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UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
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SMR/534-12

ICTP/WMO WORKSHOP ON EXTRA-TROPICAL AND TROPICAL LIMITED AREA MODELLING
22 October - 3 November 1990

"Application of an Economical Split-Explicit Time Integration Scheme to a Multi-Level LAM"

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DESCRIPTION OF THE MODEL

- The Model is a meso-scale, quasi-hydrostatic, baroclinic one incorporating boundary layer, cumulus parameterizations as well as varying topography.
- The model is in sigma coordinates ($\sigma = P/Ps$), as it is easy in handling changes in topography ($\sigma = 1$ at the surface and $\sigma = 0$ at the top of the model)
- A Staggered Arakawa's C-grid, which conserves the integral properties, is used for horizontal discretization.
- The horizontal coordinates are the general curvilinear (with horizontal grid spacing user specified) with map scale factors Hx and Hy (in spherical coordinates $Hx = a \cos\varphi$ and $Hy = a$).
- The governing equations are represented in the flux form ($Ps\,u$, $Ps\,v$ etc.)
- The closed system of the model consisting of five prognostic equations for u, v, T, q and Ps and two diagnostic equations for $\dot{\sigma}$ and Φ .
- The time integration scheme of the model is a split-explicit method, which allows to increase the time step by effectively separating various terms in the prognostic equations in to partly governing slow moving Rossby modes as opposed to the faster gravity modes.
- The model incorporates a number of physical processes such as Planetary boundary layer, dry and moist convective adjustments, deep cumulus convection and large-scale precipitation (radiation is not considered).

MODEL EQUATIONS

The model equations are written in spherical coordinates using a generalized expressions for curvilinear distances and are expressed in pressure weighted flux form. They are

Momentum Equations :

$$\frac{\partial}{\partial t}(p_s u) + \frac{1}{h_x h_y}\left[\frac{\partial}{\partial x}(p_s u h_y u) + \frac{\partial}{\partial y}(p_s v h_x u) + \right.$$
$$+ \frac{\partial}{\partial \sigma}(p_s n u) - f p_s v + \frac{p_s u v}{h_x h_y}\frac{\partial}{\partial y} h_x = -\frac{p_s}{h_x}\frac{\partial \phi}{\partial x} -$$
$$-\frac{RT}{h_x}\frac{\partial p_s}{\partial x} + p_s F_u$$

$$\frac{\partial}{\partial t}(p_s v) + \frac{1}{h_x h_y}\left[\frac{\partial}{\partial x}(p_s u h_y v) + \frac{\partial}{\partial y}(p_s v h_x v)\right] +$$
$$+ \frac{\partial}{\partial \sigma}(p_s n v) + f p_s u - \frac{p_s u^2}{h_x h_y}\frac{\partial}{\partial y} h_x = -\frac{p_s}{h_y}\frac{\partial \phi}{\partial y} -$$
$$-\frac{RT}{h_y}\frac{\partial p_s}{\partial y} + p_s F_v$$

Thermodynamic Equation :

$$\frac{\partial}{\partial t}(p_s T) + \frac{1}{h_x h_y}\left[\frac{\partial}{\partial x}(p_s u h_y T) + \frac{\partial}{\partial y}(p_s v h_x T)\right] +$$
$$+ \left(\frac{\sigma}{\sigma_0}\right)^{\kappa}\frac{\partial}{\partial \sigma}(p_s n \theta) + \frac{RT}{c_p}\tilde{D} -$$
$$-\frac{RT}{c_p}\left(\frac{u}{h_x}\frac{\partial p_s}{\partial x} + \frac{v}{h_y}\frac{\partial p_s}{\partial y}\right) = p_s H_T + p_s F_T$$

Moisture Continuity Equation :

$$\frac{\partial}{\partial t}(p_s q) + \frac{1}{h_x h_y}\left[\frac{\partial}{\partial x}(p_s u h_y q) + \frac{\partial}{\partial y}(p_s v h_x q)\right] +$$
$$+ \frac{\partial}{\partial \sigma}(p_s n q) = p_s H_q + p_s F_q$$

Surface Pressure tendency Equation :

$$\frac{\partial p_s}{\partial t} = -\tilde{D}$$

Hydrostatic Equation :

$$\frac{\partial \phi}{\partial \sigma} = -\frac{RT}{\sigma}$$

Continuity Equation :

$$\frac{\partial (p_s n)}{\partial \sigma} = \tilde{D} - D$$

Where

$$\tilde{D} = \int_0^1 \nabla \cdot (p_s v)\, d\sigma$$

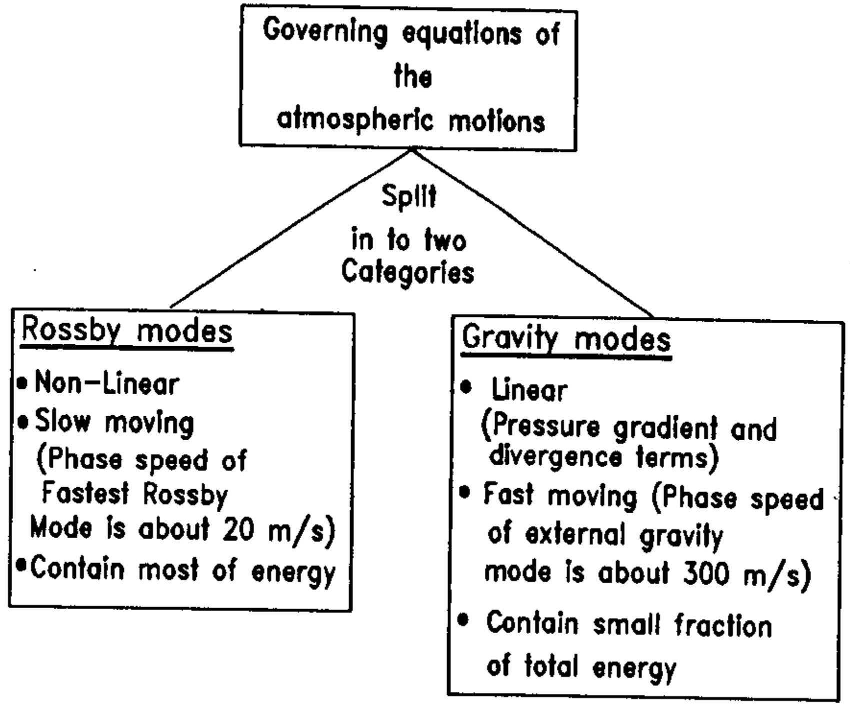
$$D = \nabla \cdot (p_s v) = \frac{1}{h_x\, h_y}\left[\frac{\partial}{\partial x}(h_y\, p_s\, u) + \frac{\partial}{\partial y}(h_x\, p_s\, v)\right]$$

and Hx and Hy are the map factors. Other symbols have their usual meaning.

AN OVERVIEW OF THE MODEL

Domain	:	15°S–45°N, 30°E–120°E : 49 x 33 (Flexible)
Independent Variables	:	λ, ϕ, σ, t
Prognostic Variables	:	u, v, T, q, Ps
Diagnostic Variables	:	ϕ, $\dot{\sigma} = \frac{d\sigma}{dt}$
Vertical Levels	:	Ten levels, non-uniform (σ = 0.1, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.925, 0.975)
Grid	:	Arakawa C-grid, Uniform in λ and ϕ ($\Delta\lambda = \Delta\phi = 1.875°$)
Finite Difference Scheme	:	Second order accuracy
Time Integration	:	Split-explicit time integration scheme; Δt = 900 Sec (15 mts)
Horizontal Diffusion	:	Linear Fourth order
Initialization	:	Non-Linear normal mode (Vertical mode scheme)
Topography	:	Envelope Orography (1 σ) based on US Navy 10' Orography
Sea Surface Temperature	:	Climatologically prescribed (Monthly mean)
Physical Processes		
Convection	:	Dry convective adjustment, Deep cumulus convection (Modified Kuo scheme of Anthes, 1977) Large-scale stratified precipitation (with RH > 95%)
Turbulent Process	:	Stability dependent vertical diffusion of u, v, T and q within surface layer (Monin-Obukhov Similarity Approach)
		Air-Sea sensible heat exchange and evaporation from Ocean.

Split-Explicit Time Integration



Based on these facts attempts have been made to carryout time integration with different time steps, each satisfying C.F.L. criteria of respective modes. Such type of schemes are called split-explicit schemes. (Gadd, 1978; Madala 1981 and Mohanty et. al., 1990).

In this scheme, each Rossby and Gravity mode is treated separately on time steps Δt_R and Δt_g which are determined from the respective phase speeds and C.F.L. criteria. In this scheme, various terms contribute additively instead of multiplicatively as in ordinary time integration methods.

For time integration: The equations in Matrix form are:

MOMENTUM EQUATIONS

$$\frac{\partial}{\partial t}(p_s u) + \frac{1}{h_x}\frac{\partial \phi}{\partial x} = A_u$$

$$\frac{\partial}{\partial t}(p_s v) + \frac{1}{h_y}\frac{\partial \phi}{\partial y} = A_v$$

THERMODYNAMIC EQUATION

$$\frac{\partial}{\partial t}(p_s T) + \underset{\approx}{M}_2 \underset{\sim}{D} = A_T$$

MOISTURE CONTINUITY EQUATION

$$\frac{\partial}{\partial t}(p_s q) = G$$

SURFACE PRESSURE TENDENCY EQUATION

$$\frac{\partial p_s}{\partial t} + \underset{\approx}{N}_2^T \underset{\sim}{D} = 0$$

HYDROSTATIC EQUATION

$$\underset{\sim}{\phi} = \underset{\approx}{M}_1 \ T$$

CONTINUITY EQUATION

$$p_s = \underset{\approx}{N}_1 \ \underset{\sim}{D}$$

where, ϕ represents geopotential height from the terrain

$$A_u = -\frac{1}{h_x h_y}\left[\frac{\partial}{\partial x}(p_s u\, h_y u) + \frac{\partial}{\partial y}(p_s v\, h_x u)\right]$$

$$-\frac{\partial}{\partial \sigma}(p_s \ \dot{\eta} u) + f p_s \frac{uv}{h_x h_y} \ \frac{\partial h_x}{\partial y}$$

$$+\frac{1}{h_x}(\phi' - RT')\frac{\partial p_s}{\partial x} - \frac{1}{h_x}\frac{\partial}{\partial x}(\phi_s p_s) + p_s F_u^H + p_s F_u^V$$

$$A_V = -\frac{1}{h_x h_y}\left[\frac{\partial}{\partial x}(p_s u\, h_y v) + \frac{\partial}{\partial y}(p_s v\, h_x v)\right]$$

$$-\frac{\partial}{\partial \sigma}(p_s \ \eta v) - f p_s u + p_s u^2 \frac{1}{h_x h_y} \ \frac{\partial h_x}{\partial y}$$

$$+\frac{1}{h_y}(\phi' - RT')\frac{\partial p_s}{\partial y} - \frac{1}{h_y}\frac{\partial}{\partial y}(\phi_s \ p_s)$$

$$A_T = -\frac{1}{h_x h_y}\left[\frac{\partial}{\partial x}(p_s u\, h_y T') + \frac{\partial}{\partial y}(p_s v\, h_x T')\right]$$

$$-\left(\frac{\sigma}{\sigma_0}\right)^{\kappa}\frac{\partial}{\partial \sigma}(p_s \eta \Theta') - \frac{RT'\tilde{D}}{C_p} + \frac{RT}{C_p}\left(\frac{u}{h_x}\frac{\partial p_s}{\partial x} + \frac{v}{h_y}\frac{\partial p_s}{\partial y}\right)$$

$$-\frac{p_s}{C_p}(Q_T^C + Q_T^S + Q_T^R) + p_s F_T^H + p_s F_T^V$$

$$G = -\frac{1}{h_x h_y}\left[\frac{\partial}{\partial x}(p_s u h_y q) + \frac{\partial}{\partial y}(p_s v h_x q)\right]$$

$$- \frac{\partial}{\partial \sigma}(p_s \eta q) + p_s(Q_q^C + Q_q^S) + p_s F_q^H + p_s F_q^V$$

$$\underset{\approx}{M}_{1_{ij}} = \begin{cases} \alpha_{j-1} + h & \text{if } i < j \\ \alpha_j & \text{if } i = j \quad i,j = 1,2,\ldots\ldots,KK \\ 0 & \text{if } i > j \end{cases}$$

where

$$\alpha_j = \begin{cases} -\frac{R}{2}\ln(\sigma^j/\sigma^{j+1}) & \text{if } j < KK \\ -R\ln\sigma^{KK} & \text{if } j = KK \end{cases}$$

$$\underset{\approx}{M}_{2_{ij}} = \underset{\approx}{N}_3 \; \underset{\sim}{N}_1 + \frac{R}{C_p} T^* N_2^T + \begin{cases} (T_i^*) & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

where

$$\underset{\approx}{N}_{1_{ij}} = -\left(\sum_{L=i+1}^{KK} \Delta\sigma^L\right)\frac{\Delta\sigma^j}{\sigma_s - \sigma_T} + \begin{cases} \Delta\sigma^j & \text{if } i < j \\ 0 & \text{if } i \geq j \end{cases}$$

for $i = 1,2,\ldots\ldots,KK$ and $j = 1,2,\ldots\ldots,(KK-1)$.

$$(\underset{\sim}{N}_2)_i = \frac{\Delta\sigma^i}{\sigma_S - \sigma_T} \quad \text{for } i = 1,2,\ldots,KK$$

and

$$\left(\frac{\sigma_i}{\sigma_o}\right)^{\kappa}\frac{1}{\Delta\sigma^i}\left|\theta^{i+1}\Delta\sigma^i + \theta^i + \Delta\sigma^{i+1}\right| / (\Delta\sigma^i + \Delta\sigma^{i+1}) \quad \text{if } i = j$$

$$\underset{\approx}{N}_{3_{ij}} = -\left(\frac{\sigma_i}{\sigma_o}\right)^{\kappa}\frac{1}{\Delta\sigma^i}\left|\theta^{i-1}\Delta\sigma^i + \theta^i\Delta\sigma^{i-1}\right| / (\Delta\sigma^i + \Delta\sigma^{i-1}) \quad \text{if } i-1 = J$$

$$= 0 \quad \text{otherwise}$$

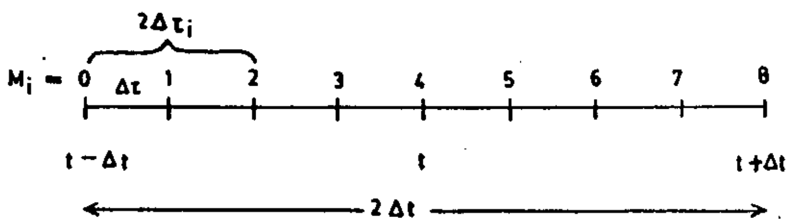
where $\kappa = R/C_p$

Time Integration Scheme

This model uses an efficient split-explicit time integration scheme (Gadd, 1978; Madala, 1981 and Mohanty et. al., 1990) which is nearly five times more economical than an explicit scheme. The scheme is designed to take care of propagation of gravity wave component with suitable small time step to satisfy C.F.L. criteria.

Basic concept of the scheme lies in the evaluation of the terms representing slow moving meteorological waves (Rossby mode) with large time step and high frequency gravity wave with sufficiently small time step. Thus, implementation of varying time steps to different modes is the basis of the split explicit time integration scheme using the present work.

Its implementation in the model is made efficient by integrating all the modes by the same time step (large time step) and introducing suitable corrections to take into account the deviations caused due to integration of fast moving gravity waves at large time steps (instead of at small time steps) so as to arrive at the same results which would have been obtained by the explicit integration at small time step.



For example integration of the zonal component of the momentum equation from $t-\Delta t$ to $t+\Delta t$ time level leads to

$$Ps\,u(t+\Delta t) - Ps\,u(t-\Delta t) + 2\Delta t \frac{1}{hx}\frac{\partial \phi}{\partial x} = 2\Delta t\ \bar{Au}$$

Where the operator (–) is defined as

$$\bar{\beta} = \frac{1}{2\Delta t}\int_{t-\Delta t}^{t+\Delta t} \beta\, dt$$

Here, we define C.F.L. time step limits Δtg and Δt_R for the external gravity mode and fast moving Rossby mode respectively and choose a time step Δt such that

$$\Delta t g < \Delta t < \Delta t_R$$

- Here, the R.H.S. terms (Au) of the u-prognostic equation vary slowly over the time scales of the Rossby mode and therefore computed at time intervals Δt instead of Δtg. Thus, R.H.S. terms are evaluated at time t.

$$Ps\,u(t+\Delta t) - Ps\,u(t-\Delta t) + 2\Delta t \frac{1}{hx}\frac{\partial \phi}{\partial x} = 2\Delta t\ Au(t)$$

- The terms on the L.H.S. namely the pressure gradient and divergence terms (in case of thermodynamic and surface pressure tendency equations) vary over the time step determined by all the modes (including fast moving external gravity modes).

Therefore, in split explicit method, time interval $2\Delta t$ is sub-divided in to m sub-intervals of length $2\Delta\tau$ ($\approx 2\Delta tg$).

- With in these sub-intervals (time step $\Delta\tau$), time integration is carriedout explicitly keeping R.H.S. terms unchanged.

  Thus, here $\bar{\beta} = \frac{1}{m}\sum_{n=1}^{m}\beta^n$ where

  $m = \Delta t / \Delta\tau$ and $\beta^n = \beta(t - \Delta t + 2m\Delta\tau)$

- Therefore, results of the future state variables obtained by using split-explicit techniques differ from the results obtained by explicit time integration technique by a deviation term (correction to the result of explicit method) as shown below.

$$p_s u\ (t+\Delta t) + 2\Delta t\ \frac{1}{h_x}\frac{\partial}{\partial x}\ [\bar{\Phi} - \phi(t)]$$

$$= p_s u\ (t - \Delta t) - 2\Delta t\ \frac{1}{h_x}\frac{\partial}{\partial x}\ \phi(t) + 2\Delta t\ A_u(t), \text{ or}$$

$$p_s u\ (t+\Delta t) + 2\Delta t\ \frac{1}{h_x}\frac{\partial}{\partial x}\ (\bar{\Phi} - \phi(t))$$

$$= p_s u^{ex}\ (t+\Delta t)$$

Where the subscript "e_x" denotes values computed using time integration about $2\Delta t$. Note that the explicit time integration of the momentum equation leads to

$$p_s u\ (t+\Delta t) \equiv p_s^{ex}\ u\ (t+\Delta t)$$

$$\equiv p_s u\ (t-\Delta t) - \frac{2\Delta t}{h_x}\frac{\partial}{\partial x}\bar{\Phi}(t) + 2\Delta t\ A_u(t)$$

and the split-explicit time integration over the same range leads to

$$p_s u(t+\Delta t)$$

$$= p_s u^{exp}(t+\Delta t) - 2\Delta t\ \frac{1}{h_x}\ \frac{\partial}{\partial x}\ [\phi - \phi(t)]$$

Thus, split-explicit time integration is found to be equivalent to explicit time integration at larger time step $(2\Delta t)$ and $-2\Delta t\ \frac{1}{hx}\frac{\partial}{\partial x}(\bar{\Phi} - \Phi(t))$ which is known as correction term. It arises due to integration of gravity modes at large time steps which is to be computed for small time step.

Similarly, for the other prognostic equations we have

$$p_s v\ (t+\Delta t) + 2\Delta t\ \frac{1}{h_y}\frac{\partial}{\partial y}\ (\bar{\Phi} - \phi(t))$$

$$= p_s v^{ex}\ (t+\Delta t)$$

$$p_s T\ (t+\Delta t) + 2\Delta t\ \underset{\sim}{M}_2\ (\bar{D} - D(t))$$

$$= p_s T^{ex}\ (t+\Delta t)$$

$$p_s q\ (t+\Delta t) = p_s q^{ex}(t+\Delta t)$$

$$p_s\ (t+\Delta t) + 2\Delta t\ \underset{\sim}{N}_2^T\ (\bar{D} - D(t))$$

$$= p_s^{ex}\ (t+\Delta t)$$

Implementation of the Split-explicit scheme in LAM

The implementation of the split-explicit time integration scheme is a limited area model is carriedout in four steps

- Linearization of the model equations and the computation of the vertical modes.
- Determination of time steps corresponding to different phase speeds (based on CFL criteria).
- Estimation of correction terms in eigen space.
- Transformation from eigen space to grid space (for integration at large time step).

First two steps are dependent only on structure of the model (no. of vertical levels) and basic state of the atmosphere. Therefore, these are computed only from these equations, it is seen that the future state of the variables Ps^u, Ps^v, PsT, Psq and Ps by split-explicit method can be obtained by applying correction terms of

$$2\Delta t \frac{1}{h_x}\frac{\partial}{\partial x}[\bar{\phi}-\phi(t)], \quad 2\Delta t \frac{1}{h_y}\frac{\partial}{\partial y}[\bar{\phi}-\phi(t)],$$

$$2\Delta t\, \underset{\sim}{M}_2[\bar{D}-D(t)], \quad 0 \text{ and } 2\Delta t\, \underset{\sim}{N}_2^T[\bar{D}-D(t)]$$

respectively to the results obtained by the explicit technique. Thus, in order to use split-explicit integration scheme, we have to evaluate $\bar{\phi}$ and $\bar{D}$ once in the beginning before the first time step of the model integration.

Linearization of model equations and computation of vertical modes

The momentum equations for Ps^u and Ps^v can be combined to give a prognostic equation for the divergence $\left(\frac{\partial D}{\partial t}\right)$ as follows :

$$\frac{\partial}{\partial t}\underset{\sim}{D} + \nabla^2 \underset{\sim}{\phi} = \frac{\partial}{\partial x}\underset{\sim}{Au} + \frac{\partial}{\partial y}\underset{\sim}{Av}$$

Similarly, thermodynamic, surface pressure tendency and the hydrostatic equations can be combined to get a prognostic equation for pseudogeopotential $\left(\frac{\partial \Phi}{\partial t}\right)$ as follows

$$\frac{\partial}{\partial t}\underset{\sim}{\phi} + M_3 \underset{\sim}{D} = M_1 \underset{\sim}{A}\, T$$

Where $\phi = Ps(\underset{\sim}{\phi} - \phi s) + (RT^* - \phi^*)\, Ps$

and $M_3 = M_1 M_2 + \underset{\sim}{N}_2^T (RT^* - \phi^*)$

Here R.H.S. contains non-linear terms representing the Rossby modes and on L.H.S., the pressure gradient and divergence terms (linear terms) give rise to gravity modes.

In principle, these two prognostic equations for D and Φ can be used to obtain the values of $\bar{D}$ and Φ and the correction terms. However, there is a practical difficulty in implementing split-explicit method using these equations in the grid point space. Because different gravity modes of the model will satisfy different C.F.L. criteria and magnitude of small time step ΔT will differ and thus values of $\bar{D}$ and $\bar{\Phi}$ will be different to corresponding different gravity modes.

In order to integrate these equations for different gravity modes separately, we have to transform grid point variables into eigen space where the modes can be treated separately.

For this purpose, the equations $\frac{\partial D}{\partial t}$ and $\frac{\partial \Phi}{\partial t}$ are linearized and vertical modes which are the natural gravity modes of the numerical model are determined. The number of these natural modes is equal to numbers of vertical layers in the model. The natural gravity mode forms a complete set of eigen functions satisfying the boundary conditions of a numerical model. Therefore, the variation of the dependant variables can be explained as a linear combination of the structure functions of these modes.

The above equations are linearized by setting R.H.S. equal to zero to get the natural modes of the system.

$$\frac{\partial}{\partial t} D + \nabla^2 \Phi = 0$$

$$\frac{\partial}{\partial t} \Phi + M_3 D = 0$$

Here, the normal modes (vertical structure of the linearized equations) of the model are found by separation of the horizontal and vertical dependences of the variables D and Φ. We define the variable Φ (Pseudogeopotential) whose derivative gives the horizontal pressure gradient in the divergent equation of the motion and the time change of this variable is related to the horizontal divergence.

This relationship allows separation of the vertical variation and results in shallow water equations (S.W.E.) for the variation of each vertical equations. These SWEs have a different mean equivalent depth (H_ℓ) for each vertical mode (which correspond to the speed of gravity waves $C_g = \sqrt{gH_\ell}$).

Here, the divergence equation displays no vertical coupling as there is no vertical derivative in this equation. However, the equation for $\frac{\partial \Phi}{\partial t}$ has vertical coupling because of M_3 matrix.

The M_3 matrix represents thickness of the different layers of the model and their mean temperature structure with non-zero diagonal elements.

Therefore, for determining the vertical modes, we may directly separate psuedo-geopotential equation.

$$\frac{\partial \hat{\Phi}}{\partial t} + gE^{-1} M_3 E \underset{\sim}{D} = 0$$

Here, E represents the eigen vector matrix of M_3 with each column representing an eigen vector.

Now, to get this equation, an uncoupled one, we can write

$$\frac{\partial \hat{\Phi}}{\partial t} + gH_\ell D = 0$$

where gH is the separation constant

$$gH_\ell = E^{-1} M \; E$$

i.e. $M_3 E = EgH_\ell$

Here, gH_l is a diagonal matrix, and the diagonal elements are eigen values of the Matrix M_3; H_ℓ is the equivalent depth and the corresponding eigen vectors E_ℓ.

$$E = (E_1, E_2, \ldots\ldots E_L)$$

is the vertical structure and L is the vertical mode index (L=no. of vertical modes=no. of layers of the model).

The eigen values of the matrix M_3 gives the phase speeds (Cg) of the vertical modes of the model

$$Cg_\ell = \sqrt{gH_\ell}, \quad \ell = 1, 2, \ldots\ldots L$$

Determination of the time step

It is seen from above that eigen values of M_3 give phase speeds of the vertical modes (natural modes of a numerical model) and there are as many natural modes as there are layers in the model.

As an example, for a ten level P.E. model in the tropics, the ten eigen modes will have the following equivalent depths and phase speeds for the given mean temperature profile.

Model level/ℓ	Sigma(σ) value	$\bar{T}(\sigma)$°K	H_ℓ (in mtrs.)	Cg_ℓ (m/s)
1	0.1	197.8	9564.0	306.1
2	0.25	230.7	606.2	77.1
3	0.35	247.9	122.5	34.6
4	0.45	260.6	32.2	17.8
5	0.55	270.1	12.4	11.0
6	0.65	277.4	5.4	7.3
7	0.75	283.9	2.5	4.9
8	0.85	290.6	1.0	3.2
9	0.925	295.6	0.3	1.8
10	0.975	298.8	0.1	1.0

Similarly, for a five layer model, the five eigen modes will have characteristic phase velocities of approximately, 300, 70, 30, 15 and 5 m/s. In principle each mode corresponds to a different time step Δt to satisfy CFL criteria ($C\Delta t < \Delta x$)

Since, the Rossby modes move at about 20 m/s, the natural modes with phase speed fall into the Rossby mode category are treated with large time interval $2\Delta t$.

Since, the pressure gradient and divergence terms in the model equations vary over the time steps determined by all the modes, time interval $2\Delta T$ is sub-divided into m sub-intervals of length $2\Delta \tau$ and in this case of 5 layer model, values of m for the gravity modes are 8, 4, 2, for the corresponding phase speed 300, 70 and 30 m/s respectively.

Within these sub-intervals (time step of $2\Delta\tau$), the time integration is carried out explicitly. Thus,

$$\beta = \frac{1}{m}\sum_{n=1}^{m}\beta^{n}$$

where $m = \frac{\Delta T}{\Delta \tau}$ and $\beta^{n} = (t - \Delta t + 2m\Delta\tau)$

Estimation of correction terms in eigen space

The correction terms in the prognostic equations for the dependent variables Ps^u, Ps^v, PsT, Ps q and Ps in the implementation of split-explicit method of time integration are presented earlier. Further, it is noted that estimation of the correction terms require the evaluation of only the two terms Φ and D from the two modified prognostic equations of $\frac{\partial \Phi}{\partial t}$ and $\frac{\partial D}{\partial t}$.

As these terms (Φ and D) vary over the time steps determined by all the natural modes, they can not be evaluated in the grid point space.

Then, for obtaining correction terms for different modes, we express the dependent variables as linear combination of structure functions (respective columns of the eigen vector matrix E) of these modes.

$$u = E \cdot a$$
$$v = E \cdot b$$
$$T = E \cdot c$$
$$D = E \cdot d$$
$$\Phi = E \cdot e$$

Where elements of the coefficient vectors a, b, c, d, and e are the amplitudes of eigen modes. Thus, multiplying the prognostic equations by E^{-1} and using above relationships between dependent variables and the amplitude of the eigen modes, we can obtain following spectral equation for the i^{th} mode.

$$\frac{\partial}{\partial t}(p_s a_i) + \frac{1}{h_x}\frac{\partial}{\partial x} e_i = (E^{-1} A_u)_i$$

$$\frac{\partial}{\partial t}(p_s b_i) + \frac{1}{h_y}\frac{\partial}{\partial y} e_i = (E^{-1} A_v)_i$$

$$\frac{\partial}{\partial t}(p_s c_i) + (\underline{M}_2 \cdot d)_i = (E^{-1} A_T)_i$$

$$\frac{\partial}{\partial t}(p_s) + N_2^T \cdot d = 0$$

Now, the correction terms for the system of equations are

$$\sum_i 2\Delta t \frac{1}{h_x} \frac{\partial}{\partial x} [\bar{e}_i - e_i(t)], \quad \sum_i 2\Delta t\, M_2\, [\bar{d}_i - d_i(t)],\ 0$$

$$\sum_i 2\Delta t \frac{1}{h_y} \frac{\partial}{\partial y} [\bar{e}_i - e_i(t)], \quad \text{and} \sum_i 2\Delta t\, N_2^T\, [\bar{d}_i - d_i(t)],$$

The summation over i is carried out for those modes for which split-explicit technique is used.

The above correction terms can be obtained from the prognostic equations for Φ and D (i.e. e and d in eigen space). The equations for e and d in the eigen space are

$$\frac{\partial}{\partial t} d_i + \nabla^2 e_i = E^{-1} (\delta x A_u + \delta y\, A_v)_i$$

$$\frac{\partial e_i}{\partial t} + \underbrace{E^{-1} M_3 E}_{\lambda_i}\ \underbrace{E^{-1} D}_{d_i} = (E^{-1} M_1 A_T)_i$$

Here i represents the i^{th} mode.

The integration of equations for $\frac{\partial d_i}{\partial t}$ and $\frac{\partial e_i}{\partial t}$ from $(t - \Delta t)$ to $(t + \Delta t)$ time levels by split-explicit method lead to

$$d_i\,(t+\Delta t) - d_i\,(t-\Delta t) + 2\Delta t\,(\delta x^2 + \delta y^2)\bar{e}$$
$$= 2\Delta t\,[E^{-1}(\delta_x A_u + \delta_y A_v)_i]$$

$$e_i(t+\Delta t) - e_i(t-\Delta t) + 2\Delta t \lambda_i \bar{d}_i$$
$$= 2\Delta t(E^{-1} M_1 A_T)$$

An explicit time integration of these equations will lead to

$$d_i^{ex}(t+\Delta t) = d_i(t-\Delta t) + 2\Delta t[E^{-1}(\delta x A_u + \delta y\, A_v)_i]_{(t)} - 2\Delta t(\delta x^2 + \delta y^2) e(t)$$

$$e_i^{ex}(t+\Delta t) = e_i(t-\Delta t) + 2\Delta t(E^{-1} M_1 A_T)_t - 2\Delta t \lambda_i d_i(t)$$

$$2\Delta t\ [E^{-1}(\delta x A_u + \delta y A_v)]_{(t)} = d_i^{ex}(t+\Delta t) - d_i(t-\Delta t) + 2\Delta t(\delta x^2 + \delta y^2) e(t)$$

and

$$2\Delta t\ [E^{-1} M_1 A_T](t) = e_i^{ex}(t+\Delta t) - e_i(t-\Delta t) + 2\Delta t \lambda_i d_i(t)$$

Introducing explit integration results into the R.H.S. of the split-explicit integration relation yields :

$$d_i\ (t+\Delta t) - d_i(t-\Delta t) + 2\Delta t(\delta x^2 + \delta y^2)[\bar{e}_i - e_i(t)] = d_i^{ex}(t+\Delta t) - d_i(t-\Delta t)$$

and

$$e_i(t+\Delta t) - e_i(t-\Delta t) + 2\Delta t \lambda_i\ [\bar{d}_i - d_i(t)] = e_i^{ex}(t+\Delta t) - e_i(t-\Delta t)$$

Here, the first step of the integration, the sub-interval $\Delta\tau$ Euler – backward time integration method is used to march from time $(t-\Delta t)$ to $(t - \Delta t + \Delta\tau)$. Then, leap-frog scheme is used to march each successive time step $\Delta\tau$ until time $t+\Delta t$ (over $2m_i$ sub – intervals). In the present example, $m_i = 8$ for first (external gravity mode) and $m_i = 4$ and 2 respectively for second and third modes.

Thus, integration of last two equations for d and e will provide the correction terms for the i-th mode.

$$2\Delta t\ \delta x[\bar{e}_i - e_i(t)]$$

$$2\Delta t \delta y\ [\bar{e}_i - e_i(t)]$$

$$2\Delta t\ M2[\bar{d}_i - d_i(t)]$$

$$2\Delta t\ N_2^T\ [\bar{d}_i - d_i(t)]$$

The solution of the equations for different modes will give net correction as the sum over different modes (here i = 1, 2, 3). The fourth and fifthh modes travel sufficiently slow enough to be incorporated into the large Rossby time step $2\Delta t$ and do not require any correction.

Transformation to grid point space

The net correction terms computed above in the eigen space for the different gravity modes are transformed back to the grid space by multiplying with the corresponding column of the eigen vector matrix (E) as given below

$$\sum_i E_i 2\Delta t \frac{1}{h_x} \frac{\partial}{\partial x} [\bar{e}_i - e_i(t)],$$

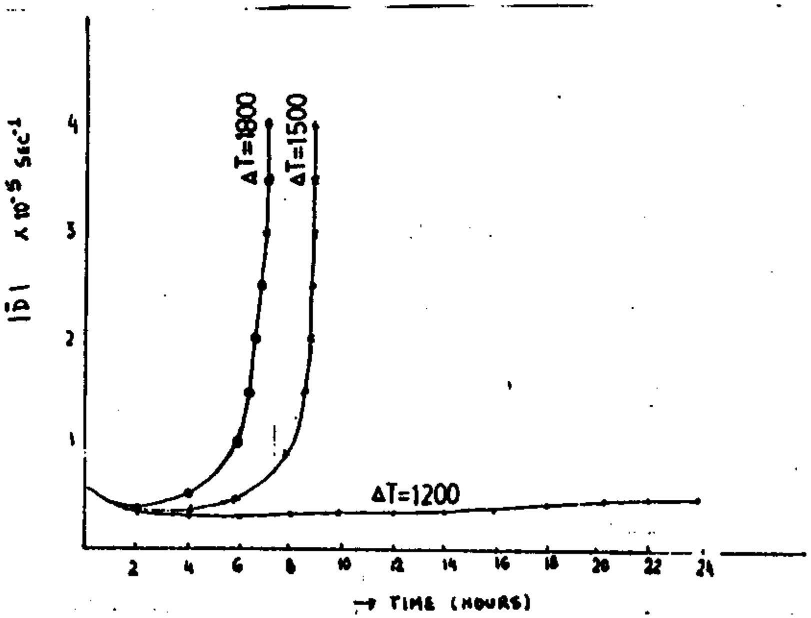
$$\sum_i E_i 2\Delta t \frac{1}{h_y} \frac{\partial}{\partial y} [\bar{e}_i - e_i(t)],$$

$$\sum_i E_i 2\Delta t\ M2[\bar{d}_i - d_i(t)],\ 0 \text{ and}$$

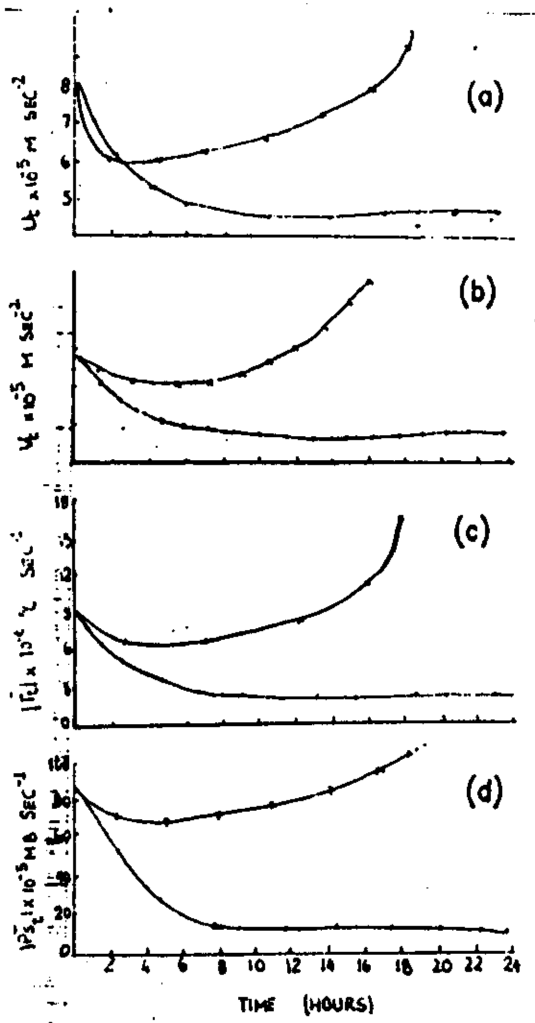
$$\sum_i E_i 2\Delta t\ N_2^T[\bar{d}_i - d_i(t)],$$

Such correction terms computed above are applied to the prognostic equations to get the future state of the variables Ps^u, Ps^v, PsT, Ps^q and Ps at a grid point by split-explicit method.

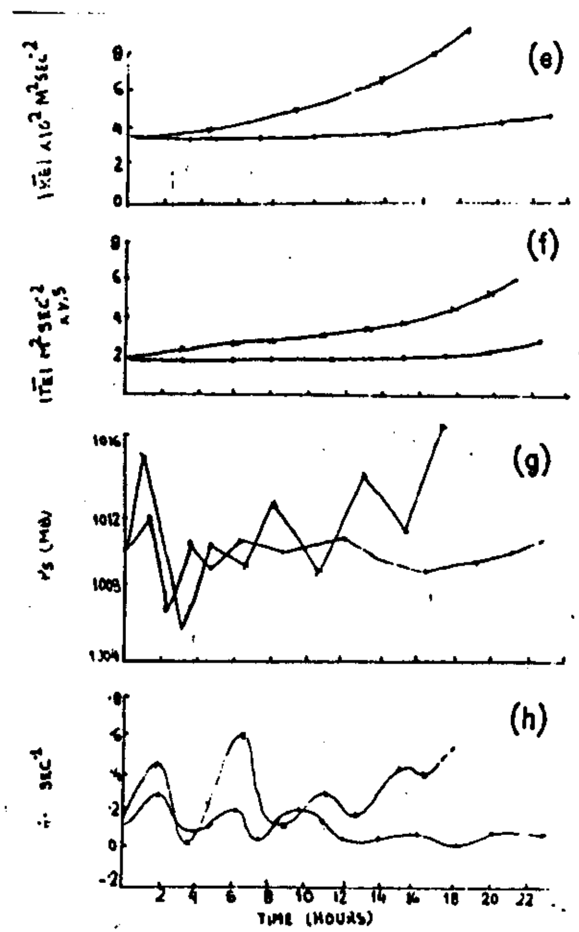
It may be noted that computation of non-linear terms $(\tilde{A}u, \tilde{A}v, \tilde{A}\tau)$ and physics is carriedout in usual grid point space at large time step $2\ \Delta t$.



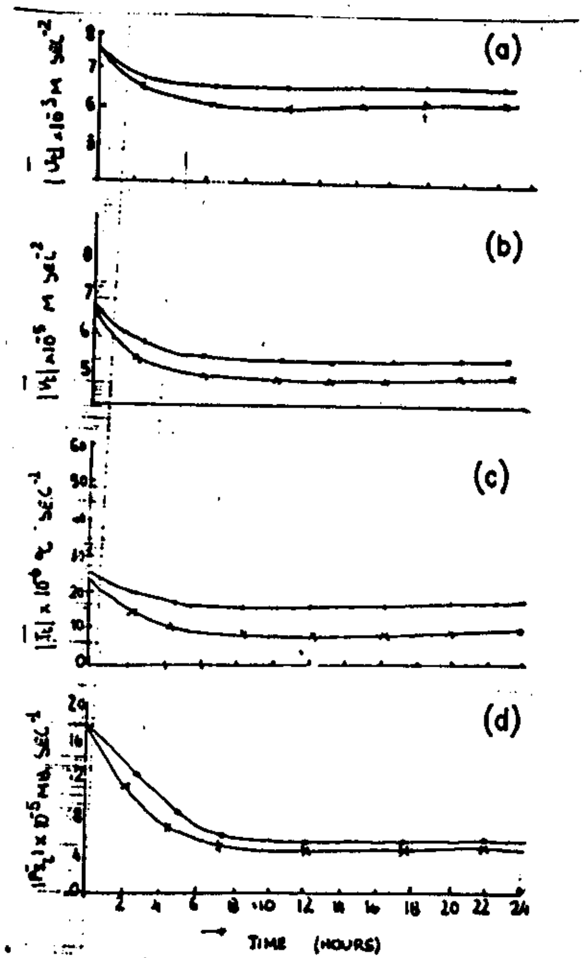
Time series of $[\bar{D}]$ during a one day forecast obtained by split-explicit method of time integration with varying time step (Δt).



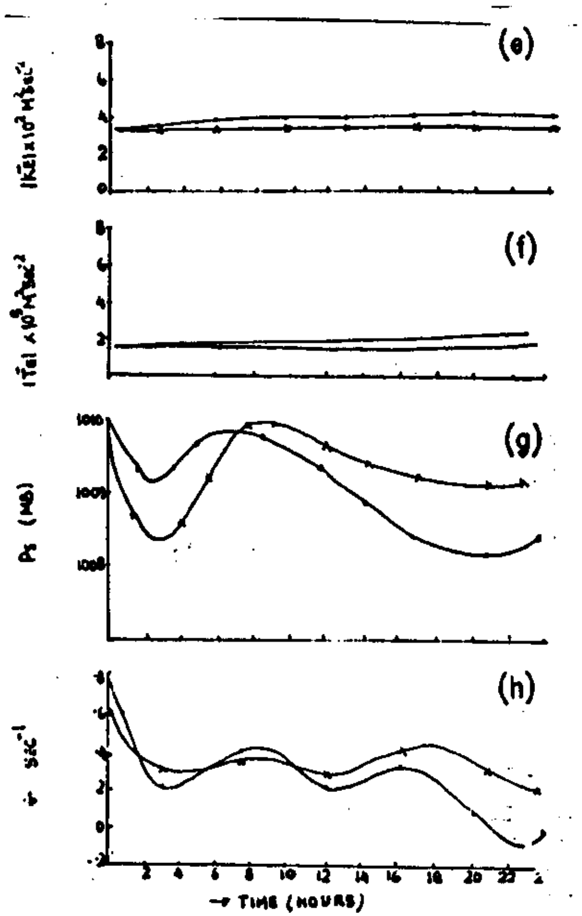
Time-series during a one day forecasts obtained by split-explicit (O——O) and explicit (X——O) methods of time integration with uninitialised initial data
(a) $[\bar{U}_t]$, (b) $[\bar{V}_t]$, (c) $[\bar{T}_t]$, (d) $[\bar{p}_{st}$]



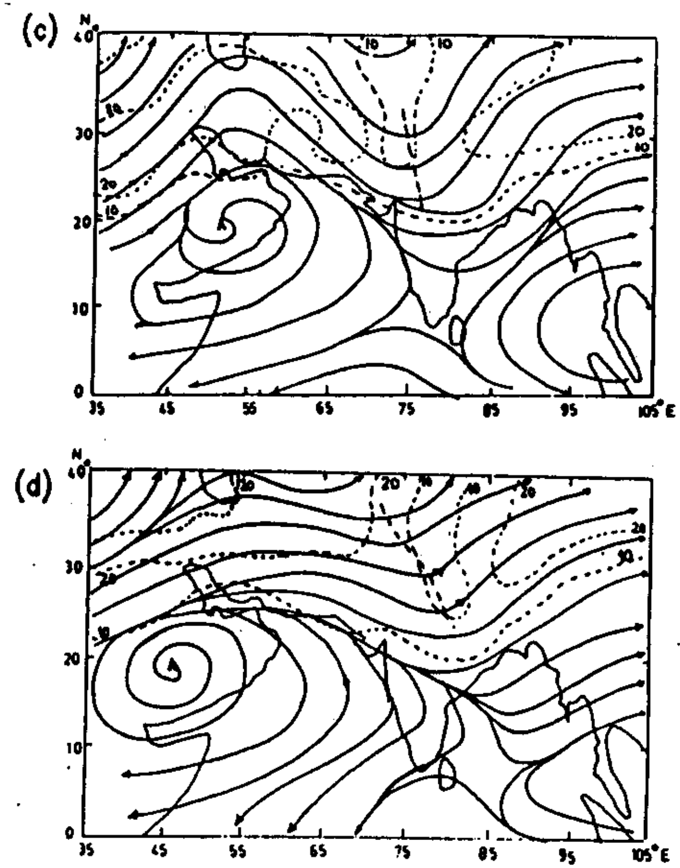
Time-series during a one day forecasts obtained by split-explicit (0——0) and explicit (X——X) methods of time integration with uninitialised initial data
(e) [$\overline{KE}$], (f) [$\overline{TE}$], (g) [$\bar{p}_s$], and (h) $\dot{\sigma}$



Time-series during a one day forecasts obtained by split-explicit (X——X) and (0——0) methods of time integration for (a) [$\bar{U}_t$], (b) [$\bar{V}_t$], (c) [$\bar{T}_t$], (d) [$\bar{p}_{st}$]

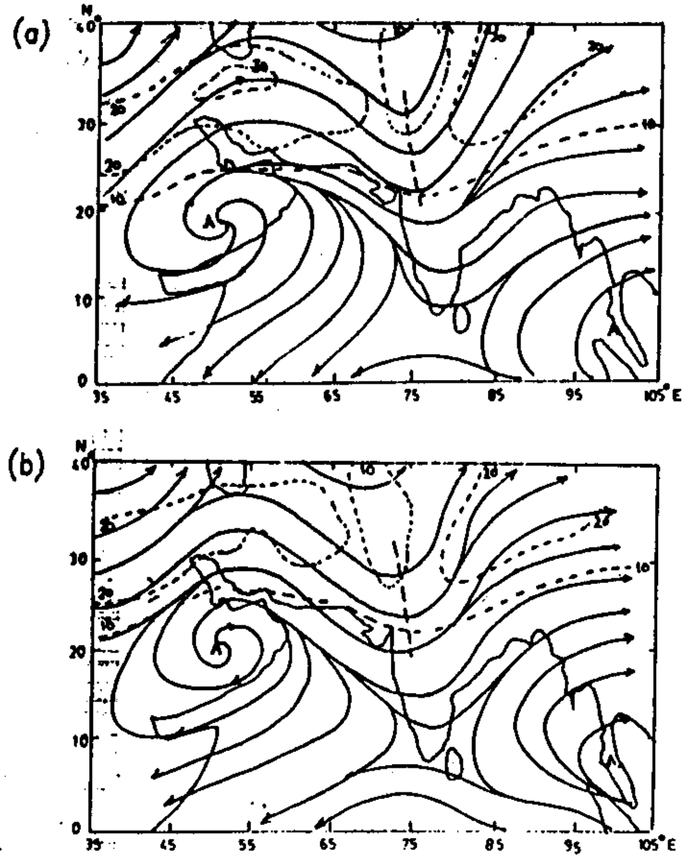


Time-series during a one day forecasts obtained by split-explicit (X——X) and (0——0) methods of time integration for (e) [$\overline{KE}$], (f) [$\overline{TE}$], (g) $\bar{p}_s$ and (h) $\dot{d}$, with initialised initial data.



Stream line and isolated analysis of 48 hours forecast wind field from the initial data of 22 May 79 (12 GMT)

(c) Split-explicit (Δt = 900 sec)
(d) Corresponding verification yield.



Stream line and isolated analysis of 48 hours forecast wind field from the initial data of 22 May 79 (12 GMT)
(a) Explicit Scheme (Δt = 180 sec)
(b) Split-explicit (Δt = 180 sec)

ROOT MEAN SQUARE ERROR – 22 MAY 1979

Hours		24 hrs			48 hrs		
Levels	Variables	E_1	E_2	E_3	E_1	E_2	E_3
300 mb	u	3.8	3.7	3.8	5.4	5.2	5.5
	v	4.5	4.2	4.4	7.0	6.7	7.0
	T	2.5	2.3	2.4	3.3	3.0	3.1
500 mb	u	2.2	2.0	2.1	7.2	6.8	7.0
	v	2.8	2.9	2.8	2.4	2.4	2.3
	T	1.1	1.0	1.0	1.5	1.4	1.4
700 mb	u	3.4	3.1	3.2	4.2	4.1	4.0
	v	2.5	2.3	2.2	3.3	3.0	3.2
	T	1.2	1.0	0.9	1.3	1.2	1.1
850 mb	u	2.8	2.7	2.9	3.3	3.1	3.1
	v	2.3	2.2	2.2	2.6	2.6	2.7
	T	1.7	1.5	1.6	1.9	1.8	1.8

E_1: Explicit scheme with 180 sec time step.
E_2: Split-Explicit Scheme with 180 sec time step.

Tentative Programme

24 October (Wednesday)

16:00 – Opening of the meeting (chairman, WMO Representative)
– Approval of the Agenda

16:10 – Report of the Chairman of the Steering Committee (chairman)

16:40 – Information on descisions of CAS-X, EC-XLI and the WMO Cg-X concerning the WMO Tropical Meteorology Research Programme (WMO Secretariat)

17:00 – Current status of NWP products in tropical countries
* African region (A.E. Okoye)
* Asian region (R.K. Datta)

25 October (Thursday)

16:00 – Current status of development in LAM activities including those of activity centres
* T.N. Krishnamurti (USA)
* R.K. Datta (India)
* A. Segami (Japan)
* Other participants in their individual countries

17:00 – Future work of the Steering Committee
18:00 – Closure of the meeting