INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4.SMR/480-9

WORKSHOP ON EARTHQUAKE SOURCES & REGIONAL LITHOSPHERIC STRUCTURES FROM SEISMIC WAVE DATA

19 - 30 November 1990

Frequency-time Analysis of Surface Waves

L. I. Ratnikova

International Inst. of Earthquake Prediction & Math. Geophysics of the USSR Moscow, U.S.S.R.

L.I.RATNIKOVA

1. FREQUENCY-TIME ANALYSIS OF SURFACE WAVES

Introduction.

Surface wave data processing includes usually two main stages:
1) detection, identification and separation of signals;
2) measurements of signal characteristics, such as phase and group velocities, attenuation, polarization, amplitude and phase spectrum, etc.

In general each of these stages requires special approach as it is impossible by means of single procedure to solve in optimal way all that tasks. Nevertheless, in many cases it is convenient to use one common procedure for both purposes. We call this procedure by FREQUENCY-TIME ANALYSIS or briefly FTAN. Computer code which implements this procedure is called SVAN. It has different options depending on the particular problem to be solved.

You will have the possibility to get acquintanted with SVAN during computer exercises. As we will see it provides representation of surface waves appropriate for their identification and separation and at the same time gives the way to mesure their parameters with reasonable accuracy.

The frequency-time representation has been in use in analog form for at least four decades (Ewing et al., 1959), while its digital form was introduced into seismology in 1969 (Dziewonski, Bloch and Landisman, 1969). Theoretical background and inherent properties of FTAN were investigated by Levshin et al,1972; Cara, 1971; Lander, 1974,1975,1978. For detailed discussion see Keilis-Borok (ed), 1986, chapter 5.

Frequency-time representation.

As everybody knows an appropriate representation of data is very important for successful processing. Short impulsive waves that are characterized by a single arrival time can frequently be studied in the time domain. Long-continued quasi-harmonic oscillations like free oscillations of the Earth are much more conveniently studied in spectral form. Surface waves are rather difficult for processing. Their principal feature, the dispersion is described by a function, rather than a single parameter. A visual picture of such a signal requires a function of two variables. This is the frequency-time representation which has the remarkable property of separating signals in accordance with their group velocity dispersion curves.

Suppose a signal is given in the form of a real-valued function, $w(t)$. We define the spectrum of the signal as

$$K(\omega) = |K(\omega)| e^{i\psi(\omega)} = \begin{cases} \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} w(t) e^{-i\omega t} dt, & \omega > 0 \\ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} w(t) dt, & \omega = 0 \\ 0, & \omega < 0 \end{cases} \quad (1.1)$$

where $K(\omega)$, $\psi(\omega)$ are spectral amplitude and phase, respectively.

The above definition allows a complex-valued signal, $W(t)$, to be constructed that corresponds to $w(t)$ and is related to $K(\omega)$ through the ordinary Fourier transform on an infinite axis:

$$W(t) = |W(t)| e^{i\varphi(t)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} K(\omega) e^{i\omega t} d\omega$$

$$K(\omega) = |K(\omega)| e^{i\psi(\omega)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} W(t) e^{-i\omega t} dt \quad (1.2)$$

where $|W(t)|$, $\varphi(t)$ are time-related (or instantaneous) amplitude and phase. In many cases the functions $|W(t)|$ and $\varphi(t)$ have the visualizable meanings of "envelope amplitude" and "carrier phase". Now let us pass the signal through a system of parallel relatively narrow-band filters $H(\omega - \omega^M)$ with varying central frequency ω^M (Figure 1.1), then the combination of all signals at the output of all the filters $H(\omega - \omega^M)$ can now be treated as a (complex) function of two variables, ω^M and t :

$$S(\omega^M, t) = \int_{-\infty}^{\infty} H(\omega - \omega^M)\, K(\omega)\, e^{i\omega t} d\omega \quad (1.3)$$

which is what we call the frequency-time representation of a signal. A contour map of $\log|S(\omega^M, t)|$ called a FTAN map is used for visual representation. For ω^M fixed, $|S(\omega^M, t)|$ is the signal

envelope at the output of the relevant filter. For this reason, corresponding to each input signal is a "mountain ridge" (increased values) in the FTAN map . We will see later that in the case of simple surface wave signal the ridge extends along the group velocity dispersion curve.

There is an important feature of the frequency-time representation that the function $S(\omega^M, t)$ is not a property of the original signal alone, but also involves the filter characteristics $H(\omega - \omega^M)$ chosen by the investigator. Different choices of $H(\omega - \omega^M)$ will transform one and the same signal to different $S(\omega^M, t)$ functions. An analogy can be drawn with measurements in quantum mechanics where the result is also affected by the "influence of the experimentalist." It does not follow nevertheless that the calculation of $S(\omega^M, t)$ loses some information contained in the signal (that might have occurred if a single filter had been used instead of a set of filters). When the shape of $H(\omega - \omega^M)$ is known, the function $W(t)$ or $K(\omega)$ can in principle be recovered from $S(\omega^M, t)$.

A complete frequency-time representation involves two functions: $S(\omega^M, t)$ proper and the law of variation of the filter shape. When we use a frequency-time representation, we are in fact dealing with a whole class of signal representations that differ in filter choice. This gives rise to the question of the choice of that representation which is the most relevant to the processing problem in hand (optimal FTAN filtering).We note that the spectrum and the record can be regarded as two extreme cases in the class of frequency-time representations: $K(\omega)$ is for infinitesimally narrow filters $H(\omega - \omega^M) = \delta(\omega - \omega^M)$, $W(t)$ is for infinitely broad filters $H(\omega - \omega^M) = 1/\sqrt{2\pi}$.

The choice of $H(\omega^M, \omega)$ in (1.3) may be guided by typical properties of the signal to be processed. As a rule, preliminary surface wave processing uses filters possessing the following simple properties: no phase distortion (a real-valued H) and the best resolution in the time-frequency domain. It is known that the gaussian filter is optimal in this sense. It is this kind of filter which will be considered further on.

To sum up, the frequency-time representation of a signal given by its spectrum $K(\omega)$ is understood as the following complex function on the (ω^M, t) - plane

$$S(\omega^H,t) = \frac{1}{\sqrt{2\pi}\,\beta(\omega^H)} \int_{-\infty}^{\infty} \exp\left(-\frac{(\omega-\omega^H)^2}{2\beta^2(\omega^H)}\right) K(\omega)\, e^{i\omega t} d\omega \qquad (1.4)$$

Optimal filtering. The function $S(\omega^H,t)$ is significantly dependent on the choice of FTAN filters or, to be more exact, on the function $\beta(\omega^H)$. We should have two problems in view when choosing $\beta(\omega^H)$: separation of signal and noise, and a satisfactory estimation of signal characteristics. The requirements that these two problems impose on $\beta(\omega^H)$ are largely contaradictory. However, practical processing of teleseismic surface waves shows that when FTAN is regarded as preliminary processing (where a high accuracy of estimation is not needed), one should give preference to the problem of signal separation. The criteria of optimal filtering will be understood in the narrow sense just indicated.

Since we do not a priori know noise models, optimality criteria are only based on the properties of the signal of interest, consisting in minimizing either the area of the frequency-time region or some section of it.It is possible to relate the choice of β with dispersive properties of surface wave. In the simplest case of a laterally homogeneous medium surface wave phase spectrum has the form

$$\psi(\omega) = -\xi(\omega)r + \psi_s(\omega) = -\frac{\omega r}{C(\omega)} + \psi_s(\omega) \qquad (1.5)$$

where $\xi(\omega)$ is the wavenumber, r the distance the wave has travelled, $C(\omega)$ is the phase velocity, $\psi_s(\omega)$ is the spectral phase at the source. $C(\omega)$ and $\xi(\omega)$ are controlled by the medium alone and do not depend on wave shape; they contain the wanted information on the medium provided by travelling waves. We shall see further that surface wave processing requires measuring another phase parameter, namely group velocity

$$U(\omega) = 1/\frac{d\xi}{d\omega}(\omega) \qquad (1.6)$$

The functions $U(\omega)$ and $C(\omega)$ are related by

$$\frac{1}{U} = \frac{1}{C} + \omega \frac{d}{d\omega}\left(\frac{1}{C}\right) \qquad (1.7)$$

Differentiation of (1.5) yields

$$\tau(\omega) = -\psi'(\omega) = \frac{r}{U(\omega)} - \psi_s'(\omega) \qquad (1.8)$$

It defines the term "group time", $\tau(\omega)$ being that particular characteristics of the signal used to determine group velocity.

It was shown by A.Lander that for many quite general assumptions optimal value of β in (1.4) can be determined from

$$\beta^2(\omega^H) = 1/|\tau'(\omega^H)| \qquad (1.9)$$

or in terms of group velocity

$$\beta_0^2(\omega^H) = U^2(\omega^H) \Big/ |r\, U'(\omega^H)|$$

that is, the FTAN optimal filter width decreases with epicentral distance increasing. No excessive accuracy is actually needed in the choice of β_o. The available information on average group velocities of surface waves in the Earth is sufficient for a satisfactory choice of β_o before processing. $U(\omega)$ and its derivatives are fixed beforehand, the only input parameter to calculate β_o being the epicentral distance r.

Near to the extremum of $\tau(\omega)$ a different estimate of β should be used

$$\beta_o^2 = \sqrt[3]{\frac{2}{|\tau''|^2}} \qquad (1.10)$$

or in terms of group velocity

$$\beta_o^2 = \left(\frac{2\,U^2}{r^2 |U''(\omega^H)|}\right)^{1/3} \qquad (1.11)$$

Some examples of signal separation by means of FTAN will be presented.

Measurements of dispersion using FTAN maps.

Maps of $|S(\omega^H,t)|$ can be directly used for determining dispersion curves. But before $C(\omega)$ and $U(\omega)$ are determined all distortions of phase spectrum not related to propagation effects should be eliminated. So it is necessary

(1) to correct observed phase specrum $\arg K(\omega)$ for the instrument response;

(2) to eliminate source phase $\psi_s(\omega)$.

To make (1) one should be able to calculate the frequency response of seismic channel from known parameters, such as values of zeros and poles ot its transfer function. Such an option exists in SVAN program.

To apply (2) to one-station measurements it is necessary to use some additional information, concerning the source, namely its depth, mechanism and surrounding structure. If this is approximately known $\psi_s(\omega)$ may be calculated by one of methods for solving forward surface wave problems. If one tries to get results from interstation measurements $\psi_s(\omega)$ could be eliminated when the phase difference between two stations' measurements is taken. Group velocity measurements are less sensitive to the source phase, as seismic sources act usually during a time that is short compared with the typical values of $\tau(\omega)$, enabling $-\psi_s'(\omega)$ in (1.8) to be replaced by a constant ("source time") to within the required accuracy. Group velocity can thus be found from known epicenter parameters, even though $\tau(\omega)$ has been measured at a single station. We shall often set $\psi_s'(\omega)\equiv 0$ in what follows.

To estimate properties of FTAN one should consider typical surface wave signals. This was done as for signals described by some simple formulas as for synthetic seismograms of surface waves in realistic Earth models. It can be shown that in many such cases the 'ridge' of FTAN amplitude map follows closely the group time curve $\tau(\omega)$ (Figure 1.2). The simplest estimate $\hat{\tau}(\hat{\omega})$ of $\tau(\omega)$ is curve

$$\hat{\tau}(\hat{\omega}) = t_m(\omega^M), \qquad (1.12)$$

where t_m is the time of conditional maximum of $|S(\omega^M,t)|$ for fixed ω^M. In some cases such an estimate may be seriously biased ,e.g. in the vicinity of frequencies where the amplitude spectrum of signal has sharp extremum or decays very rapidly. By this reason other estimate of $\tau(\omega)$ is preferable, namely

$$\hat{\tau}(\hat{\omega}) = t_m(\Omega_m), \qquad (1.13)$$

where

$$\Omega_m = \frac{\partial}{\partial t}\arg S(\omega^M,t) \qquad (1.14)$$

is an instant frequency near a local maximum of $|S(\omega^M,t)|$ for

given ω^M.

Group velocity estimate is easily obtained from $\hat{\tau}(\hat{\omega})$:

$$\hat{U}(\hat{\omega}) = r/\hat{\tau}(\hat{\omega})$$

In the case of dominant single mode the procedure of finding $t_m(\Omega_m)$ and the group velocity is easily automated.

Both maps of $|S(\omega^M,t)|$ and $\arg S(\omega^M,t)$ should be used for phase velocity estimates. In the case of one-station measurements

$$\hat{\varphi}(\hat{\omega}) = \arg S(\omega^M,t_m) - \Omega_m t_m + \varphi_0 \qquad (1.15)$$

$$\hat{C}(\hat{\omega}) = -\frac{r\,\Omega_m}{\hat{\varphi}(\hat{\omega}) + 2\pi N} \qquad (1.16)$$

where φ_0 is a small constant depending on signal properties and N is usual unknown integer to be found by keeping C inside of the prescribed range.

Let us now suppose that we observed the same wave at two stations situated along the same great circle with the epicenter. Then interstation phase velocity can be measured by using FTAN diagrams for each station

$$\hat{C}(\omega) = -\frac{\Delta r\,\omega}{\Delta\varphi(\omega) + 2\pi N} \qquad (1.17)$$

Here Δr is epicentral distance difference for two stations and $\Delta\varphi$ is a corresponding difference of phase estimates found by means of (1.15) and interpolated for prescribed values of ω.

"Floating" filtering. Phase equalization.

FTAN representation can be used as a preliminary procedure for signal enhancement by means of linear filtering. The term "filter" is frequently employed in a narrow sense for a transformation whose parameters are invariant under a time shift (frequency filtering). It is however convenient to use that term in a broader sense when applied to processing of dispersed signals. We write the general form of a linear transformation (the ~ sign above a letter denotes a result of filtering)

$$\bar{K}(\omega) = \int_{-\infty}^{\infty} F(\omega,\lambda)K(\lambda)d\lambda \qquad (1.18)$$

The requirements on $F(\omega,\lambda)$ are best expressed in the frequency-time language. The filter described by (1.18) should separate, without distortions as far as possible, the part of the plane in which the signal frequency-time region lies (Figures 1.3 and 1.4). One can describe this operation as a "frequency filter whose parameters vary in time." The filter band is "floating" along the dispersion curve. Because it is around this curve that the signal energy is concentrated, it is not significantly distorted, while the noise located far from the signal dispersion curve does not pass through a floating filter.

Below we discuss the construction of a simple floating filter whose time section width is independent of frequency. This extra requirement is technically convenient, because it enables (1.18) to be computed within the framework of ordinary spectral analysis, in particular, using the fast Fourier transform.

Ordinary time-invariant bandpass filtering separates a band paralled to the t -axis on the (ω^{H},t) - plane. Similarly, a time window $G(t)$ applied to a record to give $\bar{W}(t) = G(t)W(t)$ separates a band parallel to the ω^{H}-axis. The most that one can achieve with the two operations combined is to separate a "rectangular" region on the (ω^{H},t) plane whose sides are parallel to the axes. In contrast to this, the "band" of a floating filter may be located in any way relative to the ω^{H} and t-axes. Nonetheless, floating filtering consists in a sequence of frequency filters and time windows, provided the "bandwidth along t" is kept the same for all the ω^{H} .

The most important thing in this operation is phase equalization (otherwise called phase-consistent filtering). The dispersion curve of a signal (denoted $\hat{\tau}(\omega)$) is known approximately from FTAN results, so transforming the spectral phase of the whole record

$$\bar{K}(\omega) = K(\omega)\exp\left[-i\hat{\varphi}(\omega)\right]$$

$$\hat{\varphi}(\omega) = -\left(\int_{0}^{\omega}\hat{\tau}(\eta)d\eta + c_1\omega + c_2\right) \qquad (1.19)$$

makes the signal of interest weakly dispersed ($\tilde{q} \ll 1$). The constants c_1 and c_2 do not affect the envelope shape $|\bar{W}(t)|$, only altering the initial phase of the resulting signal and shifting it to a convenient instant of time, for instance, to the midpoint of the record. Note that the use of (1.19) may increase the slope of the noise dispersion curve. Generally speaking, corresponding to phase equalization is a deformation of the (ω^{H},t) - plane that transforms a curve $t = \hat{\tau}(\omega^{H})$ + *constant* into the straight line t = *constant* parallel to the ω^{H}-axis. This means that the distance between the axes of the frequency-time regions for different signals as measured along the t-axis is not changed, but the region of the signal of interest is stretched approximately parallel to the ω^{H}-axis. A frequency-time region of this kind is quite well separated by using a time window. If now we wish to recover the original signal shape, we should only have to use the inverse procedure of phase equalization, that is, to add the same function $\hat{\varphi}(\omega)$ to the signal phase spectrum.

The procedure of floating filtering thus separates into four successive steps (Figure 1.5 shows diagrammatically the resulting FTAN maps):

(1) bandpass filtering (Figure 1.5 ,b)

$$\bar{K}_1(\lambda) = H(\lambda)\cdot K(\lambda) \qquad (1.20)$$

where $H(\lambda)$ is a real-valued function that equals unity within the signal band and falls off to zero outside it;

(2) phase equalization (Figure 1.5,c) and the transition to the time domain

$$\bar{W}_2(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\bar{K}_1(\lambda)\, e^{i\left[\lambda t-\hat{\varphi}(\lambda)\right]}\, d\lambda \qquad (1.21)$$

As a result, an originally strongly dispersed surface wave signal becomes a short violent pulse;

(3) the use of a time window $G(t)$ (Figure 1.5,d) and the reversion to the spectral representation

$$\bar{K}_3(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(t)\, \bar{W}_2(t)\, e^{-i\omega t}\, dt \tag{1.22}$$

where $G(t)$ is a real-valued function that equals unity within the time interval of the transformed (impulsive) signal and falls off to zero outside it.

(4) the inverse phase transformation (Figure 1.5,e)

$$\bar{K}(\omega) = \bar{K}_3(\omega)\, e^{i\hat{\varphi}(\omega)} \tag{1.23}$$

the record assumes the original form, but with the noise eliminated.

Combining (1.20) through (1.23), we finally get the kernel $F(\omega,\lambda)$ in (1.18)

$$\begin{aligned} F(\omega,\lambda) &= \frac{1}{\sqrt{2\pi}} H(\lambda)\, H_t(\omega-\lambda)\, e^{i\left[\hat{\varphi}(\omega)-\hat{\varphi}(\lambda)\right]} \\ H_t(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(t)\, e^{i\omega t} dt \end{aligned} \tag{1.24}$$

Actually it is of course unnecessary to compute $F(\omega,\lambda)$. It is just the possibility of partitioning (1.18) into simple operations which makes the above version of floating filtering an efficient processing tool.

Apart from $\hat{\varphi}(\omega)$, the principal parameter of a floating filter is the bandpass width along t or equivalently, the width D_t^G of the time window $G(t)$. When D_t^G is decreased, that diminishes the noise-related error ("random error"), but enhances the filter-related distortion ("systematic error"). In other words, in actual practice it is advisable to make the window as broad as the noise allows. The optimal compromise value is record-specific.

Signals which passed floating filtering can be again processed by FTAN procedure or ordinary Fourier analysis as well.

During computer exercises we will demonstrate practical application of floating filtration to seismic records.

REFERENCES

Cara M., 1973. Filtering of dispersed wave trains. Geophys. J. Roy. Astron. Soc., vol. 33, p. 65-80.

Dziewonski A., Bloch S. and Landisman M., 1969. A technique for the analysis of transient seismic signals. Bull. Seismol. Soc. Amer., vol. 59, p. 427-444.

Dziewonski A., Mills J. and Bloch S., 1972. Residual dispersion measurement: A new method of surface wave analysis. Bull. Seismol. Soc. Amer., vol. 62, p. 129-139.

Ewing M., Mueller S., Landisman M. and Sato Y., 1959. Transient analysis of earthquake and explosion arrivals. Pure and Appl. Geophys., vol. 44, p. 83-118.

Keilis-Borok V.I. (ed), 1989. Seismic surface waves in laterally inhomogeneous Earth. Kluwer Publ.House, Dordrecht, [Originally 1986 ,Moscow,Nauka (in Russian)].

Lander A.V., 1974. On interpretation of frequency-time analysis results. In: Computer Analysis of Digital Seismic Data. Moscow, Nauka, p.279-315 (Computational Seismology, 7) (in Russian).

Lander A.V., 1975. Frequency-time representation of a linearly dispersed signal with a gaussian spectrum. In: Interpretation of Seismological and Neotectonic Evidence. Moscow, Nauka, p.122-128 (Computational Seismology, 8) (in Russian).

Lander A.V., 1978. Some methodological aspects of the measurement of spectral characteristics and the interpretation of surface waves. In: Earthquake Prediction and Structure of the Earth, (Computational Seismology, 11), Allerton Press,N Y.,p.93-110 [Originally in Russian,Moscow,Nauka]

Levshin A.L., Pisarenko V.F. and Pogrebinsky G.A., 1972. On a frequency - time analysis of oscillations. Ann. Geophys., vol. 28, p. 211-218. p. 40-55.

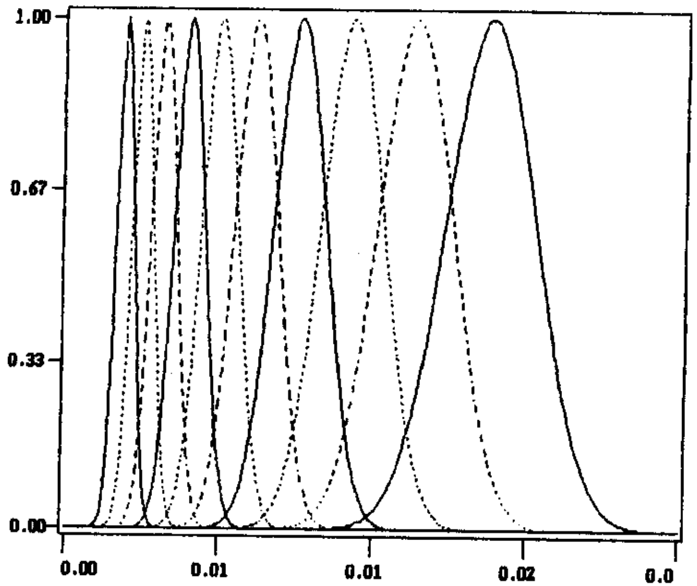

Figure 1.1. Set of narrow-band gaussian filters.

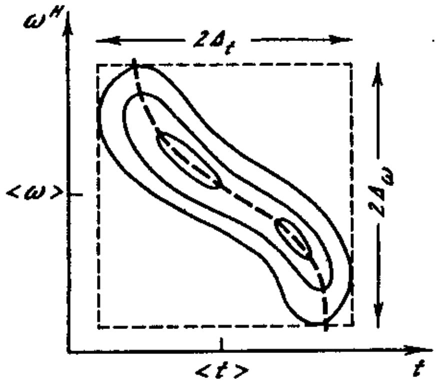

Figure 1.2. Diagrammatic representation of the FTAN map.
Dashed is the dispersion curve $t(\omega^H) = \tau(\omega^H)$

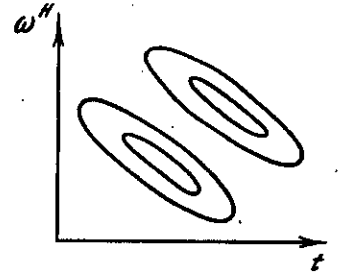

Figure 1.3. Separation of the signals in a FTAN map.
These signals are indistinguishable in the time and the frequency domain

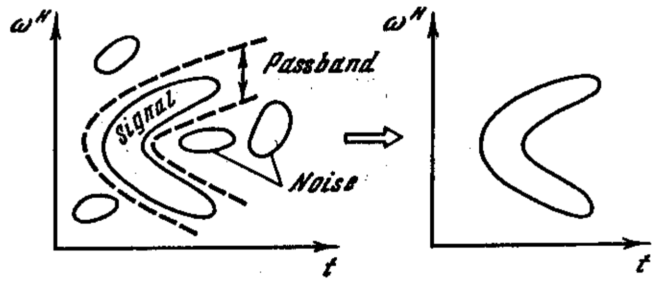

Figure 1.4. Diagrammatic representation of a floating filter in action

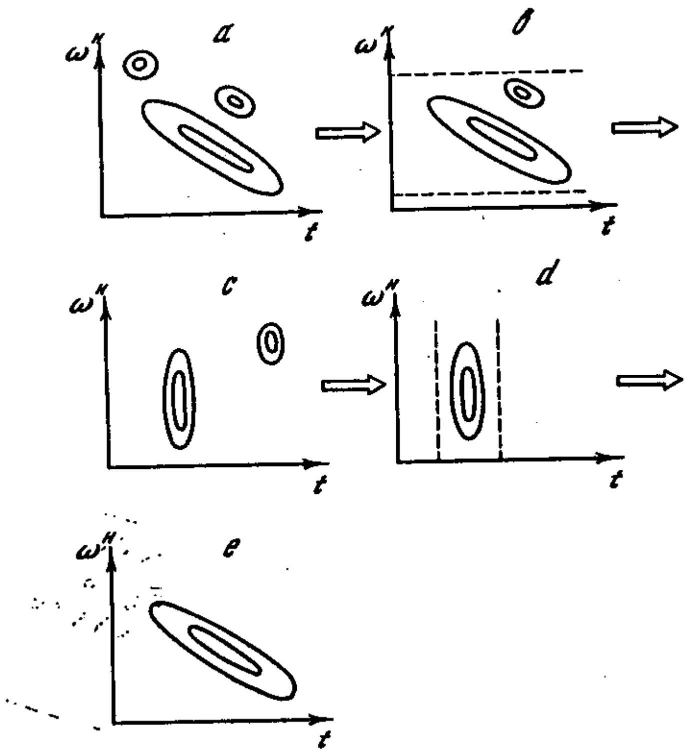

Figure 1.5. Implementation of a floating filter.

(a) FTAN map of the original signal; (b-e) FTAN maps showing the results of the sequence of operations that makes up floating filtering.