR}^{(6)} = \int_0^\infty a^2 \rho \left[\frac{dV_k^{(1)}}{dz}\right]^2 dz$$

$$I_{kL}^{(1)} = \int_0^\infty b^2 \rho \left[V_k^{(3)}\right]^2 dz$$

$$I_{kL}^{(2)} = \int_0^\infty b^2 \rho \left[\frac{dV_k^{(3)}}{dz}\right]^2 dz$$

The formulas for a radially varying sphere are similar.

b. Partial derivatives of phase velocity. Solution of inverse problems in surface wave seismology is greatly facilitated by the formalism of partial derivatives which is used to determine phase velocity perturbations due to small perturbations in the velocity and density distributions. Consider a small perturbation $\delta_x(z)$ in the parameter $x(z)$ ($x = a$, b, or ρ) that vanishes everywhere except within the interval $z_i < z < z_{i+1}$. The resulting phase velocity perturbation $\delta C_{kD}(\omega)$ for a fixed frequency ω is

$$\delta C_{kD} = \int_{z_{i-1}}^{z_i} \frac{\partial C_{kD}(\omega, z)}{\partial x} \, \delta x(z)\, dz$$

The kernel $\partial C_{kD}/\partial x$ is a partial derivative of C_{kD} with respect to x (or, to be more exact, the "response" of C_{kD} to a δ-like variation in $x(z)$ at the point z). When ω = constant, the $\partial C_{kD}/\partial x$ are given by expressions

$$\frac{\partial C_{kR}}{\partial a} = \frac{a\rho}{U_{kR} I^{(0)}_{kR}} \left[V_k^{(2)} - \frac{1}{\varepsilon_{kR}} \frac{dV_k^{(1)}}{dz} \right]^2$$

$$\frac{\partial C_{kR}}{\partial b} = \frac{b\rho}{U_{kR} I^{(0)}_{kR}} \left[\left(V_k^{(1)} + \frac{1}{\varepsilon_{kR}} \frac{dV_k^{(2)}}{dz} \right) + \frac{4}{\varepsilon_{kR}} \frac{dV_k^{(1)}}{dz} V_k^{(2)} \right]$$

$$\frac{\partial C_{kR}}{\partial \rho} = \frac{1}{2\rho} \left[\frac{\partial C_{kR}}{\partial a} a + \frac{\partial C_{kR}}{\partial b} b \right] - \frac{C_{kR}^2}{2U_{kR} I^{(0)}_{kR}} \left[\left(V_k^{(1)}\right)^2 + \left(V_k^{(2)}\right)^2 \right] \tag{1.22}$$

$$\frac{\partial C_{kL}}{\partial a} = 0$$

$$\frac{\partial C_{kL}}{\partial b} = \frac{b\rho}{U_{kL} I^{(0)}_{kL}} \left[\left(V_k^{(1)} \right)^2 + \left(\frac{1}{\varepsilon_{kL}} \frac{dV_k^{(2)}}{dz} \right)^2 \right]$$

$$\frac{\partial C_{kL}}{\partial \rho} = \frac{1}{2\rho} \frac{\partial C_{kL}}{\partial b} b - \frac{C_{kL}^2}{2U_{kL} I^{(0)}_{kL}} \left(V_k^{(3)} \right)^2$$

Similar formulas exist for the spherical case.

Partial derivatives of group velocity at frequency ω can be

found numerically from derivatives of phase velocity at two close points $\omega_1 = \omega e^{\delta}$, $\omega_2 = \omega e^{-\delta}$ (Rodi et al, 1975).

$$\left.\frac{\partial U}{\partial x}\right|_{\omega} = \frac{U|_{\omega}}{2C|_{\omega}} \left(2 - \frac{U|_{\omega}}{C|_{\omega}}\right) \left(\left.\frac{\partial C}{\partial x}\right|_{\omega_1} + \left.\frac{\partial C}{\partial x}\right|_{\omega_2} \right) + \frac{1}{2} \left[\frac{U|_{\omega}}{C|_{\omega}} \right]^2 \left(\left.\frac{\partial C}{\partial x}\right|_{\omega_1} - \left.\frac{\partial C}{\partial x}\right|_{\omega_2} \right) \delta^{-1} \tag{1.23}$$

c. Dependence of phase velocity on the depth of an interface between layers. The variational relation stated in Woodhouse (1976) may be used to obtain formulas for partial derivatives of phase velocity with respect to interface depth. Let h be the depth of some interface within a vertically varying half-space and $[\chi]_-^+$ be the jump of any function $\chi(z)$ at the interface $z=h$, i.e. $[\chi]_-^+ = \chi(h+0) - \chi(h-0)$. Then the following relation is valid for Rayleigh waves

$$\frac{\partial C_{kR}}{\partial h} = \frac{C_{kR}^3}{2\omega\left[\omega\left(I_{kR}^{(1)} + I_{kR}^{(2)}\right) + C_{kR}\left(I_{kR}^{(3)} + I_{kR}^{(4)}\right)\right]} \times$$

$$\times \left\{\omega^2\left[\left(V_k^{(2)}(h)\right)^2 + \left(V_k^{(1)}(h)\right)^2\right][\rho]_-^+ - \left(\frac{\omega}{C_{kR}}\right)^2\left(V_k^{(1)}(h)\right)^2[\mu]_-^+\right.$$

$$- \left(\frac{\omega}{C_{kR}}\right)^2\left(V_k^{(2)}(h)\right)^2[\lambda+2\mu]_-^+ + \left[(\lambda+2\mu)\left(\frac{dV_k^{(1)}}{dz}\right)^2\right]_-^+ + \tag{1.24}$$

$$\left. + \left[\mu\left(\frac{dV_k^{(2)}}{dz}\right)^2\right]_-^+\right\}$$

For Love waves we have

$$\frac{\partial C_{kL}}{\partial h} = \frac{C_{kL}^3}{2\omega^2 I_{kL}^{(1)}}\left\{\omega^2\left(V_k^{(3)}(h)\right)^2[\rho]_-^+ - \left(\frac{\omega}{C_{kL}}\right)^2\left(V_k^{(3)}(h)\right)^2[\mu]_-^+ +\right.$$

$$\left. + \left[\mu\left(\frac{dV_k^{(3)}}{dz}\right)^2\right]_-^+\right\} \tag{1.25}$$

EFFECTS OF ANELASTICITY.

The stress-strain relations in a weakly dissipating medium become integro-differential expressions (Kogan, 1966; Akopyan, Zharkov and Lyubimov, 1975; Aki and Richards, 1980; Levshin, Ratnikova and Saks, 1980). Propagation of harmonic waves can then be conveniently examined by using complex frequency-dependent

elastic moduli $\bar{K}(\omega,x)$, $\bar{\mu}(\omega,x)$ instead of frequency-independent ones $K(x)$, $\mu(x)$ for a perfectly elastic medium. Accordingly, we have complex frequency-dependent P and S wave velocities $a(\omega,x)$ and $b(\omega,x)$, respectively.

Dissipation caused by confining pressure is characterized by the quantity $Q_k = \mathrm{Re}\,\bar{K}/\mathrm{Im}\,\bar{K}$, while that caused by pure shear, by $Q_\mu = \mathrm{Re}\,\bar{\mu}/\mathrm{Im}\,\bar{\mu}$. Dissipation in propagating elastic waves is controlled by Q_a and Q_b

$$Q_b = Q_\mu = \mathrm{Re}\, b/\mathrm{Im}\, b$$

$$Q_a = \left[Q_k^{-1}\left(1 - \frac{4b^2}{3a^2}\right) + Q_\mu^{-1}\frac{4b^2}{3a^2}\right]^{-1} = \frac{\mathrm{Re}\ a}{2\ \mathrm{Im}\ a}$$

A frequent assumption is $Q_k^{-1} = 0$, giving $Q_a = Q_b\,\frac{3a^2}{4b^2}$. Denote complex velocity perturbations caused by small dissipation as $\delta a(\omega,z)$, $\delta b(\omega,z)$. The corresponding perturbations in phase velocity $C_{kD}(\omega)$ can be evaluated from the perturbation theory:

$$\delta C_{kD}(\omega) = \int_0^\infty \left[\frac{\partial C_{kD}(\omega,z)}{\partial a}\,\delta a(\omega,z) + \frac{\partial C_{kD}(\omega,z)}{\partial b}\,\delta b(\omega,z) \right] dz$$

Assuming Q_a, Q_b to be independent of frequency in a broad frequency range, the velocity pertubations $\delta a, \delta b$ become (Akopyan, Zharkov and Lyubimov, 1975; Aki and Richards, 1980; Levshin, Ratnikova and Saks, 1980)

$$\delta a(\omega) = \frac{a(\omega_0)}{Q_a}\left(\frac{1}{\pi}\ln\frac{\omega}{\omega_0} + \frac{i}{2}\right)$$

$$\tag{1.26}$$

$$\delta b(\omega) = \frac{b(\omega_0)}{Q_b}\left(\frac{1}{\pi}\ln\frac{\omega}{\omega_0} + \frac{i}{2}\right)$$

where ω_0 is the reference frequency for which we know the velocity distributions $a(z)$, $b(z)$. In that case

$$\delta C_{kD}(\omega) = \left(\frac{1}{\pi}\ln\frac{\omega}{\omega_0} + \frac{i}{2}\right)S_{kD}(\omega) \tag{1.27}$$

where

$$S_{kD} = \int^{\infty} \left[\frac{a(z)}{Q_a(z)} \frac{\partial C_{kD}}{\partial a} + \frac{b(z)}{Q_b(z)} \frac{\partial C_{kD}}{\partial b} \right] dz$$

and, for the particular case $Q_k = 0$,

$$S_{kD} = \int_0^{\infty} \left[\frac{4b(z)}{3a(z)} \frac{\partial C_{kD}}{\partial a} + \frac{\partial C_{kD}}{\partial b} \right] \frac{b}{Q_b} dz \qquad (1.28)$$

Knowing $\delta C_{kD}(\omega)$, one can easily find the attenuation coefficient for surface waves at frequency ω:

$$\alpha_{kD}(\omega) = \frac{\omega \, \mathrm{Im}(\delta C_{kD})}{C_{kD}^2} = \frac{\omega S_{kD}}{2C_{kD}^2} \qquad (1.29)$$

and the apparent $Q_{kD}(\omega)$ as determined from surface waves:

$$Q_{kD}(\omega) = \omega/[2\alpha_{kD}(\omega) U_{kD}(\omega)] \qquad (1.30)$$

With dissipation present, the second factor in surface wave displacements, (1.11) and (1.13), becomes

$$(\xi_{kD} r)^{-1/2} \exp\left[- \frac{i\omega r}{C_{kD}} \left(1 - \frac{S_{kD}}{\pi C_{kD}} \ln \frac{\omega}{\omega_0} \right) \right] \exp[-\alpha_{kD}(\omega) r]$$

or, in the spherical case ,

$$(\nu_{kD} \sin\theta)^{-1/2} \exp\left[- \frac{i\omega\bar\theta R_0}{C_{kD}} \left(1 - \frac{S_{kD}}{\pi C_{kD}} \ln \frac{\omega}{\omega_0} \right) + \right.$$

$$\left. + \, i \frac{\pi}{2} (2l+g) \right] \exp[-\alpha_{kD}(\omega) \bar\theta R_0]$$

SYNTHETIC SEISMOGRAMS

To pass from spectral to time representation of surface waves, i.e., to synthetic seismograms, one should be able to compute Fourier integrals of rapidly oscillating functions involving the factor $\exp(-i\xi_{kD}\omega r)$, where r is large. Wave forms can conveniently be evaluated by using asymptotic formulas of stationary phase (Copson, 1965;Olver, 1974) or Airy integrals (Olver, 1974; Pekeris, 1948).

We write the spectrum of a surface wave as

$$\hat{u}_q^{kD} = \frac{1}{\sqrt{r}} \Phi_q^{kD}(\omega, h, z, \varphi) \exp[-i\xi_{kD}(\omega) r],$$

where Φ_q^{kD} is a complex-valued function that slowly oscillates when ω is varied. In that case

$$u_q^{kD}(t) = \frac{1}{\pi\sqrt{r}} \mathrm{Re} \int_{\omega_{kD}}^{\infty} \Phi_q^{kD}(\omega) \exp[i(\omega t - \xi_{kD}(\omega) r)] \, d\omega \approx$$

$$\approx \sqrt{\frac{2}{\pi}} \frac{1}{r} \mathrm{Re} \sum_j \frac{U_{kD}(\omega_j) \, \Phi_{pq}^{kD}(\omega_j)}{\sqrt{|dU_{kD}(\omega)/d\omega|_{\omega=\omega_j}}} \times \qquad (1.31)$$

$$\times \exp\left[i\omega t - i\xi_{kD}(\omega) r + i \frac{\pi}{4} \mathrm{sign} \left(\frac{dU_{kD}(\omega)}{d\omega} \right) \right] + o(r^{-1}) \quad ,$$

where stationary phase points $\omega_j(t)$ are roots of the equation $t - r/U_{kD}(\omega) = 0$.

The approximation (1.56) is valid within those frequency ranges where $dU_{kD}(\omega)/d\omega$ does not vanish, the conditions being stated more precisely by Pekeris (1948). To evaluate the contribution due to the vicinity of $\bar\omega$ where $dU_{kD}(\omega)/d\omega = 0$ (called Airy phase), one needs the Airy integral

$$u_q^{kD}(t) = \frac{2^{4/3}}{\sqrt{\pi}} \frac{1}{r^{5/6}} \mathrm{Re} \frac{\Phi(\bar\omega) E(\tau)}{\left(- d^3\xi_{kD}/d\omega^3 |_{\omega=\bar\omega} \right)^{1/3}} \times \qquad (1.32)$$

$$\times \exp[i(\bar\omega t - \xi_{kD}(\bar\omega) r)] + o(r^{-2/3})$$

where $E(\tau)$ is an Airy function of τ:

$$\tau = (t - r/U_{kD}(\bar\omega)) \left(- \frac{r}{2} \frac{d^3\xi_{kD}}{d\omega^3} \bigg|_{\omega=\bar\omega} \right)^{-1/3}$$

It follows from (1.56), (1.57) that the waveform is controlled by group velocity. If we divide the dispersion curve $U_{kD}(\omega)$ into portions having the sign of $dU_{kD}(\omega)/d\omega$ constant, each such portion can be represented by a quasi-sinusoidal oscillation of varying frequency $\hat\omega(r/t)$ where $\hat\omega(r/t)$ is the inverse function of $U_{kD}(\omega)$ within that portion. The contribution of an Airy phase can be represented by an oscillation of quasi-constant frequency $\bar\omega$.

The amplitude of motion with apparent frequency ω_j falls off with increasing distance like r^{-1} ($r^{-1/2}$ is due to geometrical spreading and the other $r^{-1/2}$ is caused by the spreading of a signal with time). The decay of Airy phase amplitude is slightly slower, like $r^{-5/6}$ ($r^{-1/3}$ instead of $r^{-1/2}$ due to a slower spreading with time). The actual amplitude of a harmonic component depends on several factors; we recall that $\Phi_q^{kD}(\omega)$ depends on source mechanism and depth, source spectrum, structure of the medium.

The complete seismogram $u_p^D(t)$ is obtained by adding all the modes present inside the time interval of interest. One cannot always ascribe a physical meaning to the contribution of a particular mode, because several modes may interfere in such a way that they cannot be separated either in the time or frequency domain (Levshin, 1973). For this reason one should not attach significance to anomalies in the polarization, dispersion and other spectral characteristics of a mode, unless the mode can be separated from other modes in the time or frequency ranges studied under actual experimental conditions.

The formulas for evaluation of theoretical seismograms discussed above are not always convenient for actual computation, owing to the narrow intervals of frequency where asymptotic approximations are valid and the difficulties in joining the intervals. Numerical methods are usually employed for calculation of synthetic seismograms (Aki and Richards, 1980; Schwab et al, 1984).

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